Modified Alfvén wave of multi-ion species in the upper ionosphere of Mars

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ASP2024 Alumni Online Presentation

Outlines

- Plasma State
- Plasma Waves
- Magnetosonic Waves
- Alfven Waves
- Theoretical Model
- Application to Observed Structure
- Results
- Numerical Analysis
- Conclusions
- Future work
- References

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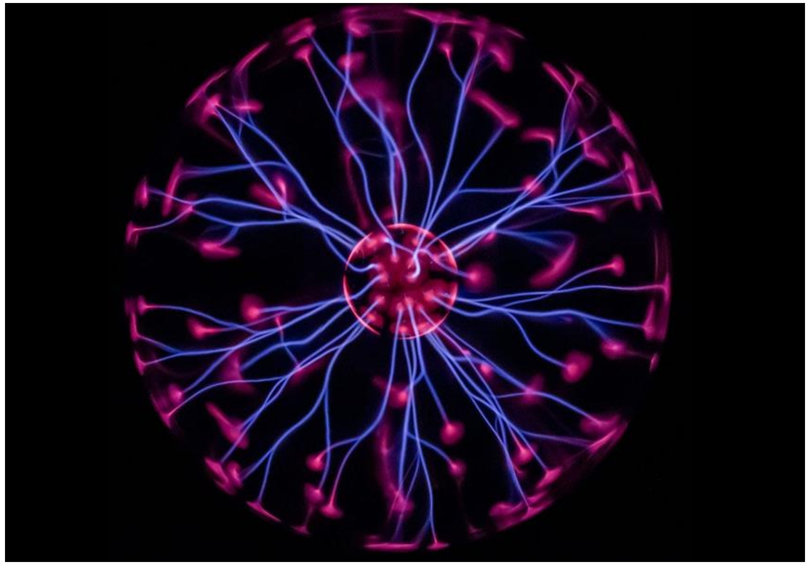
Plasma State

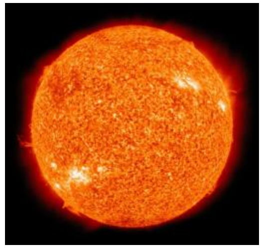
- plasma is a quasineutral gas of charged and neutral particles which exhibits collective behavior.



- Conditions: 1. $\lambda_D << L$.

  2. $N_D \ggg 1$.

  3. $\omega\tau > 1$.



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Plasma Waves

- An interconnected set of particles and fields which propagate in a periodically repeating manner.
- These particle are (e^-)and (i^+) in the simplest case.

- May contain multiple positive/negative ions and neutral particles.
- The electromagnetic fields in a plasma are assumed to have two parts, one static/equilibrium part and one oscillating/perturbation part.

Plasma Waves

- Continuity Equation: $\frac{\partial n}{\partial t} + \nabla \cdot (nv) = 0$ Eq(1)
- Momentum Equation:

$$mn\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v}\right) = -\nabla P + qn(\mathbf{E} + \boldsymbol{v} \times \boldsymbol{B}) \quad \text{Eq(2)}$$

Types of Plasma Waves:

- ***Electrostatic*** Waves: Governed by *electric fields*; particles oscillate along field lines.
- ***Electromagnetic*** Waves: Governed by both *electric* and *magnetic fields*.

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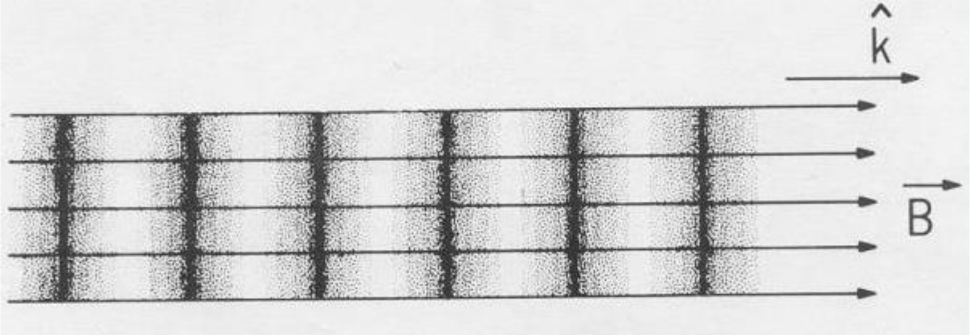
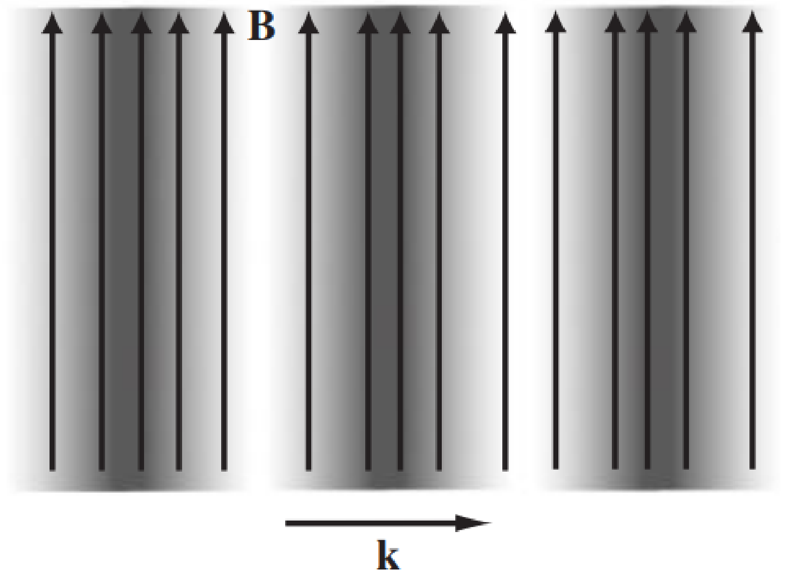
Magnetosonic Waves





- Magnetosonic Waves:
  - Compressible
  - Parallel propagation (**Slow** and **Fast**)
  - Perpendicular propagation (**Fast**)

- Magnetosonic Waves (compressional Alfven waves)

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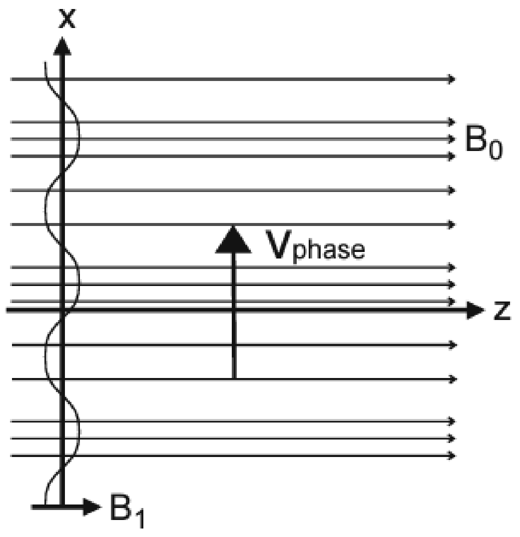
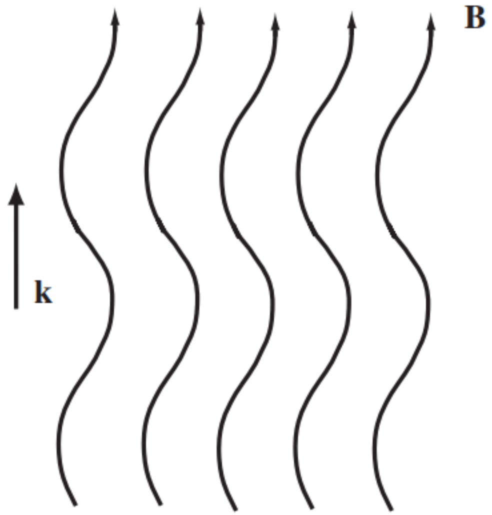
Alfven Waves



- Alfven Waves:
  - Incompressible
  - Parallel and oblique propagation



Compressional Alfven waves

Shear Alfven waves

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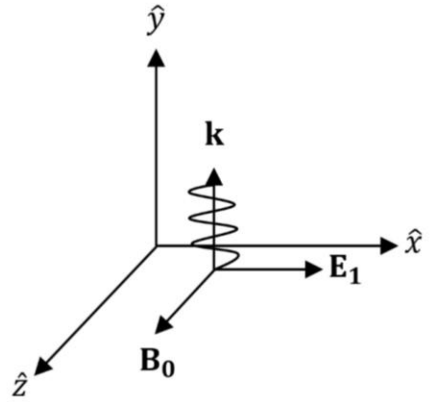
Theoretical Model

- $\mathbf{E} = E_1\, \hat{x},$ Eq(3)
- $\mathbf{B} = (B_0 + B_1)\, \hat{z},$ Eq(4)
- $\mathbf{K} = k\, \hat{y}.$ Eq(5)



where(0) and (1) stand for the unperturbed and the perturbed quantities.

- Using the multi-fluid model, the set of governing equations for each species *j are*

$$m_j n_j \frac{\partial \boldsymbol{v}_j}{\partial t} = q_j n_j(\mathbf{E} + \boldsymbol{v}_j \times \boldsymbol{B}),$$ Eq(6)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$ Eq(7)

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}_j + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t},$$ Eq(8)

The dependent variables are adopted as:

$$n_j = n_{j0} + n_{j1} e^{i(ky-\omega t)},$$ Eq(9)

$$\mathbf{E} = E_1 e^{i(ky-\omega t)} \hat{x},$$ Eq(10)

$$\mathbf{B} = (B_0 + B_1 e^{i(ky-\omega t)})\, \hat{z},$$ Eq(11)

$$\mathbf{J} = J_1 e^{i(ky-\omega t)} \hat{x},$$ Eq(12)

Theoretical Model

The Dispersion Relation:

$$\sum_j \frac{\Omega_{pj}^2}{\mathbf{1}-\frac{\omega_{cj}^2}{\omega^2}} - \mathrm{c}^2\mathrm{k}^2 \sum_j \frac{\frac{\Omega_{pj}^2}{\omega^2}}{\mathbf{1}-\frac{\omega_{cj}^2}{\omega^2}} - \left(\sum_j \frac{\Omega_{pj}^2}{\mathbf{1}-\frac{\omega_{cj}^2}{\omega^2}}\right)\left(\sum_j \frac{\frac{\Omega_{pj}^2}{\omega^2}}{\mathbf{1}-\frac{\omega_{cj}^2}{\omega^2}}\right) = 0, \qquad \text{Eq(13)}$$

where Ω_{pj} and ω_{cj} are the plasma frequency and the cyclotron frequency of the species *j*.

