Rishi Mouland (Imperial College London)
Infrared Surprises of Scattering Amplitudes, CERN, 16th May 2025

Surprise!

It’s Fermion-Monopole Scattering

Based on 2312.17746, van Beest, Boyle Smith, Delmastro, **Mouland**, Tong
See also 2306.07318, van Beest, Boyle Smith, Delmastro, Komargodski, Tong

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- **Comments** and some **further directions**

Part I: An old problem

Motivation from the UV

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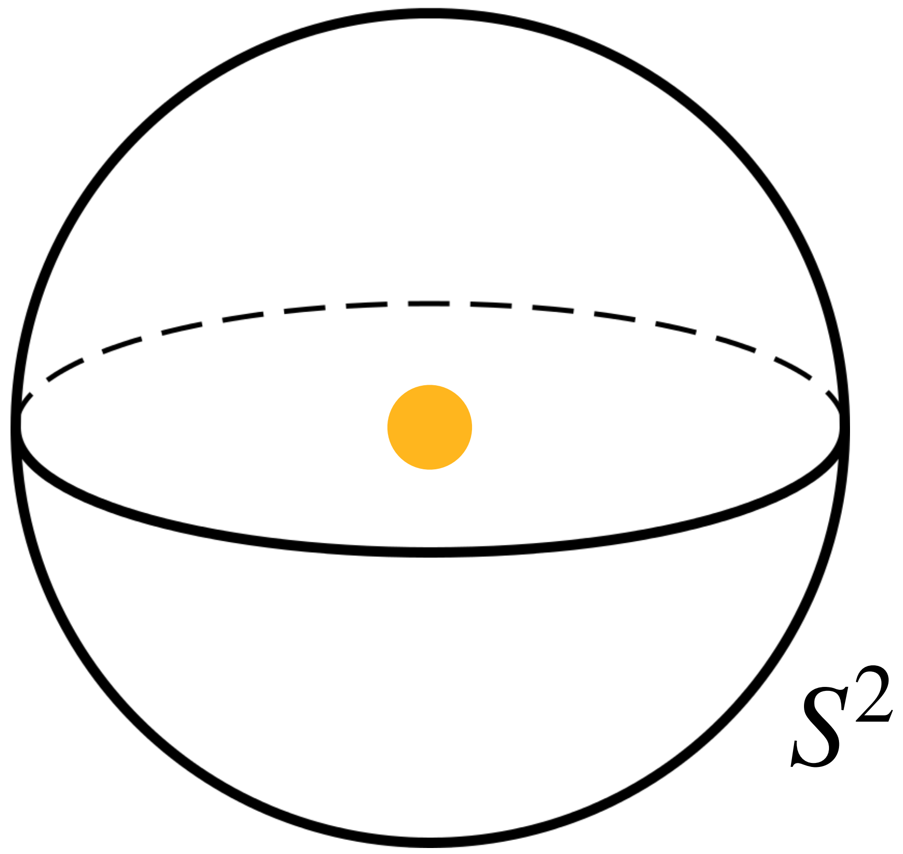
- **Low energy effective description:** massless fermions scattering off a **Dirac monopole**, i.e. a (timelike) **'t Hooft line**

$t =$ const snapshot:



$$\frac{1}{2\pi}\int F = n \in \mathbb{Z}$$

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- Work at tree level in e^2, i.e. neglect gauge field fluctuations

  $\longrightarrow$ **Free fermions** scattering off a **background monopole** (i.e. treat $U(1)$ as a **global** symmetry)

Down to a 2d chiral BCFT

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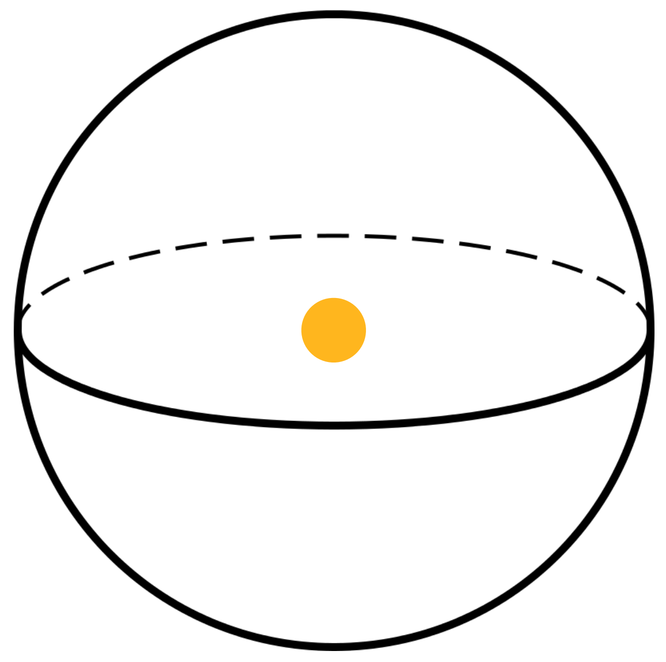
Spinor monopole harmonics

- Decompose ψ_a into angular momentum modes $|j, m\rangle$



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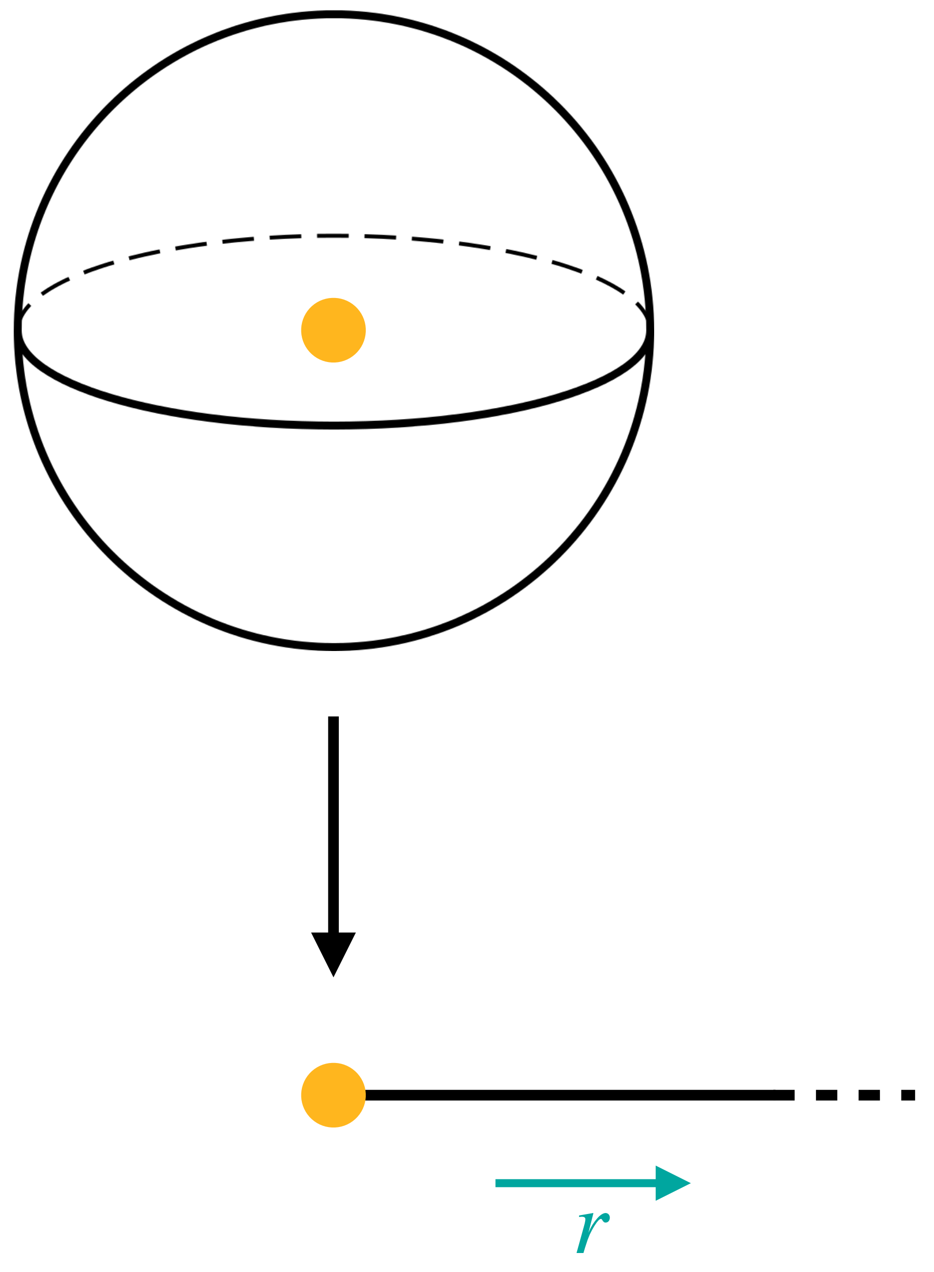

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$j_0 = \frac{1}{2}(nq_a - 1)$

	Left-movers	**Right-movers**
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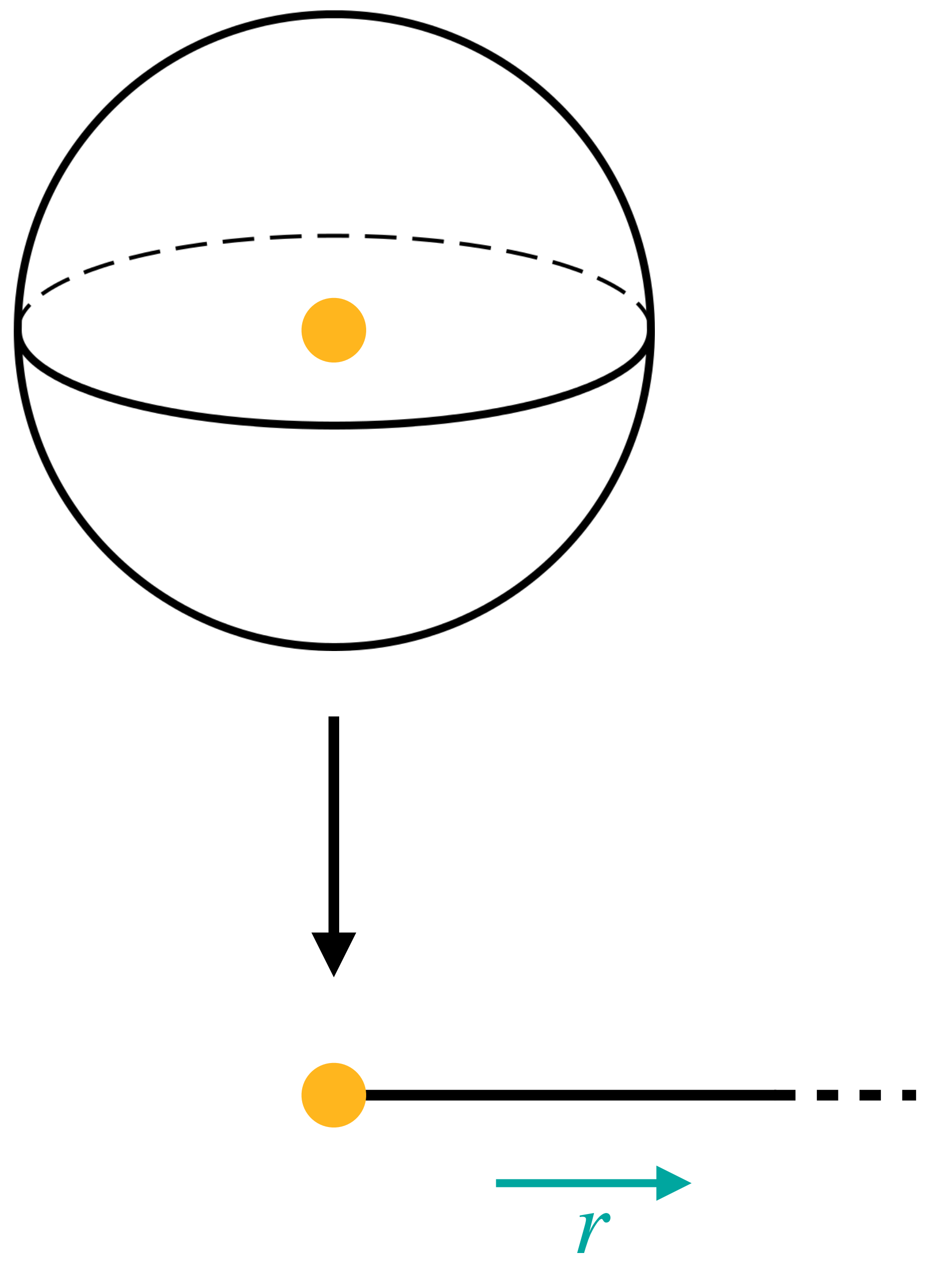

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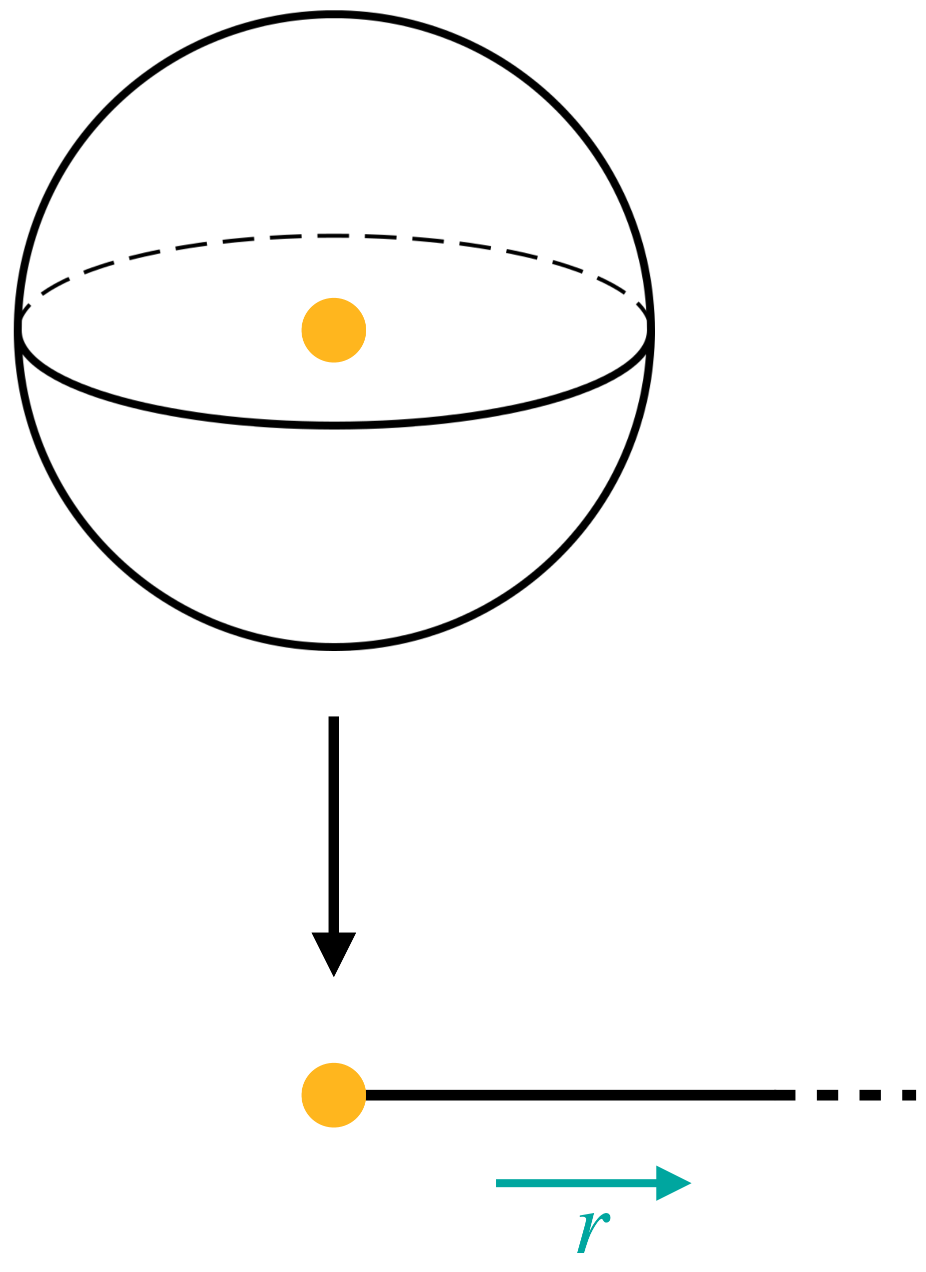

