Isospin breaking in kaon multiplicities in heavy-ion collisions: status and consequences

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+NA61/SHINE

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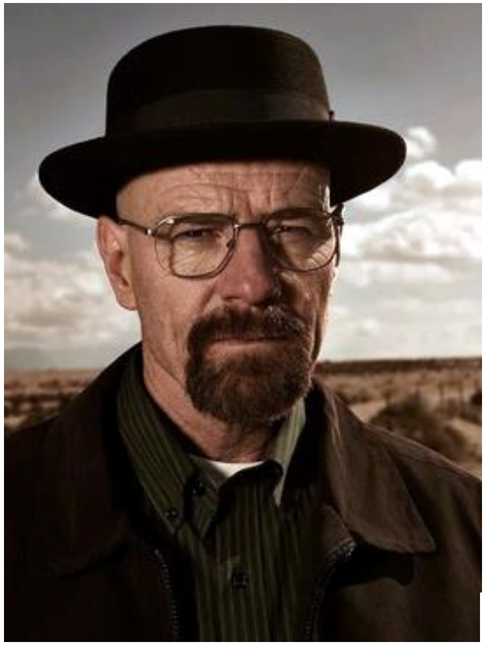
14-15/12/2024

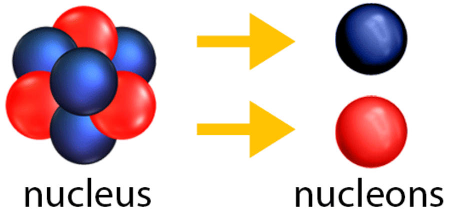
Outline

1. Isospin: brief recall
2. Kaon productions in heavy-ion collisions
3. Theory vs experiment
4. Consequences of a large isospin-breaking
5. Conclusions

Heisenberg (1932): the nucleon

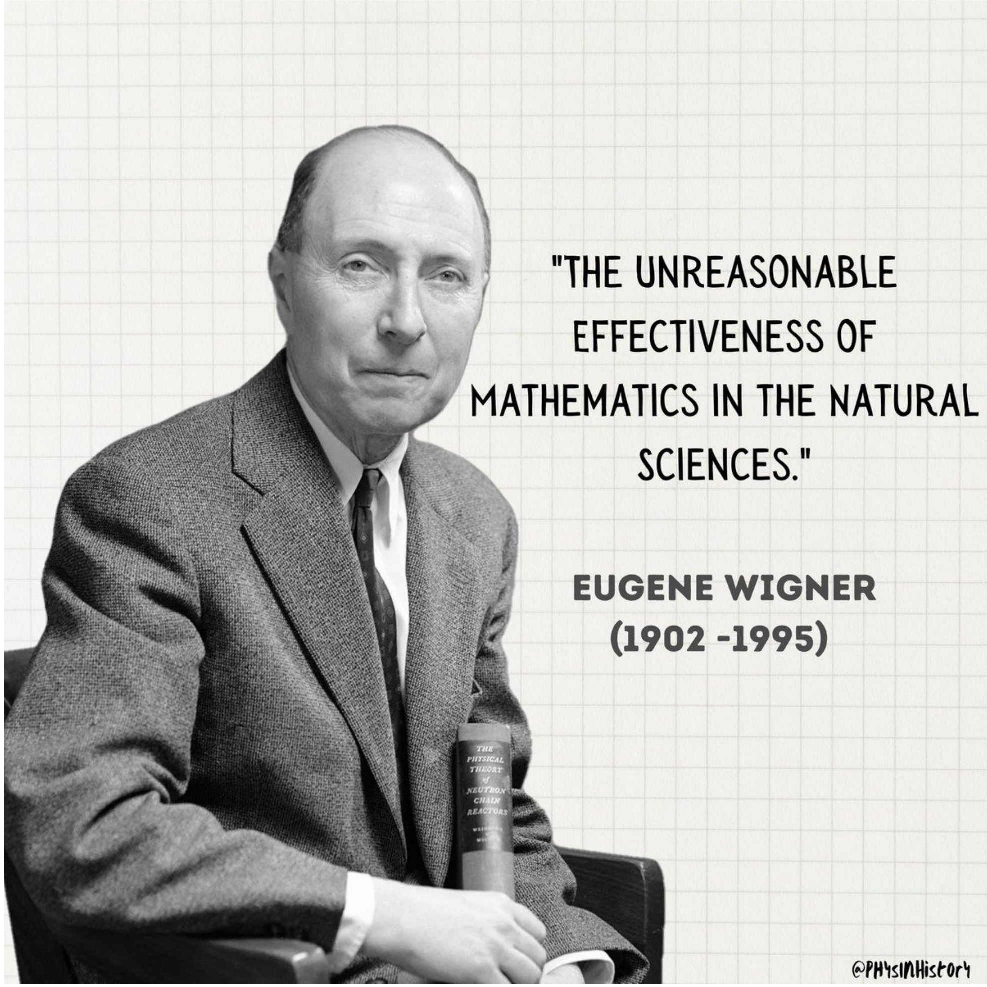


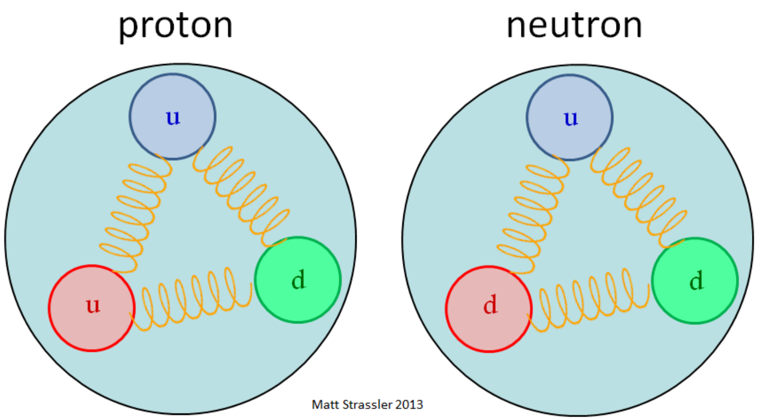
A nucleon is either a proton or a neutron as a component of an atomic nucleus

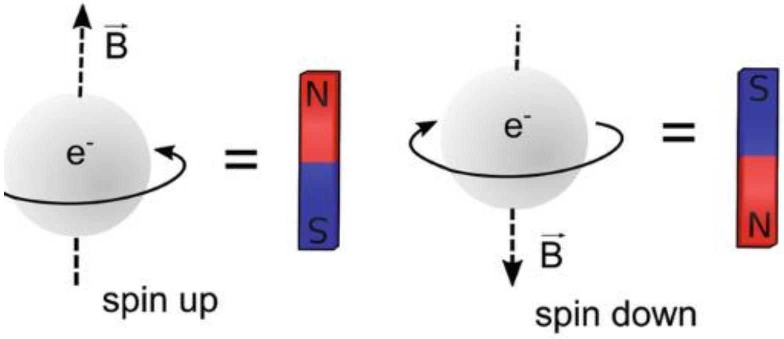


Proton and neutron merge into the nucleon
Masses very similar.

Wigner (1932): isotopic spin, thus isospin







Nucleon doublet: I=1/2

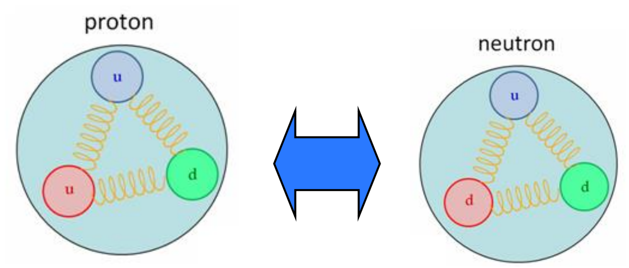
$$\begin{pmatrix} p \\ n \end{pmatrix} \rightarrow \hat{O} \begin{pmatrix} p \\ n \end{pmatrix}$$

$\hat{O}$ is a 2×2 unitary matrix. $\qquad \hat{O} = e^{i\theta_i \sigma_i/2}$

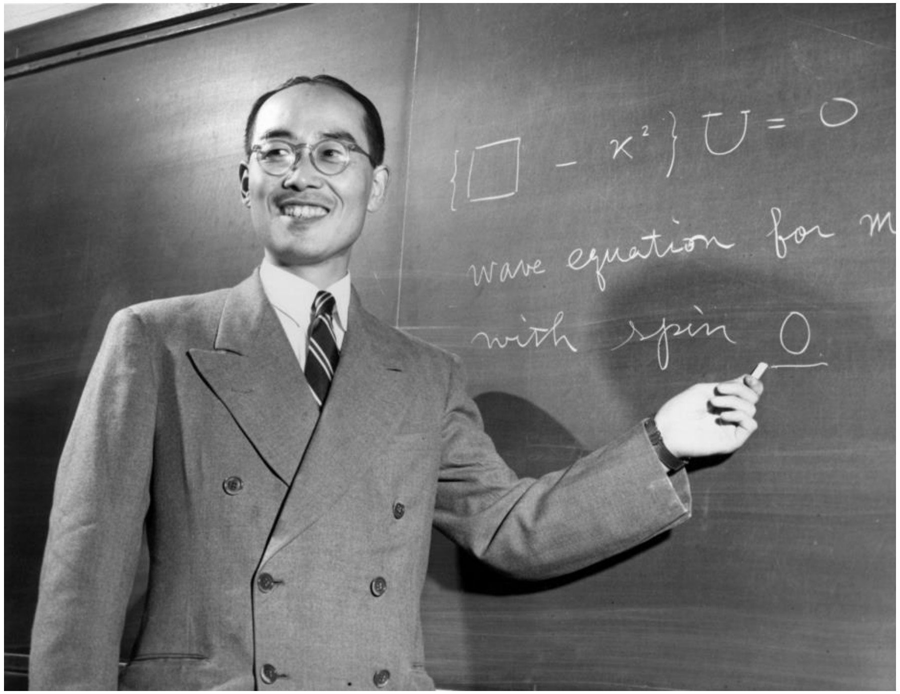
A specific isospin transformation is the so-called charge transformation:

$$\hat{C} = e^{i\pi\sigma_2/2} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Then under $\hat{C}$: $\qquad p \Longleftrightarrow n$



