Results from the MD on Schottky signals

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Acknowledgements: Tatiana Pieloni, Ivan Karpov, Theodoros Argyropoulos

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Outline

- Introduction
  - Longitudinal Schottky spectrum
  - Transverse Schottky spectrum

- Experimental Schottky spectra from the LHC
  - Longitudinal impedance effect
  - Estimate of the LHC longitudinal impedance
  - Transverse impedance effect

- Conclusion

Introduction: The Schottky Monitor

- The Schottky spectrum is based on the measurement of the beam fluctuations in the longitudinal and transverse planes.

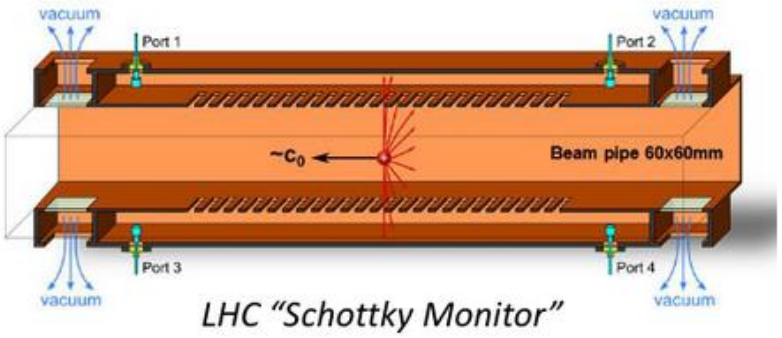
- The Schottky spectrum is the **power spectral density** of the beam current in the longitudinal plane and the dipole moment in the transverse planes.

- Important **non-invasive** method for beam diagnostics (emittance, tune, chromaticity, bunch profile, …).

- The Schottky monitor is one of the only instruments with the potential of measuring the LHC chromaticity in a non-invasive way.

- However, **impedance, non-linearities, and other collective effects** can strongly affect the Schottky spectrum, preventing the extraction of beam and machine parameters.

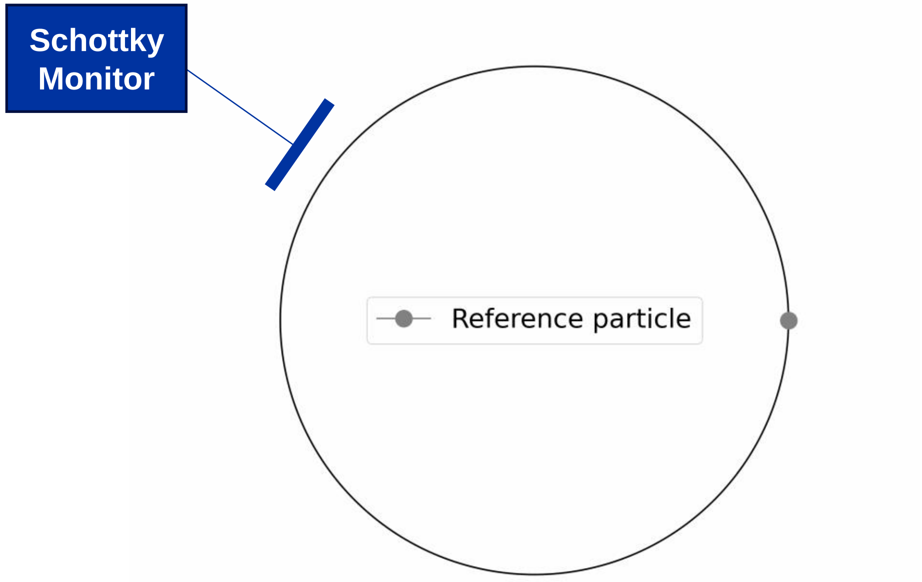
  ➔ The distortion caused by impedance can also be used to estimate the impedance itself.

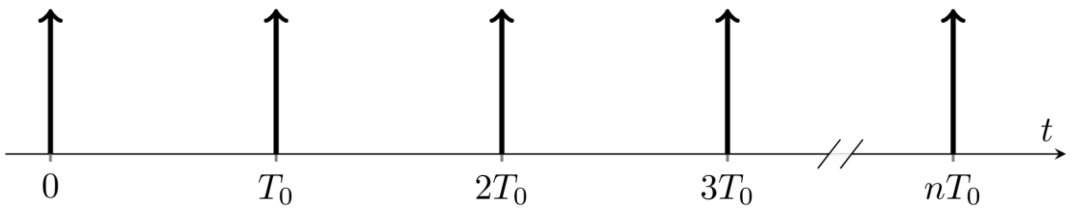


Details of the LHC Schottky system in
M. Betz et al., NIM, vol. 874, pp 113-126, 2017

Longitudinal Schottky Spectrum (synchronous particle)

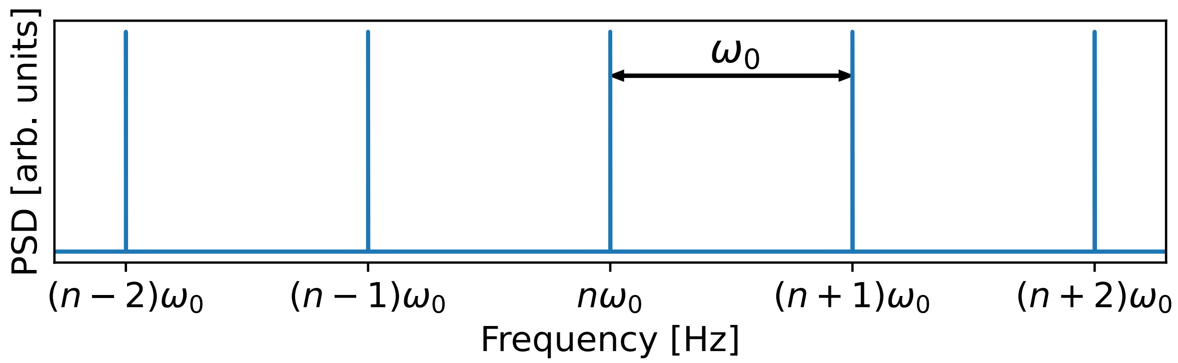
Time domain:





$$i_i(t) = q \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

Frequency domain:

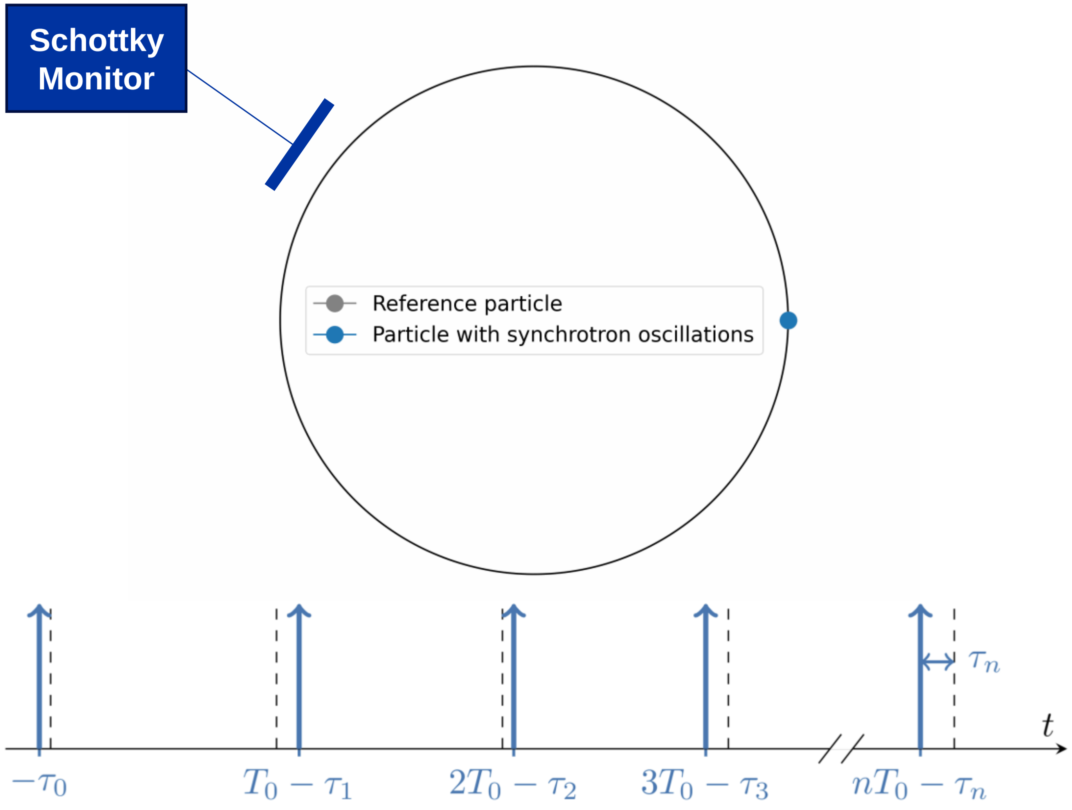


$$\widetilde{i_i(f)} = \frac{q f_0}{2\pi} \sum_{n_h=-\infty}^{\infty} \delta(f - n_h f_0)$$

