Université Mohammed V
Faculté des Sciences
Rabat

ROYAUME DU MAROC
UNIVERSITE MOHAMMED V-AGDAL
FACULTE DES SCIENCES

LPHE-MS

Weak/strong gauge duality in M-Theory on K3xK3

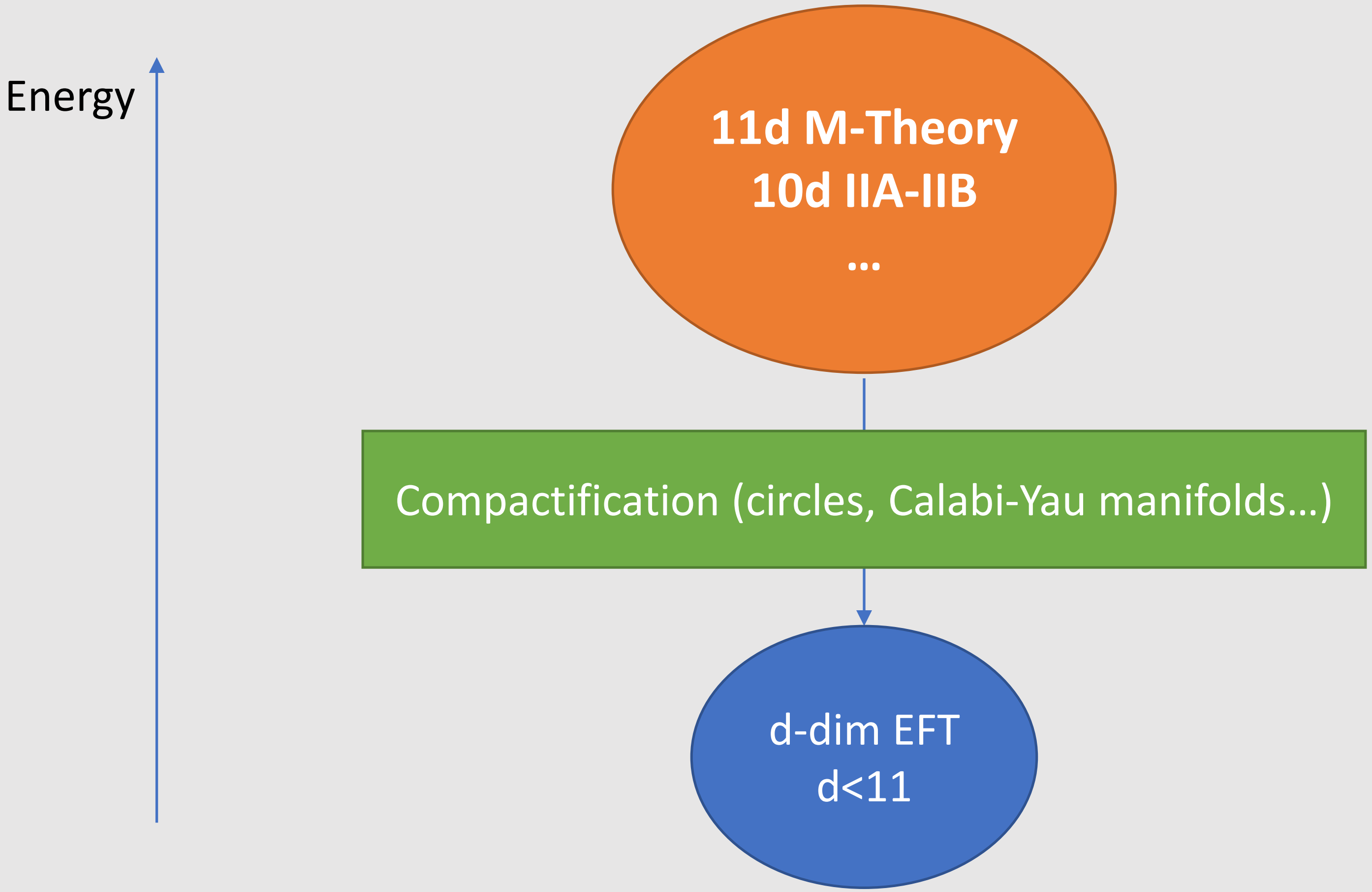
M. Charkaoui, R. Sammani, E. H. Saidi, R. Ahl Laamara