Yukawa (1932) and Kemmer (1939): isospin triplet I=1



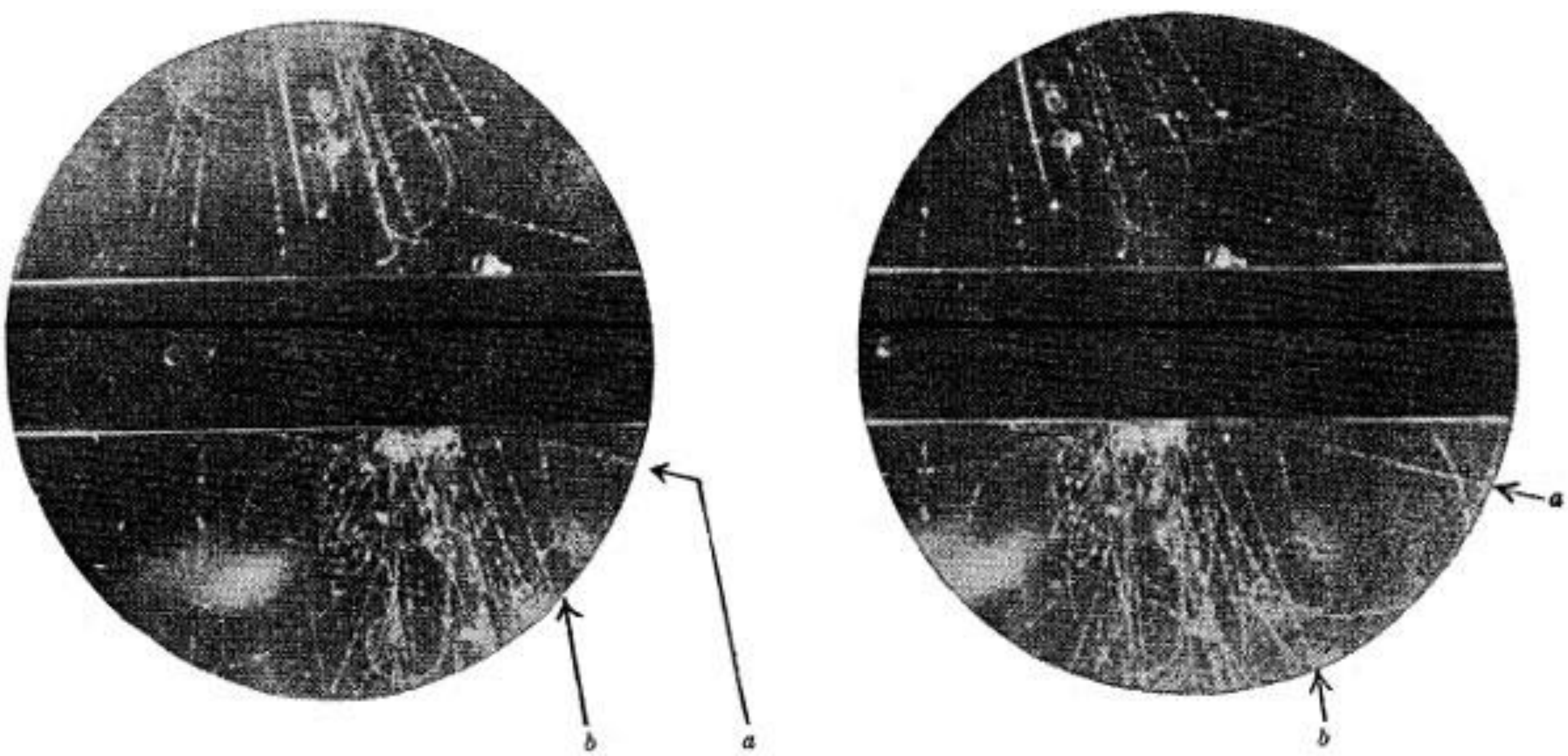
$$\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$$

under $\hat{C}$: $\pi^+ \Longleftrightarrow \pi^-$

Kaons

20 DECEMBER 1947

Clifford Butler and George Rochester discover the kaon; first strange particle



Kaons form isospin doublets, just as the nucleon

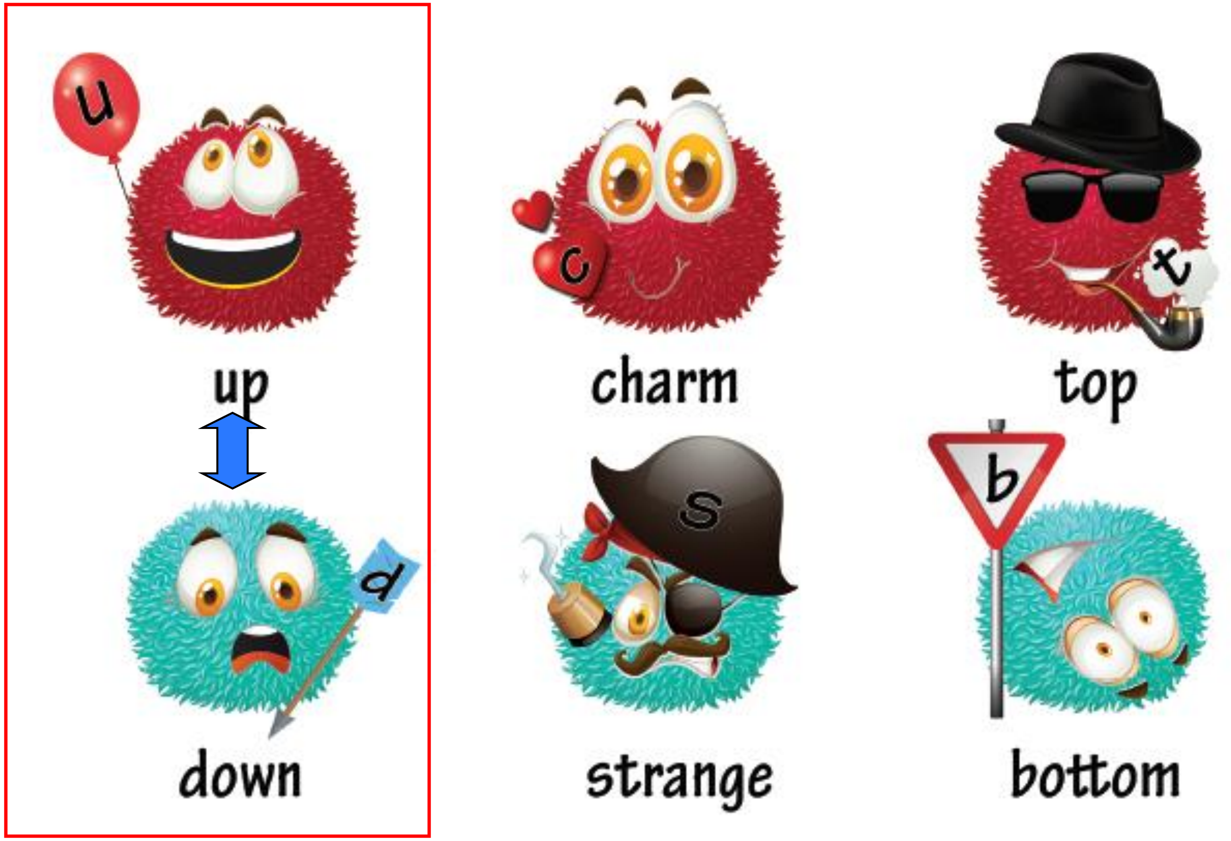
$$\begin{pmatrix} p \\ n \end{pmatrix} \quad \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \quad \begin{pmatrix} -\bar{K}^0 \\ K^- \end{pmatrix} \quad \cdots$$

under $\hat{C}$:

$$\begin{array}{ccc} p & \Longleftrightarrow & n \\ K^+ & \Longleftrightarrow & K^0 \\ \bar{K}^0 & \Longleftrightarrow & K^- \end{array}$$

Quarks and QCD

Quarks and QCD, isospin:



In terms of quarks: $\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow \hat{O} \begin{pmatrix} u \\ d \end{pmatrix}$

Then under $\hat{C}$: $u \Longleftrightarrow d$

Isospin is an approximate symmetry of QCD

- Mesonic multiplets (nucleon doublet, pion triplet, kaon doublets).

- Reactions: if an initial state has a certain (I, I_z), then the final state is also such. Indeed, pion-pion, pion-nucleon and nucleon-nulceon scattering conserve isospin (to a good level of accuracy).

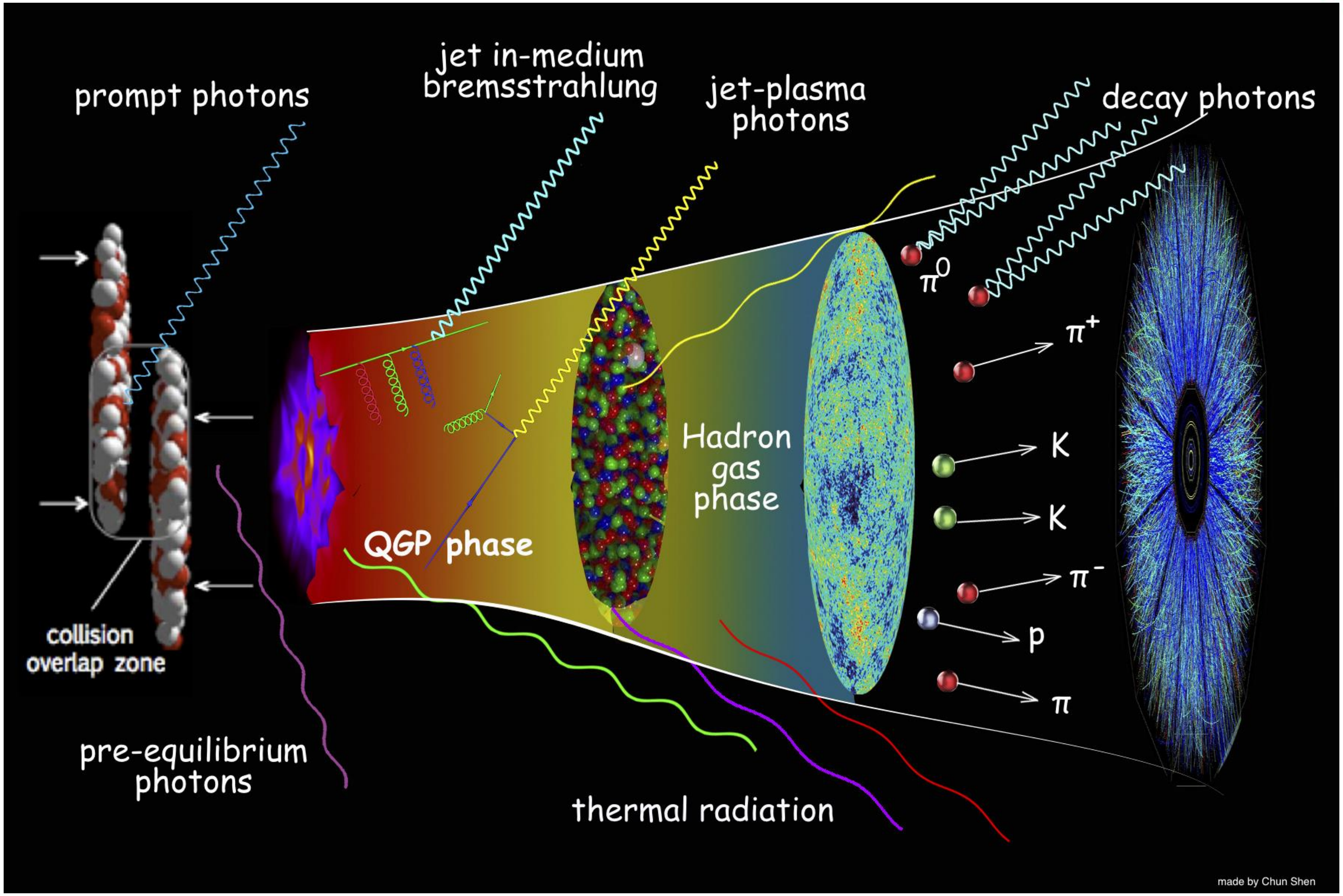
Example: ($I=I_z=1$)

$$p + p \to \Lambda + K^+ + p$$

- Isospin symmetry is good, but not exact. Masses of u and d not equal (explicit symmetry breaking).

- Isospin transformations are a subset of flavor transformations.

Heavy-ion collisions



C. Shen, U. Heinz, Nucl. Phys. News 25 (2015) 2, 6-11

At the freeze-out, the emission of hadrons is well described by e.g. thermal models.

