Spin hydrodynamics in heavy-ion collisions – recent numerical applications

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primary reference: S. K. Singh, R.R., W. Florkowski e-Print: 2411.08223 [hep-ph]

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Motivation

QGP EVOLVES HYDRODYNAMICALLY

Studies of observables constructed from **momentum-space distribution of charged hadrons**

The **hot and dense QCD matter behaves as a fluid with very low viscosity**

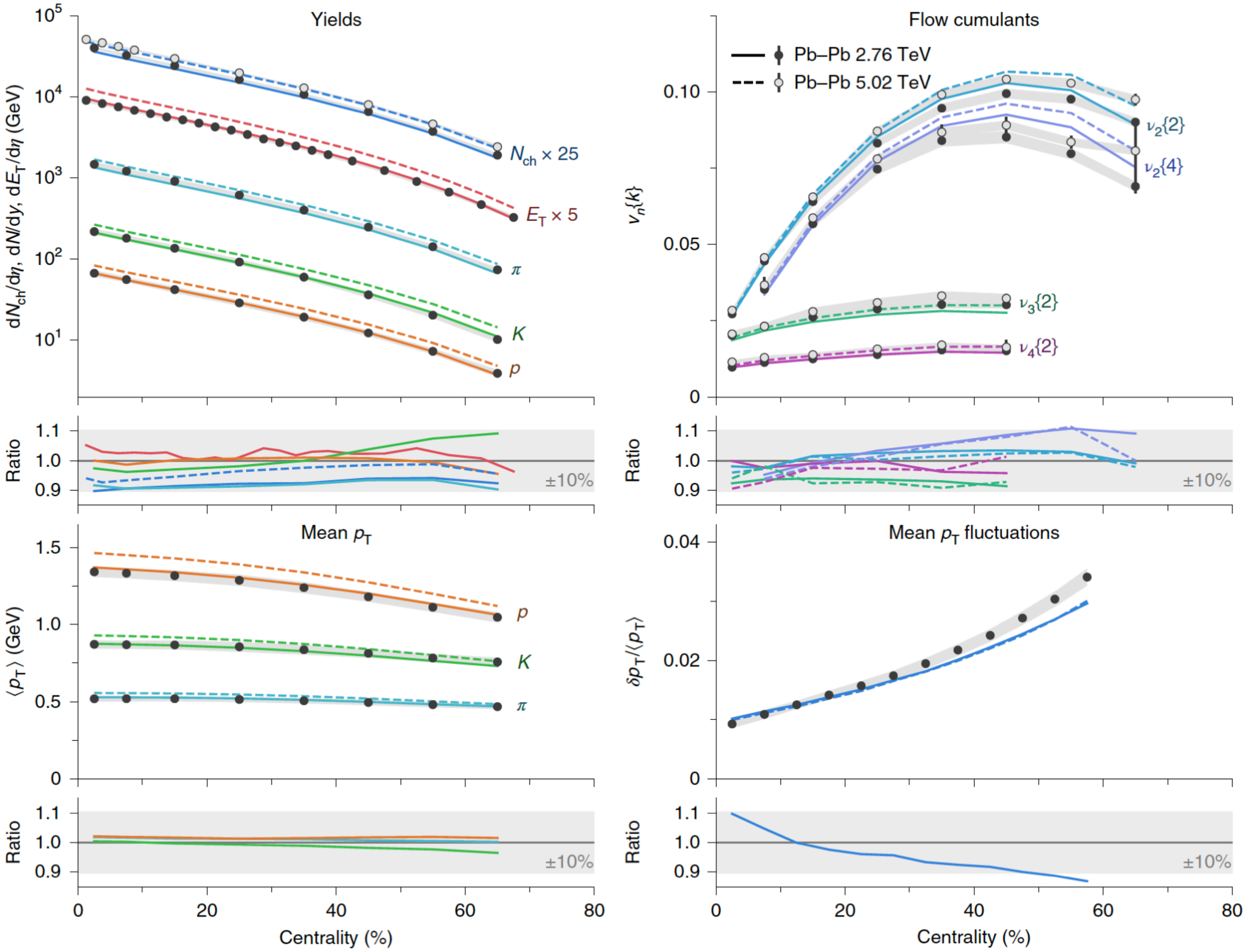
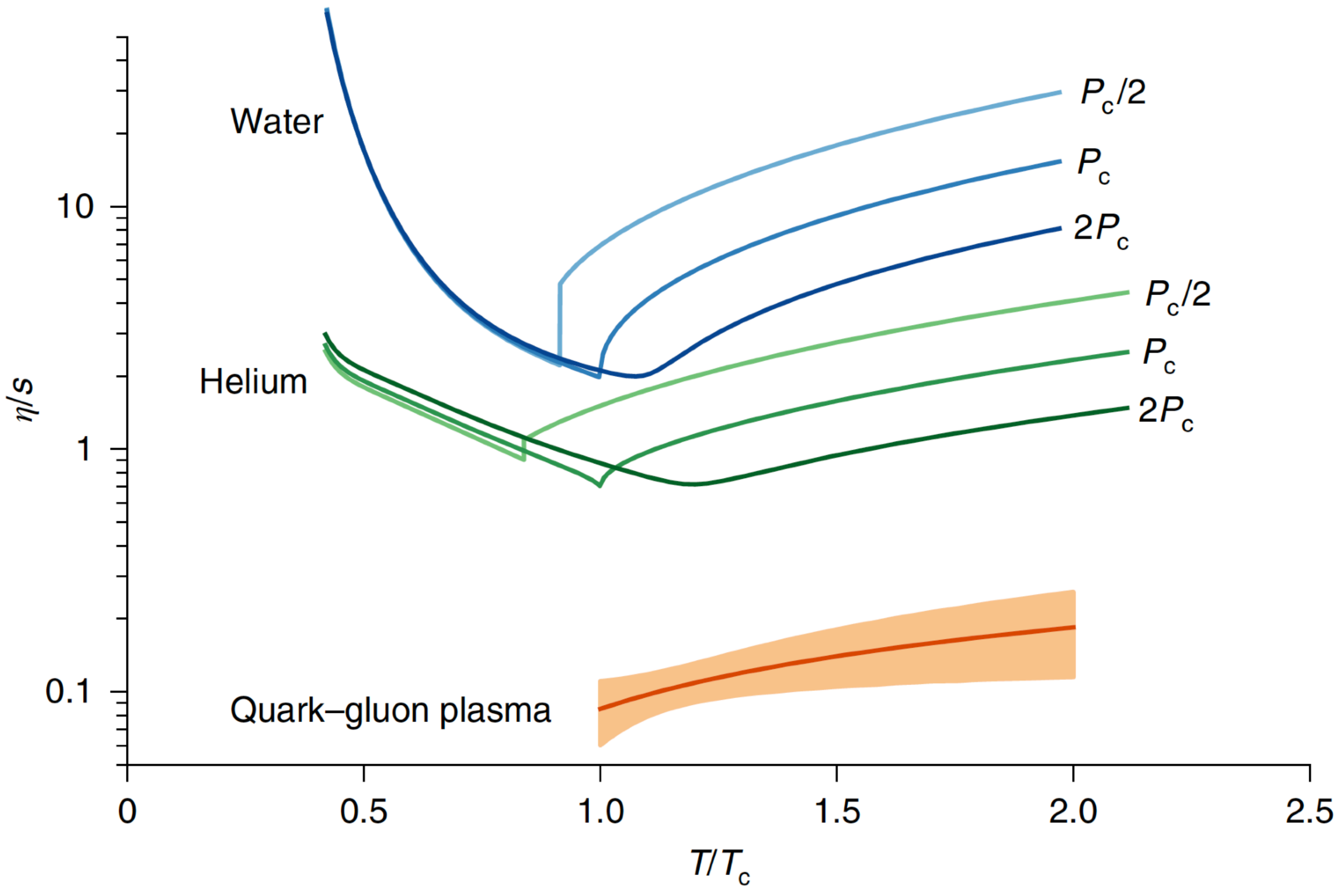


fig: Bernhard, Moreland, Bass, Nature Phys. 15, 1113–1117 (2019)



Non-central heavy-ion collisions - angular momentum and magnetic field

- **Large initial orbital angular momentum** (OAM)

  Becattini, Piccinini, Rizzo, PRC 77 (2008) 024906

orbital

$\boldsymbol{L}_{\text{init}}$

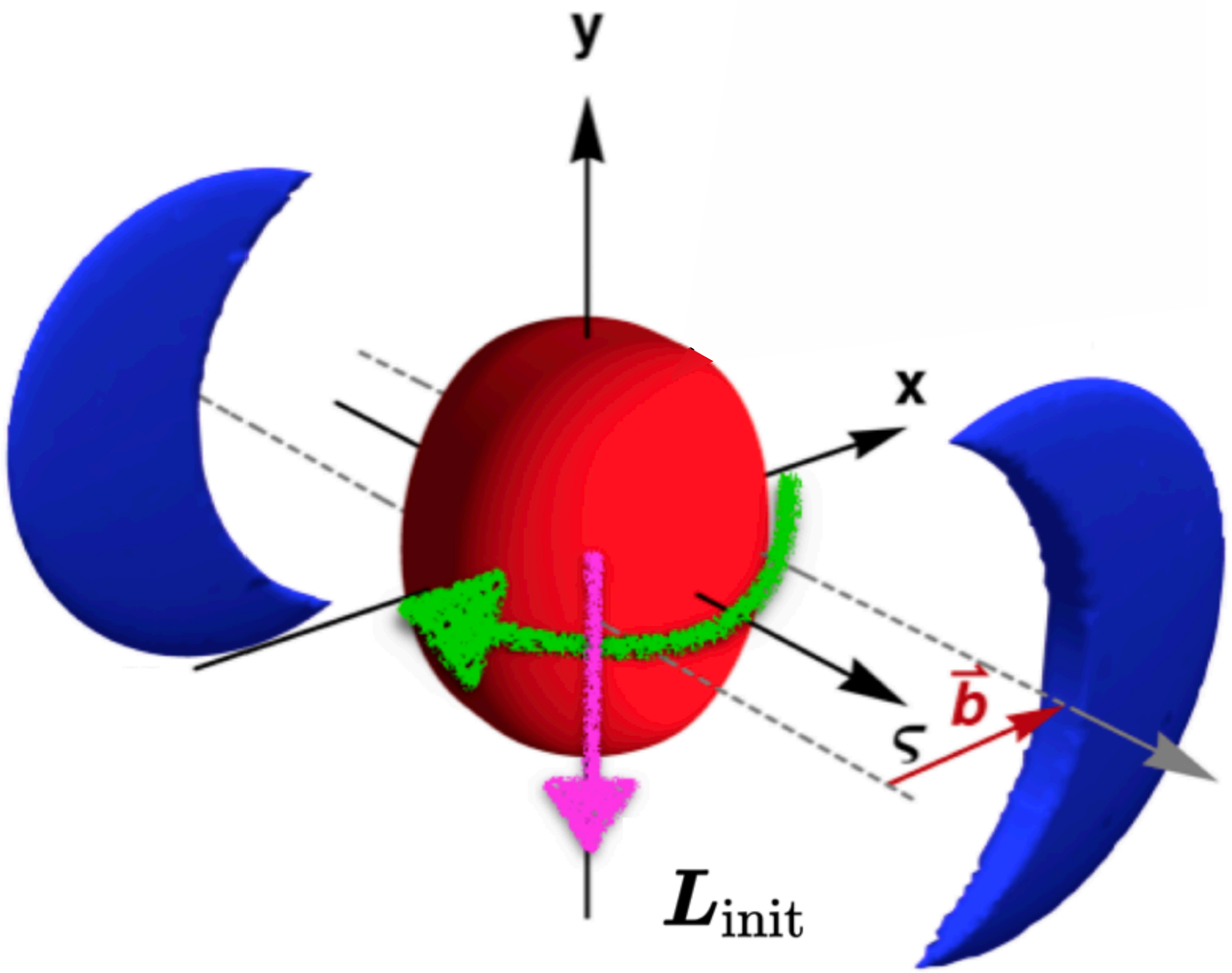


fig: R. R.

NON-CENTRAL HEAVY-ION COLLISIONS - ANGULAR MOMENTUM AND MAGNETIC FIELD

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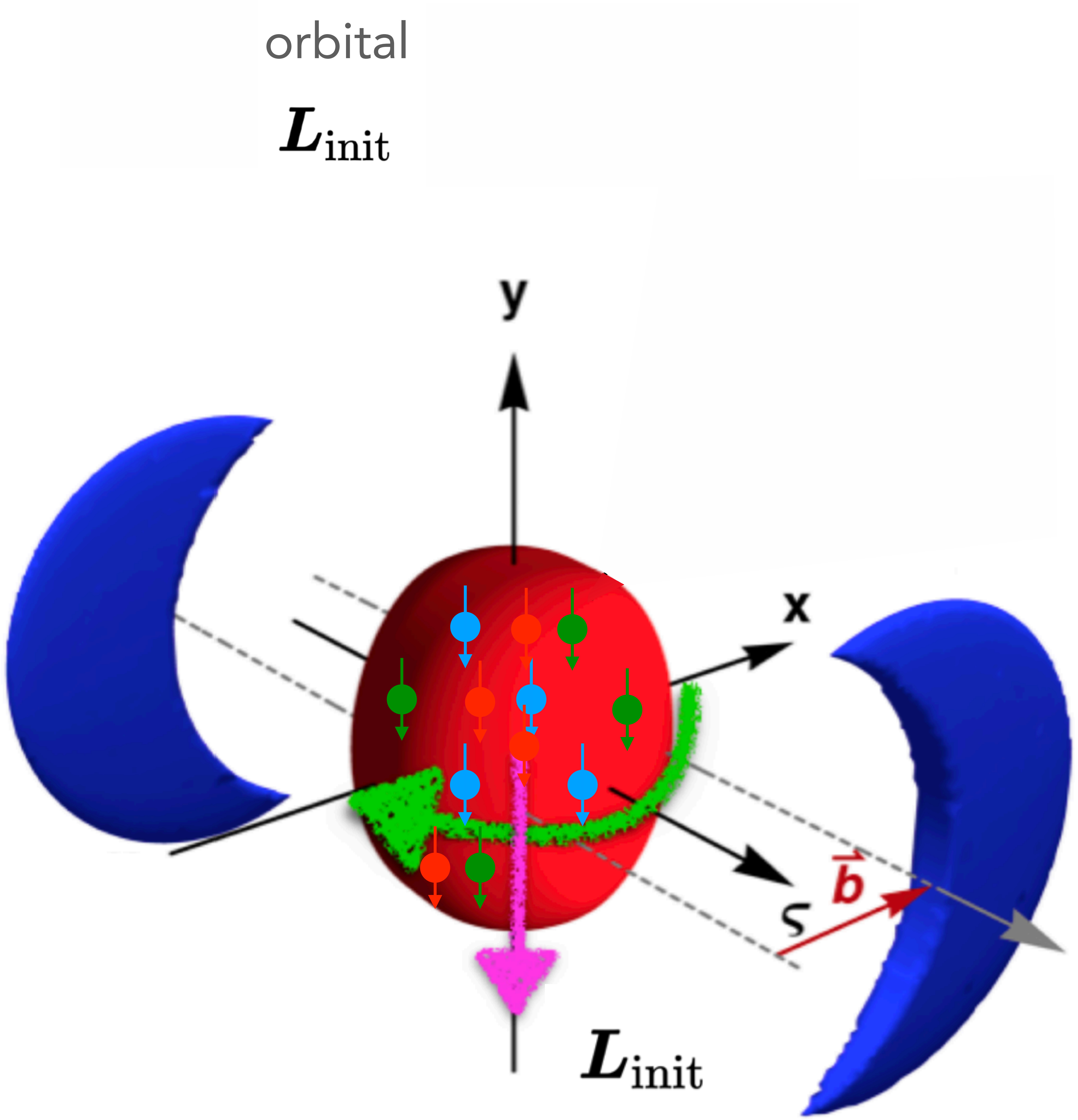
  OAM can be transferred to the spin of QGP constituents



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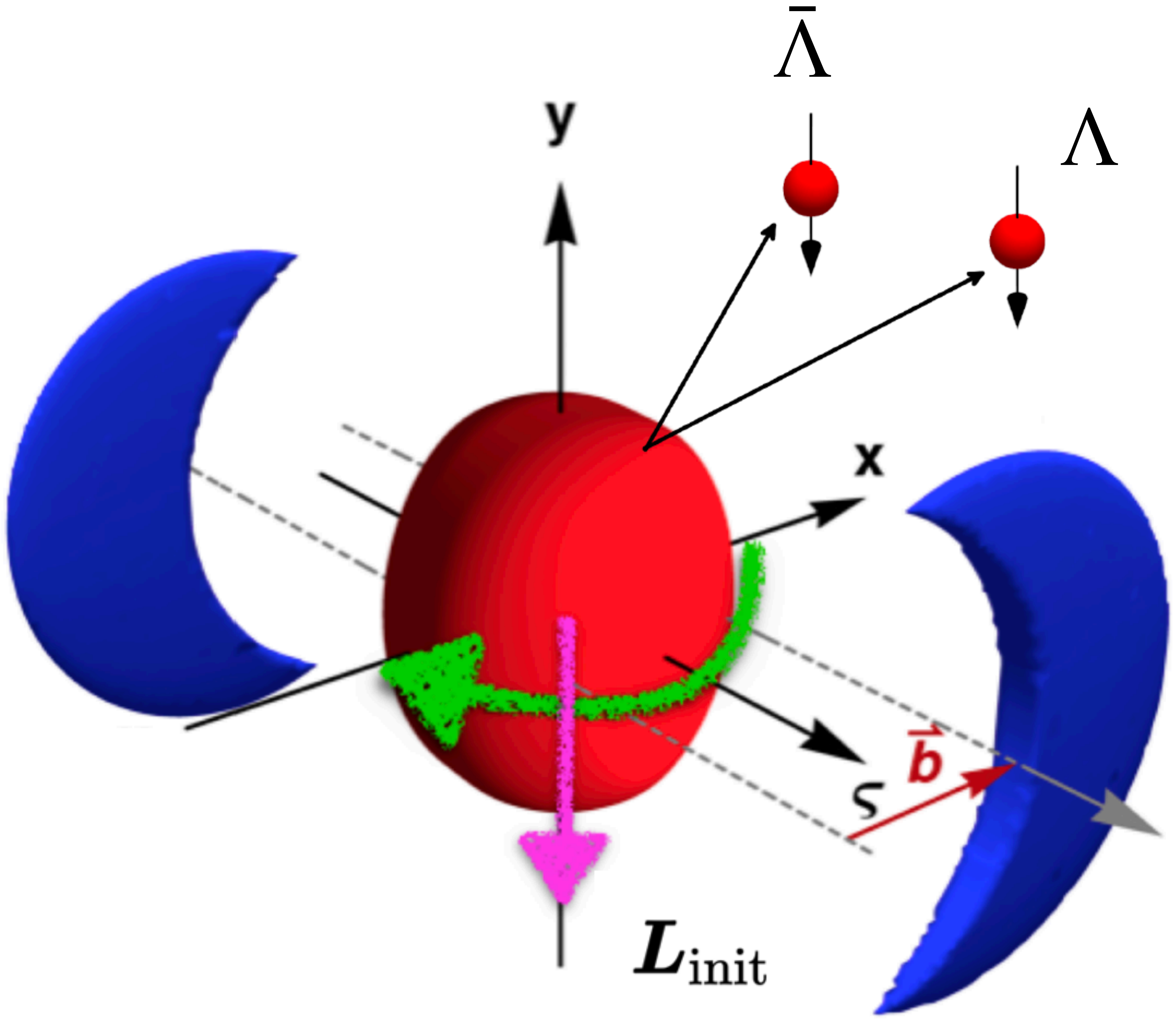
  Emitted **particles (on average) are expected to be polarized** along the fireball's global angular

  Liang, Wang PRL 94:102301 (2005)
  Betz, Gyulassy, Torrieri, PRC 76:044901 (2007)
  Gao, et al. PRC 77:044902 (2008)
  Becattini, Piccinini, et al. J. Phys. G 35:054001 (2008)

orbital orbital + spin

$$\boldsymbol{L}_{\text{init}} \rightarrow \boldsymbol{L}_{\text{final}} + \boldsymbol{S}_{\text{final}}$$



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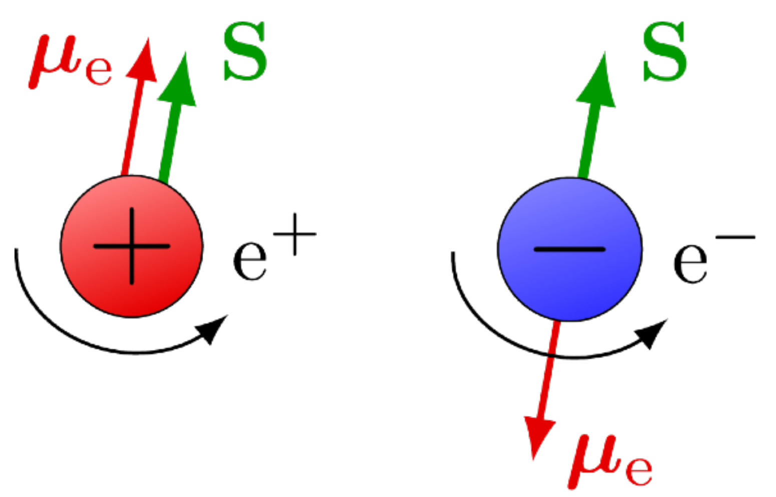
  Becattini, Piccinini, et al. J. Phys. G 35:054001 (2008)

- **Large magnetic field** may be created initially

  Bzdak and Skokov, Phys. Lett. B 710 (2012) 171-174

  Particle's magnetic moments alignment is possible



$$\boldsymbol{\mu} = \frac{g_s q}{2m}\mathbf{S}$$

$$E = -\boldsymbol{\mu} \cdot \mathbf{B}$$

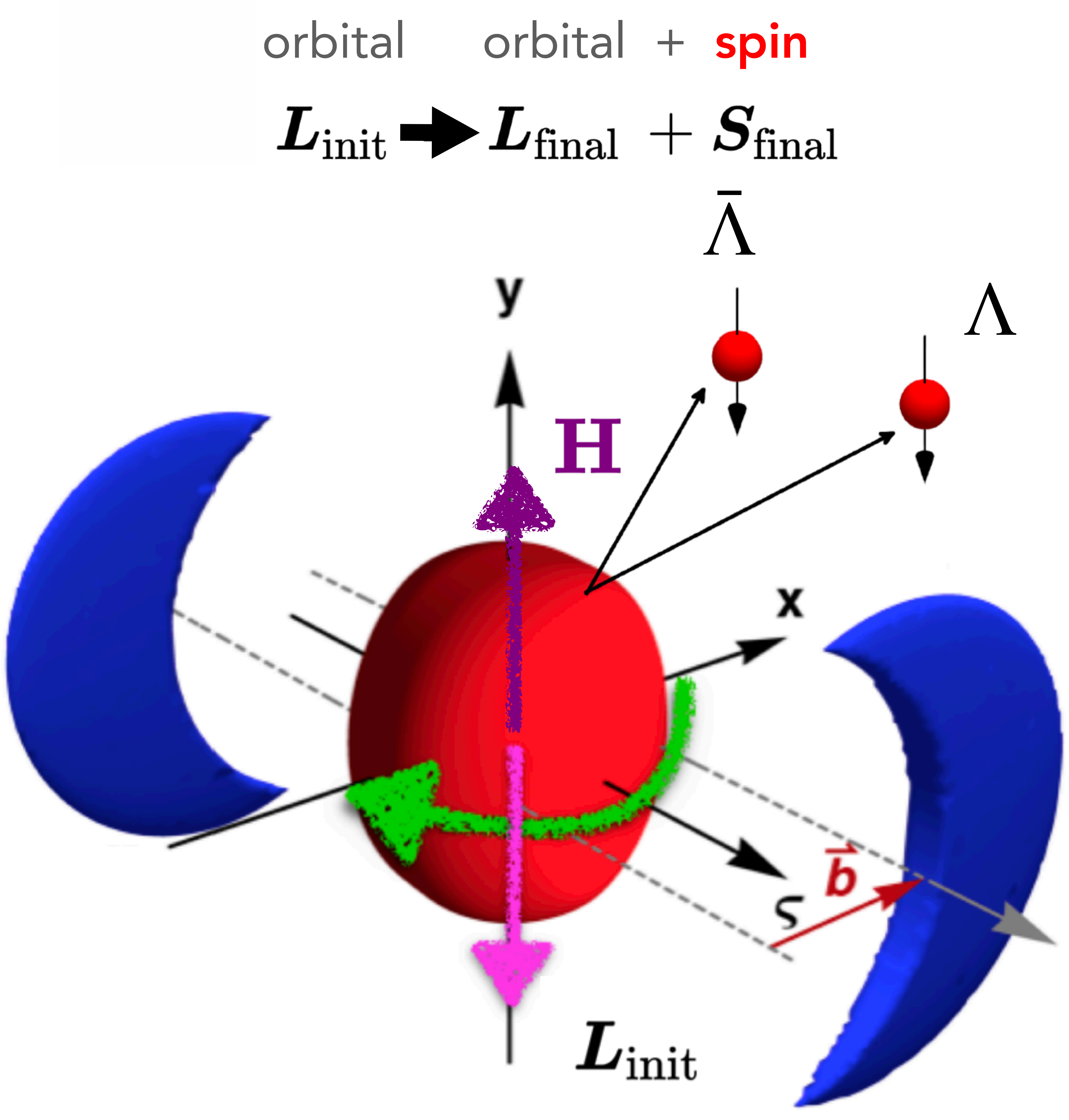
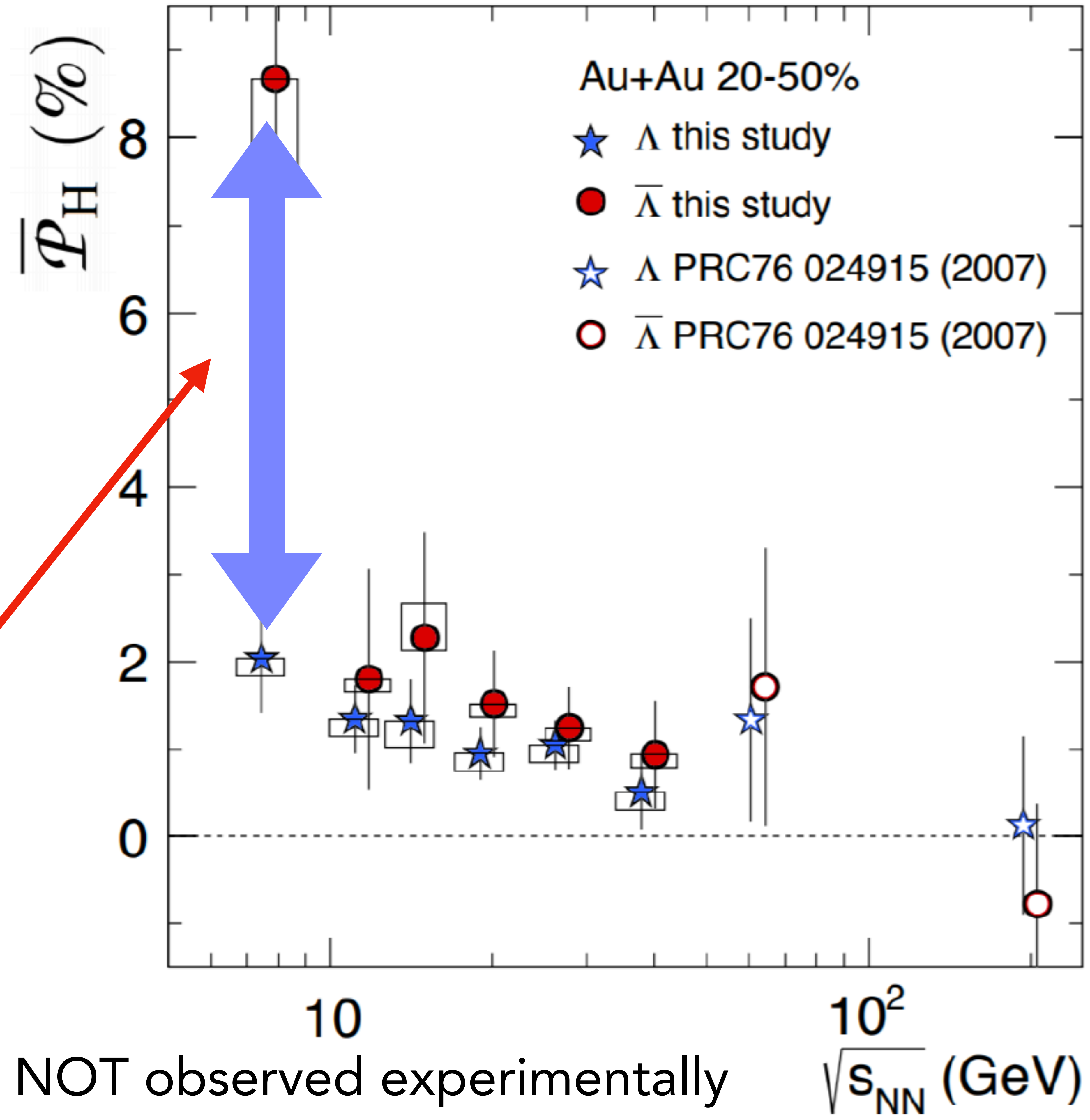


fig: R. R.

Measurement of Λ and $\bar{\Lambda}$ global spin polarization

global polarization ⟷ average over all phasespace



Update: Splitting NOT observed experimentally

Hu (STAR Collaboration), SQM 2024

Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65

INTERPRETARTION: SPIN-THERMAL APPROACH

In **thermodynamic equilibrium** one can establish a link between spin and (thermal) vorticity

Becattini, Chandra, Del Zanna, Grossi, AP 338:32 (2013)
Becattini, Csernai, and Wang, PRC 88, 034905 (2013)
Fang, Pang, Wang, Wang, PRC 94:024904 (2016)
Becattini, Karpenko, Lisa, Upsal, and Voloshin PRC 95, 054902 (2017)

The **polarization vector** of emitted particles is

$$S^{\mu}_{\varpi}(p) = -\frac{1}{8m_{\Lambda}}\epsilon^{\mu\nu\rho\sigma}p_{\sigma}\frac{\int d\Sigma \cdot p\, n_F(1-n_F)\varpi_{\nu\rho}}{\int d\Sigma \cdot p\, n_F}$$