Plan

- Introduction
  - The Swampland Program
- The Weak Gravity Conjecture (WGC).
- M-theory on Calabi-Yau fourfold
  - 3D Effective field theory
  - Fibration structure
- Weak/strong gauge duality
- Conclusion and future directions

Introduction

The Swampland Program

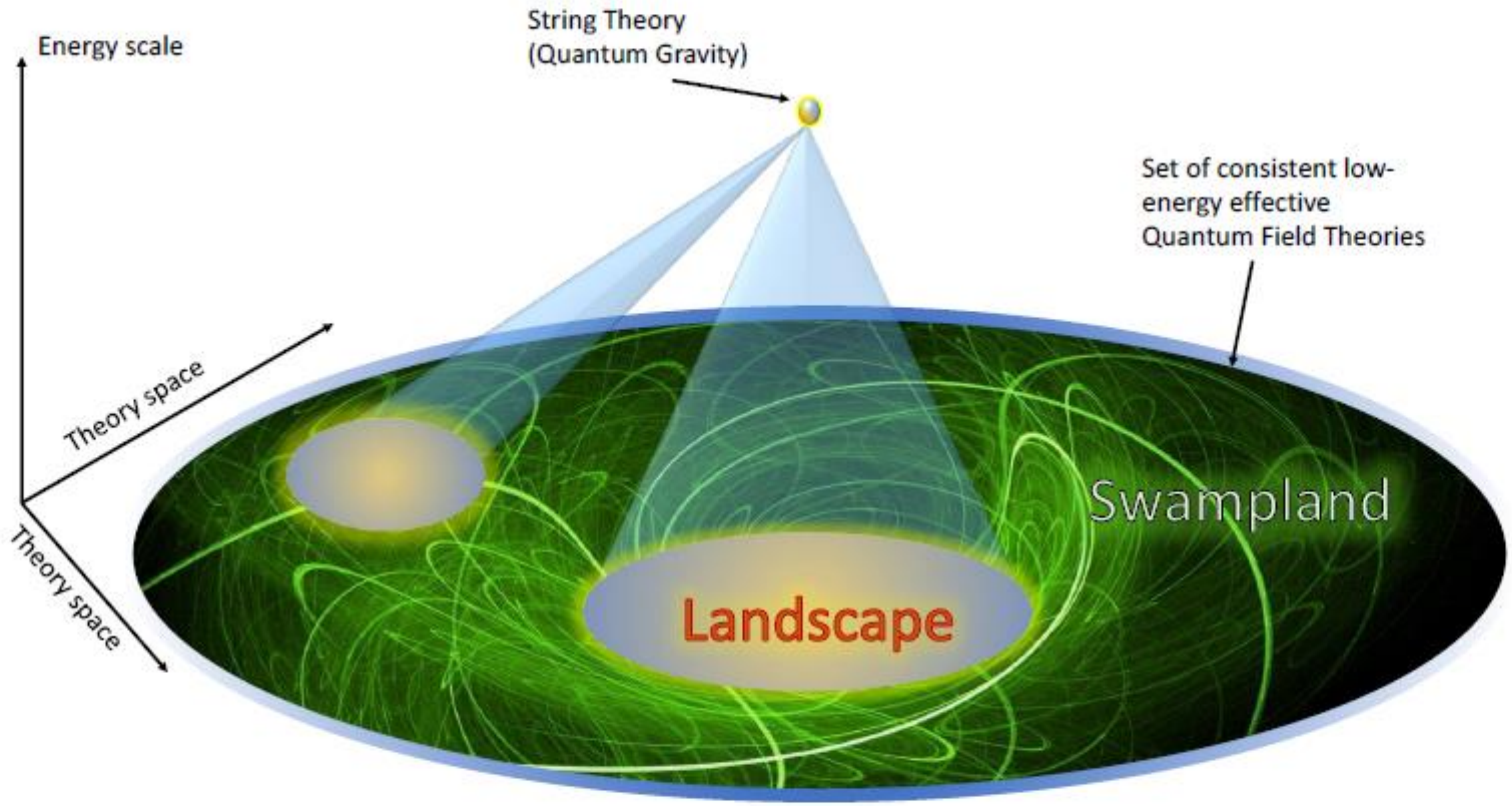


- Using different geometries for compactification we get a large spectrum of low energy EFT

Question:

If we start from a low energy EFT can we always get to a theory in the UV compatible with quantum gravity?