- Kaon production: unexpected large violation of isospin in charged to neutral kaon ratio

- Adhikary et al. [NA61/SHINE], Excess of Charged Over Neutral K Meson Production in High-Energy Collisions of Atomic Nuclei, [arXiv:2312.06572 [nucl-ex]]

  (new vs4 from October 2024 with both experiment and theory)

- …as well as to a compilation of other experiments

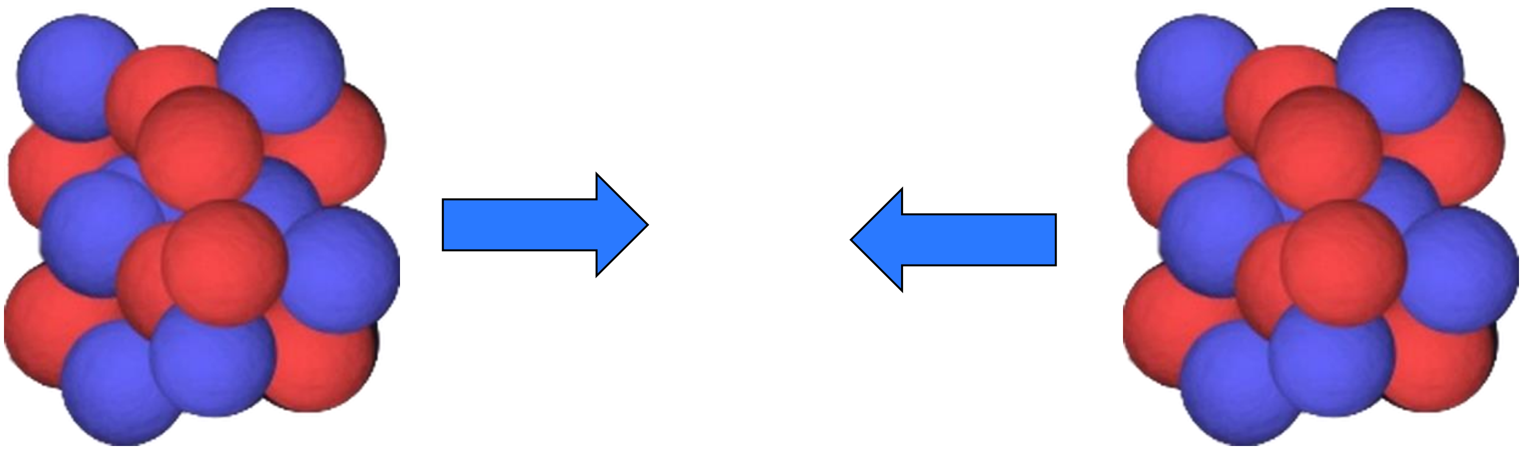
- Previous theoretical considerations:

  Brylinski et al., Large isospin symmetry breaking in kaon production at high energies," [arXiv:2312.07176 [nucl-th]].

Nucleus-nucleus collion with equal numbers of protons and neutrons

$$Z = N = A/2 \qquad\qquad Q/B = 1/2$$

$$|A + A\rangle$$



Oxygen-16 **Oxygen-16**

$I_z = 0$ (typically also I =0 for each nucleus, thus total isospin also vanishing)

Toward the general initial state

- For total initial I = 0 it is easy to show that $\langle K^+ \rangle = \langle K^0 \rangle$
- The result can be easily extended to any **fixed** total initial isospin $I=I_0$.
- It can be even generalized to initial states that are not isospin eigenstates, provided that an appropriate average is performed.

Expected kaon multiplicities

Charge symmetry means that strong interactions are invariant under the inversion of the third component of the isospin of hadron of the initial and final states.

Then:

$$\langle K^+ \rangle = \langle K^0 \rangle$$

$$\langle K^- \rangle = \langle \bar{K}^0 \rangle$$

Neutral kaons and the ratio R$_K$

$$\begin{pmatrix} |K_S^0\rangle \\ |K_L^0\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix}$$

$$\langle K_S^0\rangle = \frac{1}{2}\langle K^0\rangle + \frac{1}{2}\langle \bar{K}^0\rangle = \langle K_L^0\rangle$$

$$\langle K^+\rangle + \langle K^-\rangle = 2\langle K_S^0\rangle$$

$$Q/B = 1/2$$

+ isospin exact…

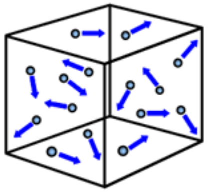
$$R_K \equiv \frac{\langle K^+\rangle + \langle K^-\rangle}{\langle K^0\rangle + \langle \bar{K}^0\rangle} = \frac{\langle K^+\rangle + \langle K^-\rangle}{2\langle K_S^0\rangle} = 1$$

Theoretical approaches

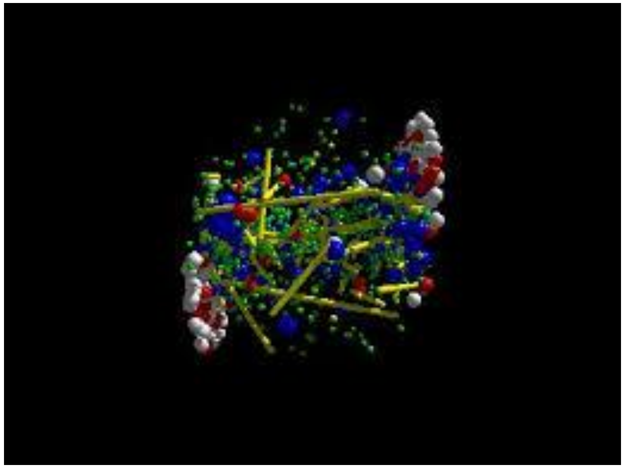
- HRG (hadron resonance gas approach)

$$\ln Z = \sum_k \ln Z_k^{\text{stable}} + \sum_k \ln Z_k^{\text{res}}$$

$$\ln Z_k^{\text{stable,}} = f_k V \int \frac{d^3p}{(2\pi)^3} \ln\left[1 \pm e^{-E_p/T}\right]^{\pm 1}$$

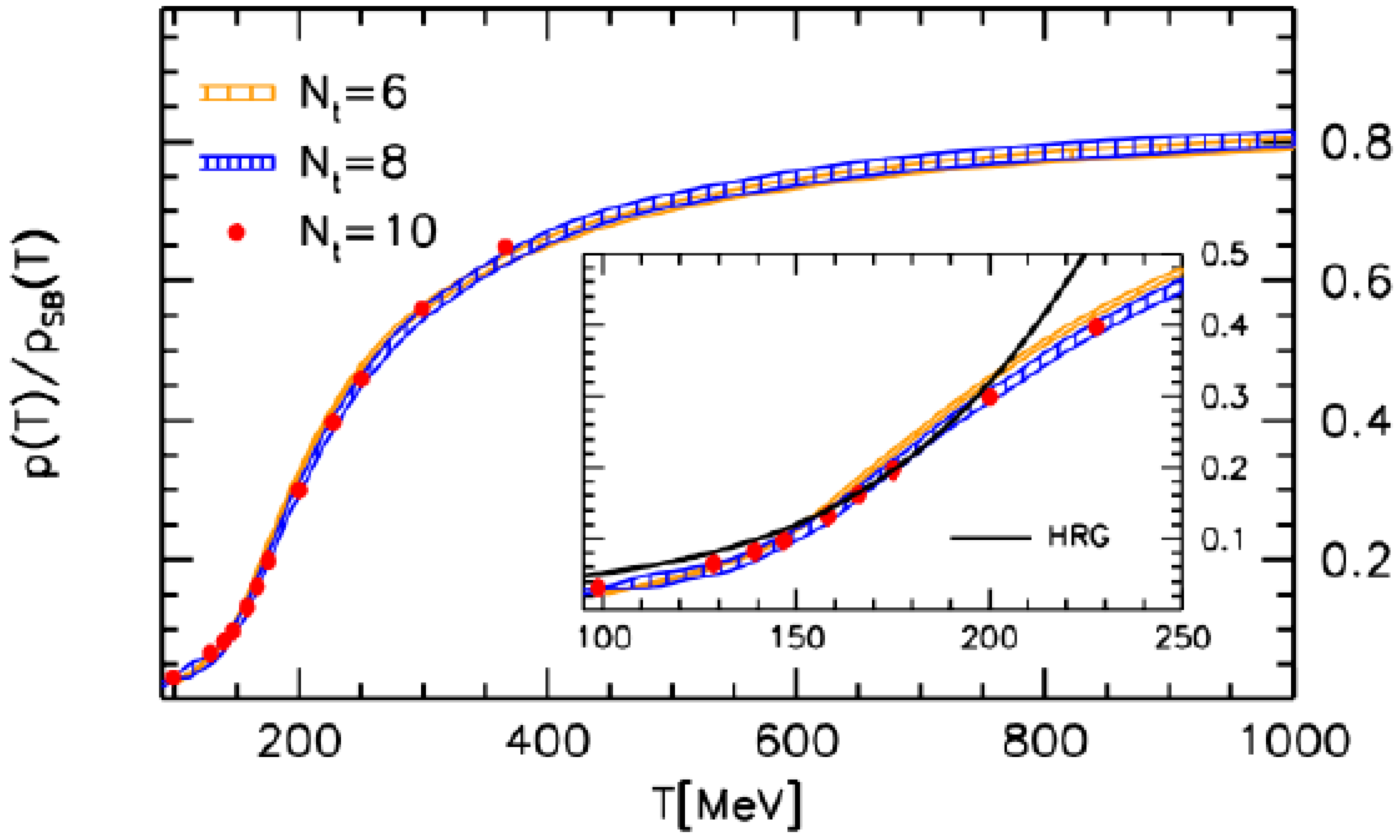


- UrQMD (Hadron-String transport model, fully integrated Monte Carlo simulation of nucleus-nucleus simulations)