Summation over harmonic n_h

Longitudinal Schottky Spectrum

Time domain:



$$i_i(t) = q \sum_{n=-\infty}^{\infty} \delta(t + \tau_i(t) - nT_0)$$

τ_i : Time difference between particle i and the synchronous particle

Frequency domain:

$\tau_i = ?$

Analytical expression for $\tau_i(t)$

$$\tau_i(t) = \hat{\tau}_i \; \sin(\Omega_{s_i} t + \varphi_{s_i})$$

1. Inserting the harmonic synchrotron motion $\tau_i(t)$in $i_i\,(t)$
2. Writing the Fourier series of this periodic signal
3. Using the Jacobi Anger relation $$e^{jz\sin\theta} = \sum_{p=-\infty}^{\infty} J_p(z) e^{jp\theta}$$

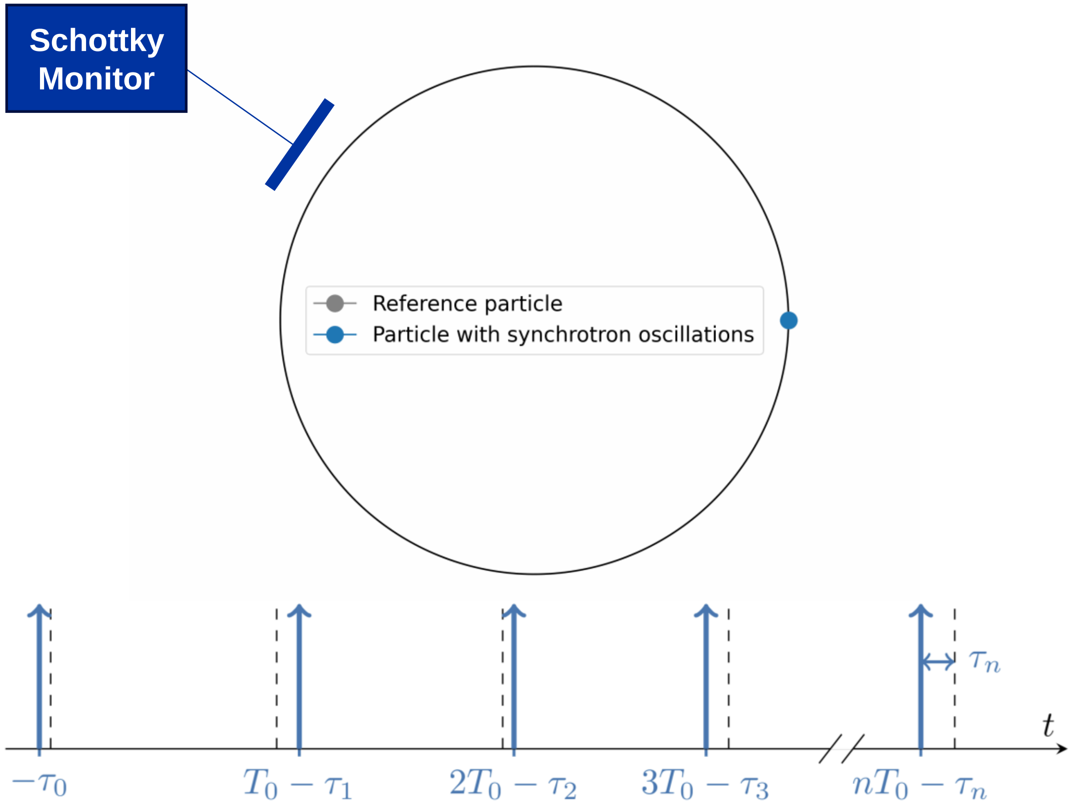
➔ The intensity signal can be written in the following form:

$$i_i(t) = q f_0 \sum_{n,p=-\infty}^{\infty} J_p(n\omega_0\hat{\tau}_i)\, e^{j\left[(n\omega_0 + p\Omega_{s_i})t + p\varphi_{s_i}\right]}$$

- $J_p(n\omega_0\hat{\tau}_i)$: Amplitude
- $(n\omega_0 + p\Omega_{s_i})$: Frequency
- $p\varphi_{s_i}$: Phase
- n: Summation over harmonic n
- p: Summation over Bessel satellites p

Longitudinal Schottky Spectrum

Time domain:



$$i_i(t) = q \sum_{n=-\infty}^{\infty} \delta(t + \tau_i(t) - nT_0)$$

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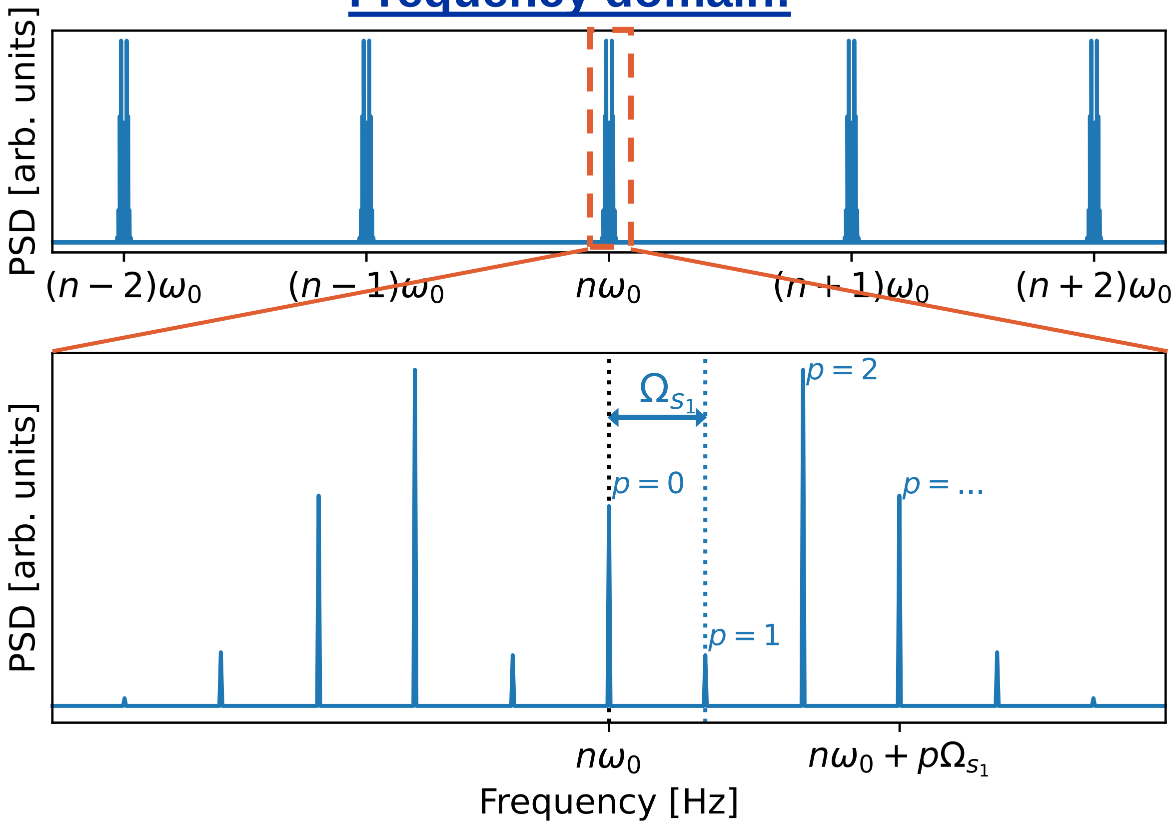


$$i_i(t) = q f_0 \sum_{n,p=-\infty}^{\infty} J_p(n\omega_0 \widehat{\tau}_i)\, e^{j\left[(n\omega_0 + p\Omega_{s_i})t + p\varphi_{s_i}\right]}$$