Introduction



This is done via conjectures, not yet all proven, but tested in many setups.

- Weak Gravity Conjecture
- Distance Conjecture
- No global symmetries Conjecture…

The asymptotic WGC relates many of the conjectures.

The Weak Gravity Conjecture

The Weak Gravity Conjecture

Consider a theory coupled to gravity, with a U(1) gauge symmetry with gauge coupling g

$$S = \int d^d X \sqrt{-G}\,[(M_d)^{d-2}\frac{R_d}{2} - \frac{1}{4g^2}F^2 + \cdots]$$

- Electric WGC: there exists ***a particle*** in the theory with mass m and charge q such that:

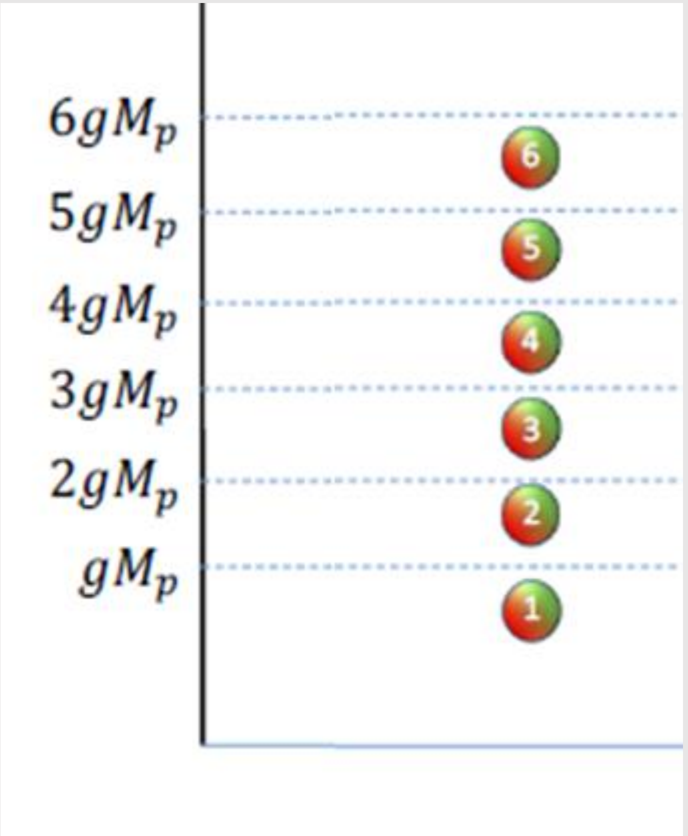
$$m \leq \sqrt{\frac{d-2}{d-3}}\, gq(M_d)^{\frac{d-2}{2}}; \text{ d>3, q=1,2...}$$

