Chemical Freeze-Out in the QCD Phase Diagram

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In collaboration with:
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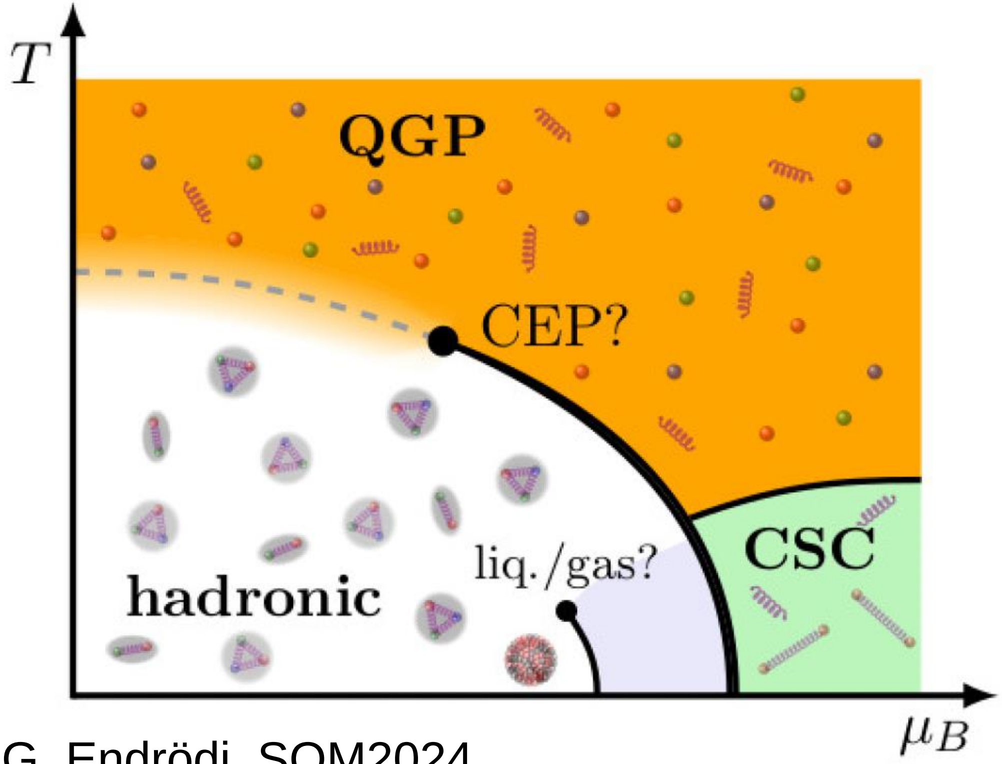
XVII Polish Workshop on Relativistic Heavy-Ion Collisions

Phase diagram and Equation of State of strongly interacting matter

Exploring the QCD Phase Diagram



G. Endrödi, SQM2024

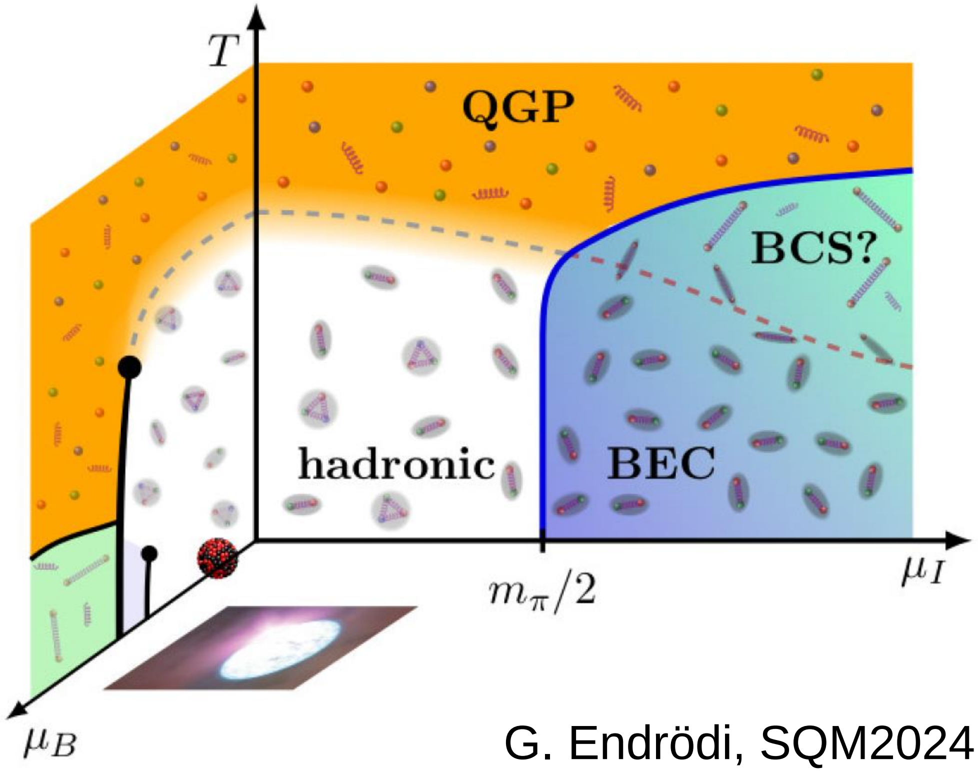
The Goal: Proving the phase structure!

Lattice QCD results only for pseudocritical temperature Tc near μ ~ 0 (sign problem)

Liquid-gas PT indicated in experiment

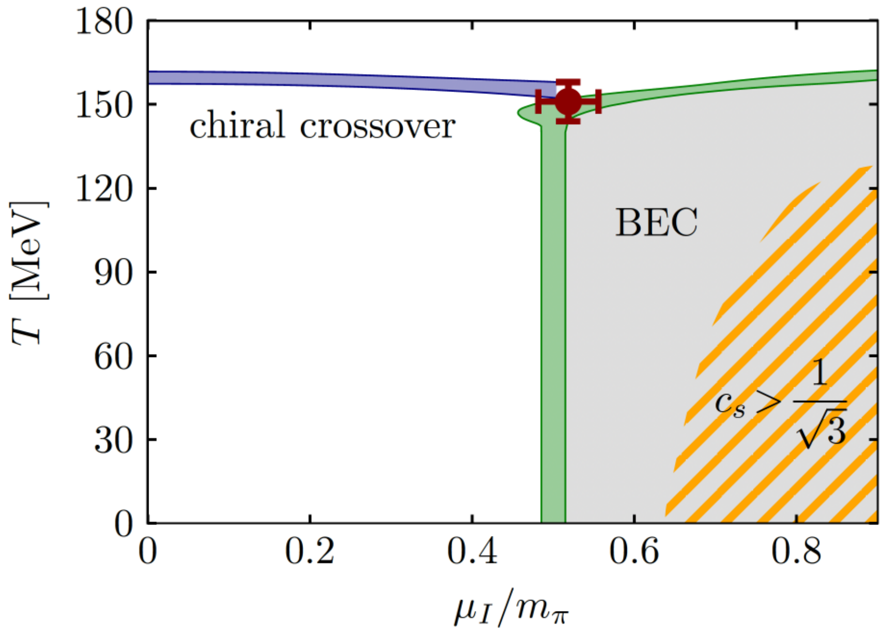
Other structures are so far model dependent conjectures!!

Exploring the QCD Phase Diagram



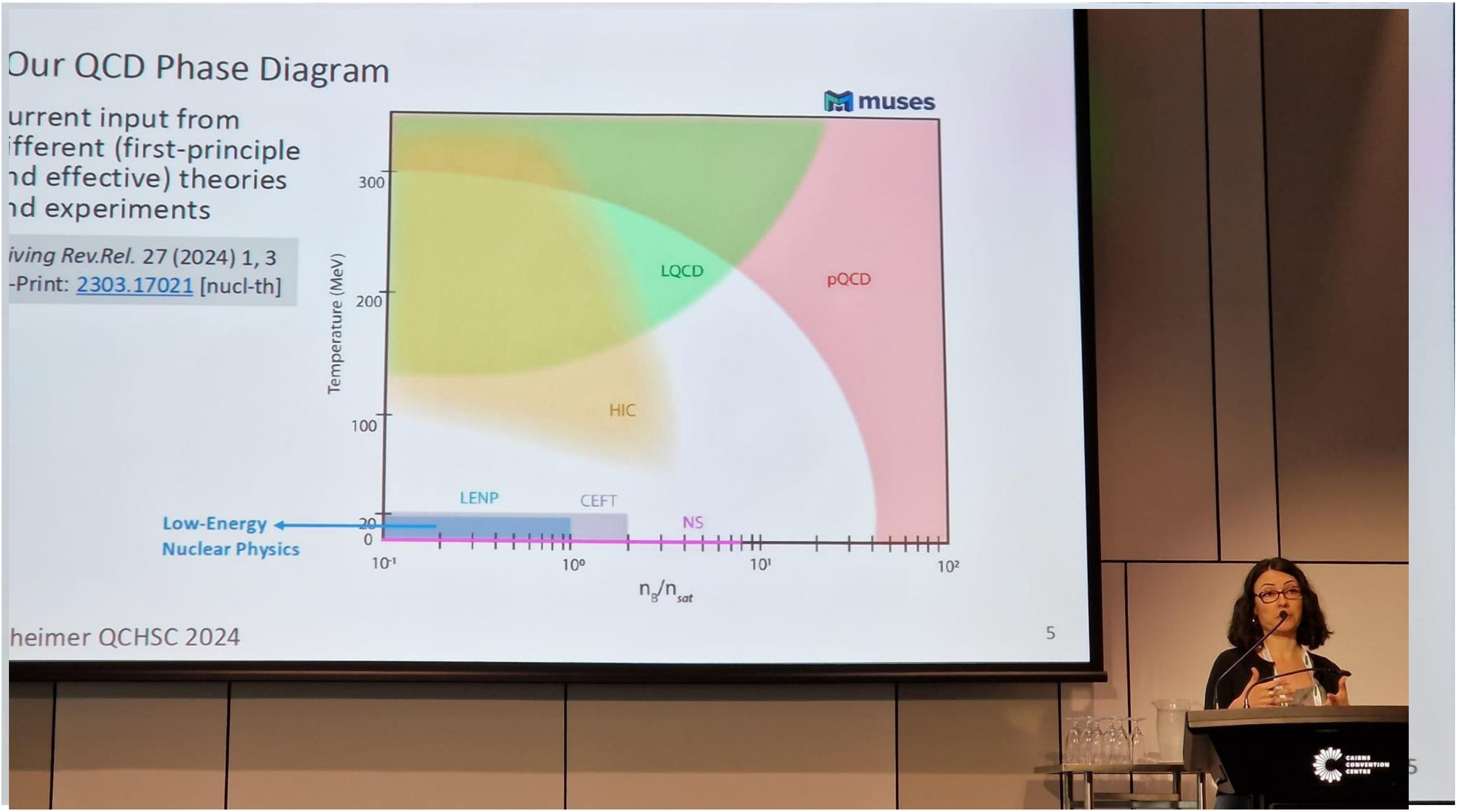
G. Endrödi, SQM2024

Isospin-QCD ? Lattice QCD results !!



