

Based on:

- J. Berman, HE, A. Herderschee 2310.10729
- J. Berman, HE: 2406.03543
- J. Berman, HE, N. Geiser, L. Lin, 2412.13368
- J. Berman, HE, C. Figueiredo, to appear

Effective Field Theory Bootstrap

Henriette Elvang University of Michigan

Leinweber Center for Theoretical Physics

Funding: Department of Energy Cottrell Star Award, Research Corporation for Science Advancement

CERN **Theory Colloquium** May 21, 2025

Conformal Bootstrap S-matrix Bootstrap Amplitude bootstrap String bootstrap Cluster-Algebra-Symbol bootstrap Soft bootstrap Wour own favoritel bootstrap Primal Bootstrap Lightcone bootstrap Dual Bootstrap Landau bootstrap Form factor bootstrap EFT bootstrap Hikingbootstrap

<mark>📈</mark> UNIVERSITY OF MICHIGAN

S-matrix Bootstrap

Theory Observables

1) "Write down" an amplitude based on symmetries, analyticity, unitarity.... Chew, Mandelstam,... 1960s

given a theory, "bootstrap" the observable.

Multiple modern incarnations, e.g. higher-loop amplitudes from mathematical properties of special functions etc.



S-matrix Bootstrap



2) Map out theory space based on symmetries, analyticity, unitarity,... of amplitudes

Use observables to "bootstrap" the space of theories.

Multiple modern incarnations, e.g. weak-coupling / non-perturbative / primal /dual... applied to pions, super Yang-Mills, gravity, scalars, ...

This talk

Bootstrap in the context of *effective field theories*

Weak-coupling

Bounds on the effective couplings (Wilson coefficients) in Effective Field Theory

Constraints on spectrum

Talk has two parts

Part 1: general ideas in bootstrap, some examples, phrased generally, but think of pions

Part 2: constraints from higher-point amplitudes on the bootstrap of 4-point amplitudes

<mark>🙀</mark> UNIVERSITY OF MICHIGAN

????? ????? ????? Scale of new physics Λ Top quark What we know Higgs and W, Z what we don't know proton, neutron pions muon electron neutrinos photon, gluons

Energy²





Key Example

Massless Adjoint scalar ϕ^a

Large rank N Weak coupling Mass gap, analyticity, unitarity





*) N_f vs. N_c : for pions, need large N_c limit. Have pion triplet for N_f = 2, octet of pions, kaons, eta for N_f = 3, etc. Keep N_f general.

Amplitudes

Color-ordered scattering amplitudes



Mandelstam variables $s = (p_1 + p_2)^2$ $t = (p_1 + p_3)^2$ $u = (p_1 + p_4)^2$ s + t + u = 0

All incoming

$$A_4(\phi^a \phi^b \to \phi^c \phi^d) = A[1234] \operatorname{Tr}(T^a T^b T^c T^d) + \operatorname{perms}(234)$$

are written in terms of the Mandelstams $A_4[1234] = A(s, u)$

Cyclicity $A_4[1234] = A_4[2341]$ gives "crossing relation"

$$A(s,u) = A(u,s)$$





EFT expansion (ignoring for

(ignoring for now massless poles)

The effective couplings $a_{k,q}$ are in one-to-one correspondence with the Wilson coefficients

$$\begin{split} A(s,u) &= a_{0,0} + a_{1,0}(s+u) + a_{2,0}(s^2+u^2) + a_{2,1}su + a_{3,0}(s^3+u^3) + \dots \\ & \\ \mathrm{Tr}(\phi^4) & \mathrm{Tr}[(\partial\phi)^2\phi^2] \\ & \\ \mathrm{Tr}[(\partial\phi)^2\phi^2] \\ \mathrm{Tr}[(\partial_\mu\partial_\nu\phi)(\partial^\mu\partial^\nu\phi)\phi^2] \\ & \\ \mathrm{Tr}[(\partial_\mu\partial_\nu\phi)(\partial^\mu\phi)(\partial^\nu\phi)\phi] \\ \end{split}$$