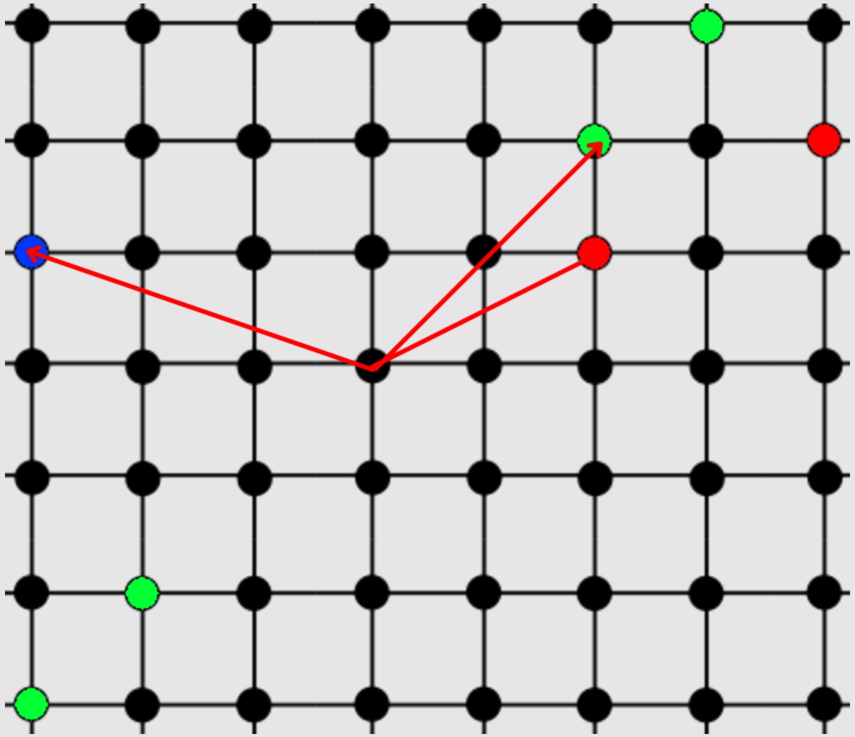
- Magnetic WGC: the cutoff scale Λ of the effective field theory is bounded from above by the gauge coupling

$$\Lambda_{WGC} \leq g(M_d)^{\frac{d-2}{2}}; \mathrm{d} > 3$$

Refinements of the WGC

- The tower WGC states that there should exist an infinite tower of superextremal states:





M-theory on Calabi-Yau fourfolds

Our main interest is the gauge theory sector, the bosonic part of the 11d M-theory effective action:

$$S_{11d} = 2\pi M_{11d}^{9} \int_{\mathcal{M}_{11}} (R \star I - \frac{1}{2} dC_3 \wedge \star dC_3) + \cdots$$

BY compactifying the gauge potential C_3 on a 3-fold we can write it as:

$$C_3 = (2\pi)^{-1} M_{11d}^{-1} A^{\alpha} \wedge J_{\alpha}$$

$\{J_\alpha\}$ $\alpha = 1, \ldots, h^{1,1}(X_3)$ is a basis of $H^{1,1}(X_3, \mathbb{Z})$.

The gauge potentials A^{α} defines a basis $\{U(1)^{\alpha}\}$ of abelian gauge groups.

The duality between the 2-forms space and the 2-cycles space implies:

$$J_\alpha C^\beta = \delta_\alpha^\beta$$

Thus we can define the linear combination:

$$U(1)_C = c_\alpha U(1)^\alpha \quad , \qquad \mathrm{C} = c_\alpha C^\alpha \in H_2(X_3)$$

The fluctuations of the Kahler form:

$$J = \lambda v^0 J_0 + \lambda^\alpha v^i J_i$$

With λ a positive parameter controlling the volume of the cycles

The action in 3d is given by:

$$S_{3d} = \frac{M_{Pl}}{2}\int_{\mathcal{M}_{3d}}(R \star I - g_{AB} d\Phi^A \wedge \star\, d\Phi^B) - \frac{1}{2g_3^2}\int_{\mathcal{M}_{3d}} f_{\alpha\beta} F^\alpha \wedge \star\, F^\beta$$

With the volume of the manifold given by:

$$\mathcal{V}_{X_4} = \frac{1}{4!}\int_{X_4} J^4$$

For gravity to remain dynamical we need $\mathcal{V}_{X_4} < \infty$.