Brandt, Endrödi, Schmalzbauer, PRD 97, 054514 (2018)

Exploring the QCD Phase Diagram



V. Dexheimer, QCHS2024

Exploring the QCD Phase Diagram

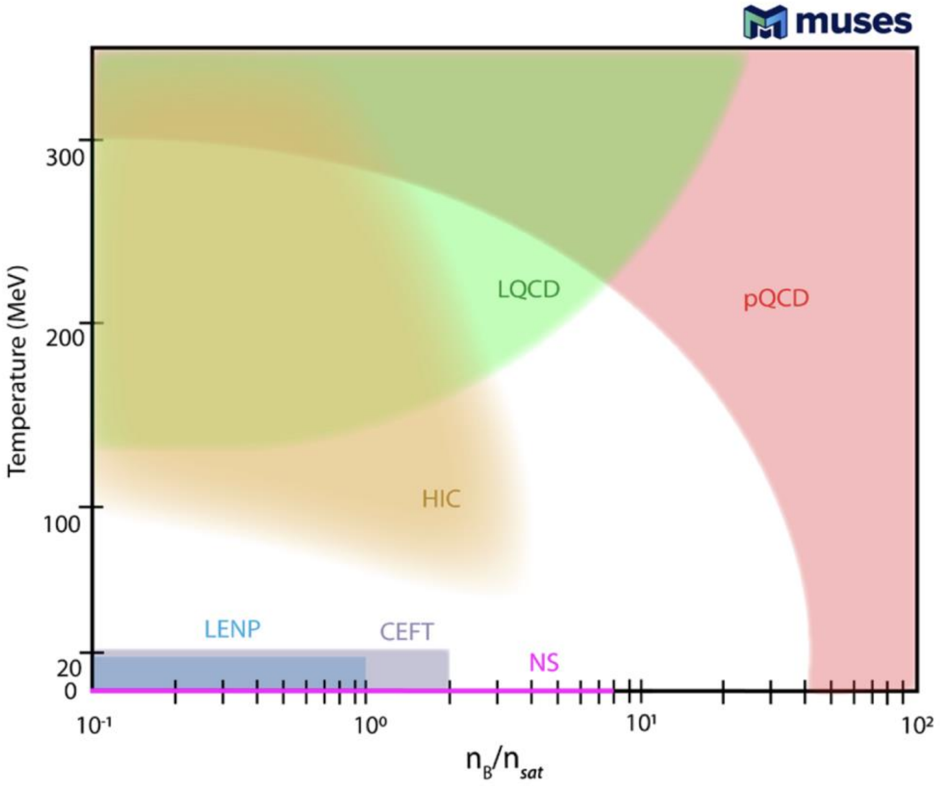
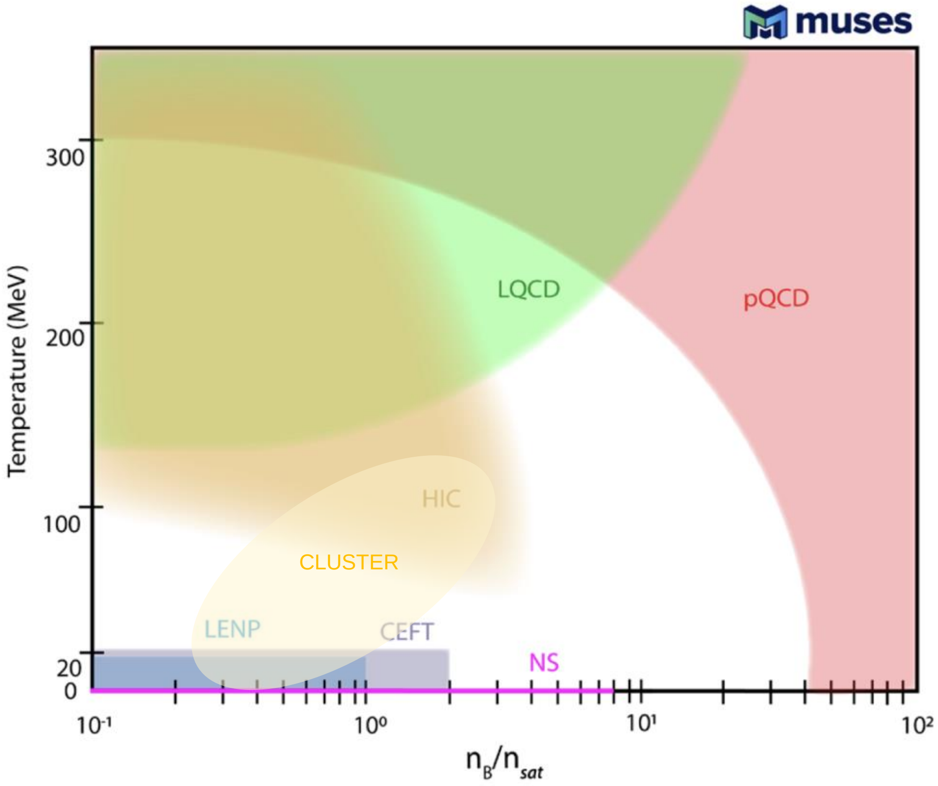


Fig. 1 Regions of the QCD phase diagram where constraints from heavy-ion collisions (HIC), lattice QCD (LQCD), perturbative QCD (pQCD), low-energy heavy-ion collisions (LENP), chiral effective field theory (χEFT), and astrophysics (neutron stars, NS) are available

Living Reviews in Relativity (2024)27:3
https://doi.org/10.1007/s41114-024-00049-6

Exploring the QCD Phase Diagram



This Talk:
Chemical Freeze-Out @ T ~ 20 – 100 MeV

- Statistical Model Fit, T – μ diagram

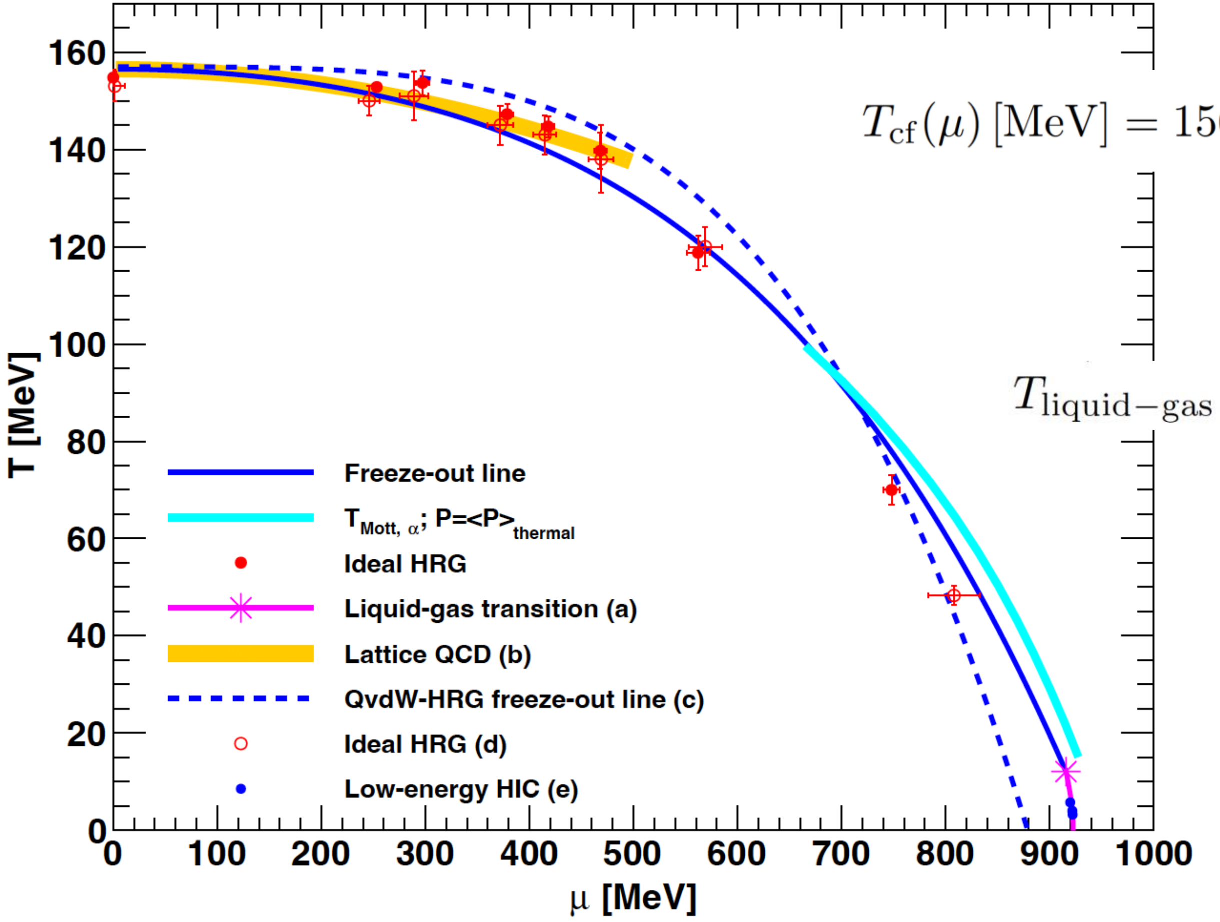
- CFO in the T – n diagram

- Mott dissociation for light clusters

- CFO as inverse Mott dissociation

- Summary & Outlook

Statistical Model Fit for CFO, T – μ Diagram



The freeze-out line (blue):

$$T_{\mathrm{cf}}(\mu)\,[\mathrm{MeV}] = 156.5 - 76.68\,(\mu\,[\mathrm{GeV}])^2 - 139.7\,(\mu\,[\mathrm{GeV}])^4$$

interpolates between lattice QCD chiral restoration crossover (yellow) and CEP of the liquid-gas transition

$$T_{\mathrm{liquid-gas}} = 12.1\,\mathrm{MeV},\ \mu_{\mathrm{liquid-gas}} = 915.61\,\mathrm{MeV}$$