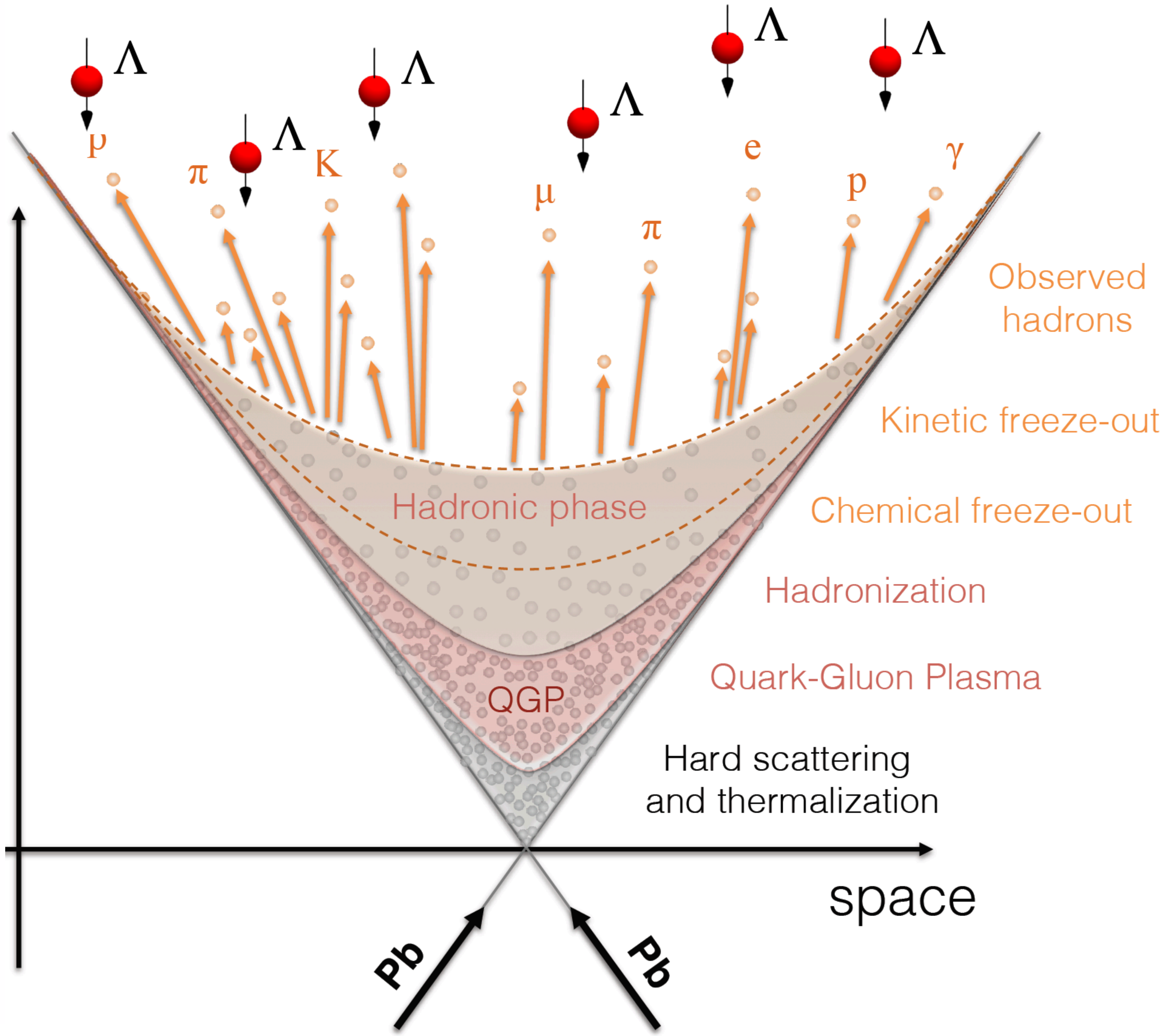


figure: D.D. Chinellato

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$$\varpi_{\mu\nu} = \partial_{[\nu}\beta_{\mu]} = -\frac{1}{2}(\partial_{\mu}\beta_{\nu} - \partial_{\nu}\beta_{\mu})$$

$$\beta^{\mu} \equiv u^{\mu}/T$$

$$T, \mu_B\, u^{\rho}$$

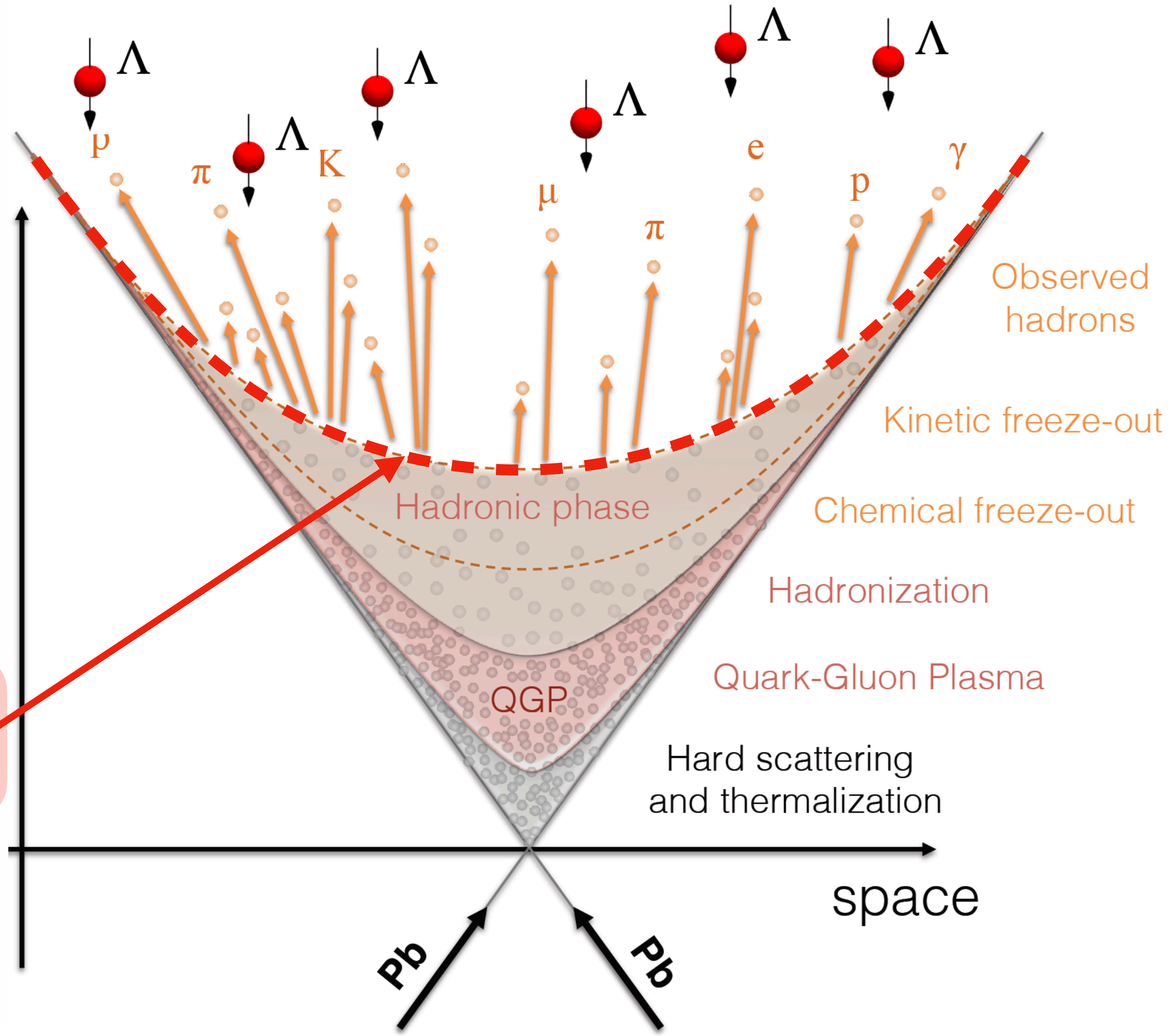


figure: D.D. Chinellato

Global polarization: measurement vs spin-thermal approach

Global polarization data supports spin-thermal approach

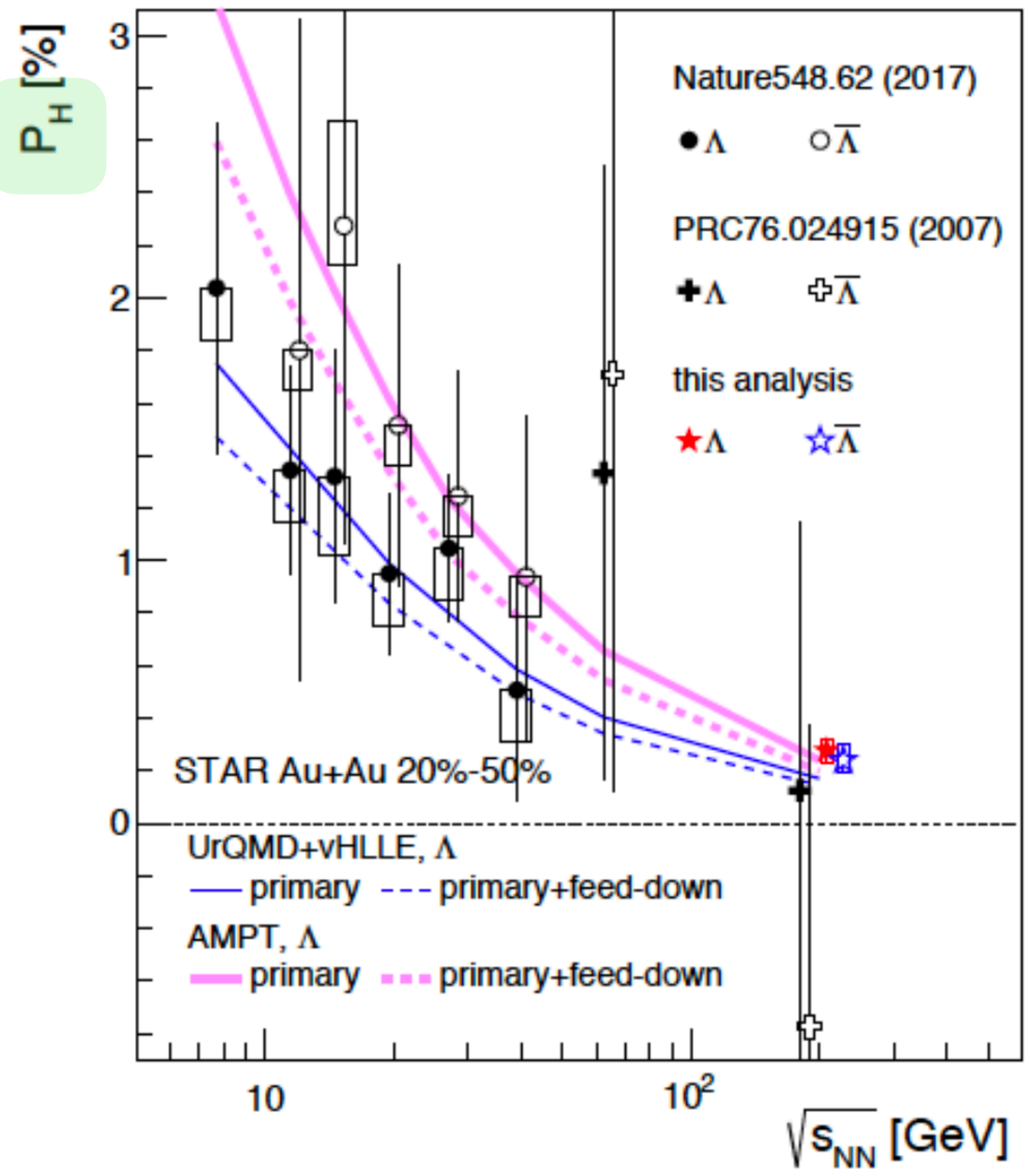
Agrees well with predictions of transport and hydrodynamic models

UrQMD+vHLLE: Karpenko, Becattini, EPJC 77, 213 (2017)
AMPT: Li, Pang, Wang, and Xia, PRC 96, 054908 (2017)

$$P_H = -S^y_\varpi$$

Many other models capture this behavior fairly well.

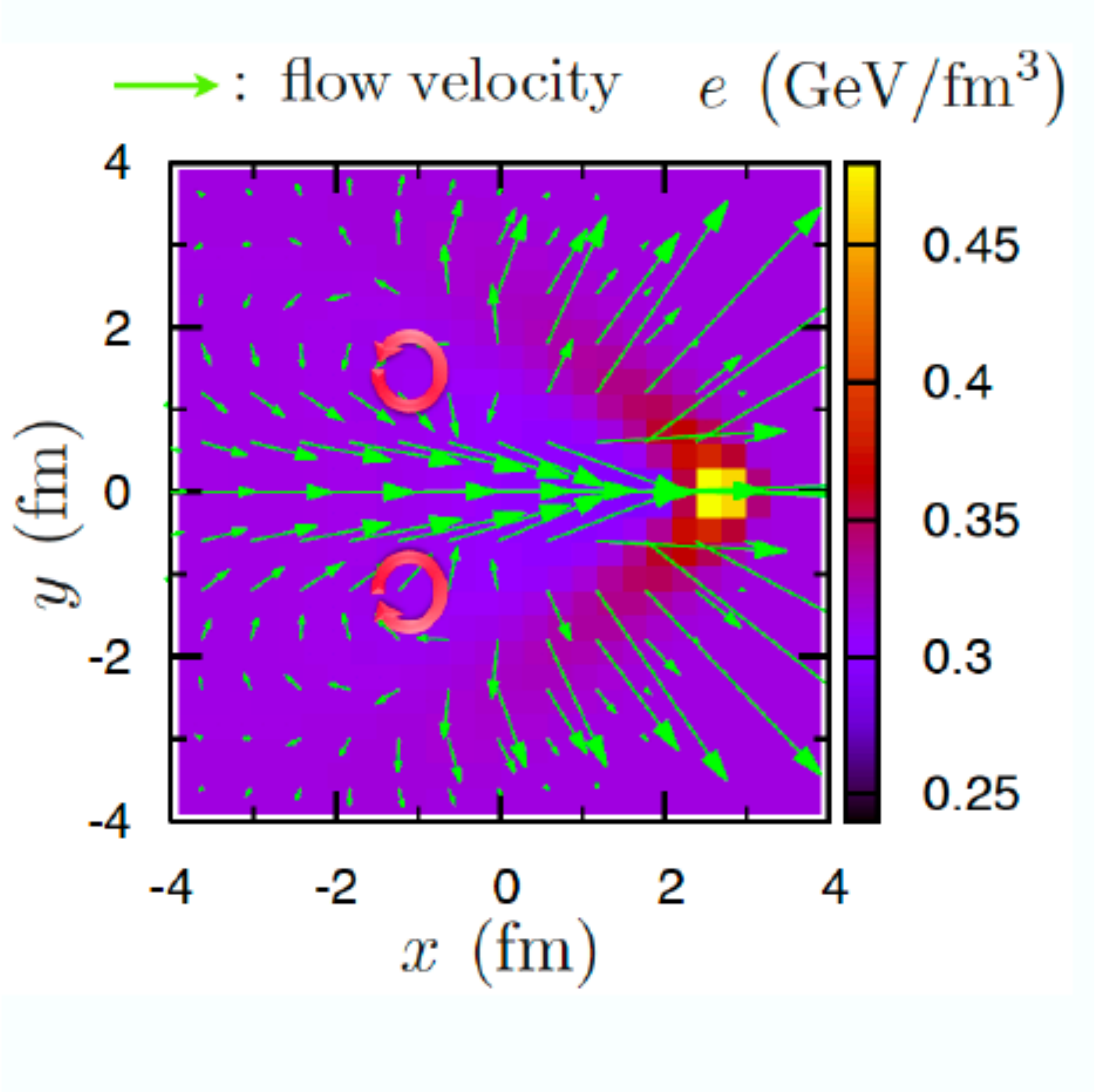
J. Adam et al. (STAR), Phys. Rev. C 98, 014910 (2018)



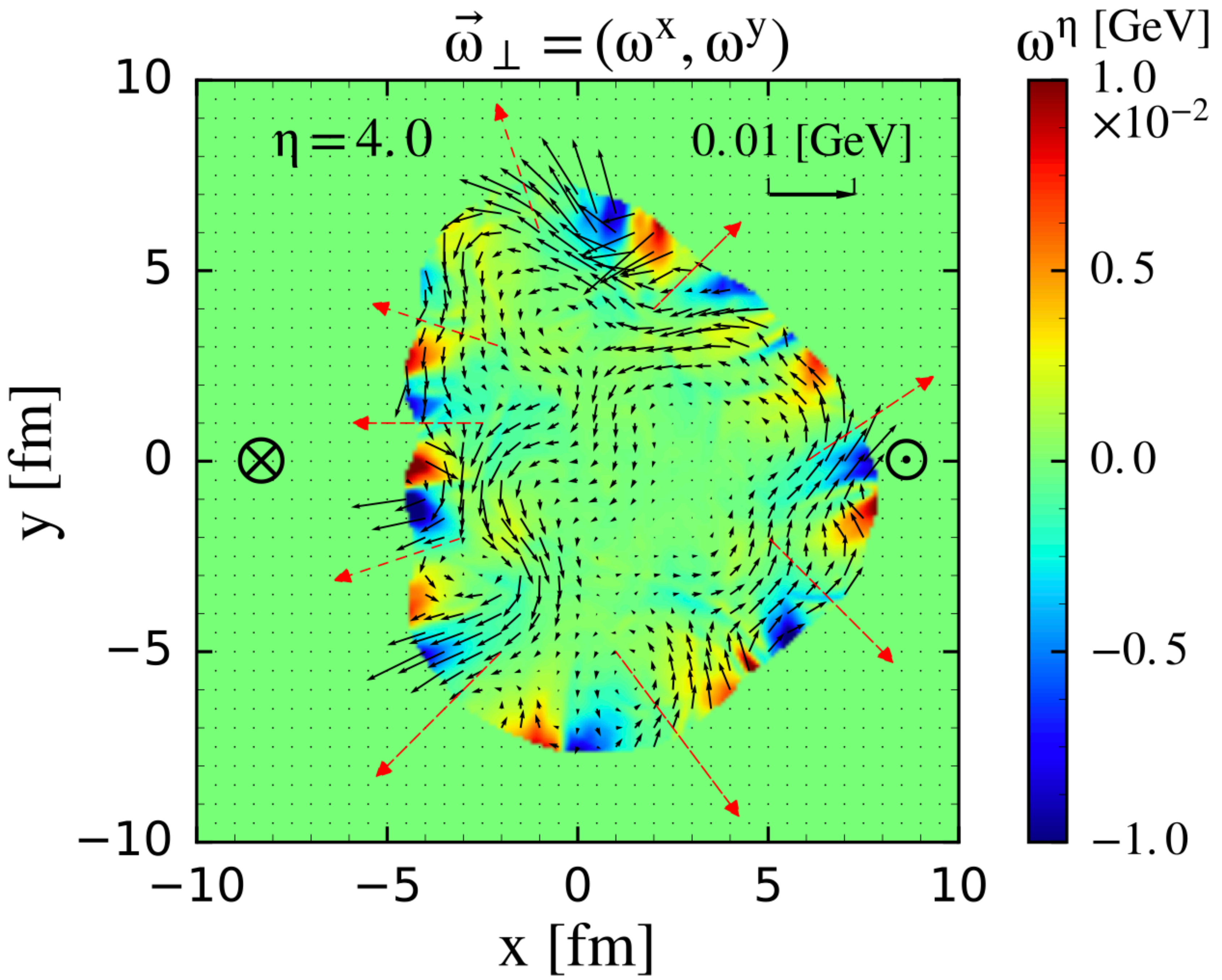
LONGITUDINAL POLARIZATION

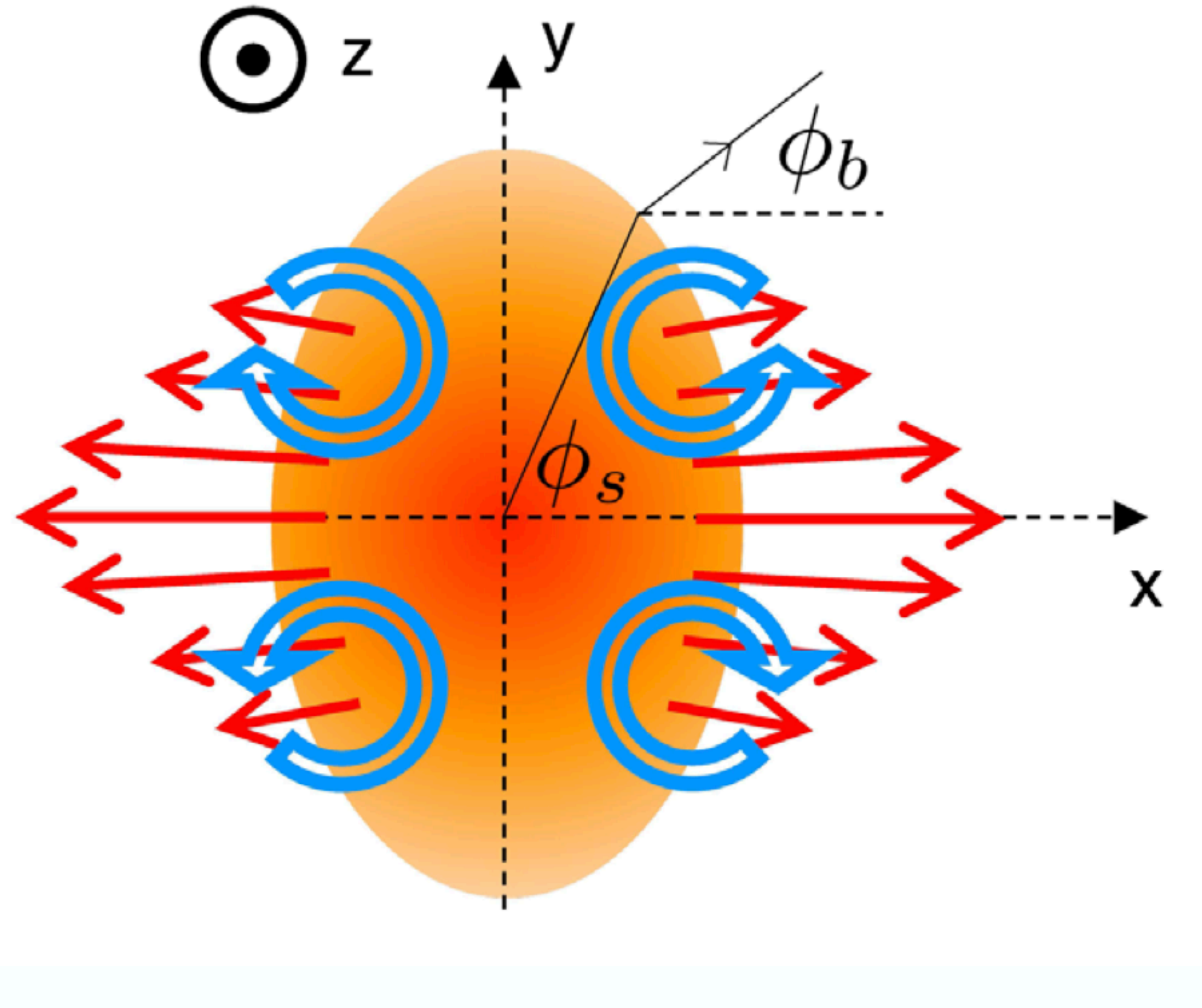
Local **flow structures** in the plane transverse to the beam (jets, ebe fluctuations, collision geometry, etc.) lead to longitudinal (beam-direction) polarization

Pang, Petersen, Wang, Wang, Phys.Rev.Lett. 117 (2016) 19, 192301



Tachibana, Hirano, Nucl.Phys.A 904-905 (2013) 1023c-1026c

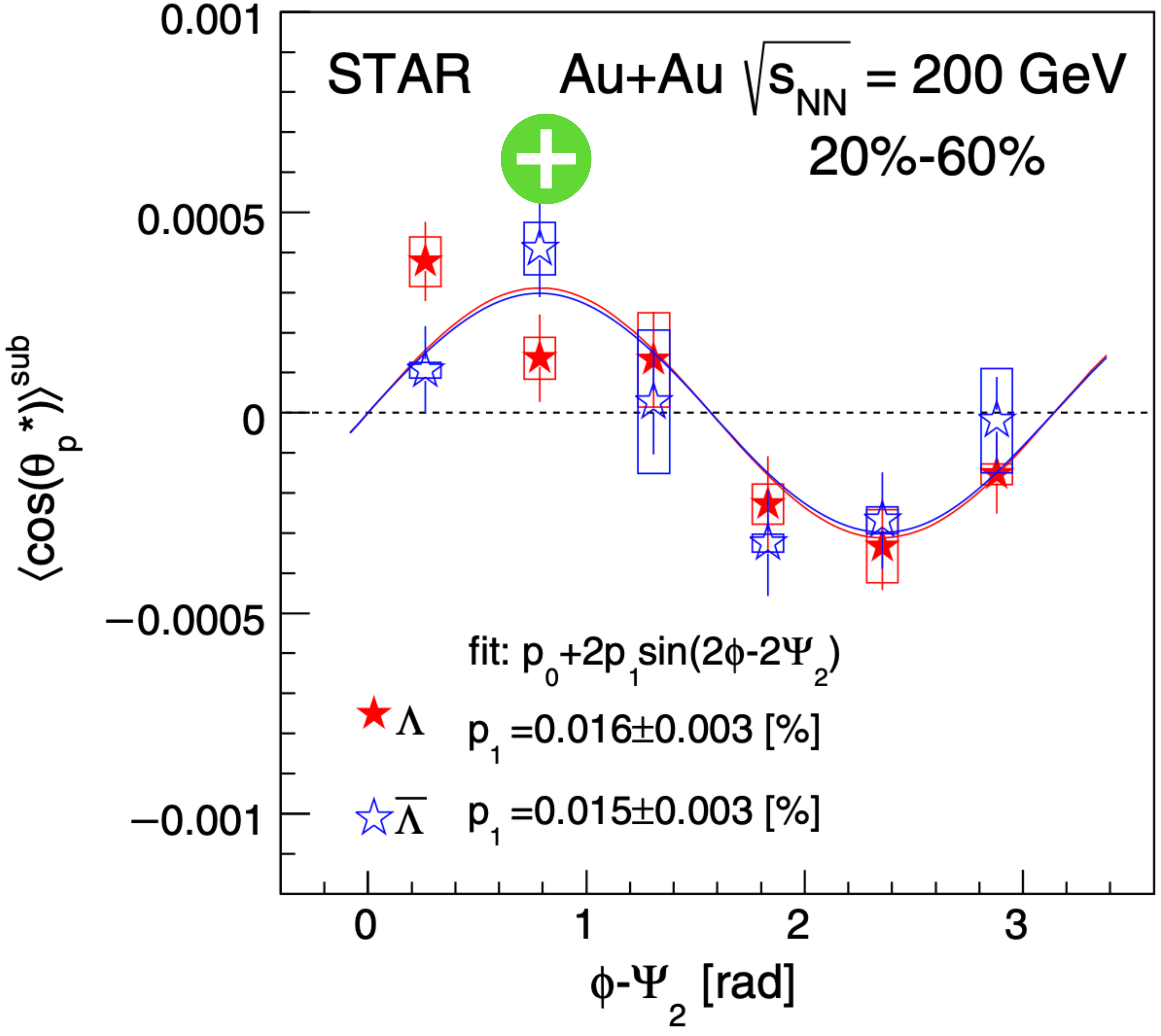


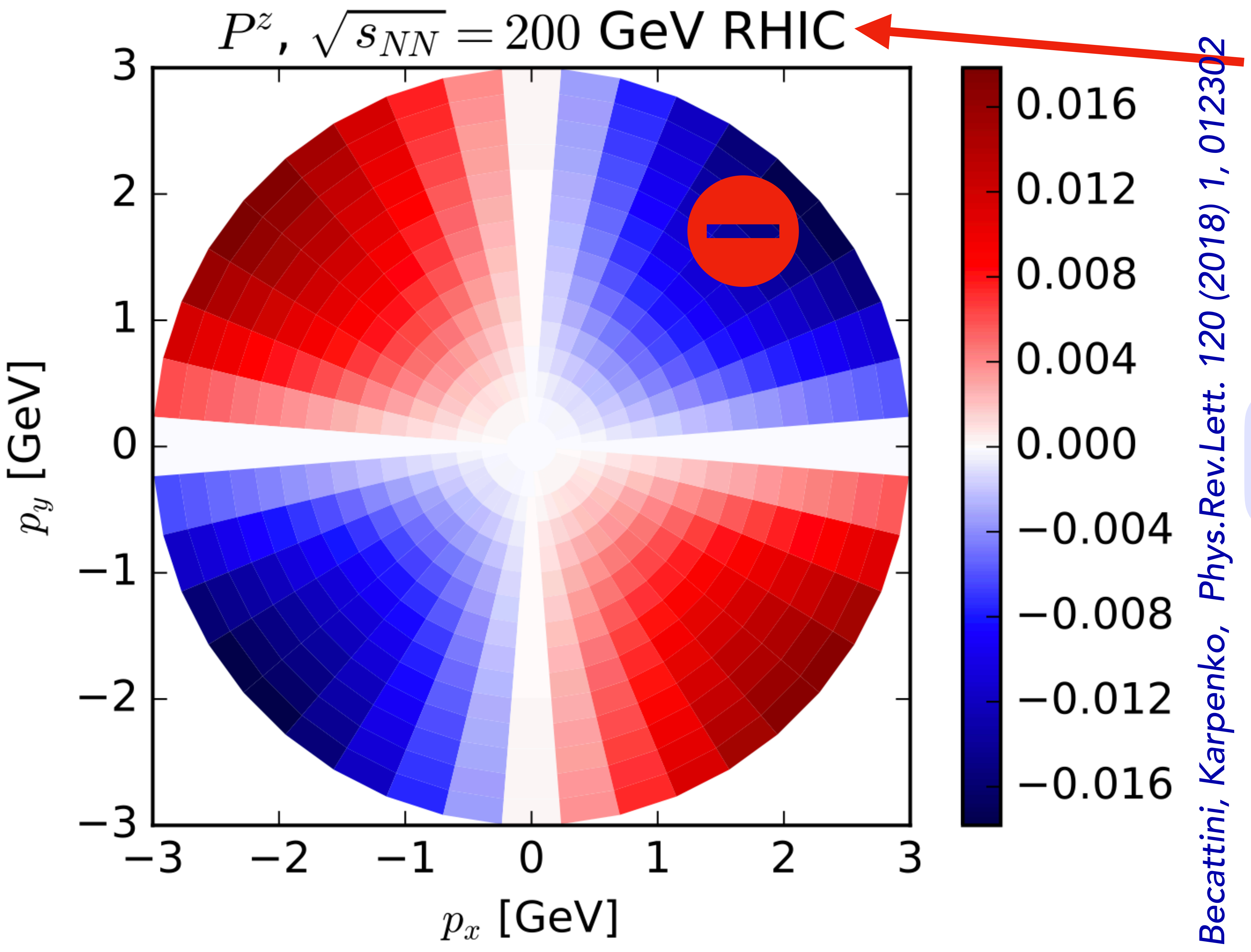


Adam et al (STAR Collaboration) Phys. Rev. Lett. 123, 132301

Longitudinal polarization: measurement vs spin-thermal approach

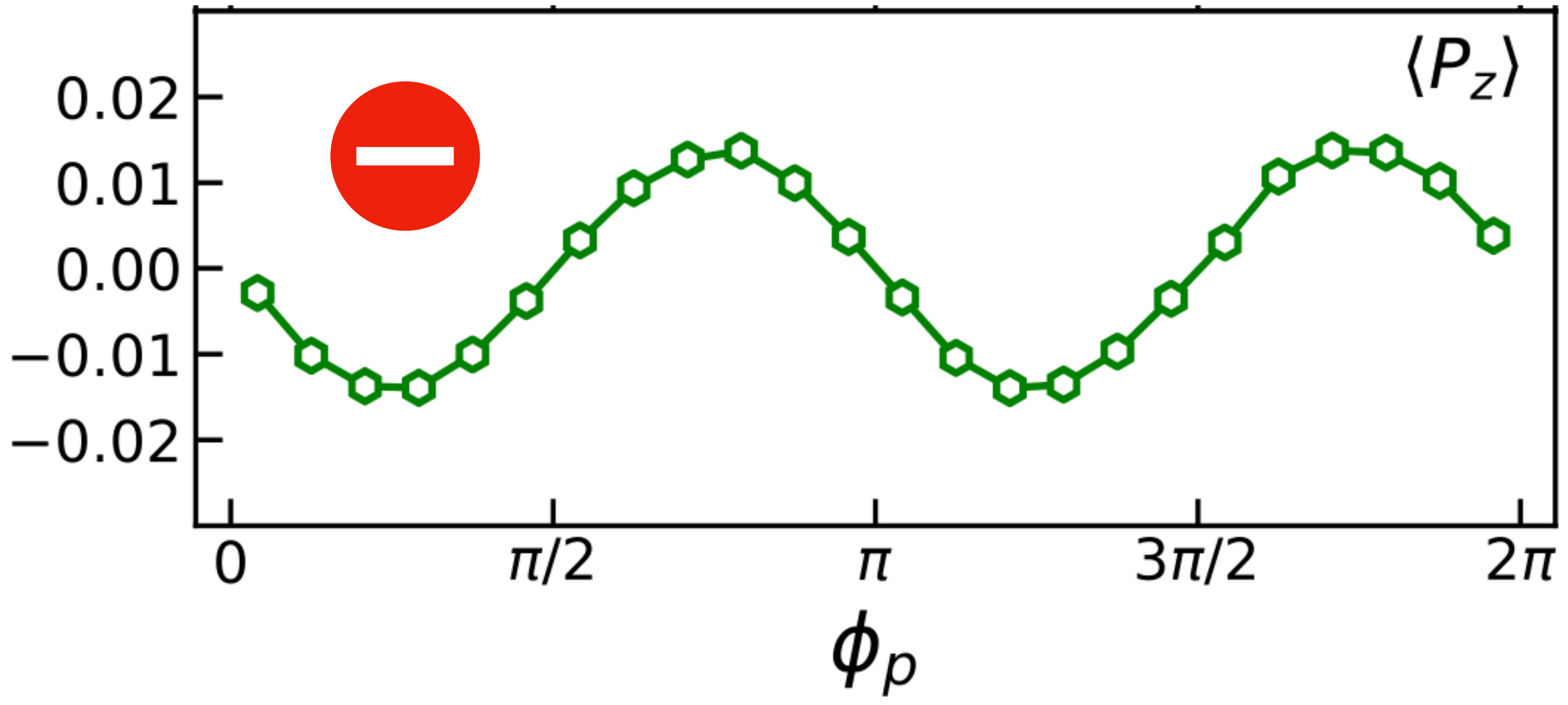
Adam et al (STAR Collaboration) Phys. Rev. Lett. 123, 132301





$$P^z = S^z_\varpi$$

Hydrodynamics



Transport

Xia, Li, Tang, Wang, Phys.Rev.C 98 (2018) 024905

SPIN HYDRODYNAMICS – CURRENT STATUS

Spin-thermal approach does not describe the data properly (or requires some ad-hoc assumptions)

Spin polarization may be trully hydrodynamic variable,
hence it should not be enslaved to thermal vorticity

Perfect spin hydrodynamics was formulated

Florkowski, Friman, Jaiswal, Speranza, Phys. Rev. C97 (4) (2018) 041901
Florkowski, Friman, Jaiswal, RR, Speranza, Phys. Rev. D97 (2018) 116017

Spin hydrodynamics is being actively developed

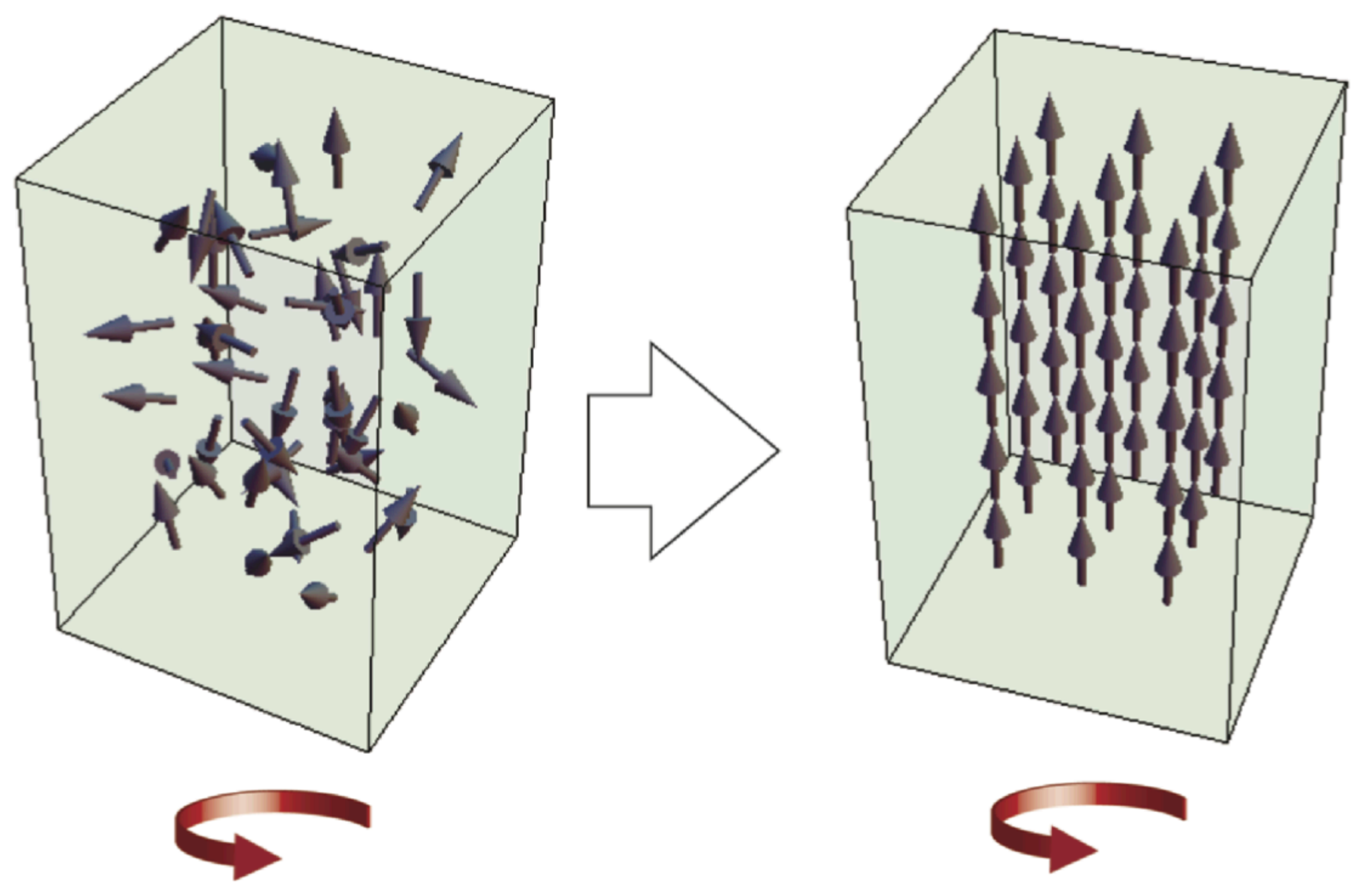
Becattini and Tinti, Annals Phys. 325, 1566 (2010)
Montenegro and Torrieri, Phys. Rev. D 100, 056011 (2019)
Bhadury, Florkowski, Jaiswal, Kumar, and R. R, Phys. Rev. Lett. 129, 192301 (2022)
Weickgenannt, Speranza, Sheng, Wang, and Rischke, Phys. Rev. Lett. 127, 052301 (2021)
Li, Stephanov, and Yee, Phys. Rev. Lett. 127, 082302 (2021)
Gallegos, Gursoy, and Yarom, JHEP 05, 139
Hongo, Huang, Kaminski, Stephanov, Yee JHEP 11, 150
Drogosz, Florkowski, Hontarenko, Phys.Rev.D 110 (2024) 9, 096018

Future measurements are planned (NA61/SHINE)

Bondar and Florkowski, Acta Phys. Polon. B 55, 9 (2024)

Spin hydrodynamics was studied in simple systems
Florkowski, Kumar, RR, Singh, Phys.Rev.C 99 (2019) 4, 044910
Drogosz, Florkowski, Łygan, RR e-Print: 2411.06154

figure: Journal of the Physical Society of Japan 90, 081003 (2021)



A realistic modelling in 3+1D is needed → **THIS TALK**

Theoretical framework

Conservation Laws and Lagrange Multipliers

conservation laws + (near) local equilibrium ➡ hydrodynamics

- conservation of charge (baryon number, electric charge, ...)

$$\partial_\mu N^\mu(x) = 0 \quad \Rightarrow \quad \mu \equiv \xi T$$

(1 eq / charge)

- conservation of energy and linear momentum

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad \Rightarrow \quad T, u^\nu$$

(4 eqs)

Conservation Laws and Lagrange Multipliers

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- conservation of charge (baryon number, electric charge, …)

$$\partial_\mu N^\mu(x) = 0 \quad \Rightarrow \quad \mu \equiv \xi T \qquad \text{(1 eq / charge)}$$

- conservation of energy and linear momentum

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad \Rightarrow \quad T, u^\nu \qquad \text{(4 eqs)}$$

- conservation of angular momentum

spin chemical potential ↓ spin polarization tensor ↓

$$\partial_\lambda J^{\lambda\mu\nu}(x) = 0 \quad \Rightarrow \quad \Omega_{\mu\nu} \equiv T\omega_{\mu\nu} \qquad \text{(6 eqs)}$$

Perfect spin hydrodynamics

Perfect spin hydrodynamics

The **conservation law for total angular momentum** is

$$D_\alpha J^{\alpha,\beta\gamma} = 0$$

PERFECT SPIN HYDRODYNAMICS

The **conservation law for total angular momentum** is

$$D_\alpha J^{\alpha,\beta\gamma} = 0$$

The total angular momentum is decomposed into **orbital angular momentum** and intrinsic **spin tensor**

$$J^{\lambda\mu\nu} = L^{\lambda\mu\nu} + S^{\lambda\mu\nu} = \left(x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}\right) + S^{\lambda\mu\nu}$$

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$$D_\lambda S^{\lambda\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

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For conserved **symmetric EMT** implies the **conservation of the spin tensor** (if the lattter is nonzero)

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For conserved **symmetric EMT** implies the **conservation of the spin tensor** (if the lattter is nonzero)

From Quantum Kinetic Theory at linear order in spin polarization tensor (small polarization limit)

Florkowski, Kumar, and RR, Phys. Rev. C98, 044906 (2018)
Florkowski, RR, and Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)

$$T^{\mu\nu} = T^{\mu\nu}[\beta, \cancel{\omega}, \xi], \quad S^{\mu,\lambda\nu} = S^{\mu,\lambda\nu}[\beta, \omega, \xi], \quad N^\mu = N^\mu[\beta, \cancel{\omega}, \xi]$$

Background hydrodynamics decouples from spin hydrodynamics!