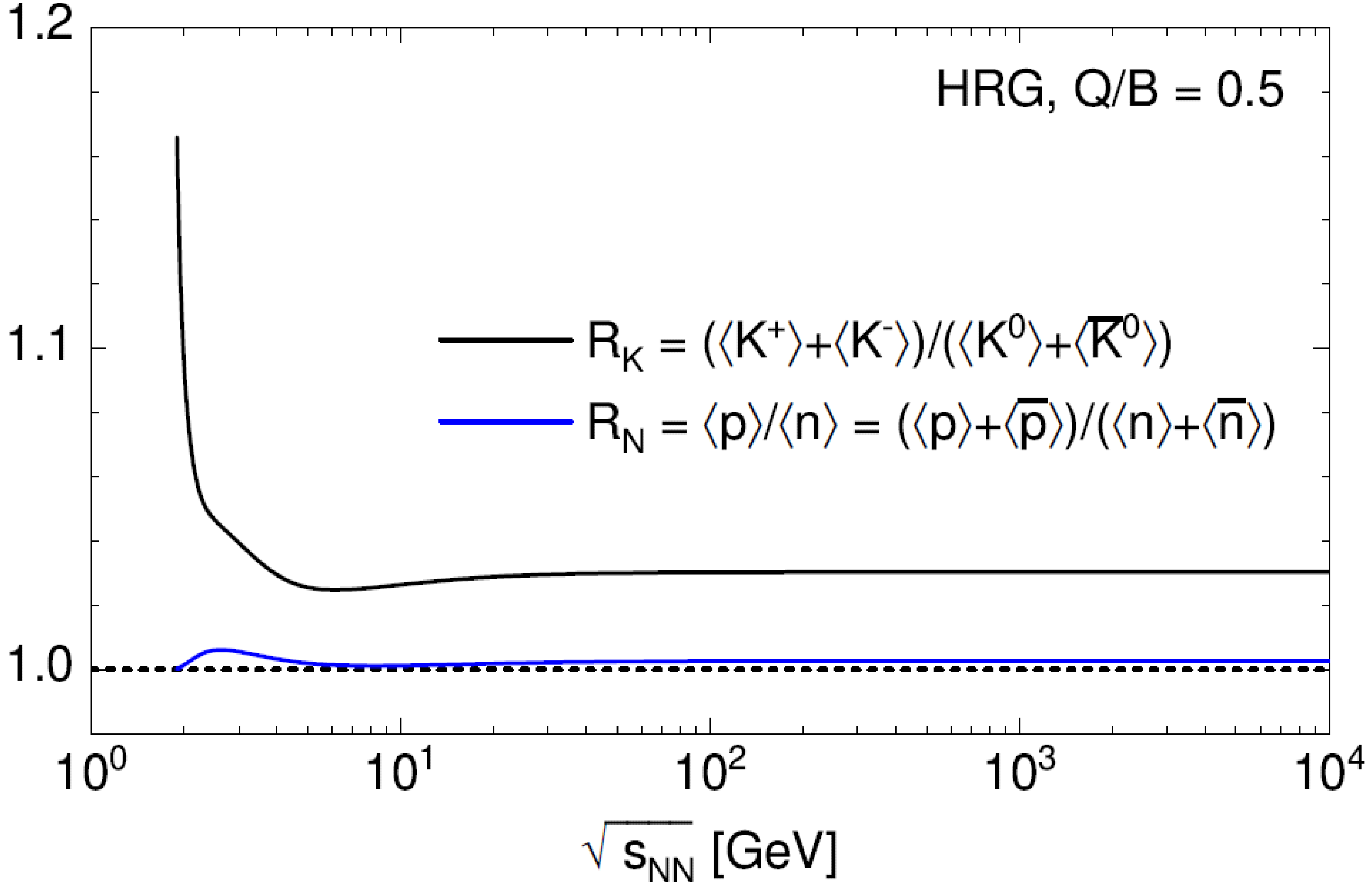


Hadron resonance gas vs lattice results

- All baryons and mesons (m < 2.5 GeV) from PDG [Borsnayi et al.JHEP11(2010)077]

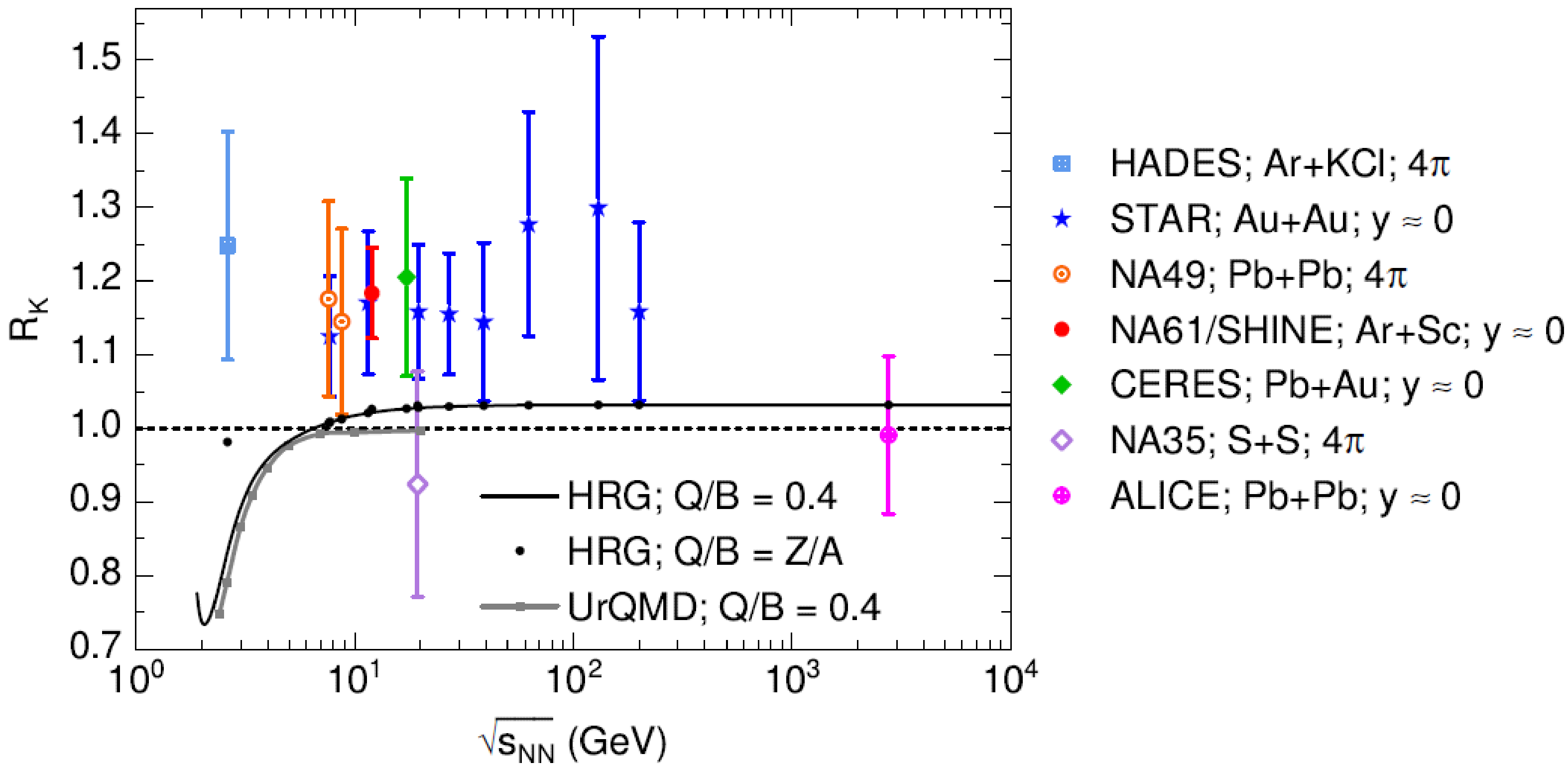


HRG for Q/B=1/2



If we enforce isospin symmetry to be exact, R_K = 1 for any energy.

Experimental results (NA61/SHINE plus others)



Latest NA61/SHINE result: $R_K = 1.184 \pm 0.061$

Note, however, most experiments have Q/B < 0.5

Experimental data: NA61/SHINE and previous exp

Experiment	Collision system	$\sqrt{s_{NN}}$ (GeV)	R_K	σ_{stat}	σ_{total}
NA61/SHINE	Ar+Sc	11.9	1.1839	0.0138	0.0615
HADES	Ar+KCl	2.6	1.2483	0.1027	0.1545
STAR (BES I)	Au+Au	7.7	1.1247	-	0.0819
STAR (BES I)	Au+Au	11.5	1.1707	-	0.0973
STAR (BES I)	Au+Au	19.6	1.1584	-	0.0910
STAR (BES I)	Au+Au	27	1.1553	-	0.0819
STAR (BES I)	Au+Au	39	1.1446	-	0.1079
NA49	Pb+Pb	7.6	1.1758	0.0198	0.1325
NA49	Pb+Pb	8.7	1.1447	0.0295	0.1263
CERES	Pb+Au	17.3	1.2052	0.0539	0.1340
NA35	S+S	19.4	0.9238	-	0.1533
STAR	Au+Au	62.4	1.2774	-	0.1525
STAR	Au+Au	130	1.2994	-	0.2331
STAR	Au+Au	200	1.1586	-	0.1214
ALICE	Pb+Pb	2760	0.9909	-	0.1071

Considerations

- HRG and UrQMD agree with each other
- Q/B <1/2 favors neutral kaons
- charged kaons are lighter than neutral ones: this favors charged kaons

Considerations/2

- Non-QCD effects: weak processes are negligible
- Non-QCD effects: electromagnetic processes are small, of the order of α^2 . However, nonperturbative effects possible for soft charged kaons?
- Decays of ϕ(1020) meson generates quite small effects.
- Role of a0(980) and f0(980) is also small.

Experiment vs theory: ratio

$$1.129 \pm 0.027.$$

$$\chi^2_{min}/\mathrm{dof} \approx 0.3$$

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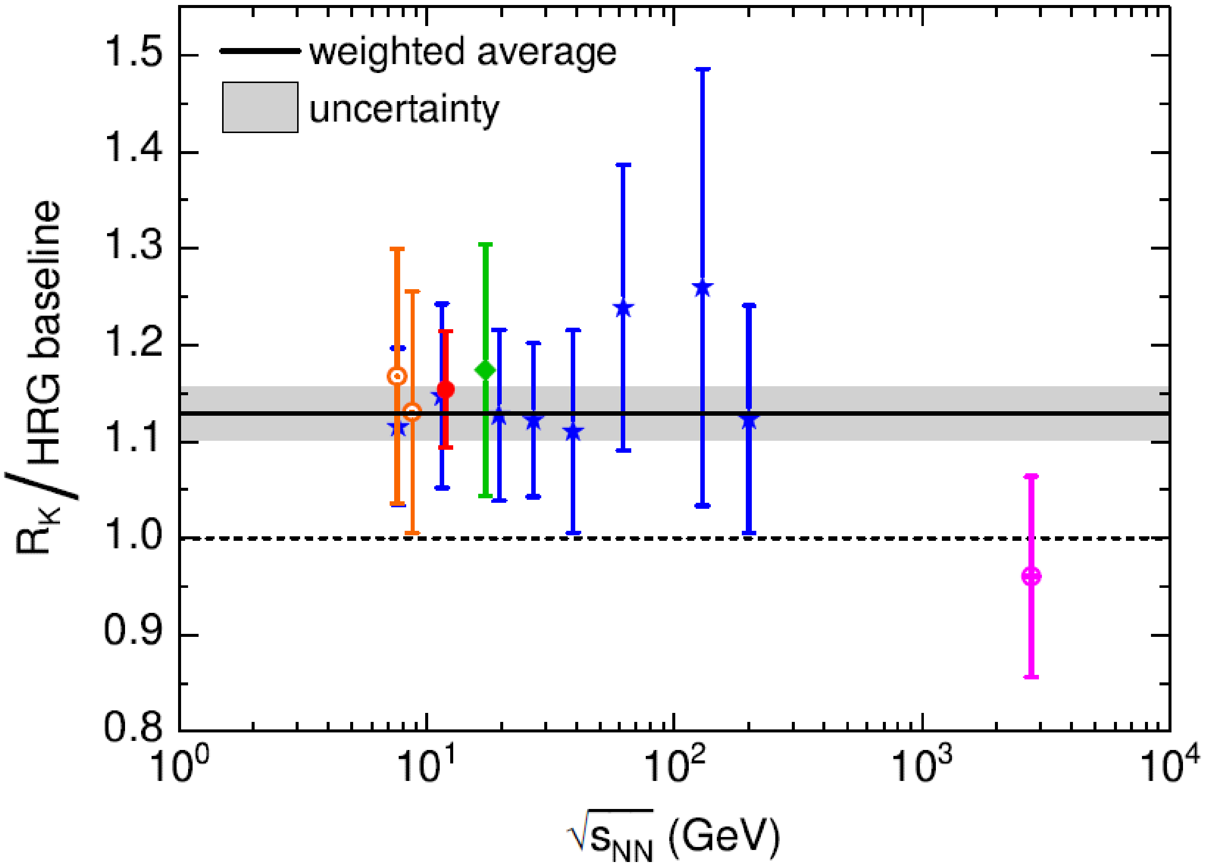

The exp/th missmatch is 4.7σ.

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More on the resonance ϕ(1020)

$\phi(1020)$ $I^G(J^{PC}) = 0^-(1^{--})$

ϕ(1020) MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
1019.461±0.016 OUR AVERAGE				

ϕ(1020) WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
4.249±0.013 OUR AVERAGE	Error includes scale factor of 1.1.			