Mott dissociation line for alphas is well correlated in baryon-dominant region

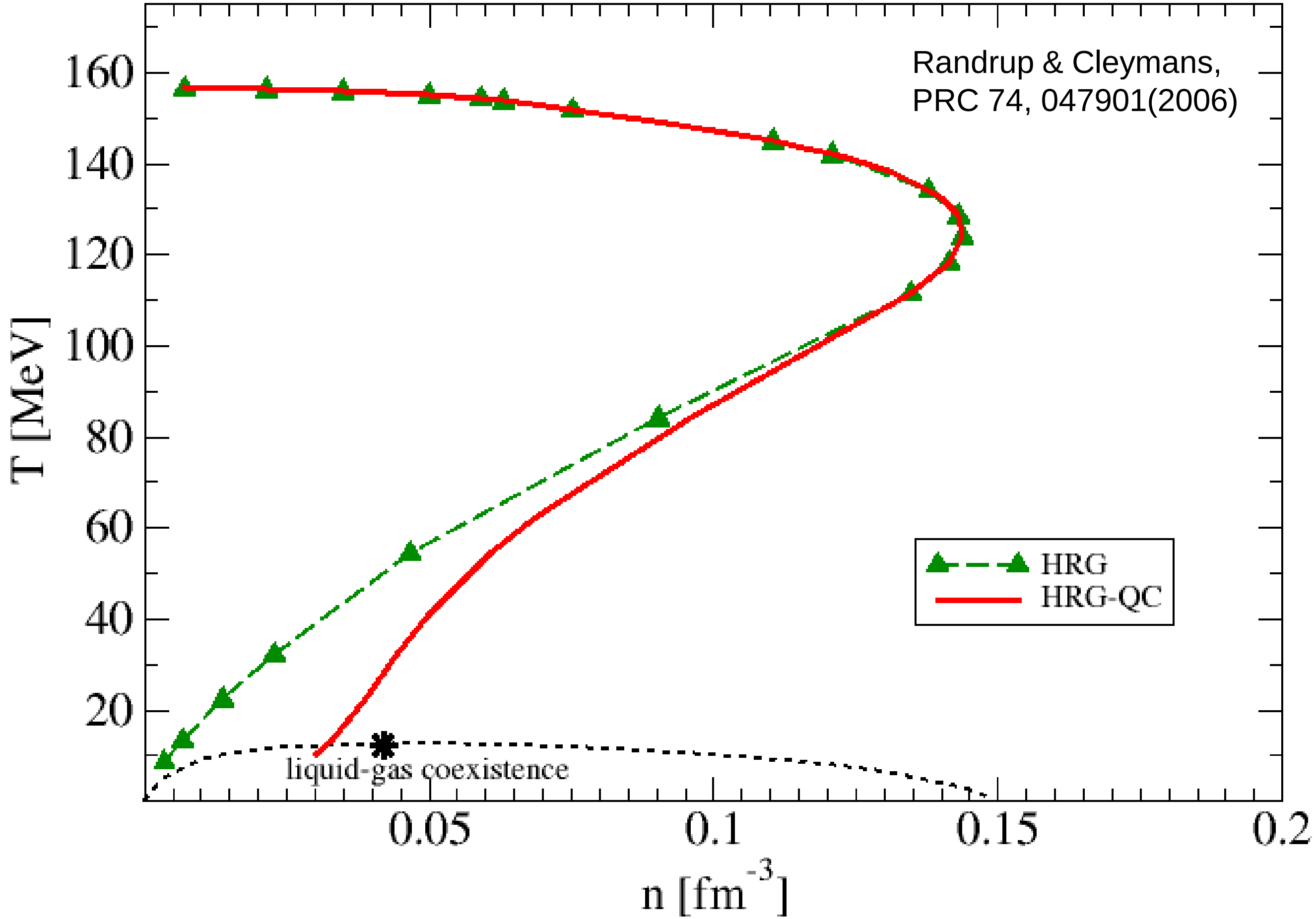
(a) Typel et al., PRC 81, 015803 (2010)

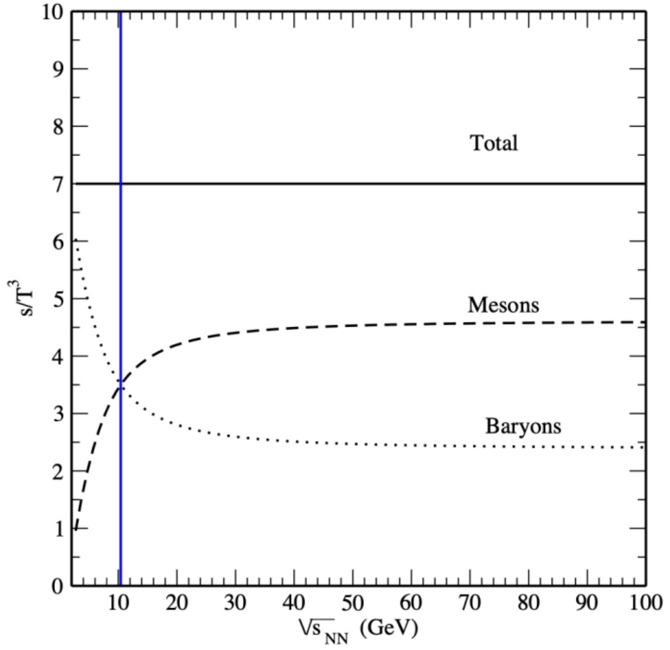
(c) Poberezhnyuk et al., PRC 100, 054904 (2019)

(e) Natowitz et al., PRL 104, 202501 (2010)

D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB accepted

Statistical Model Fit for CFO, T – n Diagram

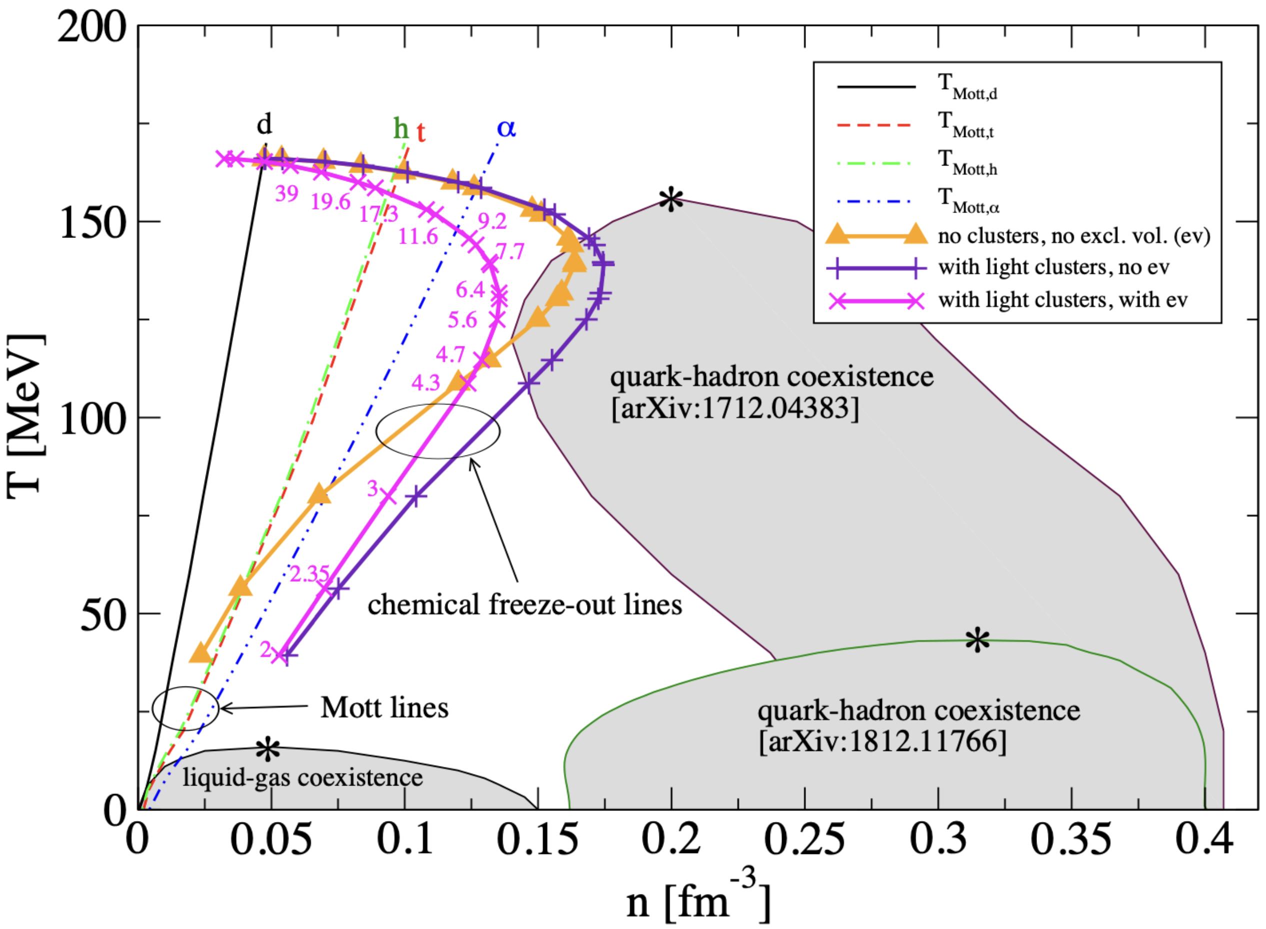




Transition meson- to baryon-dominance at T ~ 140 MeV

Andronic et al., NPA 837, 65 (2010)

Statistical Model Fit for CFO, T – n Diagram



Correlation of CFO line with chiral restoration/ deconfinement gets lost at T < 140 MeV

Is CFO in the baryon-dominated region correlated with Mott lines for dissociation of light clusters?