Perfect spin hydrodynamics

Background hydrodynamics

$$T^{\mu\nu} = T^{\mu\nu}[\beta,\xi]$$
$$N^{\mu} = N^{\mu}[\beta,\xi]$$

$$D_\alpha T^{\alpha\beta}(x) = 0$$
$$D_\alpha N^{\alpha}(x) = 0$$

$$T,\, \mu_B\, u^\rho$$

Spin hydrodynamics

$$S^{\mu,\lambda\nu} = S^{\mu,\lambda\nu}[\beta,\omega,\xi]$$

$$D_\alpha S^{\alpha,\beta\gamma}(x) = 0$$

$$\omega_{\mu\nu}$$

BACKGROUND HYDRODYNAMICS

BACKGROUND HYDRODYNAMICS

Equations of motion (EOMs) for **relativistic viscous hydrodynamics** result from the following **conservation laws**

$$\begin{aligned} D_\alpha T^{\alpha\beta}(x) &= 0 \\ D_\alpha N^{\alpha}(x) &= 0 \end{aligned}$$

We adopt **Landau's definition of flow four-velocity**

$$T^{\alpha\beta} u_\beta = \varepsilon\, u^\alpha .$$

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\mathrm{eq}} + \Pi)\, \Delta^{\alpha\beta} + \pi^{\alpha\beta} \quad \text{(energy-momentum tensor, EMT)}$$

$$N^\alpha = n\, u^\alpha + n^\alpha \quad \text{(net baryon current)}$$

BACKGROUND HYDRODYNAMICS

In this case, the **constitutive relations** read

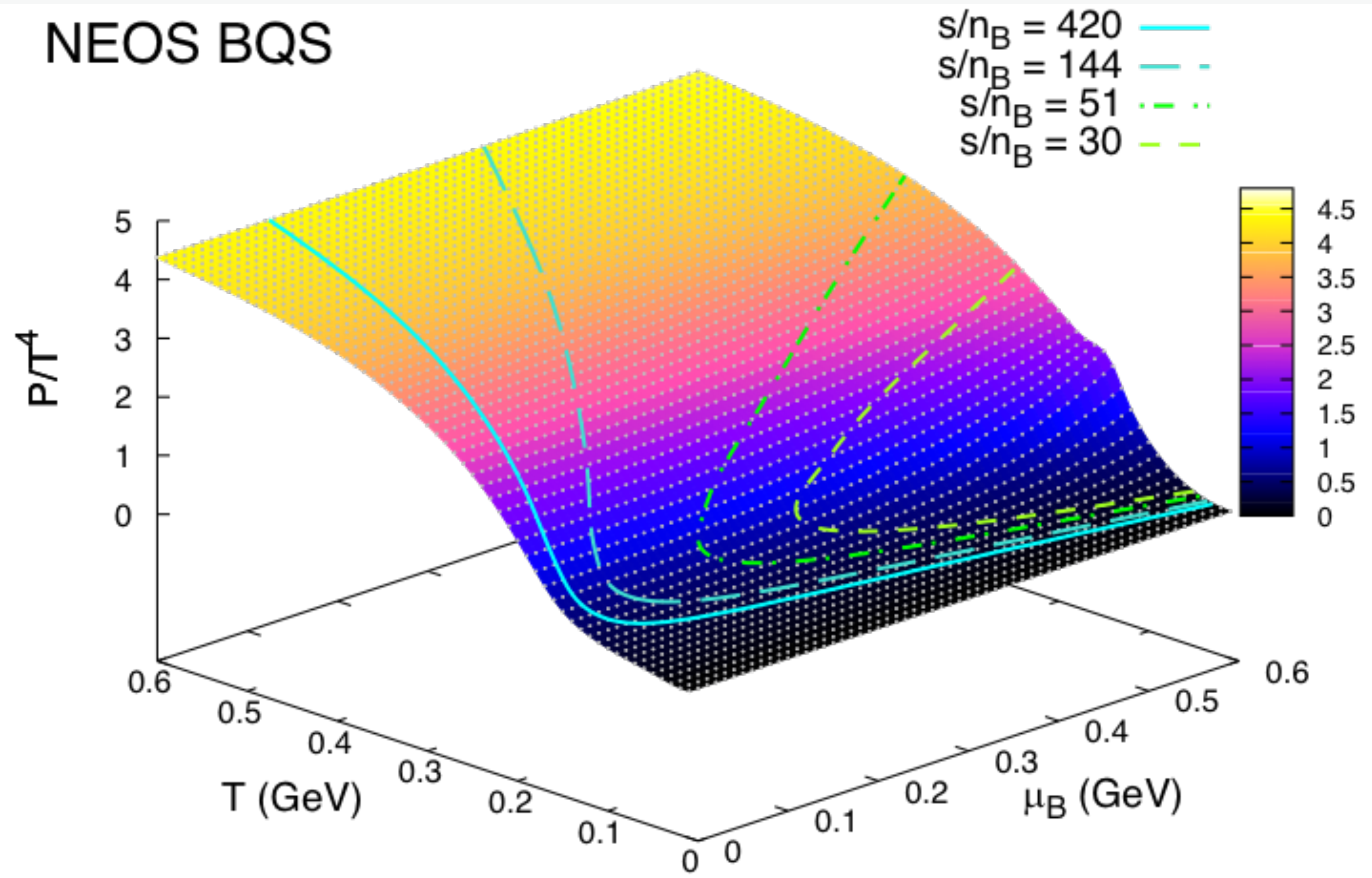
$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} + \Pi)\, \Delta^{\alpha\beta} + \pi^{\alpha\beta}$$
$$N^\alpha = n\, u^\alpha + n^\alpha$$

Must be supplemented by **equation of state (EOS)** relating pressure to energy density and baryon density

$$P_{\rm eq} = P_{\rm eq}(\varepsilon, n)$$

We use **lattice-QCD-based EOS at finite net baryon density** which exhibits a crossover phase transition across the entire parametric space of the phase diagram

Monnai, Schenke, and Shen, Phys. Rev. C 100, 024907 (2019)
Shen and Alzhrani, Phys. Rev. C 102, 014909 (2020)



Background hydrodynamics

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At **second order** in spacetime gradients, in DNMR framework, the **time evolution of the dissipative currents** is

Denicol, Niemi, Molnar, and Rischke, Phys. Rev. D 85, 114047 (2012);
Denicol, Jeon, and Gale, Phys. Rev. C 90, 024912 (2014).

$$\dot{\Pi} = \frac{\Pi_{\rm NS} - \Pi}{\tau_\Pi} - \frac{\delta_{\Pi\Pi}}{\tau_\Pi}\Pi\,\theta + \frac{\lambda_{\Pi\pi}}{\tau_\Pi}\pi^{\alpha\beta}\sigma_{\alpha\beta}$$
$$\dot{\pi}^{\langle\alpha\beta\rangle} = \frac{\pi^{\alpha\beta}_{\rm NS} - \pi^{\alpha\beta}}{\tau_\pi} - \frac{\delta_{\pi\pi}}{\tau_\pi}\pi^{\alpha\beta}\theta + \frac{\lambda_{\pi\Pi}}{\tau_\pi}\Pi\,\sigma^{\alpha\beta} - \frac{\tau_{\pi\pi}}{\tau_\pi}\pi_\gamma^{\langle\alpha}\sigma^{\beta\rangle\gamma} + \frac{\phi_7}{\tau_\pi}\pi_\gamma^{\langle\alpha}\pi^{\beta\rangle\gamma}$$

The **second-order transport coefficients** are calculable from kinetic theory

Denicol, Jeon, and Gale, Phys. Rev. C 90, 024912 (2014);

$$\frac{\delta_{\Pi\Pi}}{\tau_\Pi} = \frac{2}{3}\,, \quad \frac{\lambda_{\Pi\pi}}{\tau_\Pi} = \frac{8}{5}\left(\frac{1}{3} - c_s^2\right) \quad \frac{\delta_{\pi\pi}}{\tau_\pi} = \frac{4}{3}\,, \quad \frac{\lambda_{\pi\Pi}}{\tau_\pi} = \frac{6}{5}\,, \quad \frac{\tau_{\pi\pi}}{\tau_\pi} = \frac{10}{7}\,, \quad \frac{\phi_7}{\tau_\pi} = \frac{9}{70 P_{\rm eq}\tau_\pi}$$

SPIN HYDRODYNAMICS

Spin hydrodynamics

The **conservation law for total angular momentum** is

$$D_\alpha S^{\alpha,\beta\gamma}(x) = 0$$

In the **de Groot—van Leeuven—van Weert (GLW) pseudogauge** the spin tensor for spin 1/2 is expressed as

Florkowski, Kumar, and R. R., Phys. Rev. C98, 044906 (2018)
de Groot, van Leeuwen, and van Weert, Relativistic Kinetic Theory: Principles and Applications

$$S^{\alpha,\beta\gamma} = \mathcal{A}_1 u^\alpha \omega^{\beta\gamma} + \mathcal{A}_2 u^\alpha u^{[\beta}\omega^{\gamma]\delta} u_\delta + \mathcal{A}_3 \left(u^{[\beta}\omega^{\gamma]\alpha} + g^{\alpha[\beta}\omega^{\gamma]\delta} u_\delta \right)$$

SPIN HYDRODYNAMICS

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de Groot, van Leeuwen, and van Weert, Relativistic Kinetic Theory: Principles and Applications

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$$\mathcal{A}_1 = \cosh\left(\frac{\mu_B}{T}\right)\frac{T^3}{\pi^2}\left[\left(4+\frac{z^2}{2}\right)K_2(z) + zK_1(z)\right],$$

$$\mathcal{A}_2 = 2\cosh\left(\frac{\mu_B}{T}\right)\frac{T^3}{\pi^2}\left[\left(12+\frac{z^2}{2}\right)K_2(z) + 3zK_1(z)\right], \qquad z \equiv m/T$$

$$\mathcal{A}_3 = \frac{1}{2}\left(\mathcal{A}_1 - \frac{\mathcal{A}_2}{2}\right),$$

$$\mathcal{A}_i = \mathcal{A}_i(\mu_B, T; m)$$

NUMERICAL FRAMEWORK

BACKGROUND HYDRODYNAMICS

BACKGROUND HYDRODYNAMICS

We focus on **Au+Au** collisions at the **top RHIC energy** of $\sqrt{s_{\mathrm{NN}}} = 200\ \mathrm{GeV}$

Initialize the background evolution at the proper time $\tau_0 = 1\ \mathrm{fm}$

The initial **energy density and baryon density profiles** are set according to the model based on the Glauber collision geometry with local energy-momentum conservation

Shen and Alzhrani, Phys. Rev. C 102, 014909 (2020)
Ryu, Jupic, and Shen, Phys. Rev. C 104, 054908 (2021)

However, to compute the thickness functions and wounded nucleon densities we use the **optical limit of the Glauber model**

The **longitudinal flow** is numerically determined from the initial energy-momentum tensor components

The **initial transverse flow components are zero**

The **dissipative corrections initially are zero**

$v^{\eta_s}(\tau_0; x, y, \eta_s)$

$n_B(\tau_0; x, y, \eta_s)$

$\varepsilon(\tau_0; x, y, \eta_s)$

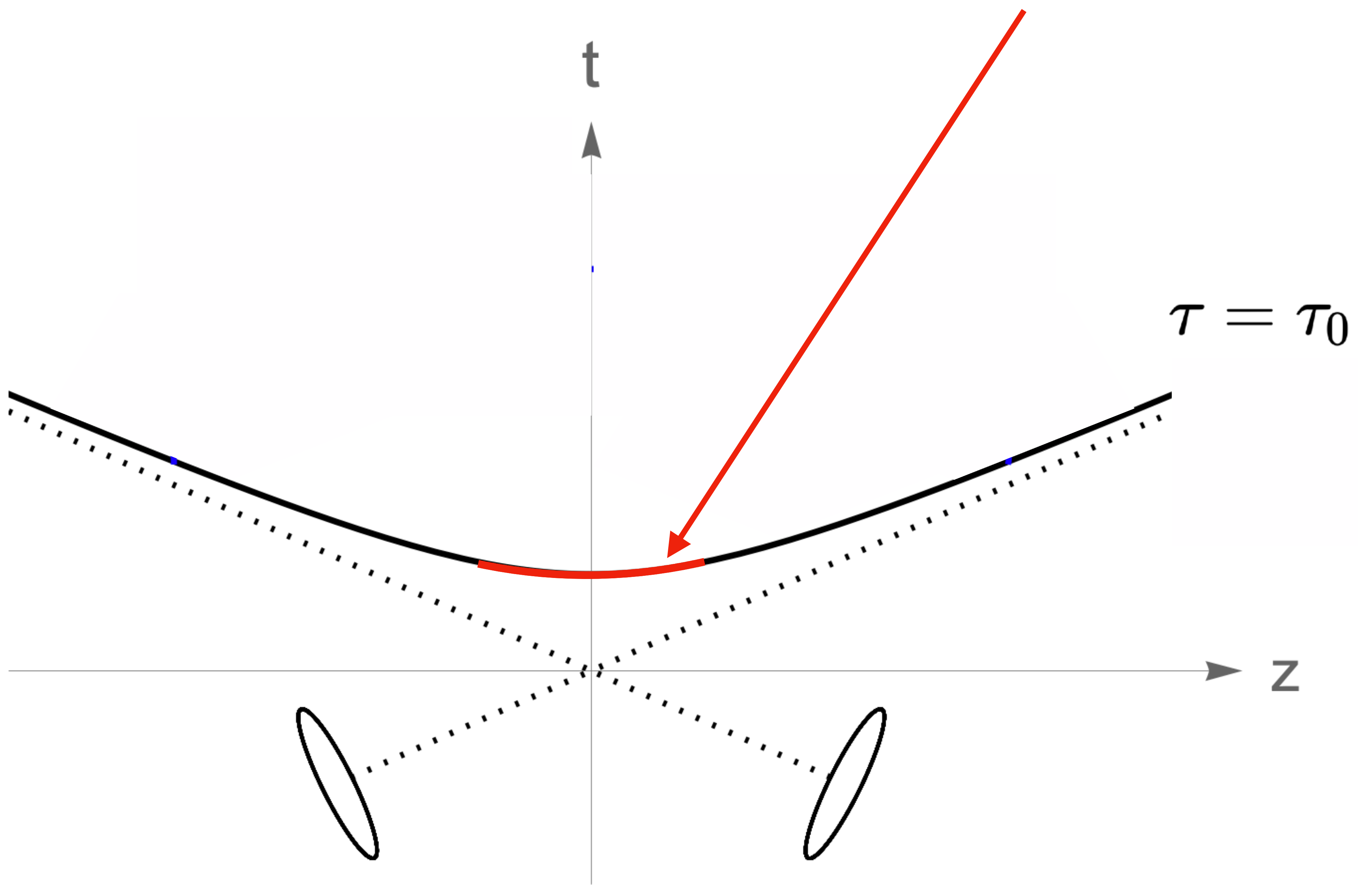

fig: https://arxiv.org/pdf/2407.12130 (modified)

Background hydrodynamics

Evolve background EOMs in 3+1 dimensions in τ

EOMs constitute **11 PDEs** for **11 DOFs**

Use Godunov-type relativistic Harten-Lax-van Leer-Einfeldt (HLLE) approximate Riemann solver

Karpenko, Huovinen, and Bleicher, Comput. Phys. Commun. 185, 3016 (2014)
Singh and Alam, The European Physical Journal C 83, 585 (2023)

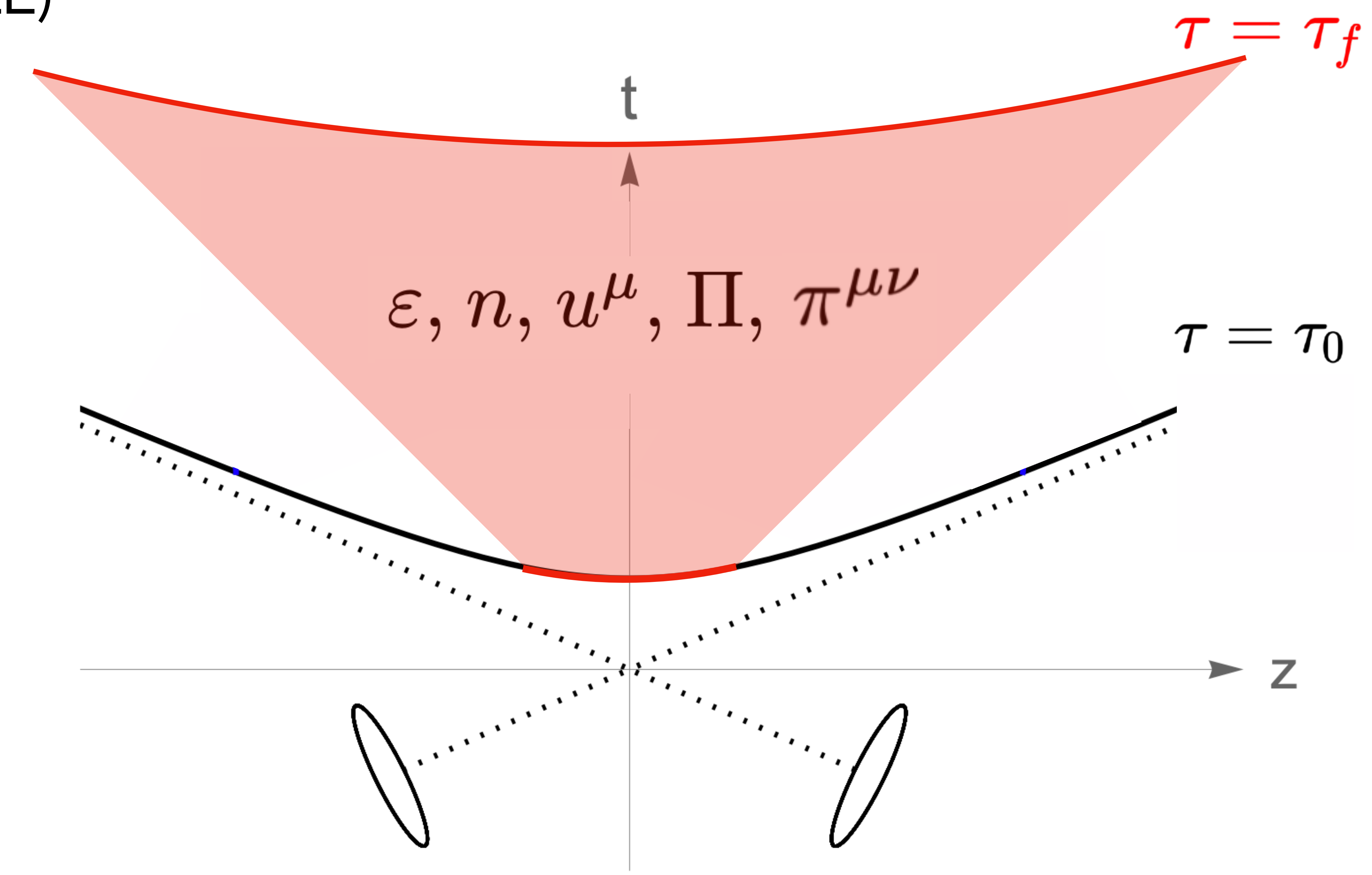

fig: https://arxiv.org/pdf/2407.12130 (modified)

Background hydrodynamics

Evolve until the energy density in the system decreases everywhere below the threshold value $\varepsilon_{\rm sw} = 0.5\ {\rm GeV/fm^3}$

The switching hypersurface Σ is extracted with the CORNELIUS code using the condition $\varepsilon(T, \mu_B) = \varepsilon_{\rm sw}$

Huovinen, Petersen, Eur.Phys.J.A 48 (2012) 171

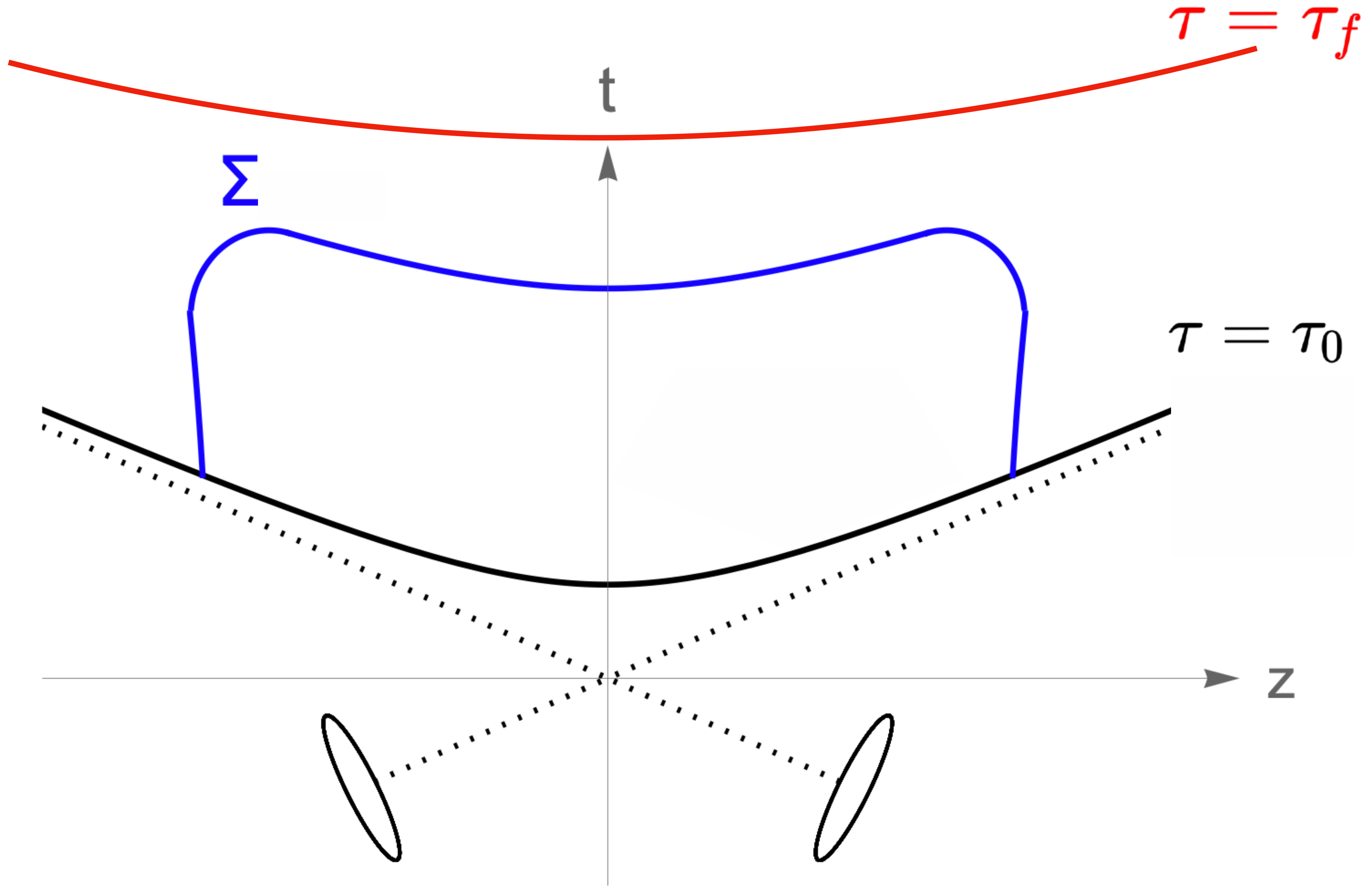

fig: https://arxiv.org/pdf/2407.12130 (modified)

Background hydrodynamics

Evolve until the energy density in the system decreases everywhere below the threshold value $\varepsilon_{\rm sw} = 0.5~{\rm GeV/fm^3}$

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Huovinen, Petersen, Eur.Phys.J.A 48 (2012) 171

The hydrodynamic fields on Σ are passed to a hadron sampler

Karpenko, Huovinen, Petersen, and Bleicher, Phys. Rev. C 91, 064901 (2015),
Schafer, Karpenko, Wu, Hammelmann, and Elfner, The European Physical Journal A 58, 230 (2022).

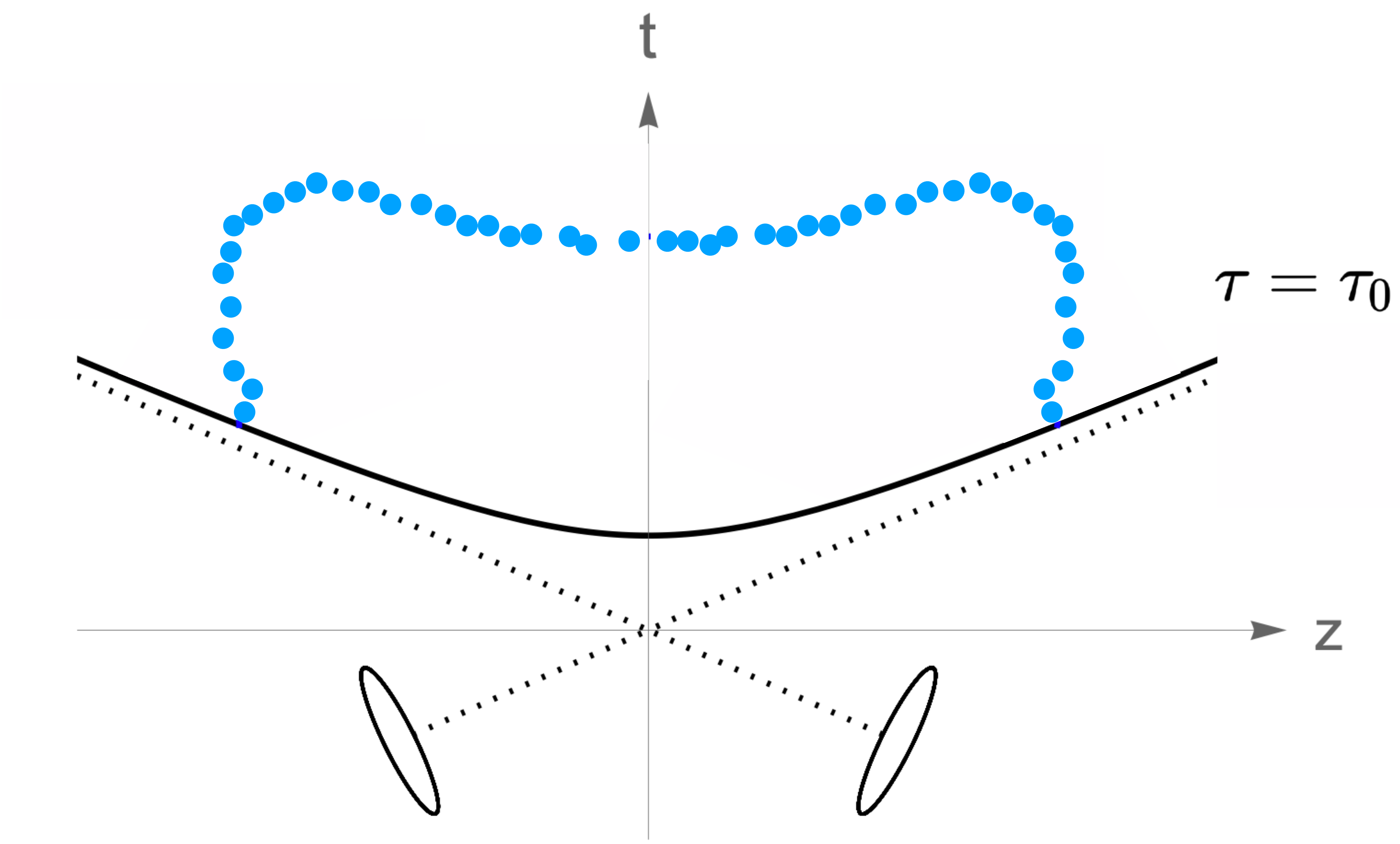

fig: https://arxiv.org/pdf/2407.12130 (modified)

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Evolve until the energy density in the system decreases everywhere below the threshold value $\varepsilon_{\text{sw}} = 0.5\ \text{GeV}/\text{fm}^3$

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Schafer, Karpenko, Wu, Hammelmann, and Elfner, The European Physical Journal A 58, 230 (2022).

