Entropy production and dissipation in spin hydrodynamics

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Outline

1. **What is spin hydrodynamics?**
2. **Covariant thermodynamics approach: results and outlook**
3. **Relativistic quantum-statistical approach: Results and future directions**
4. **Conclusion and outlooks**

What is spin hydrodynamics?

- **Spin hydrodynamics, emerging as an effective limit of quantum field theory, is applicable to a wide range of systems, including the evolution of the quark-gluon plasma produced in heavy-ion collision experiments.**

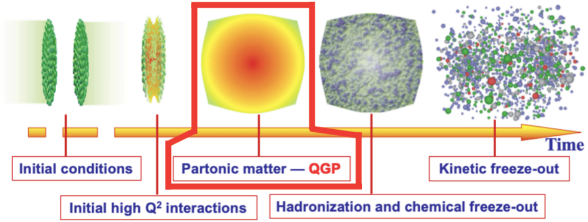

Illustration of the evolution of a heavy ion collision [1912.07822]

- **While spin hydrodynamics holds theoretical promise, no experimental evidence has yet emerged to demonstrate its relevance for QGP description.**

Evolution of:

- $\mathbf{T}^{\mu\nu} \equiv$ **Energy-momentum current,**

$$\mathbf{P}^{\nu} = \int \mathbf{d}\,\Sigma_{\mu}\,\mathbf{T}^{\mu\nu}\ , \quad \partial_{\mu}\mathbf{T}^{\mu\nu} = 0\ , \quad \mathbf{T}^{\mu\nu} = \mathbf{T}_0^{\mu\nu} + \boldsymbol{\delta}\mathbf{T}^{\mu\nu}.$$

- $\mathbf{j}^{\mu} \equiv$ **Particle number current,**

$$\mathbf{J} = \int \mathbf{d}\,\Sigma_{\mu}\,\mathbf{j}^{\mu}\ , \quad \partial_{\mu}\,\mathbf{j}^{\mu} = 0\ , \quad \mathbf{j}^{\mu} = \mathbf{j}_0^{\mu} + \boldsymbol{\delta}\mathbf{j}^{\mu}.$$

- $\mathbf{S}^{\lambda\mu\nu} \equiv$ **Spin current,**

$$\mathbf{J}^{\lambda\mu\nu} = \mathbf{L}^{\lambda\mu\nu} + \mathbf{S}^{\lambda\mu\nu}\ , \quad \boldsymbol{\partial}_{\lambda}\mathbf{J}^{\lambda\mu\nu} = 0,$$

$$\mathbf{S}^{\mu\nu} = \int \mathbf{d}\Sigma_{\lambda}\mathbf{S}^{\lambda\mu\nu}\ , \quad \partial_{\lambda}\mathbf{S}^{\lambda\mu\nu} = \mathbf{T}^{\nu\mu} - \mathbf{T}^{\mu\nu}.$$

The primary objective in the formulation of spin hydrodynamics is to

- **Identify and characterize novel transport coefficients that arise in addition to those present in standard relativistic hydrodynamics.**

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Covariant thermodynamics approach: results and outlook

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Toolkit

- **Euler equation with spin (for simplicity we set $\mu = 0$) :**

$$\varepsilon + \mathbf{P} = \mathbf{Ts} + \boldsymbol{\omega}_{\mu\nu}\mathbf{S}^{\mu\nu}$$

- **Hydrodynamic currents :**

$$\mathbf{T}^{\mu\nu} = (\varepsilon + \mathbf{P})\mathbf{u}^{\mu}\mathbf{u}^{\nu} - \mathbf{pg}^{\mu\nu} + \mathbf{T}_{S}^{\mu\nu} + \mathbf{T}_{A}^{\mu\nu}\ , \quad \mathbf{S}^{\lambda\mu\nu} = \mathbf{u}^{\nu} S^{\mu\nu} + \mathbf{S}_{1}^{\lambda\mu\nu}$$

[Xu-Huang et al. Phys.Lett.B 795(2019)100-106]

Navier-Stokes-Like formulation

10 dynamical variables $\varepsilon, \mathbf{u}^{\mu}, \mathbf{S}^{\mu\nu}$ with 10 evolution equations ,