ϕ(1020) DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	K^+K^-	(49.1 ±0.5) %	S=1.3
Γ_2	$K_L^0 K_S^0$	(33.9 ±0.4) %	S=1.2
Γ_3	$\rho\pi + \pi^+\pi^-\pi^0$	(15.4 ±0.4) %	S=1.2

$$\frac{\Gamma_{K^+K^-}}{\Gamma_{K^0\bar{K}^0}} = \frac{g^2_{K^+K^-}}{g^2_{K^0\bar{K}^0}} \frac{\left(\frac{m_\phi^2}{4} - m_{K^+}^2\right)^{3/2}}{\left(\frac{m_\phi^2}{4} - m_{K^0}^2\right)^{3/2}} = \frac{g^2_{K^+K^-}}{g^2_{K^0\bar{K}^0}} 1.52 \overset{\mathrm{PDG}}{=} 1.45 \pm 0.03$$

$$\frac{g_{K^+K^-}}{g_{K^0\bar{K}^0}} = 0.98 \pm 0.01$$

More on the resonance a0(980)

$a_0(980)$

$I^G(J^{PC}) = 1^-(0^{++})$

See the related review(s):
Scalar Mesons below 1 GeV

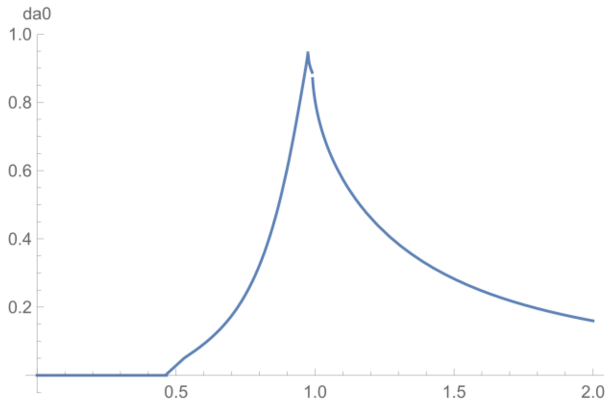
$a_0(980)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma = -2\,\text{Im}(\sqrt{s})$.

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
(970–1020) − i (30–70) OUR ESTIMATE	(see Fig. 64.2 in the review)		

$a_0(980)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\eta\pi$	seen
Γ_2	$K\overline{K}$	seen
Γ_3	$\eta'\pi$	seen



Using the PDG average $\bar{K}K/\pi\eta = 0.172 \pm 0.019$ amounts to the following $\bar{K}K$ overall branching ratio :

$$\frac{\Gamma_{\bar{K}K}}{\Gamma_{tot}} = \frac{\Gamma_{\bar{K}K}}{\Gamma_{\bar{K}K} + \Gamma_{\pi\eta} + \Gamma_{\pi\eta'}} \simeq \frac{\Gamma_{\bar{K}K}}{\Gamma_{\bar{K}K} + \Gamma_{\pi\eta}}$$
$$= \frac{1}{1 + \frac{\Gamma_{\pi\eta}}{\Gamma_{\bar{K}K}}} = \frac{1}{1 + \frac{1}{\Gamma_{\pi\eta}/\Gamma_{\bar{K}K}}} = 0.15 \pm 0.01\ .$$

HRG: at first, equal amount for charged and neutral kaons.
Including threshold effects, leads to the branching ratio $K^+K^-/K^0\bar{K}^0 \simeq 1.1$
No significant effect on RK

More on the resonances f0(980)

$f_0(980)$ $\qquad I^G(J^{PC}) = 0^+(0^{++})$

See the related review(s):
Scalar Mesons below 1 GeV

$f_0(980)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma = -2\ \mathrm{Im}(\sqrt{s})$.

VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
(980–1010) − *i* (20–35) OUR ESTIMATE (see Fig. 64.4 in the review)			

$f_0(980)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi$	seen
Γ_2	$K\overline{K}$	seen

$\Gamma(\pi\pi)/[\Gamma(\pi\pi)+\Gamma(K\overline{K})]$

VALUE	*EVTS*
• • • We do not use the followin	
0.52 ± 0.12	9.9k
$0.75^{+0.11}_{-0.13}$	
0.84 ± 0.02	
~ 0.68	
0.67 ± 0.09	
$0.81^{+0.09}_{-0.04}$	
0.78 ± 0.03	

The $\pi\pi$ mode dominates.
Similar consideration as for the a0(980) mesons.

Even including threshold effects,
no significant change of RK.

Toward a simple ‘quark counting’ model

- Provided the large-isospin symmetry is true, two questions can be asked: why and which are its consequences.
- ‘Why’ is, as usual, a difficult question. Can electromagnetic interaction enhance K+K-? We argued that this is not the case. But…
- What about a sum over many small effects? All phi-f0-a0 etc effects would lead to the measured results.
- Eventually a combination of both QED and many small contributions…

A simple 'quark counting' model

$$\alpha = N_u^{vac} = N_{\bar{u}}^{vac}$$
$$\beta = N_d^{vac} = N_{\bar{d}}^{vac}$$
$$\gamma = N_s^{vac} = N_{\bar{s}}^{vac}$$

$$r = \frac{\alpha}{\beta} \sim 1.2$$

Preliminary!!!

Ratio	large $\sqrt{s_{NN}}$ result
$R_K = \frac{K^+ + K^-}{K^0 + \bar{K}^0}$	$r \sim 1.2$
$\frac{p}{n}$	$r \sim 1.2$
$\frac{\pi^+}{\pi^0}$	$\frac{2r}{1+r^2} \sim 0.98$
$\frac{\Sigma^+}{\Sigma^0}$	$r \sim 1.2$
$\frac{\Sigma^+}{\Sigma^-}$	$r^2 \sim 1.4$

Inspired by **Joanna Stepaniak Damian Pszczel** Eur. Phys. J. C (2023) 83:928

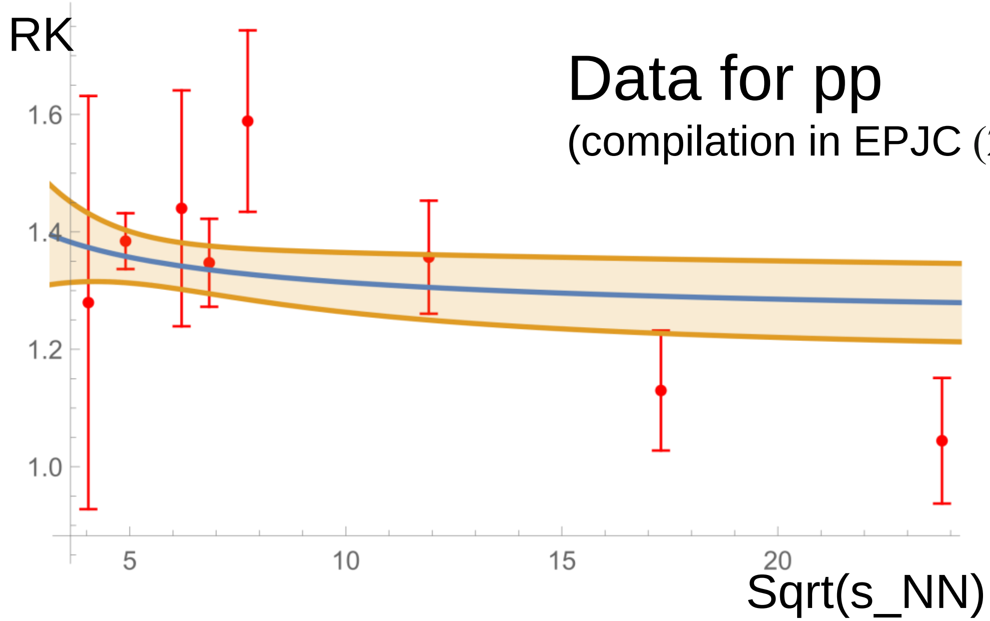
RK as function of Energy

$$R_K = \frac{N_u^{initial} + 2\alpha}{N_d^{initial} + 2\beta} \qquad r = \frac{\alpha}{\beta} \sim 1.2$$

$$R_K = \frac{1 + 2\lambda\left(\sqrt{s_{NN}}\right)^k}{\frac{2-Q/A}{1+Q/A} + \frac{2\lambda}{r}\left(\sqrt{s_{NN}}\right)^k}$$

Data for pp

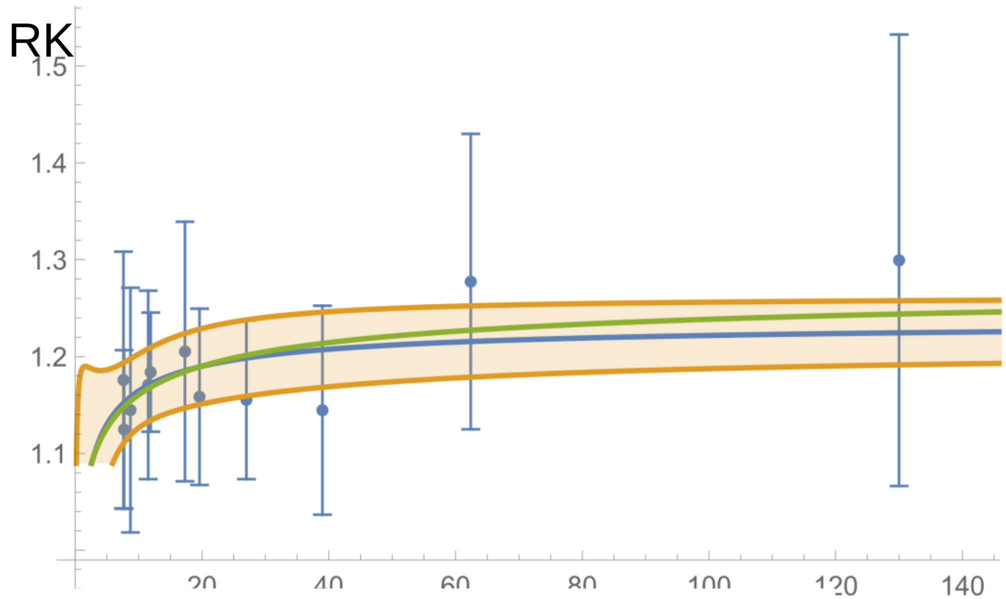
(compilation in EPJC (2023) 83:928)



Preliminary!!!

$$R_K = \frac{1 + 2\lambda\left(\sqrt{s_{NN}}\right)^k}{\frac{2-Q/A}{1+Q/A} + \frac{2\lambda}{r}\left(\sqrt{s_{NN}}\right)^k}$$

	Estimate	Standard Error
r	1.23788	0.0362554
λ	0.510271	0.84899
k	0.73791	0.811282



Sqrt(s_NN)