The resulting particle set serves as input to the SMASH transport model, which describes subsequent hadron interactions and decays

Weil et al., Phys. Rev. C 94, 054905 (2016)

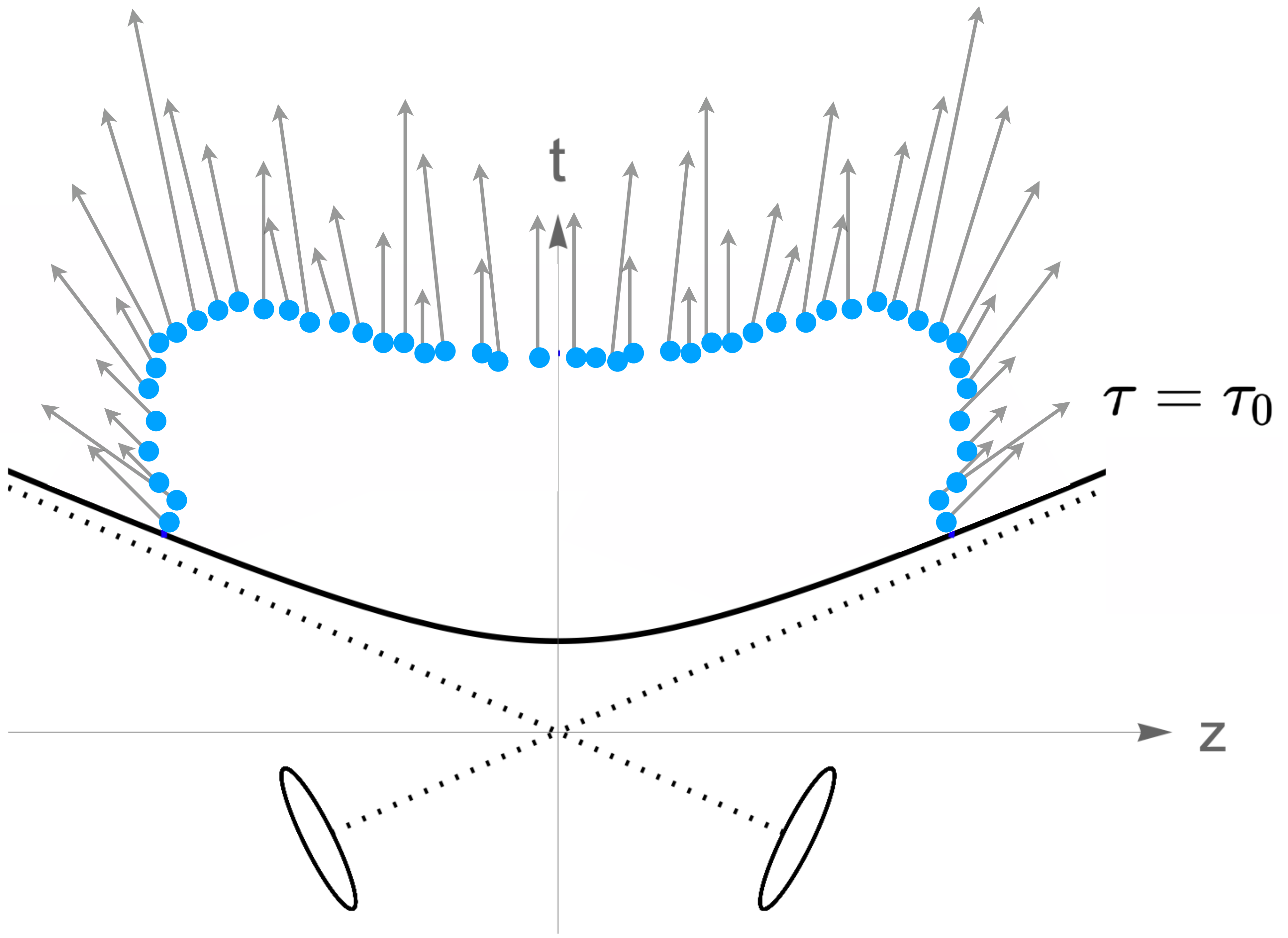
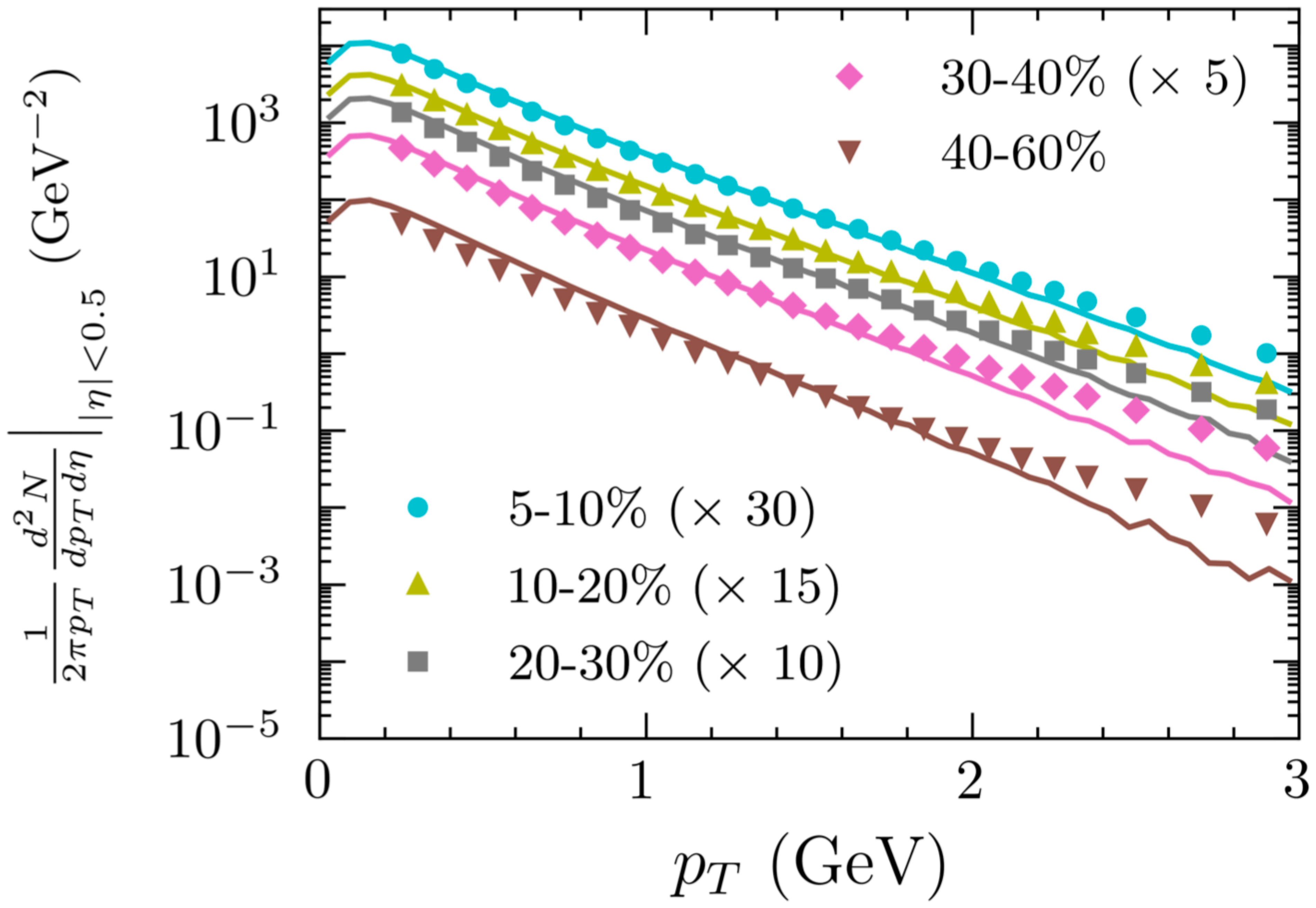


fig: https://arxiv.org/pdf/2407.12130 (modified)

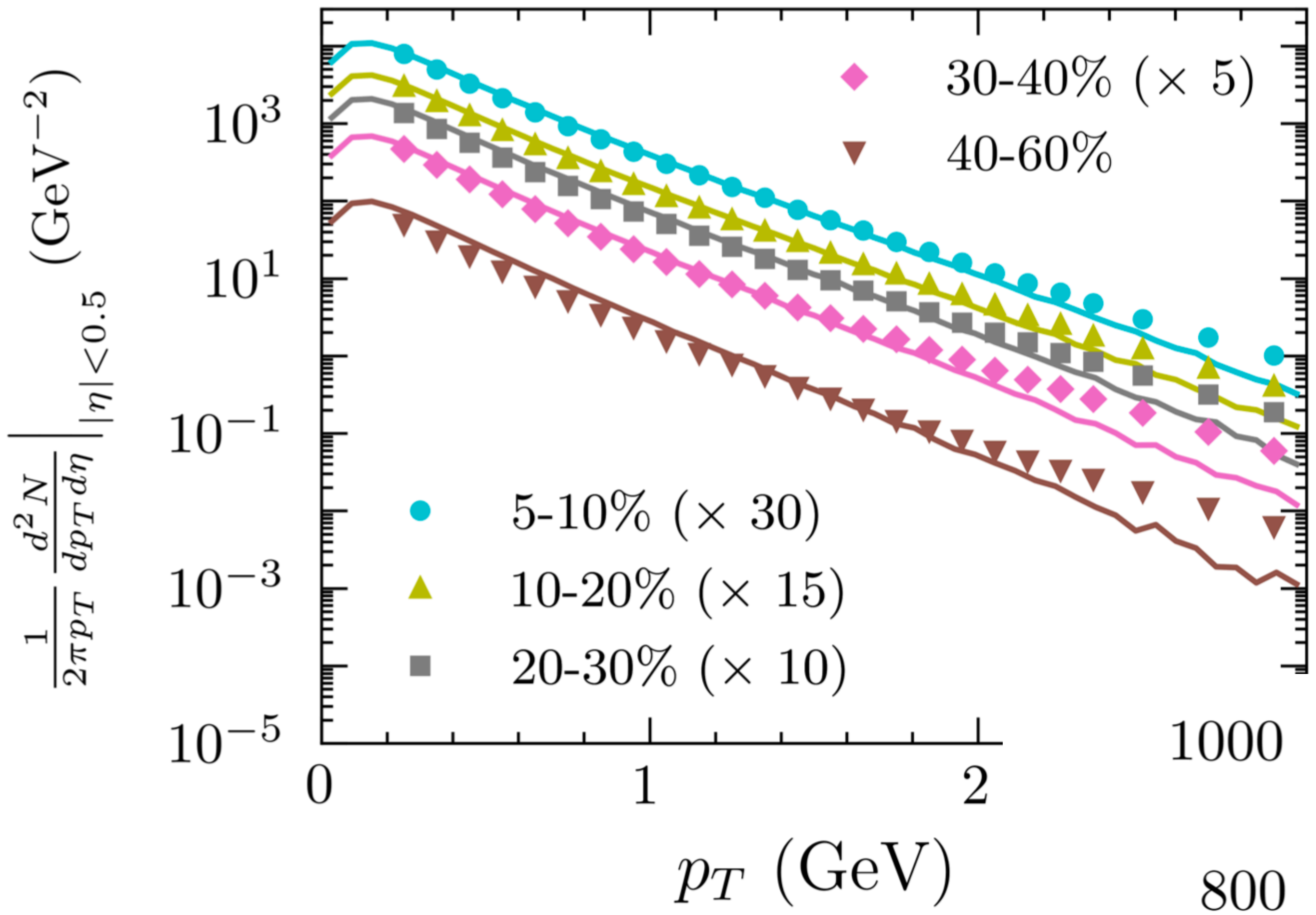
Results for background

Results for background hydrodynamics



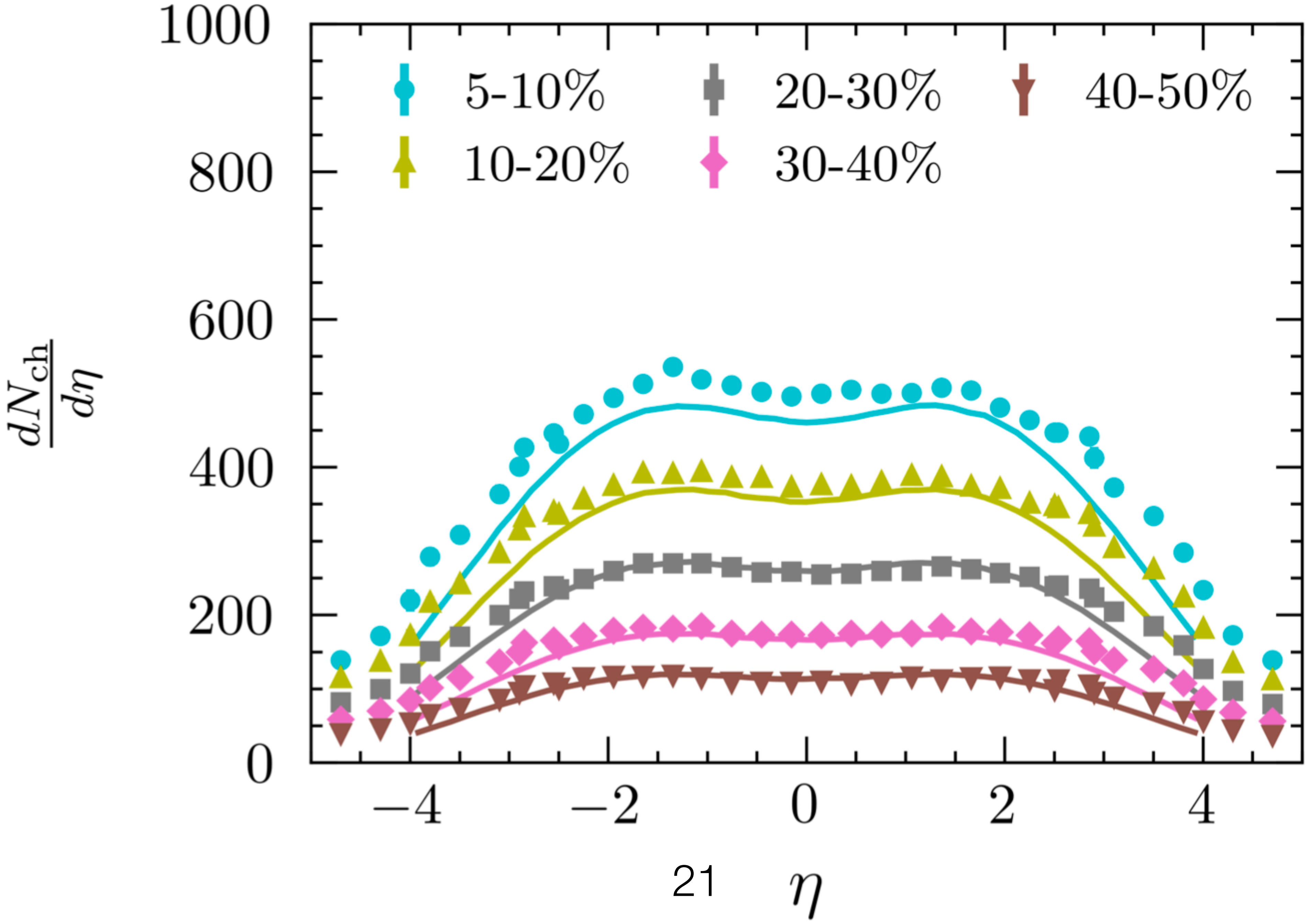
Adams et al. (STAR Collaboration),
Phys. Rev. Lett. 91, 172302 (2003)

Results for background hydrodynamics



Adams et al. (STAR Collaboration), Phys. Rev. Lett. 91, 172302 (2003)

Bearden et al. (BRAHMS Collaboration), Phys. Rev. Lett. 88, 202301 (2002).



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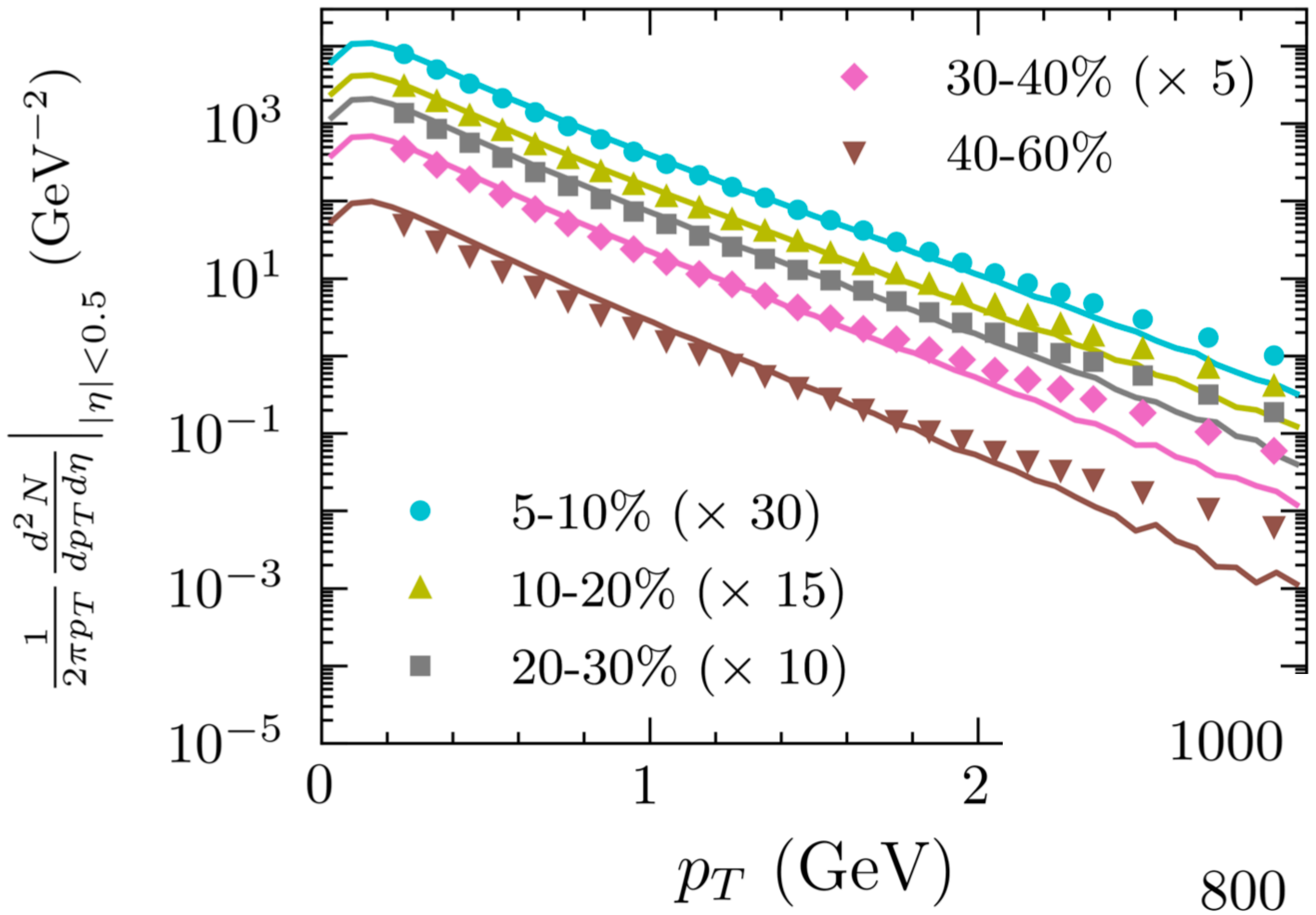

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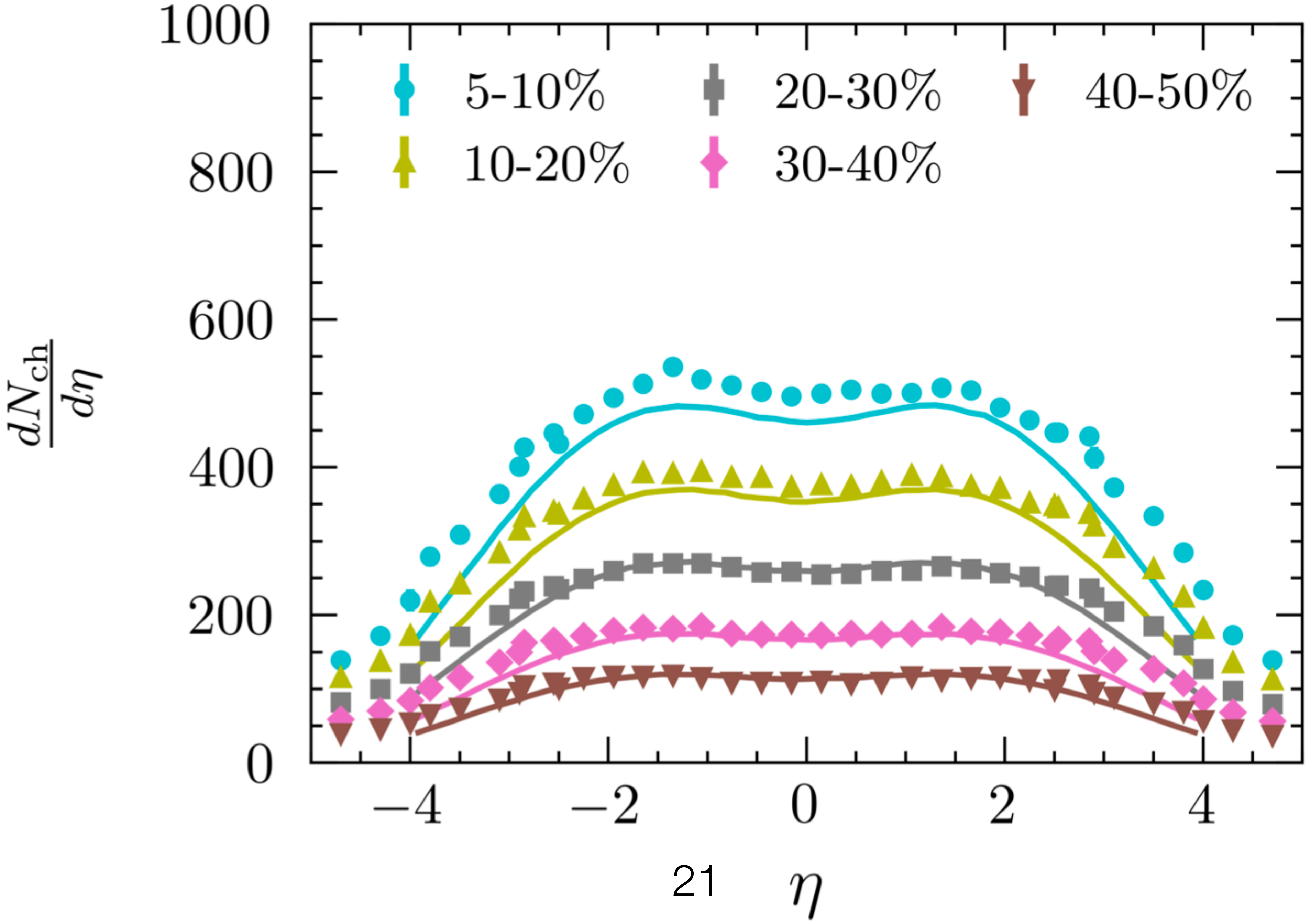
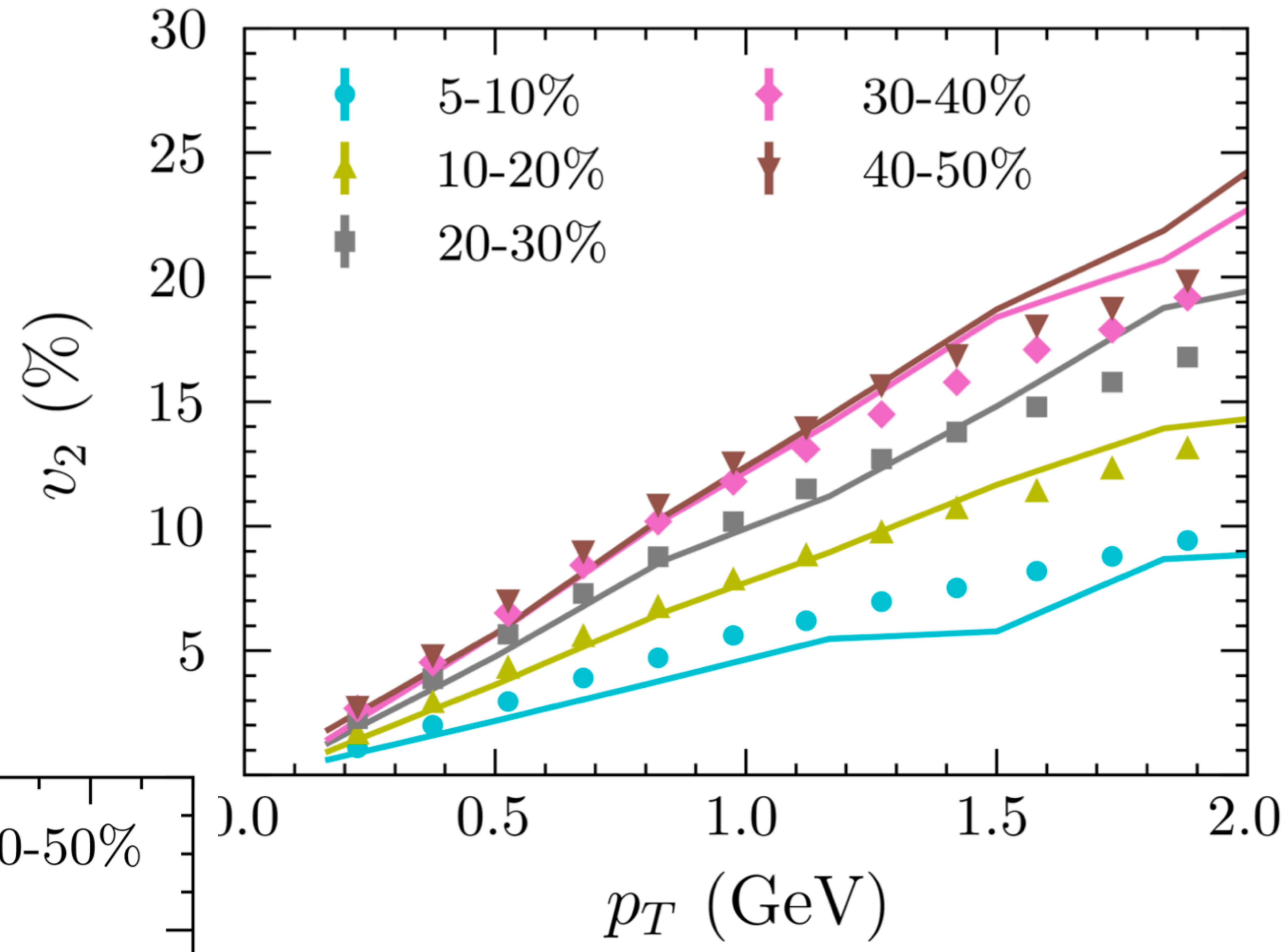




Adams et al. (STAR and STAR-RICH Collaboration),
Phys. Rev. C 72, 014904 (2005)

Numerical framework

Spin hydrodynamics

SPIN HYDRODYNAMICS

Initialize the spin evolution at the proper time $\tau_0^s \geq \tau_0$

$T, \mu_B\, u^\rho$ → $\omega_{\mu\nu}(\tau_0; x, y, \eta_s)$

We intend to account for equilibration of spin DOFs resulting from strong spin-orbit interactions occurring in the early stages before the system reaches perfect spin hydrodynamics regime

Initial **spin polarization tensor** is set as follows

Liu and Yin, JHEP 07, 188,
Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)

$$\omega_{\mu\nu}(\tau_0^s) = \varpi_{\mu\nu} + 4\hat{\tau}_{[\mu}\xi_{\nu]\rho}u^\rho$$

$$\varpi_{\mu\nu} = \varpi_{\mu\nu}^{\rm iso} + \varpi_{\mu\nu}^{\rm T} \qquad \xi_{\mu\nu} = \xi_{\mu\nu}^{\rm iso} + \xi_{\mu\nu}^{\rm T}$$

$$\varpi_{\mu\nu}^{\rm iso} = \frac{1}{T}\partial_{[\nu}u_{\mu]} \qquad \varpi_{\mu\nu}^{\rm T} = \frac{1}{T}u_{[\nu}\partial_{\mu]}\ln T$$

$$\hat{\tau}^\mu = (1,0,0,0)$$

$$\omega_{\mu\nu}(\tau_0^s) = \varpi_{\mu\nu}^{\rm iso} + 4\hat{\tau}_{[\mu}\xi_{\nu]\rho}^{\rm iso}u^\rho$$

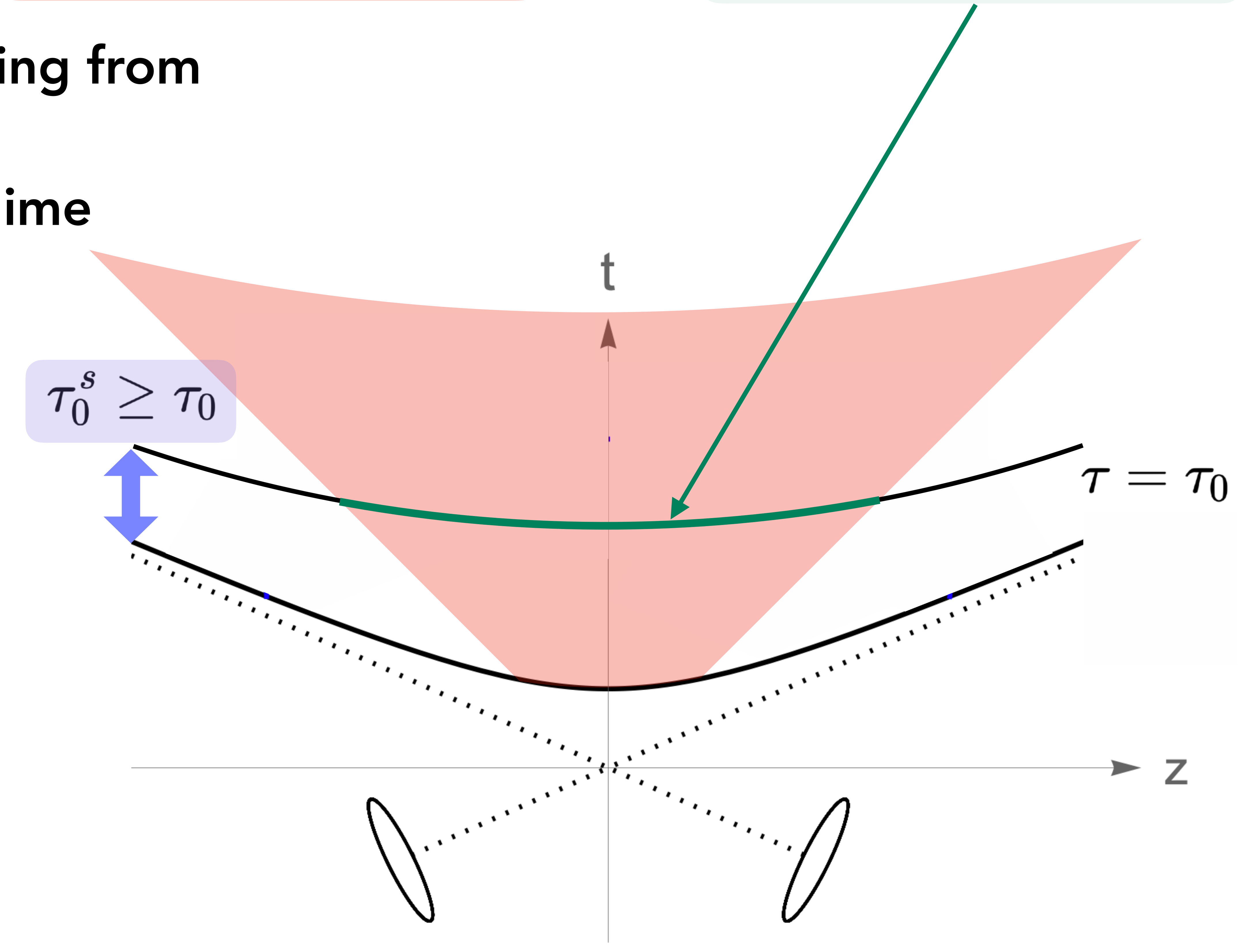


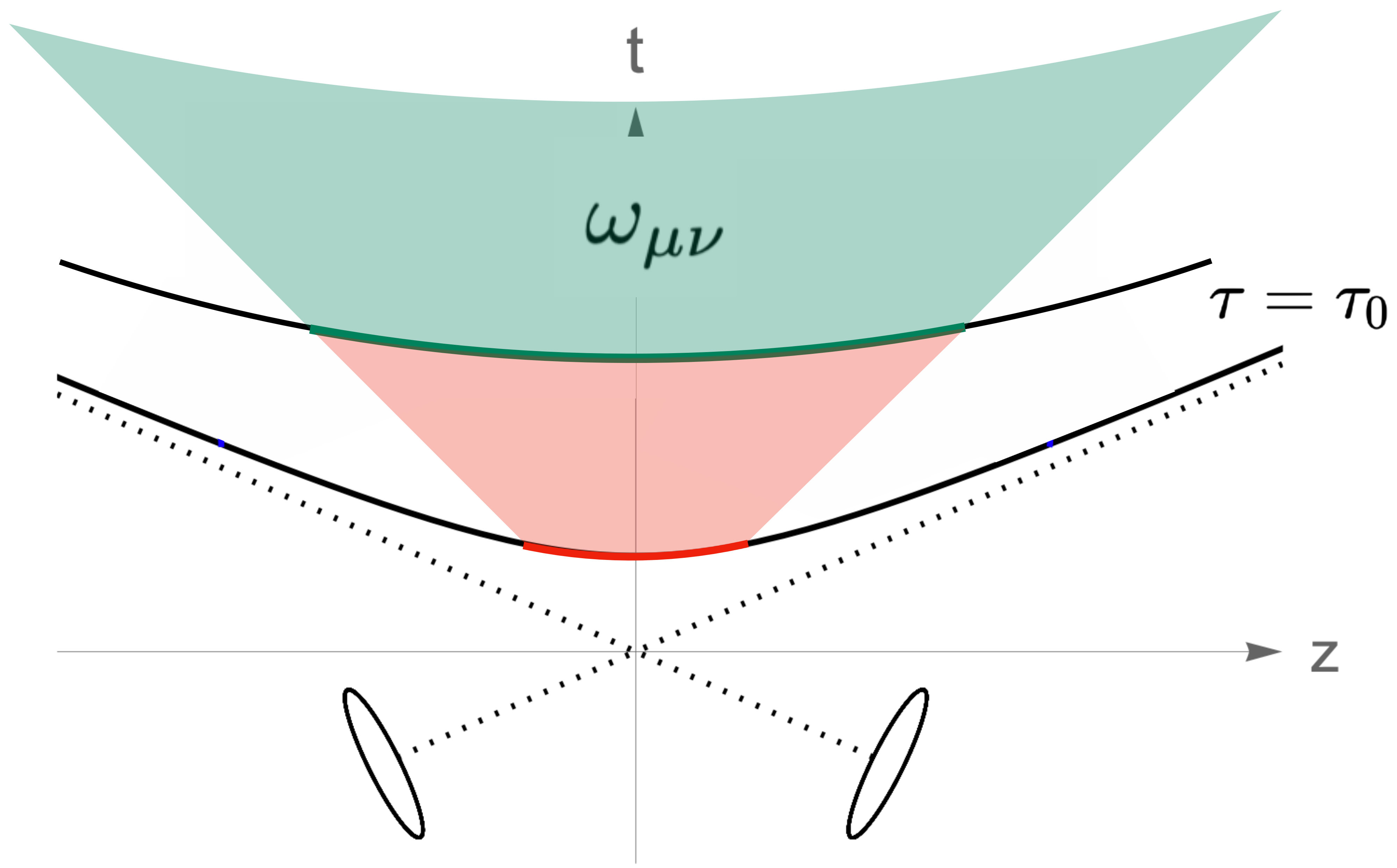
fig: https://arxiv.org/pdf/2407.12130 (modified)

Spin hydrodynamics

Spin EOMs constitute **6 PDEs** for **6 DOFs:** $\omega_{\mu\nu}$

We extended the code (using also HLLE algorithm) to incorporate the spin EOMs

Evolve spin EOMs in 3+1 dimensions in τ



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Evolve spin EOMs in 3+1 dimensions in τ

