Effects of jet-medium interactions versus vacuum like emissions on jet azimuthal angular decorrelations

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Collaborators:

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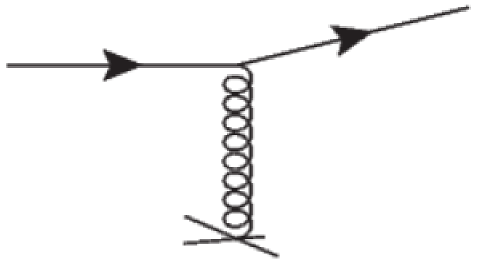
[S. Adhya, K. Kutak, W. Płaczek, MR, K. Tywoniuk, arxiv: 2409.06675]

Processes in jets in the medium

Scattering:



Momentum transfer!

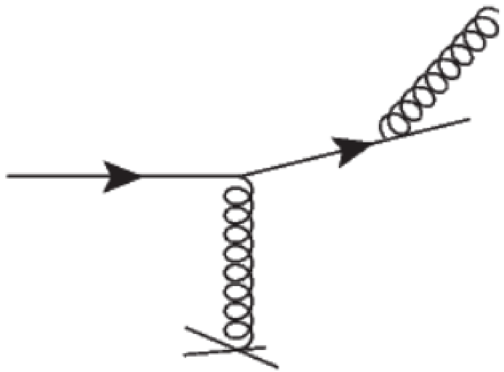
$$p \to p + Q$$

Scattering Kernel:

$$\frac{\partial^3 \mathcal{P}_{\rm scat}}{\partial t\, \partial^2 \mathbf{Q}} = \frac{1}{(2\pi)^2} w(\mathbf{Q})$$

Average transfer: $\hat{q}$

Induced radiation:



Momentum distribution:

$$p \to zp$$

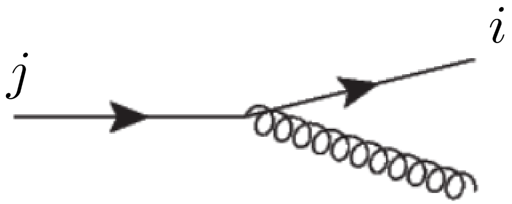
+Momentum transfer:

$$p \to zp + Q$$

Kernel: $$\frac{\partial^4 \mathcal{P}_{\rm split}}{\partial t\, \partial z\, \partial^2 \mathbf{Q}} = \frac{\alpha_s}{(2\pi)^2} \mathcal{K}(\mathbf{Q}, z, p_+)$$

Jet-Medium interactions

Splitting:



Bremsstrahlung as
in vacuum.

Momentum distribution:

$$p \to zp$$

DGLAP-Kernel:

$$\frac{\partial^2 \mathcal{P}_{\rm split}}{\partial \hat{t}\, \partial z} \propto \frac{1}{\hat{t}} P_{ij}(z)$$

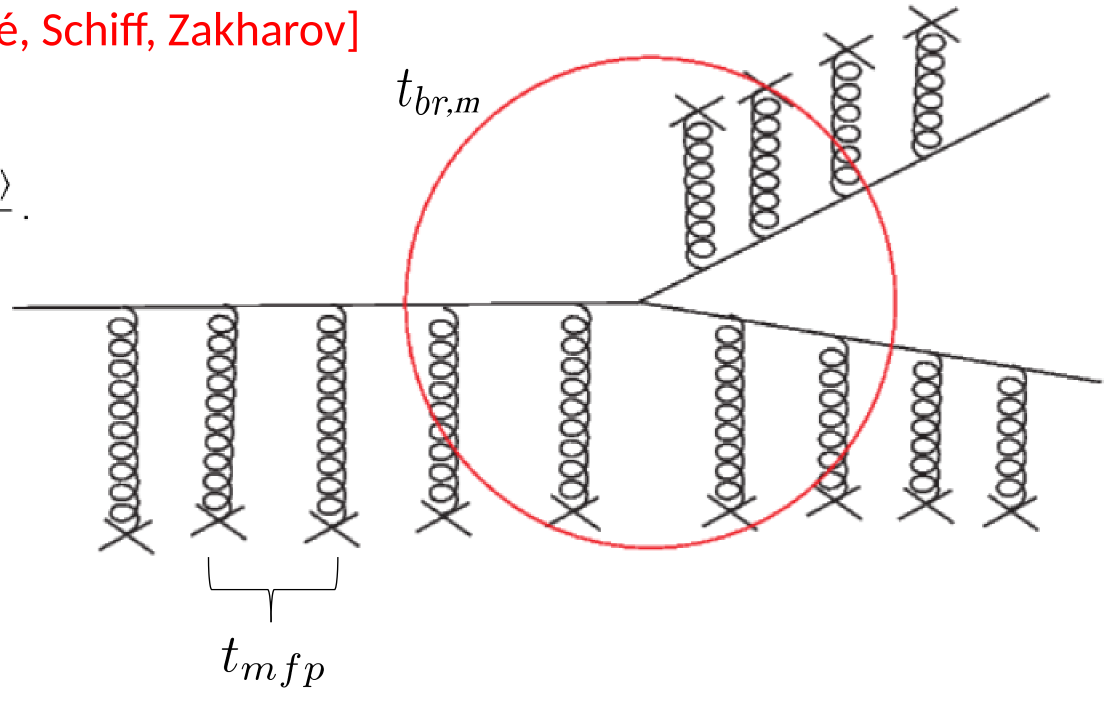
Emissions in Vacuum,
Vacuum Like
Emissions in medium

Coherent emission

…à la BDMPS-Z [Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov]

$$t_{br,m} \sim \sqrt{\frac{2\omega}{\hat{q}}}$$

$$\hat{q} = \frac{\mathrm{d}\langle k_\perp^2 \rangle}{\mathrm{d}t} .$$



$t_{br,m} \sim t_{mfp}$: one scattering + radiation
…Bethe-Heitler spectrum

$t_{br,m} \gg t_{mfp}$: coherent radiation

$$\omega \frac{dI}{d\omega} \sim \alpha_s \frac{L}{t_{br,m}} = \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$

Look at range: $\omega_{BH} < \omega < \omega_c$

need effective kernel: $\mathcal{K}(z, k_T)$

cf. [Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1301 (2013) 143]
[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

Splitting Kernels for Quarks and Gluons

[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918], [Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1301 (2013) 143]

$$\mathcal{K}_{ij}(\boldsymbol{Q}, z, p_0^+) = \frac{2P_{ij}(z)}{z(1-z)p_0^+} \sin\left(\frac{\boldsymbol{Q}^2}{2k_{\rm br}^2}\right) \exp\left(-\frac{\boldsymbol{Q}^2}{2k_{\rm br}^2}\right)$$

$$k_{\rm br}^2 = \sqrt{z(1-z)p_0^+ f_{ij}(z)\frac{\hat{q}}{N_c}}$$

$$f_{gg}(z) = (1-z)C_A + z^2 C_A$$

$$f_{qg}(z) = C_F - z(1-z)C_A\,,$$

$$f_{gq}(z) = (1-z)C_A + z^2 C_F$$

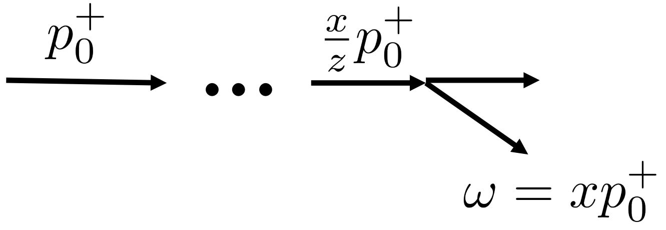
$$f_{qq}(z) = zC_A + (1-z)^2 C_F$$

$$\frac{\partial^4 \mathcal{P}_{\rm split}}{\partial t \partial z \partial^2 \mathbf{Q}} = \frac{\alpha_s}{(2\pi)^2}\mathcal{K}(\mathbf{Q}, z, p_+)$$

$$\frac{\partial^5 \mathcal{P}_{\rm split}}{\partial t \partial x \partial z \partial^2 \mathbf{Q}} = \frac{\alpha_s}{(2\pi)^2}\mathcal{K}\left(\mathbf{Q}, z, \frac{x}{z}p_0^+\right)$$

$$\Big\downarrow \int_0^\infty d^2\mathbf{Q} \times$$

$$\frac{\partial^3 \mathcal{P}_{\rm split}}{\partial t \partial x \partial z} = \frac{2\pi}{\sqrt{x}t^*}\sqrt{z}\mathcal{K}(z)$$



$$\frac{\partial^2 \mathcal{P}_{\rm split}}{\partial t \partial x} = 2\pi \frac{1}{\sqrt{x}t^*} \int dz \sqrt{z}\mathcal{K}(z)$$

$$\frac{1}{t^*} = \frac{\alpha_s}{\pi}\sqrt{\frac{\hat{q}}{p_0^+}}$$

$$\sqrt{x}t^* \propto t_{{\rm br},m}$$

Generalization of BDMPS-Z approach

Scattering Kernels

Used right now:

$$w_g(\mathbf{q}) = \frac{16\pi^2\alpha_s^2 N_c n_{\rm med}}{\mathbf{q}^4}$$

$$w_g(\mathbf{q}) = \frac{g^2 m_D^2 T}{\mathbf{q}^2(\mathbf{q}^2 + m_D^2)}\,, \qquad g^2 = 4\pi\alpha_s$$

