

Flavor Hierarchies from $SU(2)$ Flavor and Quark-Lepton Unification

Based on Greljo, Thomsen, Tiblom; [2406.02687]

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- Naively these couplings should be similar, but instead there is a hierarchical structure between generations
- This is known as the *Flavor Puzzle*

Producing the flavor hierarchies

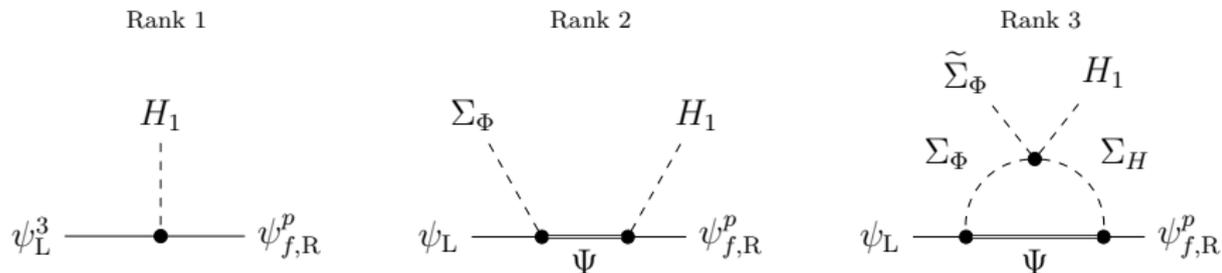


Figure: To provide suitable flavor hierarchies, we make use of three independent rank-one contributions to the Yukawa matrices.

Model Content

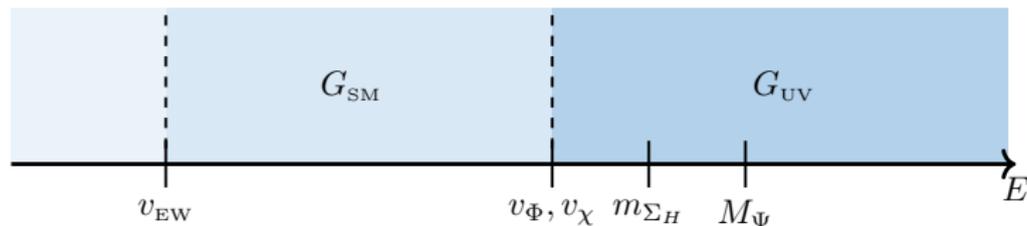
Field	SU(4)	SU(2) _L	U(1) _R	SU(2) _{q+l}
ψ_L	4	2	0	2
ψ_L^3	4	2	0	1
$\psi_{u,R}^p$	4	1	1/2	1
$\psi_{d,R}^p$	4	1	-1/2	1
$\Psi_{L,R}$	4	2	0	1
χ	4	1	1/2	1
H_1	1	2	1/2	1
Σ_H	15	2	1/2	1
Σ_Φ	15	1	0	2

Table: The matter field content of the model and their representations under the gauge group. χ is used to break the gauge group down to $G_{\text{SM}} \times \text{SU}(2)_{q+l}$, and Σ_Φ is used to break the flavor symmetry.

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- Rank 1:

$$\mathcal{L}_{UV} \supset -x_d^p \bar{\psi}_L^3 H_1 \psi_{d,R}^p - X_d^p \bar{\psi}_L^3 \Sigma_H \psi_{d,R}^p + \text{h.c.}$$

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- After integrating out the VLF:

$$\mathcal{L}_{\text{EFT}} \supset \frac{Y_\Phi}{M_\Psi} \left(y_d^p \bar{\psi}_L \Sigma_\Phi H_1 \psi_{d,R}^p + Y_d^p \bar{\psi}_L \Sigma_\Phi \Sigma_H \psi_{d,R}^p + \text{h.c.} \right)$$

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- Rank 3 (calculated with Matchete):

$$\mathcal{L}_{\text{EFT}} \supset \frac{15}{32} \frac{1}{16\pi^2} \frac{1}{M_\Psi} \left[\log \left(\frac{M_\Psi^2}{\mu^2} \right) - 1 \right] Y_u^p Y_\Phi \lambda_{45}^* \text{Tr} \left[\tilde{\Sigma}_H \tilde{\Sigma}_\Phi \right] \bar{\psi}_L \psi_{u,R}^p$$

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- We give VEVs to both Higgses, defining the angle between them as

$$\tan \beta \equiv \frac{v_2}{v_1}, \quad v_1^2 + v_2^2 = v_{\text{EW}}^2 = (174 \text{ GeV})^2.$$