Relativistic *Navier-Stokes evolution equations* with spin current
$D\varepsilon + (\varepsilon + p)\theta = 2\,h^{\mu} D u_{\mu} - \nabla \cdot (q + h) + \pi^{\mu\nu}\partial_{\mu}u_{\nu} + \Pi\Delta^{\mu\nu}\partial_{\mu}u_{\nu} + \phi^{\mu\nu}\partial_{\mu}u_{\nu}$
$(\varepsilon + p)Du^{\alpha} - \nabla^{\alpha}p =$ $-(q + h)\cdot\nabla u^{\alpha} + (q^{\alpha} - h^{\alpha})\theta + \Delta^{\alpha}{}_{\nu}Dq^{\nu} - \Delta^{\alpha}{}_{\nu}Dh^{\nu} - \Delta^{\alpha}{}_{\nu}\partial_{\mu}(\pi^{\mu\nu} + \Pi\Delta^{\mu\nu}) - \Delta^{\alpha}{}_{\nu}\partial_{\mu}\phi^{\mu\nu}$
$\partial_{\lambda}(u^{\lambda}S^{\mu\nu}) + \partial_{\lambda}S^{\lambda\mu\nu}_{(1)} = -2(q^{\mu}u^{\nu} - q^{\nu}u^{\mu} + \phi^{\mu\nu})$

Dissipative currents and transport coefficients

Using entropy current analysis $\partial_\mu s^\mu \geq 0$,

$$\Pi = \zeta\theta,$$

$$h^\mu = -\kappa\left(Du^\mu - \beta\nabla^\mu T\right),$$

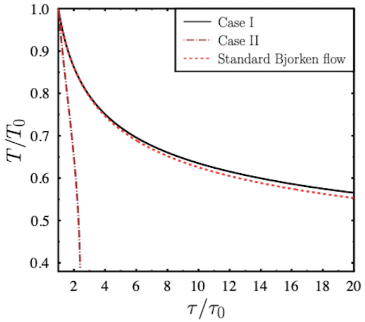
$$q^\mu = \lambda\left(Du^\mu + \beta\nabla^\mu T - 4\omega^{\mu\nu}u_\nu\right),$$

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu},$$

$$\phi^{\mu\nu} = \tilde{\gamma}\left(2\nabla^{[\mu}u^{\nu]} + 4\Delta^{\mu\alpha}\Delta^{\nu\beta}\omega_{\alpha\beta}\right).$$

Stability, causality, and boost invariant studies

- **The system tends to be unstable and acausal in rest frame and boosted frame.**
- **Temperature evolution ,**



Red (dashed) line represents the variation of temperature for Bjorken flow in relativistic hydro without spin. Brown (dashed-dotted) line represents the variation evolution in Navier-stokes theory with spin. We consider $T_0 = 200$ MeV and $\tau_0 = 0.5$ fm.

Muller-Israel-Stewart-Like formulation

- **One of the possible scenarios is to allow for every variable in $\mathbf{T}^{\mu\nu}$ and $\mathbf{S}^{\lambda\mu\nu}$ to be dynamical, i.e.,**

$$\begin{cases} \mathbf{T}^{\mu\nu} & \longrightarrow \textbf{16 evolution equations}, \\ \mathbf{S}^{\lambda\mu\nu} & \longrightarrow \textbf{24 evolution equations.} \end{cases}$$

- **Muller-Israel-Stewart entropy production rate with spin,**

$$\partial_\mu \mathbf{s}^\mu_{\mathbf{IS}} = (\partial_\mu \boldsymbol{\beta}_\nu + 2\boldsymbol{\omega}_{\mu\nu}\boldsymbol{\beta})\delta \mathbf{T}^{\mu\nu}_A + \delta \mathbf{T}^{\mu\nu}_S \partial_\mu \boldsymbol{\beta}_\nu - \delta \mathbf{S}^{\mu\alpha\beta}\partial_\mu(\boldsymbol{\beta}\boldsymbol{\omega}_{\alpha\beta}) + \partial_\mu \mathbf{Q}^\mu \quad \geq \quad 0.$$