$n_{\rm med}\ldots$ density of scatterers
$m_D\ldots$ Debye mass
$T\ldots$ medium temperature

$$w_q(\mathbf{q}) = \frac{C_F}{C_A} w_g(\mathbf{q})$$

Monte-Carlo algorithms

Probabilities of interaction:

$$\Phi_g(x) = \alpha_s \int_{\epsilon}^{1-\epsilon} dz \int_{q>0} \frac{d^2\mathbf{q}}{(2\pi)^2} \left[\mathcal{K}_{gg}(\mathbf{q}, z, xp_+) + \mathcal{K}_{qg}(\mathbf{q}, z, xp_+) \right] + \int_{q>q_{\rm min}} \frac{d^2\mathbf{q}}{(2\pi)^2} w_g(\mathbf{q}) \,,$$

$$\Phi_q(x) = \alpha_s \int_{\epsilon}^{1-\epsilon} dz \int_{q>0} \frac{d^2\mathbf{q}}{(2\pi)^2} \mathcal{K}_{qq}(\mathbf{q}, z, xp_+) + \int_{q>q_{\rm min}} \frac{d^2\mathbf{q}}{(2\pi)^2} w_q(\mathbf{q}) \,,$$

Probability of no interaction for particle A over time (t_2-t_1):

$$\Delta_A(x, t_2 - t_1) = \exp\left(-\Phi_A(x)(t_2 - t_1)\right)$$

… Sudakov factor

TMDICE code:
- Written in C++
- Source code available at

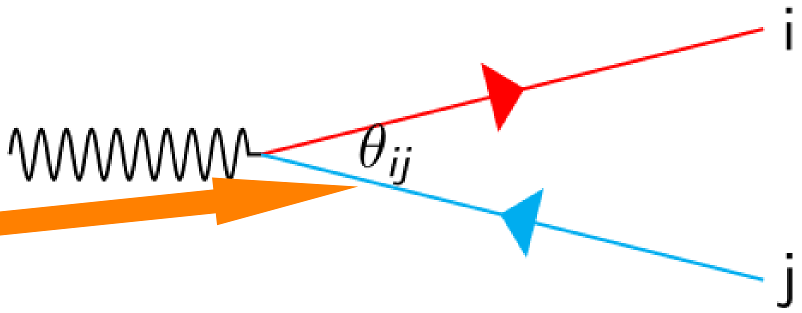
https://github.com/Rohrmoser/TMDICE

Other codes implementing BDMPS-Z spectra:

MARTINI, JEWEL, QPYTHIA, …

Emission of Bremsstrahlung

- In vacuum → parton cascades in vacuum
- In medium?

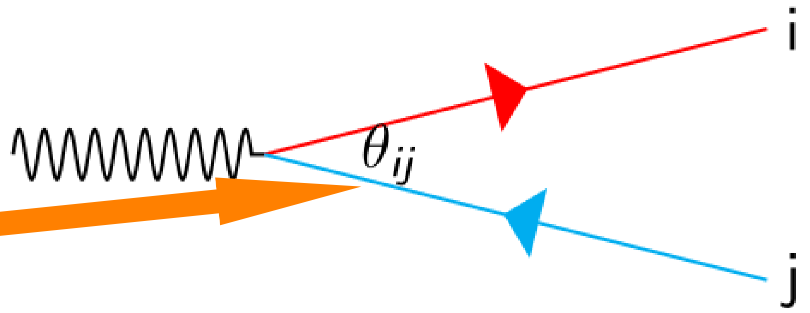


Individual colors of partons may not be resolvable by medium particles

Color Resolution

$$\hat{q} = \frac{\mathrm{d}\langle k_\perp^2 \rangle}{\mathrm{d}t} .$$

- Transverse length of dipole: $r_t \sim \theta_{ij} t$
- Momentum transfer from medium: $Q_s = \sqrt{\hat{q}\, t}$



Individual colors of partons may not be resolvable by medium particles

$$t_{decoh} \approx \left(\frac{12}{\hat{q}\,\theta_{ij}^2}\right)^{1/3}$$

[Mehtar-Tani, Salgado, Tywoniuk, JHEP 10 (2012)197]

Branching time

Uncertainty principle in particle rest-frame:

$$\Delta m \Delta t_{\mathrm{rest}} \geq \frac{\hbar}{2}$$

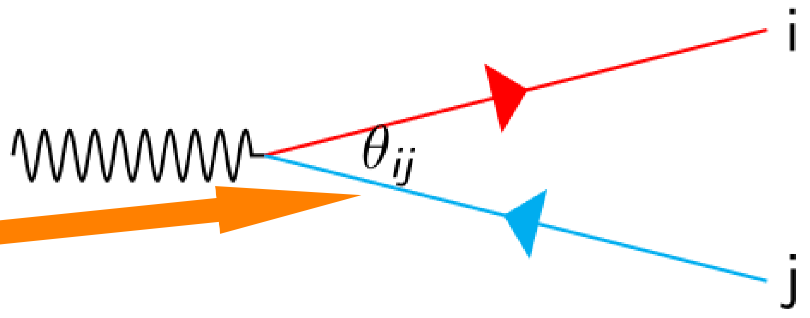
$\Rightarrow$ Estimation of particle life-time in the rest frame:

$$\Delta t_{\mathrm{rest}} \sim \frac{\hbar}{2\Delta m} \sim \frac{1}{Q}$$

Branching time, boosted to Lab-frame:

$$\boxed{t_{br} \approx \frac{2E}{Q^2}}$$

Color Coherence



Individual colors of partons may not be resolvable by medium particles

Phenomenological estimation of color resolution via Decoherence time:

$$t_{\mathrm{decoh}} \approx \left(\frac{12}{\hat{q}\theta_{ij}^2}\right)^{1/3} , \qquad \hat{q} = \frac{\mathrm{d}\langle k_\perp^2 \rangle}{\mathrm{d}t} .$$

and Branching time: $t_{\mathrm{br}} \approx 2E_i/Q_i^2 .$

No color resolution if $t_{\mathrm{br}} < t_{\mathrm{decoh}}$
=> Branching as in Vacuum

Vacuum Like Emissions (VLE)

DGLAP-Evolution as in Vacuum
Branching probability:

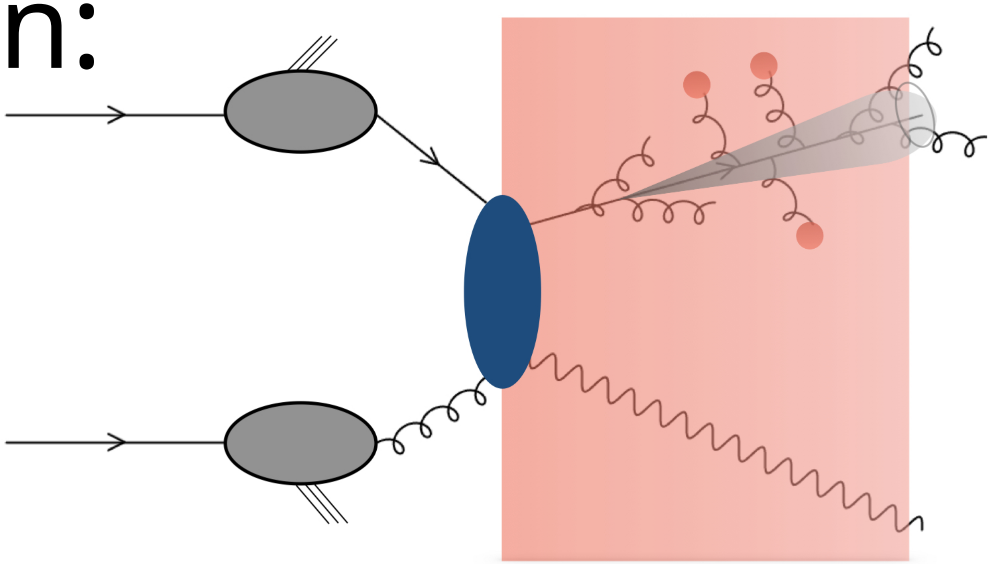
$$P_{qq}(z) = C_F \frac{1+z^2}{1-z}\,,$$
$$P_{gq}(z) = P_{qq}(1-z)\,,$$
$$P_{qg}(z) = T_R\left[z^2 + (1-z)^2\right]\,,$$
$$P_{gg}(z) = C_A\left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right]\,.$$

$$\frac{\mathrm{d}^2\mathcal{P}_{ji}}{\mathrm{d}Q^2\mathrm{d}z} = \frac{\alpha_s}{2\pi}\frac{1}{Q^2}P_{ji}(z)\,,$$