We **calculate spin observables for Λ hyperons at Σ**

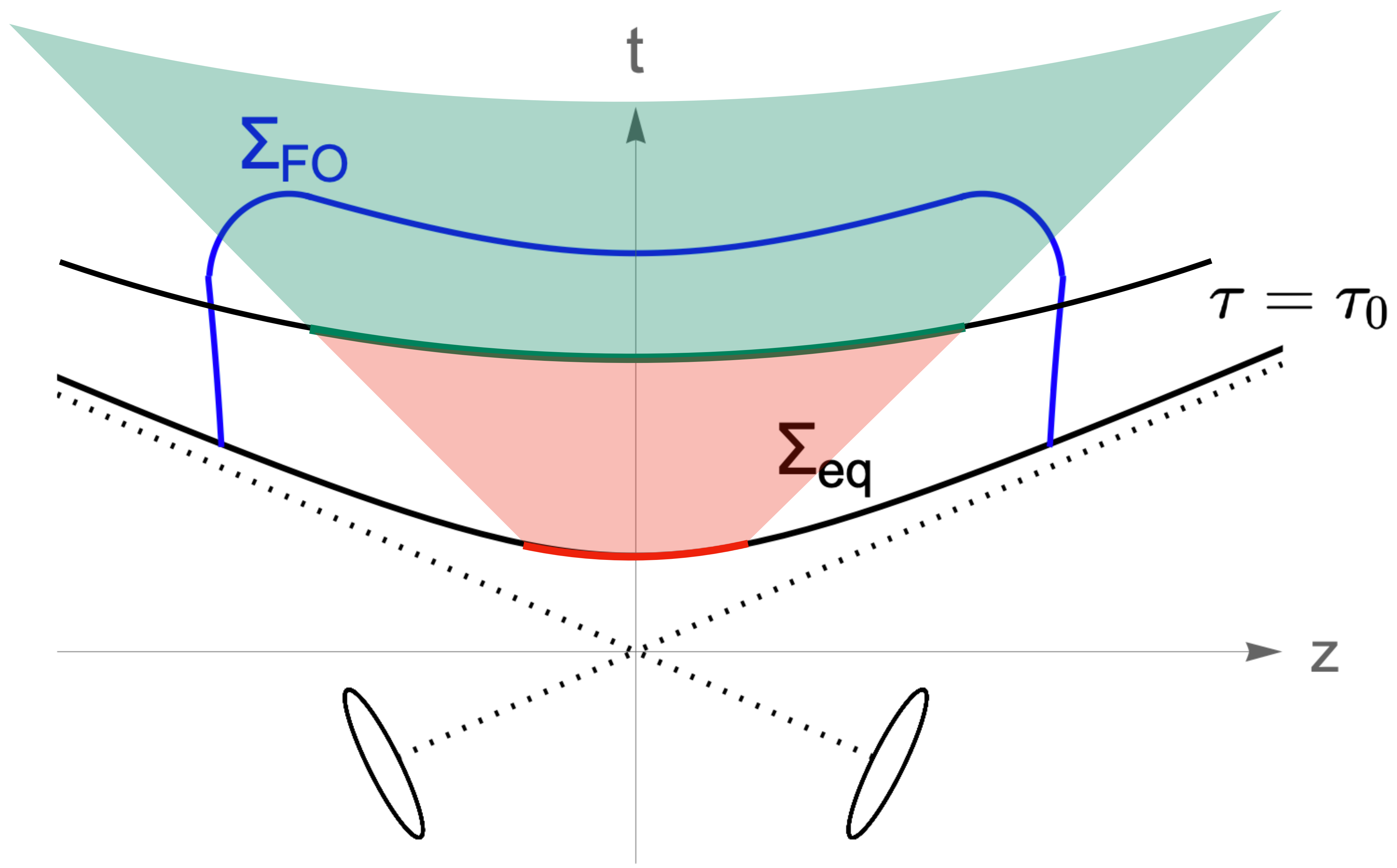

fig: https://arxiv.org/pdf/2407.12130 (modified)

Results for spin

POLARIZATION VECTOR FROM SPIN HYDRODYNAMICS

We calculate the components of the **polarization vector** for Λ hyperons

Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^\mu(p) = -\frac{1}{8m_\Lambda}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int d\Sigma\cdot p\, n_F(1-n_F)\omega_{\nu\rho}}{\int d\Sigma\cdot p\, n_F},$$

POLARIZATION VECTOR FROM SPIN-THERMAL APROACH

Our results are compared with those obtained using the spin polarization formula obtained at first order of thermodynamic gradients

Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)
Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^\mu(p) = S^\mu_\varpi(p) + S^\mu_\xi(p)$$

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$$S^\mu(p) = S^\mu_\varpi(p) + S^\mu_\xi(p)$$

Where the first term is

Becattini, Inghirami, Rolando, Beraudo, DelZanna, DePace, Nardi, Pagliara, and Chandra, Eur. Phys. J. C 75, 406 (2015),

$$S^\mu_\varpi(p) \; = \; -\frac{1}{8m_\Lambda}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int d\Sigma\cdot p\; n_F(1-n_F)\varpi_{\nu\rho}}{\int d\Sigma\cdot p\; n_F}$$

$$\varpi_{\mu\nu} = \partial_{[\nu}\beta_{\mu]} \; = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right)$$

Polarization vector from spin-thermal aproach

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Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)
Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^\mu(p) = S^\mu_\varpi(p) + S^\mu_\xi(p)$$

For the second term there are currently two prescriptions

First is **BBP**

Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)

$$S^\mu_{\xi,\mathrm{BBP}}(p) = -\frac{\epsilon^{\mu\nu\rho\sigma}}{4m_\Lambda}\frac{p_\sigma p^\lambda}{p\cdot\hat{t}}\frac{\int d\Sigma\cdot p\; n_F(1-n_F)\hat{t}_\nu\xi_{\lambda\rho}}{\int d\Sigma\cdot p\; n_F}$$

$$\xi_{\mu\nu} = \partial_{(\nu}\beta_{\mu)} = \frac{1}{2}\left(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu\right)$$

$$\hat{t} = (1,0,0,0)$$

Polarization vector from spin-thermal aproach

Our results are compared with those obtained using the spin polarization formula obtained at first order of thermodynamic gradients

Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)
Buzzegoli, Phys. Rev. C 105, 044907 (2022)

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For the second term there are currently two prescriptions

Second is **LY**

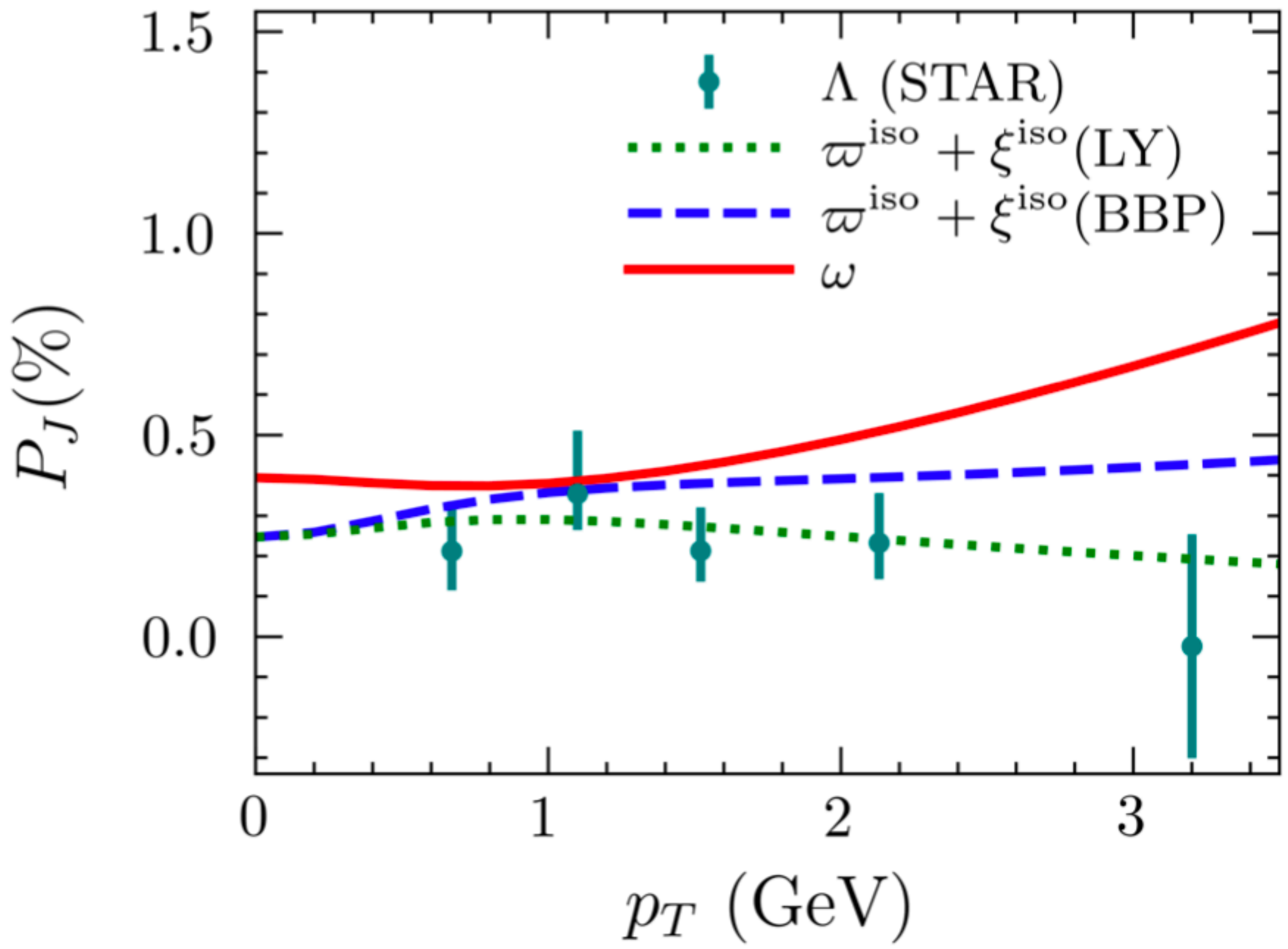
Liu and Yin, JHEP 07, 188,

$$S^\mu_{\xi,\mathrm{LY}}(p) \;=\; -\frac{\epsilon^{\mu\nu\rho\sigma}}{4m_\Lambda}p_\sigma \frac{\int d\Sigma\cdot p\; n_F(1-n_F)\frac{p^\lambda_\perp u_\nu}{p\cdot u}\xi_{\rho\lambda}}{\int d\Sigma\cdot p\; n_F}$$

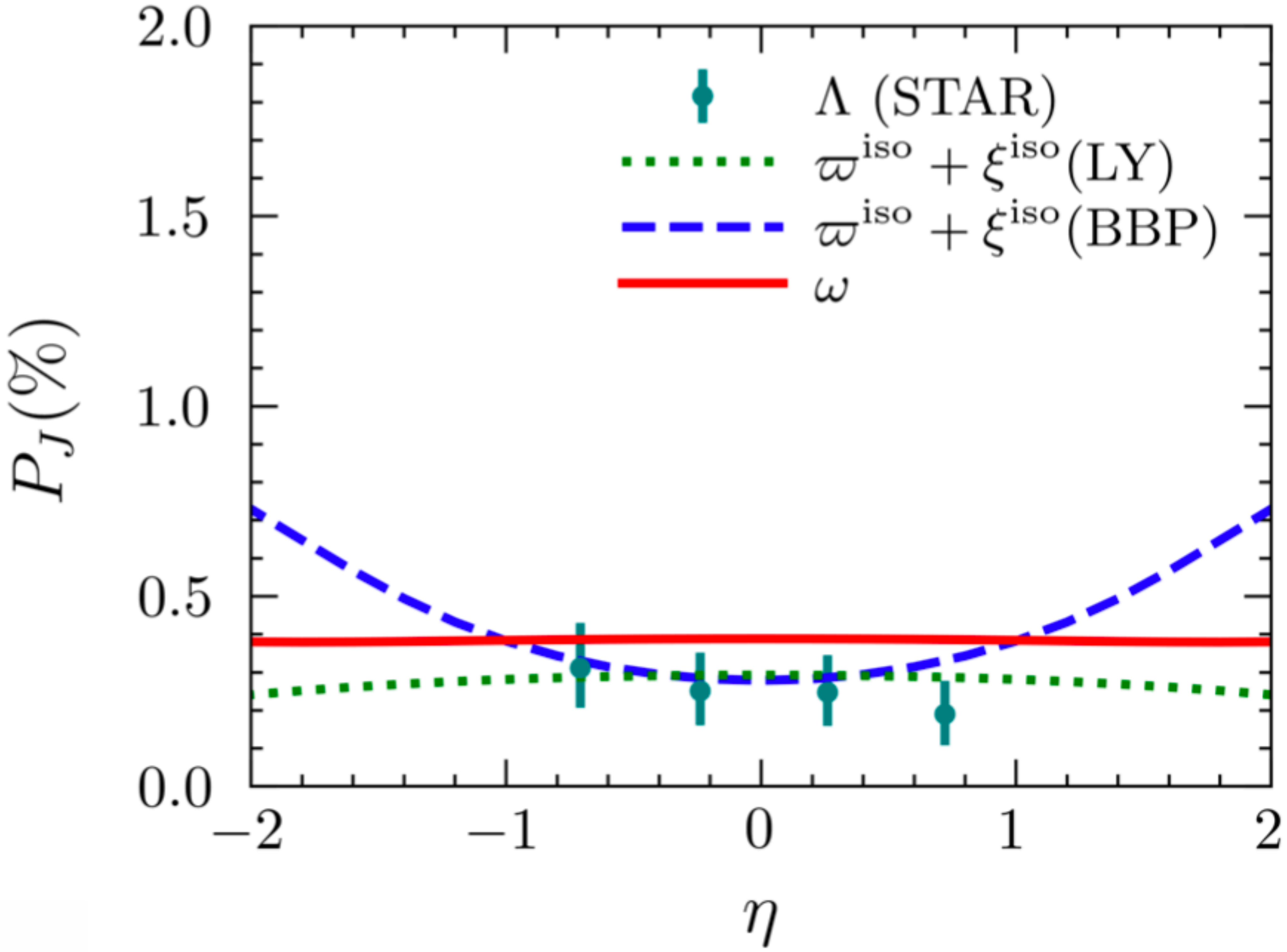
$$p^\perp_\mu \equiv \Delta_\mu^{\ \nu} p_\nu$$

RESULTS FOR SPIN HYDRODYNAMICS

Adam et al. (STAR Collaboration), Phys. Rev. C 98, 014910 (2018)



Adam et al. (STAR Collaboration), Phys. Rev. C 98, 014910 (2018)

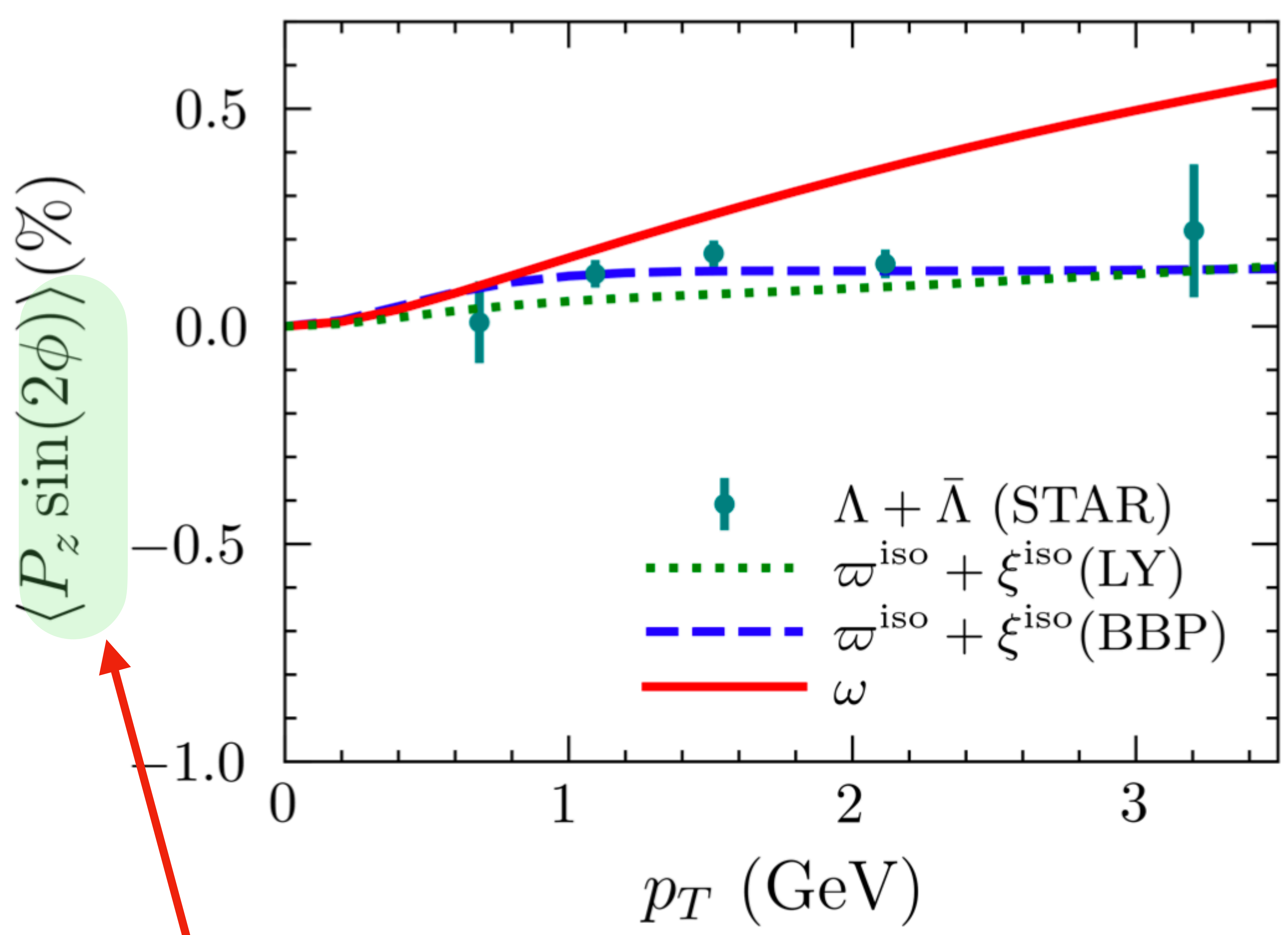


$$\tau_0^s = 4 \text{ fm} \qquad m = m_\Lambda$$

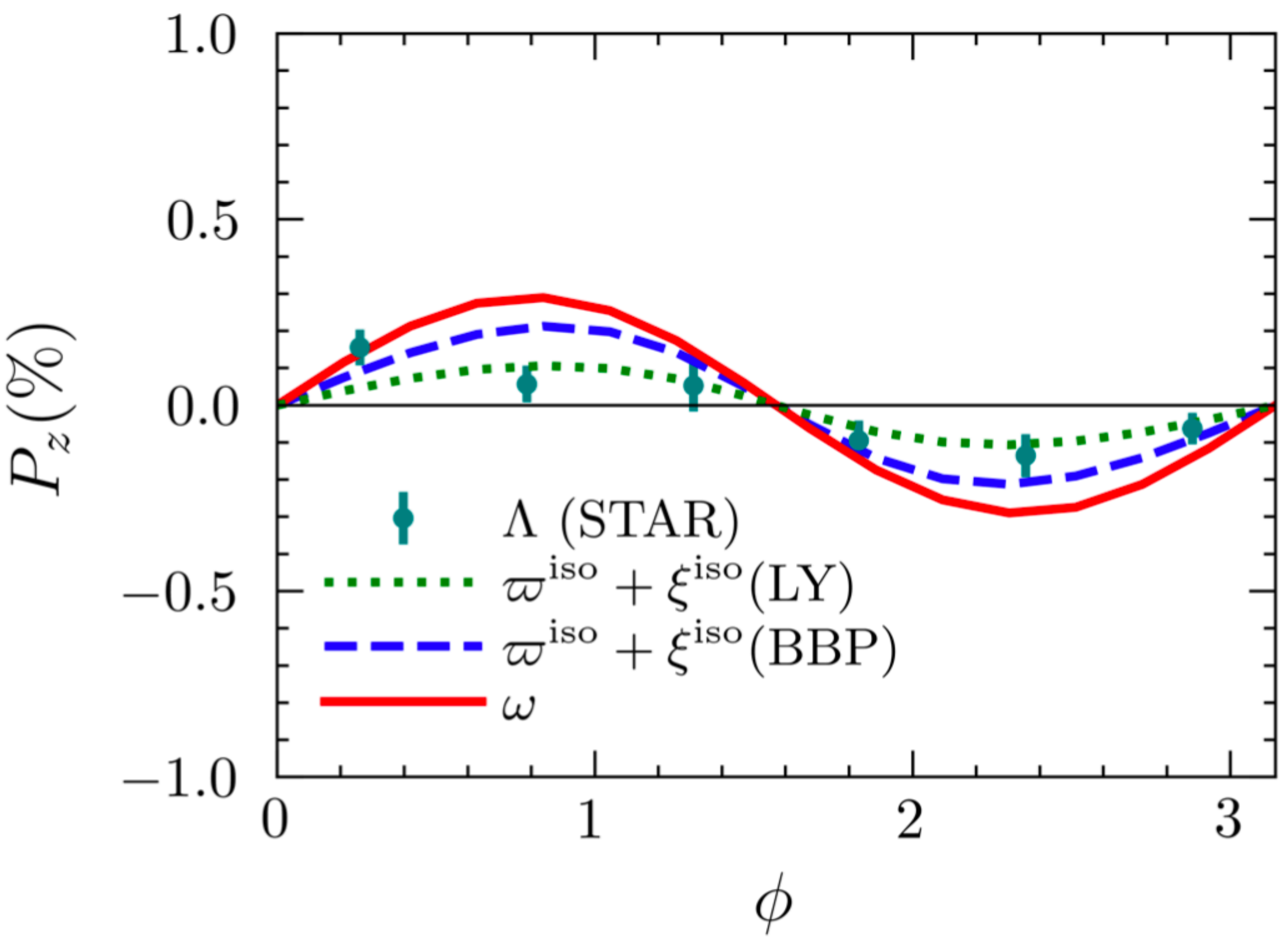
$$\varpi_{\mu\nu} = \varpi^{\rm iso}_{\mu\nu} + \varpi^{\rm T}_{\mu\nu} \qquad \xi_{\mu\nu} = \xi^{\rm iso}_{\mu\nu} + \xi^{\rm T}_{\mu\nu}$$

RESULTS FOR SPIN HYDRODYNAMICS

Adam et al. (STAR Collaboration), Phys. Rev. Lett. 123, 132301 (2019)



Adam et al. (STAR Collaboration), Phys. Rev. Lett. 123, 132301 (2019)



$$\langle P_z \sin(2\phi) \rangle \equiv \frac{\int P_z \sin(2\phi) d\phi dy}{\int E \frac{dN}{d^3p} d\phi} dy$$

$$\tau_0^s = 4 \text{ fm} \qquad m = m_\Lambda$$

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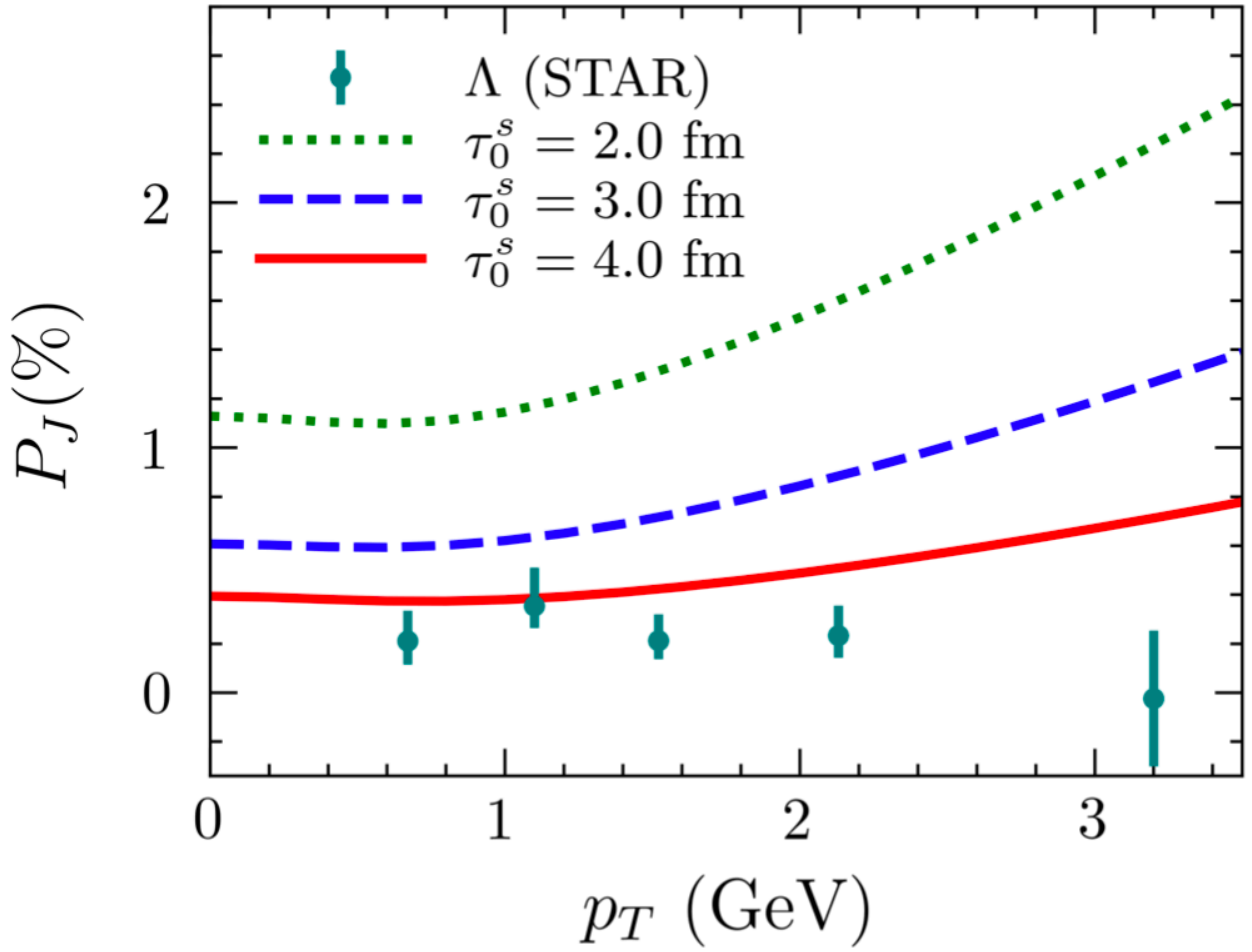
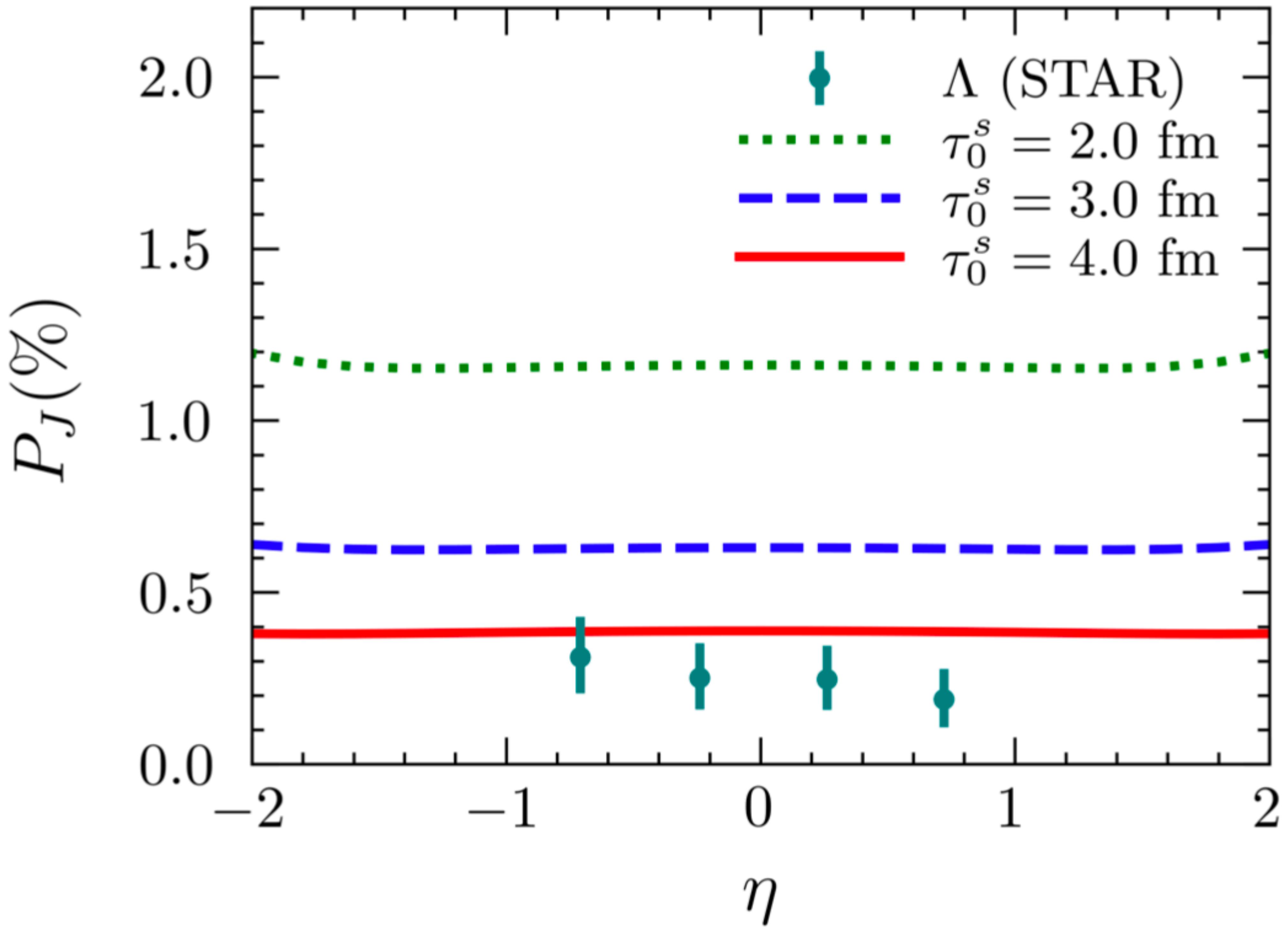
Results for spin hydrodynamics - initial time extraction





$$m = m_\Lambda$$

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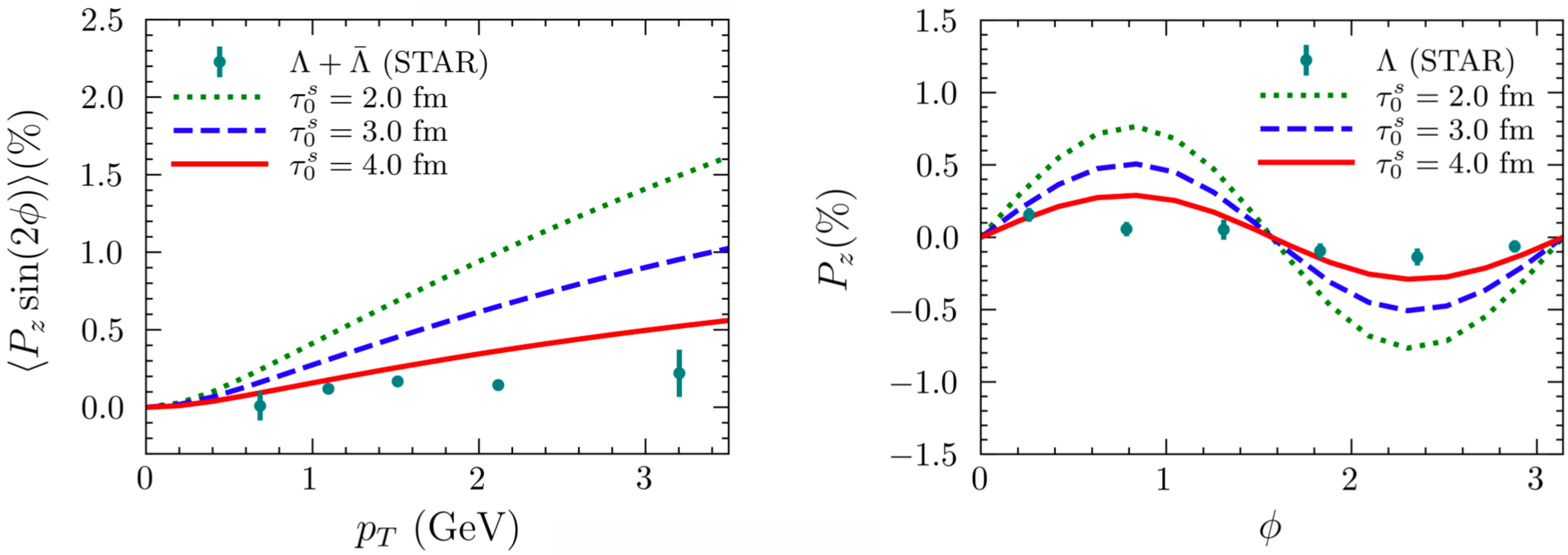

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Summary

We **developed a complete numerical framework** for perfect spin hydrodynamics

We solved perfect spin hydrodynamics in **realistic 3+1 dimensional case**

We **tuned the background** to describe basic hadronic observables

We determined polarization vector for Lambda hyperons and compared with data and other frameworks

Acceptable agreement is obtained with delayed initialization time for spin evolution

Our results suggest that the spin-orbit dissipative interaction plays a significant role only in the very early stages of the system evolution.