Amplitude

Frequency

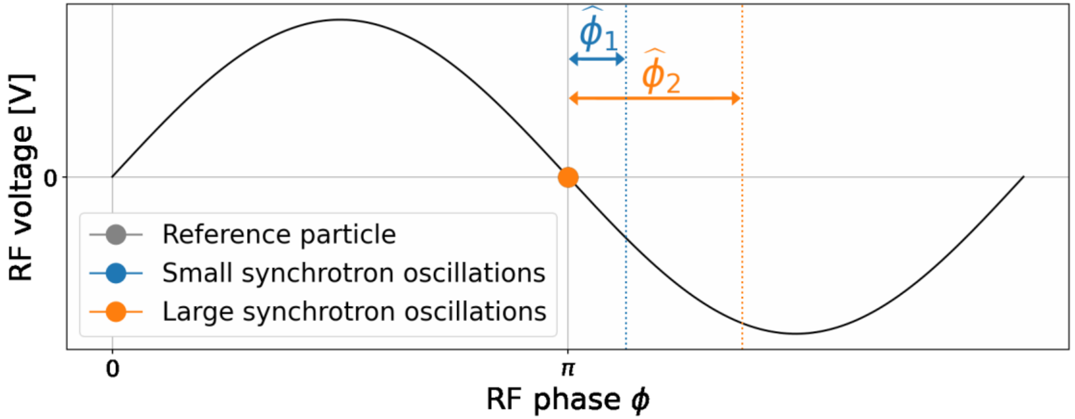
Phase

Summation over harmonic n

Summation over Bessel satellites p

Longitudinal Schottky Spectrum

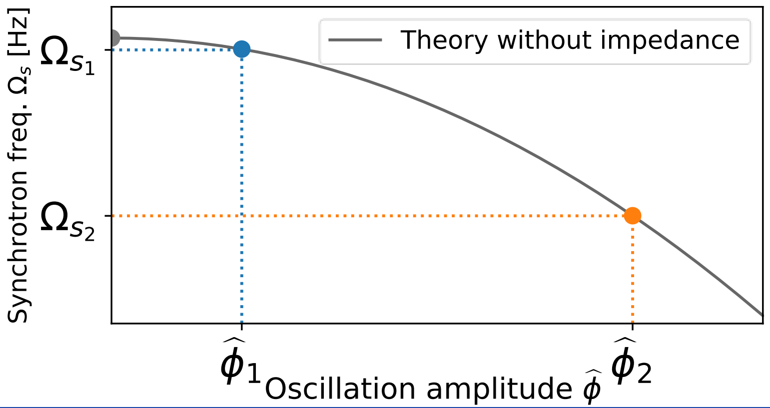
Synchrotron oscillations:



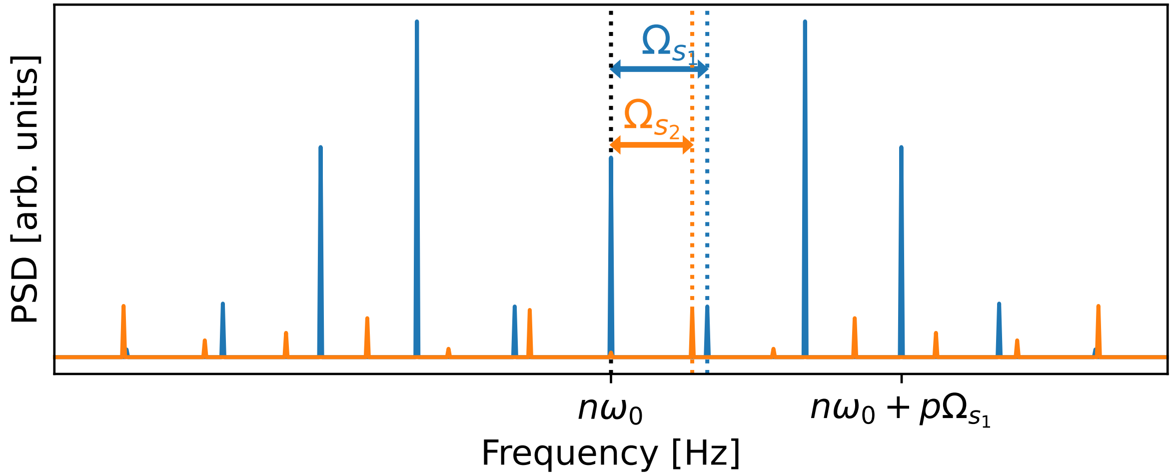
From the theory of the non-linear pendulum:

$$\frac{d^2\phi}{dt^2} + \Omega_0^2 \ \sin\phi = 0 \quad \Rightarrow \quad \Omega_s(\widehat{\phi}) = \frac{\pi}{2\mathcal{K}\left[\sin\left(\frac{\widehat{\phi}}{2}\right)\right]}\Omega_0$$

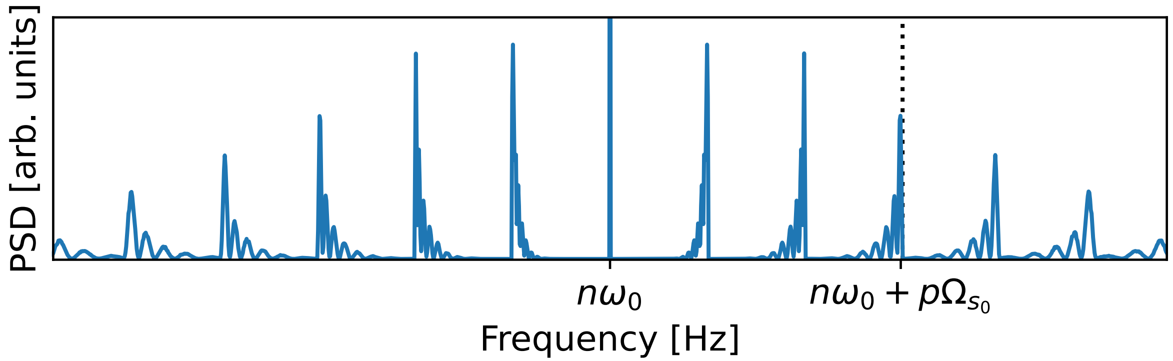
K: Elliptic integral of the first kind



Spectrum for **two particles** with different synchrotron frequencies



Spectrum for a **bunch of particles**



Transverse Schottky spectrum

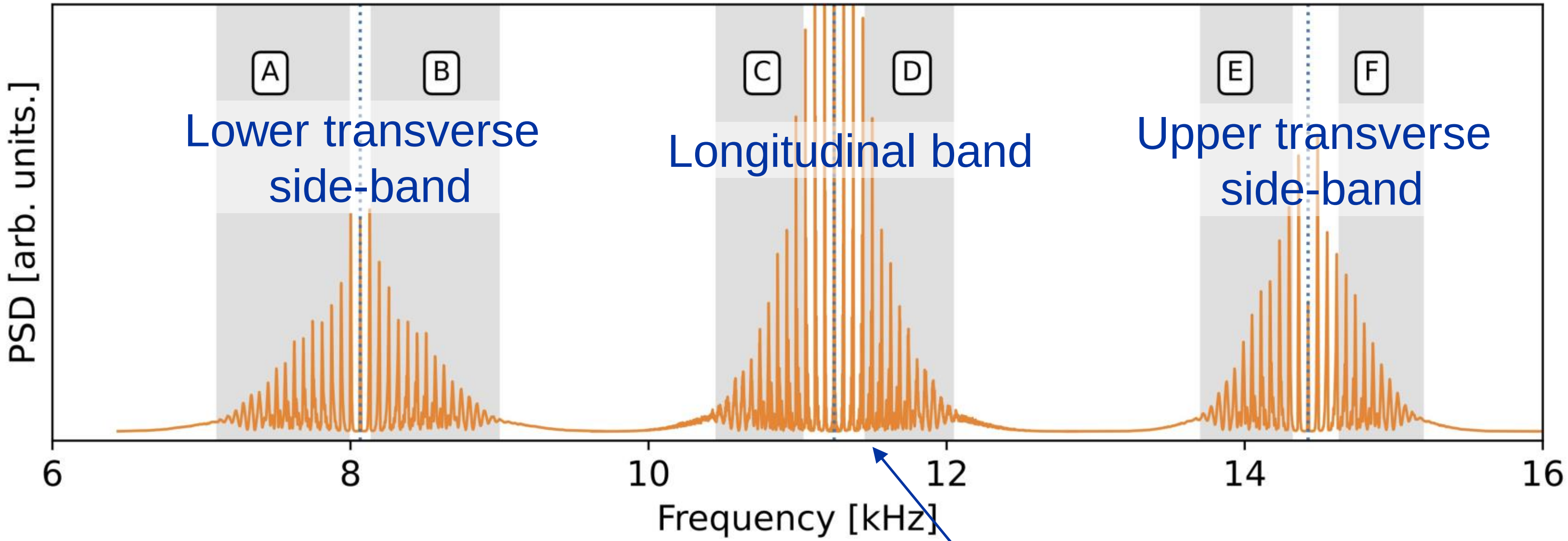
Frequency

Amplitude

Phase

$$d_i(t) = q f_0 \frac{\widehat{x}}{2} \sum_{n,p=-\infty}^{\infty} J_p(X_{i,n}^{\pm}) e^{j(t[(n \pm Q)\omega_0 + p\Omega_{s_i}] \pm \varphi_{\beta_i} + p\varphi_{s_i})}$$