Evolve as long as $t_{\mathrm{br}} < t_{\mathrm{decoh}}$ or $t_{\mathrm{decoh}} < t_{\mathrm{L}}$,

Then: In-Medium Evolution

Jet-photon Production:



Cross section =
PDF*TMD
*hard cross section
*fragmentation of jet

Here via KATIE

[van Hameren: Comput.Phys.Commun. 224 (2018) 371-380]

k_T-factorization:

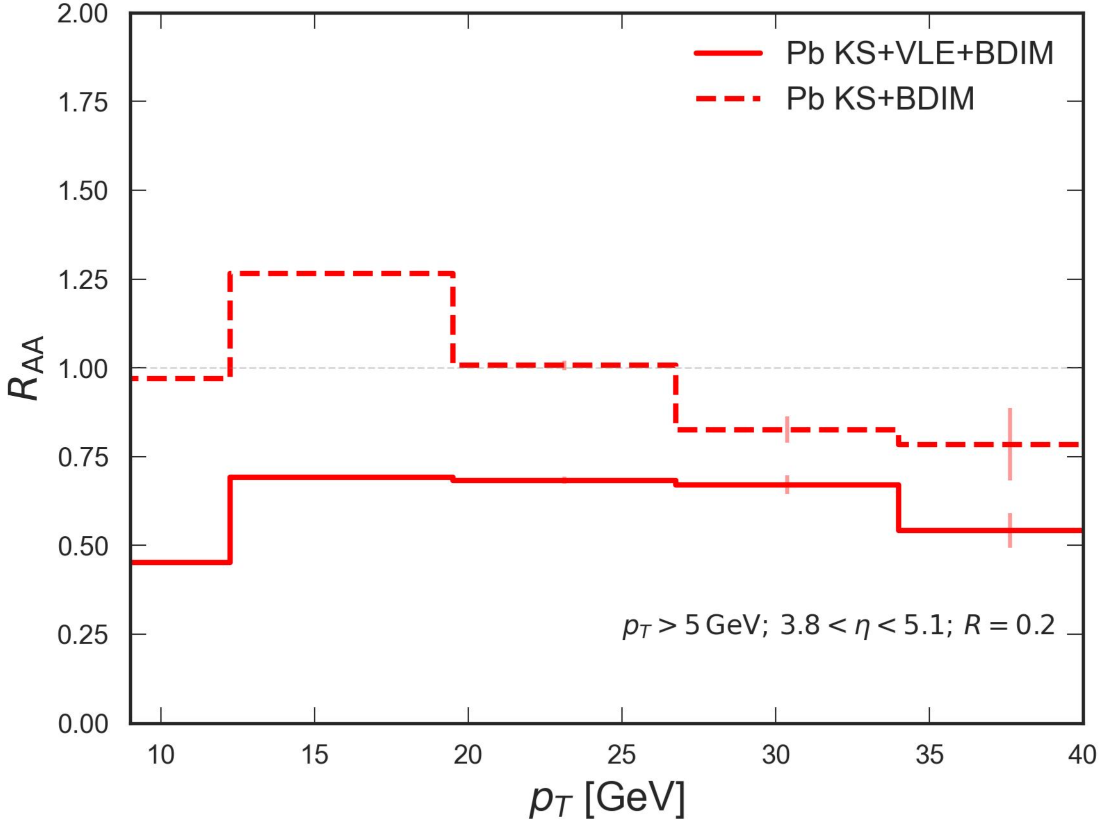
$$\frac{d\sigma^{AA\to\gamma+\text{jet}+X}}{dy_1 dy_2 dp_{1T} dp_{2T} d\Delta\phi} = \frac{p_{1T}p_{2T}}{8\pi^2(x_1x_2s)^2}\sum_a x_1 f_{a/A}(x_1,\mu_F^2)\,|\mathcal{M}^{\text{off-shell}}_{ag^*\to\gamma a}|^2\mathcal{F}(x_2,k_{2T}^2,\mu_F^2)$$

cf. [I. Ganguli, A. van Hameren, P. Kotko, K. Kutak, Eur.Phys.J.C 83 (2023) 9, 868]

TMDs used:

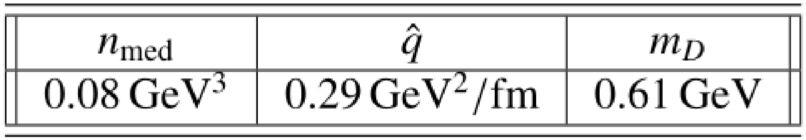
- NCTEQ [K. Kovarik, et al.,Phys. Rev. D 93 (8)(2016) 085037]
- Pb KS [K. Kutak, S.Sapeta: Phys. Rev. D 86 (2012) 094043, M. A. Al-Mashad, A. van Hameren, H. Kakkad, P. Kotko, K. Kutak,P. van Mechelen, S. Sapeta, JHEP 12 (2022) 131]
- p KS [K. Kutak, S.Sapeta: Phys. Rev. D 86 (2012) 094043, M. A. Al-Mashad, A. van Hameren, H. Kakkad, P. Kotko, K. Kutak,P. van Mechelen, S. Sapeta, JHEP 12 (2022) 131]

Photon-jet production



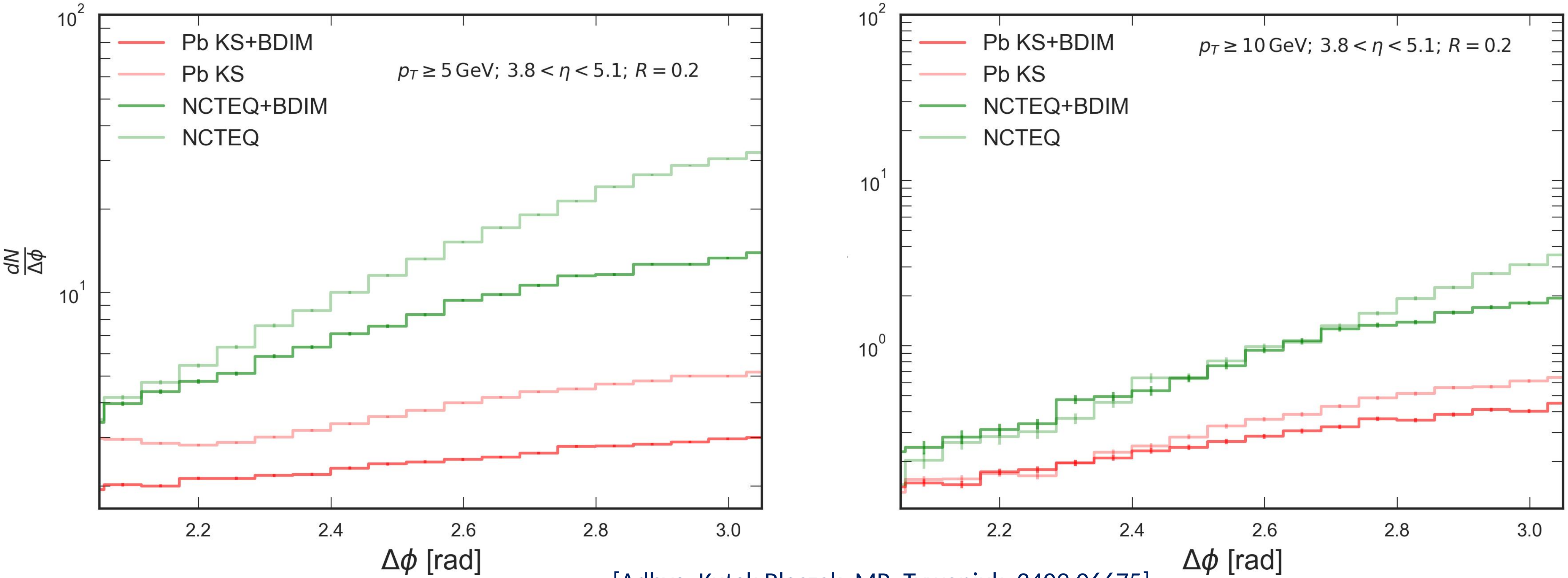
[Adhya, Kutak,Płaczek, MR, Tywoniuk: 2409.06675]

L= 5fm T=250MeV :



n_{med}	$\hat{q}$	m_D
$0.08\,\mathrm{GeV}^3$	$0.29\,\mathrm{GeV}^2/\mathrm{fm}$	$0.61\,\mathrm{GeV}$

Azimuthal decorrelations (1/2)