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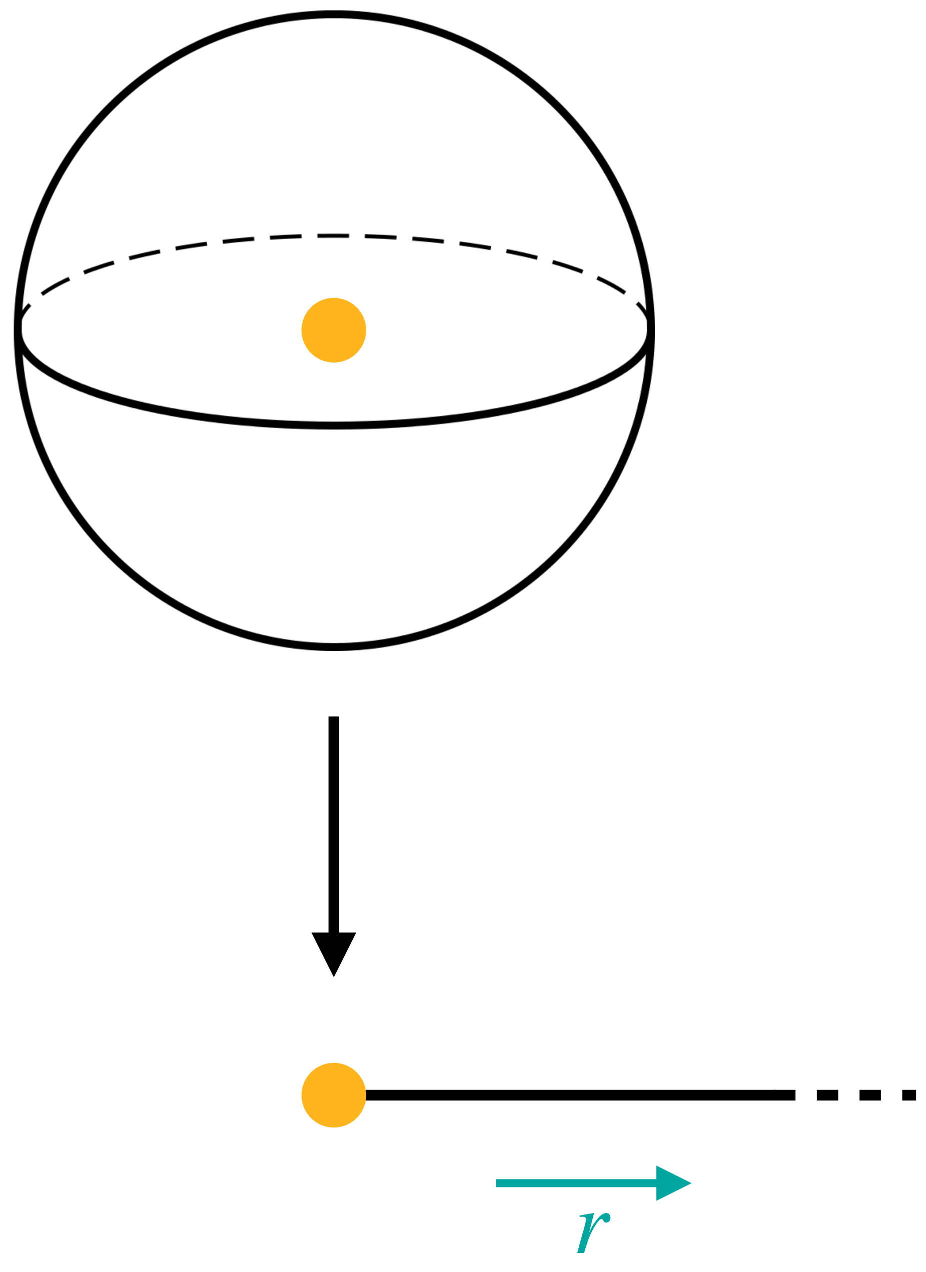

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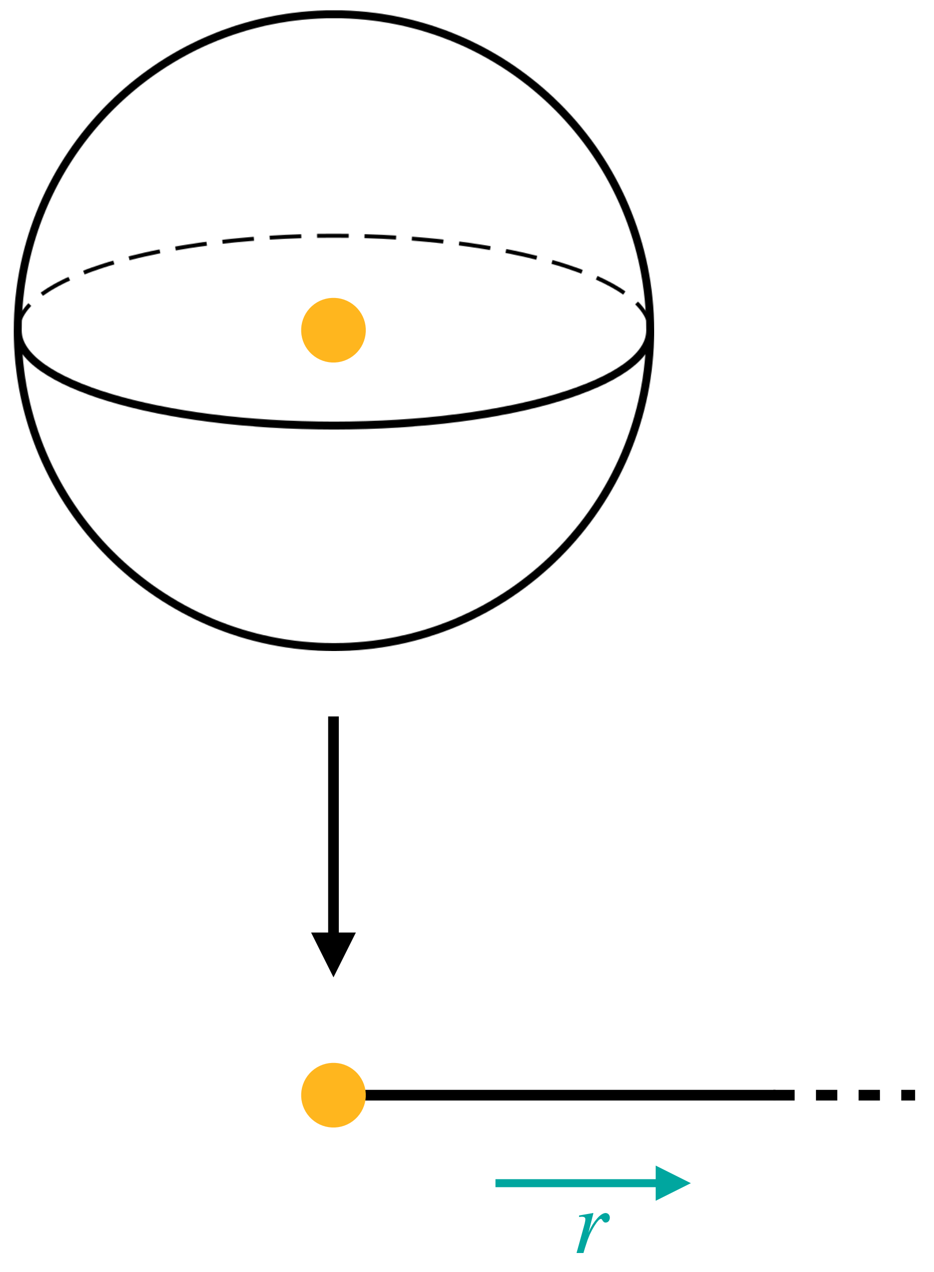

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- Left with **2d free chiral fermion BCFT** of the **lowest-lying modes**, and we need **boundary conditions**!

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- **Takeaway:** The symmetries preserved by the monopole are precisely those with **vanishing 't Hooft anomaly in 2d**

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- **In general:** 4d symmetry preserved $\longleftrightarrow$ Does not participate in 2-group with $U(1)^{(1)}_{\text{mag}}$

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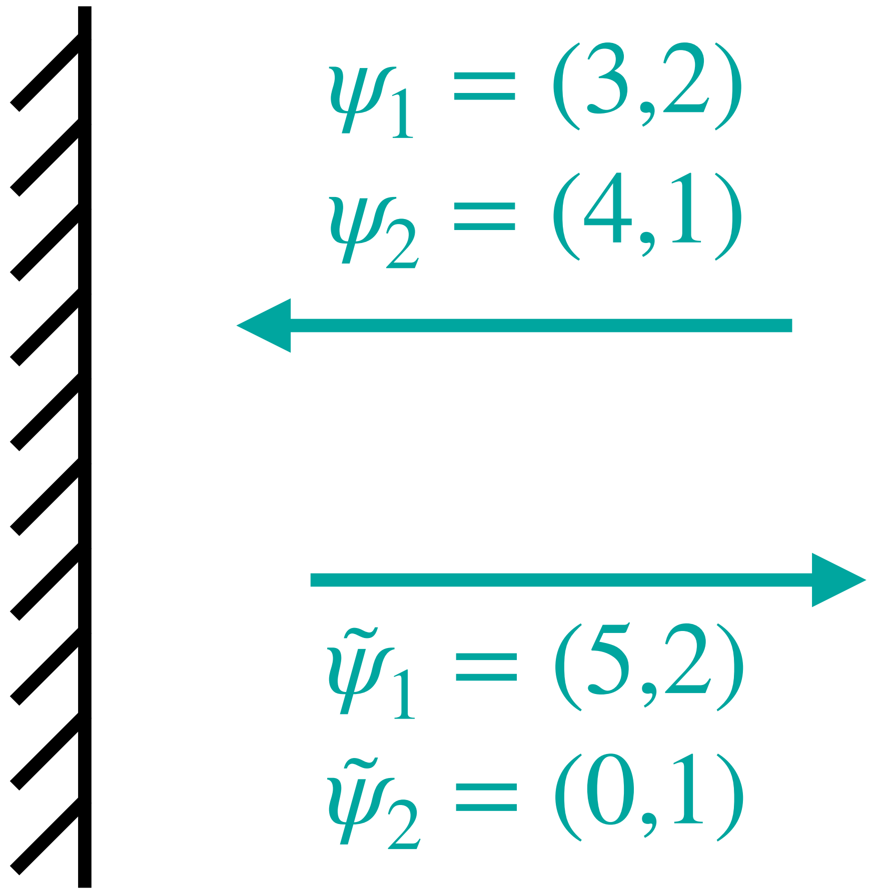

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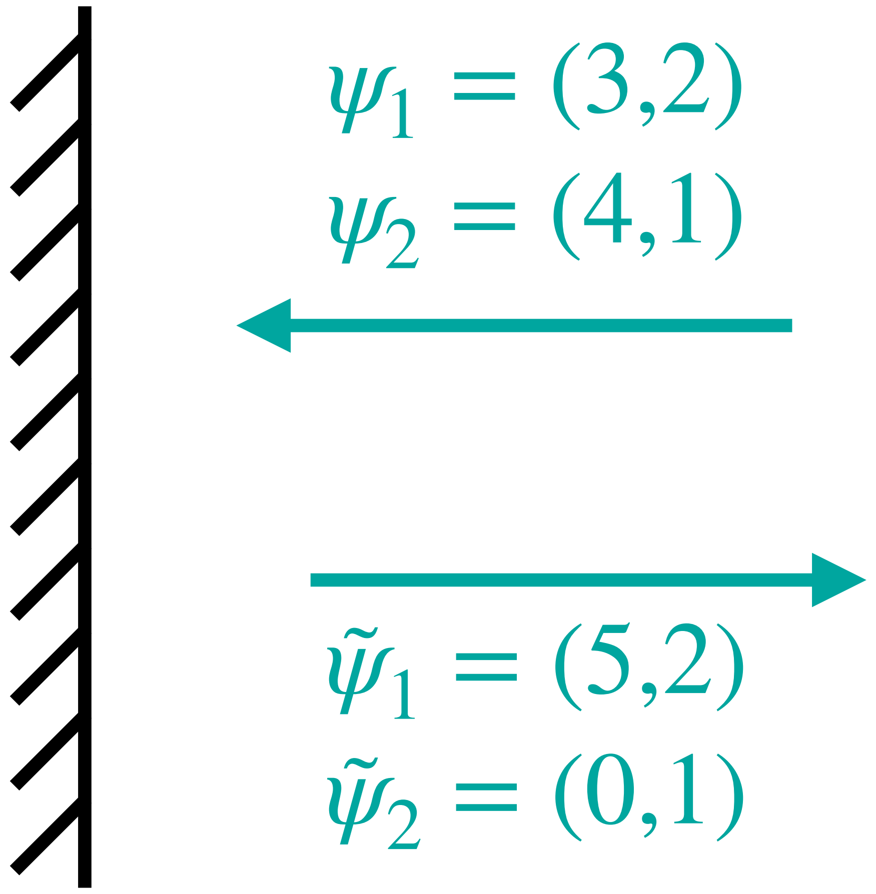