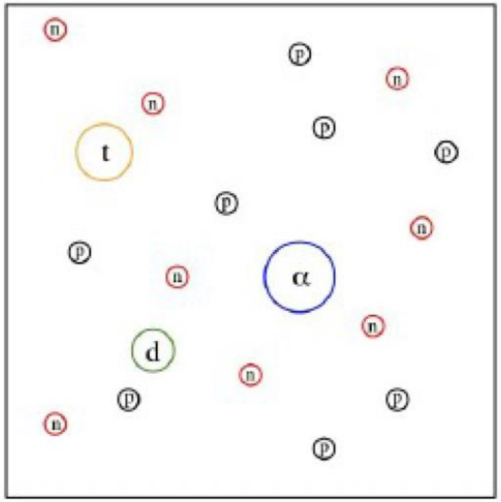
Blaschke et al., Springer Proc. Phys. 250, 183 (2020); arXiv:2001.02156 (SQM 2019)

Mott dissociation for bound states in a plasma

Chemical picture:

Ideal mixture of reacting components

Mass action law

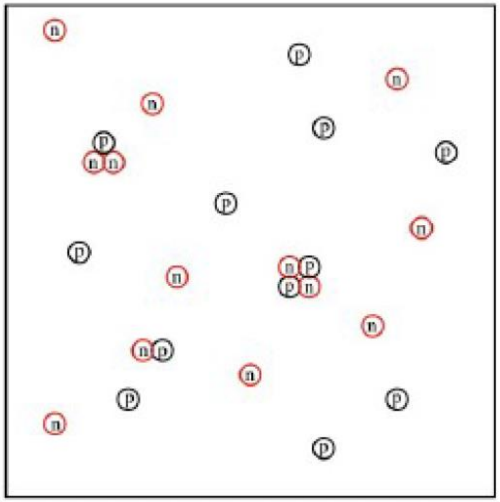


Interaction between the components internal structure: Pauli principle

Physical picture:

"elementary" constituents

and their interaction



Quantum statistical (QS) approach, quasiparticle concept, virial expansion

Mott dissociation for bound states in a plasma

Effective wave equation for deuterons in nuclear matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Thouless criterion

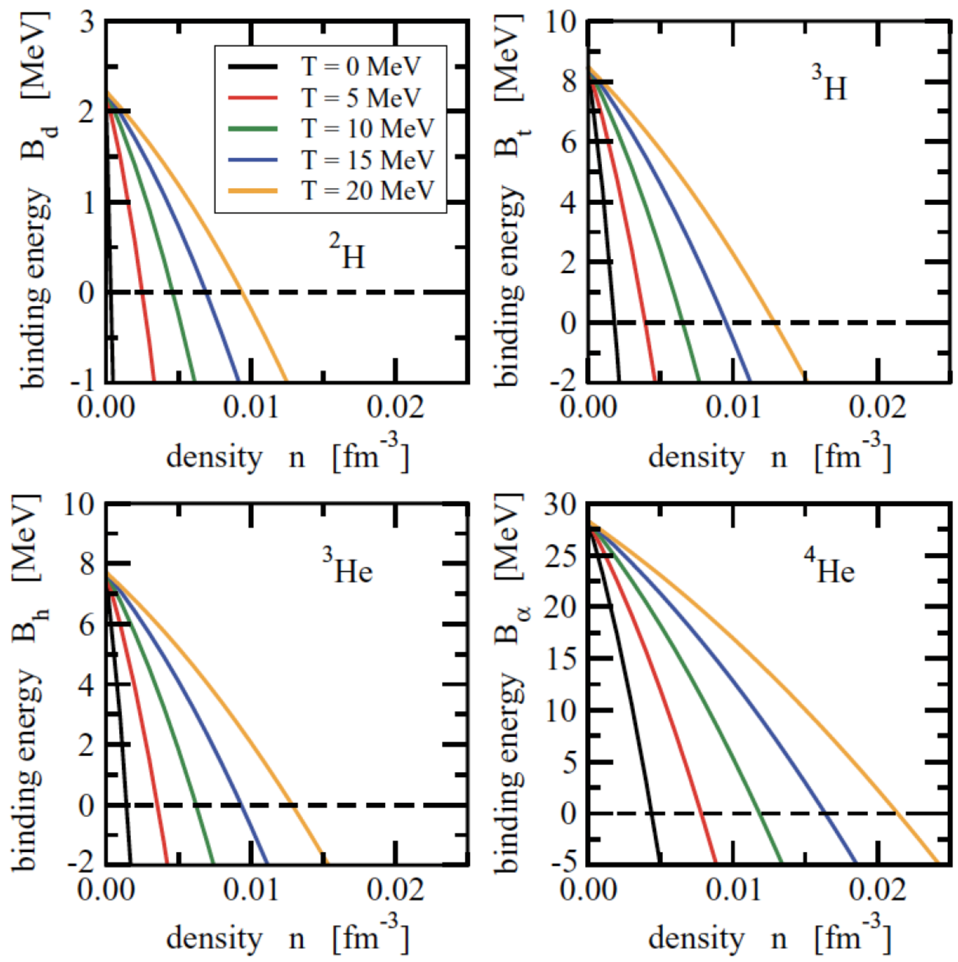
$$E_d(T,\mu)=2\mu$$

BEC-BCS crossover:
Alm et al.,1993

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

Binding energies for light clusters in T – n plane



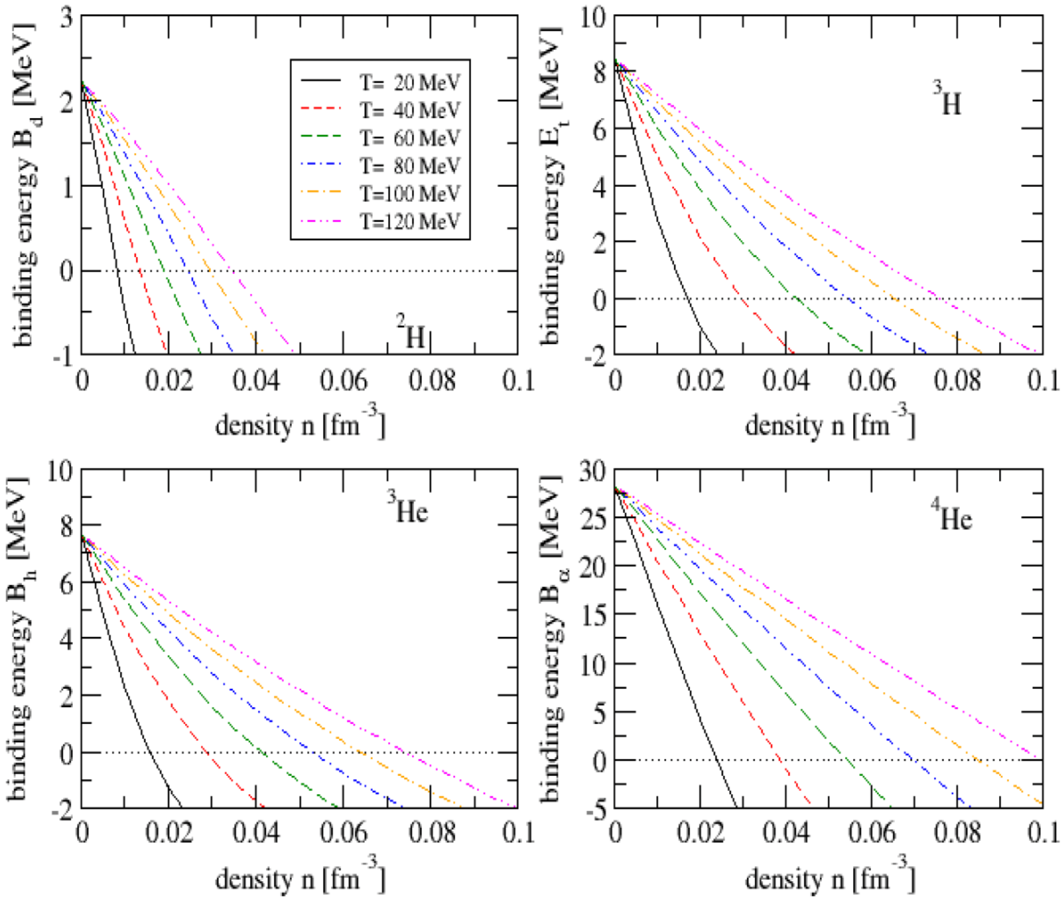
Vanishing binding energies
Indicate Mott effect for the
Light clusters!

Mott-lines in the T-μ plane
can be extracted, where the
Binding energy vanishes

Here lower temperatures:
0 < T[MeV] < 20

S. Typel et al., PRC 81, 015803 (2010)

Binding energies for light clusters in T – n plane

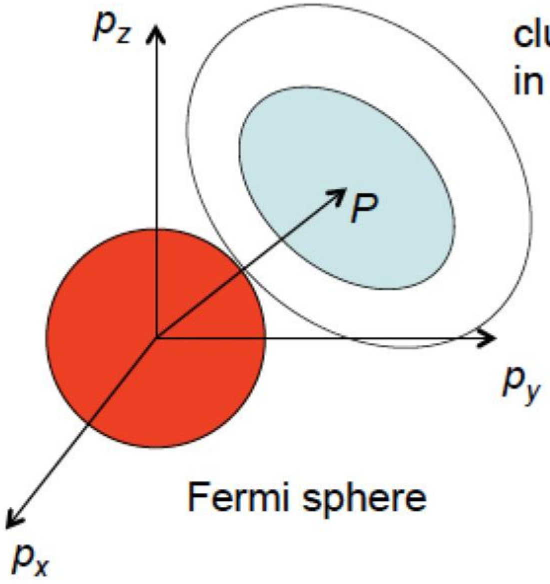


Mott-lines in the T-μ plane can be extracted, where The binding energy vanishes

Here higher temperatures:

20 < T[MeV] < 120

Pauli blocking: phase space occupation



cluster wave function (deuteron, alpha,…) in momentum space

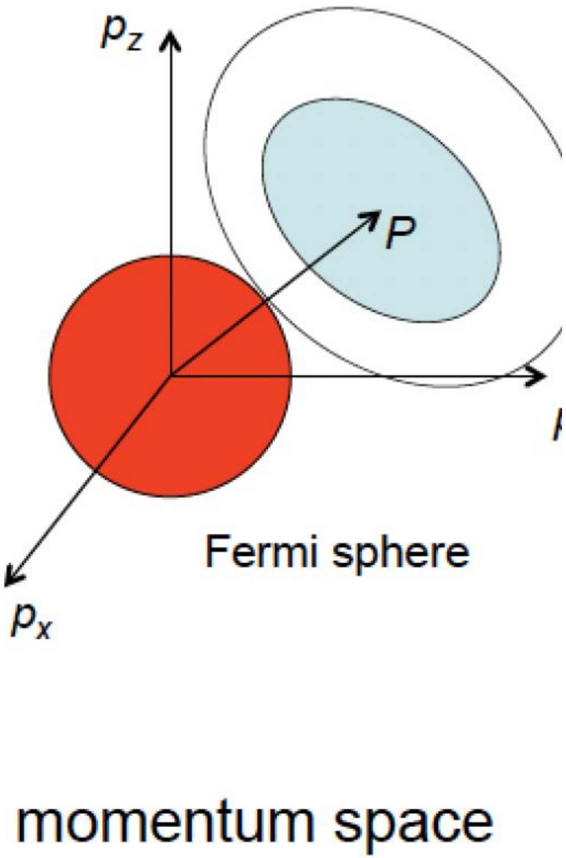
P - center of mass momentum

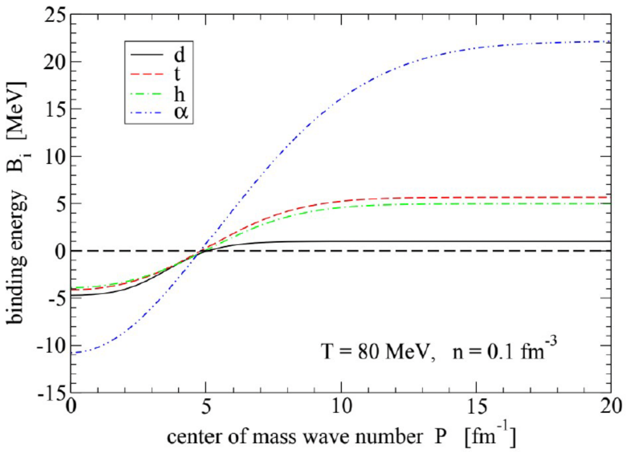
The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum P