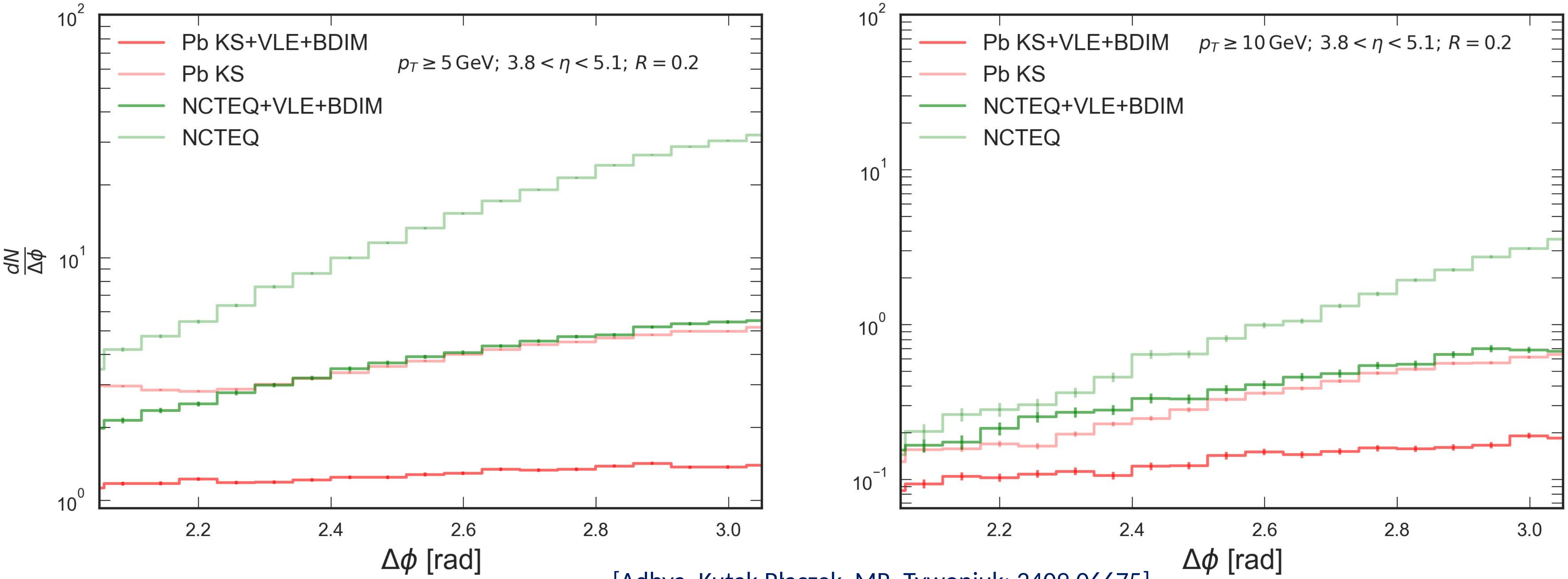


[Adhya, Kutak,Płaczek, MR, Tywoniuk: 2409.06675]

L= 5fm T=250MeV :

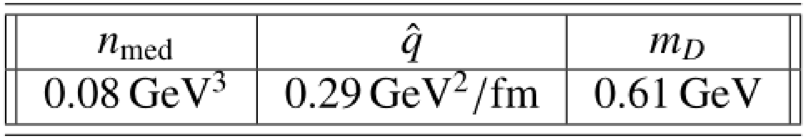
n_{med}	$\hat{q}$	m_D
$0.08\,\mathrm{GeV}^3$	$0.29\,\mathrm{GeV}^2/\mathrm{fm}$	$0.61\,\mathrm{GeV}$

Azimuthal decorrelations (2/2)



[Adhya, Kutak,Płaczek, MR, Tywoniuk: 2409.06675]

L= 5fm T=250MeV :



n_{med}	$\hat{q}$	m_D
$0.08\,\mathrm{GeV}^3$	$0.29\,\mathrm{GeV}^2/\mathrm{fm}$	$0.61\,\mathrm{GeV}$

Summary & Outlook:

- Estimation of photon-jet events in forward direction (FOCAL-range) via Monte-Carlo algorithms
- Quenching: k_T Broadening and jet suppression.

  → Emissions at low energies
- Inclusion of VLE in quenching:

  → VLE yield strong suppression broadening effects
- Gluon-saturation effects survive after jet quenching.

Outlook:

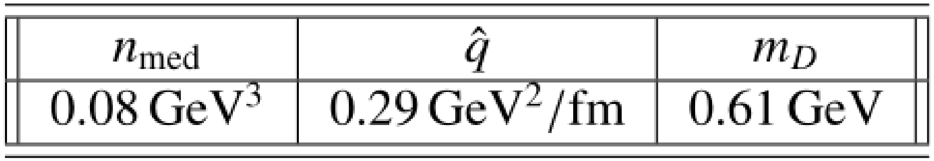
- More realistic Media (e.g.: expanding media; Temperature profiles)
- Study dijets.

Thank you for your attention!

Back-up slides

Parameters

- Medium: continuous field of length L=5fm and constant jet medium interactions:



n_{med}	$\hat{q}$	m_D
$0.08\,\text{GeV}^3$	$0.29\,\text{GeV}^2/\text{fm}$	$0.61\,\text{GeV}$

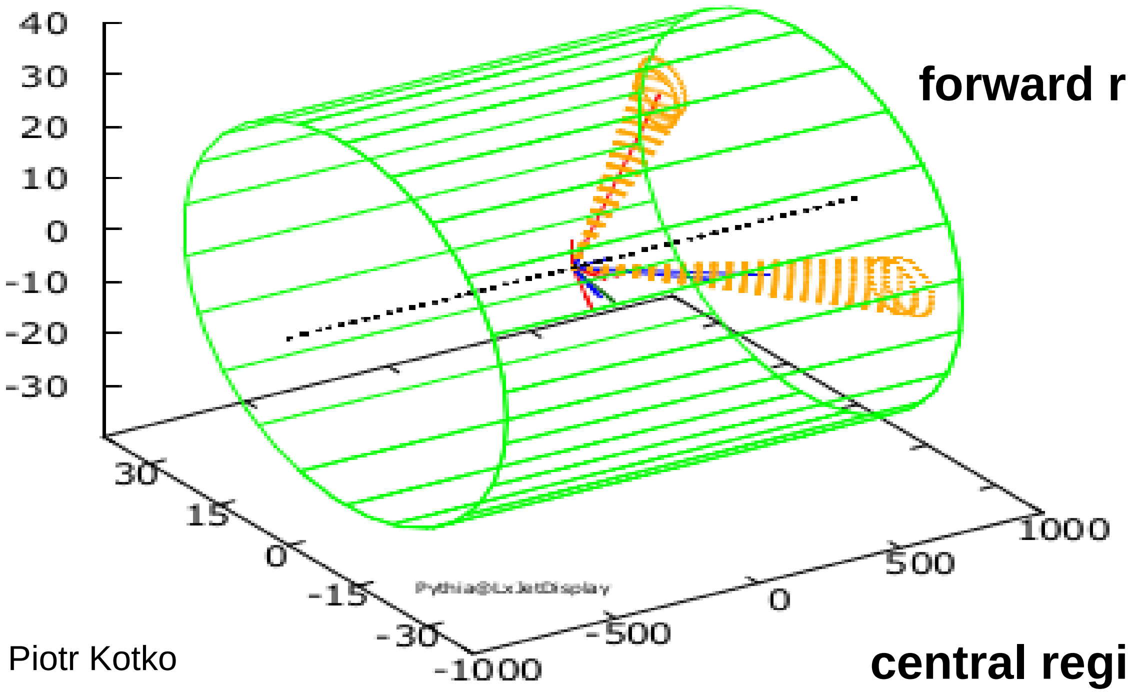
- TMD without saturation effects: NCTEQ

[E. Blanco, A. van Hameren, H. Jung, A. Kusina, K. Kutak, Phys. Rev. D 100 (5) (2019) 054023]

- TMD with saturation effects: Pb KS

[K. Kutak, S. Sapeta, Phys. Rev. D 86 (2012) 094043]

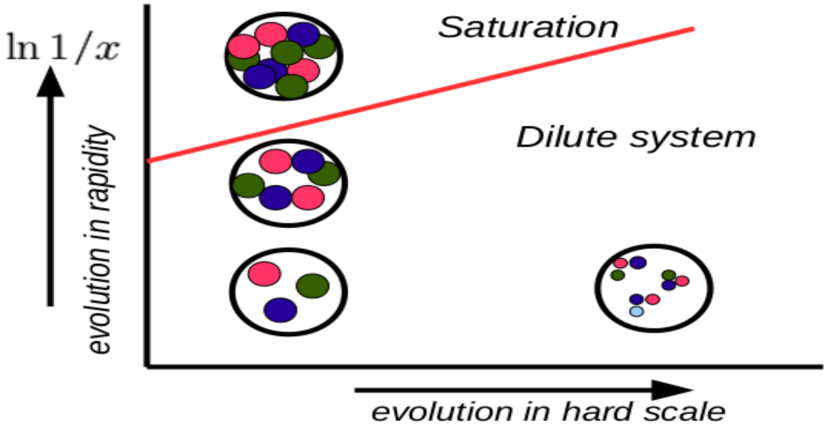
p – A (dilute-dense) forward-forward di-jets



forward region

central region

From: Piotr Kotko
LxJet



It originated from the aim to provide predictions for forward-forward jet production at the LHC