This is the dispersion relation of a modified Alfven waves in collisionless, cold plasma medium that propagate perpendicularly to the ambient magnetic field.

The derived mathematical model could be applied to a wide range of application in space and astrophysical plasmas.

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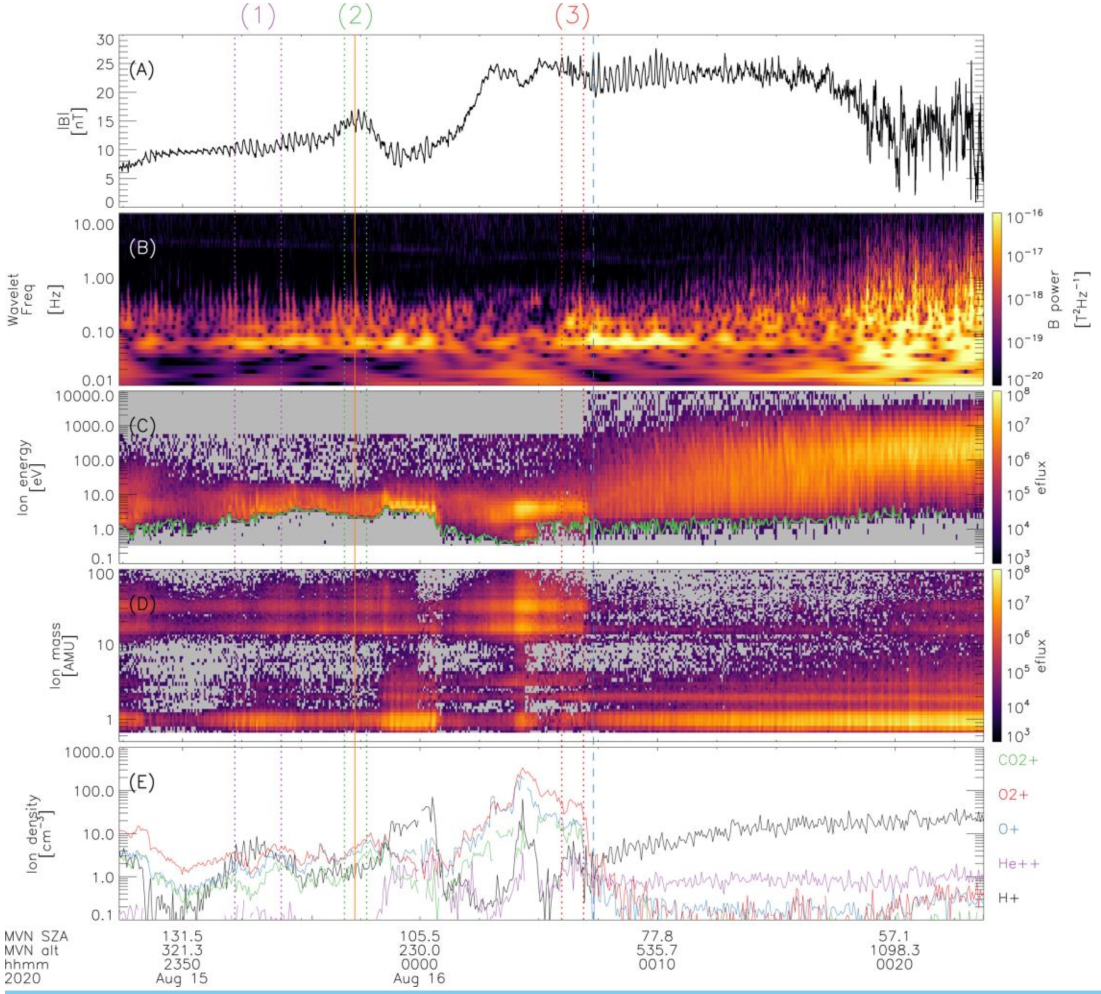
Application to Observed Structure



$$j = CO_2^+, O_2^+, O^+, He^{+2}, H_2^+, e^-$$

The Dispersion Relation becomes:

$$c_{14}\omega^{14} + c_{12}\omega^{12} + c_{10}\omega^{10} + c_8\omega^8 + c_6\omega^6 + c_4\omega^4 + c_2\omega^2 + c_0\omega^0 = 0, \quad \text{Eq(14)}$$

Where $c_{14}, c_{12}, \ldots, c_0$ are the coefficients of ω.

Solving this equation numerically relying on the plasma parameters observed on the Martian upper ionosphere, results in 7 symmetrical roots.

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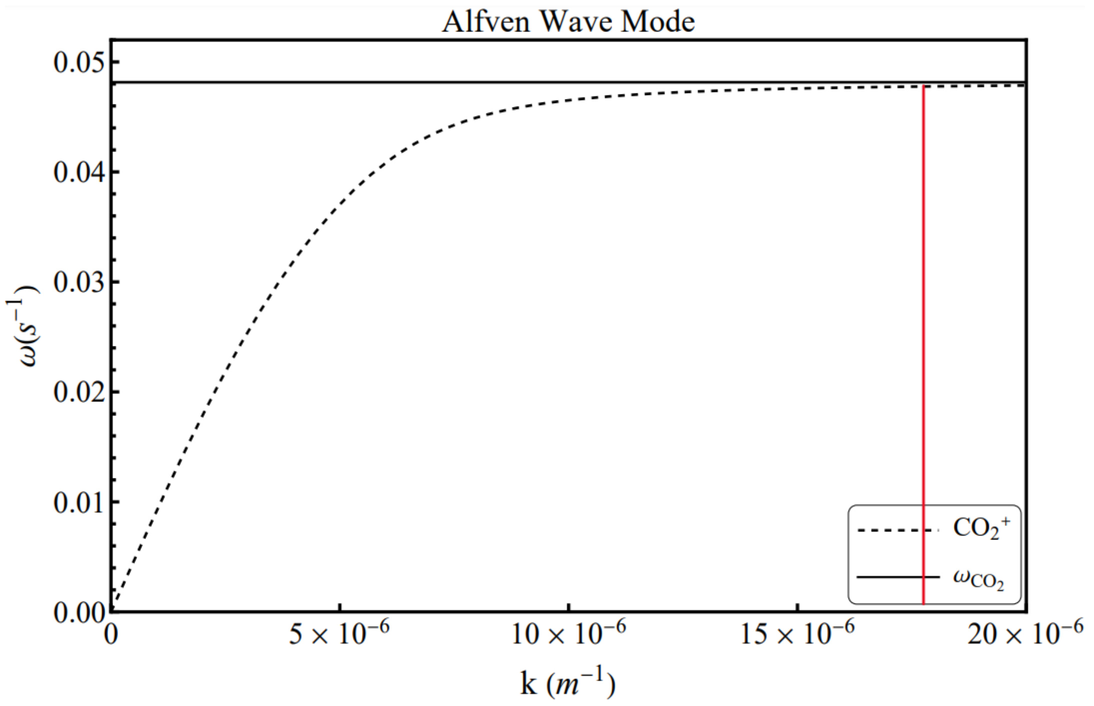
Results

- A root corresponds to each species.
- Each root is divided into two wave modes.
- Three different wave modes:
  - Alfven mode.
  - Whistler mode.
  - Magnetized Plasma Analog of Langmuir mode.
- For each species' mode, there are:
  - Long wavelength (Magnetized mode).
  - Short wavelength (cyclotron mode).