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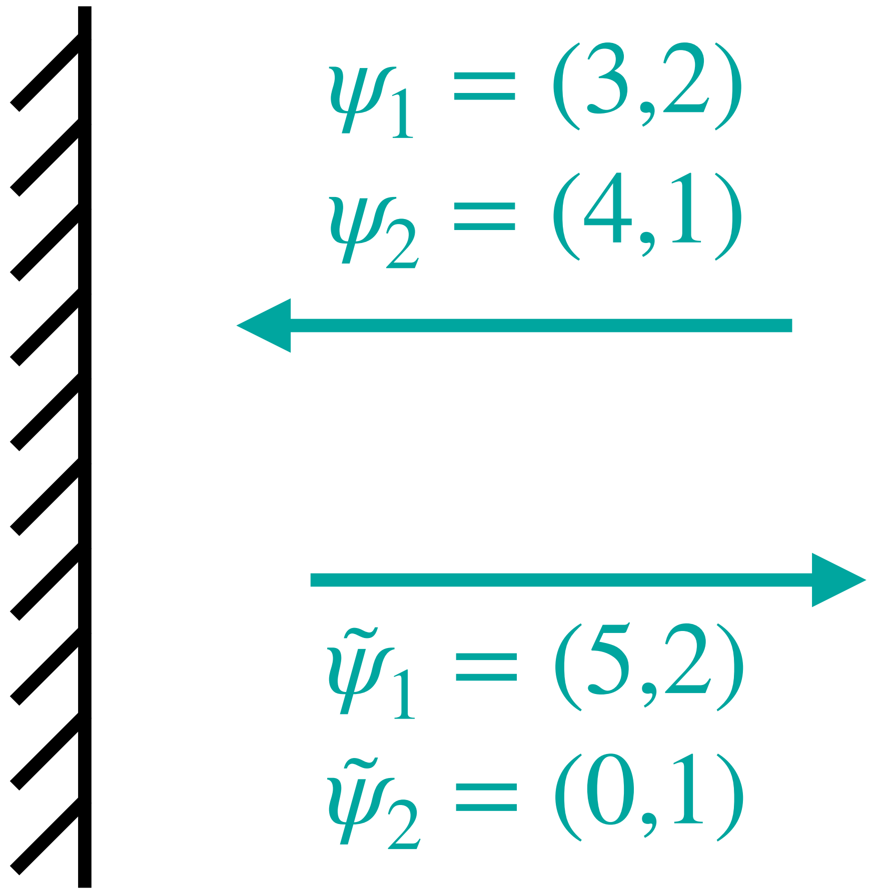

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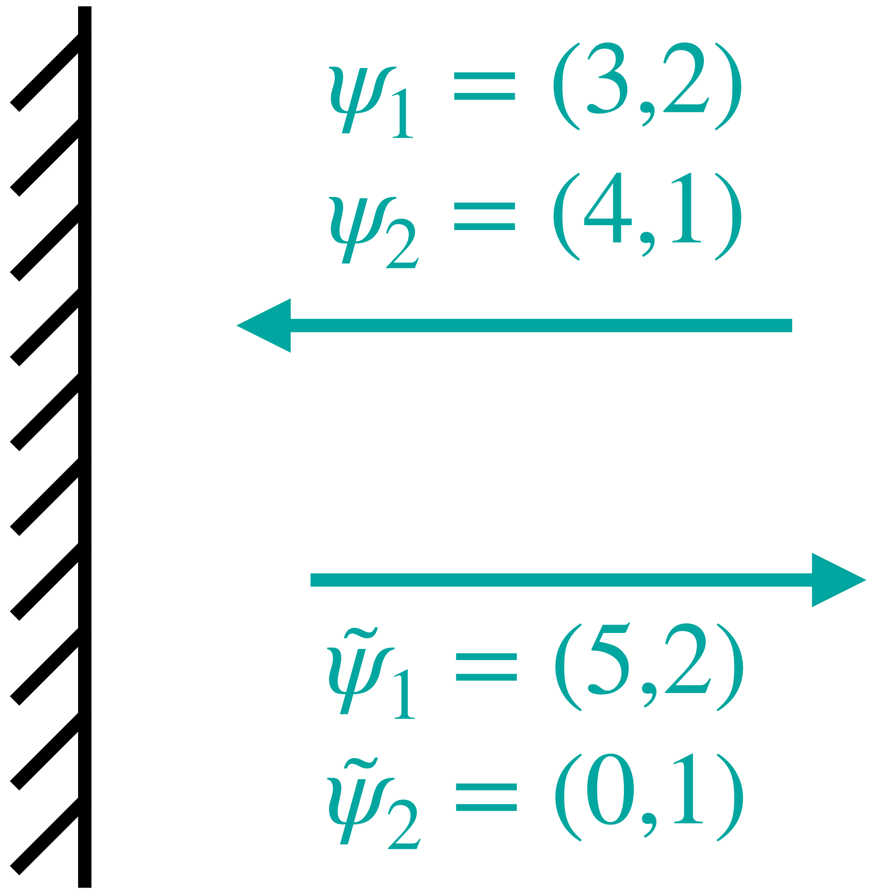

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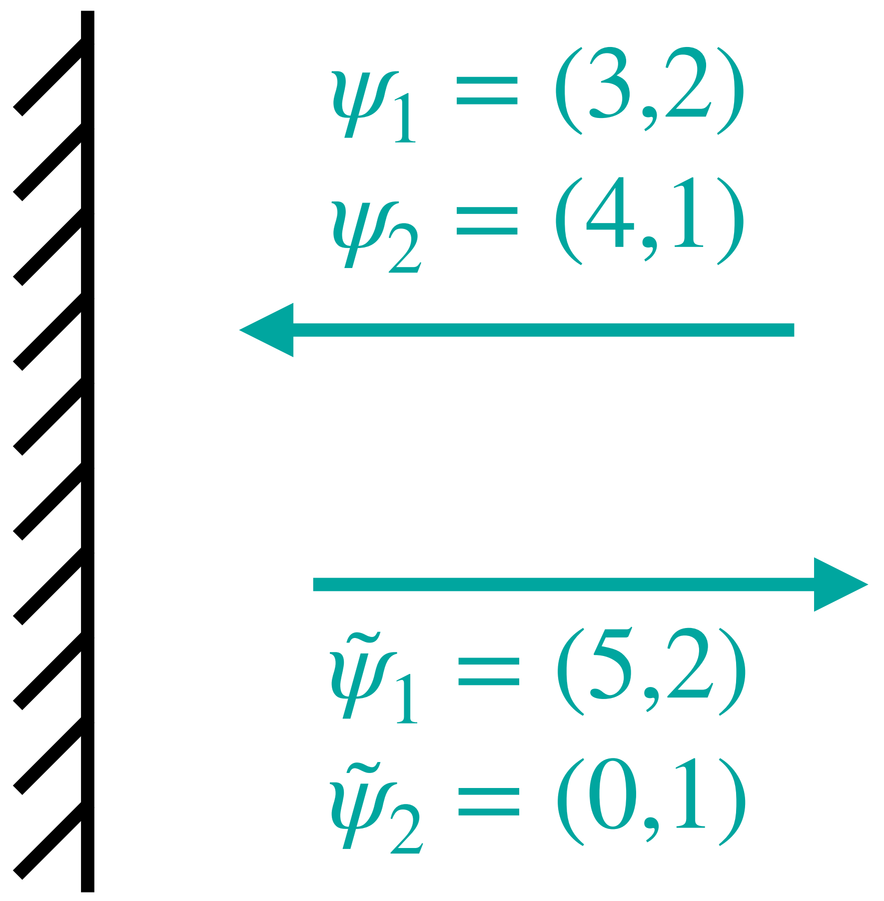

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$$\psi_1 \longrightarrow \frac{3}{5}\tilde{\psi}_1 + \frac{4}{5}\tilde{\psi}_2$$

Part II: A new solution

Gaining intuition with bosons

- Bosonise: $\psi_a \to e^{iX_a}$, $\tilde{\psi}_A \to e^{i\tilde{X}_a}$. Scattering ψ_1 becomes: $\boxed{e^{iX_1} \quad \longrightarrow \quad e^{\frac{3}{5}i\tilde{X}_1+\frac{4}{5}i\tilde{X}_2}}$

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$\implies$ $e^{\frac{3}{5}i\tilde{X}_1+\frac{4}{5}i\tilde{X}_2}$ is a **twist operator**, living at the end of a **topological line**

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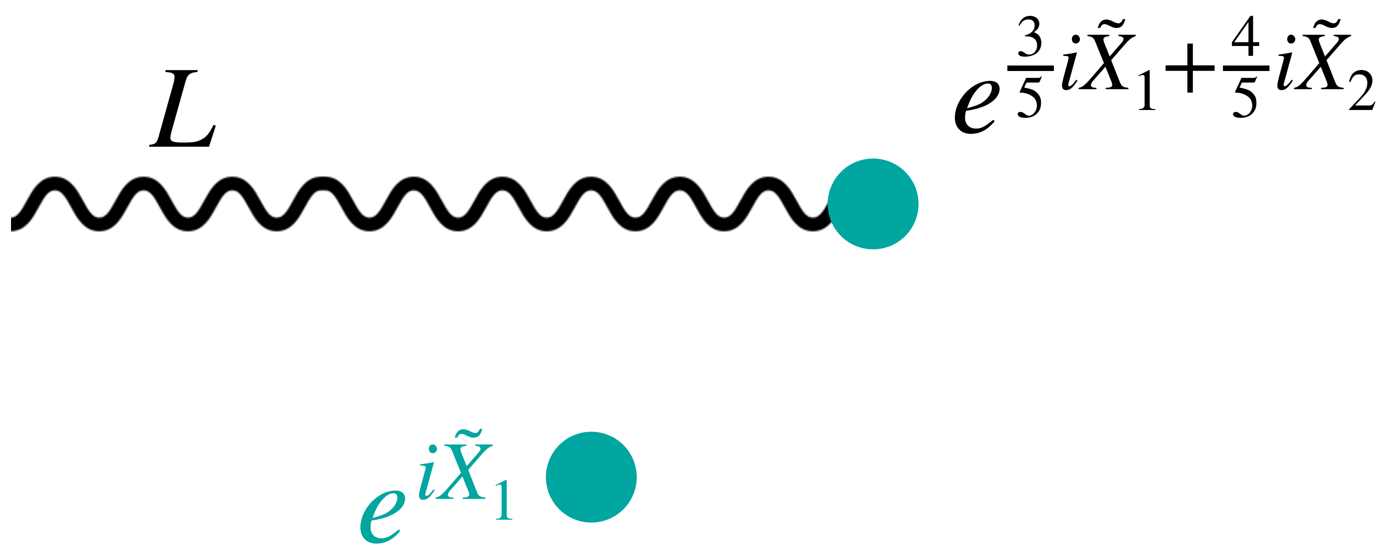

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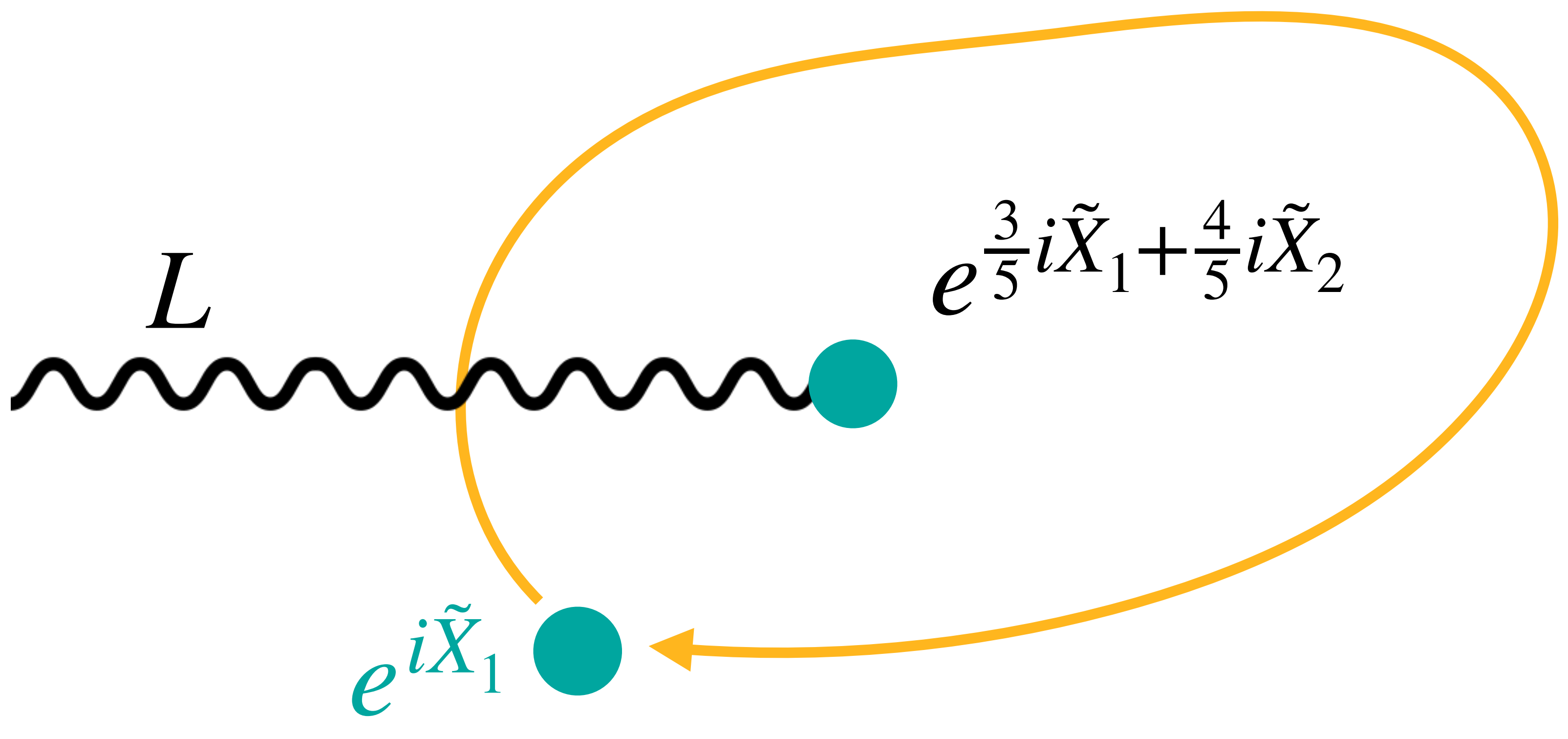