momentum space

The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

Momentum-dependent binding energies

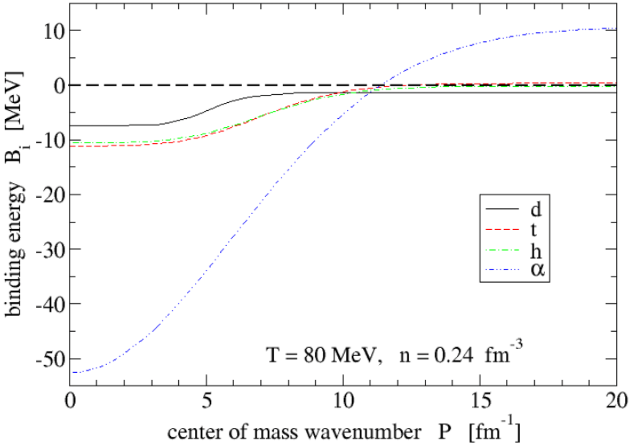


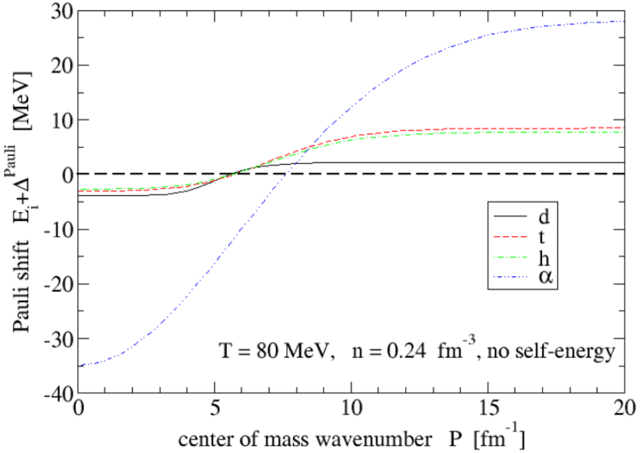


The light clusters that underwent a Mott Dissociation for low momenta become "resurrected" at high momenta relative to the medium !

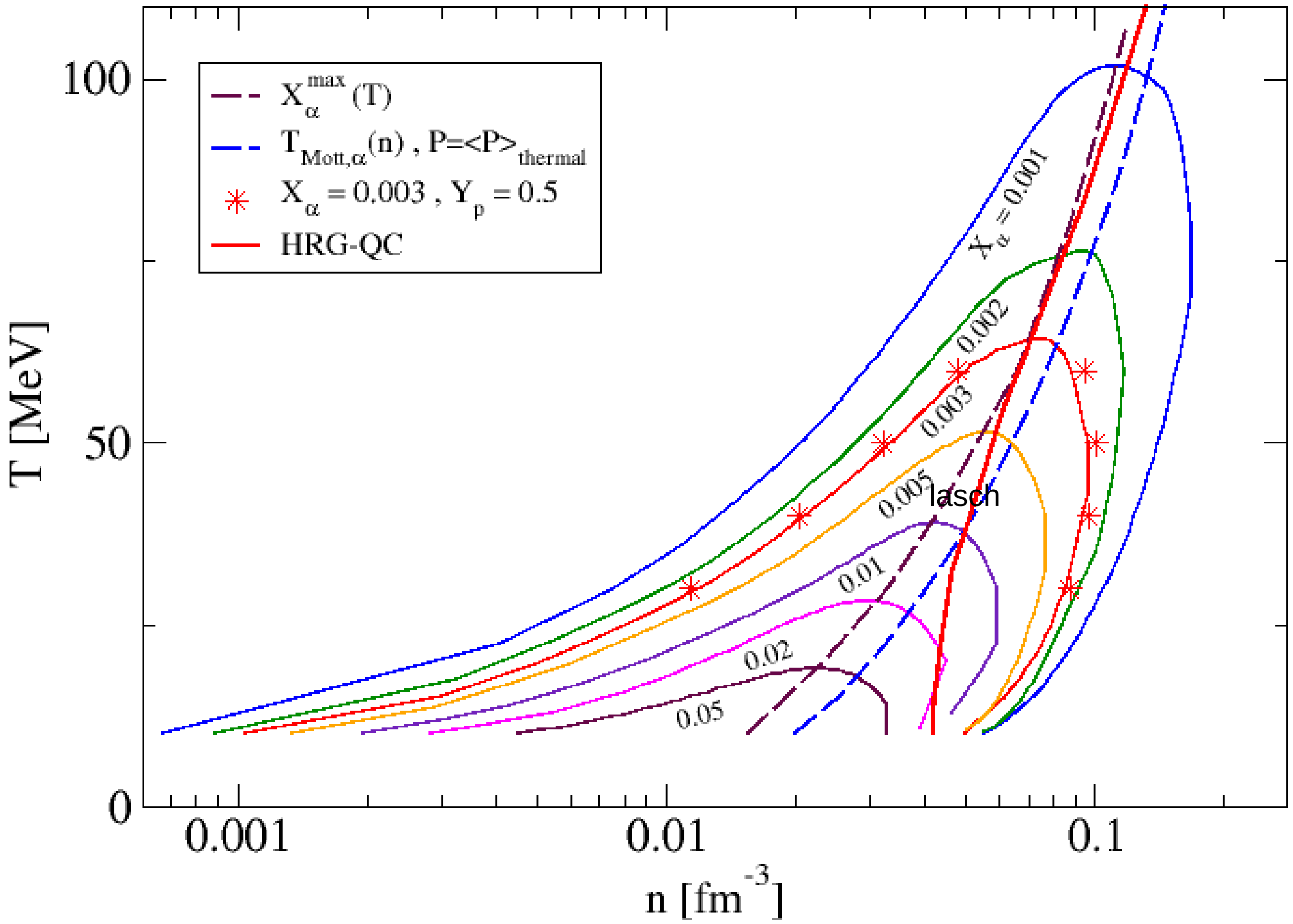
The minimal momentum where this Occurs is called "Mott momentum";
It depends on temperature and density

Binding energies without selfenergy shift,
Only Pauli blocking shift accounted for





CFO in the Temperature – density plane



Mass action law of the chemical picture (nuclear statistical equilibrium) is modified by quantum effects (compositeness):

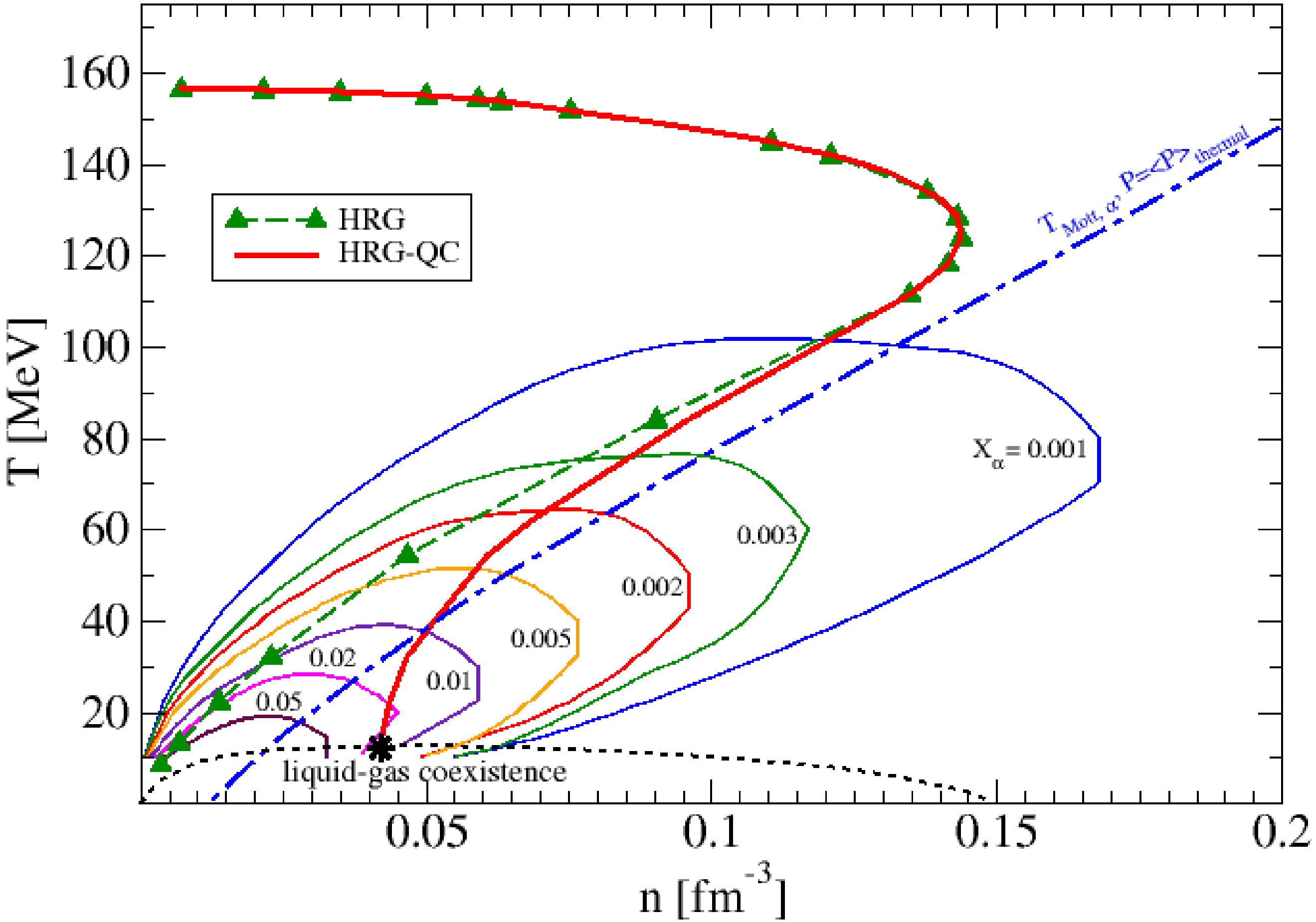
Pauli blocking →

→ Mott dissociation

Mott-line for alpha clusters (equivalent to the line of maximum alpha fraction) is well correlated with the Chemical Freeze-Out line