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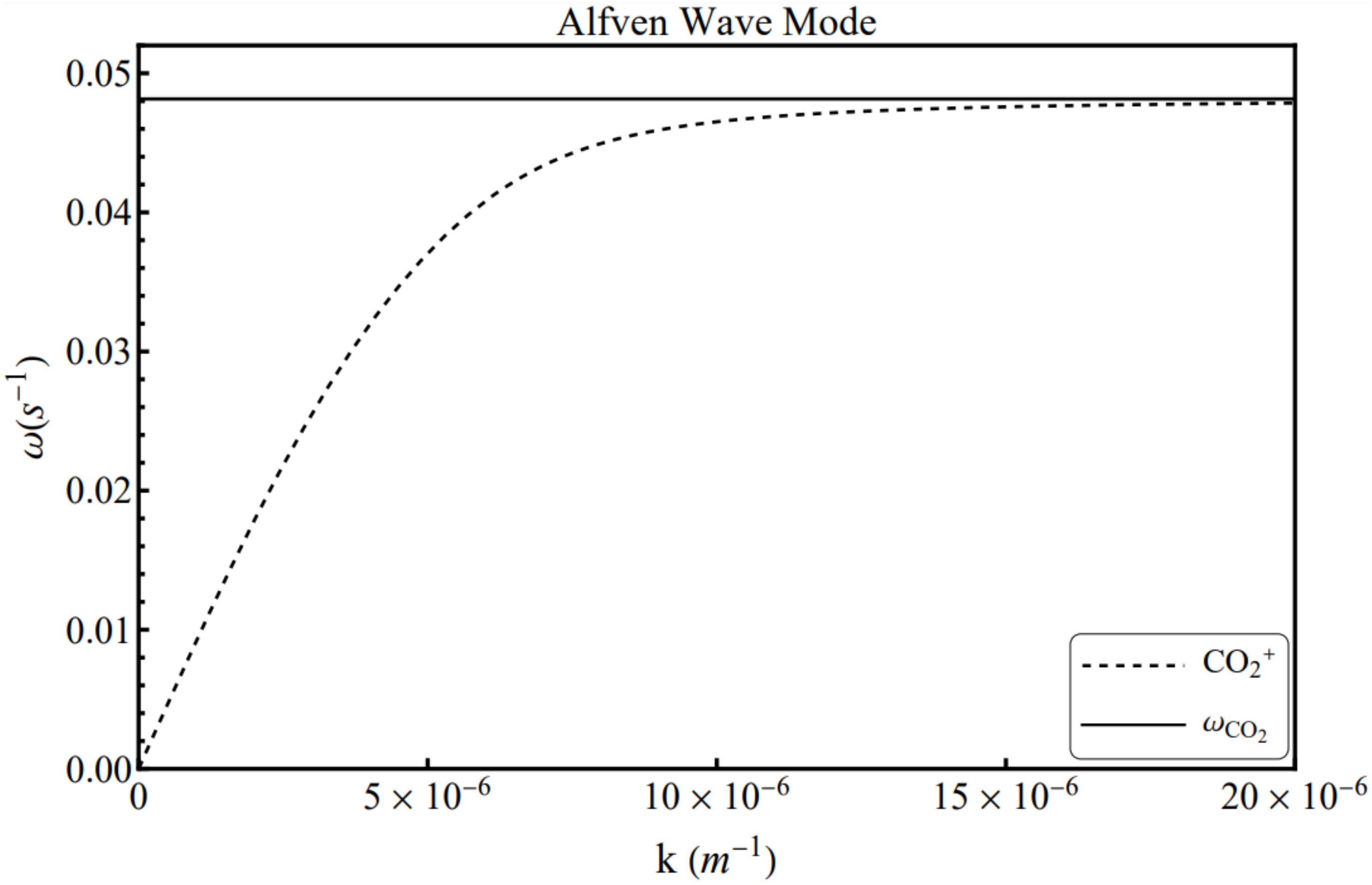
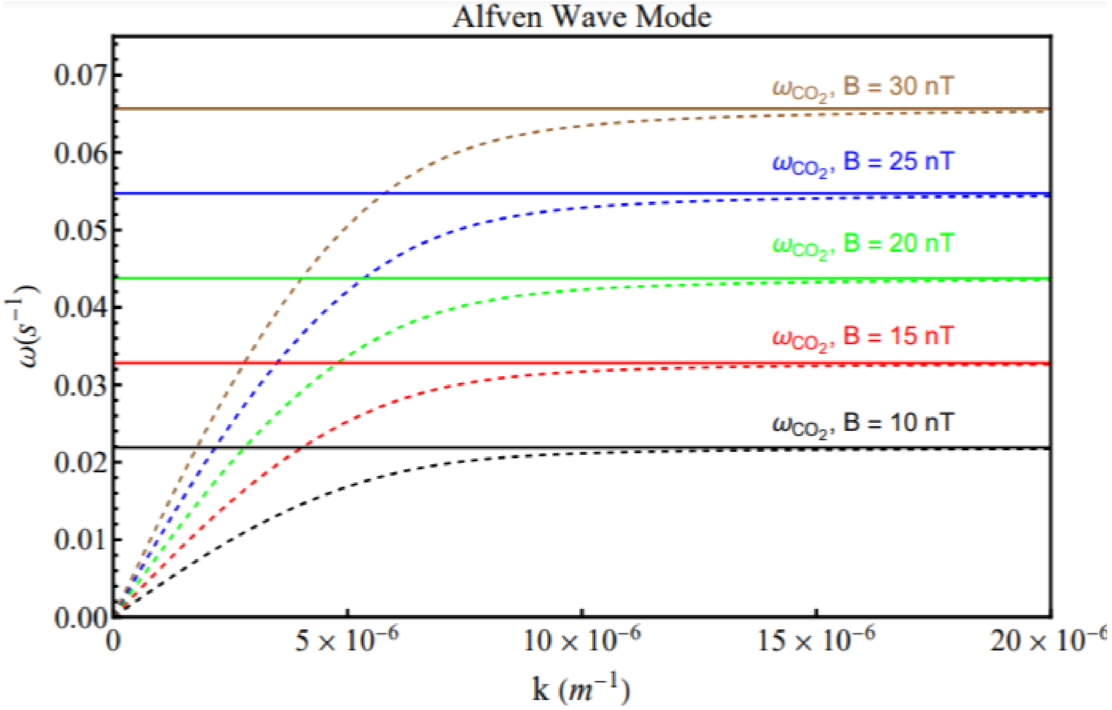
Numerical Analysis

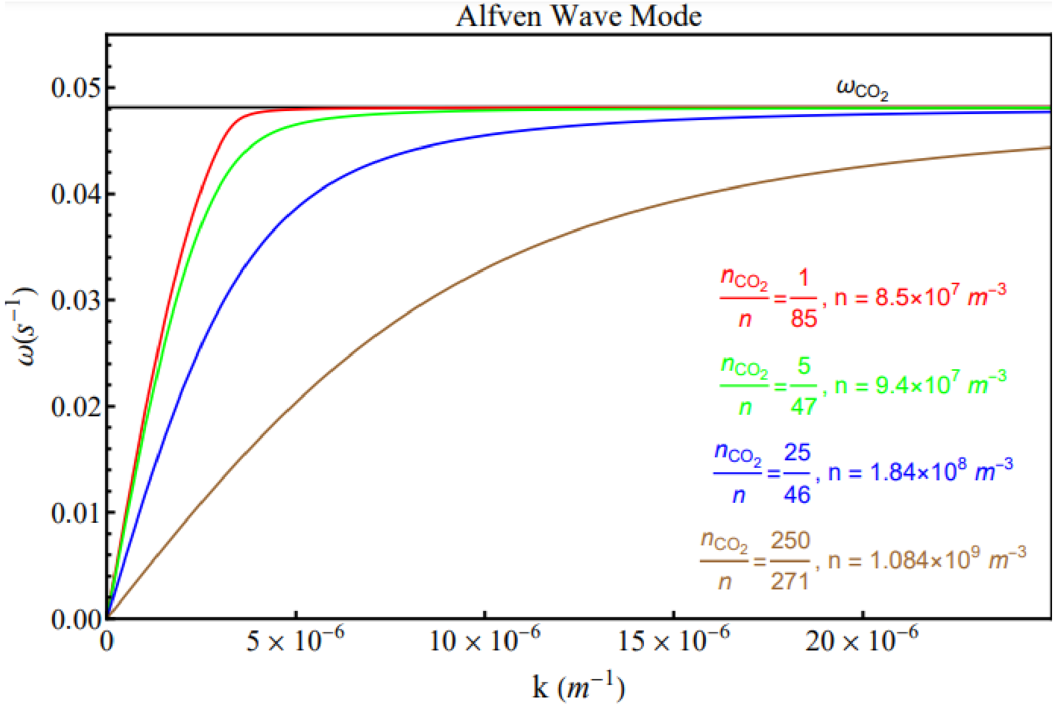
- Alfven Mode:



Numerical Analysis

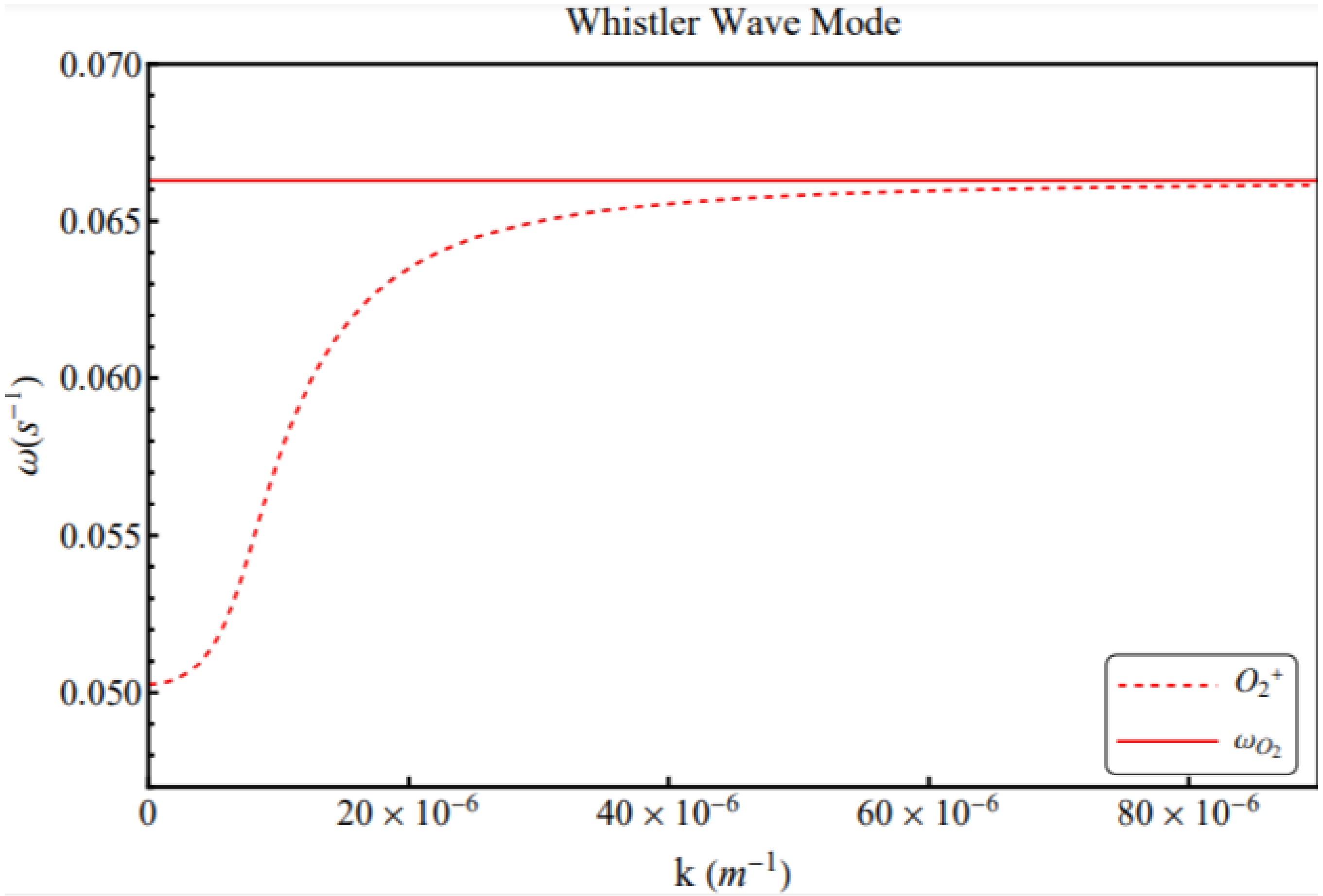
- Effects of varying the magnetic field and the total plasma density and the mixing ratios on the Alfven Mode:





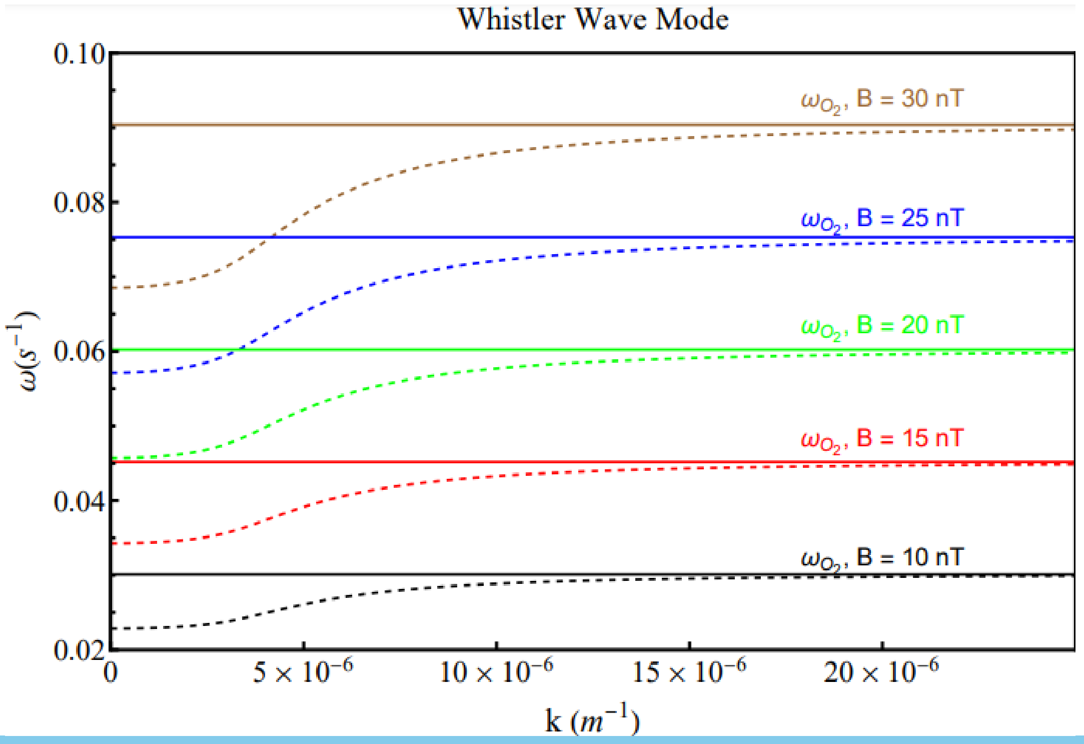
Numerical Analysis

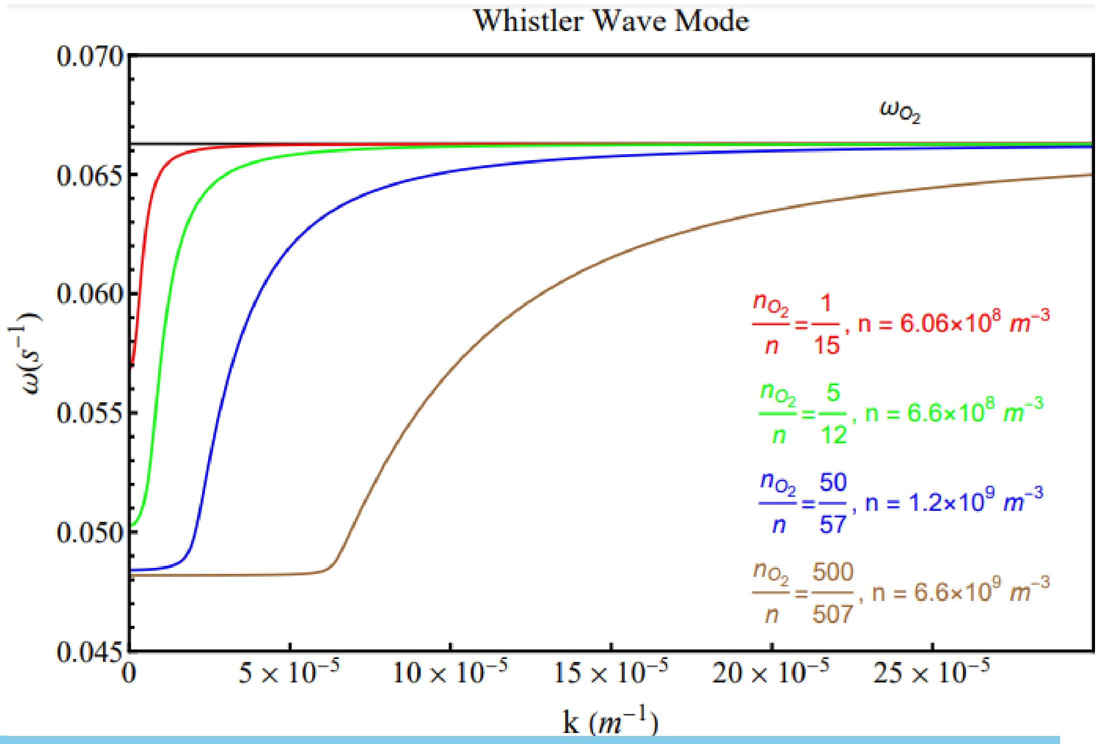
- Whistler Mode:



Numerical Analysis

- Effects of varying the magnetic field and the total plasma density and the mixing ratios on the Whistler Mode:





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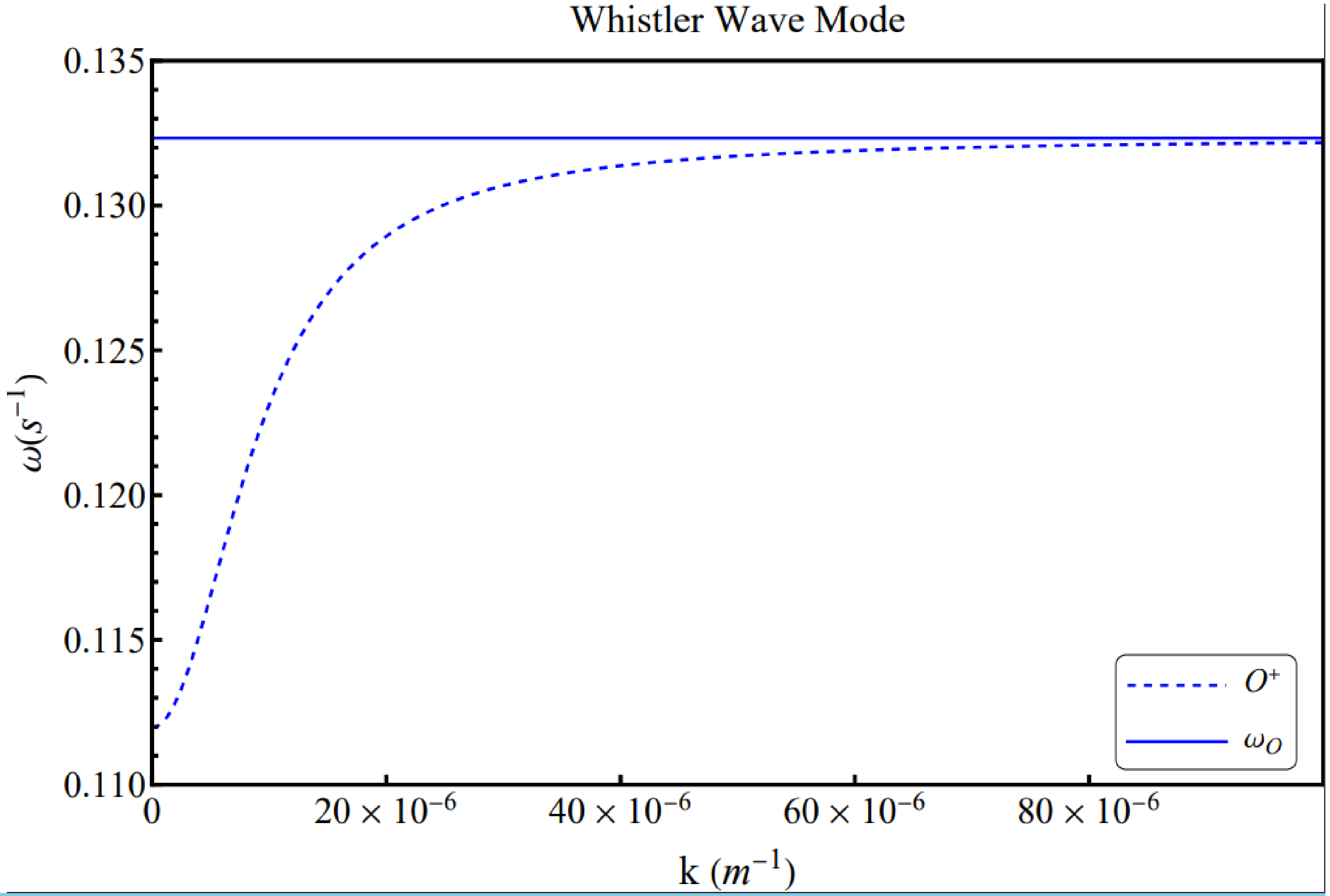