The saturation problem: supressing gluons below Qs

Originally formulated in coordinate space
Balitsky '96, Kovchegov '99

Fit AAMQS '10

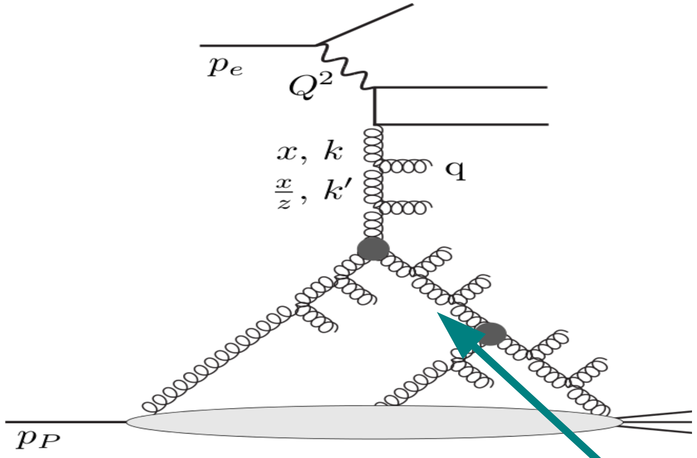
NLO accuracy
Balitsky, Chirilli '07

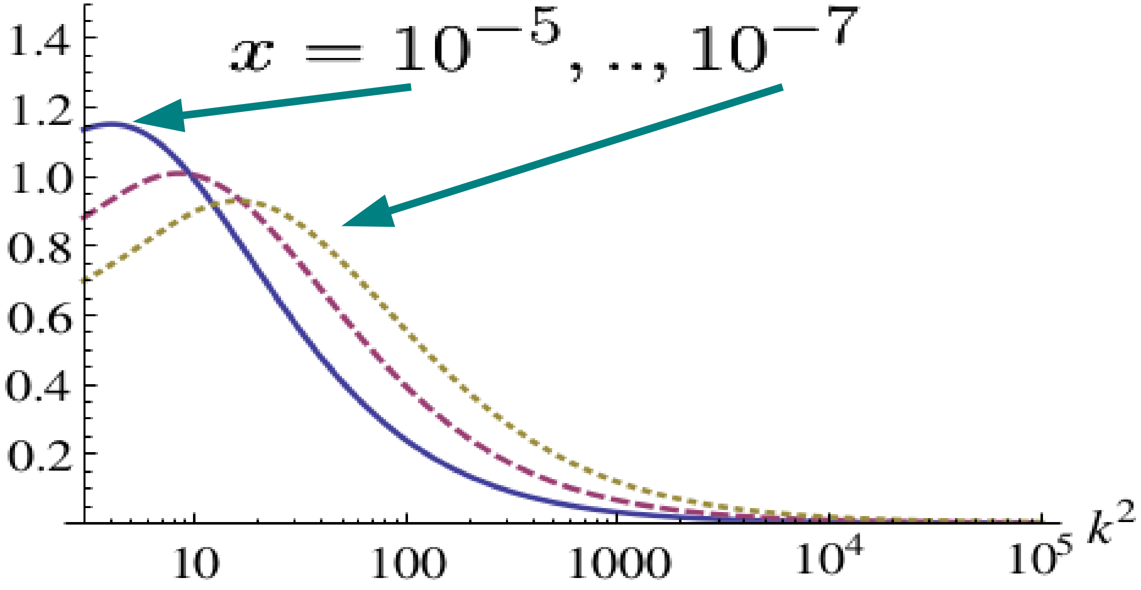
and solved
Lappi, Mantysaari '15

Kinematic corrections
Iancu at el

Solved b dependent
Stasto, Golec-Biernat '02

with kinematic
corrections and b
Cepila, Contrares, Matas '18





solution of Balitsky-Kovchegov directly for dipole gluon density

Kwiecinski, Kutak '02
Nikolaev, Schafer '06

Fit to F_2 data
KK. Sapeta '12

The momentum space BK equation for dipole gluon density

$$\mathcal{F} = \mathcal{F}_0 + K \otimes \mathcal{F} - \frac{1}{R^2} V \otimes \mathcal{F}^2$$

hadron's radius

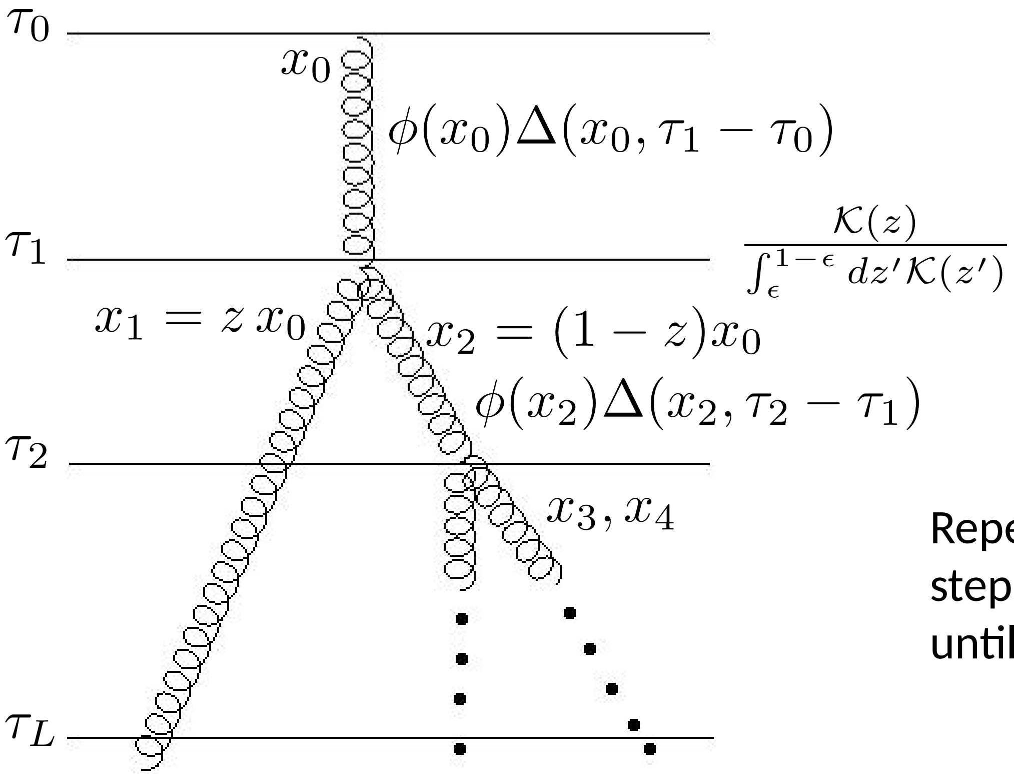
Accounts for kinematical constraint,

Nonsingular parts of splitting function, running coupling

Monte-Carlo algorithms

Other codes implementing BDMPS-Z spectra:

MARTINI, JEWEL, QPYTHIA, …



Analogous for the k_T dependent equation in x, k_T, and, τ and system of equations!

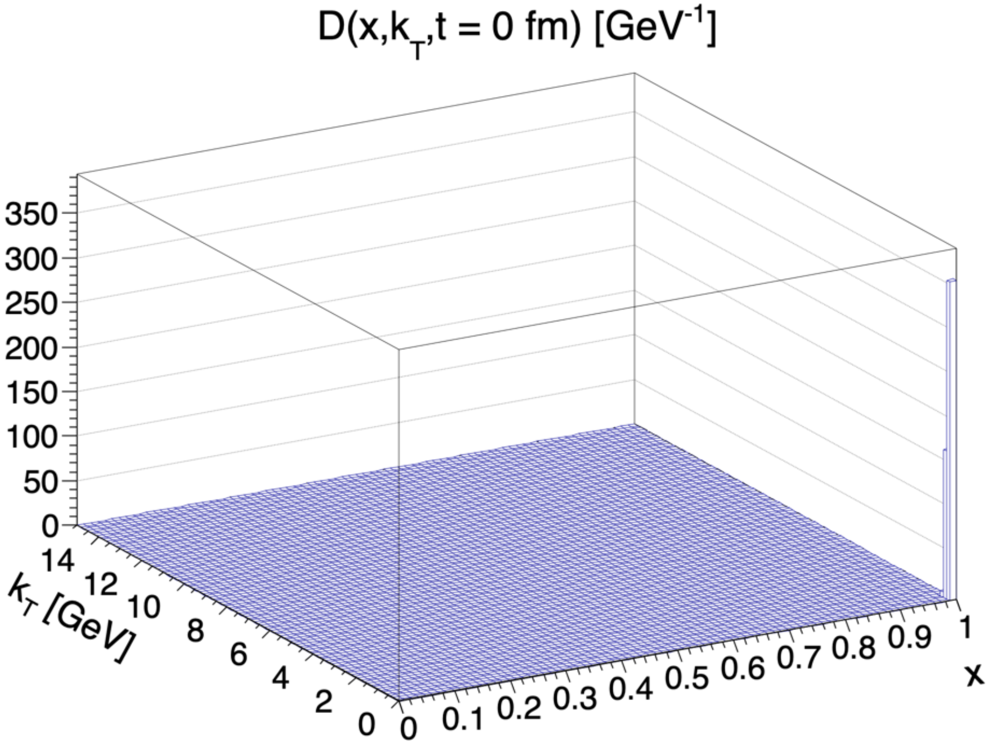
Repeat for all steps in τ and x until $\tau > \tau_L$