Results

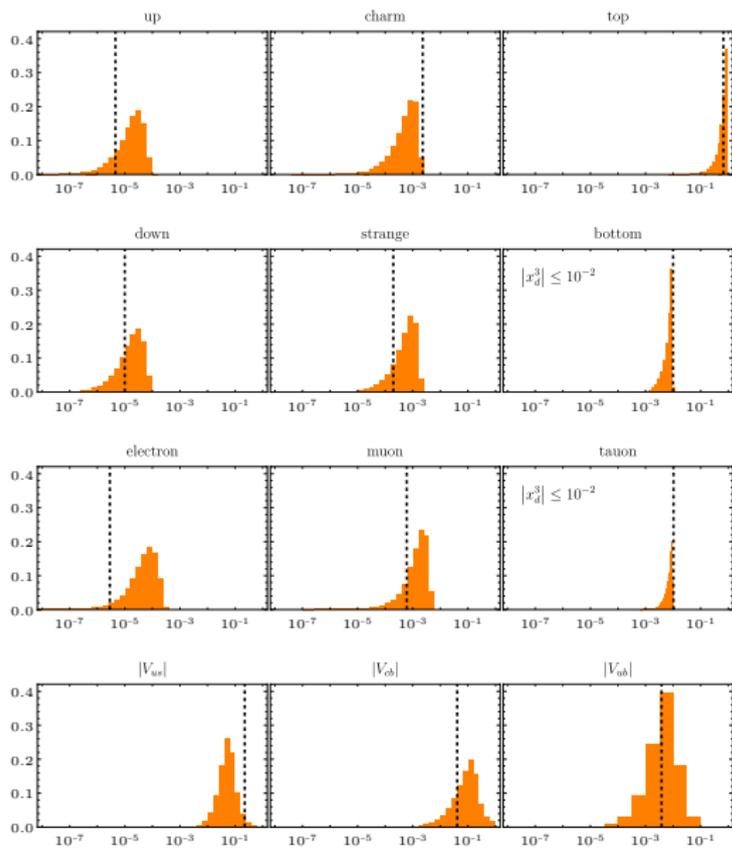


Figure: Histogram showing the probability of obtaining the observed flavor hierarchies when the UV parameters are drawn randomly from a flat distribution between -1 and 1. Only the parameter x_d^3 needs to be fine-tuned.

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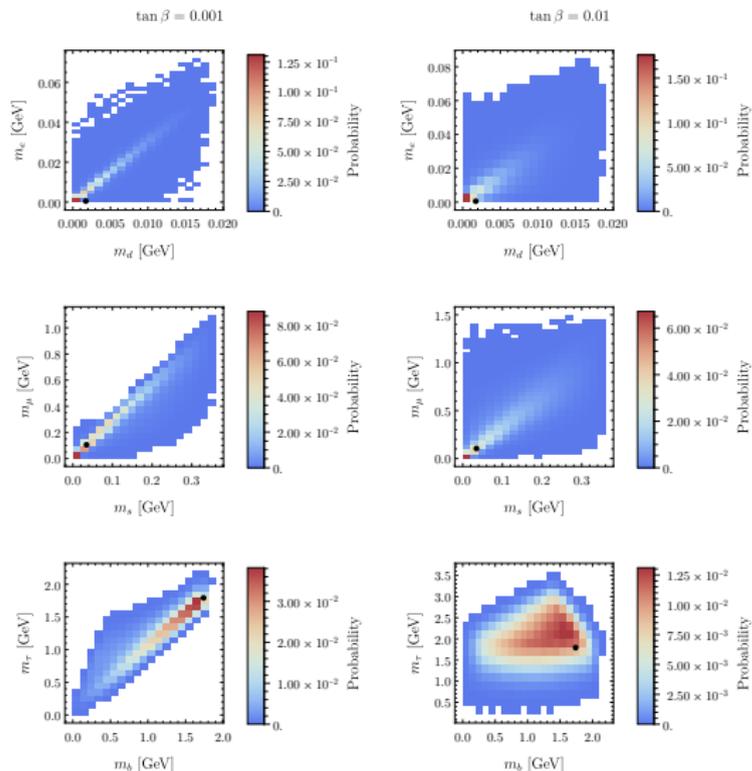


Figure: Correlation between the masses of the down quarks and charged leptons for two different values of β .

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- The model allows for small neutrino masses through the use of the inverse seesaw mechanism, but to predict large PMNS mixing angles additional structure is needed

- In Pati-Salam type models vector leptoquarks contribute to $K_L \rightarrow \mu e$
- This constrains the breaking scale to a few PeV
- Muon conversion on heavy nuclei also places bounds on the model
- Best present bound is from SINDRUM-II for the process $\mu Au \rightarrow e Au$
- Is expected to improve with future experiments such as Mu2e and COMET