16+24 dynamical equations

$$D\varepsilon + (\varepsilon + p)\theta = \pi^{\mu\nu}\partial_\mu u_\nu + \Pi\theta - \nabla\cdot q + \phi^{\mu\nu}\partial_\mu u_\nu \,,$$

$$\begin{aligned}(\varepsilon + p)Du^\alpha - \nabla^\alpha p = &- \Delta^\alpha_\nu\partial_\mu\pi^{\mu\nu} - \Delta^{\mu\alpha}\partial_\mu\pi + \pi Du^\alpha - q^\mu\partial_\mu u^\alpha \\ &+ \Delta^\alpha_\nu Dq^\nu + q^\alpha\theta - \Delta^\alpha_\nu\partial_\mu\phi^{\mu\nu} \,,\end{aligned}$$

$$\tau_\Pi D\Pi + \Pi = \zeta\left[\theta + Ta_1\Pi\theta + T\Pi Da_1\right],$$

$$\tau_\pi\Delta^{\mu\nu}_{\alpha\beta}D\pi^{\alpha\beta} + \pi^{\mu\nu} = 2\eta\left[(\nabla^{(\mu}u^{\nu)} - \frac{1}{3}\theta\Delta^{\mu\nu}) + Ta_2\theta\pi^{\mu\nu} + T\pi^{\mu\nu}Da_2\right],$$

$$\tau_q\Delta^\mu_\nu Dq^\nu + q^\mu = \lambda\left[(\beta\nabla^\mu T + Du^\mu - 4\omega^{\mu\nu}u_\nu) - Ta_4q^\mu\theta - Tq^\mu Da_4\right],$$

$$\tau_\phi\Delta^{[\mu\nu]}_{[\alpha\beta]}D\phi^{\alpha\beta} + \phi^{\mu\nu} = \gamma\left[(\beta\nabla^{[\mu}u^{\nu]} + 2\beta\Delta^{\mu\alpha}\Delta^{\nu\beta}\omega_{\alpha\beta}) + a_5\theta\phi^{\mu\nu} + \phi^{\mu\nu}Da_5\right],$$

$$DS^{\alpha\beta} + S^{\alpha\beta}\theta + \partial_\mu S_1^{\mu\alpha\beta} = -2(q^\alpha u^\beta - q^\beta u^\alpha + \phi^{\alpha\beta}),$$

$$\tau_\Phi D\Phi + \Phi = \chi_1\left[-2u^\alpha\nabla^\beta(\beta\omega_{\alpha\beta}) + \tilde{a}_1\theta\Phi + \Phi D\tilde{a}_1\right],$$

$$\tau_{\tau_s}\Delta^{\mu\nu}_{\alpha\beta}D\tau_s^{\alpha\beta} + \tau_s^{\mu\nu} = \chi_2\left[-u^\alpha(\Delta^{\gamma\mu}\Delta^{\rho\nu} + \Delta^{\gamma\nu}\Delta^{\rho\mu} - \frac{2}{3}\Delta^{\gamma\rho}\Delta^{\mu\nu})\nabla_\gamma(\beta\omega_{\alpha\rho}) + \tilde{a}_2\theta\tau_s^{\mu\nu} + \tau_s^{\mu\nu}D\tilde{a}_2\right],$$

$$\tau_{\tau_a}\Delta^{[\mu\nu]}_{[\alpha\beta]}D\tau_a^{\alpha\beta} + \tau_a^{\mu\nu} = \chi_3\left[-u^\alpha(\Delta^{\gamma\mu}\Delta^{\rho\nu} - \Delta^{\gamma\nu}\Delta^{\rho\mu})\nabla_\gamma(\beta\omega_{\alpha\rho}) + \tilde{a}_3\theta\tau_a^{\mu\nu} + \tau_a^{\mu\nu}D\tilde{a}_3\right],$$

$$\tau_\Theta\Delta^\alpha_\lambda\Delta^\mu_\sigma\Delta^\nu_\beta D\Theta^{\lambda\sigma\beta} + \Theta^{\alpha\mu\nu} = -\chi_4\left[-\Delta^{\delta\mu}\Delta^{\rho\nu}\Delta^{\gamma\alpha}\nabla_\gamma(\beta\omega_{\delta\rho}) + \tilde{a}_4\theta\Theta^{\alpha\mu\nu} + \Theta^{\alpha\mu\nu}D\tilde{a}_4\right],$$