With $X_{i,n}^{\pm} = \left(n\widehat{\tau}_i \pm \frac{\widehat{Q}_i}{\Omega_{s_i}} \right) \omega_0$

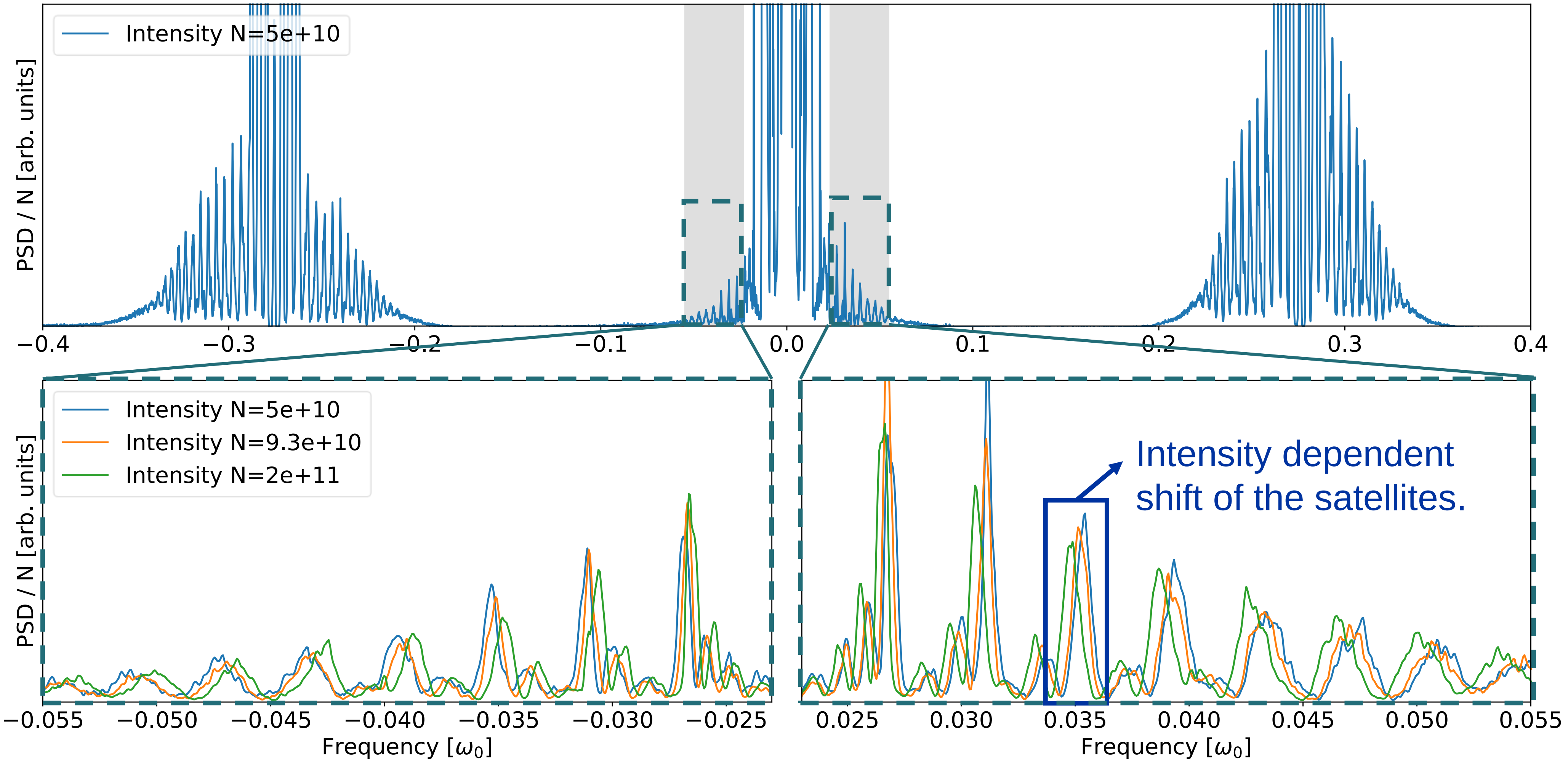


$$i_i(t) = q f_0 \sum_{n,p=-\infty}^{\infty} J_p(n\omega_0 \widehat{\tau}_i)\, e^{j\left[(n\omega_0 + p\Omega_{s_i})t + p\varphi_{s_i}\right]}$$

Experimental LHC spectra: Longitudinal Impedance Effects

- Longitudinal **impedance** significantly affects proton Schottky spectra.
- Spectra of bunches of different intensities (0.5e11 to 2e11 ppb) acquired at injection during MD block 1.

Schottky spectra from MD 11723



Longitudinal impedance

↓

Synchrotron tune shift

↓

Shift of Schottky satellites

Longitudinal Impedance Effects on LHC Schottky spectra

- Impedance can be source of instabilities and can cause intensity limitation.
  ➔ Important to have a good knowledge of the machine impedance.

- Current LHC **longitudinal impedance model** is a **byproduct of the transverse impedance model.**
  N. Mounet, PhD thesis, *The LHC Transverse Coupled-Bunch Instability*

- Could be inaccurate. Main source of longitudinal impedance not necessarily the same as for transverse impedance.

- Model re-evaluation is in progress (RF team, Michail Zampetakis, Ivan Karpov).

Can we extract information about the machine (longitudinal) impedance from the shift of the Schottky satellites?

1. Understand how impedance affects the dynamic of the particles (amplitude dependent tune shift).
2. How this new dynamic will be reflected in the Schottky spectrum.

Longitudinal Impedance Effects on LHC Schottky spectra

Without impedance

Longitudinal equation of motion.

$$\frac{d^2\phi}{dt^2} + \Omega_0^2 \ \sin\phi = 0$$

From the theory of the non-linear pendulum.

$$\Omega_s(\widehat{\phi}) = \frac{\pi}{2\mathcal{K}\left[\sin\left(\frac{\widehat{\phi}}{2}\right)\right]}\Omega_0$$

K: Elliptic integral of the first kind.

With impedance

Longitudinal equation of motion including forces coming from impedance:

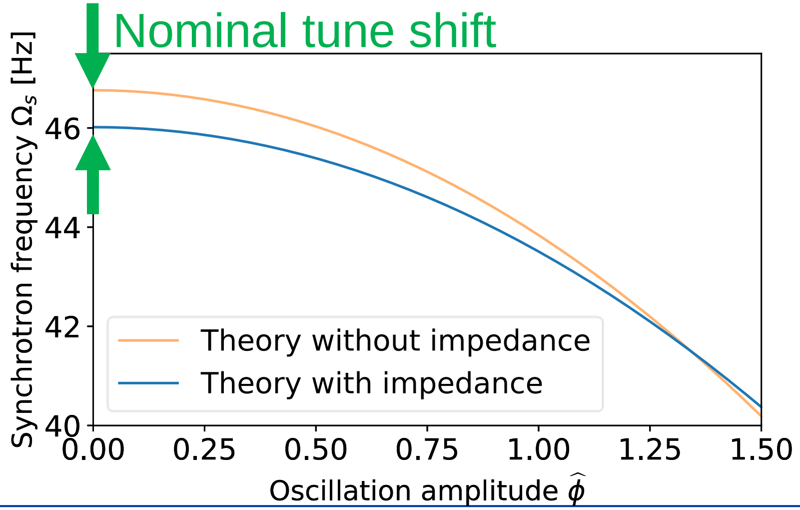
$$\ddot{\phi} + \Omega_0^2 \sin\phi = \frac{\eta h \omega_0}{p_0} F_{Imp}(t)$$

$$\Omega_s(\widehat{\phi}) = \Omega_0\sqrt{S_1}\left(1 + \frac{3S_3}{8S_1}\widehat{\phi}^2\right)$$

With some approximations, a **relation between synchrotron frequency and oscillation amplitude** can be derived.

Where the S_n coefficients account for the effect of impedance and are defined from the bunch spectrum $\widehat{\lambda(\omega)}$ and the impedance function $Z(\omega)$.