Fibration structure

This implies that the manifold must be fibered as:

$$X_4 = F_1 \times B_3$$

Or

$$X_4 = F_2 \times B_2$$

Or

$$X_4 = F_3 \times B_1$$

With the fiber shrinking while the base expanding to keep the voume finite, more precisely the fibers are given by:

Fibration structure

- Limits of type T²: $J_0^4 = 0, J_0^3 \neq 0$

$$\pi: X_4 \to B_3$$

$$\mathcal{V}_{T^2} \sim \frac{1}{\lambda^3}, \qquad \mathcal{V}_{B_3} \sim \lambda^3, \qquad \lambda \to \infty$$

- Limits of type S (K3/T^4): $J_0^4 = 0, J_0^3 = 0, J_0^2 \neq 0$

$$\rho: X_4 \to B_2$$

$$\mathcal{V}_{K3/T^4} \sim \frac{1}{\lambda^2}, \qquad \mathcal{V}_{B_2} \sim \lambda^2, \qquad \lambda \to \infty$$

- Limits of type V: $J_0^4 = 0, J_0^3 = 0, J_0^2 = 0$

$$\sigma: X_4 \to B_1$$

$$\mathcal{V}_{CY3} \sim \frac{1}{\lambda}, \qquad \mathcal{V}_{B_1} \sim \lambda, \qquad \lambda \to \infty$$

Fibration structure

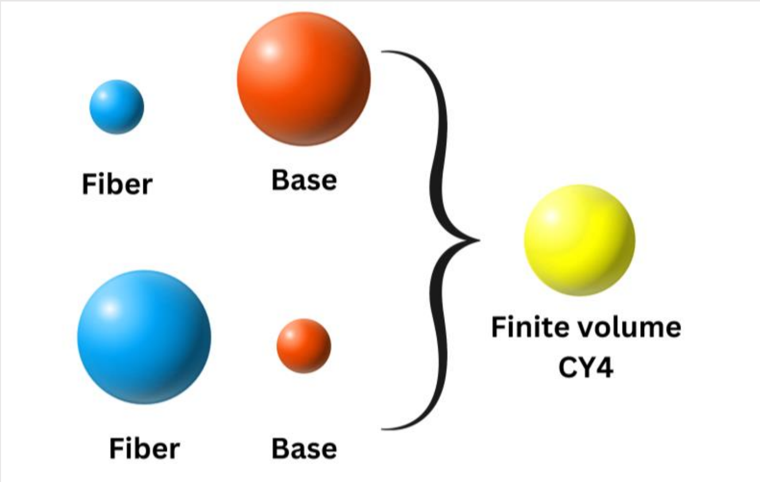
We focus on the case of type S:

$$X_4 = S_{\parallel} \times S_{\perp}$$

More precisely we take S=K3.

We find a duality exchanging the fiber and the base, given by the mapping:

$$\lambda \rightarrow 1/\lambda$$



Weak/Strong gauge duality

Weak/Strong gauge duality

Given a curve $\mathrm{C} = c_\alpha C^\alpha \in H_2(X_3)$:

The gauge coupling in that direction arising from reducing the three-form C_3 is given by:

$$g^2{}_{YM,C} = g_3^2 (c_\alpha f^{\alpha\beta} c_\beta)$$

If we take curves in the shrinking fiber $K3_{||}$ the shrinking of the curves $\mathrm{C}_{||}$ gives a weak coupling limit:

$$g^2{}_{YM,C_{||}} \sim \frac{1}{\lambda^2}$$

Similarly, we get the strong coupling limit from the expanding curves in the surface $K3_\perp$:

$$g^2{}_{YM,C_\perp} \sim \lambda^2$$

Weak/Strong gauge duality

The duality exchanging the fiber and the base , links also weak and strong gauge couplings:

λ	↔	1/λ
$K3_{\|\|}$	↔	$K3_{\perp}$
g_{weak}	↔	g_{strong}

We then show the existence of towers of states satisfying the WGC both in weak and strong couplings which is in line with the minimal WGC.

These towers of states are given by the BPS states: M2/C_2, or non-BPS excitations of M5/K3.

Conclusion

- We generalized mathematical results on the fibration of Calabi-Yau manifolds.

- We found a duality relating gauge coupling regimes, as well as the fiber and the base of the manifold

- We generalized the scope of validity of the Asymptotic WGC to probe weak and strong gauge coupling limits, this is in line with a new refined version of the WGC labeled the minimal WGC and tested it for the case of S=K3

Future directions

- Can we extend the Weak/strong gauge duality to an S-duality, or possibly S-Selfduality.
- Test the conjecture for type T^2 and type V, and find a duality relating them as well.

Thank you for your attention