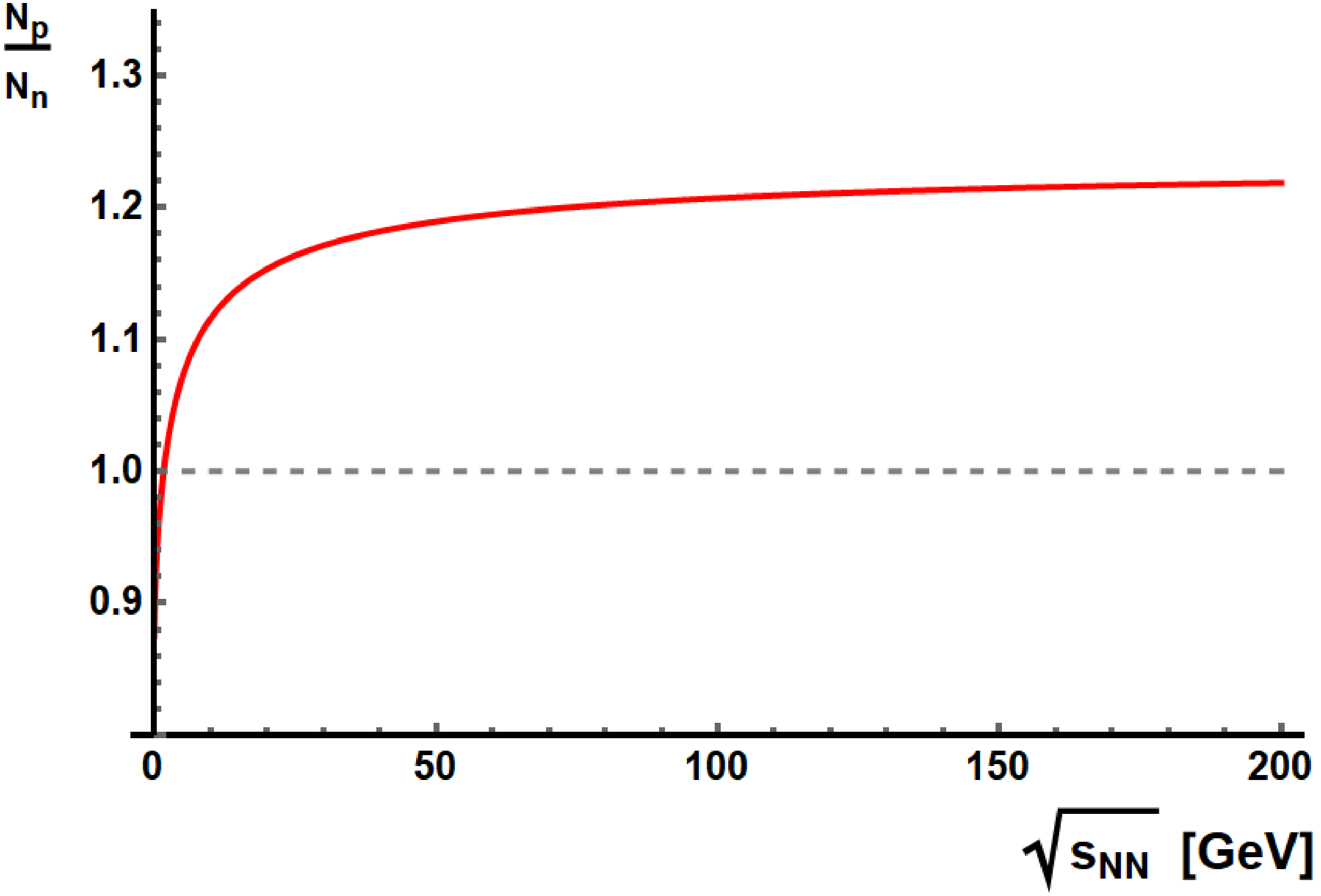
For k=0.5

	Estimate	Standard Error
r	1.24526	0.0290553
λ	0.855653	0.337169

Data for RK

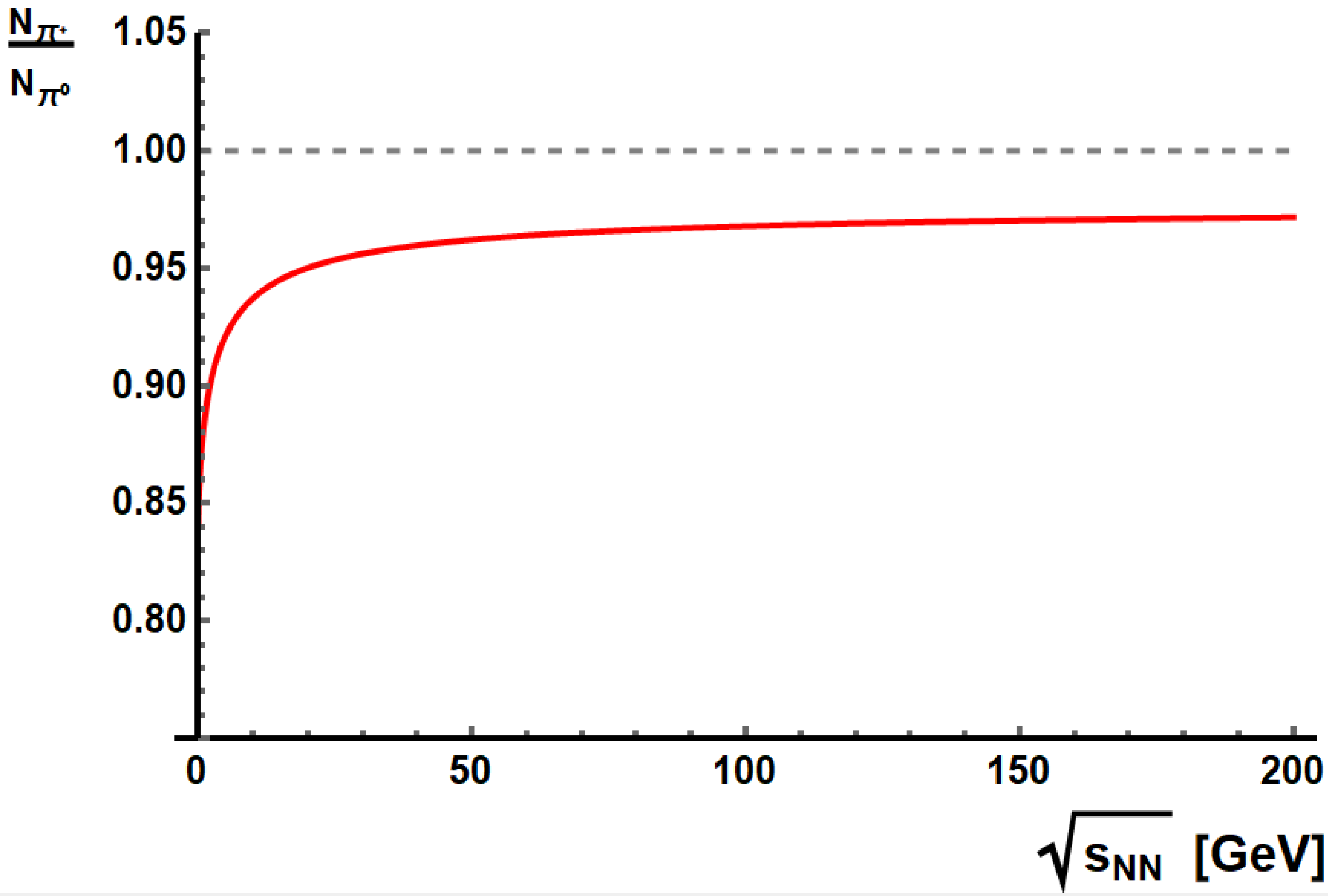
(compilation in NA61/SHINE-paper)

p/n, Q/B = 0.4



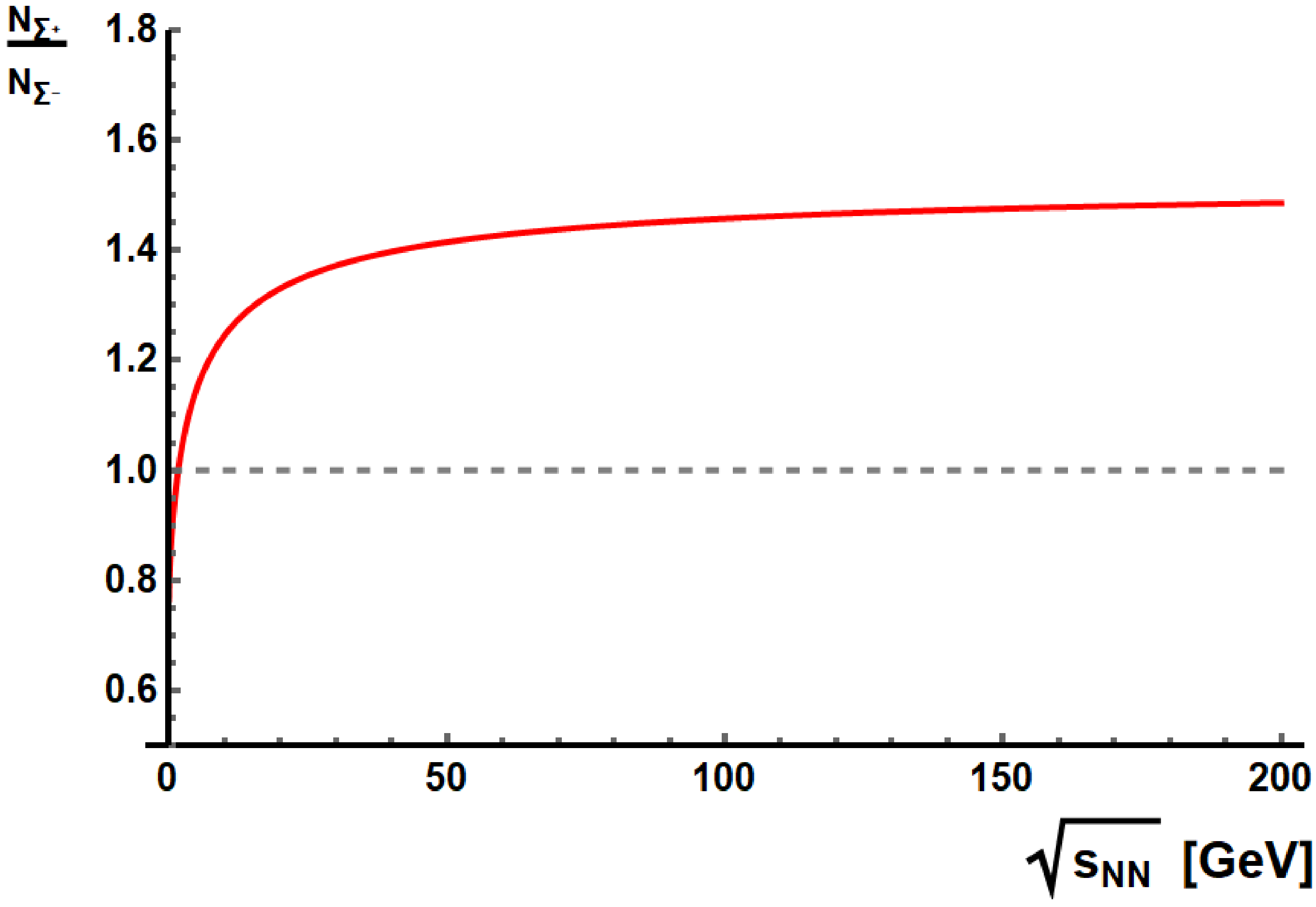
Preliminary!!!

$\pi+/\pi0$, Q/B =0.4



$\frac{N_{\pi^+}}{N_{\pi^0}}$
1.05
1.00
0.95
0.90
0.85
0.80
0
50
100
150
200
$\sqrt{s_{NN}}$ [GeV]

Σ+/Σ-, Q/B=0.4



$\frac{N_{\Sigma^+}}{N_{\Sigma^-}}$
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0
50
100
150
200
$\sqrt{s_{NN}}$ [GeV]

Pion-Carbon

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$$R_K = \frac{N_u^{initial} + N_{\bar{u}}^{initial} + 2\alpha}{N_d^{initial} + N_{\bar{d}}^{initial} + 2\beta}$$

For $\pi^+ C$ and $\pi^- C$ we have:

$$R_K^{\pi^+ C} = \frac{19 + 2\alpha}{19 + 2\beta} = R_K^{\pi^- C}$$

and in the isospin-symmetric limit

$$R_K^{\pi^+ C} = R_K^{\pi^- C} = 1$$

Summary and conclusions

- Theory (HRG,UrQMD) cannot explain experiment
- Scattering of nuclei with Z=N=A/2 highly desired…
- Easier but equally good? Average over: $\pi^- + C$ and $\pi^+ + C$

NA61/SHINE
PRD 107 (2003) 062004

- Study other isospin multiplets
- Non-perturbative effects? Chiral anomaly, QED, …

Thanks!