D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB accepted

CFO in the Temperature – density plane



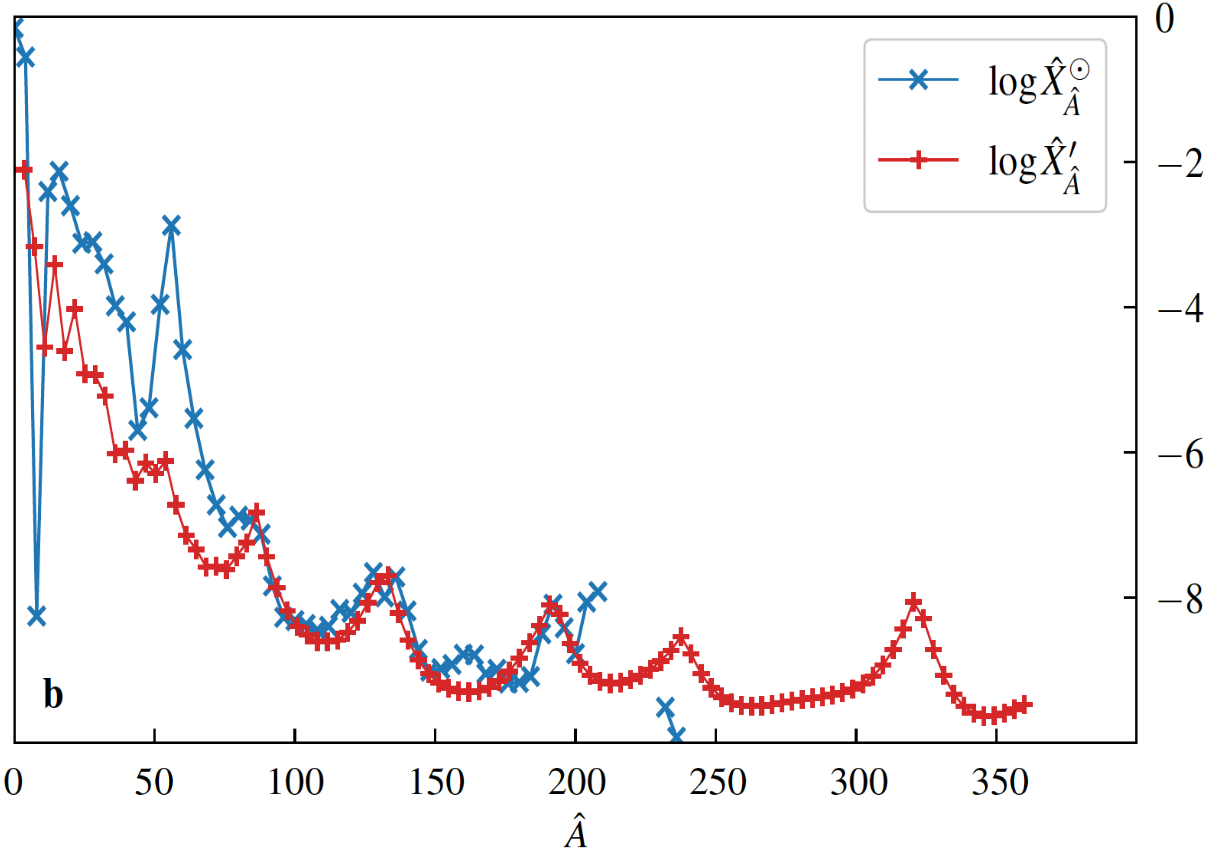
Main result:

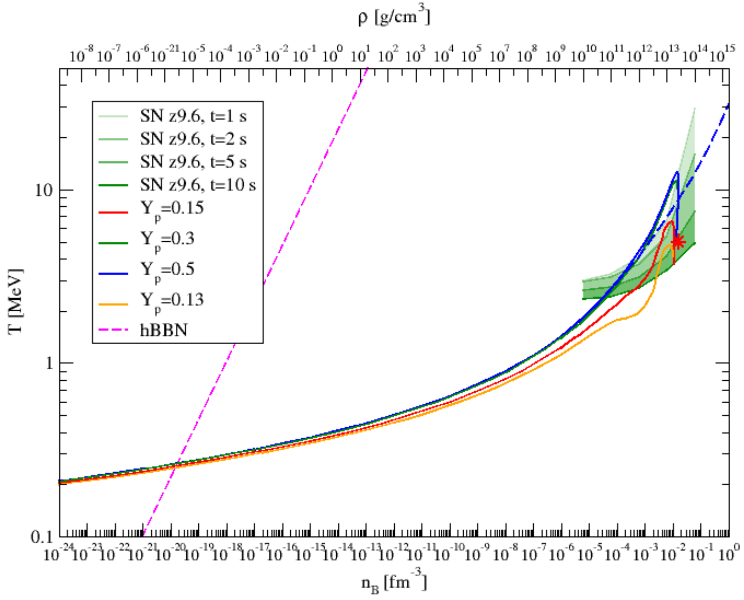
Chemical freeze-out may be interpreted as „inverse“ Mott transition:

Strong localization effect of nucleon-nucleon correlations in bound states (clusters) entails freeze out of the nuclear composition

D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB accepted

Outlook: Primordial CFO of heavy elements?

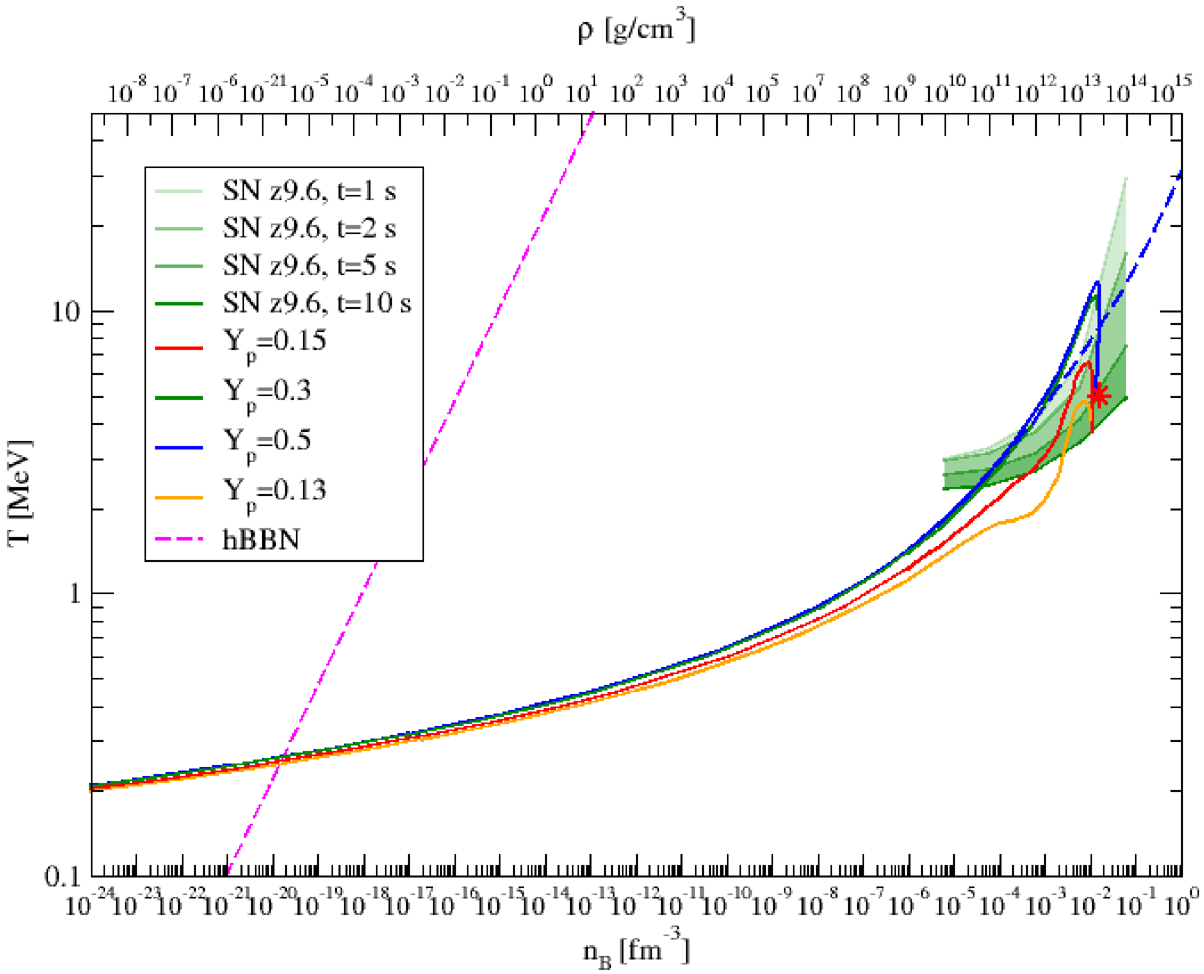




Accumulated mass fraction vs. mass number Â for solar element abundances compared with freeze-out model after neutron evaporation for T=5 MeV, μ_n=940.317 MeV, μ_p= 845.069 MeV

G. Röpke, D. Blaschke, F. Röpke, arXiv:2411.00535

Outlook: Primordial CFO of heavy elements?



Can the primordial evolution of the Universe lead to these freeze-out Parameters (red star):

T=5 MeV,
μ_n=940.317 MeV,
μ_p= 845.069 MeV

Maybe inhomogeneous Big Bang?