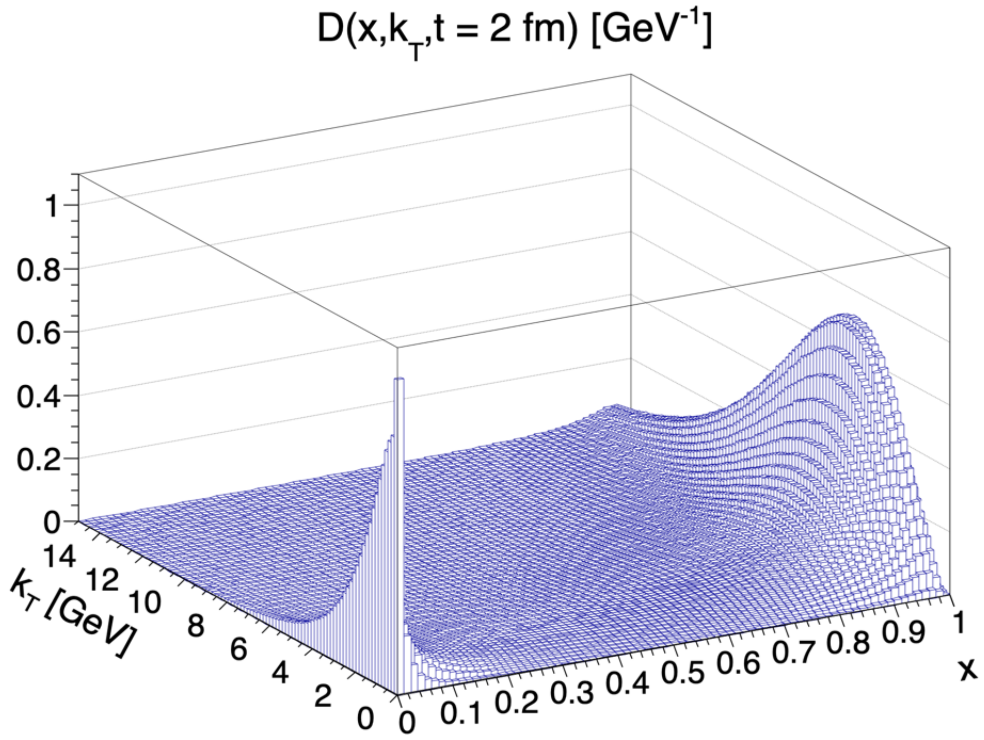
TMDICE code: [MR, Comput.Phys.Commun. 276 (2022) 108343]

- Written in C++
- Source code available at https://github.com/Rohrmoser/TMDICE

Evolution of $D(x,k_T,t)$

$$D(x,k_T,t) = x\frac{\partial^2 N(t)}{\partial x \partial k_T}$$

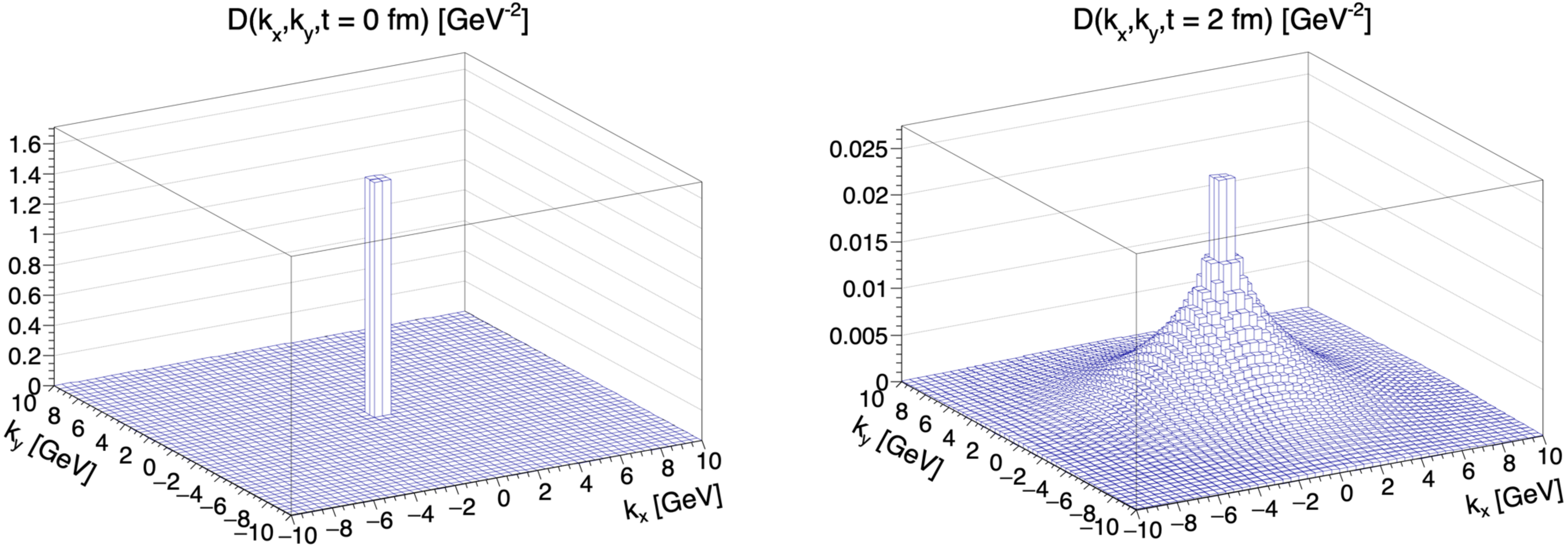




[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Evolution of D(x,k_T,t)

$$\mathcal{K}(z) \qquad w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2+m_D^2)}$$



[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Departure from Gaussian broadening

Both:

- Parton Branching
- Scattering

Central Limit Theorem does not necessarily apply.

$$p \to z_1 p \to z_1 p + \boldsymbol{q}_1$$
$$\to z_1 p + \boldsymbol{q}_1 + \boldsymbol{q}_2$$
$$\to z_2(z_1 + \boldsymbol{q}_1 + \boldsymbol{q}_2) \to \ldots$$