Bootstrap => bounds on the 4-point Wilson coefficients $a_{k,q}$



 $\frac{a_{2,0}}{a_{1,0}} = \frac{1}{M^2}$

$$A(s,u) = \frac{2\lambda^2}{M^2} + \frac{\lambda^2}{M^4}(s+u) + \frac{\lambda^2}{M^6}(s^2+u^2) + \frac{\lambda^2}{M^8}(s^3+u^3) + \dots \qquad \frac{a_{3,0}}{a_{1,0}} = \frac{1}{M^4}$$



 $\frac{a_{2,0}}{a_{1,0}} = \frac{1}{M^2}$

$$A(s,u) = \frac{2\lambda^2}{M^2} + \frac{\lambda^2}{M^4}(s+u) + \frac{\lambda^2}{M^6}(s^2+u^2) + \frac{\lambda^2}{M^8}(s^3+u^3) + \dots \qquad \frac{a_{3,0}}{a_{1,0}} = \frac{1}{M^4}$$



 $\frac{a_{2,0}}{a_{1,0}} = \frac{1}{M^2}$

$$A(s,u) = \frac{2\lambda^2}{M^2} + \frac{\lambda^2}{M^4}(s+u) + \frac{\lambda^2}{M^6}(s^2+u^2) + \frac{\lambda^2}{M^8}(s^3+u^3) + \dots \qquad \frac{a_{3,0}}{a_{1,0}} = \frac{1}{M^4}$$



 $\frac{a_{2,0}}{a_{1,0}} = \frac{1}{M^2}$

$$A(s,u) = \frac{2\lambda^2}{M^2} + \frac{\lambda^2}{M^4}(s+u) + \frac{\lambda^2}{M^6}(s^2+u^2) + \frac{\lambda^2}{M^8}(s^3+u^3) + \dots \qquad \frac{a_{3,0}}{a_{1,0}} = \frac{1}{M^4}$$



I VI UNIVERSITY OF MICHIGAN

 $A(s,u) = \frac{2\lambda^2}{M^2} + \frac{\lambda^2}{M^4}(s+u) + \frac{\lambda^2}{M^6}(s^2+u^2) + \frac{\lambda^2}{M^8}(s^3+u^3) + \dots$



$$A(s,u) = \frac{2\lambda^2}{M^2} + \frac{\lambda^2}{M^4}(s+u) + \frac{\lambda^2}{M^6}(s^2+u^2) + \frac{\lambda^2}{M^8}(s^3+u^3) + \dots$$



$$A(s,u) = a_{0,0} + a_{1,0}(s+u) + a_{2,0}(s^2 + u^2) + a_{2,1}su + a_{3,0}(s^3 + u^3) + \dots$$

Claim: this is the only allowed region of any UV completion with

- Mass gap M_{gap}=1
- Positive spectral density (unitarity)
- "UV boundedness" $\frac{A(s,u)}{s} \to 0 \text{ for } |s| \to \infty$

for *u* fixed



Dispersion relation for $a_{k,q}$ with $k-q \ge 1$



$$\frac{A(s,u)}{s} \to 0 \text{ for } |s| \to \infty$$



Basic consequences

1) $a_{k,q} \ge 0$ $k-q \ge 1$

$$a_{k,q} = \sum_{\ell=0}^{\infty} \int_{1}^{\infty} dy f_{\ell}(y) y^{-k} v_{\ell,q}$$

2)
$$a_{1,0} \ge a_{2,0} \ge a_{3,0} \ge \dots$$

3) Hankel matrix constraint
$$det \begin{pmatrix} a_{1,0} & a_{2,0} \\ a_{2,0} & a_{3,0} \end{pmatrix} \ge 0$$

It follows that
$$\left(\frac{a_{2,0}}{a_{1,0}}\right)^2 \le \frac{a_{3,0}}{a_{1,0}} \le \frac{a_{2,0}}{a_{1,0}}$$

3) 2)

$$A(s,u) = a_{0,0} + a_{1,0}(s+u) + a_{2,0}(s^2 + u^2) + a_{2,1}su + a_{3,0}(s^3 + u^3) + \dots$$