Details in: C. Lannoy et al 2024, JINST 19 P03017



➔ Impedance is responsible of **an amplitude-dependent synchrotron tune shift**.

Longitudinal Impedance Effects on LHC Schottky spectra

- The new relation between synchrotron frequency and amplitude can be inserted in the original theoretical expression of the Schottky spectrum:

$$i_i(t) = q f_0 \sum_{n,p=-\infty}^{\infty} J_p(n\omega_0 \widehat{\tau}_i)\ e^{j\left[(n\omega_0 + p\Omega_{s_i})t + p\varphi_{s_i}\right]}$$

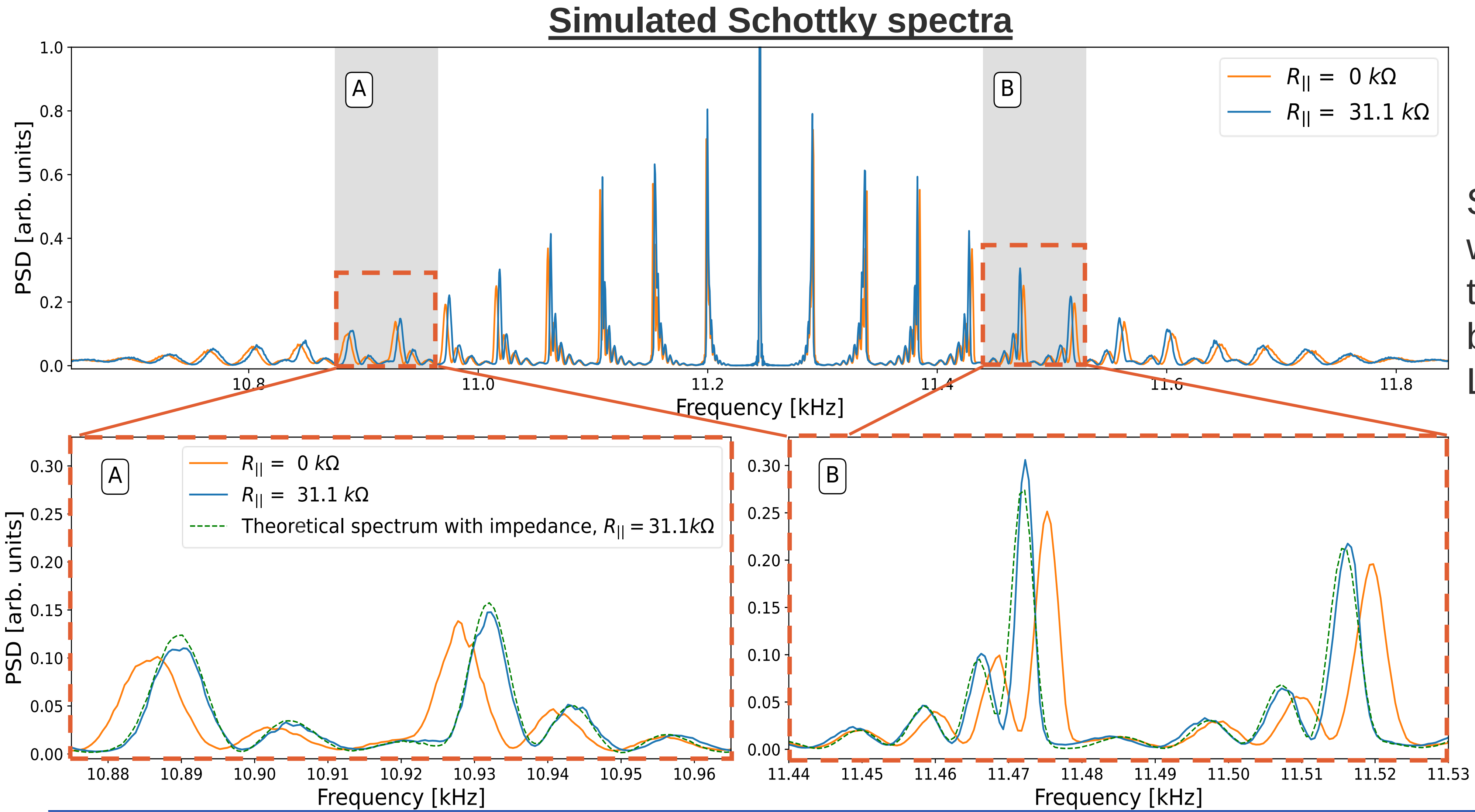
Synchrotron frequency with impedance

$$\Omega_s(\widehat{\tau}) = \Omega_0 \sqrt{S_1}\left(1 - \frac{3S_3}{8S_1}(h\omega_0\widehat{\tau})^2\right)$$

- This last expression allows to extend theoretical frameworks such as the Monte Carlo approach or the matrix formalism (K. Lasocha and D. Alves).

Longitudinal Impedance Effects on LHC Schottky spectra

Benchmark of the theory against macro-particle simulation (PyHEADTAIL).



Simulation conducted with a first estimate of the (longitudinal) broad-band part of the LHC impedance:

- $R_{||} = 31\ k\Omega$
- $f_r = 5\ GHz$
- $Q = 1$

$R_{||}$ equivalent to:
$\boldsymbol{Im(Z_{||})/n = 70\ m\Omega}$

Fitting of experimental LHC Schottky spectra

- Fitting of Schottky spectrum is not trivial as it **depends on many parameters**:

Longitudinal band

- RF voltage
- Long. bunch profile
- Long. impedance
- Intensity

Transverse bands

- **All longitudinal parameters**
- Betatron tune
- Chromaticity
- Transverse profile
- Transverse impedance
- Lattice non-linearities
- …

➔ The longitudinal band is easier to fit as it depends on less parameters.

Fitting of experimental LHC Schottky spectra

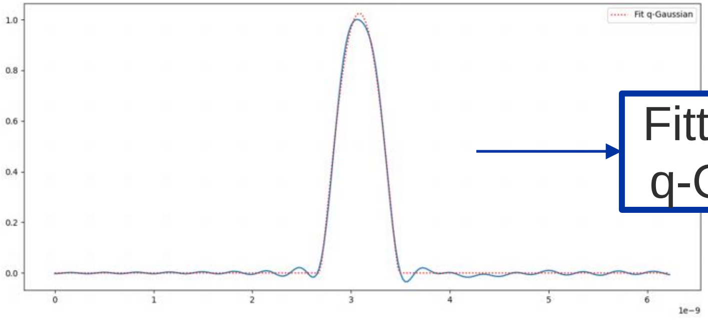
- Fitting of Schottky spectrum is not trivial as it **depends on many parameters**:

<u>Longitudinal band</u>

- RF voltage
- Long. bunch profile
- Long. impedance
- Intensity

RF voltage → Fitted once on low intensity bunch where impedance effect is low. Agree with measured RF voltage within 0.5%.

Long. bunch profile → Measured precisely with the WCM (if measurement available, else supposed Gaussian with length from BQM):



Fitted with a q-Gaussian → Analytical calculation →

- Bunch spectrum (impedance calculation)
- Synchr. amplitude distribution (Schottky theory)

Intensity → Measured with BCT.