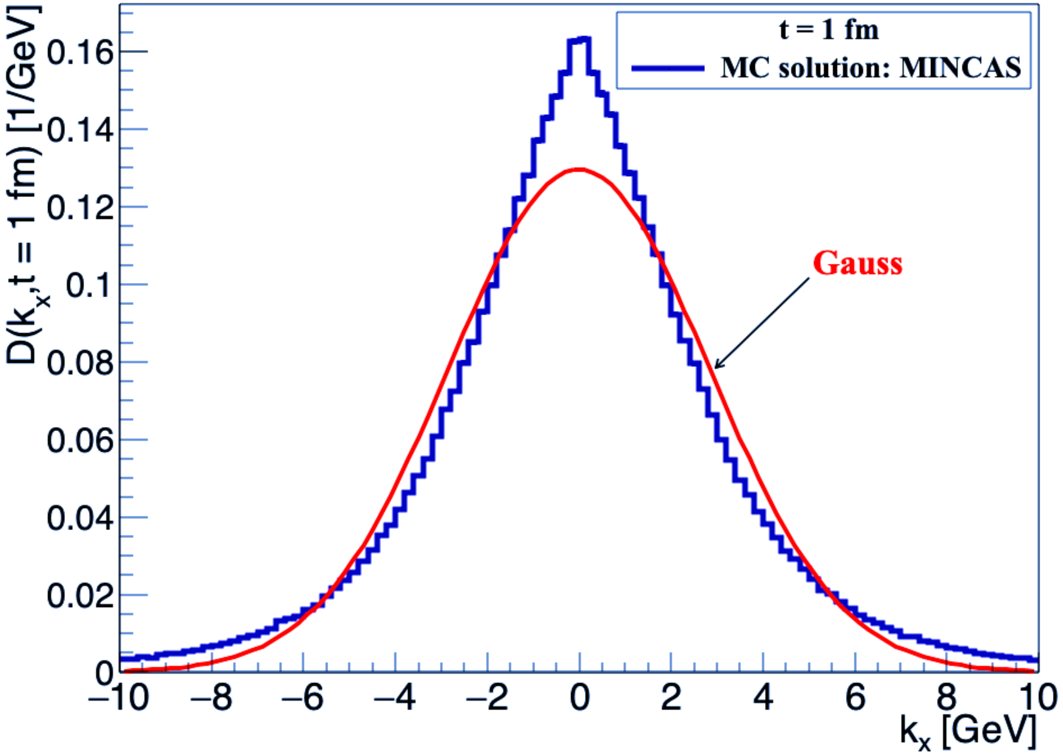
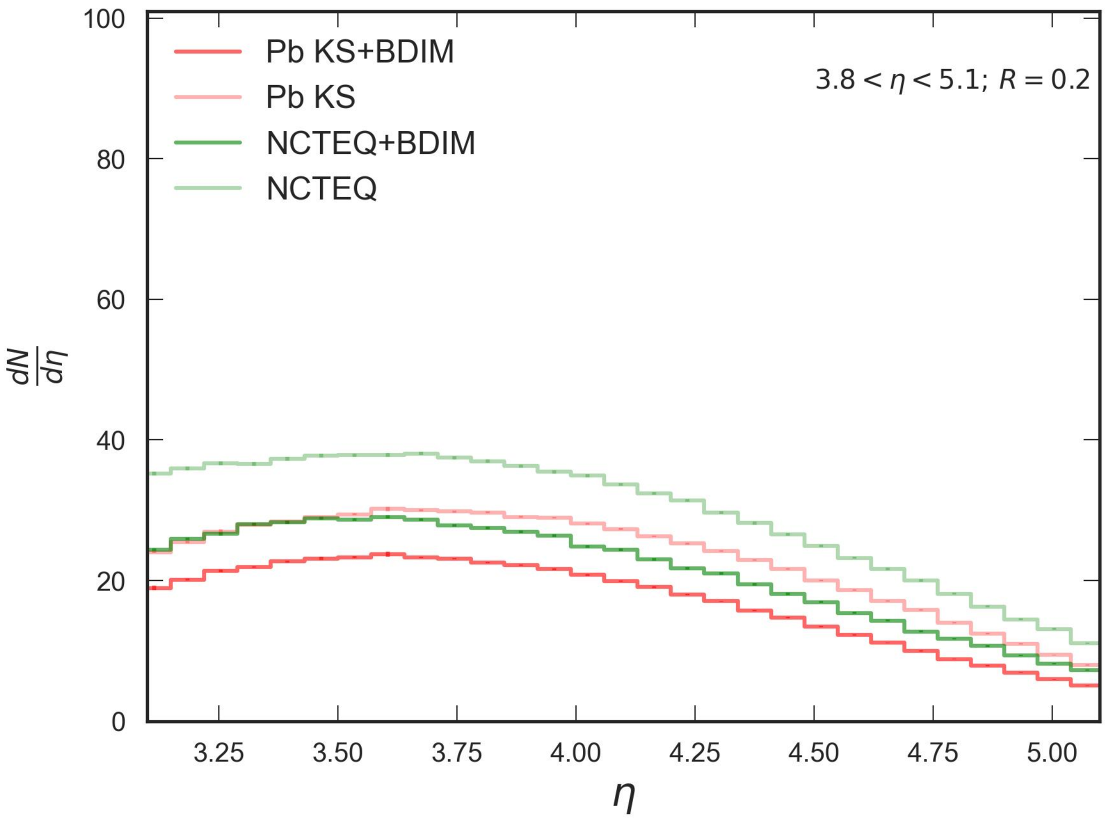
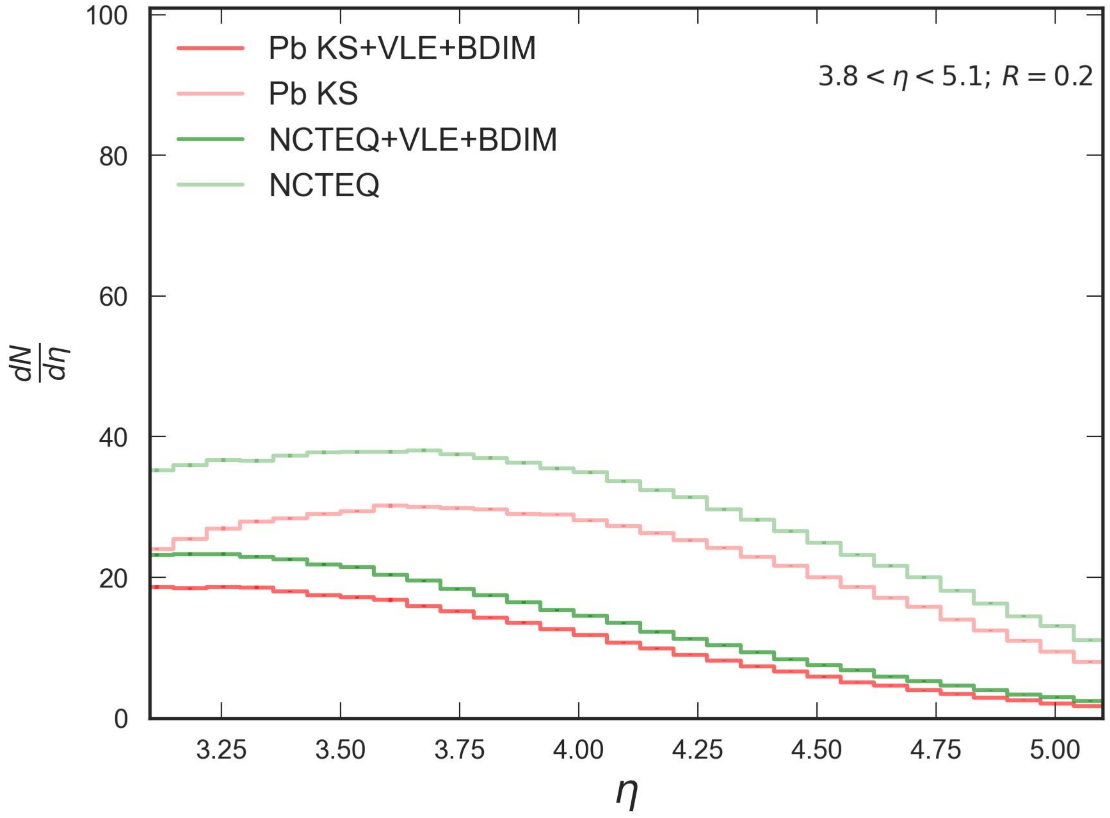


Figure: [Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Rapidity spectra





[Adhya, Kutak,Płaczek, MR, Tywoniuk: 2409.06675]

Jet Production

Factorization for AA collisions:

$$\frac{d\sigma_{AA}}{d\Omega_p} = \int d\Omega_q \int d^2\boldsymbol{l} \int_0^1 \frac{d\tilde{x}}{\tilde{x}} \delta(p^+ - \tilde{x}q^+)\delta^{(2)}(\boldsymbol{p} - \boldsymbol{l} - \boldsymbol{q})D(\tilde{x}, \boldsymbol{l}, \tau(q^+))\frac{d\sigma_{pp}}{d\Omega_q}$$

$$d\Omega_q = dq^+ d^2\boldsymbol{q}$$

$$\tau(q^+) = \tfrac{\alpha_s N_c}{\pi}\sqrt{\tfrac{\hat{q}}{q^+}}L$$

↓

$$\frac{d^2\sigma_{AA}}{d\Omega_{p_1} d\Omega_{p_2}} = \int d\Omega_{q_1} \int d\Omega_{q_2} \int d^2\boldsymbol{l}_1 \int d^2\boldsymbol{l}_2 \int_0^1 \frac{d\tilde{x}_1}{\tilde{x}_1}\delta(p_1^+ - \tilde{x}_1 q_1^+) \int_0^1 \frac{d\tilde{x}_2}{\tilde{x}_2}\delta(p_2^+ - \tilde{x}_2 q_2^+)$$

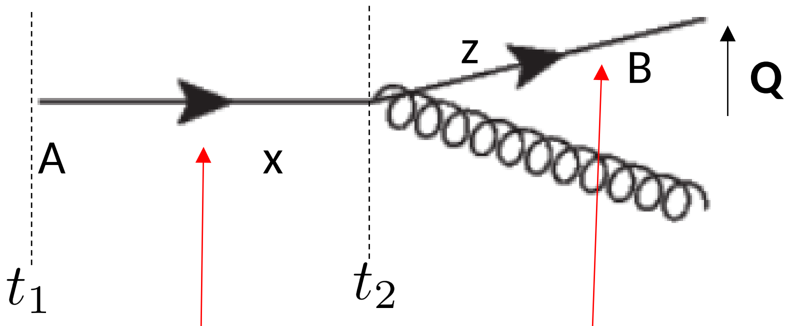
$$\delta^{(2)}(\boldsymbol{p}_1 - \boldsymbol{l}_1 - \boldsymbol{q}_1)\delta^{(2)}(\boldsymbol{p}_2 - \boldsymbol{l}_2 - \boldsymbol{q}_2)D(\tilde{x}_1, \boldsymbol{l}_1, \tau(q_1^+))D(\tilde{x}_2, \boldsymbol{l}_2, \tau(q_2^+))\frac{d^2\sigma_{pp}}{d\Omega_{q_1} d\Omega_{q_2}}$$