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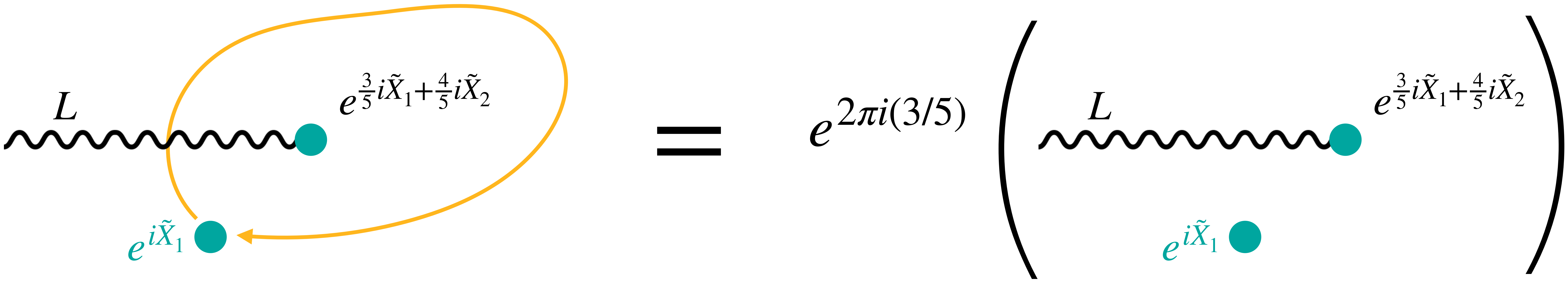

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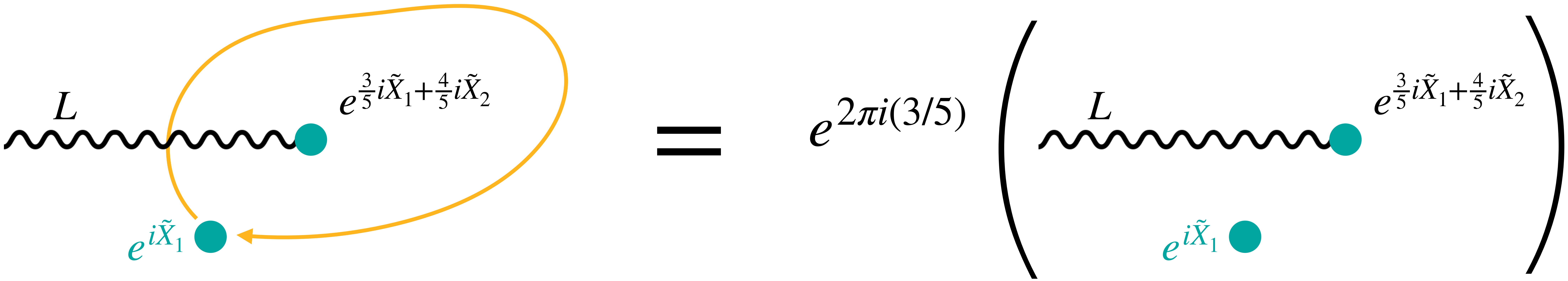

- Topological line L implements $(e^{-2\pi i/5}, e^{-2\pi i/5}) \in U(1) \times U(1)$
  - In general, scattering $e^{in_1X_1+in_2X_2}$ generates a $\mathbb{Z}_5$ twist! i.e. one of $1, L, L^2, L^3, L^4$

Fermions and the twisted Hilbert space

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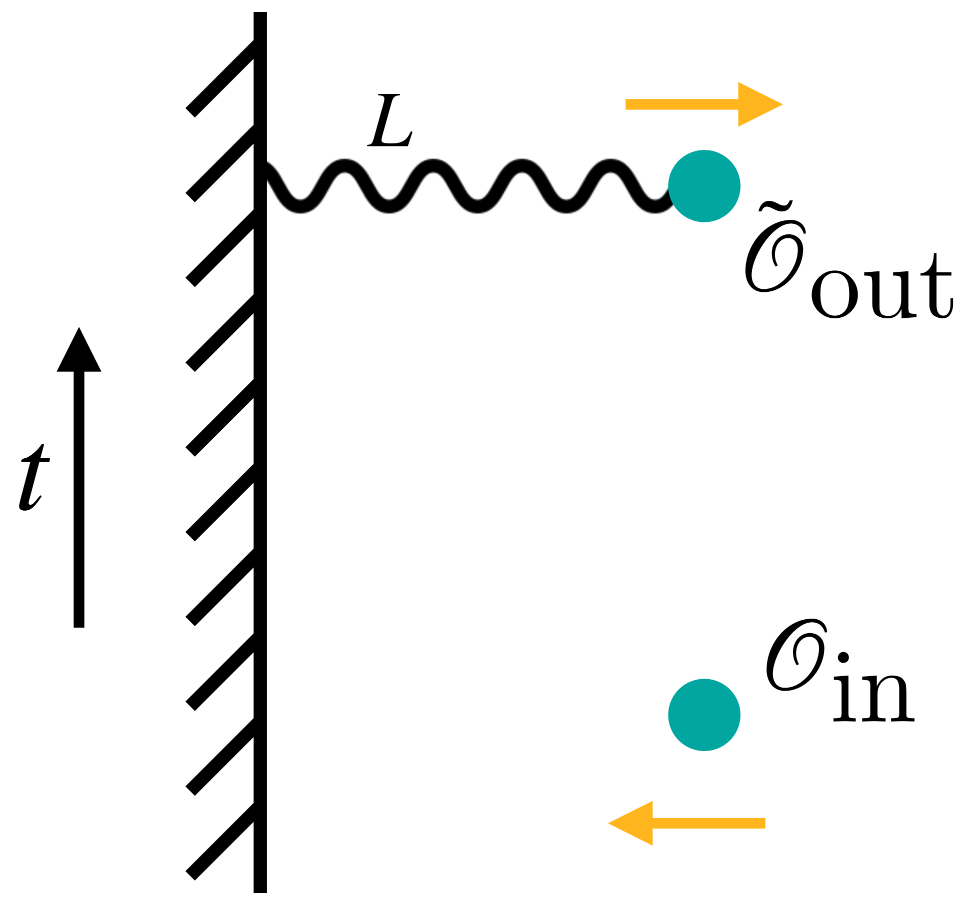

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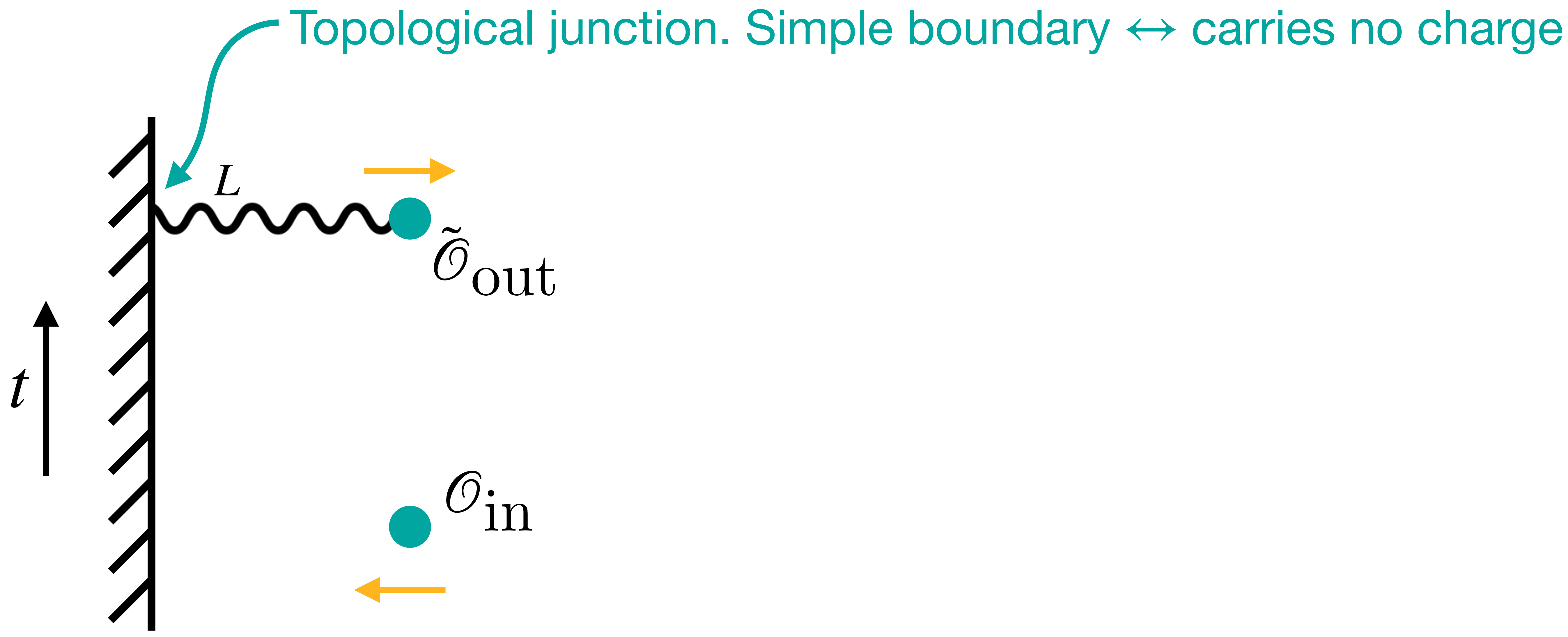
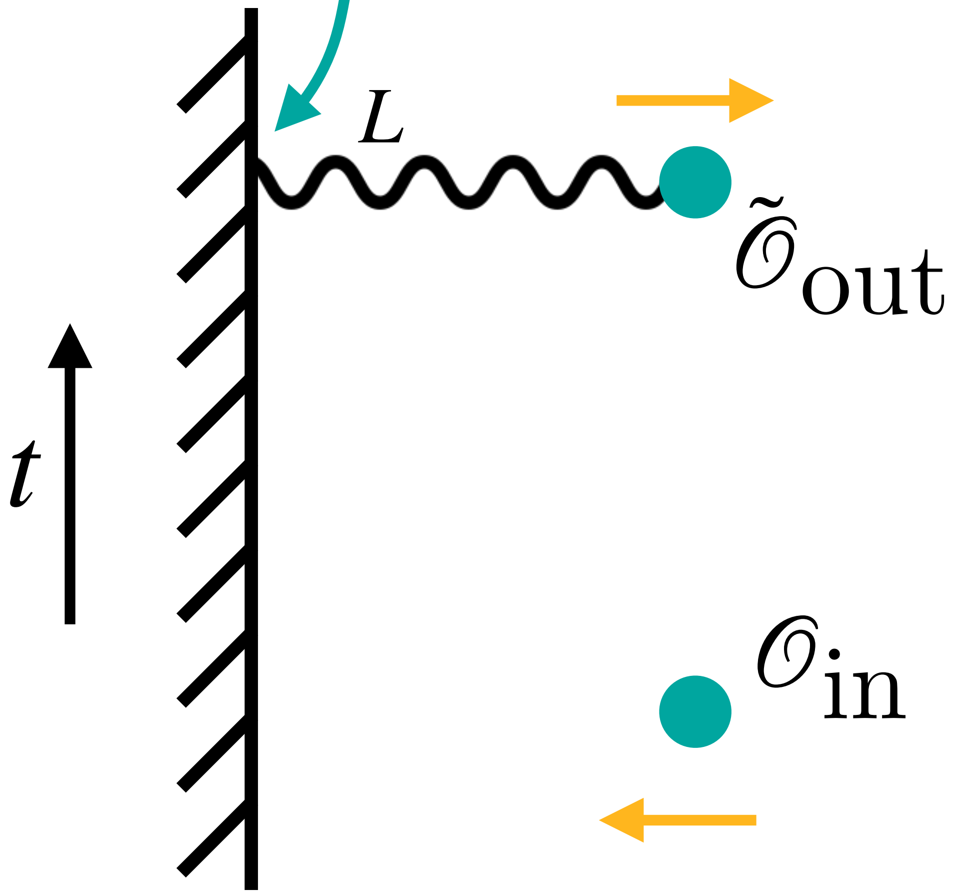
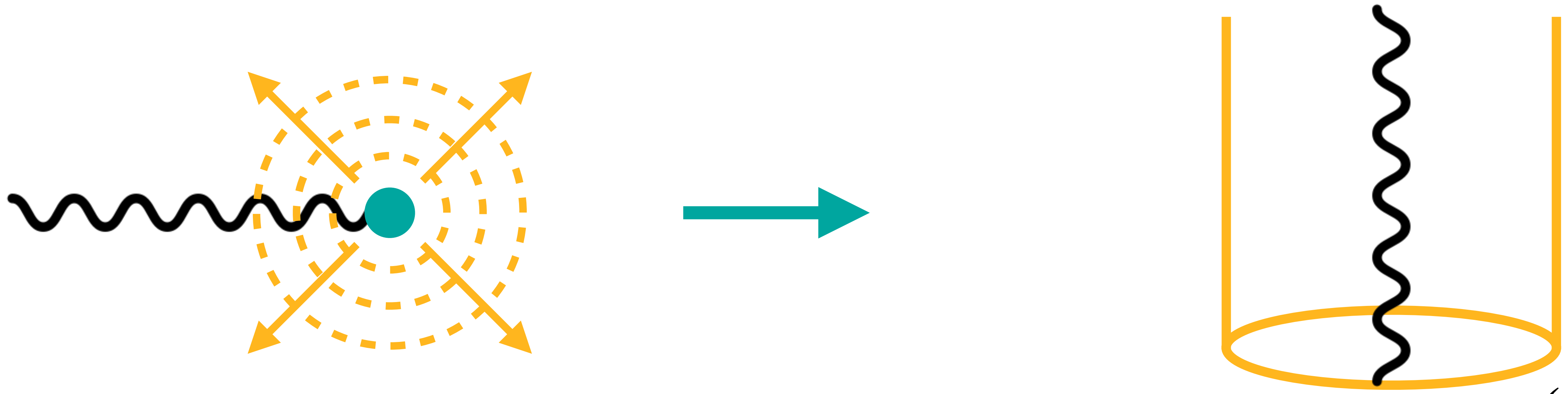


Fermions and the twisted Hilbert space

Topological junction. Simple boundary $\leftrightarrow$ carries no charge



- Possible twist operators $\tilde{\mathcal{O}}_{\text{out}}$ classified using operator-state map