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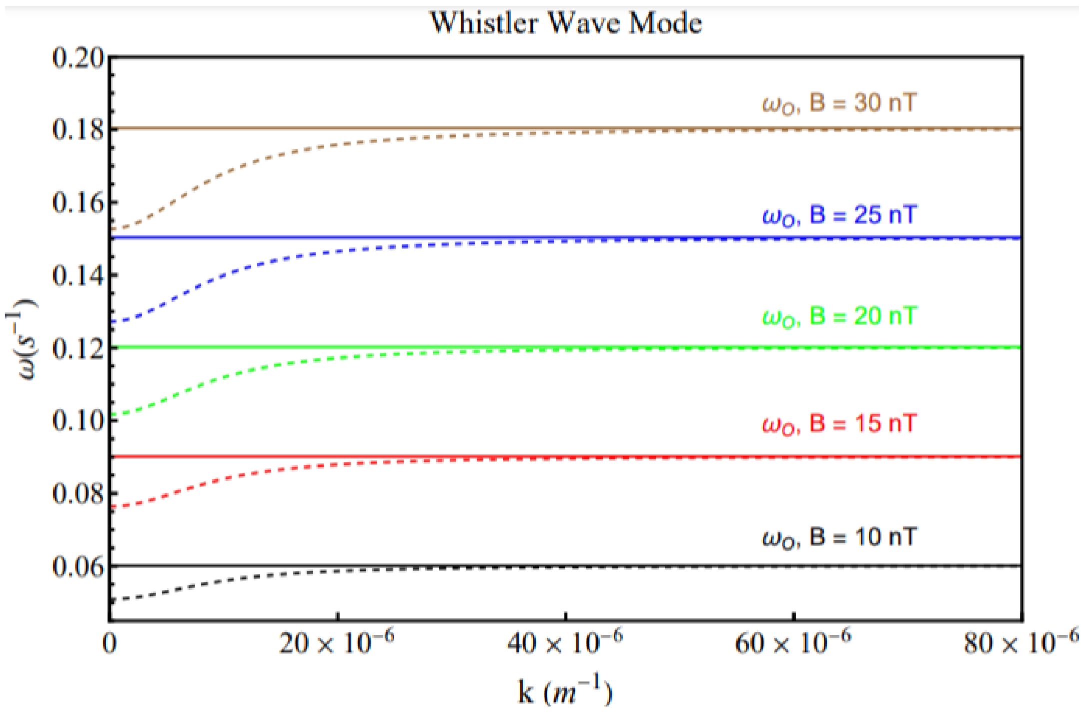

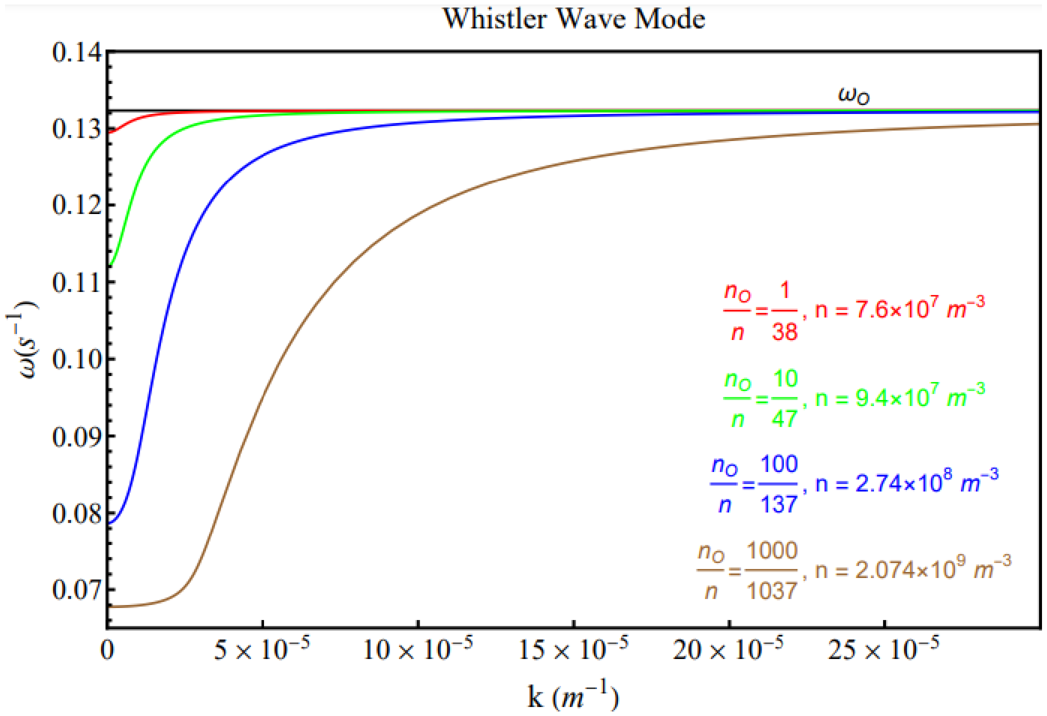

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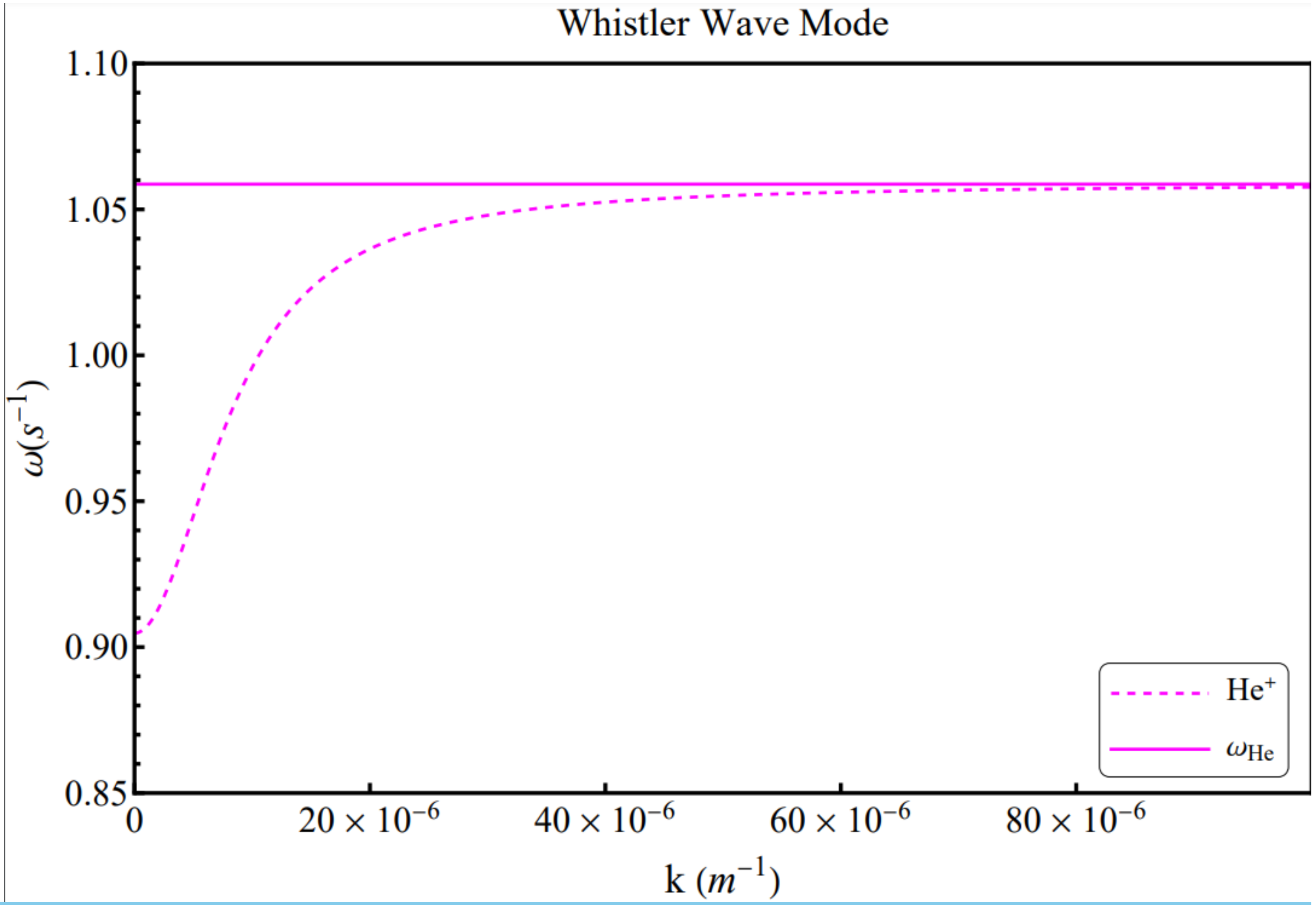

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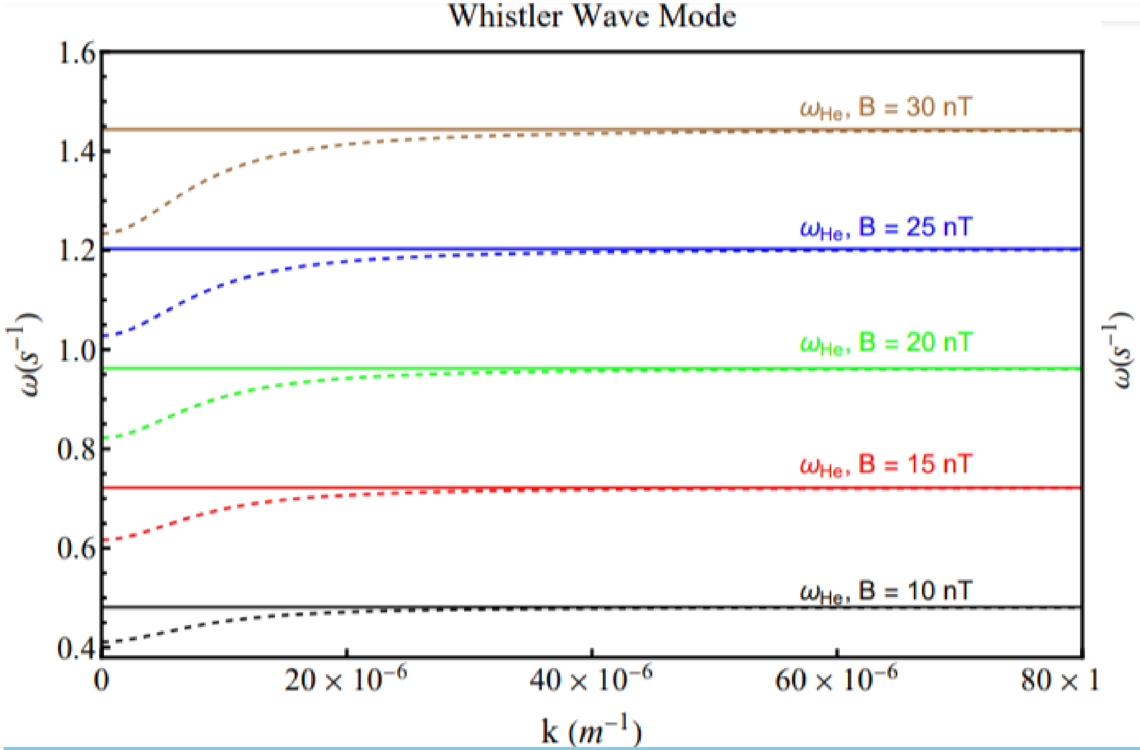