The freeze-out point lies in the domain of supernova explosions and binary neutron star mergers

G. Röpke, D. Blaschke, F. Röpke, arXiv:2411.00535

Backup Slides

Mott-Anderson localization model for sudden CFO

The basic idea: Localization of (certain) multiquark states ("cluster") = hadronization;
Reverse process = delocalization by quark exchange between hadrons

Freeze-out criterion:

$$H_{\rm exp}(\tau) = \frac{\dot{R}(\tau)}{R(\tau)} = \tau^{-1}_{{\rm coll},i}(T,\mu) \ ,$$

$$\tau^{-1}_{{\rm coll},i}(T,\mu) = \sum_j \sigma_{ij} v n_j(T,\mu)$$

Povh-Huefner law,
PRC 46 (1992) 990
→ total x-section

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle$$

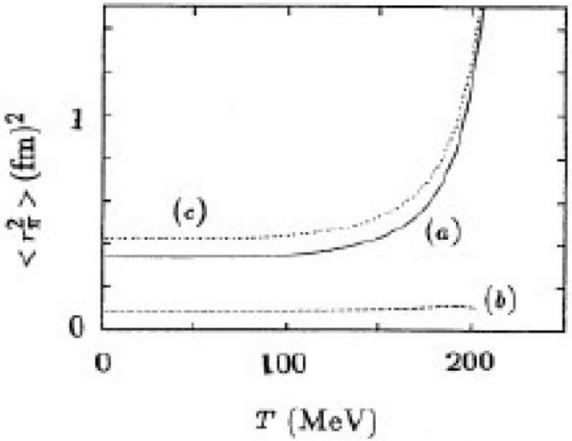
$$r_\pi^2(T,\mu) = \frac{3}{4\pi^2} f_\pi^{-2}(T,\mu)$$

$$f_\pi^2(T,\mu) = -m_0 \langle \bar{q}q \rangle_{T,\mu} / M_\pi^2$$

$$r_\pi^2(T,\mu) = \frac{3M_\pi^2}{4\pi^2 m_q} \left| \langle \bar{q}q \rangle_{T,\mu} \right|^{-1}$$

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_{\rm MF} \left[1 - \frac{T^2}{8 f_\pi^2(T,\mu)} - \frac{\sigma_N n_{s,N}(T,\mu)}{M_\pi^2 f_\pi^2(T,\mu)} \right]$$

Hippe & Klevansky, PRC 52 (1995) 2172



DB, J. Berdermann, J. Cleymans, K. Redlich, Phys. Part. Nucl. Lett. 8 (2011) 811

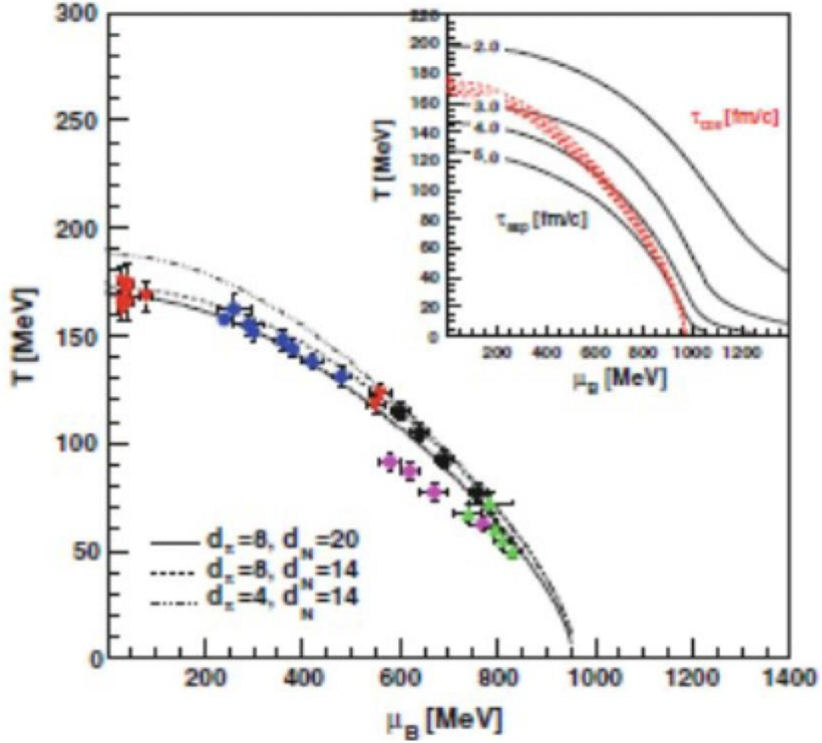
Mott-Anderson localization model for sudden CFO

Model results:

$$\tau_{exp}(T, \mu) = \tau_{coll}(T, \mu)$$

Collision time strongly T, mu dependent !

Schematic resonance gas: dπ pions, dN nucleons

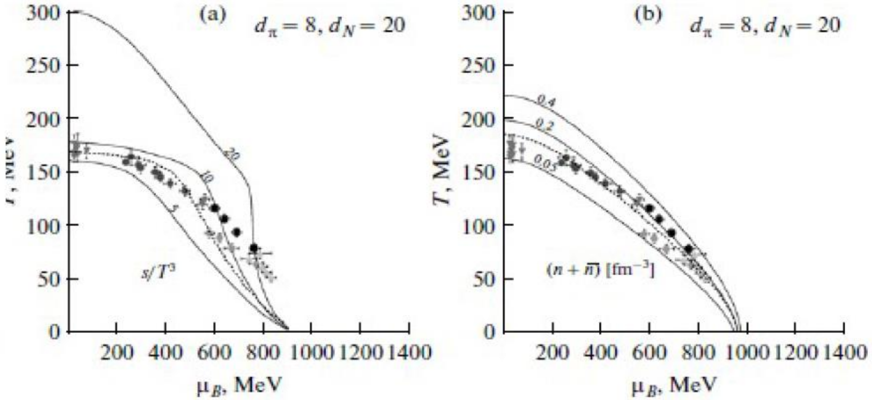


Expansion time scale from entropy conservation:

$$s(T, \mu)\ V(\tau_{exp}) = \mathrm{const}$$

$$\tau_{exp}(T, \mu) = a s^{-1/3}(T, \mu),$$

Thermodynamics consistent with phenomenological Freeze-out rules:



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Mott-Anderson localization model - refined

A) Chiral condensate for the full hadron resonance gas model → radii of hadrons

- nonstrange hadrons:

$$\langle r_\pi^2 \rangle_{T,\mu} = \frac{3}{4\pi^2 f_\pi^2} \qquad f_\pi^2(T,\mu) = \frac{-m_q \langle \bar{q}q \rangle_{T,\mu}}{m_\pi^2} ,$$

$$\langle r_\pi^2 \rangle_{T,\mu} = \frac{3 m_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T,\mu}|^{-1} \qquad \langle r_{\mathrm{N}}^2 \rangle_{T,\mu} = r_0^2 + \langle r_\pi^2 \rangle_{T,\mu}$$

- strange hadrons:

$$f_K^2 m_K^2 = -\frac{\langle \bar{q}q \rangle_{T,\mu} + \langle \bar{s}s \rangle_{T,\mu}}{2} (m_q + m_s)$$

$$\langle r_{\mathrm{K}}^2 \rangle_{T,\mu} = \frac{3}{4\pi^2 f_{\mathrm{K}}^2} = \frac{3}{2\pi^2} \frac{m_{\mathrm{K}}^2}{|\langle \bar{q}q \rangle_{T,\mu} + \langle \bar{s}s \rangle_{T,\mu}|(m_q + m_s)} \qquad \langle r_\Lambda^2 \rangle_{T,\mu} = r_0^2 + \langle r_{\mathrm{K}}^2 \rangle_{T,\mu}$$

B) Chemical freeze-out: only “reactive” cross section, flavor equilibration

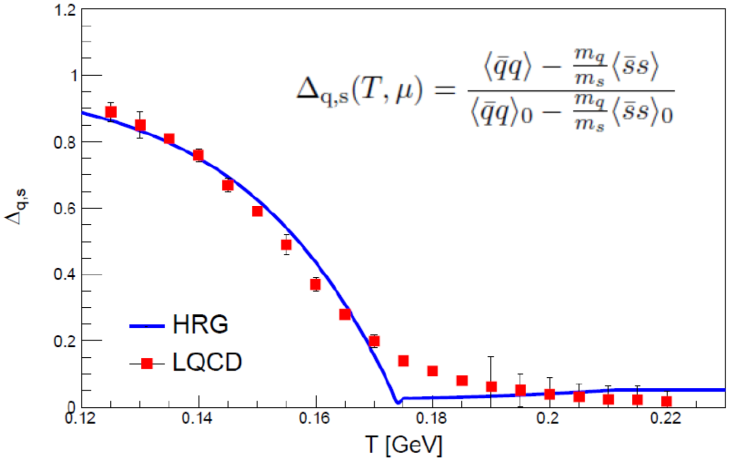
Some flavor changing processes involve reaction thresholds and need activation energy, like in the Eyring theory of chemical processes with activation:

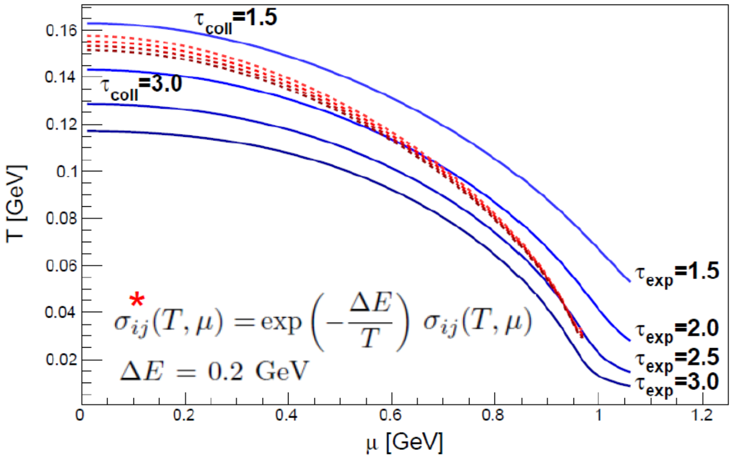
$$\sigma_{ij}^{*}(T,\mu) = \exp\left(-\frac{\Delta E}{T}\right) \sigma_{ij}(T,\mu) \qquad \sigma_{ij}(T,\mu) = \lambda \langle r_i^2 \rangle_{T,\mu} \langle r_j^2 \rangle_{T,\mu}$$

Assumption: average activation threshold for reactive processes: $\Delta E = 0.2\ \mathrm{GeV}$
(to be refined, account for all individual processes, e.g., SMASH)