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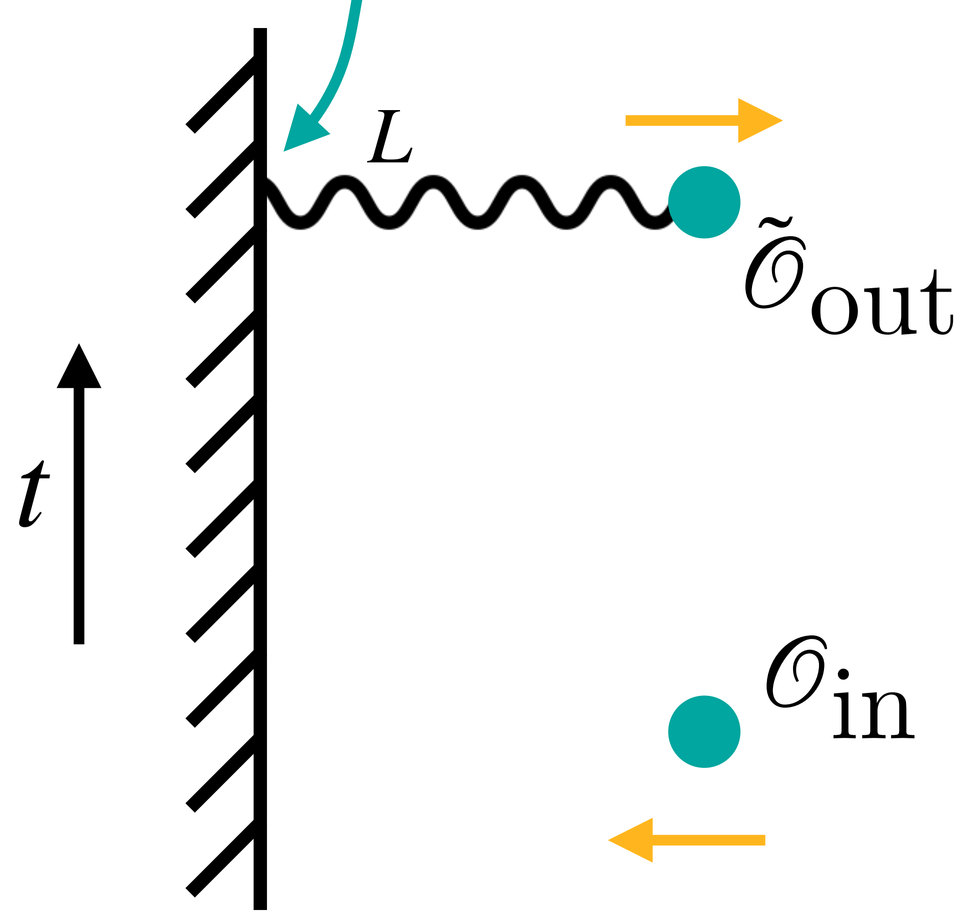

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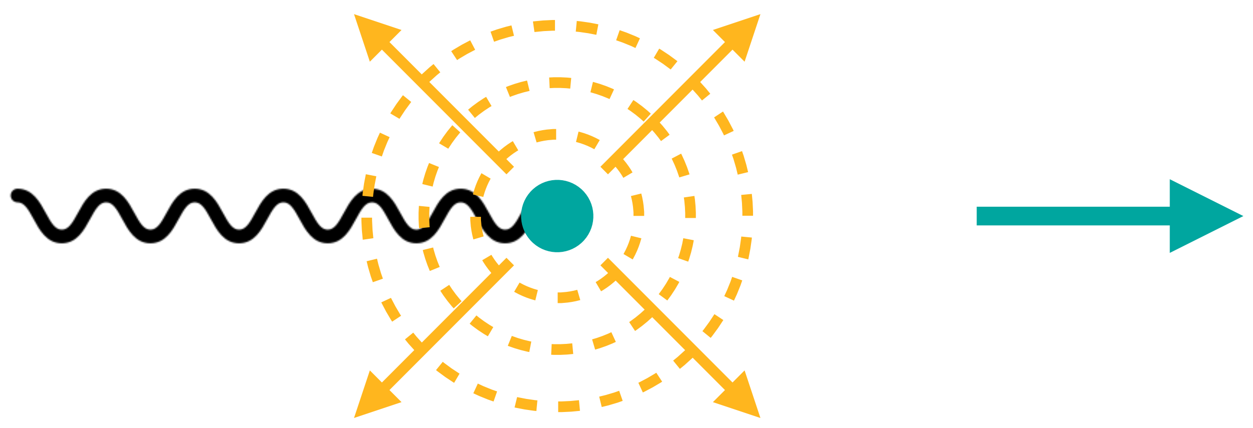

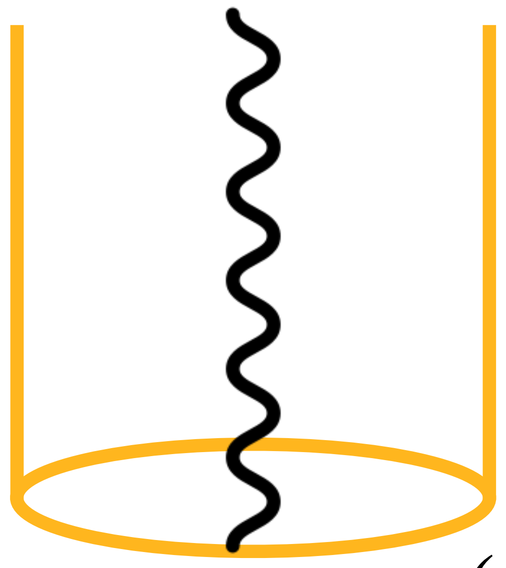

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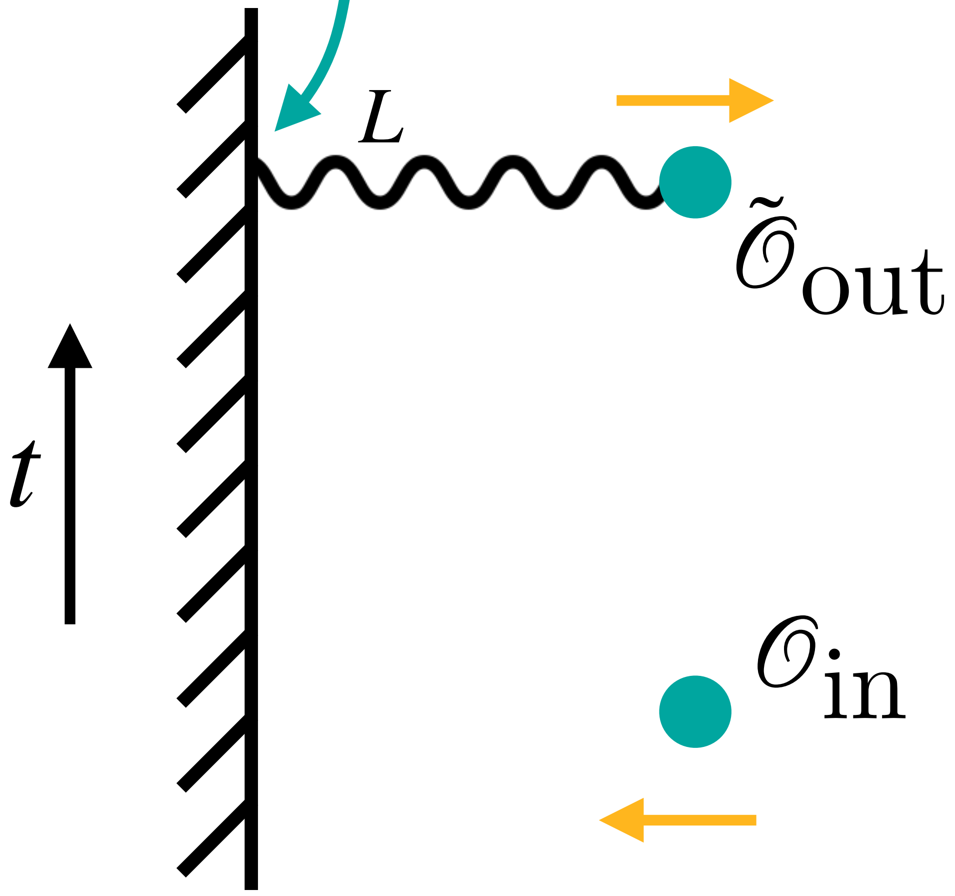

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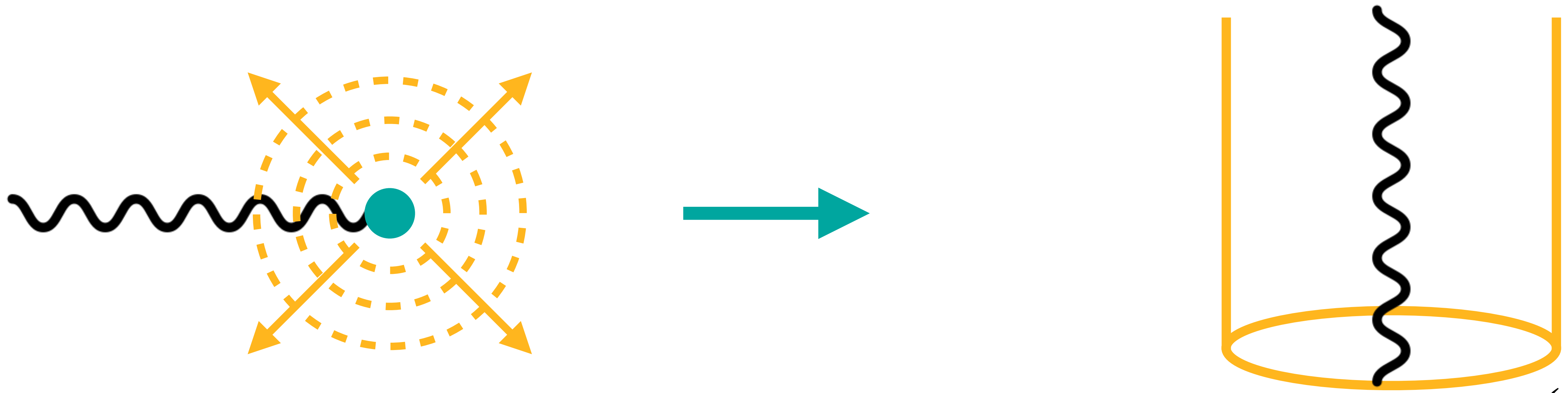

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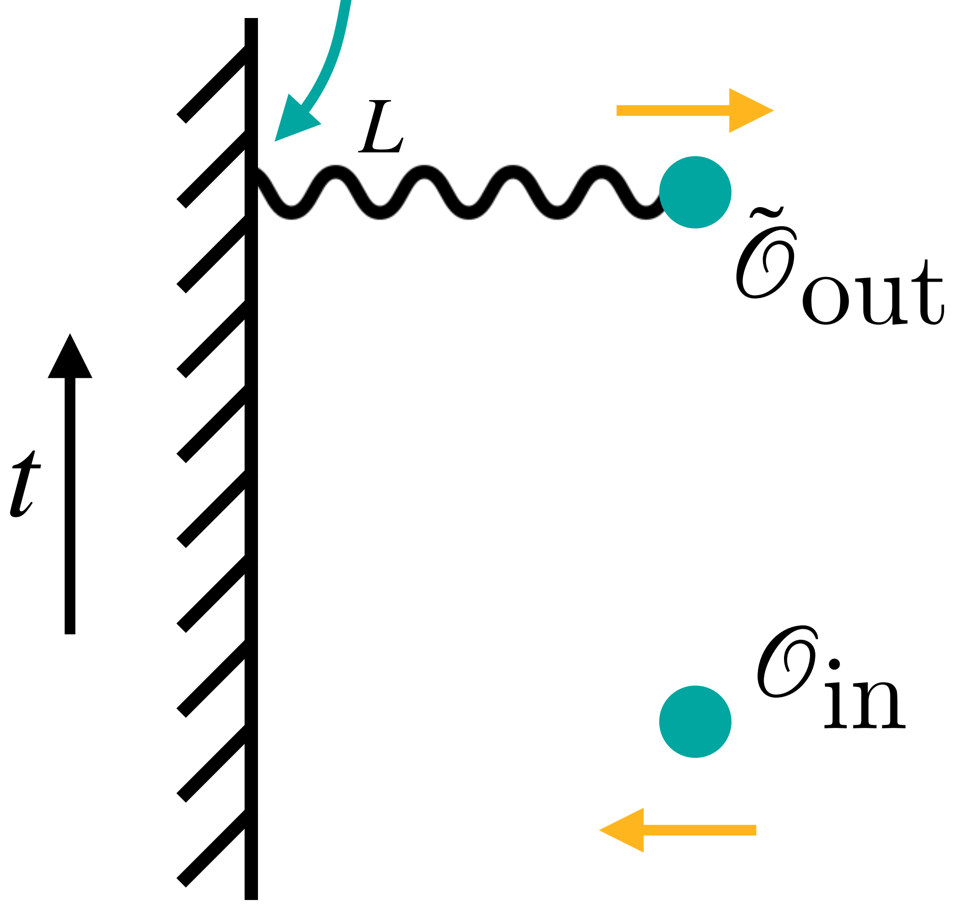

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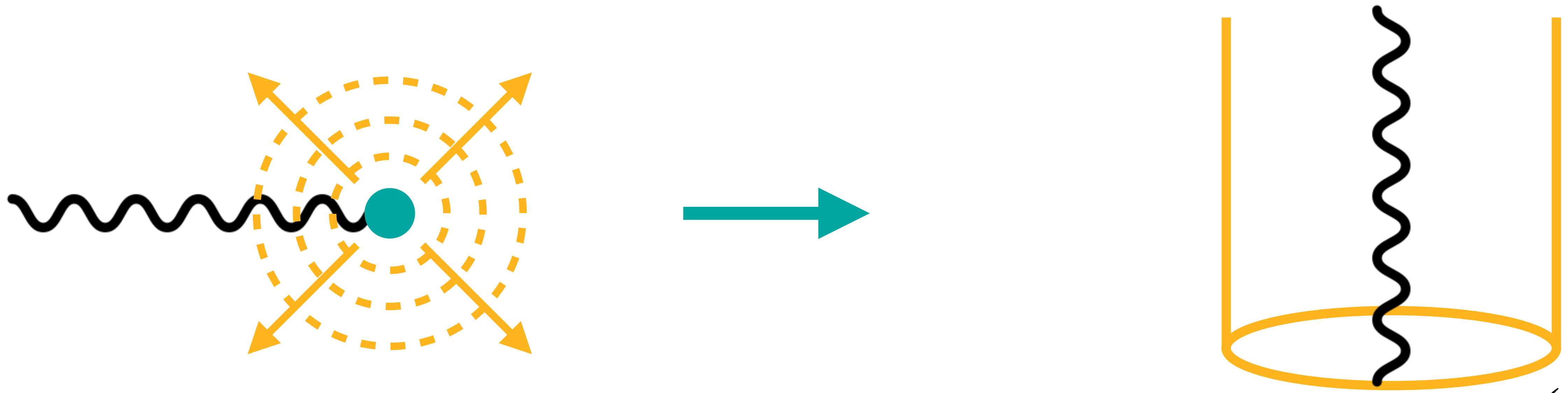