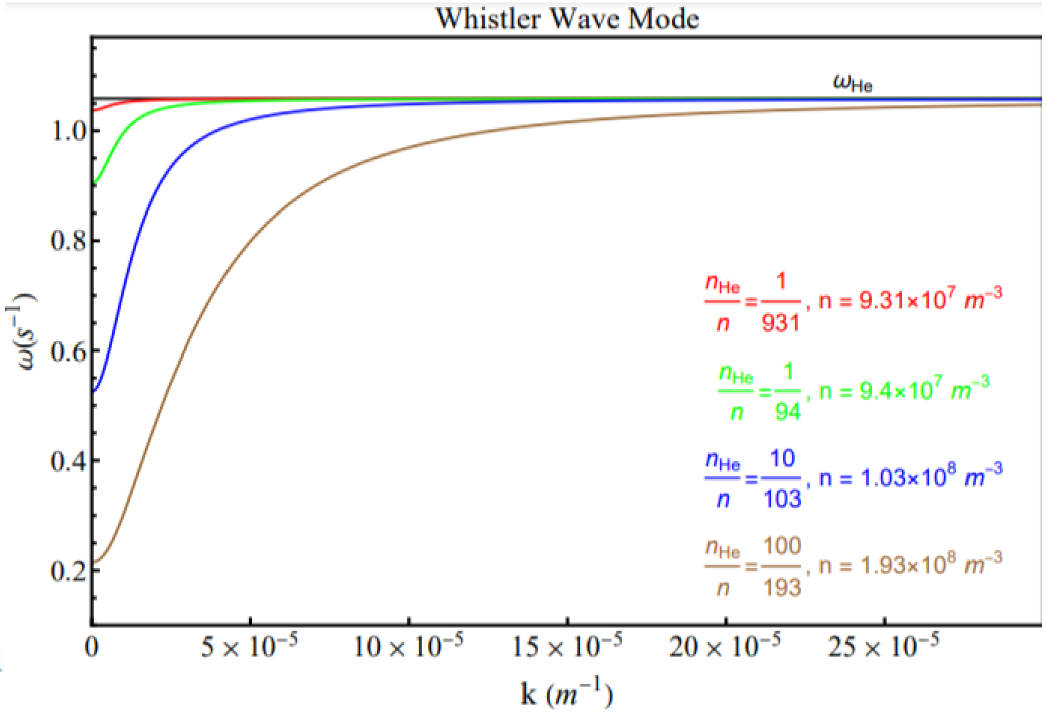

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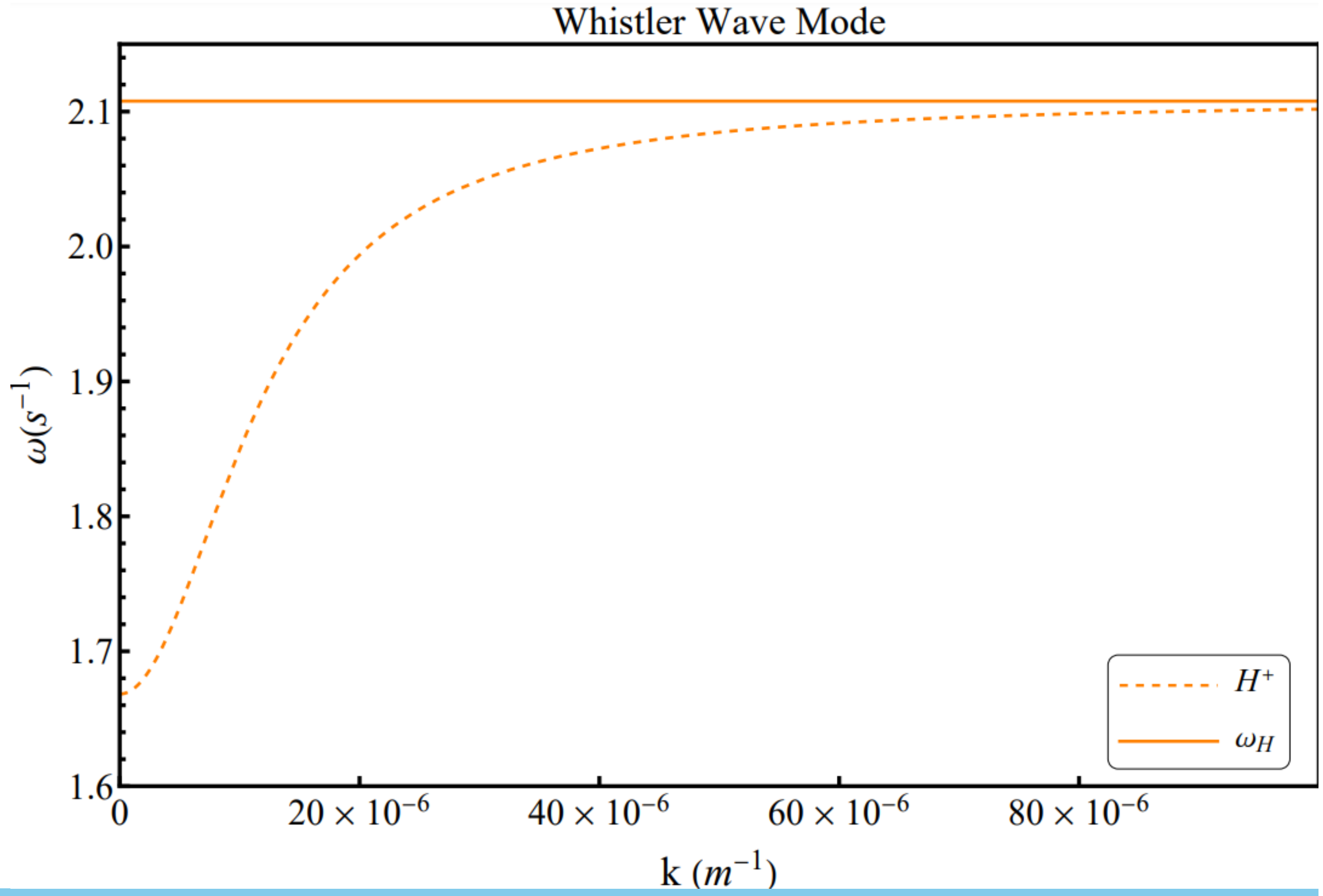

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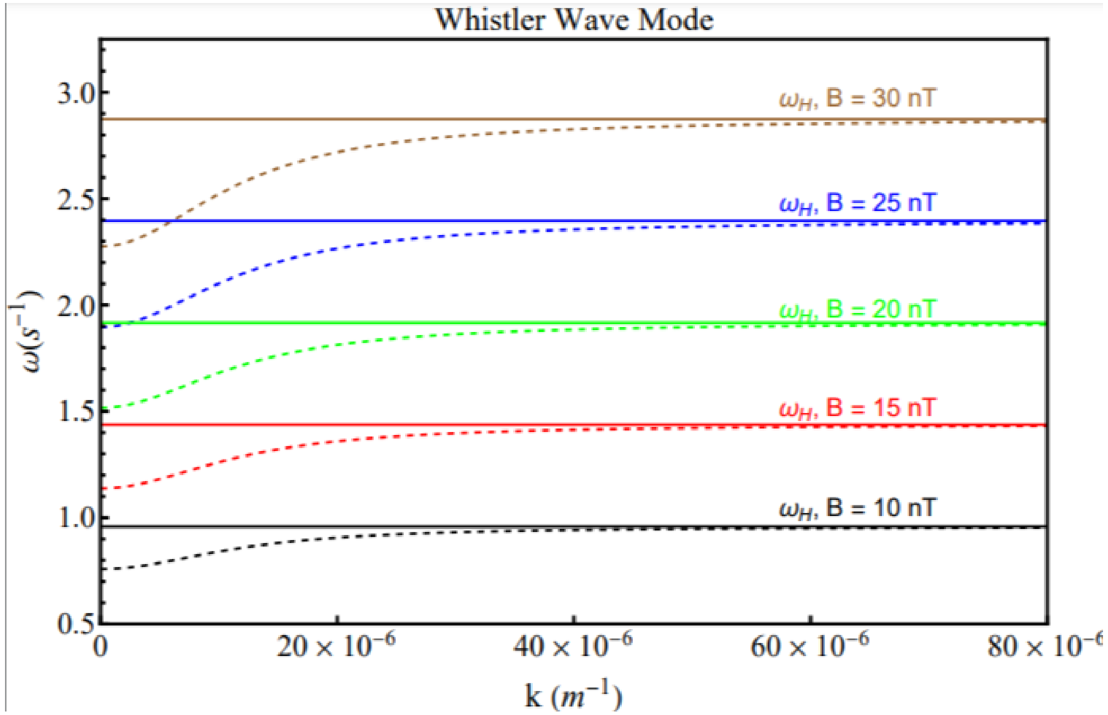