Directions for further numerical developments:
medium-dependent effective masses, BES results analysis,
dissipative effects in spin sector, initial polarization modelling, …

Thank you for your attention

Backup

Measurement of Λ and $\bar{\Lambda}$ global spin polarization

Self-analysing parity-violating weak decay allows to measure polarization of Λ hyperon

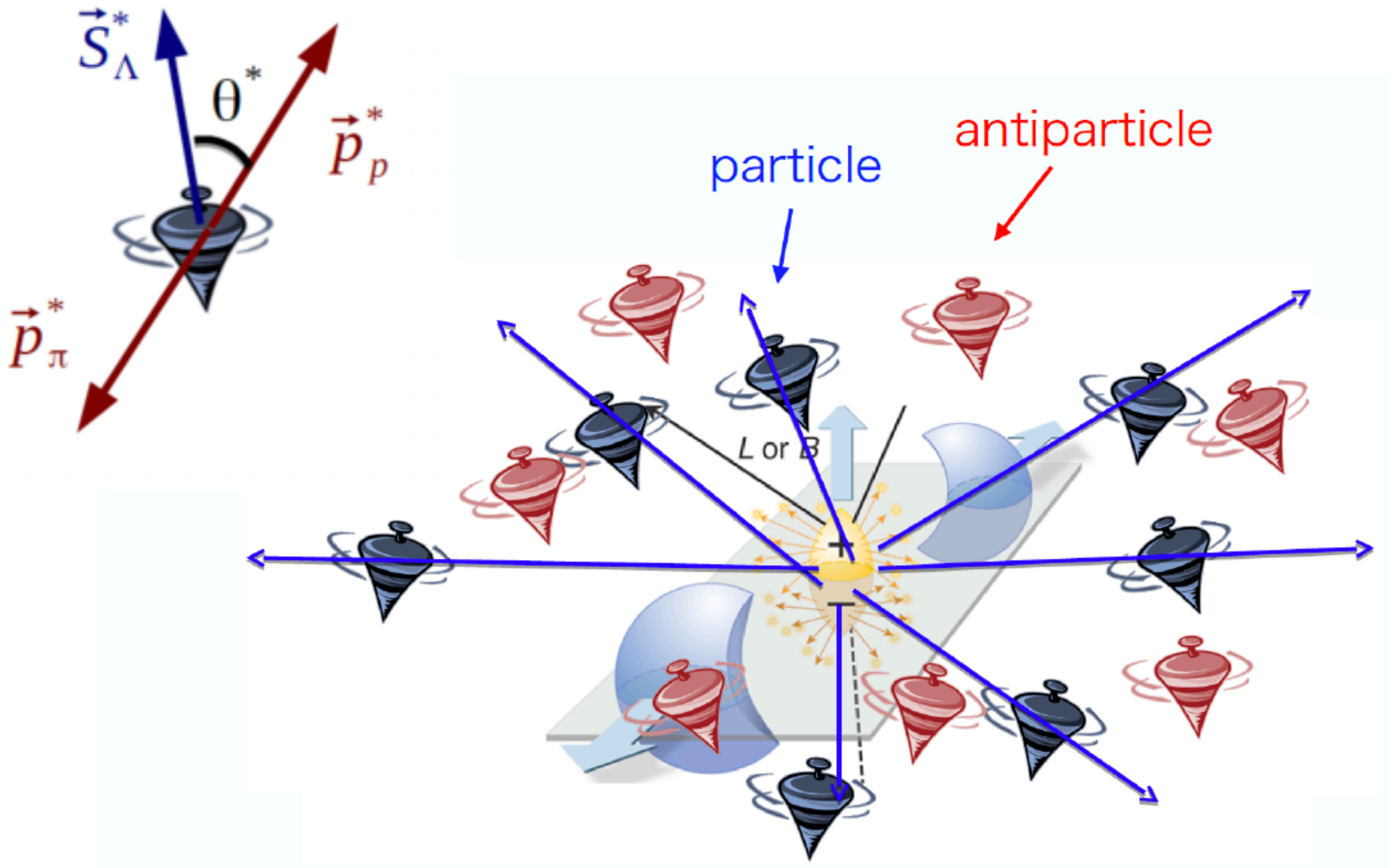


figure: T. Niida

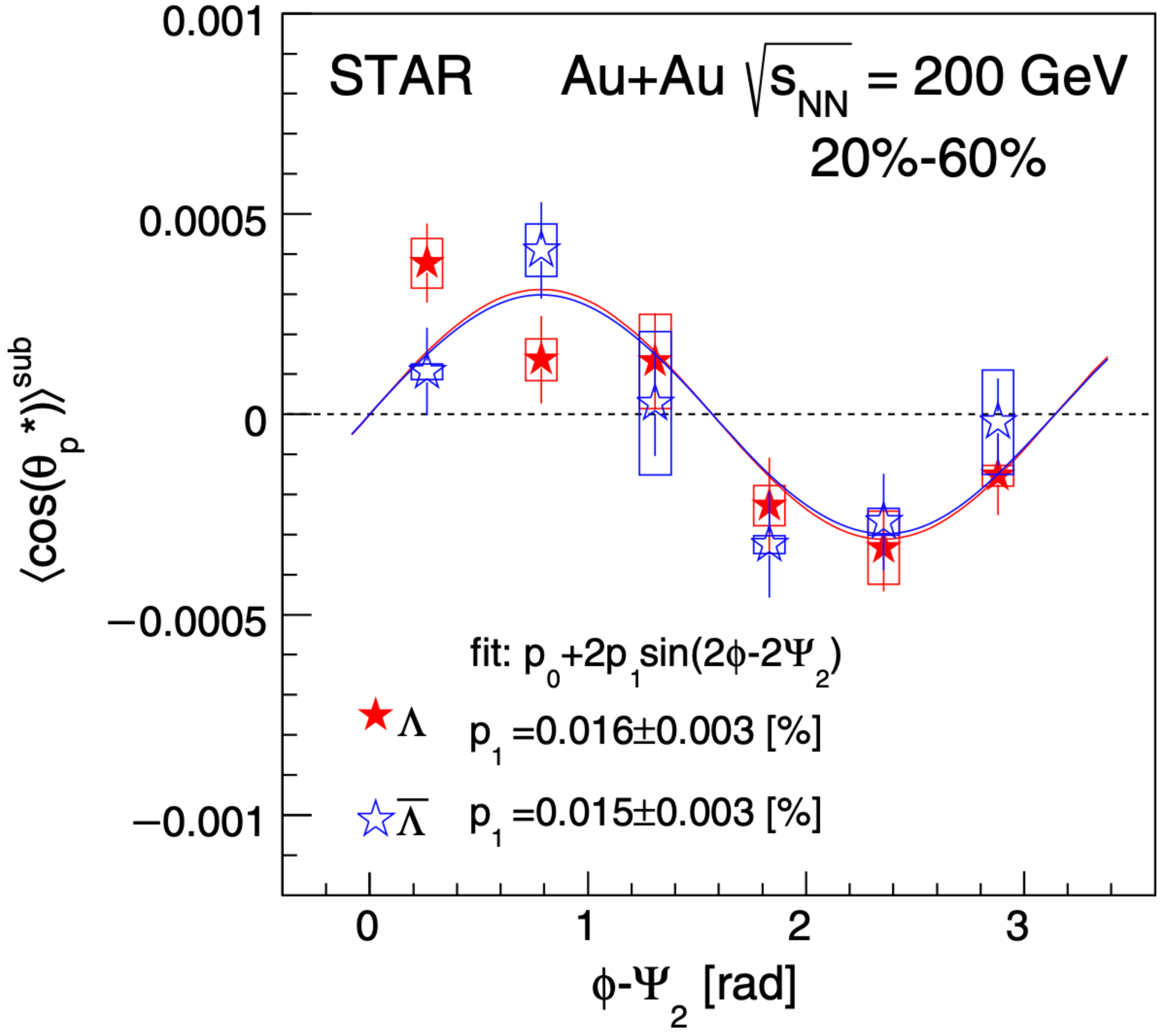
$$\frac{dN}{d\cos\theta^*} = \frac{1}{2}\left(1 + \alpha_{\rm H}|\vec{\mathcal{P}}_{\rm H}|\cos\theta^*\right)$$

$$(\alpha_\Lambda = 0.732)$$

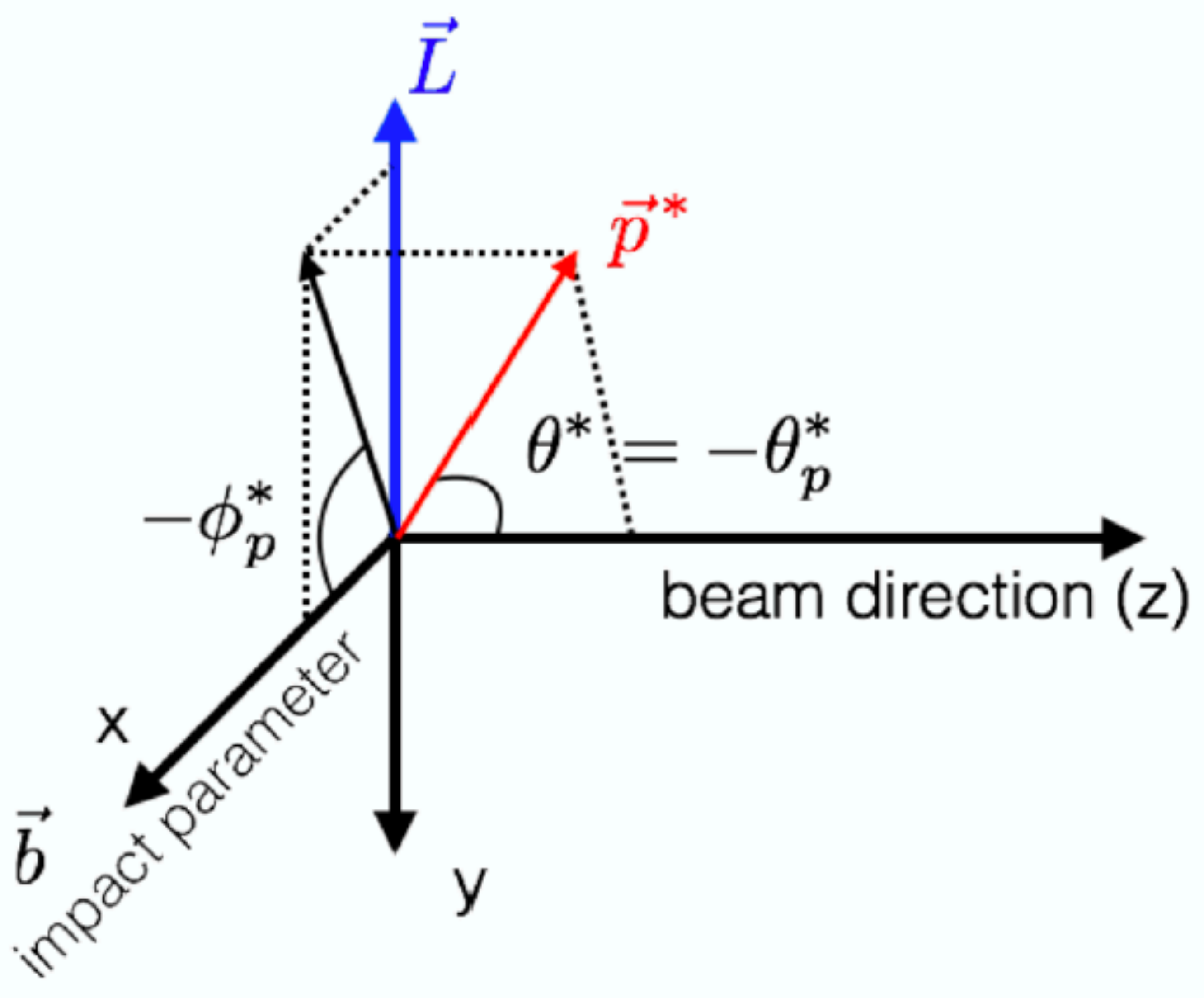
$$\overline{\mathcal{P}}_{\rm H} \equiv \langle \vec{\mathcal{P}}_{\rm H} \cdot \hat{J}_{\rm sys} \rangle = \frac{8}{\pi\alpha_{\rm H}} \frac{\left\langle \cos\left(\phi^*_p - \phi_{\hat{J}_{\rm sys}}\right)\right\rangle}{R^{(1)}_{\rm EP}}$$

Measurement of Λ and $\bar{\Lambda}$ longitudinal spin polarization

Adam et al (STAR Collaboration) Phys. Rev. Lett. 123, 132301



Niida, The 5th Workshop on Chirality, Vorticity, and Magnetic Field in HIC, '19



$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_H \mathbf{P_H} \cdot \mathbf{p}_p^*)$$

$$\langle\cos\theta_p^*\rangle = \int \frac{dN}{d\Omega^*}\cos\theta_p^* d\Omega^*$$

$$= \alpha_H P_z \langle(\cos\theta_p^*)^2\rangle$$

$$\therefore\ P_z = \frac{\langle\cos\theta_p^*\rangle}{\alpha_H\langle(\cos\theta_p^*)^2\rangle}$$

$$= \frac{3\langle\cos\theta_p^*\rangle}{\alpha_H} \quad \text{(if perfect detector)}$$

α_H: hyperon decay parameter

θ_p^*: θ of daughter proton in Λ rest frame

POLARIZATION VECTOR

One calculates the components of the **polarization vector** for Λ hyperons

Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^{\mu}_{\varpi}(p) \;=\; -\frac{1}{8m_{\Lambda}}\epsilon^{\mu\nu\rho\sigma}p_{\sigma}\frac{\int d\Sigma\cdot p\; n_F(1-n_F)\varpi_{\nu\rho}}{\int d\Sigma\cdot p\; n_F}$$

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Here one uses Fermi-Dirac distribution

$$n_F = n_F(T, \mu_B, p\cdot u; m_{\Lambda}) \qquad n_F(z) = \frac{1}{\mathrm{e}^z + 1}$$

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Temperature, chemical potential and flow are found from **standard viscous hydrodynamics (or any other model)**

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Lambda mass has the PDG fixed value ~ 1.1 GeV

Background hydrodynamics

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} + \Pi)\,\Delta^{\alpha\beta} + \pi^{\alpha\beta}$$
$$N^\alpha = n\, u^\alpha + n^\alpha$$

The **first-order Navier-Stokes (NS) forms of the dissipative currents** are

$$\Pi_{\rm NS} = -\zeta\theta \qquad \pi^{\alpha\beta}_{\rm NS} = 2\eta\,\sigma^{\alpha\beta}$$

They are entirely determined by the spacetime gradients of the flow

$$\theta \equiv D\cdot u$$ (expansion scalar)

$$\sigma^{\alpha\beta} \equiv D^{\langle\gamma}u^{\delta\rangle} \equiv \Delta^{\alpha\beta}_{\gamma\delta} D^\gamma u^\delta$$ (shear-flow tensor)

$$\Delta^{\alpha\beta}_{\gamma\delta} \equiv \frac{1}{2}\left[\Delta^\alpha_\gamma\Delta^\beta_\delta + \Delta^\alpha_\delta\Delta^\beta_\gamma - (2/3)\Delta^{\alpha\beta}\Delta_{\gamma\delta}\right]$$ (projector selecting symmetric, traceless, and orthogonal part relative to flow)

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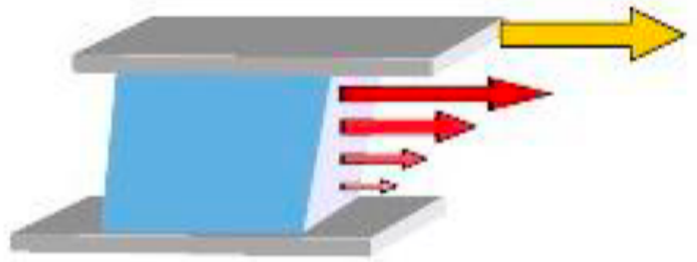
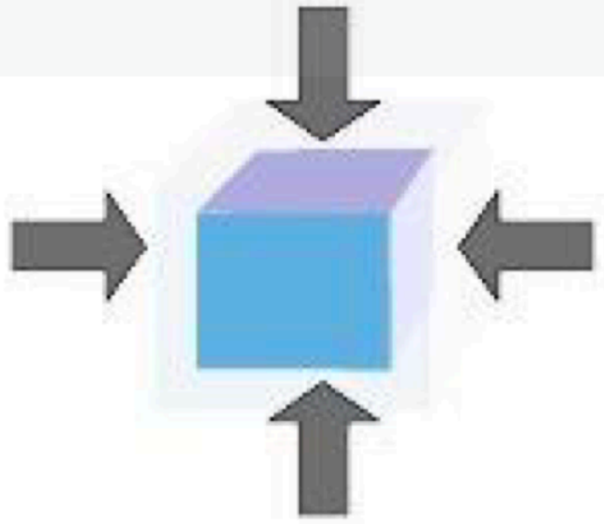
The **bulk and shear viscosities** are given by

Denicol, Jeon, and Gale, Phys. Rev. C 90, 024912 (2014);
Denicol, Gale, Jeon, Monnai, Schenke, and Shen, Phys. Rev. C 98, 034916 (2018)

$$\eta = C_\eta \frac{\varepsilon_{\rm eq} + P_{\rm eq}}{T}, \quad \zeta = 75\eta\left(\frac{1}{3} - c_s^2\right)^2 \qquad C_\eta = 0.12$$





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$$\eta = C_\eta \frac{\varepsilon_{\rm eq} + P_{\rm eq}}{T}, \quad \zeta = 75\eta\left(\frac{1}{3} - c_s^2\right)^2 \qquad C_\eta = 0.12$$

The **speed of sound** is obtained from

Monnai, Schenke, and Shen, Phys. Rev. C 100, 024907 (2019)

$$c_s^2 = \left.\frac{\partial P_{\rm eq}}{\partial \varepsilon_{\rm eq}}\right|_{n_{\rm eq}} + \frac{n_{\rm eq}}{\varepsilon_{\rm eq} + P_{\rm eq}} \left.\frac{\partial P_{\rm eq}}{\partial n_{\rm eq}}\right|_{\varepsilon_{\rm eq}}$$

BACKGROUND HYDRODYNAMICS

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^{\alpha}\, u^{\beta} - (P_{\text{eq}} + \Pi)\, \Delta^{\alpha\beta} + \pi^{\alpha\beta}$$

$$N^{\alpha} = n\, u^{\alpha} + n^{\alpha}$$

The **first-order Navier-Stokes (NS) forms of the dissipative currents** are

$$\Pi_{\text{NS}} = -\zeta\theta \qquad \pi^{\alpha\beta}_{\text{NS}} = 2\eta\, \sigma^{\alpha\beta}$$

Landau (relativistic Navier-Stokes) theory is acausal!

Hiscock and Lindblom, Annals Phys. 151, 466 (1983).
Denicol, Kodama, Koide, and Mota, J. Phys. G 35, 115102 (2008

Lev Landau, MIPT History Museum

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At **second order** in spacetime gradients, in DNMR framework, the **time evolution of the dissipative currents** is

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$$\dot{\Pi} = \frac{\Pi_{\rm NS} - \Pi}{\tau_\Pi} - \frac{\delta_{\Pi\Pi}}{\tau_\Pi}\Pi\,\theta + \frac{\lambda_{\Pi\pi}}{\tau_\Pi}\pi^{\alpha\beta}\sigma_{\alpha\beta}$$

$$\dot{\pi}^{\langle\alpha\beta\rangle} = \frac{\pi^{\alpha\beta}_{\rm NS} - \pi^{\alpha\beta}}{\tau_\pi} - \frac{\delta_{\pi\pi}}{\tau_\pi}\pi^{\alpha\beta}\theta + \frac{\lambda_{\pi\Pi}}{\tau_\pi}\Pi\,\sigma^{\alpha\beta} - \frac{\tau_{\pi\pi}}{\tau_\pi}\pi_\gamma^{\langle\alpha}\sigma^{\beta\rangle\gamma} + \frac{\phi_7}{\tau_\pi}\pi_\gamma^{\langle\alpha}\pi^{\beta\rangle\gamma}$$

Where the comoving derivative is

$$\dot{(\,)} \equiv u \cdot D$$

BACKGROUND HYDRODYNAMICS

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Relaxation times are chosen as follows

$$\tau_\pi = \tau_\Pi = \frac{5C_\eta}{T}$$

BACKGROUND HYDRODYNAMICS

We use **(Milne) coordinates**

$$(t, x, y, z) \quad \rightarrow \quad (\tau, x, y, \eta_s)$$

$$\begin{aligned} t &= \tau \cosh \eta_s \\ z &= \tau \sinh \eta_s \end{aligned} \quad \rightarrow \quad \begin{aligned} \tau &= \sqrt{t^2 - z^2} \\ \eta_s &= \frac{1}{2} \log\left(\frac{t+z}{t-z}\right) \end{aligned}$$

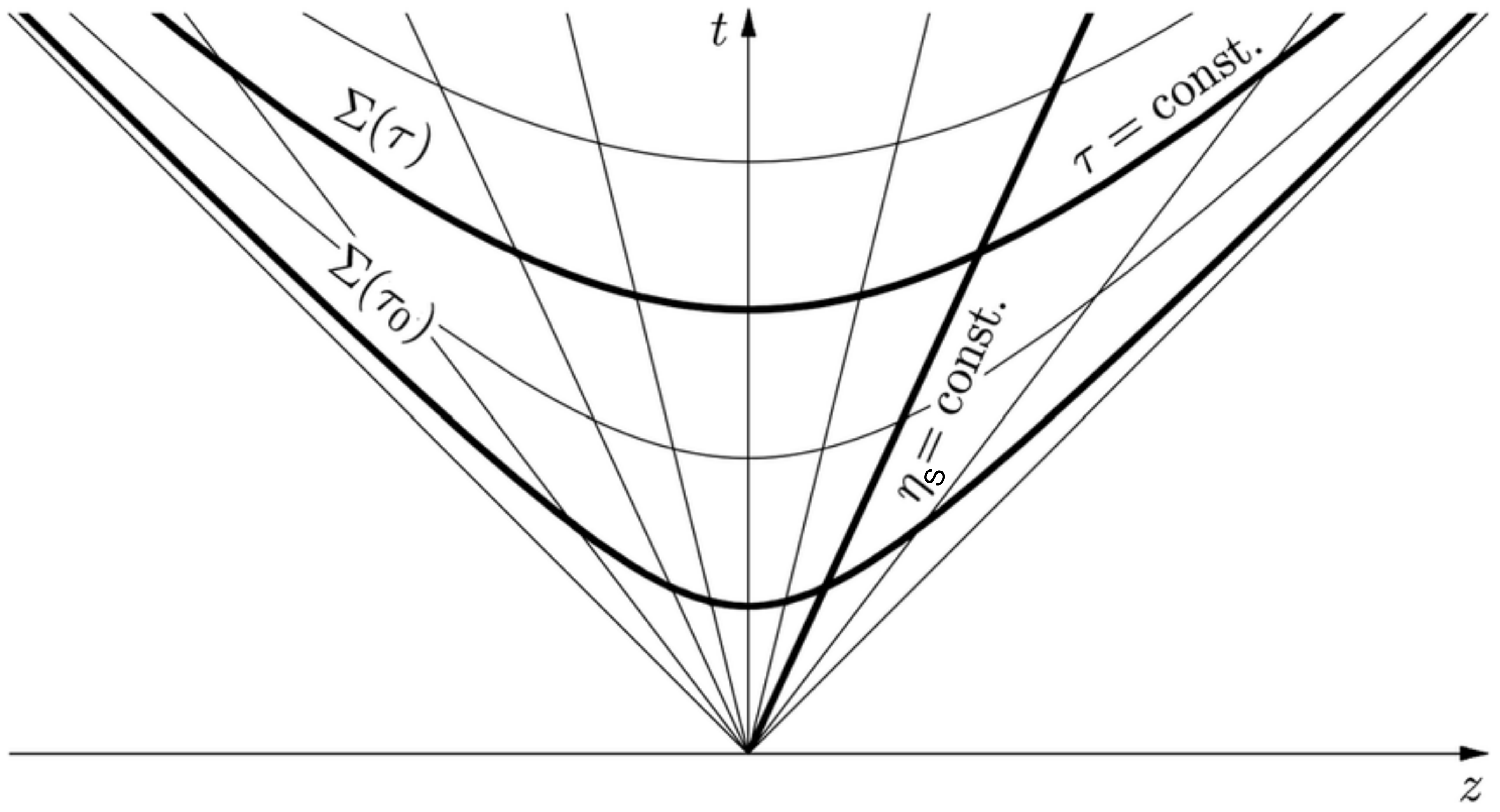

fig: Rindori, Tinti, Becattini, Rischke, Phys.Rev.D 105 (2022) 5, 056003

BACKGROUND HYDRODYNAMICS

We focus on **Au+Au** collisions at the **top RHIC energy** of $\sqrt{s_{\mathrm{NN}}} = 200\ \mathrm{GeV}$

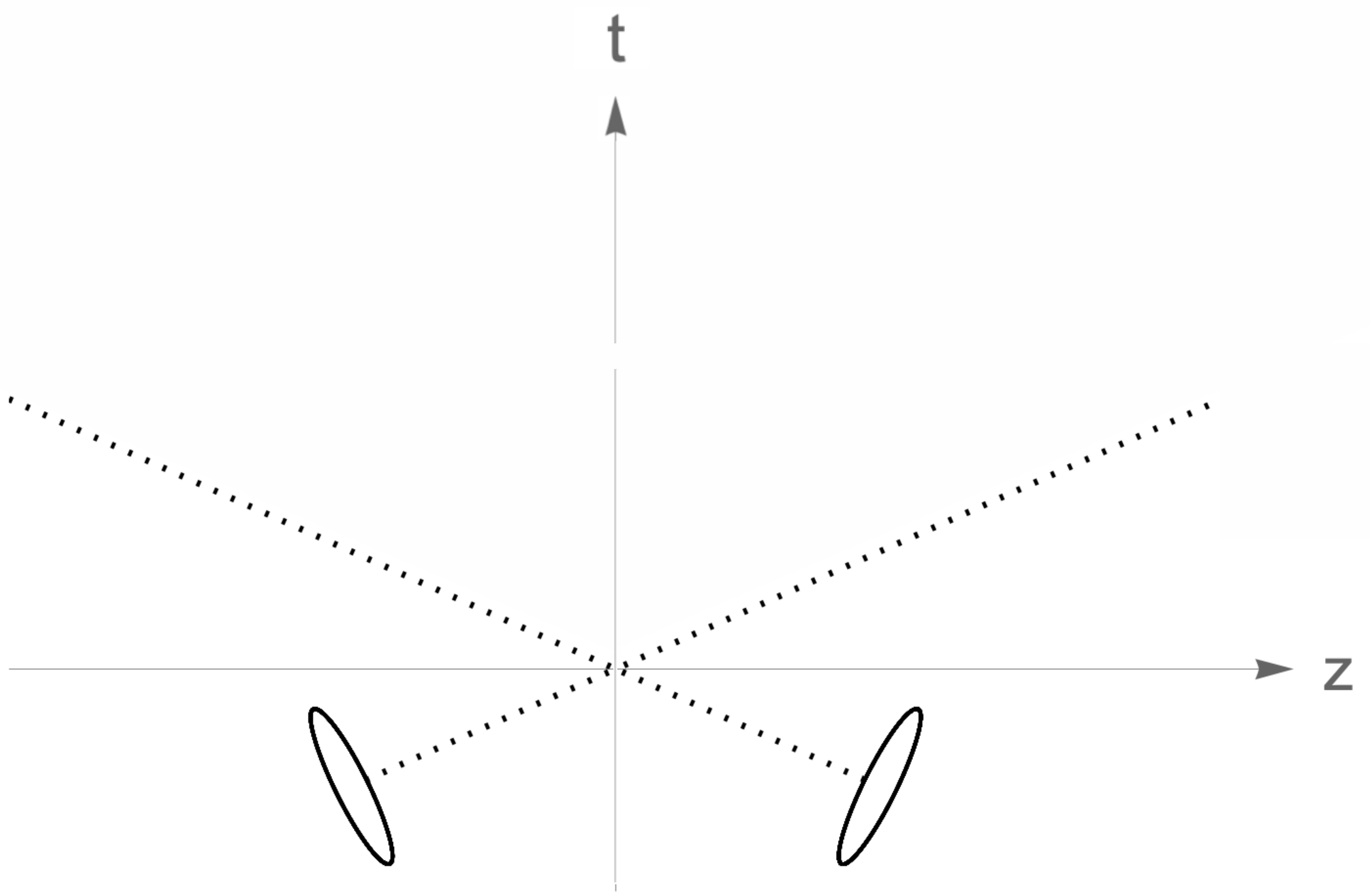

fig: https://arxiv.org/pdf/2407.12130 (modified)

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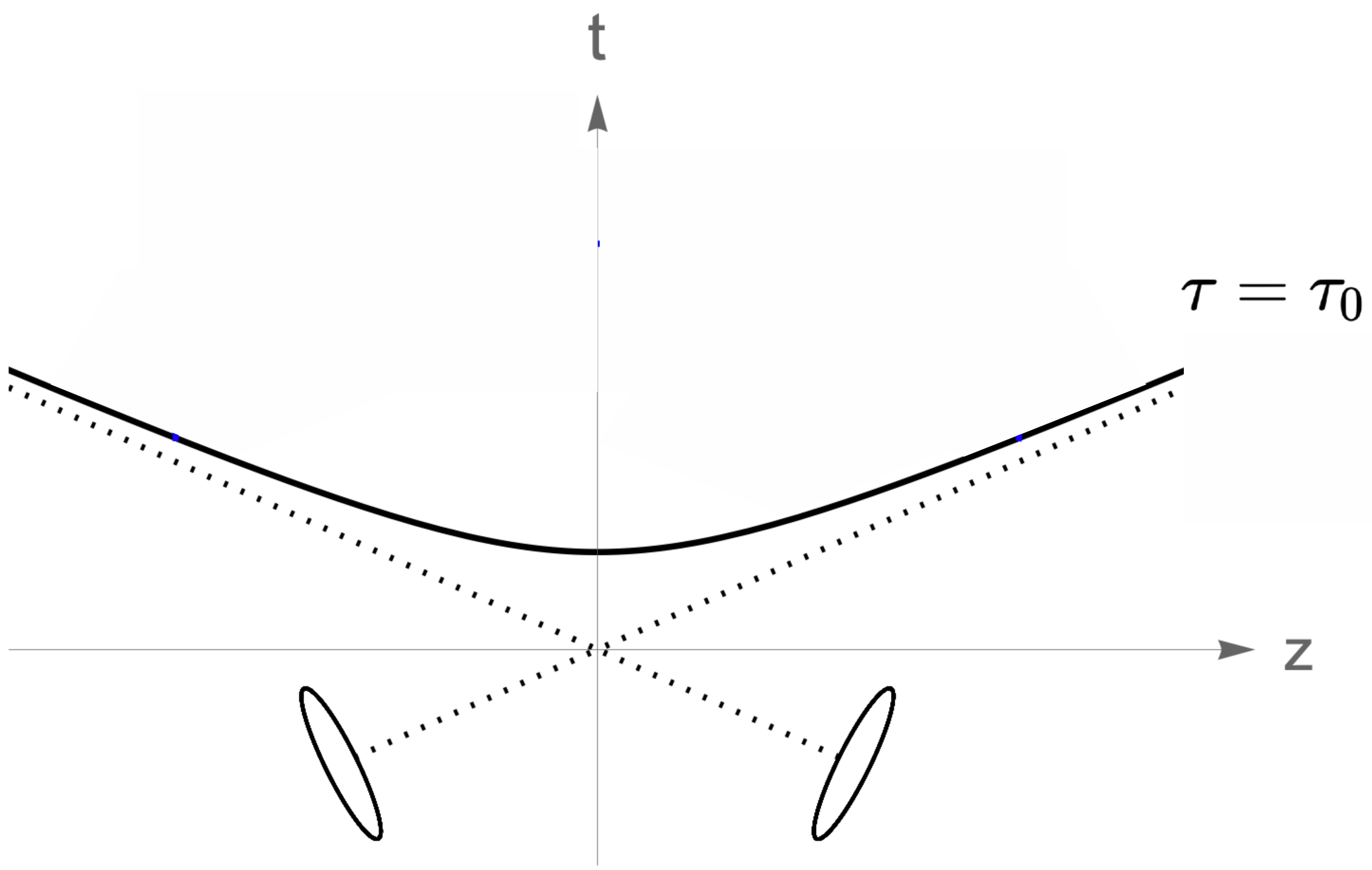
We focus on **Au+Au** collisions at the **top RHIC energy** of $\sqrt{s_{\rm NN}} = 200~\rm GeV$

Initialize the background evolution at the proper time $\tau_0 = 1~\rm fm$



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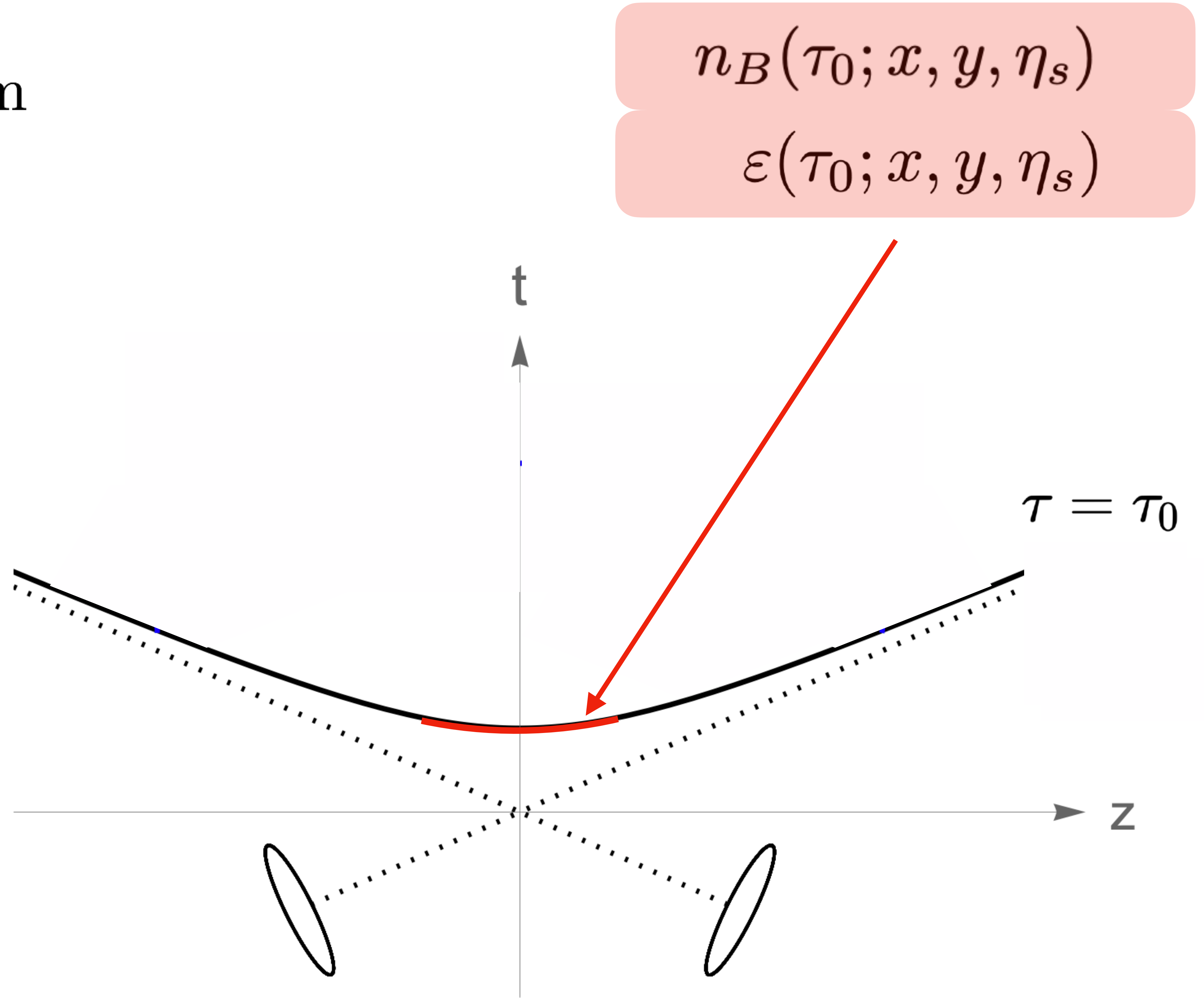
However, to compute the thickness functions and wounded nucleon densities we use the **optical limit of the Glauber model**