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- Use **symmetries** to fix $\tilde{\mathcal{O}}_{\mathrm{out}}$, given $\mathcal{O}_{\mathrm{in}}$
  - If (rank of preserved symmetry) $\geq c_L = c_R$, then **scattering completely fixed**

The view from four dimensions

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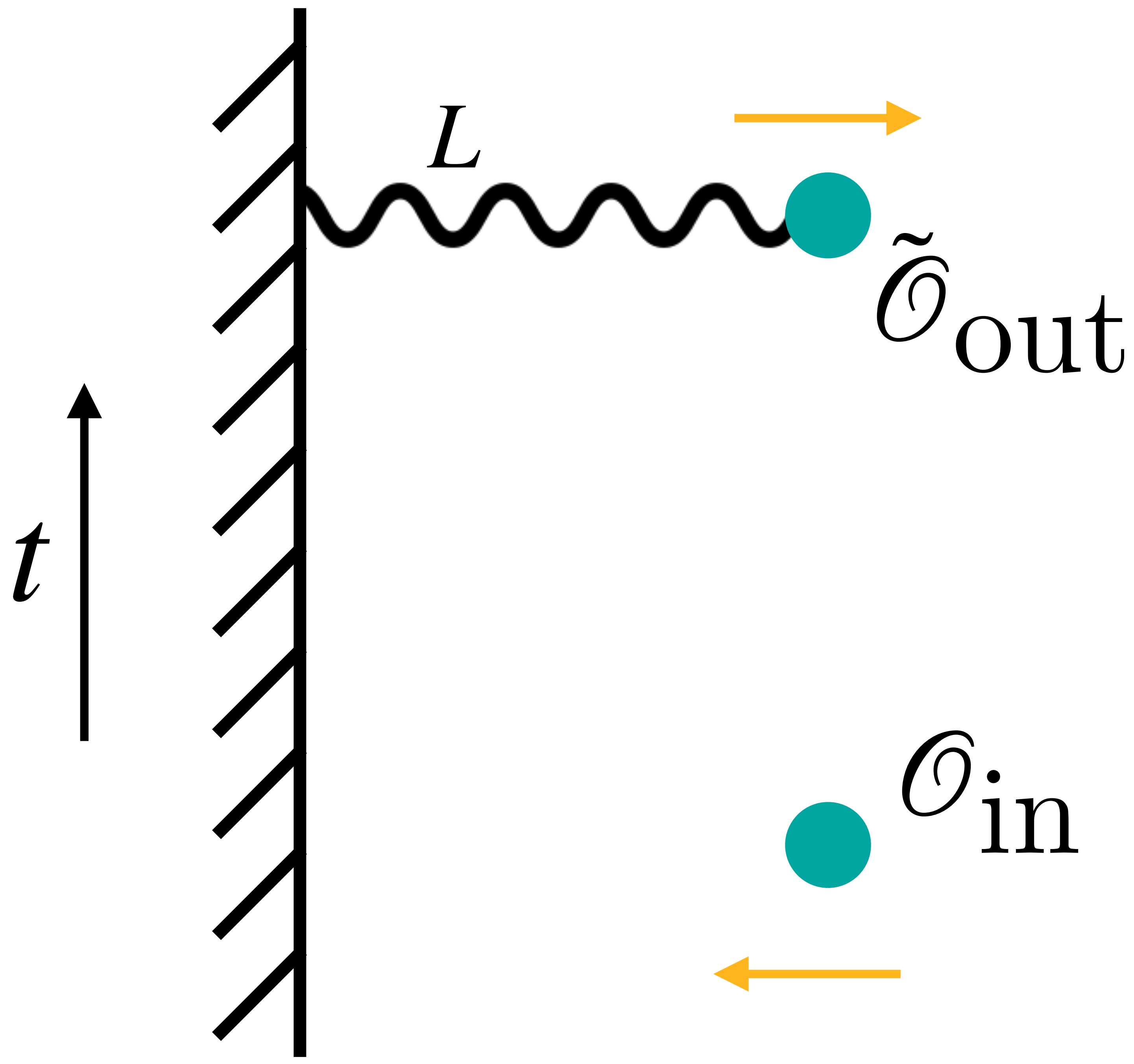

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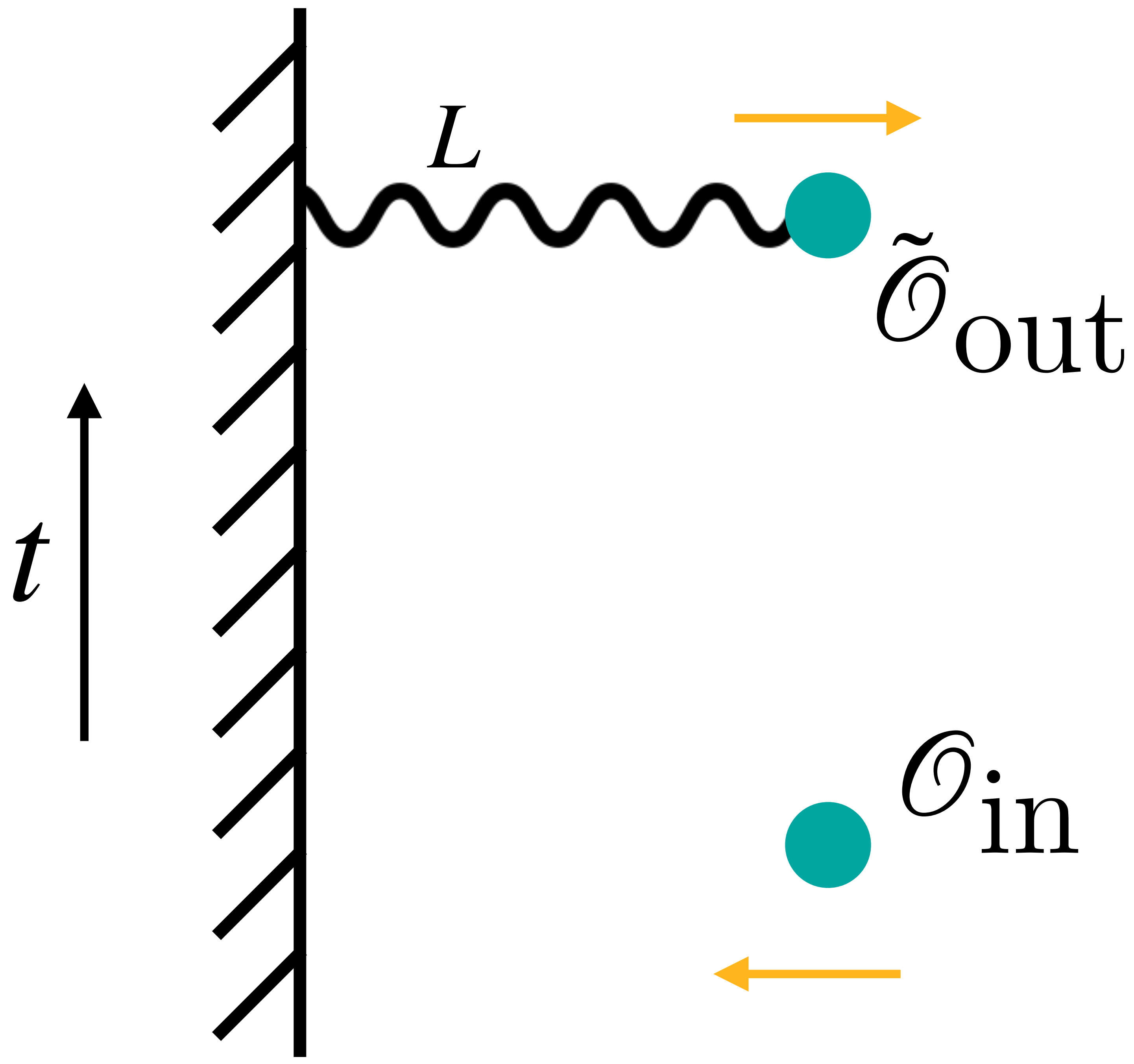

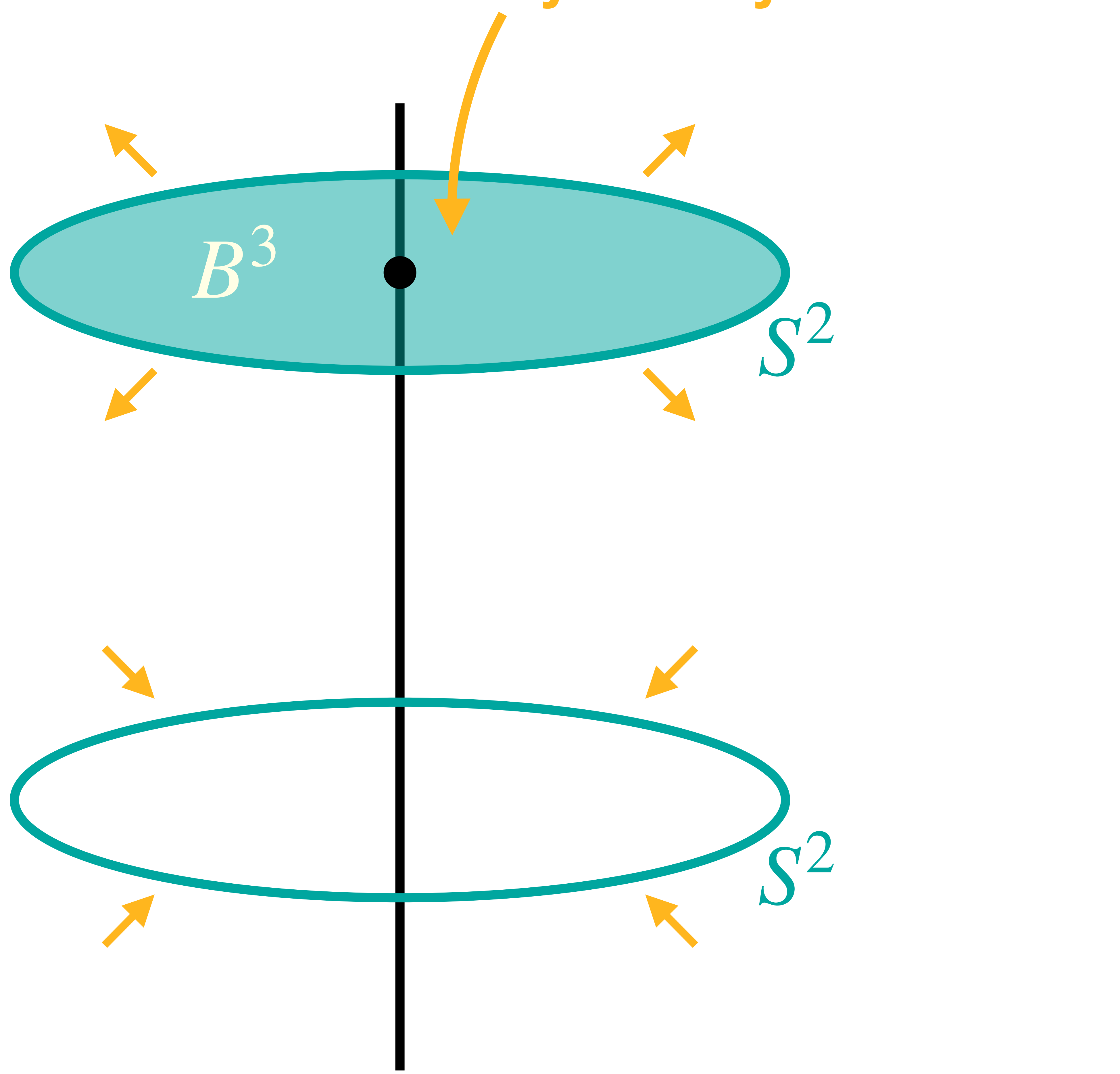

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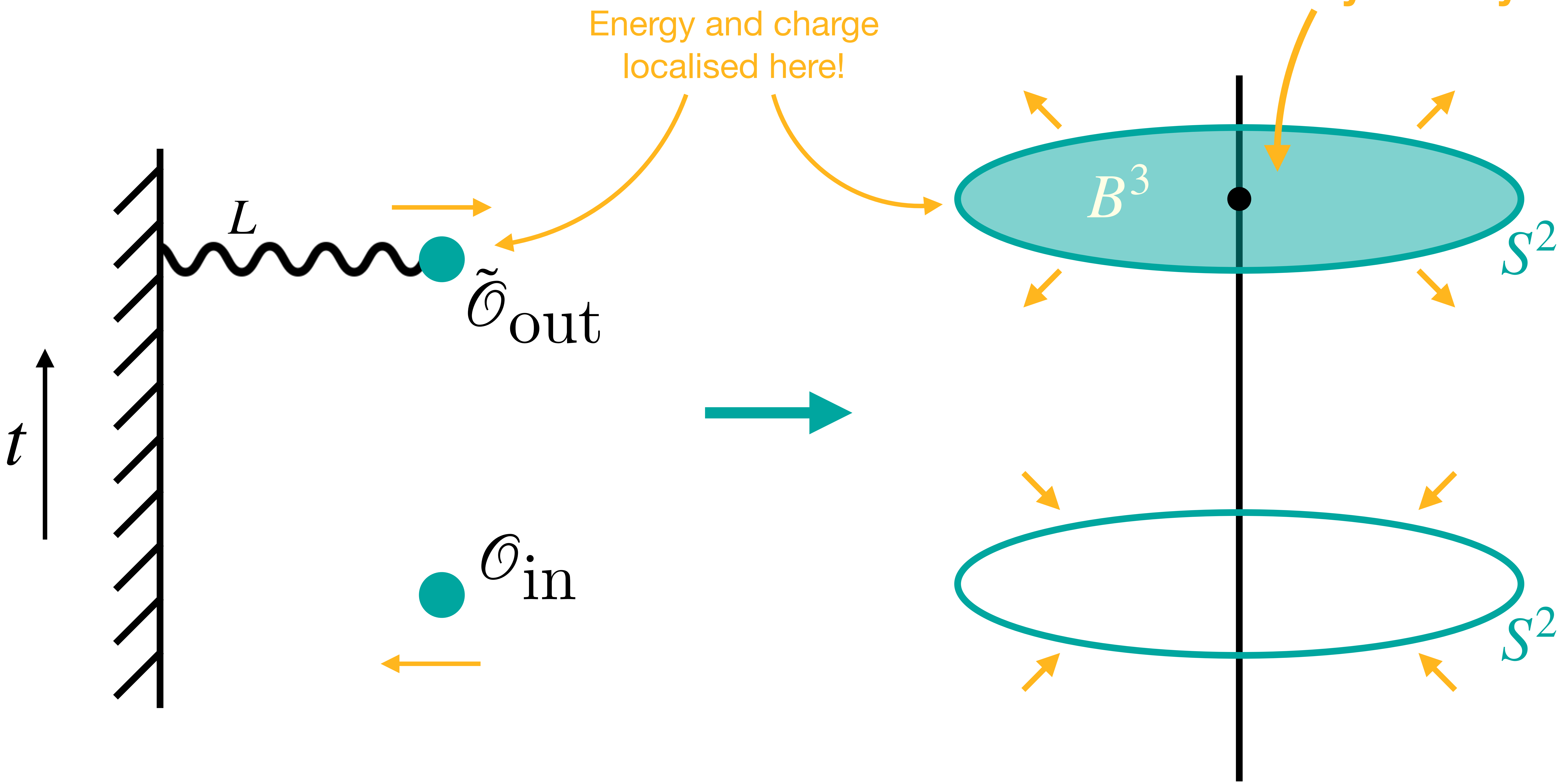
Topological surface for relevant 4d 0-form symmetry