I VI UNIVERSITY OF MICHIGAN

$$A(s,u) = a_{0,0} + a_{1,0}(s+u) + a_{2,0}(s^2 + u^2) + a_{2,1}su + a_{3,0}(s^3 + u^3) + \dots$$



$$A(s,u) = a_{0,0} + a_{1,0}(s+u) + a_{2,0}(s^2 + u^2) + a_{2,1}su + a_{3,0}(s^3 + u^3) + \dots$$



Large |s| behavior at fixed u

$$A(s,u) \sim O(s^J)$$

Assuming $\frac{A(s,u)}{s} \to 0$ for $|s| \to \infty$ Having only spin 0 is OK Having only spin 1 is borderline Only spin J > 1 is not allowed



But... what about pion scattering (at large N)?



The rho resonance is a spin 1 , next is f_2 with spin 2,....

The ``good'' UV behavior is achieved by having in infinite sum over spin J states such that all the $A(s, u) \sim O(s^J)$ re-sum.

Need an infinite tower of states





Leading Regge trajectory









Lowest massive state has to be a scalar or spin 1

So, the (large N) meson spectrum could not have started with spin higher than 1!



$$|\bar{g}_0|^2 = \frac{1}{\zeta_2}$$
$$\ell_1 = 0$$

⁰ Analytic spectrum constraints Berman (2024)

1)

$$\int J = 0 \quad \text{for} \quad A(s,u) \to 0 \quad \text{for} \quad |s| \to \infty$$

$$J = 0 \quad \text{or } 1 \quad \text{for} \quad \frac{A(s,u)}{s} \to 0 \quad \text{for} \quad |s| \to \infty$$

2) Consecutive spin bound: consecutive spin states can grow by at most 1 in spin at each mass level, i.e. going up in the mass spectrum, there has to be a spin 1 state before a spin 2 state, a spin 2 before a spin 3, etc.

Hence



$$|\bar{g}_0|^2 = \frac{1}{\zeta_2}$$
$$\ell_1 = 0$$

⁰ Analytic spectrum constraints Berman (2024)

1)

$$\int J = 0 \quad \text{for} \quad A(s, u) \to 0 \quad \text{for} \quad |s| \to \infty$$

$$J = 0 \quad \text{or } 1 \quad \text{for} \quad \frac{A(s, u)}{s} \to 0 \quad \text{for} \quad |s| \to \infty$$

- 2) Consecutive spin bound: consecutive spin states can grow by at most 1 in spin at each mass level, i.e. going up in the mass spectrum, there has to be a spin 1 state before a spin 2 state, a spin 2 before a spin 3, etc.
- **3)** Mass bounds: given the masses, M_J and M_{J+1} , of a spin J and J+1 states, the spin J+2 state has to appear before the upper mass bound

$$M_{J+2} \le \frac{M_{J+1}^2}{M_J}$$

Apply to pions (massless pions, large N limit)

3) Mass bounds: given the masses, M_J and M_{J+1} , of a spin J and J+1 states, the spin J+2 state has to appear before the upper mass bound

$$M_{J+2} \le rac{M_{J+1}^2}{M_J}$$
 Berman (2024)

State	J	Maximum Mass (MeV)	Measured Mass (MeV) $[33]$
$\rho(770)$	1	-	775.3 ± 0.2
$f_2(1270)$	2	-	1275 ± 1
$ \rho_3(1690) $	3	$m_{f_2}^2/m_ ho = 2098$	1689 ± 2
$f_4(2050)$	4	$m_{ ho_3}^2/m_{f_2} = 2237$	2018 ± 11
$ \rho_5(2350) $	5	$m_{f_4}^2/m_{ ho_3} = 2411$	2330 ± 35
$f_6(2510)$	6	$m_{ ho_5}^2/m_{f_4} = 2690$	2470 ± 50
$ ho_7$	7	$m_{f_6}^2/m_{ ho_5} = 2618$??