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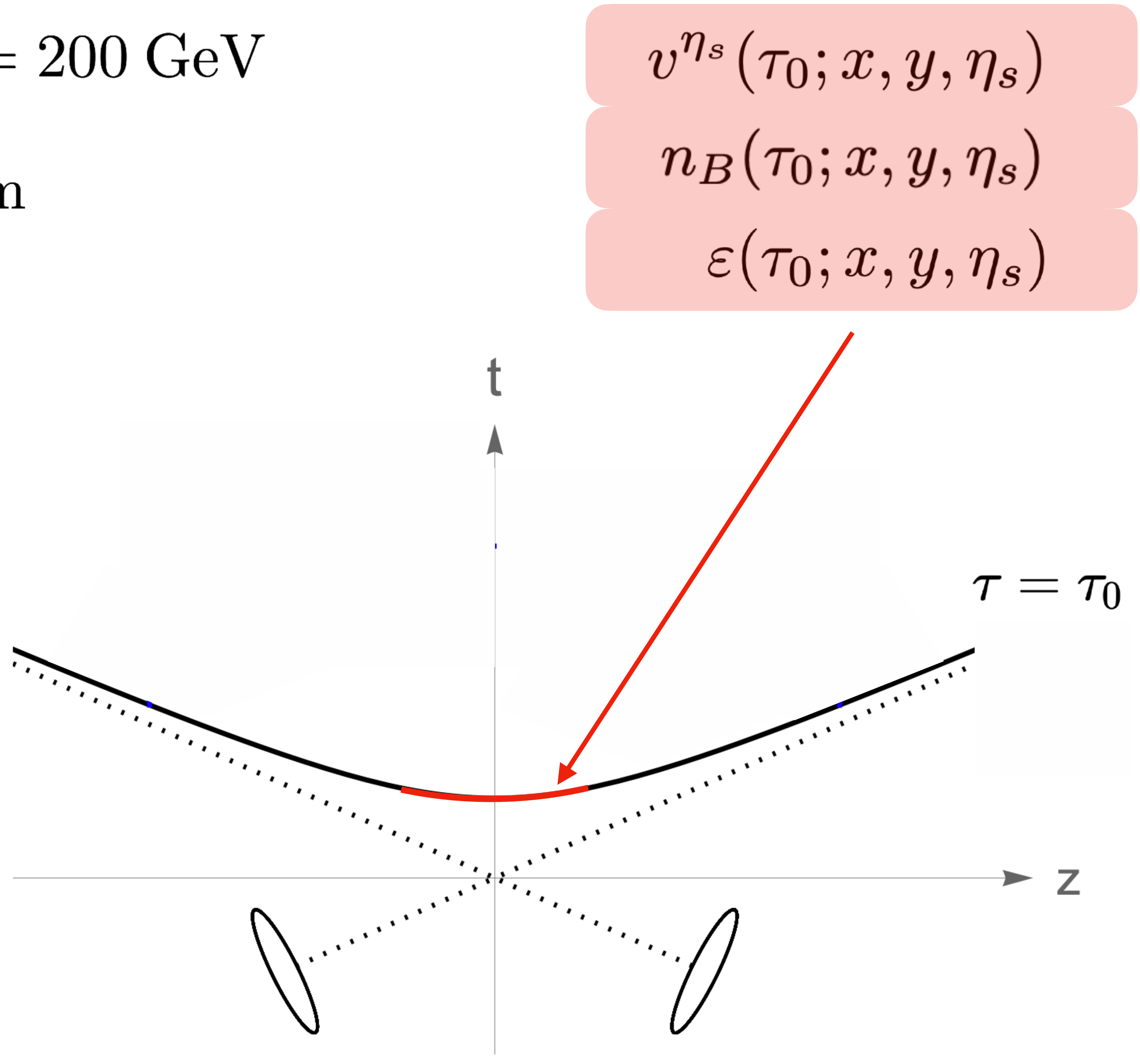
The **longitudinal flow** is numerically determined from the initial energy-momentum tensor components

The **initial transverse flow components are zero**



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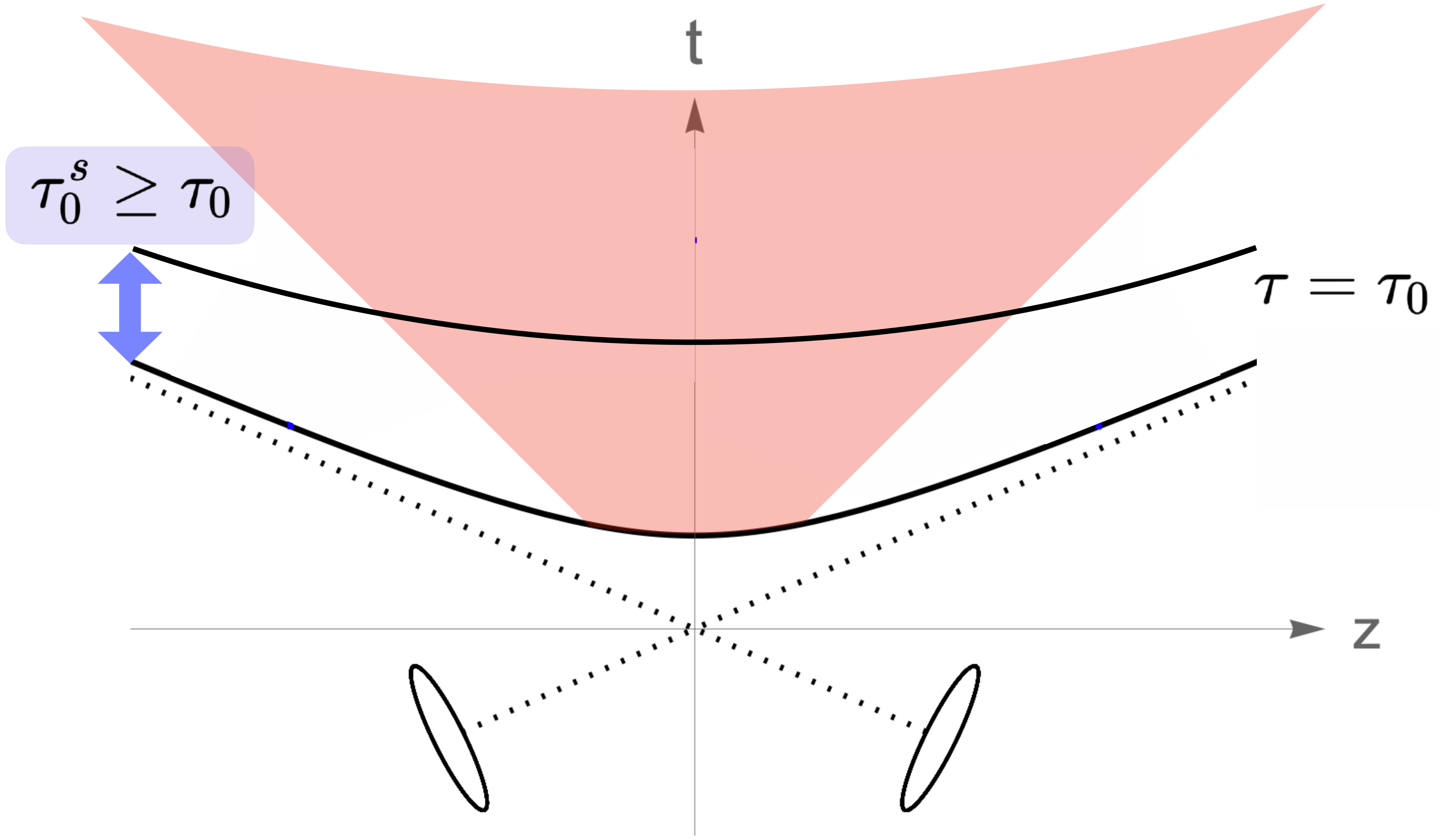
Spin hydrodynamics

Initialize the spin evolution at the proper time $\tau_0^s \geq \tau_0$

We intend to account for equilibration of spin DOFs resulting from strong spin-orbit interactions occurring in the early stages before the system reaches perfect spin hydrodynamics regime



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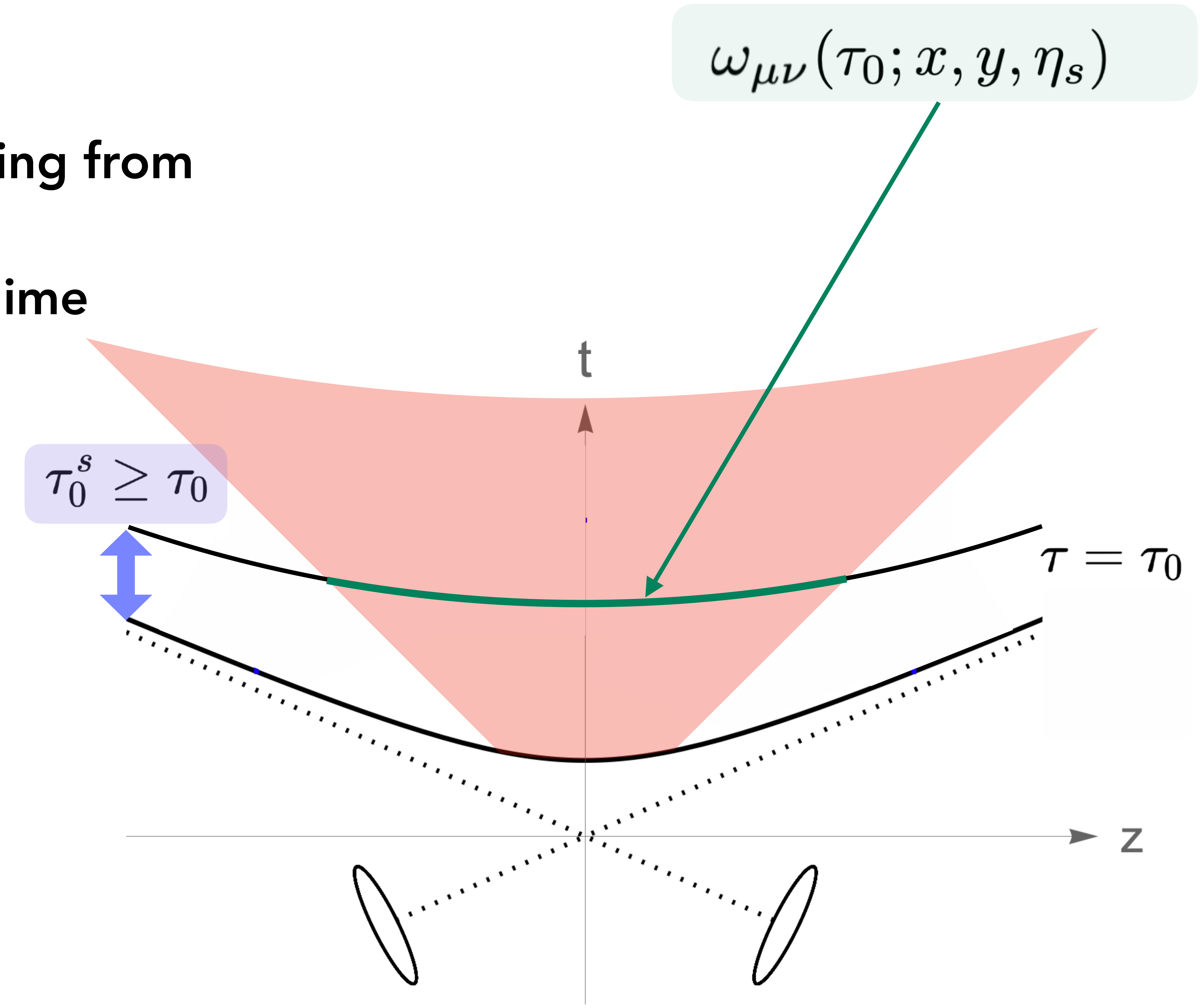

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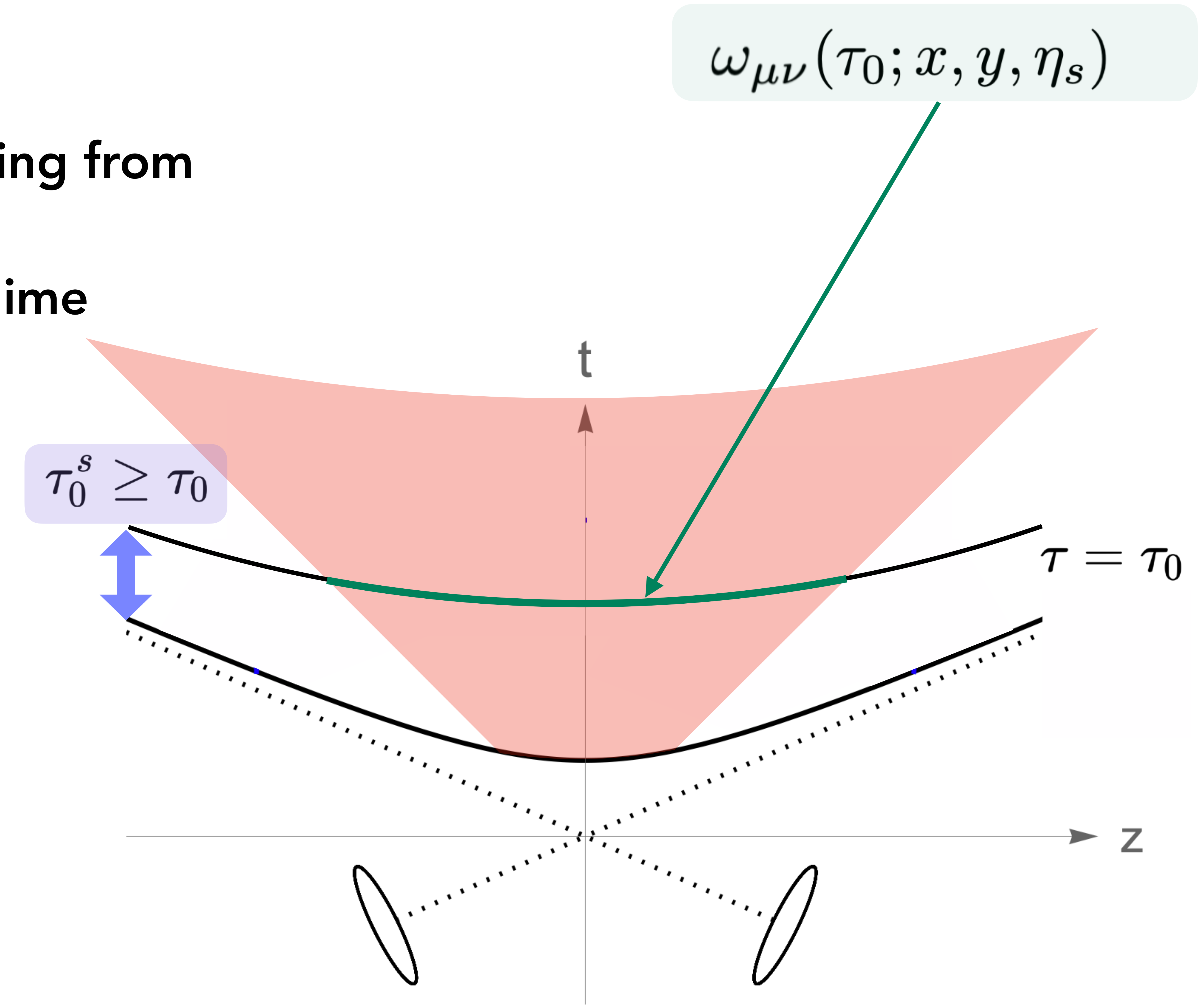

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$T, \mu_B\, u^\rho$ → $\omega_{\mu\nu}(\tau_0; x, y, \eta_s)$

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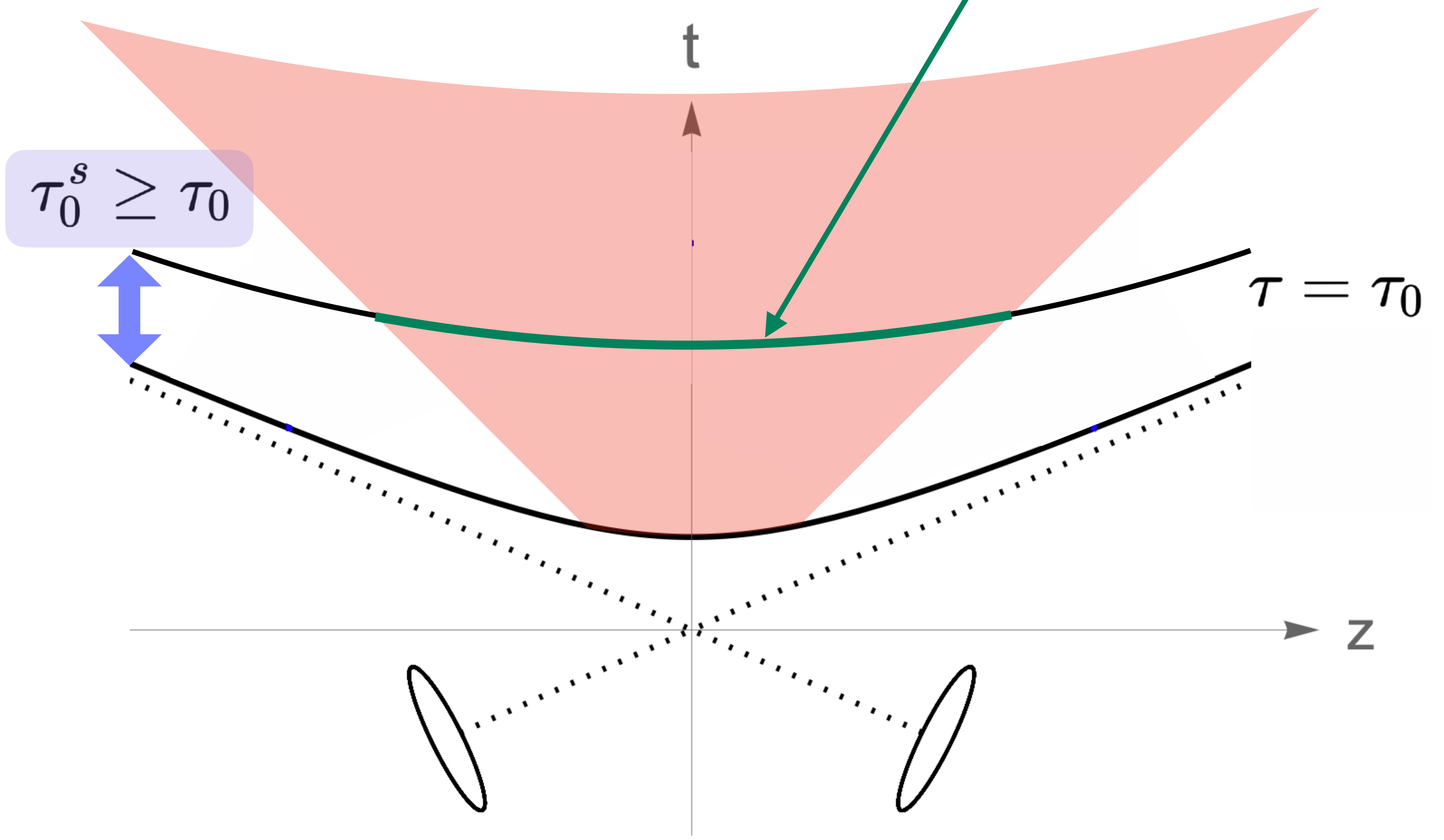

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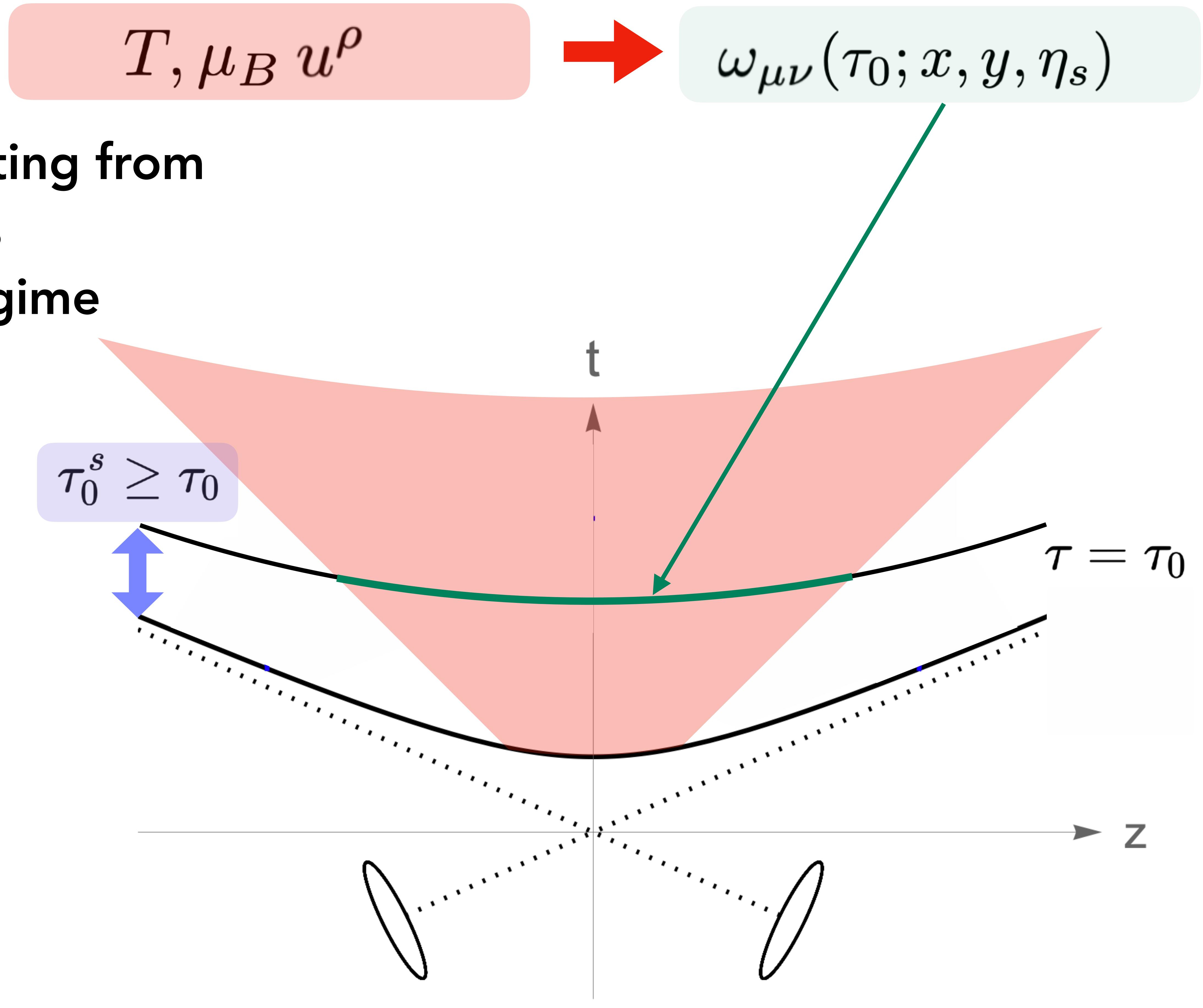
$$\hat{\tau}^\mu = (1,0,0,0)$$



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ORBITAL AND SPIN ANGULAR MOMENTUM

$$\mathcal{J}_C^{\lambda\mu\nu} = \underbrace{x^\mu T_C^{\lambda\nu} - x^\nu T_C^{\lambda\mu}}_{\text{orbital part}} + \underbrace{\mathcal{S}_C^{\lambda\mu\nu}}_{\text{spin part}}$$

orbital angular momentum of a point particle

$$\vec{L} = \vec{x} \times \vec{p} \quad \Rightarrow \quad L_i = \varepsilon_{ijk}\, x_i\, p_j$$

its dual is

$$L_{ij} \equiv \varepsilon_{ijk}\, L_k \quad \Rightarrow \quad L_{ij} = x_i\, p_j - x_j\, p_i$$

relativistic generalization is

$$L^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu$$

for relativistic fluid one has

$$L^{\lambda,\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}$$

Background hydrodynamics

Equations of motion (EOMs) for **relativistic viscous hydrodynamics** result from the following **conservation laws**

$$D_\alpha T^{\alpha\beta}(x) = 0$$
$$D_\alpha N^{\alpha}(x) = 0$$

In local fluid rest frame:

$$T^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

We adopt **Landau's definition of flow four-velocity**

$$T^{\alpha\beta} u_\beta = \varepsilon\, u^\alpha$$

In this case, the **constitutive relations** read

$$T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} \qquad)\,\Delta^{\alpha\beta}$$ (energy-momentum tensor, EMT)

$$N^\alpha = n\, u^\alpha$$ (net baryon current)

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\end{aligned}
$$

In local fluid rest frame:

$$
T^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P+\Pi+\pi^{11} & \pi^{12} & \pi^{13} \\ 0 & \pi^{21} & P+\Pi+\pi^{22} & \pi^{23} \\ 0 & \pi^{31} & \pi^{32} & P+\Pi+\pi^{33} \end{pmatrix}
$$

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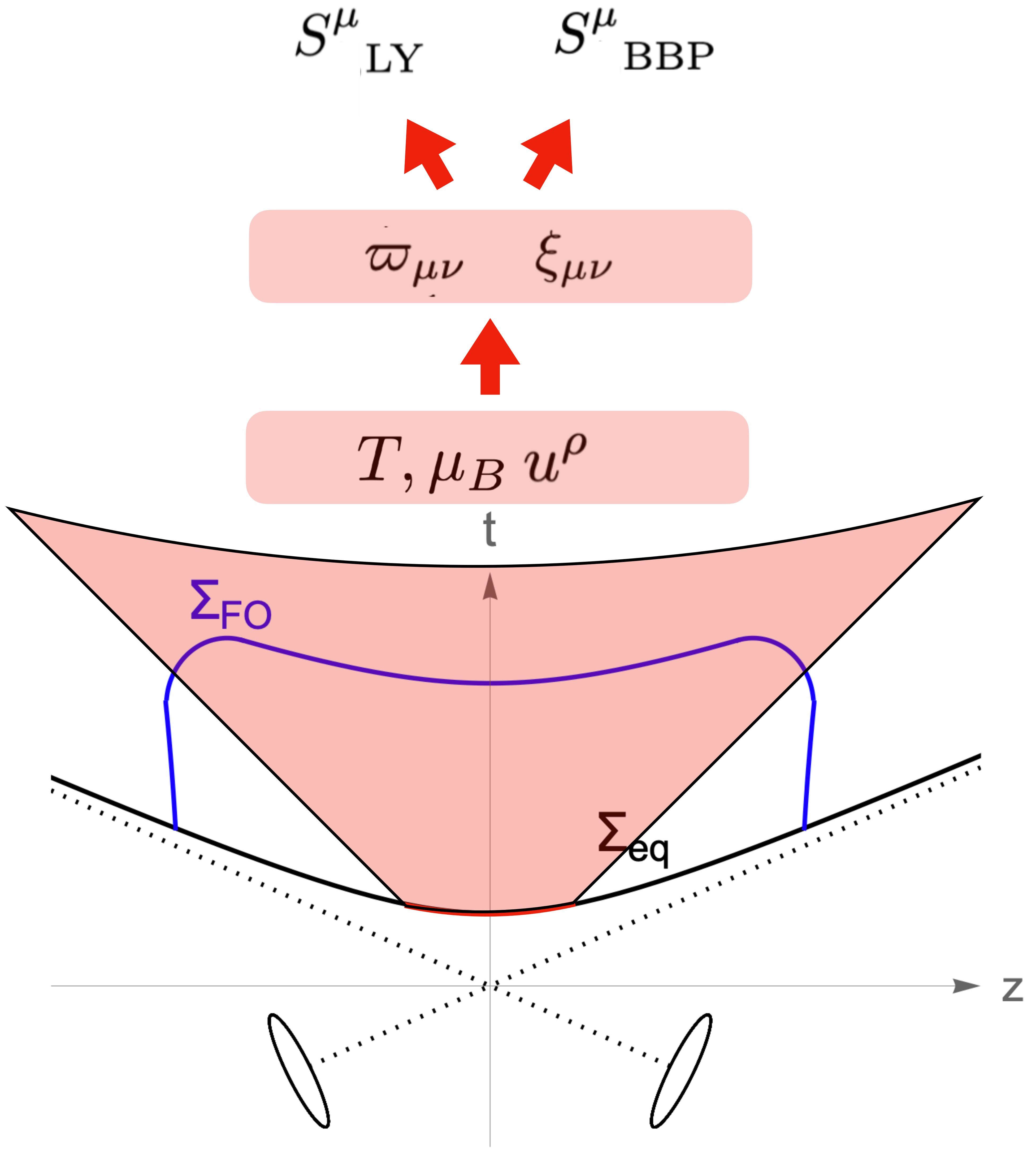
$$

In this case, the **constitutive relations** read

$$
T^{\alpha\beta} = \varepsilon\, u^\alpha\, u^\beta - (P_{\rm eq} + \Pi)\, \Delta^{\alpha\beta} + \pi^{\alpha\beta} \quad \text{(energy-momentum tensor, EMT)}
$$

$$
N^\alpha = n\, u^\alpha + n^\alpha \quad \text{(net baryon current)}
$$

Standard hydrodynamics vs Spin hydrodynamics



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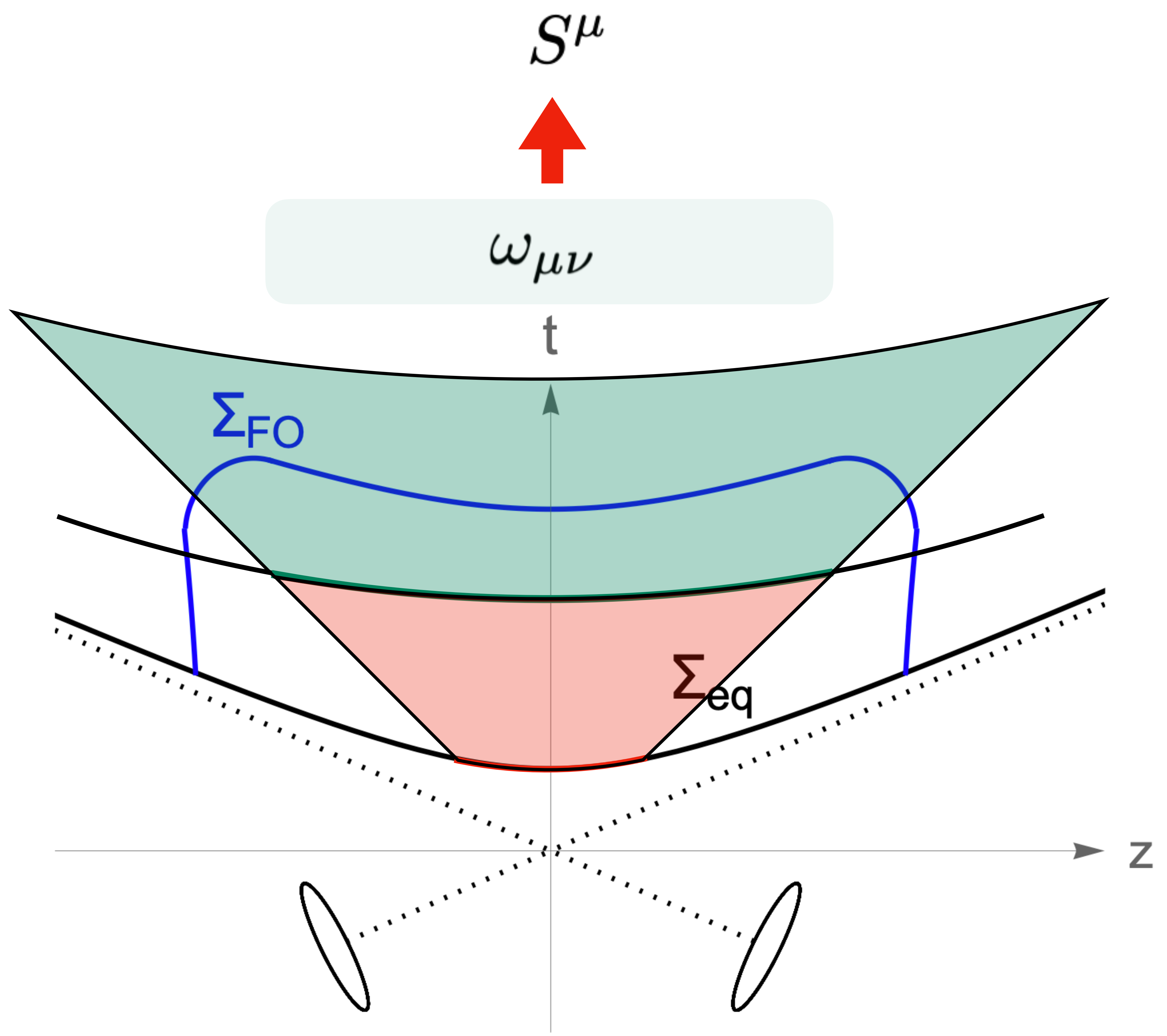