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5. **Come and talk at CERN about it**

Part III: The Standard Model

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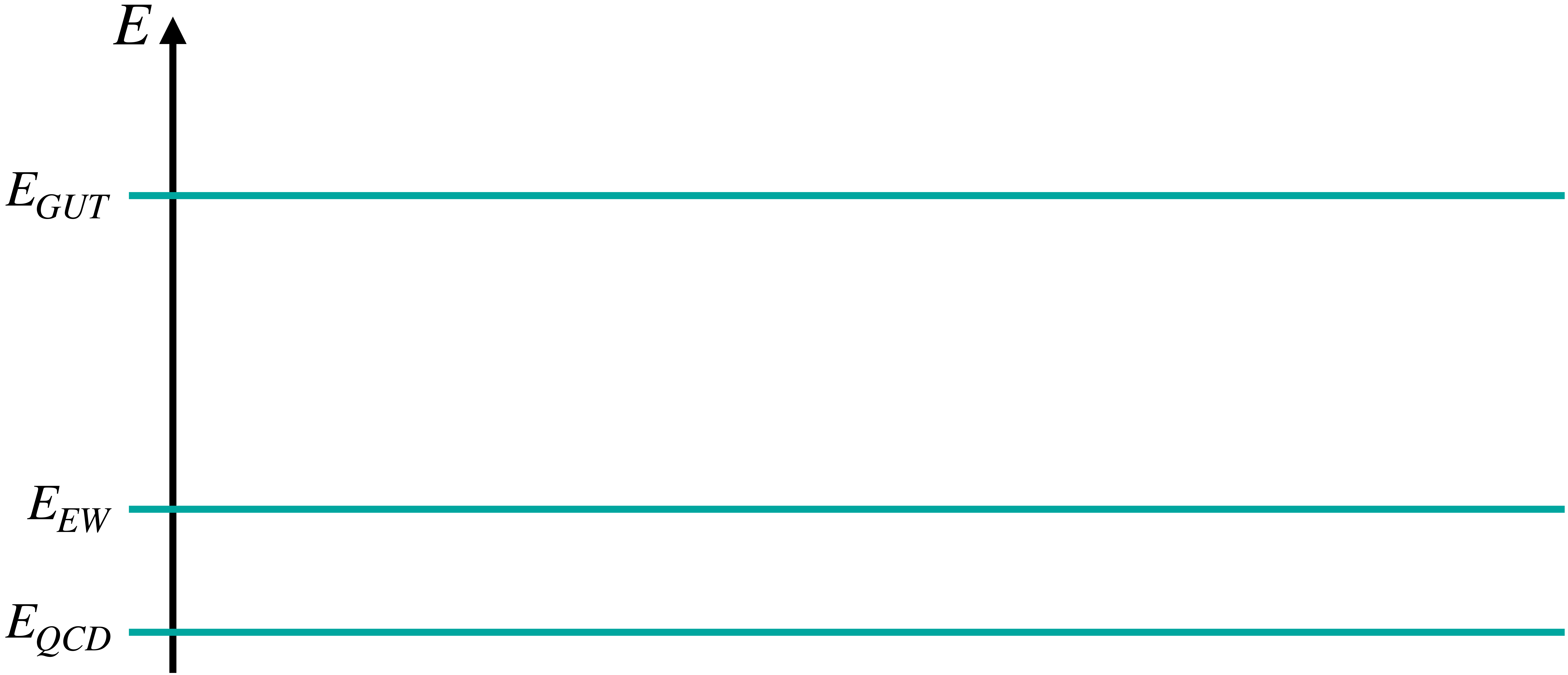
- A rotation of e_R has only a mixed anomaly with $U(1)_Y \longrightarrow$ **Non-invertible $\mathbb{Z}_n$** symmetry!

The regime of interest



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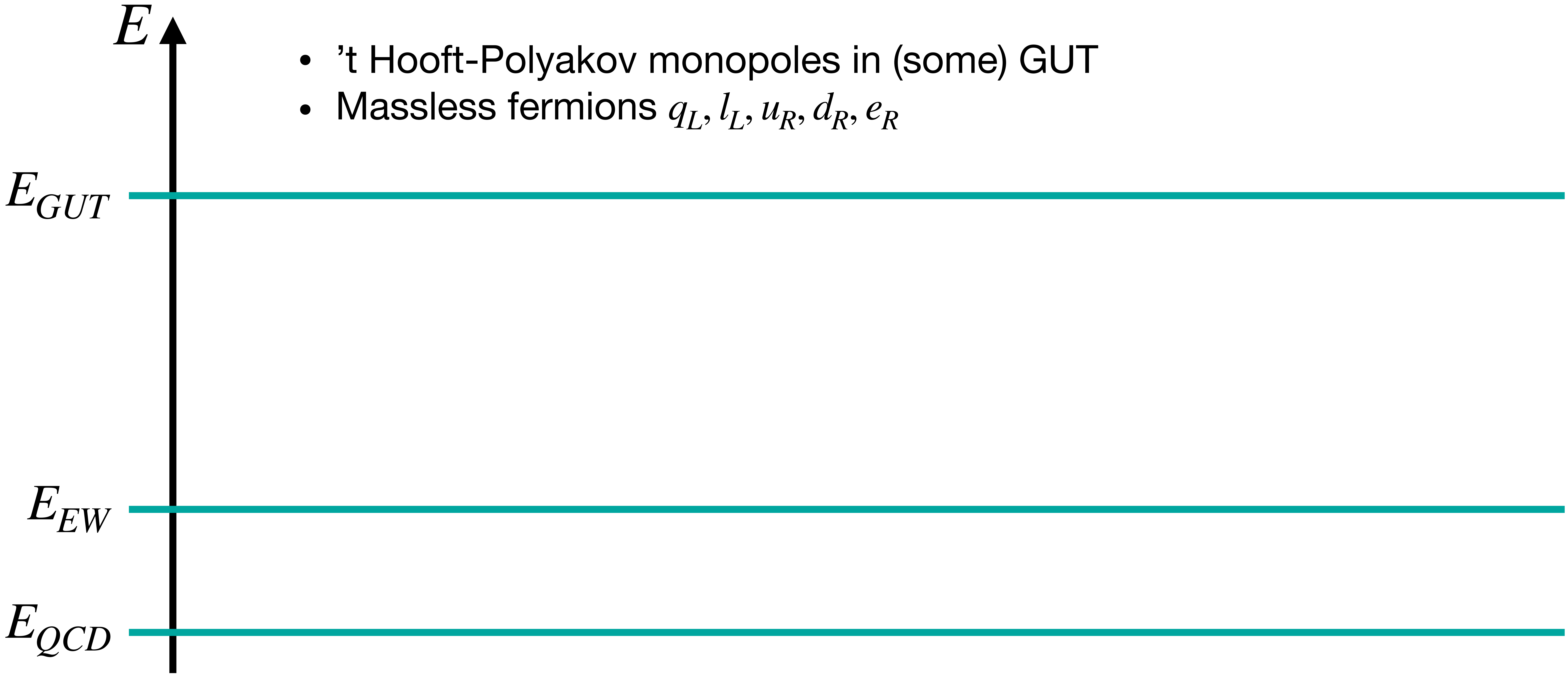
- ’t Hooft-Polyakov monopoles in (some) GUT
- Massless fermions q_L, l_L, u_R, d_R, e_R



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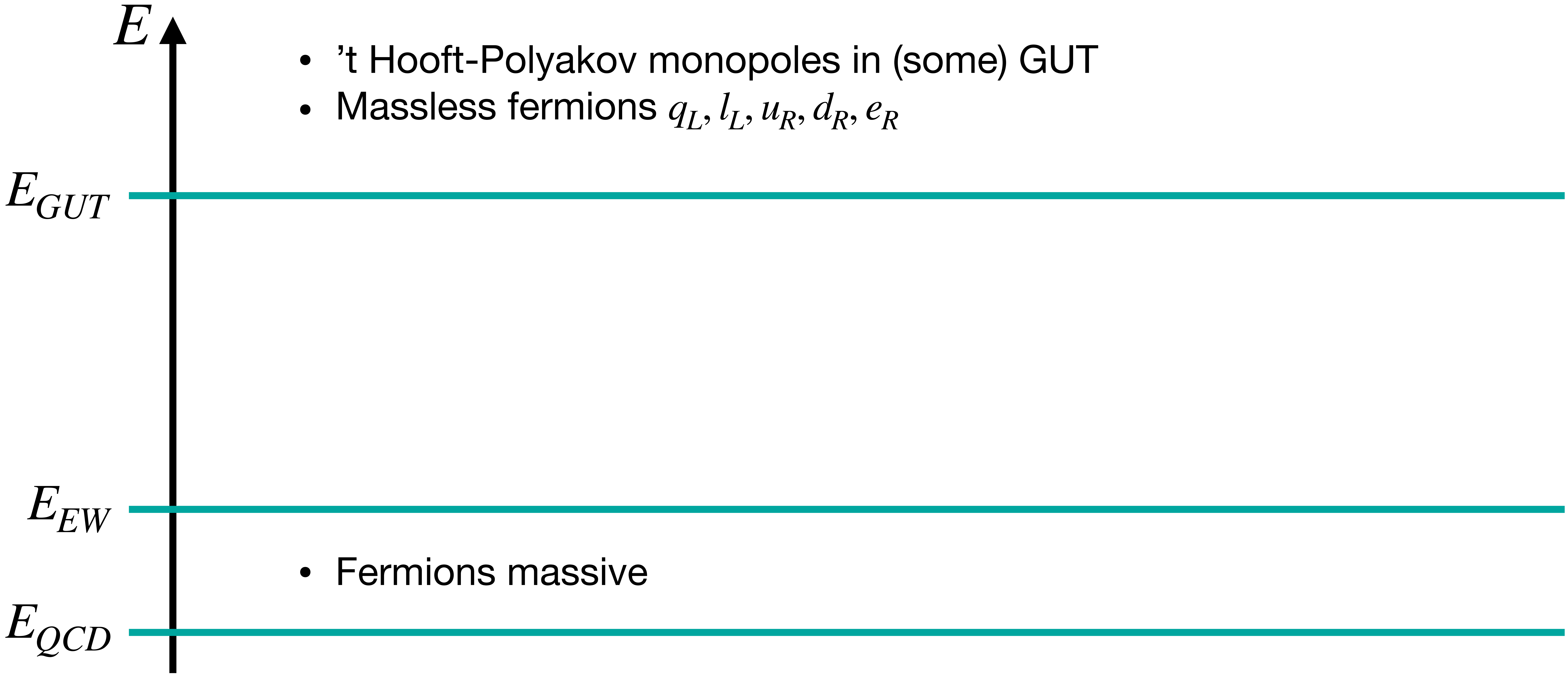

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E

- ’t Hooft-Polyakov monopoles in (some) GUT
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E_{GUT}

- Monopoles very heavy $\rightarrow$ ’t Hooft lines in G
- Massless fermions q_L, l_L, u_R, d_R, e_R
- Neglect gauge field fluctuations $\rightarrow$ G treated as global symmetry

E_{EW}

- Fermions massive

E_{QCD}

Spectrum of ’t Hooft lines

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- Spherically-symmetric Dirac monopoles in G admit **GNO-like classification**
  - Different **embeddings** of $U(1)$ into the Cartan subgroup of G

$$F = \begin{pmatrix} n_1 - \frac{N}{3} & & \\ & n_2 - \frac{N}{3} & \\ & & -\frac{N}{3} \end{pmatrix} \oplus \begin{pmatrix} \frac{1}{2}m & \\ & -\frac{1}{2}m \end{pmatrix} \oplus \left(g + \frac{N}{3} + \frac{m}{2}\right)\left(\frac{1}{2}\sin\theta\, d\theta \wedge d\phi\right)$$

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- In full gauge theory: Unless $n_1, n_2, m \leq 1$, monopole **unstable to W-boson condensation**. Precisely one stable monopole for each p. **Ask me about this!**

Preserved symmetries and solvable monopoles

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- **Upshot:** Can **uniquely solve scattering** for monopoles with $c_L = c_R \leq 7$
    - Turns out there's nine of these!