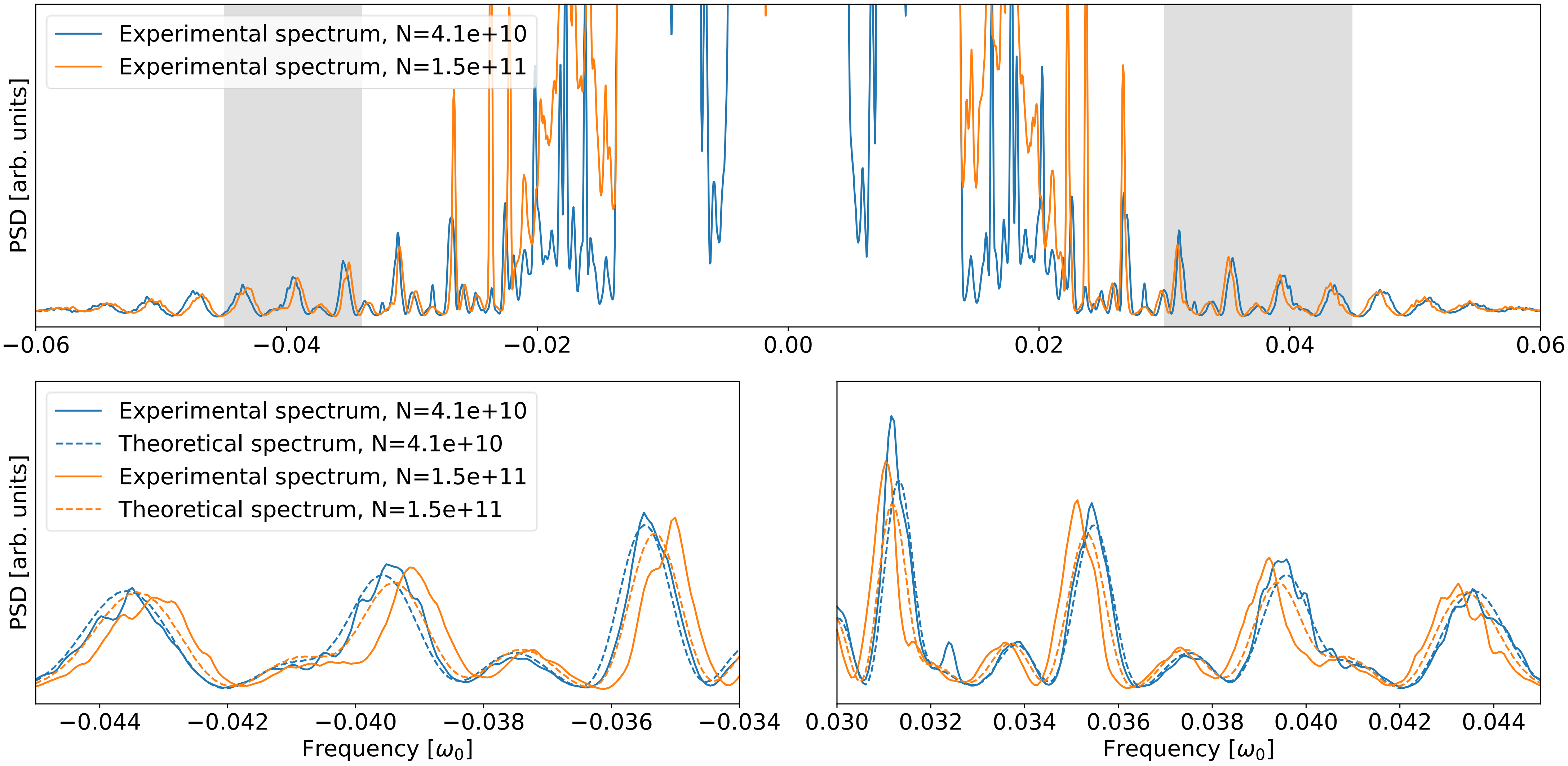
Long. impedance → Only variable left:

Impedance - Modeled as a broadband resonator of parameters:

- $Im(Z_{||})/n$: Scanned variable
- $f_r = 5\ GHz$
- $Q = 1$

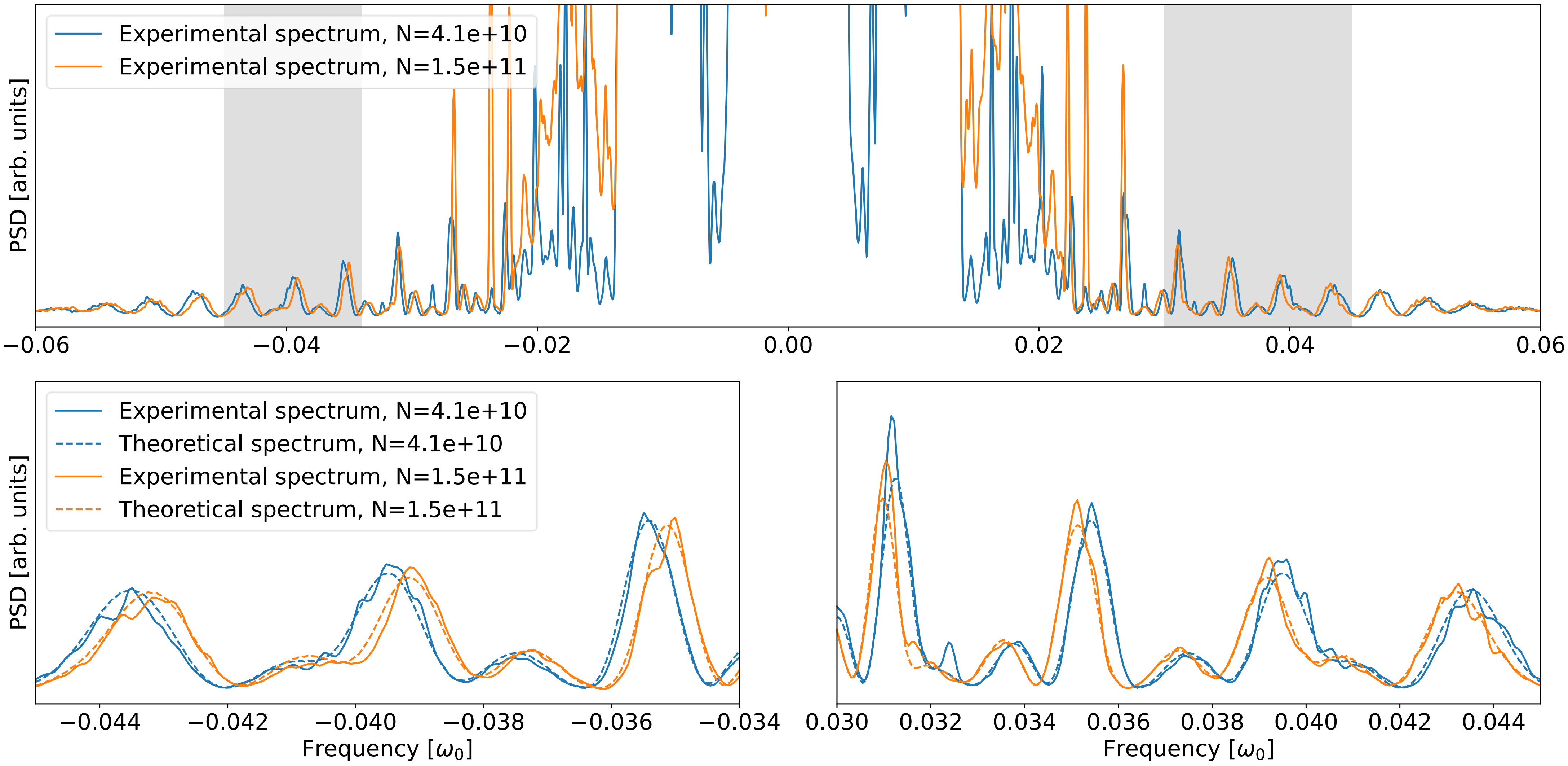
Fitting of experimental LHC Schottky spectra

Schottky spectra from MD 11723: Nominal Gaussian bunch, fit with $Im(Z_{||})/n = 70\ \mathrm{m\Omega}$