DB, J. Jankowski, M. Naskret, arxiv:1705.00169

Mott-Anderson localization model - refined





$$\langle\bar{q}q\rangle_{T,\mu} = \langle\bar{q}q\rangle^{MF}_{T,\mu} + \sum_{h=M,B} \frac{\sigma_q^h}{m_q} n_h(T,\mu) \; ,$$

$$n_h(T,\mu) = \frac{d_h}{2\pi^2}\int_0^\infty dk k^2 \frac{m_h}{E_h}\frac{1}{\mathrm{e}^{(E_h-\mu_h)/T} \mp 1} \; .$$

$$\tau^{-1}_{\mathrm{coll},i}(T,\mu) = \sum_j \sigma^*_{ij} v n_j(T,\mu) \; ; \; \sigma_{ij} = \lambda \langle r_i^2\rangle \langle r_j^2\rangle$$

$$\langle r_\pi^2\rangle_{T,\mu} \simeq \frac{3}{4\pi^2} f_\pi^{-2}(T,\mu) = \frac{3M_\pi^2}{4\pi^2 m_q}\left|\langle\bar{q}q\rangle_{T,\mu}\right|^{-1}$$

$$\langle r_K^2\rangle_{T,\mu} \simeq \frac{3M_K^2}{\pi^2(m_q+m_s)}\left|\langle\bar{q}q\rangle_{T,\mu} + \langle\bar{s}s\rangle_{T,\mu}\right|^{-1}$$

The factor a stands for the inverse system size in the formula

$$\tau_{\exp}(T,\mu) = \tau_{\mathrm{coll}}(T,\mu)$$

for the 3D expansion time scale assuming entropy conservation

Full HRG model condensate;
J. Jankowski et al., Phys. Rev. D (2013)

DB, J. Jankowski, M. Naskret, arxiv:1705.00169

Mott-Anderson localization model - refined

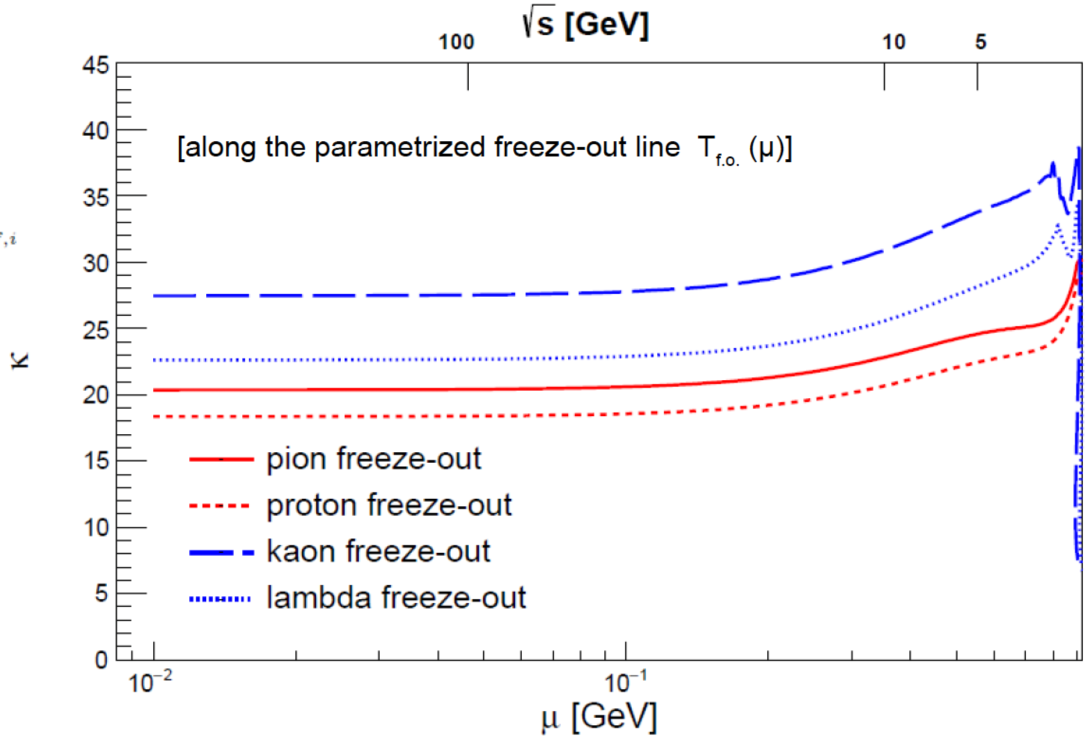
Inelastic collision rate $\tau_{\rm coll} \propto T^{\kappa}, \quad \kappa \gtrsim 20$ from fit to STAR data

U. Heinz and G. Kestin, PoS CPOD **2006**, 038 (2006) [nucl-th/0612105]

Species-dependent exponent of the power law,

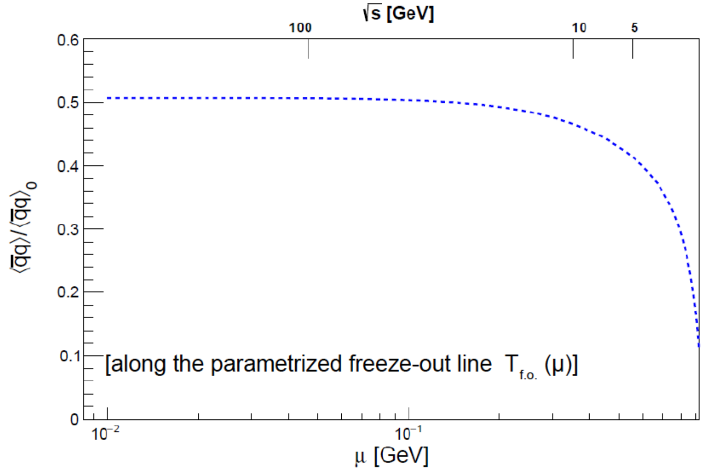
$$\kappa_i = -\frac{d\ln\tau_{{\rm coll},i}(T,\mu)}{d\ln T}\Bigg|_{T_{f,i};\mu_{f,i}}$$

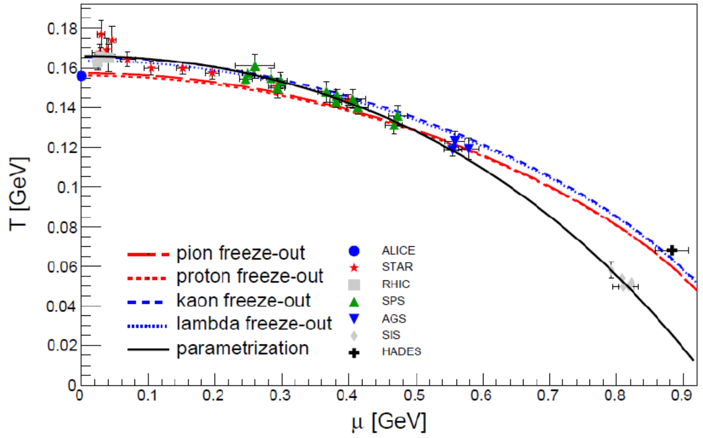
extracted from the model for the collision rate.



DB, J. Jankowski, M. Naskret, arxiv:1705.00169

Mott-Anderson localization model - refined





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The factor a stands for the inverse system size in the formula

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for the 3D expansion time scale assuming entropy conservation

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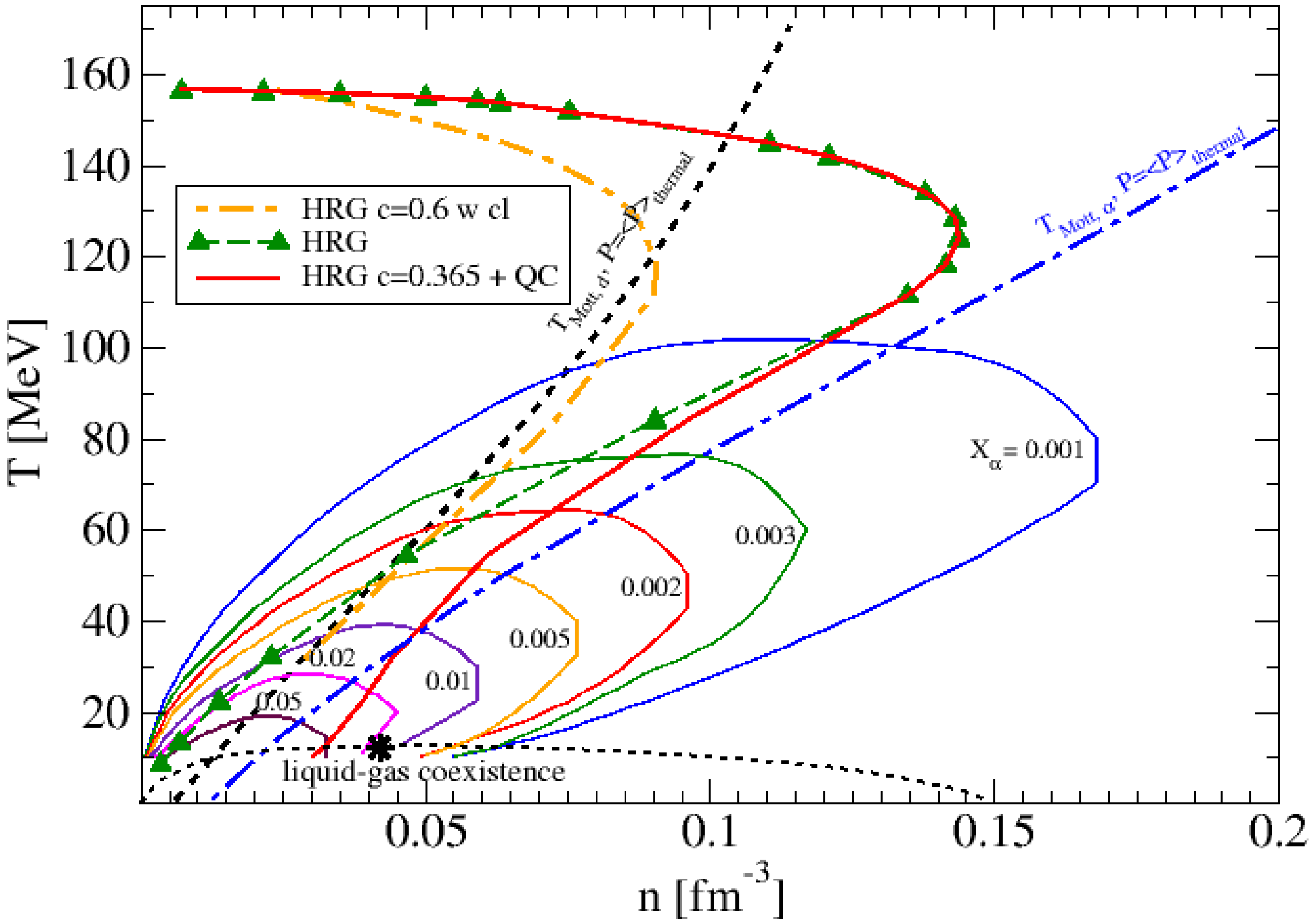
CFO in the Temperature – density plane



CFO in the Temperature – density plane