Quarks and QCD, flavor symmetry:

Flavor transformation is a rotation in the (u,d,s) space.
Isospin is a subgroup of flavor.

Example of isospin breaking/1

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-EP/84-27

March 8th, 1984

THE ISOSPIN-VIOLATING DECAY $\eta' \rightarrow 3\pi^0$

IHEP[1]-IISN[2]-LAPP[3] Collaboration

$$BR(\eta' \rightarrow 3\pi^0) = 5.2 \left(1 - \frac{m_u}{m_d}\right)^2 10^{-3}$$

Example of isospin breaking/2

$\phi(1020)$

$$I^G(J^{PC}) = 0^-(1^{--})$$

ϕ(1020) MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
1019.461±0.016 OUR AVERAGE				

ϕ(1020) DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	K^+K^-	(49.1 ± 0.5) %	S=1.3
Γ_2	$K^0_L K^0_S$	(33.9 ± 0.4) %	S=1.2

Example of isospin breaking/3

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Citation: R.L. Workman *et al.* (Particle Data Group), Prog.Theor.Exp.Phys. **2022**, 083C01 (2022) and 2023 update

$D^*(2007)^0$

$I(J^P) = \frac{1}{2}(1^-)$
I, J, P need confirmation.

J consistent with 1, value 0 ruled out (NGUYEN 77).

$D^*(2007)^0$ DECAY MODES

$\overline{D}^*(2007)^0$ modes are charge conjugates of modes below.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$D^0\pi^0$	(64.7 ± 0.9) %
Γ_2	$D^0\gamma$	(35.3 ± 0.9) %
Γ_3	$D^0 e^+ e^-$	$(3.91 \pm 0.33) \times 10^{-3}$

Citation: R.L. Workman *et al.* (Particle Data Group), Prog.Theor.Exp.Phys. **2022**, 083C01 (2022) and 2023 update

$D^*(2010)^\pm$

$I(J^P) = \frac{1}{2}(1^-)$
I, J, P need confirmation.

$D^*(2010)^\pm$ DECAY MODES

$D^*(2010)^-$ modes are charge conjugates of the modes below.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$D^0\pi^+$	(67.7 ± 0.5) %
Γ_2	$D^+\pi^0$	(30.7 ± 0.5) %
Γ_3	$D^+\gamma$	(1.6 ± 0.4) %

Historical recall: „Shmushkevich” rule

An initial ‘uniform’ ensemble of hadronic state (that is, one with an equal mean number of each member of any isospin multiplet, such as the scattering of two isosinglet nuclei) evolves into a uniform final-state ensemble.

Uniform stays uniform

Shmushkevich, I.: . Dokl. Akad. Nauk SSSR **103**, 235 (1955)

Dushin, N., Shmushkevich, I.: . Dokl. Akad. Nauk SSSR **106**, 801 (1956)

MacFarlane, A.J., Pinski, G., Sudarshan, G.: Shmushkevich's method for a charge independent theory. Phys. Rev. **140**, 1045 (1965) https://doi.org/10.1103/PhysRev.140.B1045

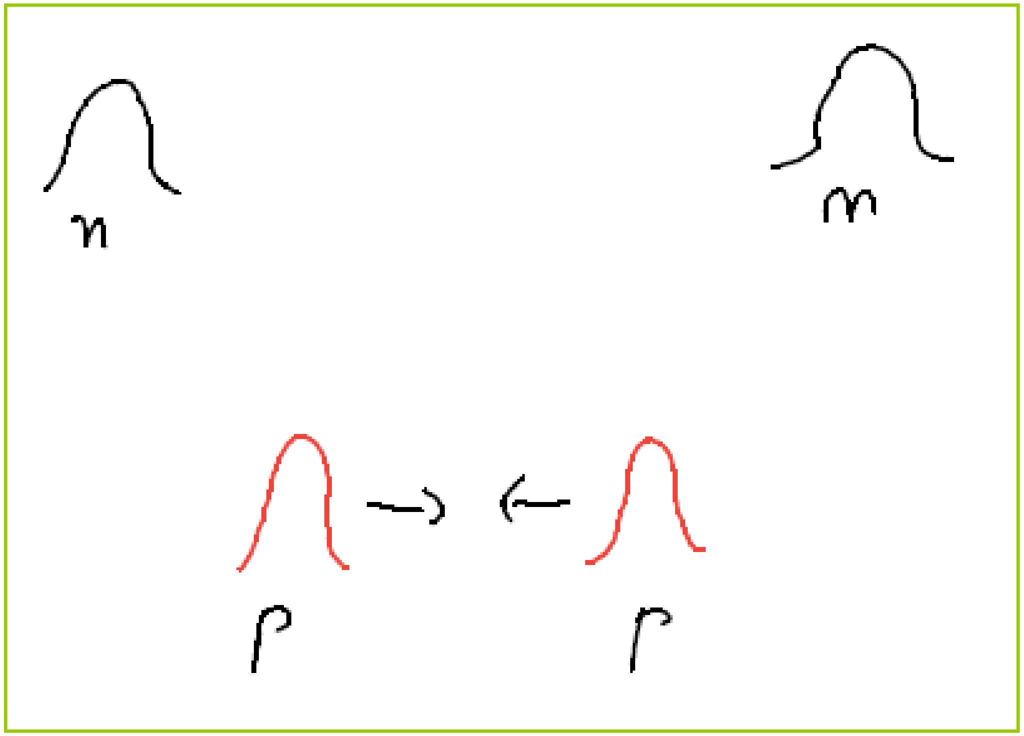
Wohl, C.G.: Isospin relations by counting. American Journal of Physics **50**(8), 748–753 (1982) https://doi.org/10.1119/1.12743

Pal, P.: An Introductory Course of Particle Physics -CRC Press, (2014)

Comment:

Let us consider an ensemble of initial states being invariant under the charge transformation - probabilities of having initial states related by this transformation are equal. This is the case of nucleus-nucleus collisions where each nucleus has an equal number of protons and neutrons (thus, $I_z = 0$). Then, the invariance under C-transformation holds also for the final state ensemble:

ppmm ⟼ ?



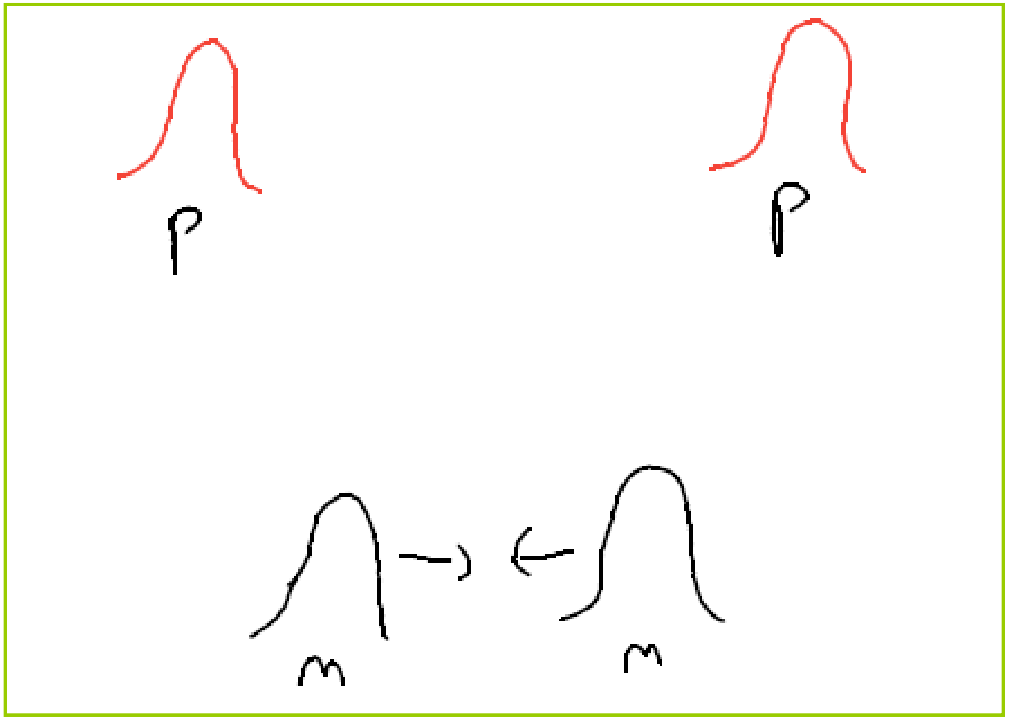
Just as pp!

More K^+ than K^0

Is then the previous argumentation wrong?

No.
One needs to average.

But... $\hat{C}$ transform



This is the C-transformed version fo the previous reaction.

Here, the protons are spectactors and the neutrons interact.

Just as nn scattering!

More K^0 than K^+

Averaging leads to…

If both initial states are equally probable

⬇

$$\langle K^+ \rangle = \langle K^0 \rangle$$

holds!

Formally:

$$\hat{\rho} = \sum_n p_n \left|\Psi_n\right\rangle \left\langle\Psi_n\right|$$

$$\hat{C}\hat{\rho}\hat{C}^\dagger = \hat{\rho}$$

This is a general result!

Chat-GPT and e.m. interaction

1. **Strong Interaction with Isospin Breaking**:
   - Quark mass differences m_u and m_d break isospin symmetry, leading to slightly different couplings for u-quark and d-quark production rates.
   - The effective strong interaction rates now include a dependence on quark masses:
   $$\alpha_s^u = \alpha_s(1-\delta), \quad \alpha_s^d = \alpha_s(1+\delta),$$
   where δ is a small parameter quantifying the isospin-breaking effect due to $m_d > m_u$.
2. **Electromagnetic Contribution**:
   - The electromagnetic terms remain as before:
   $$\alpha_{\mathrm{em}} Q_u^2 \quad \text{and} \quad \alpha_{\mathrm{em}} Q_d^2.$$
3. **Total Rates**:
   - The total rates now include both effects:
   $$\mathrm{Rate}(u\bar{u}) = \alpha_s^u + \alpha_{\mathrm{em}} Q_u^2,$$
   $$\mathrm{Rate}(d\bar{d}) = \alpha_s^d + \alpha_{\mathrm{em}} Q_d^2.$$
4. **Ratio of Effective Rates**:
   - Incorporating isospin breaking, the ratio α/β becomes:
   $$\frac{\alpha}{\beta} = \frac{\mathrm{Rate}(u\bar{u})}{\text{Rate(}} = \frac{\alpha_s(1-\delta) + \alpha_{\mathrm{em}} Q_u^2}{\alpha_s(1+\delta) + \alpha_{\mathrm{em}} Q_d^2}.$$

Chat-GPT and e.m. interaction /2

Numerical Calculation:

Using the same parameters as before:

- $\alpha_s = 0.1$,
- $\alpha_{\mathrm{em}} = 1/137$,
- $Q_u^2 = 4/9$, $Q_d^2 = 1/9$,
- For isospin breaking: $\delta \approx 0.003$ (a typical estimate reflecting $m_d - m_u \sim 2 - 3$ MeV).