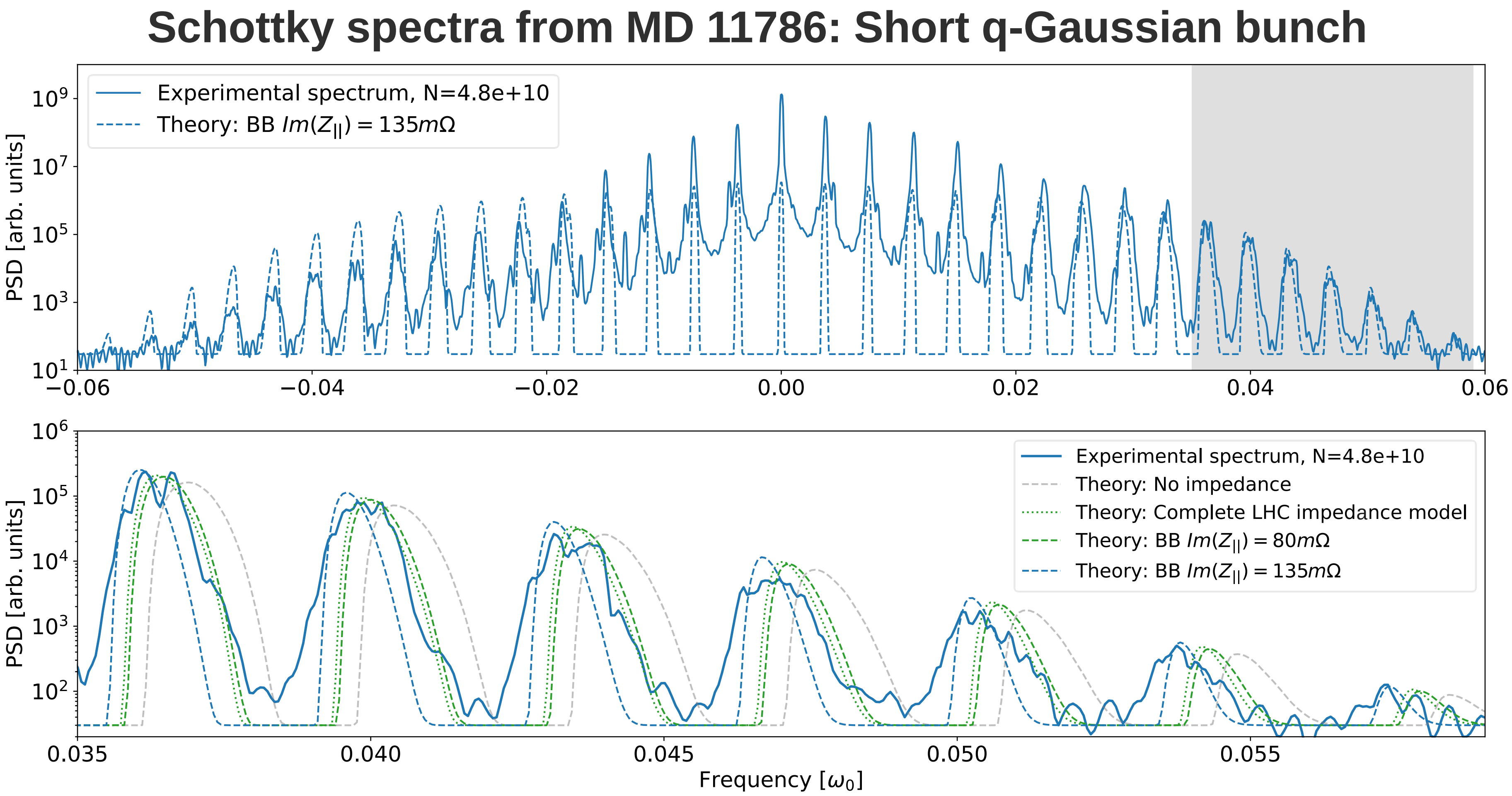


Fitting of experimental LHC Schottky spectra

Schottky spectra from MD 11723: Nominal Gaussian bunch, fit with $Im(Z_{||})/n = 135\ \mathrm{m\Omega}$



Fitting of experimental LHC Schottky spectra



Short q-Gaussian bunch:

- $\sigma_{rms} = 0.67\ ns$
- $q = 0.25$
- $N = 4.8e10$ ppb

Fitting less obvious ➔ Neither the full LHC impedance model nor a BB resonator can reproduce closely the measurement.

Better agreement might be obtained by fitting both **shunt impedance** and **cutoff frequency** (study ongoing).

Fitting of experimental LHC Schottky spectra

- Overall, best fitting obtained with: $\boldsymbol{Im(Z_{||})/n} = \mathbf{135\ m\Omega}$

- Increase of 70% compared with the current longitudinal impedance model: $\boldsymbol{Im(Z_{||})/n} = \mathbf{80\ m\Omega}$

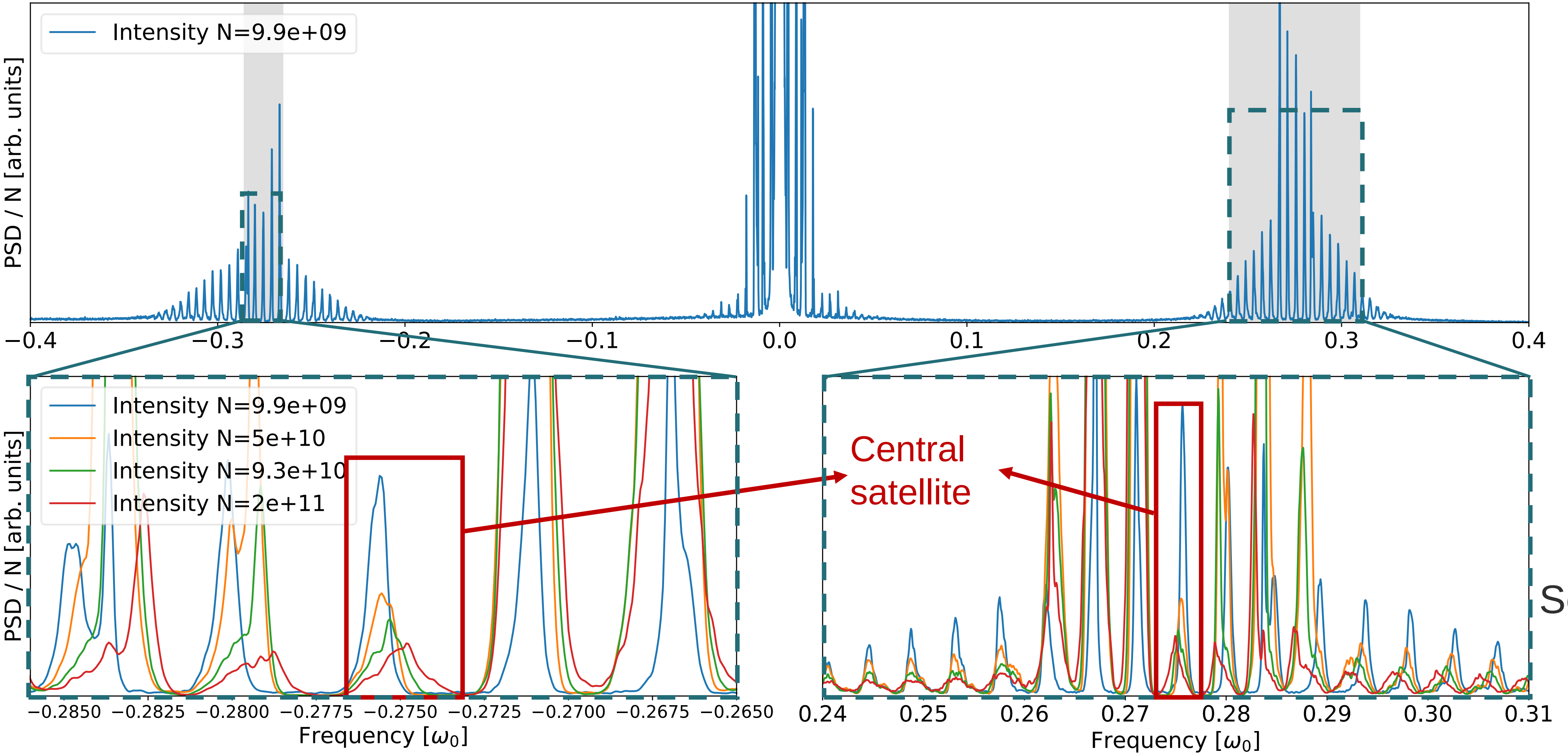
N. Mounet, PhD thesis, *The LHC Transverse Coupled-Bunch Instability*
I. Karpov and L. Giacomel IWG talks, https://indico.cern.ch/event/1422663/

- **Still preliminary result,** impact of the cut-off frequency of the broadband model to be analysed.

Experimental LHC spectra: Transverse impedance effects

- Transverse **impedance** significantly affects proton Schottky spectra.
- Spectra of bunches of different intensities (0.1e11 to 2e11 ppb) acquired at injection during MD block 1.

Schottky spectra from MD 11723



Transverse impedance

↓

Betatron tune shift and tune spread

↓

Shift and spreading of Schottky satellites

Conclusion

Summary of the talk:

- Longitudinal impedance.
  - Shift of synchrotron satellites observed experimentally.
  - Theory available allowing fitting of impedance.
  - First measurements seem to indicate an increased impedance w.r.t. current model.
  - Further studies needed to analyse the impact of cut-off frequency.