Vacuum like emissions

$$\frac{\mathrm{d}^2\mathcal{P}_{ji}}{\mathrm{d}Q^2\mathrm{d}z} = \frac{\alpha_s}{2\pi}\frac{1}{Q^2}P_{ji}(z)\,,$$

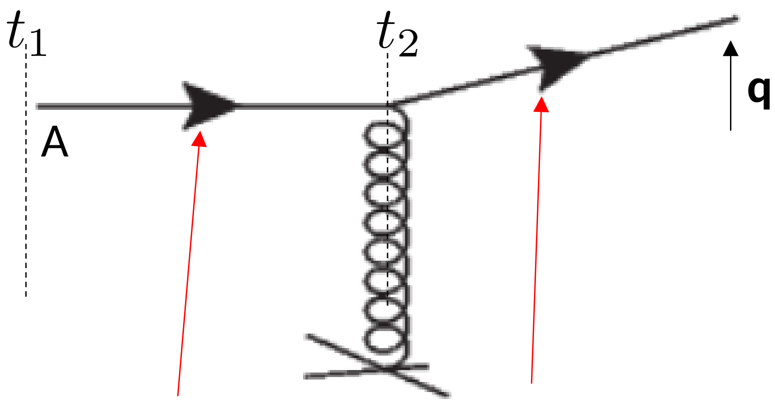
$$\begin{aligned}
P_{qq}(z) &= C_F\,\frac{1+z^2}{1-z}\,,\\
P_{gq}(z) &= P_{qq}(1-z)\,,\\
P_{qg}(z) &= T_R\left[z^2+(1-z)^2\right]\,,\\
P_{gg}(z) &= C_A\left[\frac{1-z}{z}+\frac{z}{1-z}+z(1-z)\right]\,.
\end{aligned}$$

Probabilities for interactions



$$\Delta_A(x,t_2-t_1)\left(\frac{\alpha_s}{(2\pi)^2}\mathcal{K}_{BA}(\mathbf{Q},z,xp_+)\right)$$

$$(\Delta_A(x,t_2-t_1)\phi_A(x))\underbrace{\left(\frac{\sum_B\rho_{BA}(x)}{\phi_A(x)}\right)}_{\text{Splitting probability}}\underbrace{\left(\frac{\rho_{BA}(x)}{\sum_B\rho_{BA}(x)}\right)}_{\text{Probability for splitting into B}}\left(\frac{1}{\rho_{BA}(x)}\frac{\alpha_s}{(2\pi)^2}\mathcal{K}_{BA}(\mathbf{Q},z,xp_+)\right)$$



$$\Delta_A(x,t_2-t_1)\left(\frac{1}{(2\pi)^2}w_A(\mathbf{q})\right)$$

$$(\Delta_A(x,t_2-t_1)\phi_A(x))\underbrace{\left(\frac{W_A}{\phi_A(x)}\right)}_{\text{Scattering probability}}\left(\frac{1}{W_A}\frac{1}{(2\pi)^2}w_A(\mathbf{q})\right)$$

$$\rho_{BA}(x)=\alpha_s\int_\epsilon^{1-\epsilon}dz\int_{q>0}\frac{d^2q}{(2\pi)^2}\mathcal{K}_{BA}(\mathbf{q},z,xp_+)$$

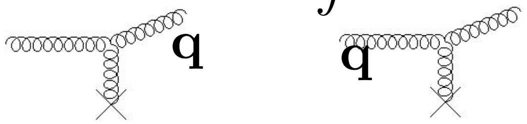
$$W_A=\int\frac{d^2\mathbf{q}}{(2\pi)^2}w_A(\mathbf{q})$$

BDIM Equation for Gluons

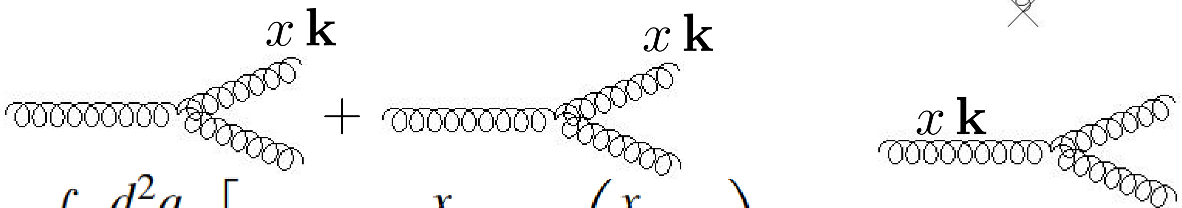
[Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1406 (2014) 075]

Scattering:

$$C(\boldsymbol{q}) = w(\boldsymbol{q}) - \delta(\boldsymbol{q}) \int d^2\boldsymbol{q}' w(\boldsymbol{q}')$$



$$D(x, \mathbf{k}, t) = x \frac{\partial^3 N(x,\mathbf{k},t)}{\partial x \partial^2 \mathbf{k}}$$



For gluon-jets:

$$\frac{\partial}{\partial t} D(x,\mathbf{k},t) = \alpha_s \int_0^1 dz \int \frac{d^2q}{(2\pi)^2} \left[2\mathcal{K}(\mathbf{Q}, z, \frac{x}{z} p_0^+) D\left(\frac{x}{z}, \mathbf{q}, t\right) - \mathcal{K}(\mathbf{q}, z, x p_0^+) D(x, \mathbf{k}, t) \right] + \int \frac{d^2\mathbf{l}}{(2\pi)^2} C(\mathbf{l}) D(x, \mathbf{k} - \mathbf{l}, t).$$

↓ Average Kernels over $\mathbf{Q}$

$$\frac{\partial}{\partial t} D(x, \boldsymbol{k}, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\boldsymbol{k}}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, \boldsymbol{k}, t) \right] + \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} C(\boldsymbol{q}) D(x, \boldsymbol{k} - \boldsymbol{q}, t)$$

↓ Integrate over $\mathbf{k}$

$$\frac{\partial}{\partial t} D(x, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) - \frac{z}{\sqrt{x}} D(x, t) \right]$$

$$D(x,t) = \int d^2\mathbf{k} D(x, \mathbf{k}, t)$$

System of Equations for quarks and gluons

[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

$$\begin{aligned}\frac{\partial}{\partial t}D_g(x,\boldsymbol{k},t) = &\int_0^1 \mathrm{d}z \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}\alpha_s\Big\{2\mathcal{K}_{gg}\left(\boldsymbol{Q},z,\frac{x}{z}p_0^+\right)D_g\left(\frac{x}{z},\boldsymbol{q},t\right)+\mathcal{K}_{gq}\left(\boldsymbol{Q},z,\frac{x}{z}p_0^+\right)\sum_i D_{q_i}\left(\frac{x}{z},\boldsymbol{q},t\right)\\ &-\left[\mathcal{K}_{gg}(\boldsymbol{q},z,xp_0^+)+\mathcal{K}_{qg}(\boldsymbol{q},z,xp_0^+)\right]D_g(x,\boldsymbol{k},t)\Big\}+\int\frac{\mathrm{d}^2\boldsymbol{l}}{(2\pi)^2}\,C_g(\boldsymbol{l})\,D_g(x,\boldsymbol{k}-\boldsymbol{l},t),\\ \frac{\partial}{\partial t}D_{q_i}(x,\boldsymbol{k},t) = &\int_0^1 \mathrm{d}z \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}\alpha_s\Big\{\mathcal{K}_{qq}\left(\boldsymbol{Q},z,\frac{x}{z}p_0^+\right)D_{q_i}\left(\frac{x}{z},\boldsymbol{q},t\right)+\frac{1}{N_F}\mathcal{K}_{qg}\left(\boldsymbol{Q},z,\frac{x}{z}p_0^+\right)D_g\left(\frac{x}{z},\boldsymbol{q},t\right)\\ &-\mathcal{K}_{qq}(\boldsymbol{q},z,xp_0^+)\,D_{q_i}(x,\boldsymbol{k},t)\Big\}+\int\frac{\mathrm{d}^2\boldsymbol{l}}{(2\pi)^2}\,C_q(\boldsymbol{l})\,D_{q_i}(x,\boldsymbol{k}-\boldsymbol{l},t)\,,\end{aligned}$$

$$C_{q(g)}(\boldsymbol{l}) = w_{q(g)}(\boldsymbol{l}) - \delta(\boldsymbol{l})\int \mathrm{d}^2\boldsymbol{l}'\,w_{q(g)}(\boldsymbol{l}')$$