We compute:

$$\text{Numerator: } \alpha_s(1-\delta) + \alpha_{\mathrm{em}} Q_u^2 = 0.1 \cdot (1 - 0.003) + \frac{1}{137} \cdot \frac{4}{9} \approx 0.10075.$$

$$\text{Denominator: } \alpha_s(1+\delta) + \alpha_{\mathrm{em}} Q_d^2 = 0.1 \cdot (1 + 0.003) + \frac{1}{137} \cdot \frac{1}{9} \approx 0.10055.$$

The ratio α/β becomes:

$$\frac{\alpha}{\beta} = \frac{0.10075}{0.10055} \approx 1.002.$$

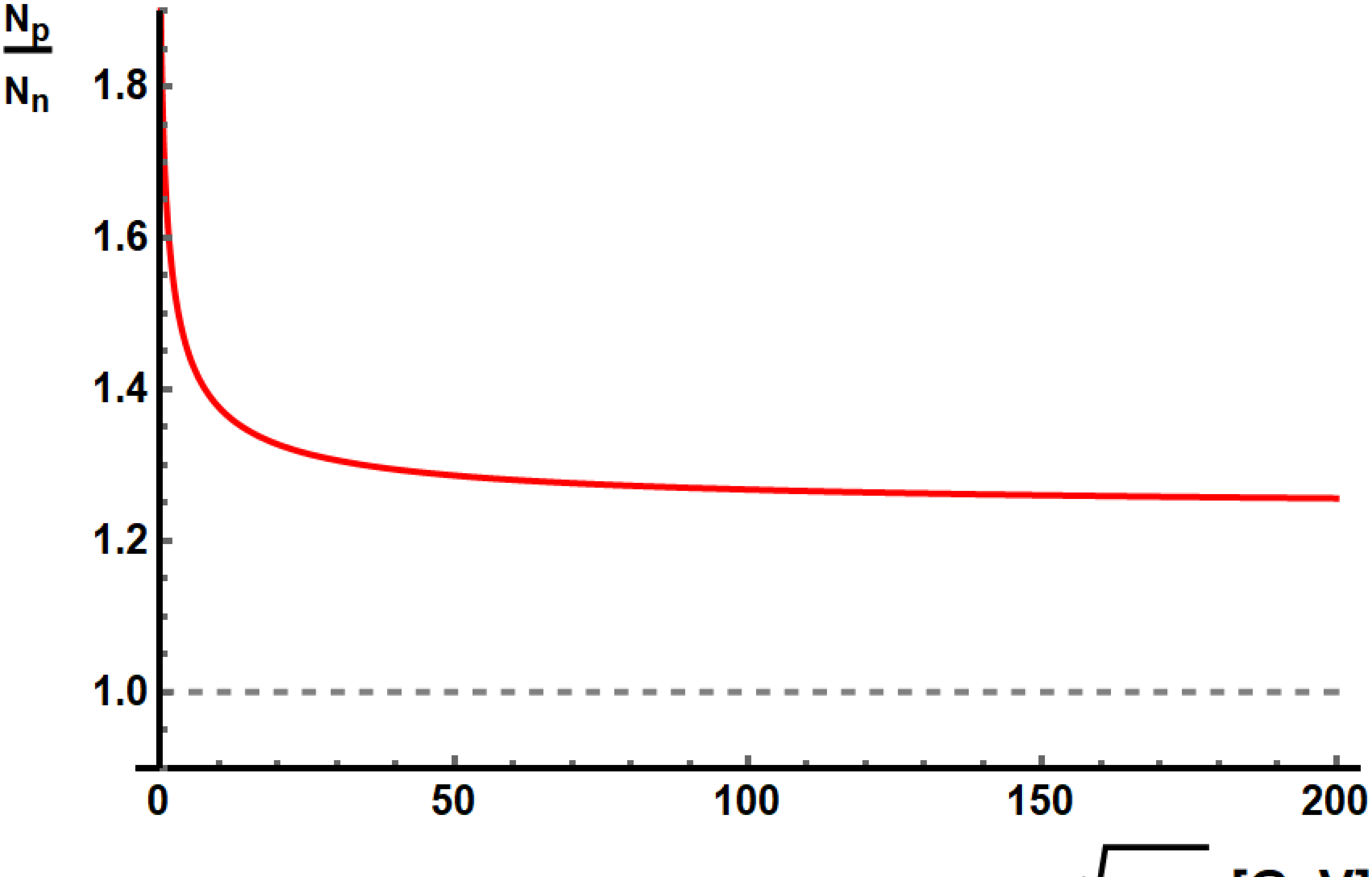
NA61/SHINE experiment					
Ar+Sc collisions at $\sqrt{s_{NN}} = 11.9$ GeV					
hadron	Yields $(y \approx 0) \pm \sigma_{stat} \pm \sigma_{sys}$	σ_{total}	Centrality	y ranges	Ref.
K^+	$3.732 \pm 0.016 \pm 0.148$	0.15	0–10%	$0.0 < y < 0.2$	[18]
K^-	$2.029 \pm 0.012 \pm 0.069$	0.070	0–10%	$0.0 < y < 0.2$	[18]
K^0_S	$2.433 \pm 0.027 \pm 0.102$	0.11	0–10%	$y = 0$	this analysis
HADES experiment					
Ar+KCl collisions at $\sqrt{s_{NN}} = 2.6$ GeV					
hadron	Yields $(4\pi) \pm \sigma_{stat} \pm \sigma_{sys}$	σ_{total}	Centrality	y ranges	Ref.
K^+	$0.028 \pm 0.002 \pm 0.0014$ $^{(*)}$	0.0024	0–35%	extrapolated to 4π	[43]
K^-	$0.00071 \pm 0.00015 \pm 0.000032$ $^{(*)}$	0.00015	0–35%	extrapolated to 4π	[43]
K^0_S	$0.0115 \pm 0.0005 \pm 0.0009$	0.0010	0–35%	extrapolated to 4π	[44]
STAR (BES I) experiment					
Au+Au collisions at $\sqrt{s_{NN}} = 7.7$ GeV					
hadron	Yields $(y \approx 0) \pm \sigma_{stat} \pm \sigma_{sys}$	σ_{total}	Centrality	y ranges	Ref.
K^+	20.8	1.7	0–5%	$-0.1 < y < 0.1$	[30]
K^-	7.7	0.6	0–5%	$-0.1 < y < 0.1$	[30]
K^0_S	$12.67 \pm 0.12 \pm 0.44$	0.46	0–5%	$-0.5 < y < 0.5$	[31]

STAR (BES I) experiment					
Au+Au collisions at $\sqrt{s_{NN}} = 11.5$ GeV					
hadron	Yields $(y \approx 0) \pm \sigma_{stat} \pm \sigma_{sys}$	σ_{total}	Centrality	y ranges	Ref.
K^+	25.0	2.5	0–5%	$-0.1 < y < 0.1$	[30]
K^-	12.3	1.2	0–5%	$-0.1 < y < 0.1$	[30]
K^0_S	$15.93 \pm 0.12 \pm 0.58$	0.59	0–5%	$-0.5 < y < 0.5$	[31]

STAR (BES I) experiment					
Au+Au collisions at $\sqrt{s_{NN}} = 19.6$ GeV					
hadron	Yields ($y \approx 0$) $\pm \sigma_{stat} \pm \sigma_{sys}$	σ_{total}	Centrality	y ranges	Ref.
K^+	29.6	2.9	0–5%	$-0.1 < y < 0.1$	[30]
K^-	18.8	1.9	0–5%	$-0.1 < y < 0.1$	[30]
K_S^0	$20.89 \pm 0.08 \pm 0.67$	0.67	0–5%	$-0.5 < y < 0.5$	[31]
STAR (BES I) experiment					
Au+Au collisions at $\sqrt{s_{NN}} = 27$ GeV					
hadron	Yields ($y \approx 0$) $\pm \sigma_{stat} \pm \sigma_{sys}$	σ_{total}	Centrality	y ranges	Ref.
K^+	31.1	2.8	0–5%	$-0.1 < y < 0.1$	[30]
K^-	22.6	2.0	0–5%	$-0.1 < y < 0.1$	[30]
K_S^0	$23.24 \pm 0.09 \pm 0.70$	0.71	0–5%	$-0.5 < y < 0.5$	[31]
STAR (BES I) experiment					
Au+Au collisions at $\sqrt{s_{NN}} = 39$ GeV					
hadron	Yields ($y \approx 0$) $\pm \sigma_{stat} \pm \sigma_{sys}$	σ_{total}	Centrality	y ranges	Ref.
K^+	32.0	2.9	0–5%	$-0.1 < y < 0.1$	[30]
K^-	25.0	2.3	0–5%	$-0.1 < y < 0.1$	[30]
K_S^0	$24.9 \pm 0.1 \pm 1.7$	1.7	0–5%	$-0.5 < y < 0.5$	[31]
NA49 experiment					
Pb+Pb collisions at $\sqrt{s_{NN}} = 7.6$ GeV					
hadron	Yields (4π) $\pm \sigma_{stat} \pm \sigma_{sys}$	σ_{total}	Centrality	y ranges	Ref.
K^+	$52.9 \pm 0.9 \pm 3.5$ (*)	3.6	0–7.2%	extrapolated to 4π	[40]
K^-	$16.0 \pm 0.2 \pm 0.4$	0.45	0–7.2%	extrapolated to 4π	[40]
K_S^0	$29.3 \pm 0.3 \pm 2.9$	2.9	0–7.2%	extrapolated to 4π	[42]
NA49 experiment					
Pb+Pb collisions at $\sqrt{s_{NN}} = 8.7$ GeV					
hadron	Yields (4π) $\pm \sigma_{stat} \pm \sigma_{sys}$	σ_{total}	Centrality	y ranges	Ref.
K^+	$59.1 \pm 1.9 \pm 3$	3.6	0–7.2%	extrapolated to 4π	[41]
K^-	$19.2 \pm 0.5 \pm 1.0$	1.1	0–7.2%	extrapolated to 4π	[41]
K_S^0	$34.2 \pm 0.2 \pm 3.4$	3.4	0–7.2%	extrapolated to 4π	[42]
CERES experiment					
Pb+Au collisions at $\sqrt{s_{NN}} = 17.3$ GeV					
hadron	Yields ($y \approx 0$) $\pm \sigma_{stat} \pm \sigma_{sys}$	σ_{total}	Centrality	y ranges	Ref.
K^+	$31.8 \pm 0.6 \pm 2.5$	2.6	0–7%	$y = 0$	[27]
K^-	$19.3 \pm 0.4 \pm 2.0$	2.0	0–7%	$y = 0$	[27]
K_S^0	$21.2 \pm 0.9 \pm 1.7$	1.9	0–7%	$y = 0$	[28, 29]
NA35 experiment					
S+S collisions at $\sqrt{s_{NN}} = 19.4$ GeV					
hadron	Yields (4π) $\pm \sigma_{stat} \pm \sigma_{sys}$	σ_{total}	Centrality	y ranges	Ref.
K^+	$12.5 \pm 0.4 \pm 0.375$ (*)	0.55	0–2%	extrapolated to 4π	[38]
K^-	$6.9 \pm 0.4 \pm 0.207$ (*)	0.45	0–2%	extrapolated to 4π	[38]
K_S^0	10.5	1.7	0–2%	extrapolated to 4π	[39]

Uniwersytet Jana Kochanowskiego w Kielcach

p/n, Q/B =1



a0(980) and f0(980) data

Radiative phi decays with derivative interactions

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Apr, 2008

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$$A_{f_0\pi\pi} = 2.88 \pm 0.22 \text{ GeV}, \; A_{f_0KK} = 5.91 \pm 0.77 \text{ GeV}.$$

$$A_{a_0\pi\eta} = 3.33 \pm 0.15 \text{ GeV}, \; A_{a_0KK} = 3.59 \pm 0.44 \text{ GeV},$$

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