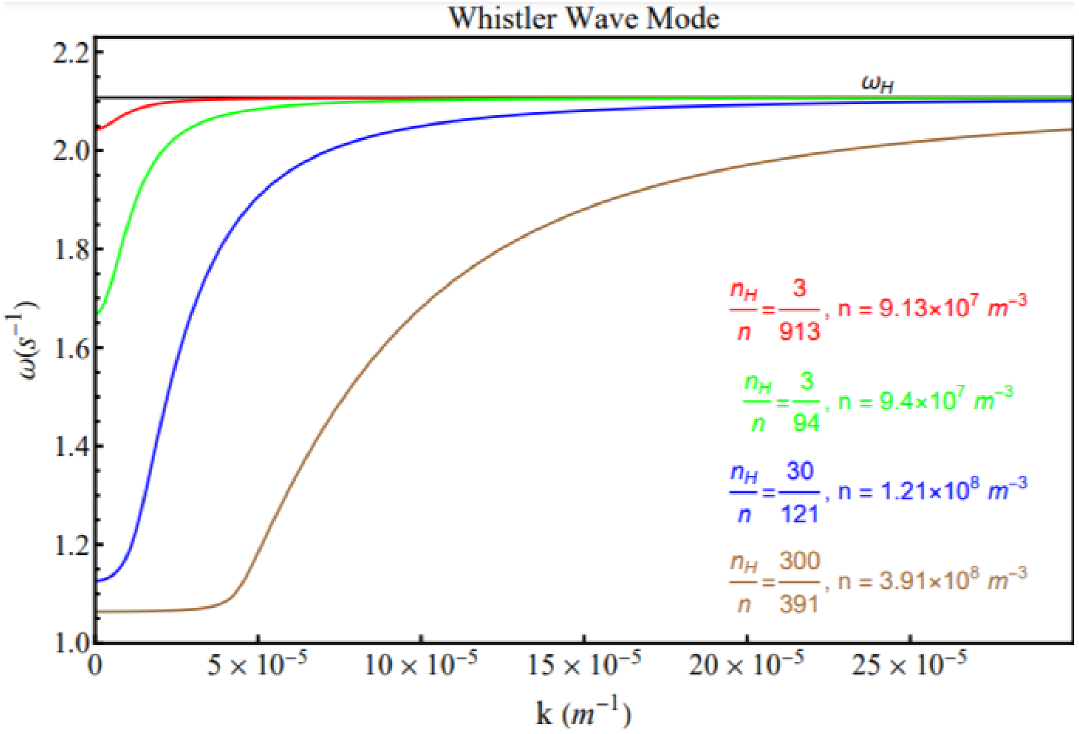

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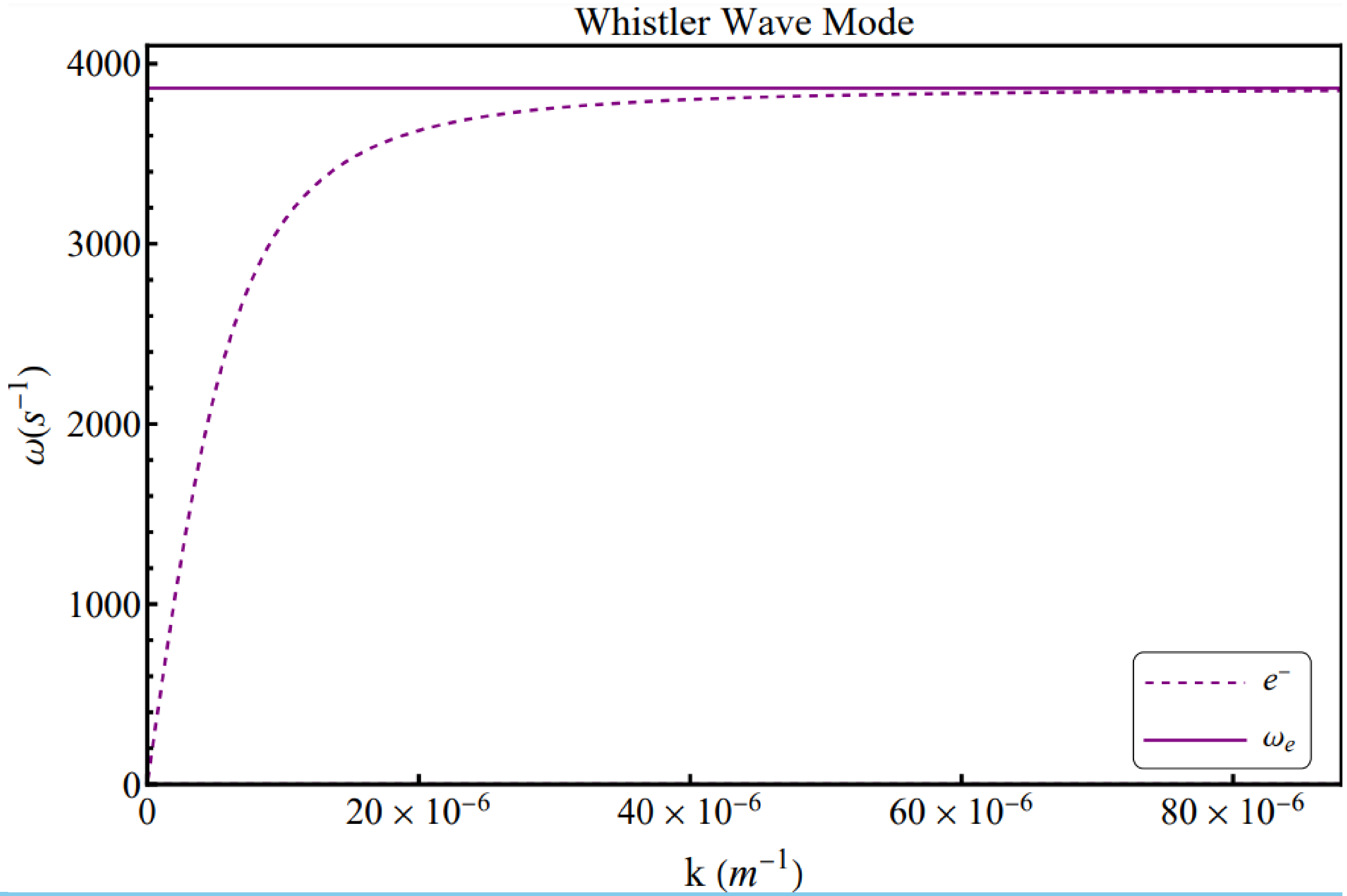

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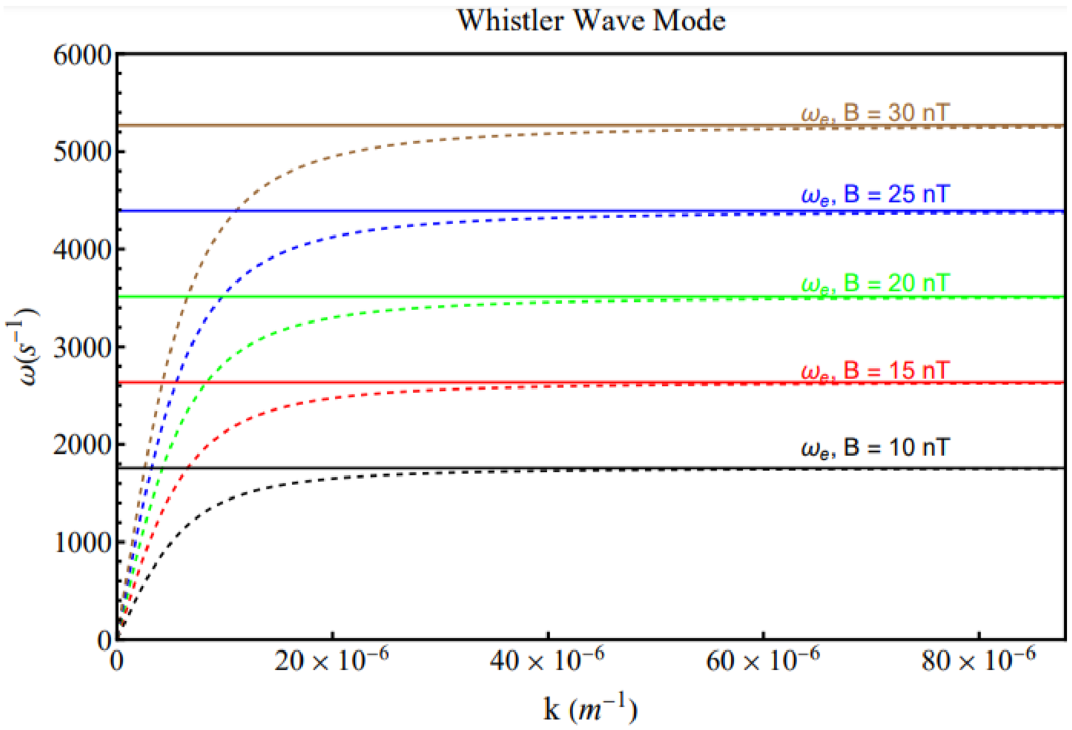
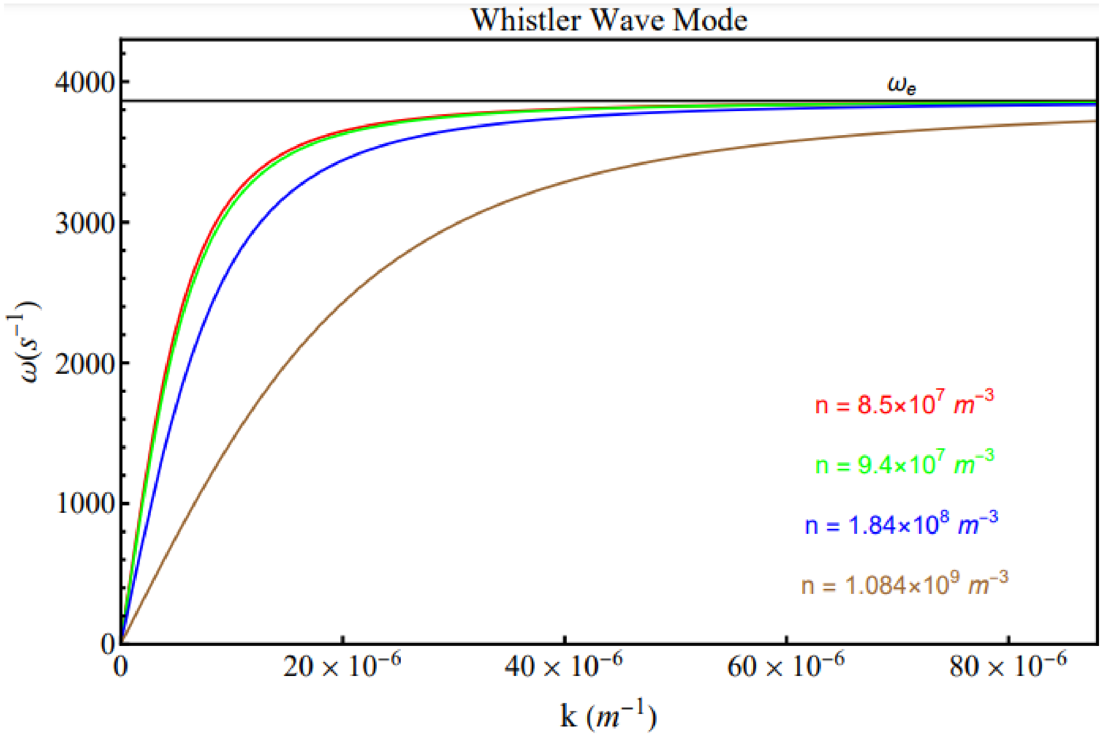
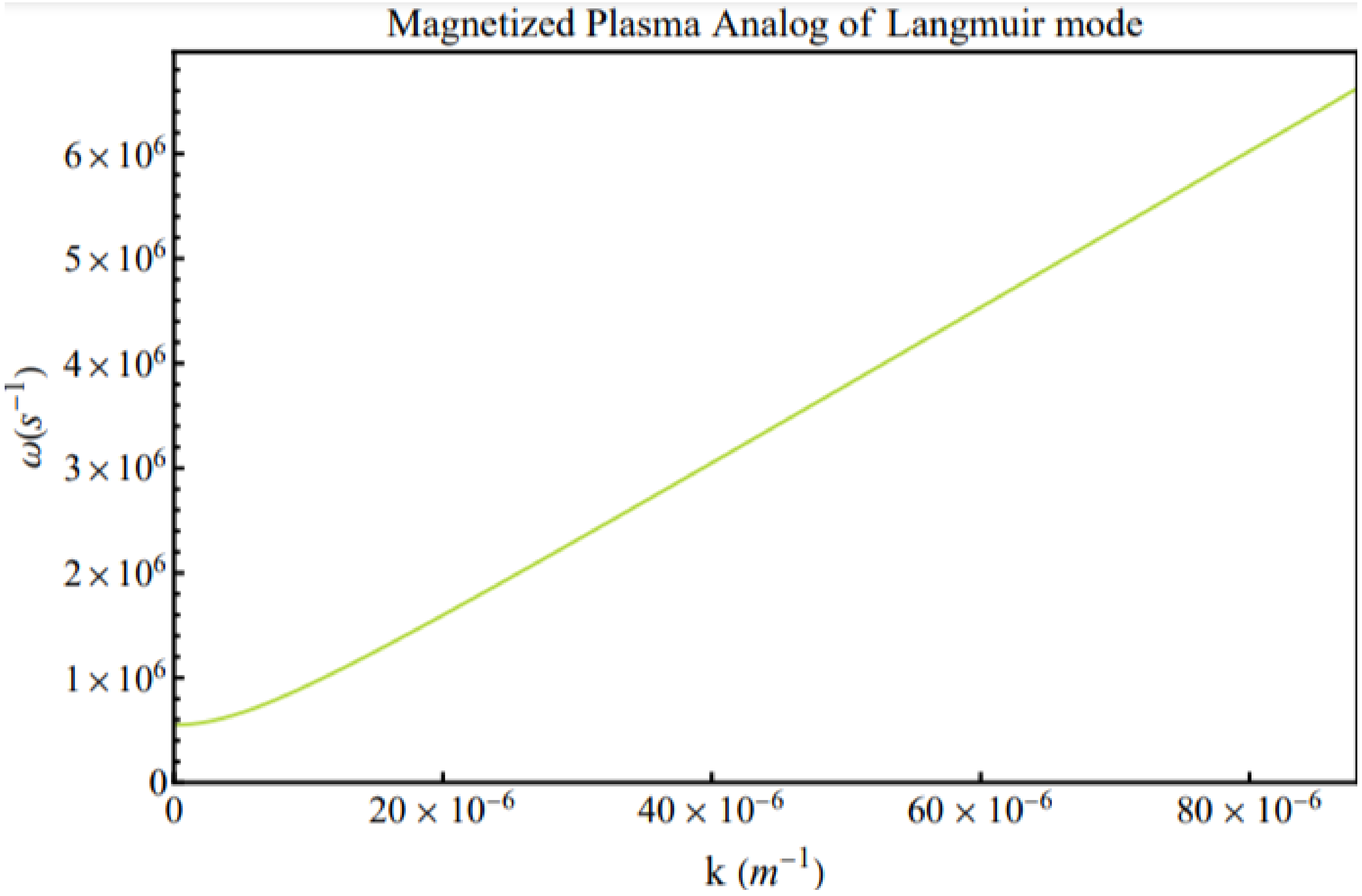