"Predicts"

 $2470 \,\mathrm{MeV} \lesssim M_{
ho_7} \lesssim 2618 \,\mathrm{MeV}$



So far

Dispersion relations

1) Bound the Wilson coefficients $a_{k,q} \Rightarrow CONVEX$ allowed regions 2) Derive spectrum constraints

Talk has two parts

Part 1: general ideas in bootstrap, some examples, phrased generally, but think of pions-

Part 2: constraints from higher-point amplitudes on the bootstrap of 4-point amplitudes

Based on upcoming work with Carolina Figueiredo and Justin Berman



Setting the stage: Rule out. But not rule in?



In this case, we could span the allowed region by the basic scalar exchange amplitudes.

That is not always the case.



Rule out. But not ruled in?

Infinite Spin Tower

$$A^{\rm IST} = \frac{1}{(s - M^2)(u - M^2)}$$



Rule out. But not ruled in?

 $a_{2,1}$

 $a_{1,0}$

In general, we don't know if

- a set of Wilson coeffs that are "not ruled out" can be "ruled in"
- --- or ---
- If any "allowed" Wilson coeffs from the 4-point actually arise in a sensible theory that also has 5- and higher-point amplitudes who correctly factorize into lower-point amplitudes, including the 4-point amplitude.

ssIST 3.0 **Ruled Out** 2.5 Not Ruled Out 2.0 J=1 1.5 IST 1.0 0.5 J=0 0.2 0.4 0.6 0.8 1.0 $a_{2,0}$ $a_{1.0}$

Infinite Spin Tower



Higher-point amplitudes

$$\operatorname{Res}_{s_{ij..}=0} A_n = A_{n_1} \times A_{n_2}$$

This makes it straightforward to reconstruct higher-point scalar theory EFT amplitudes from the bottom-up, e.g.



However, a bootstrap directly on higher-point amplitudes is elucive



Higher-point amplitudes

But!!

If we want the *n*-point amplitudes to have specific properties

e.g. supersymmetry, soft theorems, KK & BCJ relations, A. Chen, HE, A. Herderschee (2022)

then imposing the symmetries at higher-point may generate *nonlinear relationships* among the 4-point Wilson coefficients.

Then simply adding amplitudes may no longer be possible -- and hence the allowed bootstrap region need **not be convex**.

Hidden Zeros

Arkani-Hamed, Cao, Dong, Figueiredo (2023) Arkani-Hamed, Figueiredo (2024)

The scattering amplitudes of $Tr(\phi^3)$ have a peculiar property:

they have zeros in certain kinematic configurations

$$A_4 = -\frac{1}{s} - \frac{1}{u} = \frac{t}{su} = 0 \quad \text{for} \quad t = 0$$
$$A_5 = \frac{1}{s_{12}s_{45}} + \text{cyclic} = 0 \quad \text{for} \quad s_{13} = s_{14} = 0 \quad \text{and cyclic}$$

and similarly for higher points.

Hidden Zeros

Moreover, starting at 5-pt, the tree amplitudes have non-trivial ``splitting'' properties

$$\mathcal{A}_5\Big|_{s_{13}=0} = \mathcal{A}_4(s_{12}, s_{15}) \times \mathcal{A}_4(s_{23}, s_{34})$$
$$\mathcal{A}_5\Big|_{s_{14}=0} = \mathcal{A}_4(s_{12}, s_{15}) \times \mathcal{A}_4(s_{45}, s_{34})$$

and similarly for higher points

Splits => Hidden Zeros



$$s_{ijk} = (p_i + p_j + p_k)^2$$
$$s_{ij} = (p_i + p_j)^2$$

I VI UNIVERSITY OF MICHIGAN



$$s_{ijk} = (p_i + p_j + p_k)^2$$
$$s_{ij} = (p_i + p_j)^2$$



EFTs with Hidden Zero Splitting

We are going to bootstrap unitary EFTs with Hidden Zero Splits

This is a *proto-type example* to demonstrate how information from higher-point amplitudes can further carve out the allowed space of Wilson coefficients in the 4-point amplitude



4-point

$$| \text{Impose } A_4 |_{t=0} = 0 \implies \left(A_4(s, u) = -t f(s, u) \quad \text{w/} \quad f(s, u) = f(u, s) \right)$$

Most general EFT ansatz
$$f(s, u) = -\frac{g^2}{su} + \sum_{k=0}^{\infty} \sum_{0 \le q \le k} a_{k,q} s^{k-q} u^q$$

Assume
$$\frac{A_4(s,u)}{s} \to 0$$
 as $|s| \to \infty$

Zero-subtracted dispersion relations for **all** $a_{k,q}$







Imposing hidden zeros (A₅ vanishes for $s_{13}=s_{14}=0$) does not constrain 4-point Wilson coefficients $a_{k,q}$, but...