Relaxation-time dynamical equations

- **We obtain relaxation-type dynamical equations for each dynamical variables.**
- **The resultant transport coefficients are,**

$$\kappa, \quad \eta, \quad \zeta, \quad \boldsymbol{\lambda}, \quad \gamma, \quad \chi_1, \quad \chi_2, \quad \chi_3, \quad \chi_4.$$

- **The system tends to be stable and causal in rest frame provided the used relaxation times are sufficiently large.**

Outlook

Formulate analytically

Solve numerically

We are here ←

Calculate Observables at freeze-out

Compare with experiments

Relativistic quantum-statistical approach: Results and future directions

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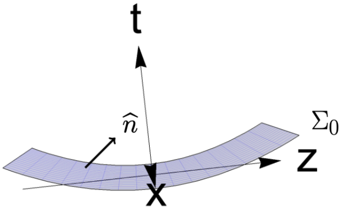
+ work in progress

Local equilibrium is achieved at initial hypersurface Σ_0, where entropy is maximum provided that the mean vales of energy, momentum, particle number, and spin currents are their actual values,

$$S = -\operatorname{Tr}(\widehat{\rho}\log\widehat{\rho})$$



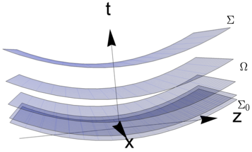
$$F[\widehat{\rho}] = -Tr[\widehat{\rho}\log\widehat{\rho}] - \int d\Sigma_0\, n_\mu\,(T^{\mu\nu}_{\rm LE} - T^{\mu\nu})\,\beta_\nu(x) - \int d\Sigma_0\, n_\mu\,(j^\mu_{\rm LE} - j^\mu)\,\zeta(x)$$
$$- \int d\Sigma_0\, n_\mu \left(S^{\mu\lambda\nu}_{\rm LE} - S^{\mu\lambda\nu}\right)\Omega_{\lambda\nu}(x)$$

$$T^{\mu\nu}_{\rm LE} \sim Tr\left[\widehat{\rho}\,\widehat{T}^{\mu\nu}\right]$$
$$T^{\mu\nu} \equiv \text{Actual Value}$$

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z}\exp\left[-\int_{\Sigma_0} {\rm d}\,\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{S}^{\mu\lambda\nu}\right)\right]$$

Near local equilibrium at the hypersuface Σ, the entropy is defined as,

$$
\begin{aligned}
S &= -\,\mathrm{Tr}(\widehat{\rho}_{\mathrm{LE}} \log \widehat{\rho}_{\mathrm{LE}}) \\
&= \int_{\Sigma} \mathrm{d}\Sigma_{\mu} \left(\phi^{\mu} + T^{\mu\nu}_{\mathrm{LE}}\beta_{\nu} - \zeta j^{\mu}_{\mathrm{LE}} - \frac{1}{2}\Omega_{\lambda\nu}\mathcal{S}^{\mu\lambda\nu}_{\mathrm{LE}} \right)
\end{aligned}
$$



$$
\partial_{\mu} s^{\mu} = \delta T^{\mu\nu}_{S}\xi_{\mu\nu} - \delta j^{\mu}\partial_{\mu}\zeta + \delta T^{\mu\nu}_{A}(\Omega_{\mu\nu} - \varpi_{\mu\nu}) - \frac{1}{2}\delta\mathcal{S}^{\mu\lambda\nu}\partial_{\mu}\Omega_{\lambda\nu}
$$

The goal is to determine,

$$
\delta \mathbf{T}^{\mu\nu}_{S}\ ,\quad \delta \mathbf{T}^{\mu\nu}_{A}\ ,\quad \delta \mathbf{j}^{\mu}\ ,\quad \delta \mathbf{S}^{\lambda\mu\nu}
$$