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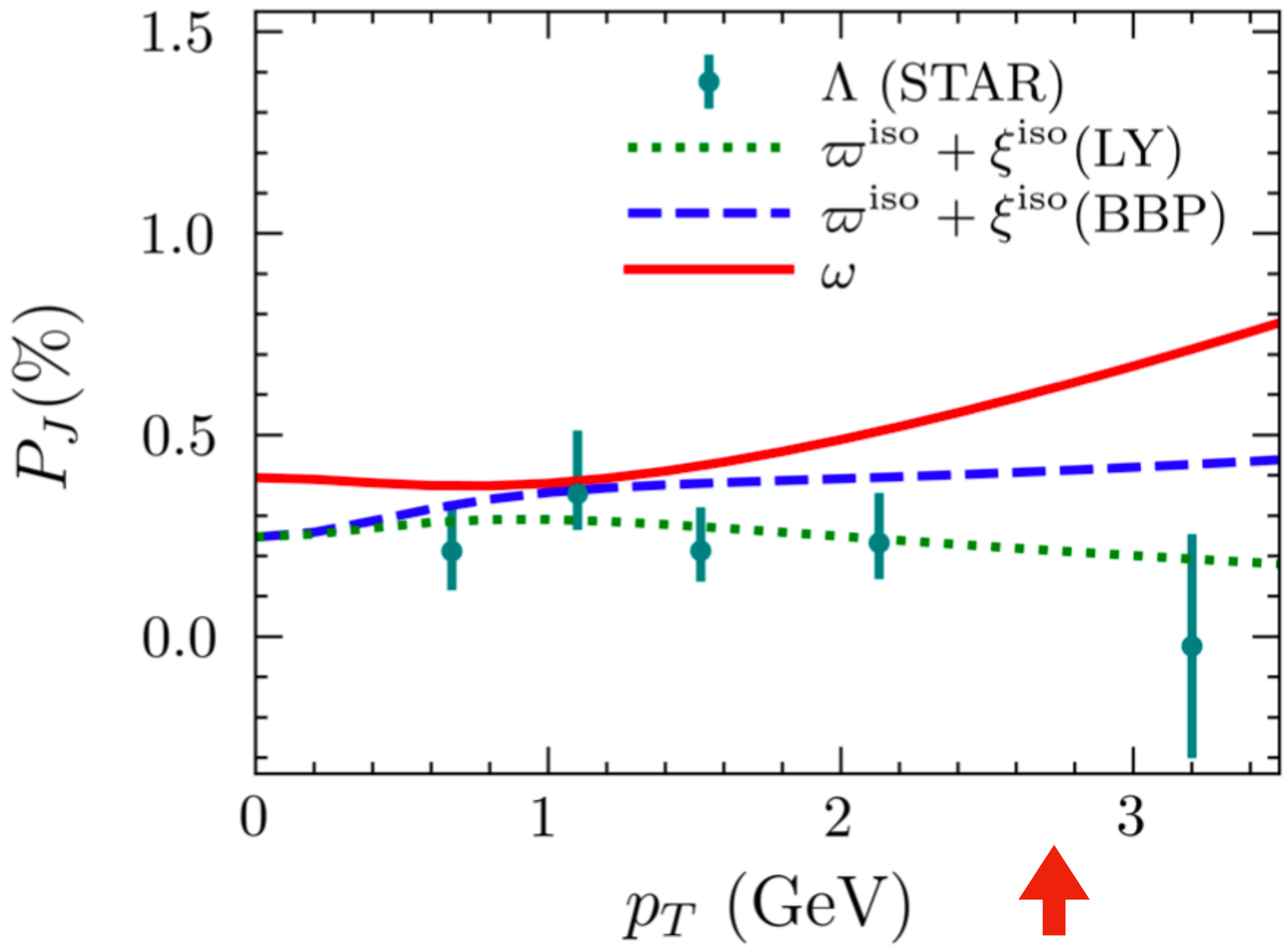
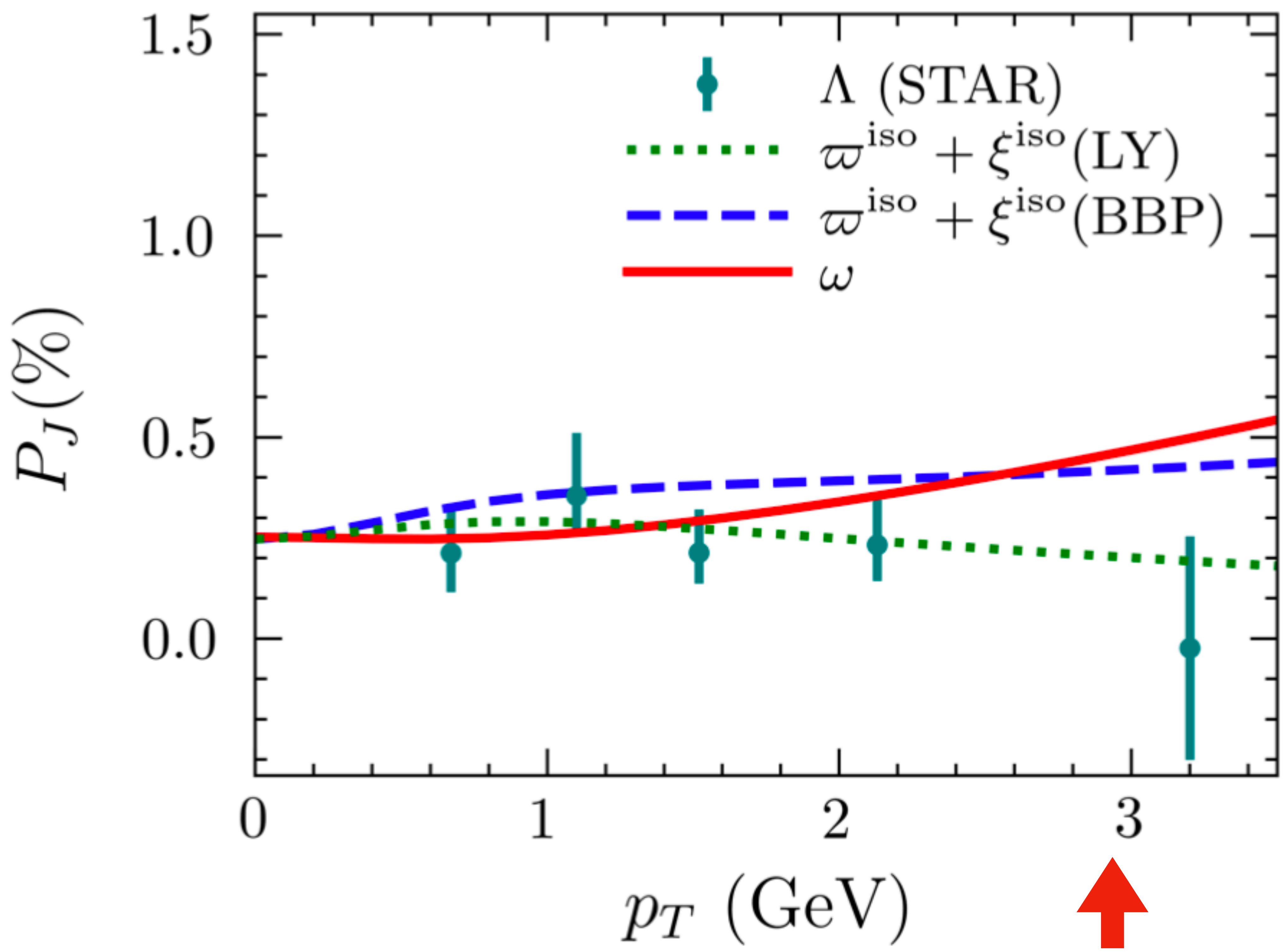
Results for spin hydrodynamics - effective mass



$\tau_0^s = 4$ fm and $m = m_\Lambda$



$\tau_0^s = 4$ fm and $m = 300$ MeV

$\varpi_{\mu\nu} = \varpi^{\rm iso}_{\mu\nu} + \varpi^{\rm T}_{\mu\nu}$ $\quad \xi_{\mu\nu} = \xi^{\rm iso}_{\mu\nu} + \xi^{\rm T}_{\mu\nu}$

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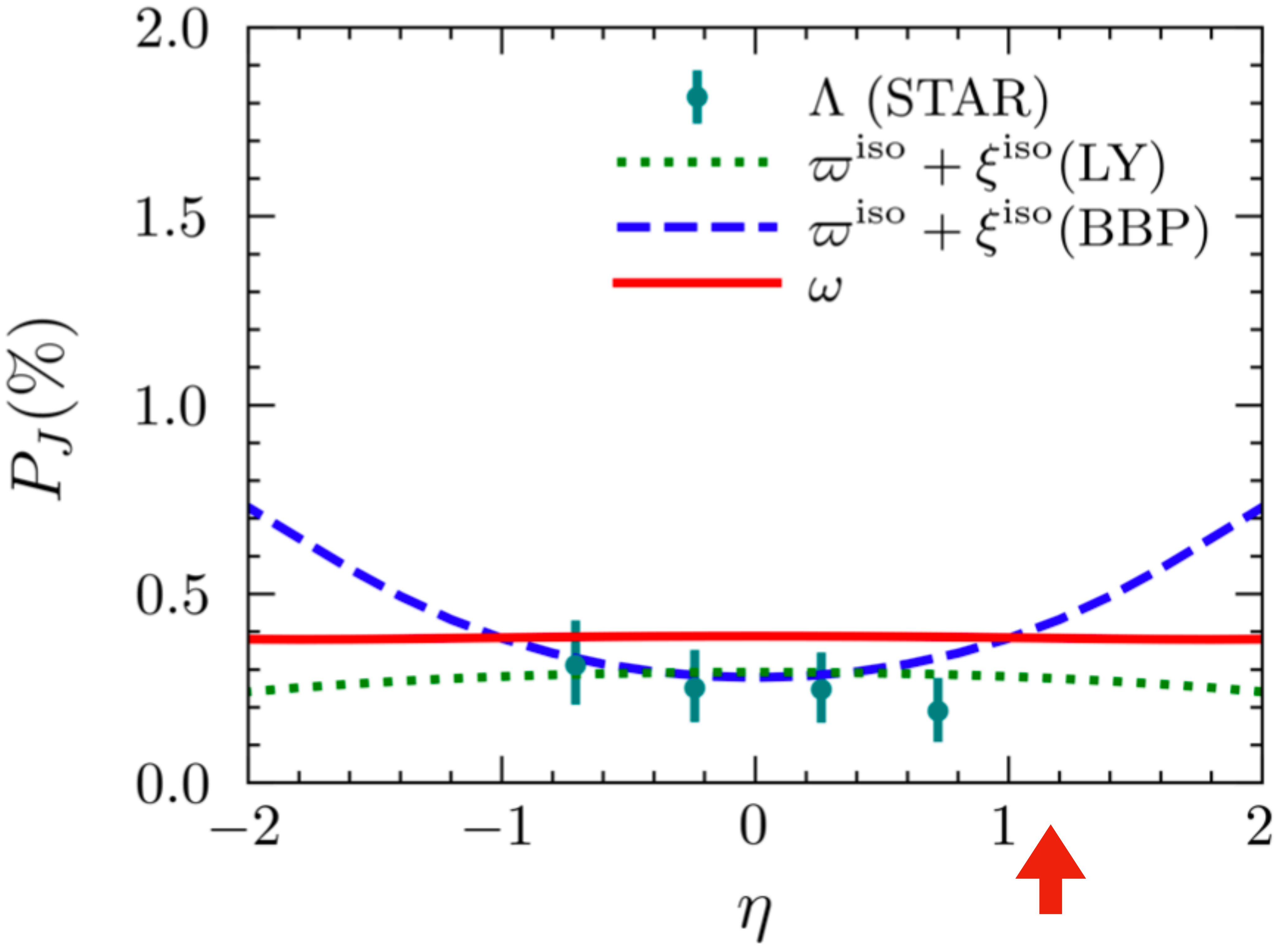

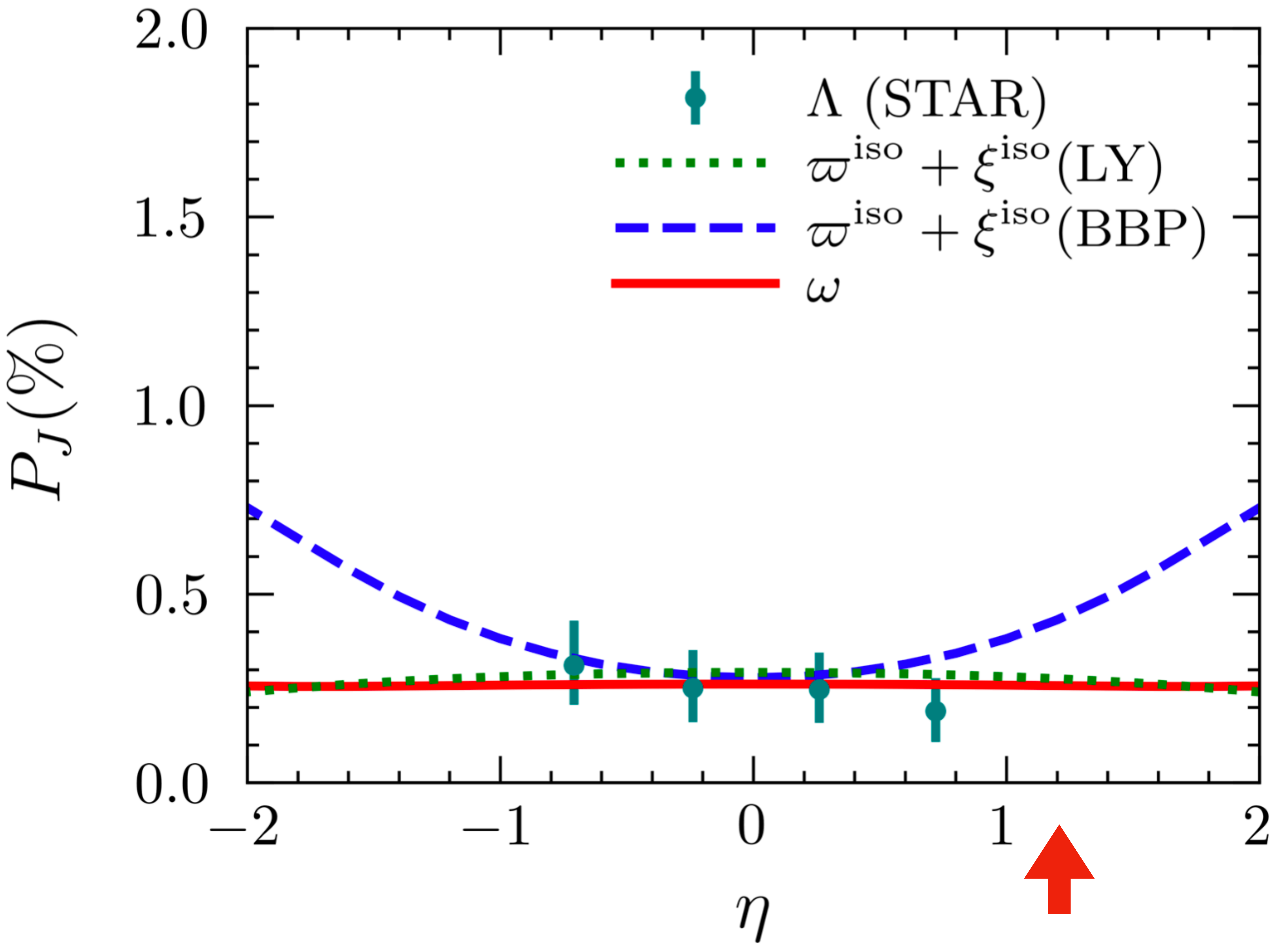

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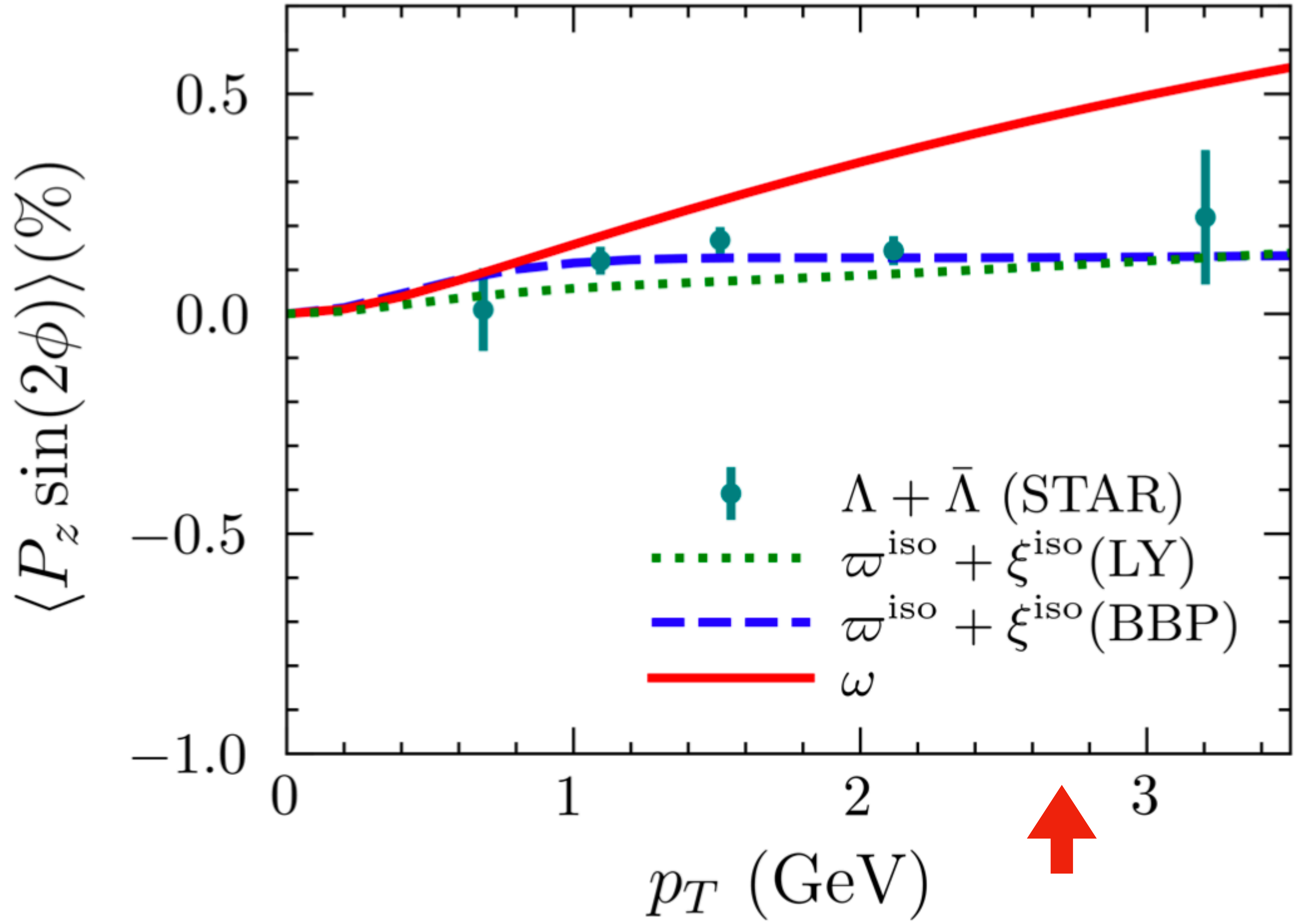

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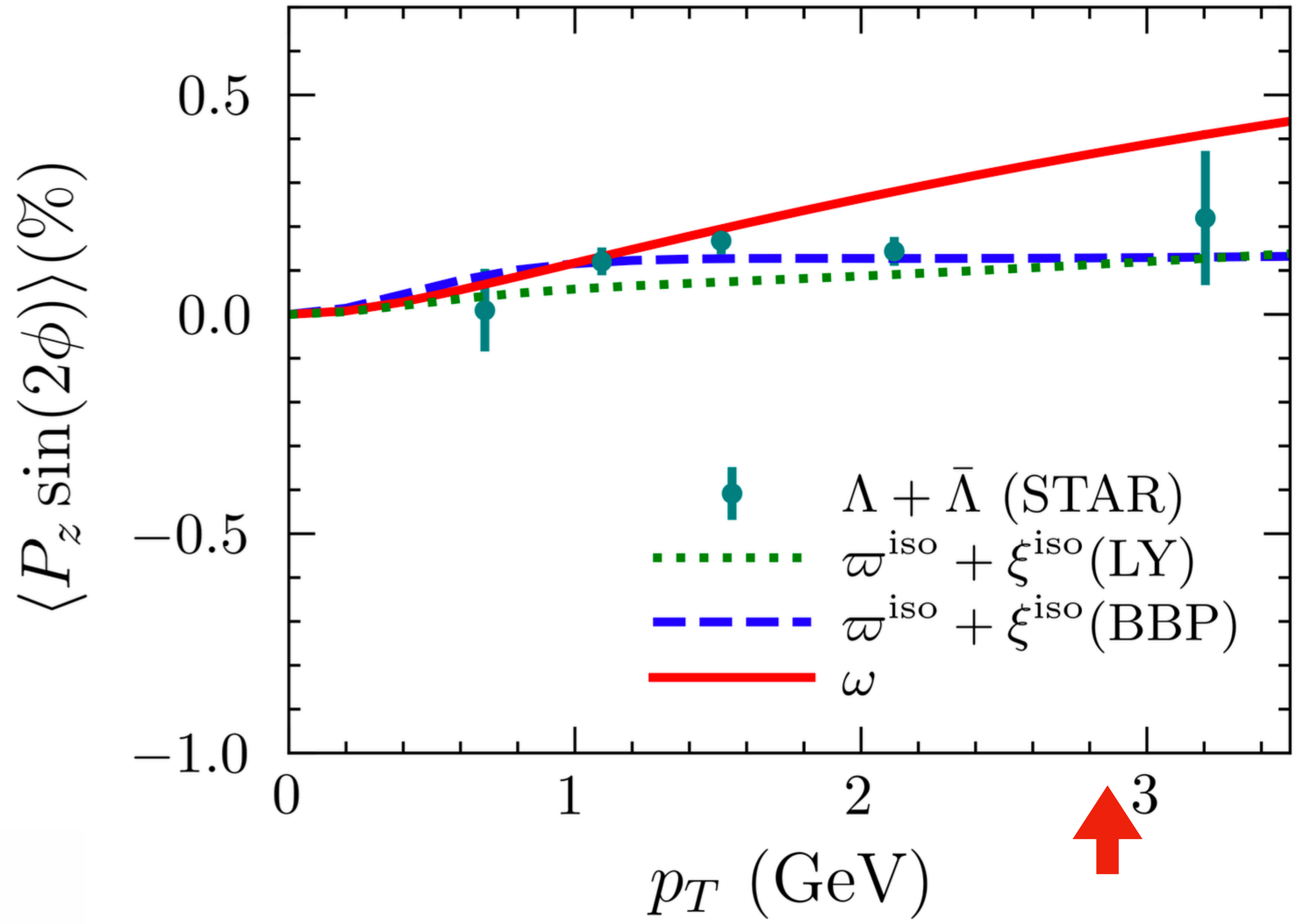

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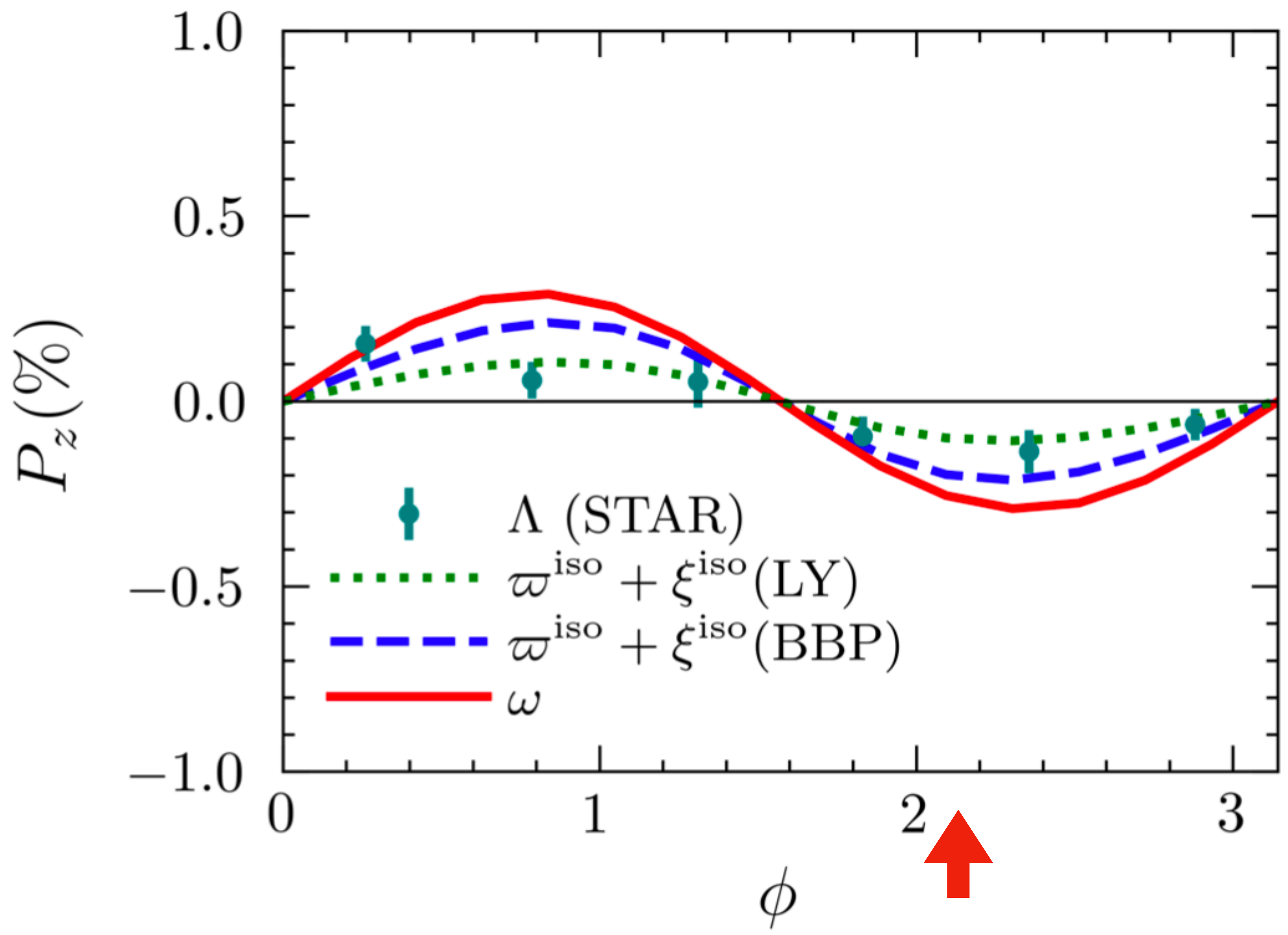
RESULTS FOR SPIN HYDRODYNAMICS - EFFECTIVE MASS



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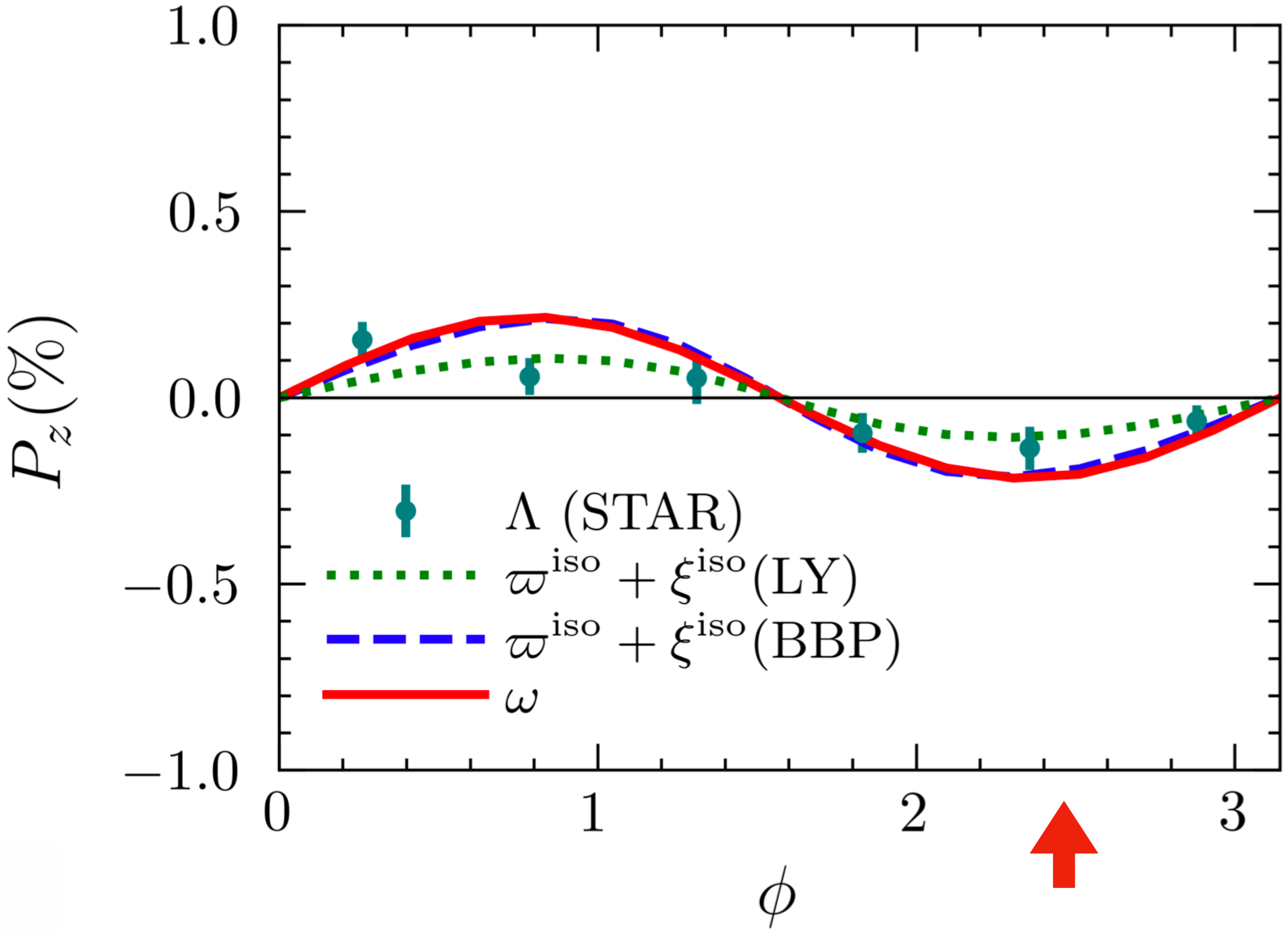

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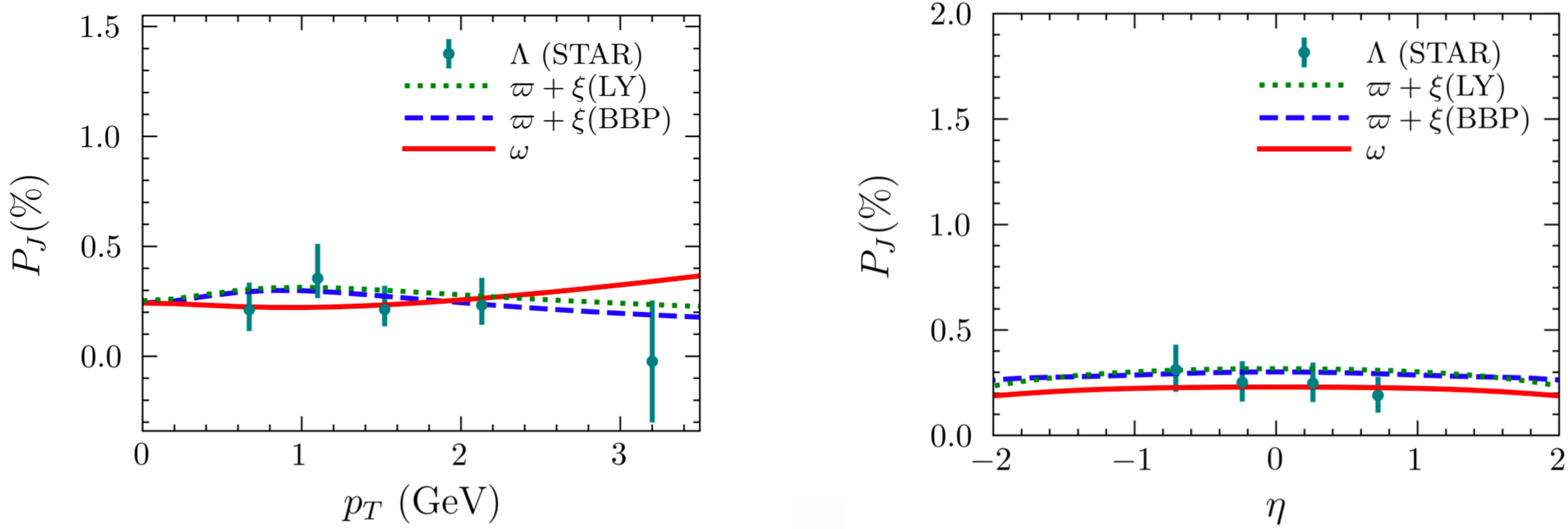
Results for spin hydrodynamics - with T gradients



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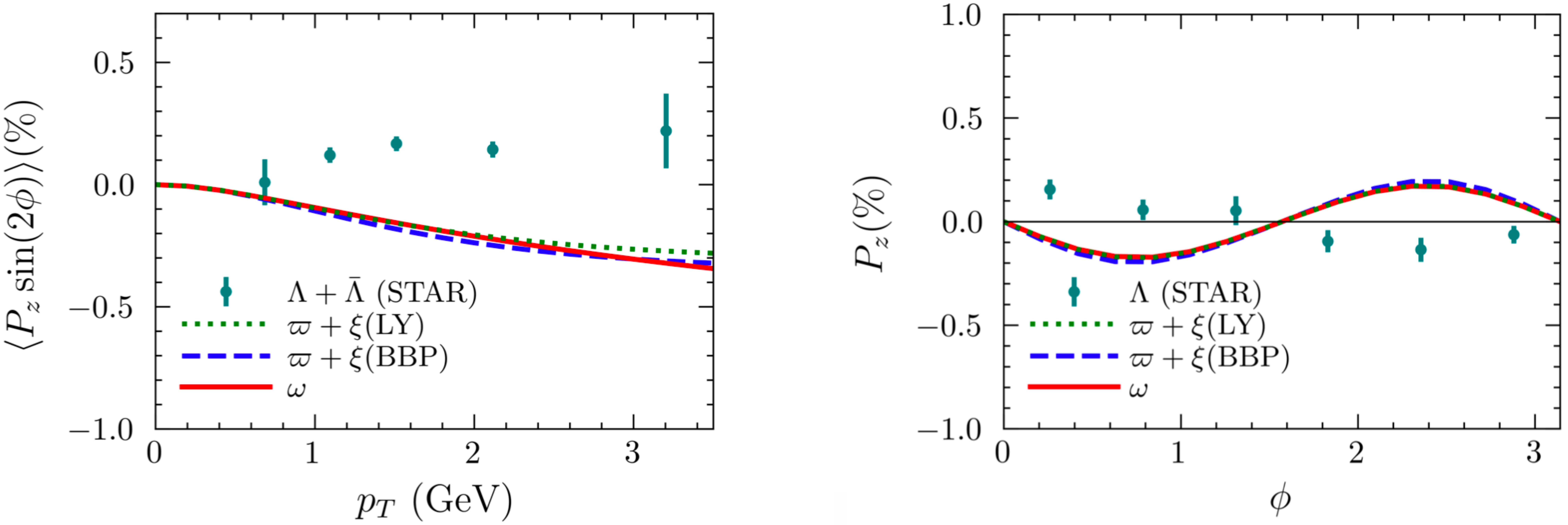
RESULTS FOR SPIN HYDRODYNAMICS - WITH T GRADIENTS



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