Impose the splits
$$A_5 \Big|_{s_{13}=0} = A_4(s_{12}, s_{15}) \times A_4(s_{23}, s_{34})$$

 $A_5 \Big|_{s_{14}=0} = A_4(s_{12}, s_{15}) \times A_4(s_{45}, s_{34})$

order-by-order in the Mandelstam expansion

5-point

This fixes all the 5-point contact terms completely up to $O(s^4)$ and in addition we find nonlinear constraints among the 4-point Wilson coefficients

$$a_{2,1} = \frac{3}{2} a_{2,0} - \frac{1}{2g^2} a_{0,0}^2$$

$$a_{3,1} = 2a_{3,0} - \frac{1}{g^2}a_{1,0}a_{0,0}$$

etc. In fact, only the $a_{k,0}$ remain free.

Berman, HE, Figueiredo (to appear)



5-point

This fixes all the 5-point contact terms completely up to $O(s^4)$ and in addition we find nonlinear constraints among the 4-point Wilson coefficients

etc. In fact, only the $a_{k,0}$ remain free.

To implement this into the bootstrap, solve for g^2 to get

$$0 = \frac{3a_{2,0} - 2a_{2,1}}{a_{0,0}}a_{1,0} - 2a_{3,0} + a_{3,1}$$

Berman, HE, Figueiredo (to appear)



Allowed region in
$$\left(\frac{a_{1,0}}{a_{0,0}}, \frac{3a_{2,0}-2a_{2,1}}{a_{0,0}}\right)$$
-plane, now with nonlinear constraints
 $A_4(s, u) = -t f(s, u)$



Berman, HE, Figueiredo (to appear)





$$0 \qquad M_{\rm gap}^2 \quad \mu_c M_{\rm gap}^2$$

$$|\bar{g}_0|^2 = \frac{1}{\zeta_2}$$
$$\ell_1 = 0$$

0 Speactr.um

Recall from earlier in the talk

$$\int_{0}^{f_1} J = 0 \quad \text{for} \quad A(s, u) \to 0 \text{ for } |s| \to \infty$$

So, for the bootstrap of f(s,u), the lowest mass state can only be a scalar

Let's assume then that we have a scalar at the mass gap and then a cutoff up to the next possible massive state.

Inspired by the string, take the cutoff to be 2.

Berman, HE, Figueiredo (to appear)



 $\ell_1 = 0$

 $M_{\rm gap}^2 = \mu_c M_{\rm gap}^2$



$$\mu_c = 2 \qquad D = 10$$









 $\ell_1 = 0$

 $M_{\rm gap}^2 = \mu_c M_{\rm gap}^2$





Berman, HE, Figueiredo (to appear)

 $\ell_1 = 0$

 $M_{\rm gap}^2 = \mu_c M_{\rm gap}^2$





Berman, HE, Figueiredo (to appear)

Upshot

The shrinking island indicates that

the string amplitude is the unique unitary UV completion

of the class of EFTs with hidden zero splits (without an infinite spin tower at the mass gap)

Strong outcome of basic higher-point input

Is something similar perhaps true if instead of hidden zeros we assume supersymmetry?

Under investigation (HE, Herderschee, Morales)



Summary

Two parts

Part 1: general ideas in bootstrap, some examples, phrased generally, but think of pions

Part 2: constraints from higher-point amplitudes on the bootstrap of 4-point amplitudes



Collaborators



Aidan Herderschee (Michigan PhD 2023, now postdoc at the IAS)



Justin Berman (4th year)



Alan Shih-Kuan Chen (PhD 2025)

Thank you





Nick Geiser (Michigan postdoc)





Aditi Chandra (1st year)

Roger Morales



(NBI PhD student & future Michigan postdoc)

Extras