The minimal monopole, solved

Symmetries and chiral modes

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- From all 15 fermions, only 4 + 4 chiral modes (i.e. $c_L = c_R = 4$). All spinless ($j_0 = 0$)

Ingoing (i.e. left-movers)	**Outgoing (i.e. right-movers)**
$q_L^{a,+}, d_R^3, e_R$	q_L^-, u_R^a, l_L^-

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- Note, T carries charges $B = L = -\frac{1}{2}$. Should be **quantised** in $\frac{1}{3}\mathbb{Z}$, $\mathbb{Z}$ respectively!
  - Non-anomalous $B - L$ preserved. But **anomalous B and L fractionalised**
  - Generic feature of scattering into topological lines

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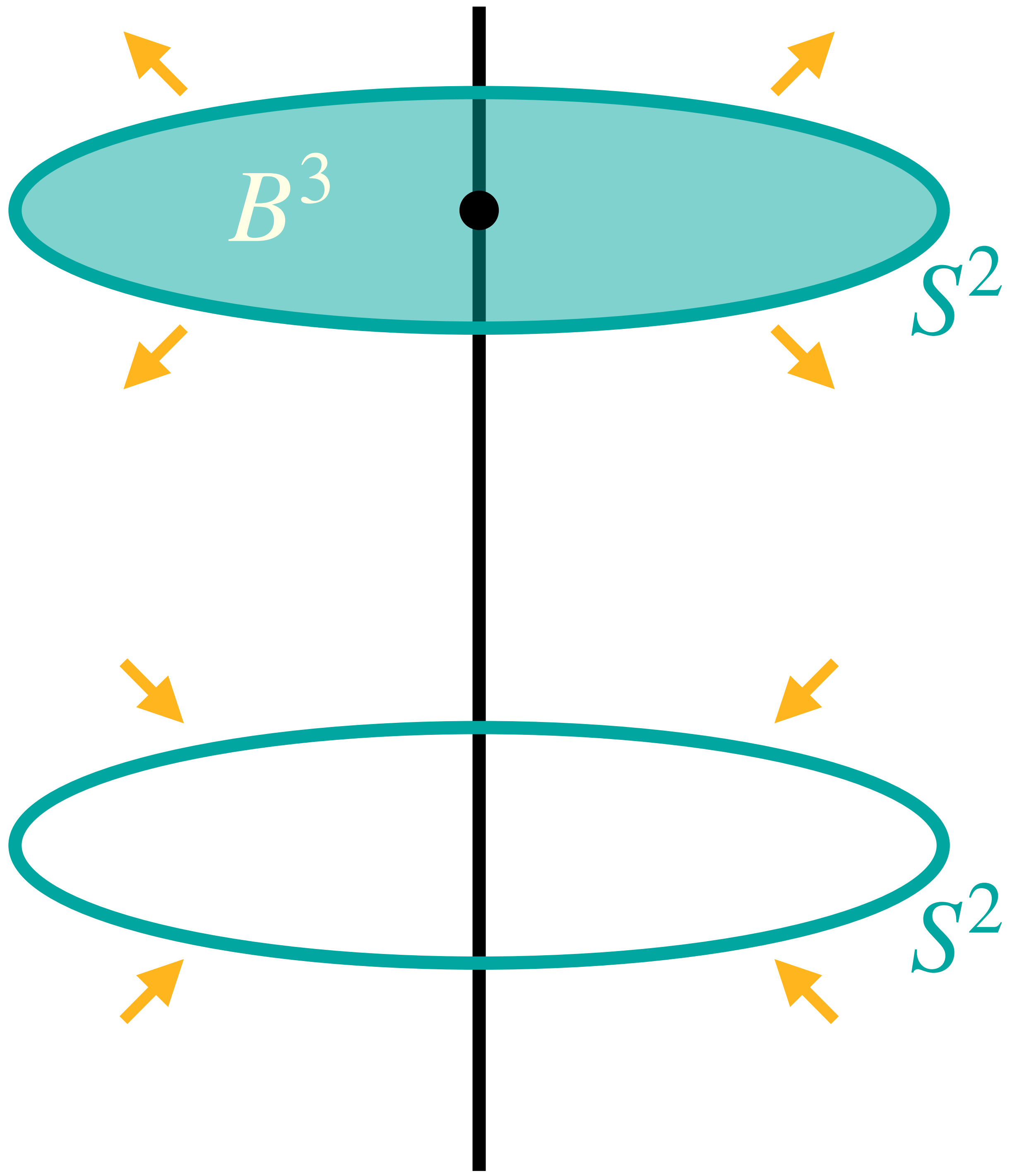

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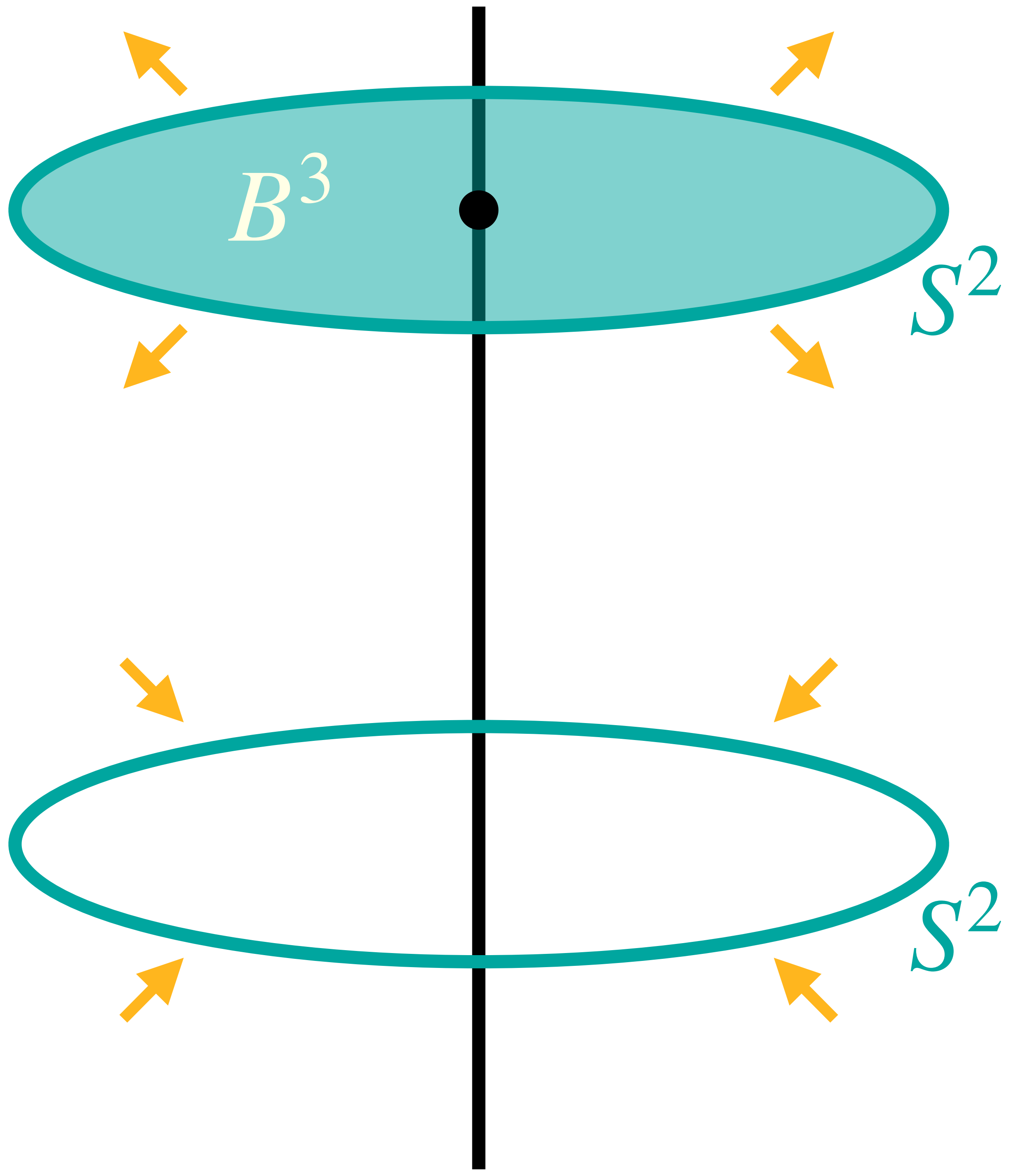

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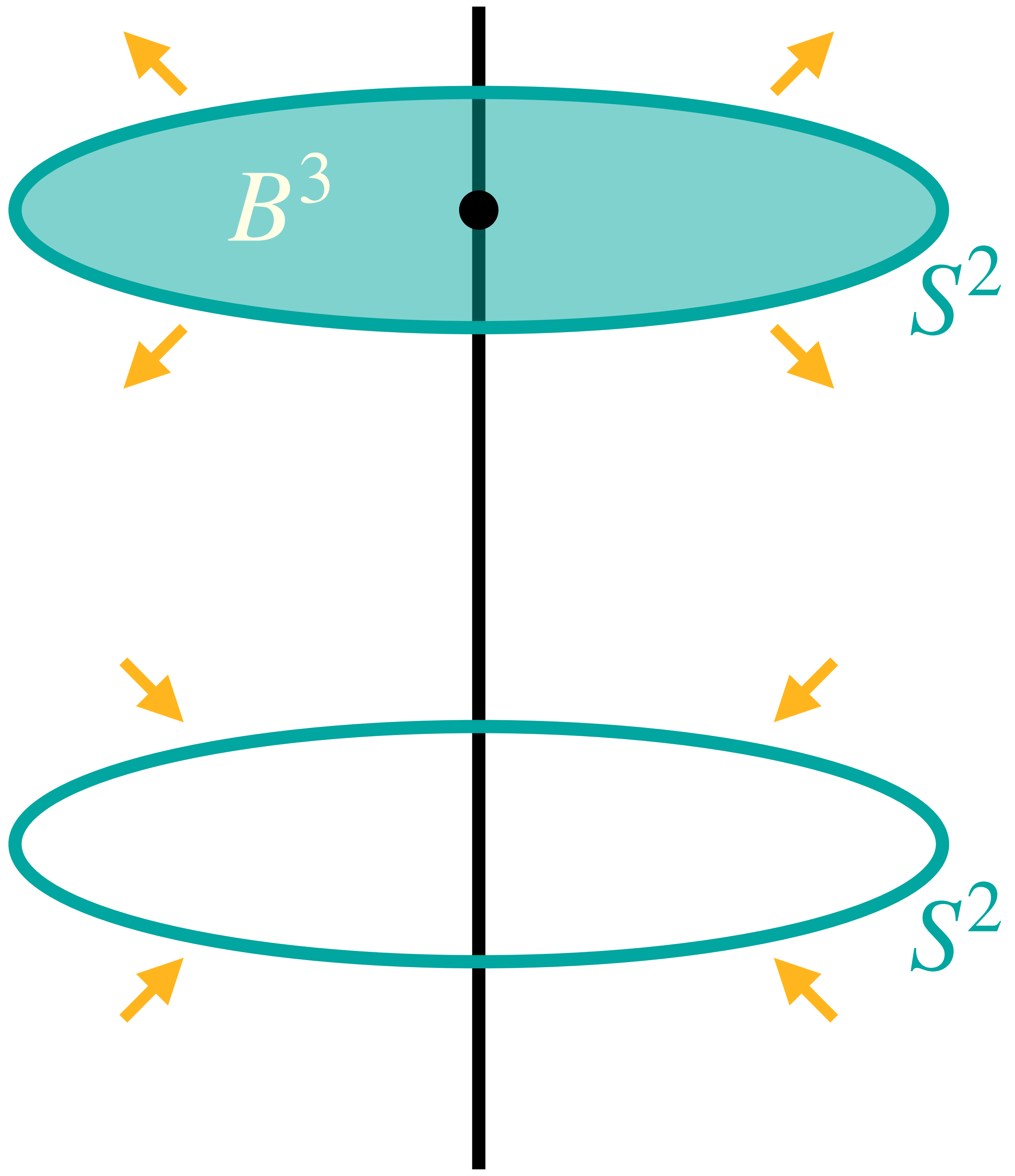

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- Only higher orders in e^2 can single out the correct one

Comments and further directions

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- **Punchline:** If we have **magnetic monopoles**, then we have **exotic matter with fractional charges**

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- Examples where the only viable 4d surface is **non-invertible**
- E.g. N fermions in QED scattering off a charge $n > 2$ monopole preserving an $SU(N)$

  $\longrightarrow$ Twist lives in a chiral $U(1)$ with a non-trivial **ABJ anomaly**!
- Expected for general scattering in Standard Model

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Surprise!

- Sometimes even this **fails**: no (invertible) 2d line does the trick
- Generically arises whenever left- and right-movers carry **different N-ality** under some non-Abelian symmetry we want to preserve
- E.g. the charge 3 monopole of the $SU(5)$ GUT

  $\longrightarrow$ Find an ingoer of spin $1/2$. But all outgoers have integer spin!

Some more further directions

- Effect of **gauge field fluctuations**
  - Generically generates a centrifugal barrier for some naively barrier-less modes
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- Remaining **scattering puzzles**. Two scenarios:
  - **Boring:** Symmetric boundary conditions exist, and just require **fancier 2d twist operators**
  - **Exciting:** UV theory somehow breaks symmetry at some scale

    $\longrightarrow$ **Non-simple** boundary condition needed, corresponding to **additional UV fermions**!

Thanks!

Backup slides

New routes to tricky 2d boundary conditions

Symmetric mass generation

- Suppose we have come UV theory with **tuneable parameter**
  - For one value, flow in the IR to a **trivially gapped phase**
  - For another value, flow to N **free Dirac fermions**
- If we can do this while preserving some (necessarily non-anomalous!) symmetry G, this is **symmetric mass generation**
- Simply tune parameter to one value in $x < 0$ and the other in $x > 0$

  $\longrightarrow$ **Dynamically generates G-preserving boundary conditions!**
- Example: Single Dirac fermion. Can preserve $U(1)_V$ or $U(1)_A$ with simple mass terms
- **Know:** How to do SMG for generic symmetries of Dirac fermions
- **Don't know:** How to "read off" boundary conditions (i.e. topological line attachments)

New routes to tricky 2d boundary conditions

Theory self-maps

- Start with N **left-moving Weyl fermions**, and gauge some (necessarily non-anomalous, and thus discrete!) symmetry
  - Chosen correctly $\longrightarrow$ Theory just mapped **back to itself**
- Do this gauging **just in** $x < 0$

  $\longrightarrow$ Generate a **non-invertible topological interface** at $x = 0$ (Tambara-Yamagami)
- Local operators pick up a **topological line attachment** as they pass through
- Now **fold** to get a theory of N Dirac fermions
  - **Interface becomes interesting boundary condition**. Can e.g. get 3450 boundary conditions precisely this way!
- **Know:** Exactly which discrete gauging gives G-preserving b.c.s for Abelian G
- **Don't know:** What to do for non-Abelian G!

Unstable monopoles

- How do fields feel about the monopole?
  - **Scalars** (spin 0): **Not interested!** Get bounced back by centrifugal barrier
  - **Fermions** (spin $\frac{1}{2}$): **Friendly.** Lowest-lying mode feels no potential
  - **Gauge fields** (spin 1): **They love it!** For $n \geq 2$, lowest-lying mode experiences **attractive potential** $\longrightarrow$ signals **instability**

- Only have to worry in non-Abelian theory: have charged off-diagonal W-bosons
- **Physics:** monopole surrounded by W-boson condensate, magnetic charge screened
- Only protected quantity: **magnetic 1-form symmetry**. In Standard Model, precisely **1 stable monopole** for each 1-form charge $p = 3N = 2m + 6g$!
  - Reasonable assumption: all unstable monopoles in this class flow to the stable one in the IR

More exotic 4d surfaces

- Got **lucky** in the Standard Model: there was a 4d surface (in fact two!) that worked
- Consider N_f Dirac fermions of unit charge, and 't Hooft line of magnetic charge n
- Mild additional assumptions $\longrightarrow$ **Natural candidate for 2d twist**
- **Problem:** Twist acts in 4d as

$$\Psi_i \longrightarrow e^{\frac{2\pi i}{nN_f}\gamma^5}\Psi_i$$

- Has ABJ anomaly! But fear not: it's a discrete rotation in $U(1)_A$

$\longrightarrow$ Required open topological surface generates a **non-invertible symmetry**!

Thanks again!