In general and without imposing any physical assumption, we can express dissipative currents in terms of linear responses to all fields in the system,

$$\delta T_S^{\mu\nu} = H^{\mu\nu\rho\sigma}\xi_{\rho\sigma} + K^{\mu\nu\rho}\partial_\rho\zeta + L^{\mu\nu\rho\sigma}\left(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}\right) + M^{\mu\nu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau},$$

$$\delta T_A^{\mu\nu} = N^{\mu\nu\rho\sigma}\xi_{\rho\sigma} + P^{\mu\nu\rho}\partial_\rho\zeta + Q^{\mu\nu\rho\sigma}\left(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}\right) + R^{\mu\nu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau},$$

$$\delta j^\mu = G^{\mu\rho\sigma}\xi_{\rho\sigma} + I^{\mu\rho}\partial_\rho\zeta + O^{\mu\rho\sigma}(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}) + F^{\mu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau},$$

$$\delta S^{\mu\lambda\nu} = T^{\mu\lambda\nu\rho\sigma}\xi_{\rho\sigma} + U^{\mu\lambda\nu\rho}\partial_\rho\zeta + V^{\mu\lambda\nu\rho\sigma}\left(\Omega_{\rho\sigma} - \varpi_{\rho\sigma}\right) + W^{\mu\lambda\nu\rho\sigma\tau}\partial_\rho\Omega_{\sigma\tau}.$$

Hence the goal reduces to determining the coefficient tensors,

$$H^{\mu\nu,\rho,\sigma}\ ,\ \ K^{\mu\nu\rho}\ ,\ \ L^{\mu\nu\rho\sigma}\ ,\ \ M^{\mu\nu\rho\sigma\tau}$$

$$N^{\mu\nu\rho\sigma}\ ,\ \ P^{\mu\nu\rho}\ ,\ \ Q^{\mu\nu\rho\sigma}\ ,\ \ R^{\mu\nu\rho\sigma\tau}$$

$$G^{\mu\rho\sigma}\ ,\ \ I^{\mu\rho}\ ,\ \ O^{\mu\rho\sigma}\ ,\ \ F^{\mu\rho\sigma\tau}$$

$$T^{\mu\lambda\nu\rho\sigma}\ ,\ \ U^{\mu\lambda\nu\rho}\ ,\ \ V^{\mu\lambda\nu\rho\sigma}\ ,\ \ W^{\mu\lambda\nu\rho\sigma\tau}$$

The minimum physical requirement is that, the tensor coefficients should be invariant under spatial rotations in the local co-moving frame of the fluid.

For that, we decompose the tensor coefficients in terms of irreducible representations of the rotation group $\mathbb{SO}(3)$. The irreducible basis are constructed by the fluid 4-velocity $\mathbf{u}^{\mu}$, the projector $\boldsymbol{\Delta}_{\mu\nu}$, and the rank-3 tensor $\epsilon^{\mu\nu\lambda\gamma}\mathbf{u}_{\gamma}$,

$$\text{Vector:}\;\; V^{\mu} = (u^{\mu} \oplus \Delta^{\mu}_{\alpha})$$

$$\text{Symmetric 2-tensor:}\;\; B^{\mu\nu} = (u^{\mu}u^{\nu} \oplus \Delta^{\mu\nu} \oplus u^{\mu}\Delta^{\nu}_{\alpha} + u^{\nu}\Delta^{\mu}_{\alpha} \oplus \Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta} + \Delta^{\nu}_{\alpha}\Delta^{\mu}_{\beta}),$$

$$\text{Antisymmetric 2-tensor:}\;\; A^{\mu\nu} = (u^{\mu}\Delta^{\nu}_{\alpha} - u^{\nu}\Delta^{\mu}_{\alpha} \oplus \epsilon^{\mu\nu\tau\alpha}u_{\tau}).$$

Imposing the below physical conditions allows us to cancel out all the non-physical coefficients,

- **Matching conditions,**

$$n_\mu(\delta T_S^{\mu\nu} + \delta T_A^{\mu\nu}) = 0, \quad n_\mu \delta j^\mu = 0, \quad n_\mu \delta S^{\mu\lambda\nu} = 0.$$

- **Entropy production semi-positivity condition** $\partial_\mu s^\mu \geq 0$.

Outlook

Formulate analytically

We are here ⟵

Solve numerically

s

Calculate Observables at freeze-out

Compare with experiments

Conclusion and outlooks

Conclusion

1. **We develop Navier-Stokes-like and Muller-Israel-Stewart-like formulations to study a relativistic fluid of particles with spin such as the QGP produced in heavy-ion collision experiments.**

2. **We used a first-principle quantum-statistical methods to derive the entropy current, entropy production rate, and obtain the transport coefficients (Work in progress).**

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