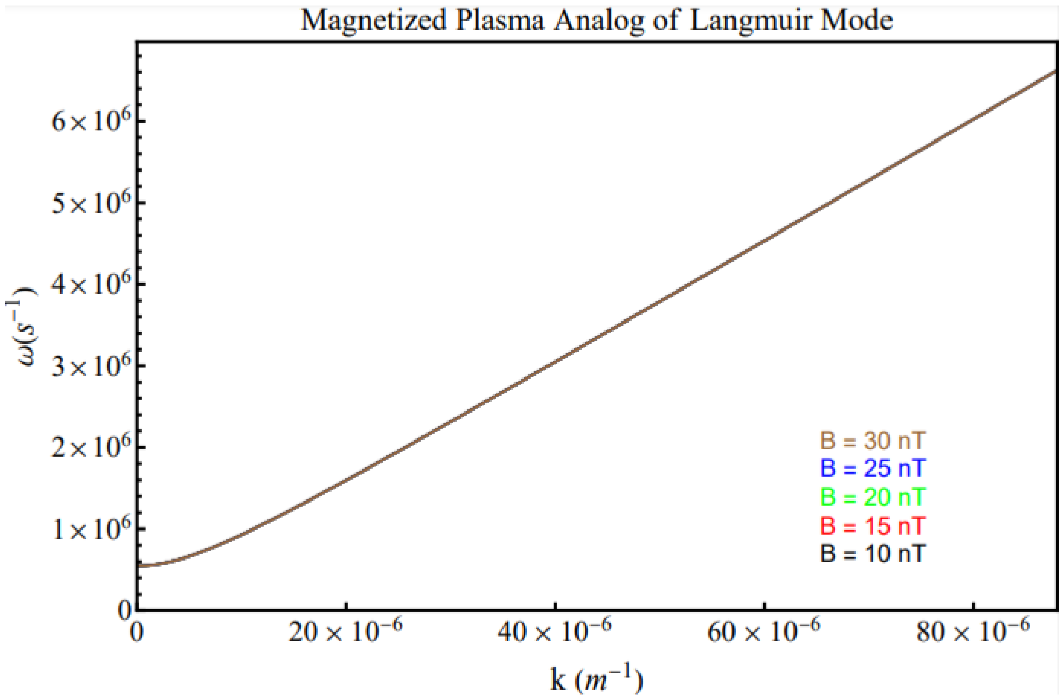
Numerical Analysis

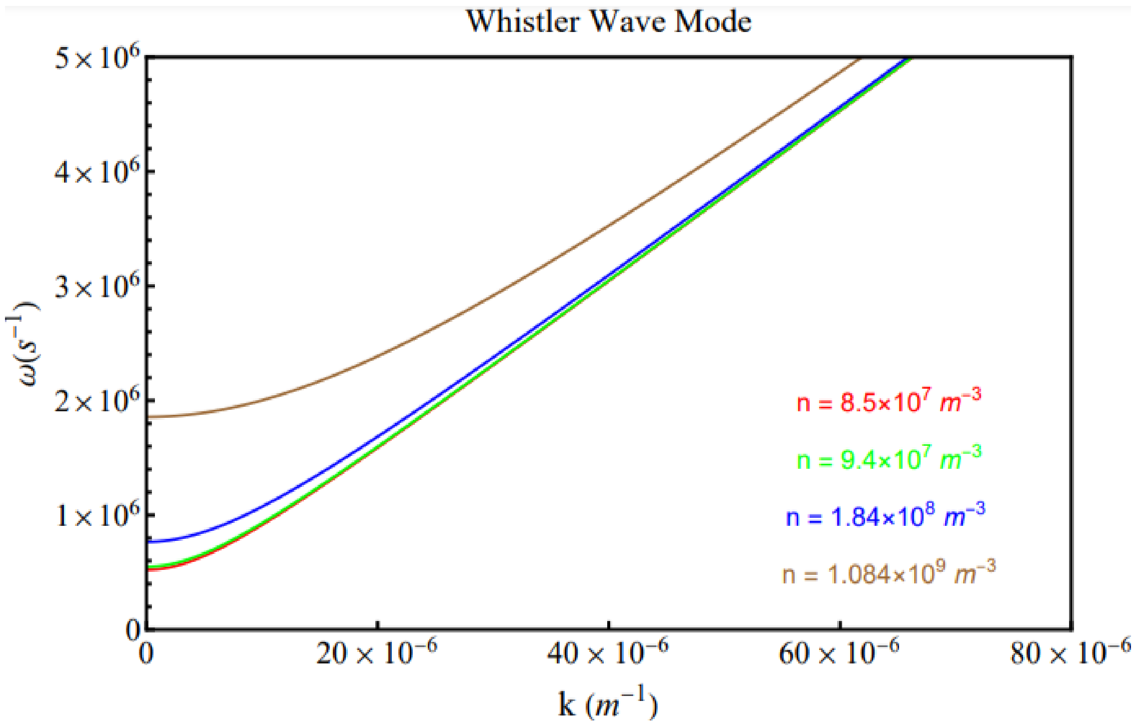
- Magnetized Plasma Analog of Langmuir mode



Numerical Analysis

- Effects of varying the magnetic field and the total plasma density and the mixing ratios on the Whistler Mode:





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Conclusions

- The derived mathematical model could be applied to a wide range of application in space and astrophysical plasmas.
- A Modified Alfven wave has been studied in conditions relevant to the upper ionosphere of Mars.
- Every species has a propagating wave mode that saturates at the specie's cyclotron frequency.
- An Alfven mode exists in the ion species with the heaviest mass.
- A Whistler mode exists in all of the less massive species.
- A Magnetized Analog of Langmuir mode exists due to the displacement current.

Conclusions

- Increasing the magnetic field increases the resonance frequency of the Alfven mode.
- Increasing the magnetic field increases the difference between the cutoff and resonance frequencies of the Whistler mode and causes nothing to the magnetized analog of Langmuir mode.
- Varying the total number density affects the value of the critical wavenumber at the Alfven and Whistler modes.
- Varying the total number density causes a shift in the magnetized analog of Langmuir mode.

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Future Work

- The observation expects a Magnetosonic wave to be steepened.
- We suggest further parameters to be considered:
  - Inhomogeneity of the medium in the magnetic field, density or temperature.
  - Oblique propagation of Magnetosonic waves. (on going)
  - Streaming conditions relevant to the effects of the solar wind.

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Thank You!