- Transverse impedance.
  - Tune shift observed in experimental spectra.
  - Theory available for quadrupolar impedance and still to be developed for dipolar impedance.

home.cern

Experimental LHC spectra: Transverse impedance effects

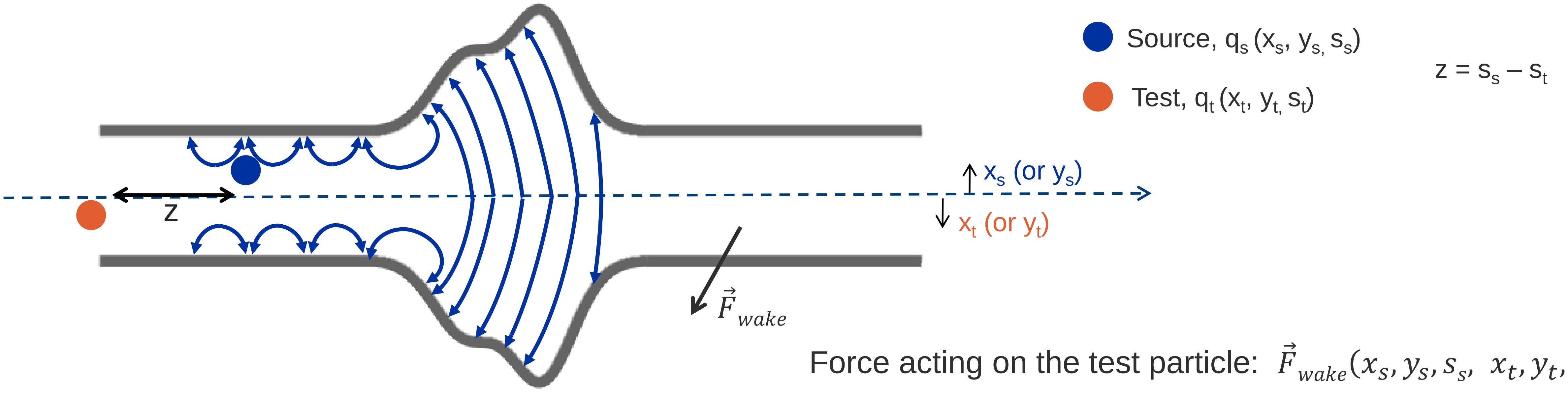
- Transverse **impedance** ➔ Shift and spreading of the central transverse satellites.

- Current development and understanding:

  - **Quadrupolar impedance** ➔ Incoherent tune shift.
    - Theory developed and validated with simulations for a transverse broad-band resonator.
    - Expanding theory to arbitrary impedance function.

  - **Dipolar impedance** ➔ Coherent tune shift.
    - Not clear how coherent tune shift is reflected in the Schottky spectrum, study ongoing.

Wake function and impedance



Force acting on the test particle: $\vec{F}_{wake}(x_s, y_s, s_s, \ x_t, y_t, s_t)$

Wake function: integrated force on the test particle.

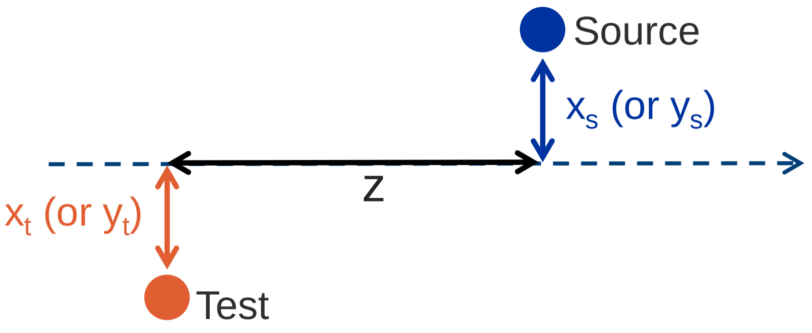
$$\int \vec{F}_{wake}(x_s, y_s, s_s, x_t, y_t, s_t)\,ds_s = -q_s q_t \vec{w}(x_s, x_t, y_s, y_t, z)$$

Impedance : Fourier transform of the wake function

$$Z_{\parallel}(\omega) = \frac{1}{\beta c}\int_{-\infty}^{\infty} W_{\parallel}(z)\, e^{\frac{-j\omega z}{\beta c}}\, dz$$

[1] Wakefields and Impedances, CAS 2022, K. Li

Transverse Impedance Effects



$$\int \vec{F}_{wake}(x_s, y_s, s_s, x_t, y_t, s_t)ds_s = -q_s q_t \vec{w}(x_s, x_t, y_s, y_t, z)$$

$$w_x(x_s, x_t, y_s, y_t, z) = \boxed{W_{C_x}(z)} + \boxed{x_s\, W_{D_x}(z)} + \boxed{x_t\, W_{Q_x}(z)} + O(y_s, y_t, x_s^2, x_t^2)$$

Zero order for asymmetric structures

Dipolar wakes – depends on **source particle**

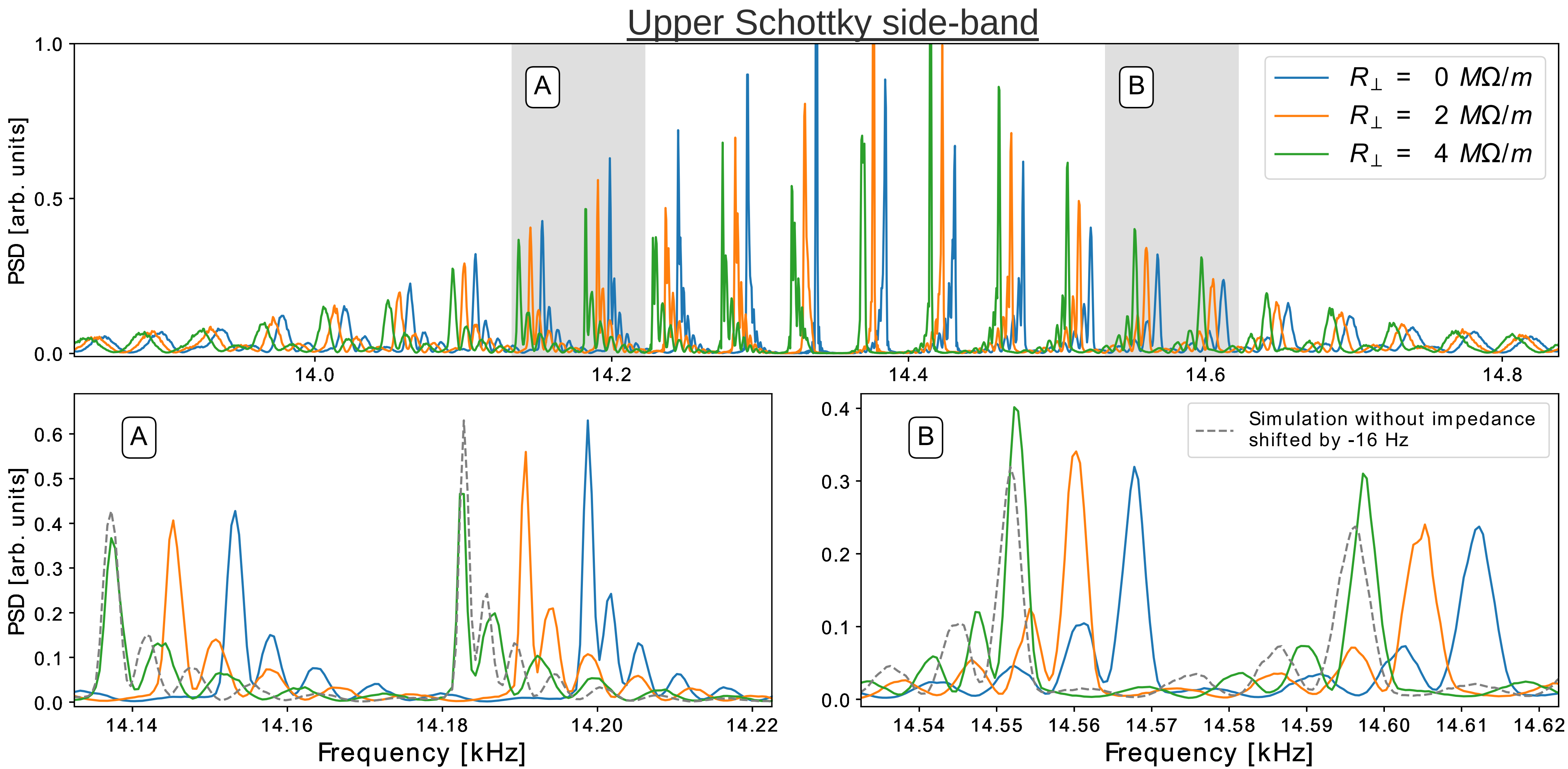
Quadrupolar wakes – depends on **test particle**

- For a stable beam where the bunch is centered and symmetric around the orbit, the dipolar wake contributions will cancel out.
  ➔ For similar value of impedances, we except the effects of the dipolar wake to be negligeable compared to the quadrupolar one (on the Schottky spectrum) .

- The quadrupolar wake will contribute to an additional linear focusing (or defocusing) force.

Transverse Impedance Effects (simulation)

We include a **transverse** broad-band resonator in the simulation (with **dipolar and quadrupolar** wakes).



➔ Overall shift of the spectrum

➔ Satellites are not simply shifted but their shape are also modified

Transverse Impedance Effects (theory vs simulation)

Benchmarking: theory and simulation