

# *Differential study of $\Lambda$ -hyperon polarization in central heavy-ion collisions*

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# Outline

- Introduction
- $\Lambda$  polarization in spin-thermal approach
- Central collisions
- UrQMD
- $\Lambda$  polarization in UrQMD
- Summary

# Introduction

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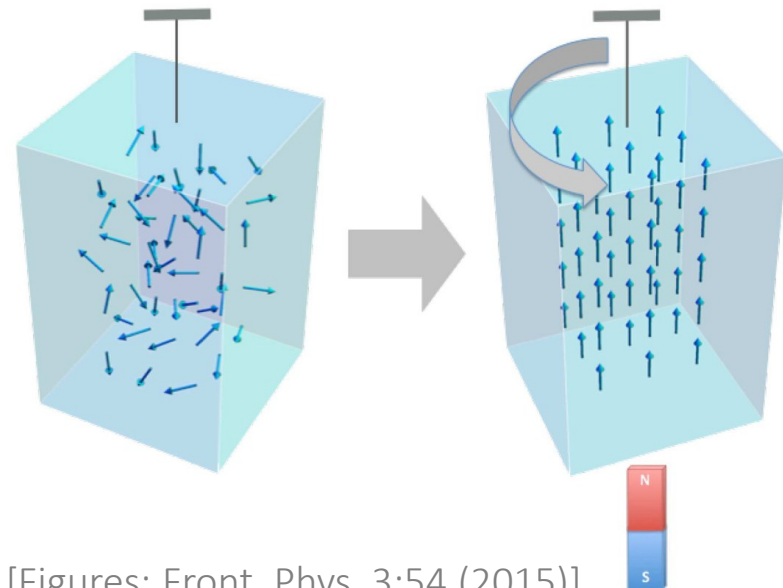
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[Figures: Front. Phys. 3:54 (2015)]

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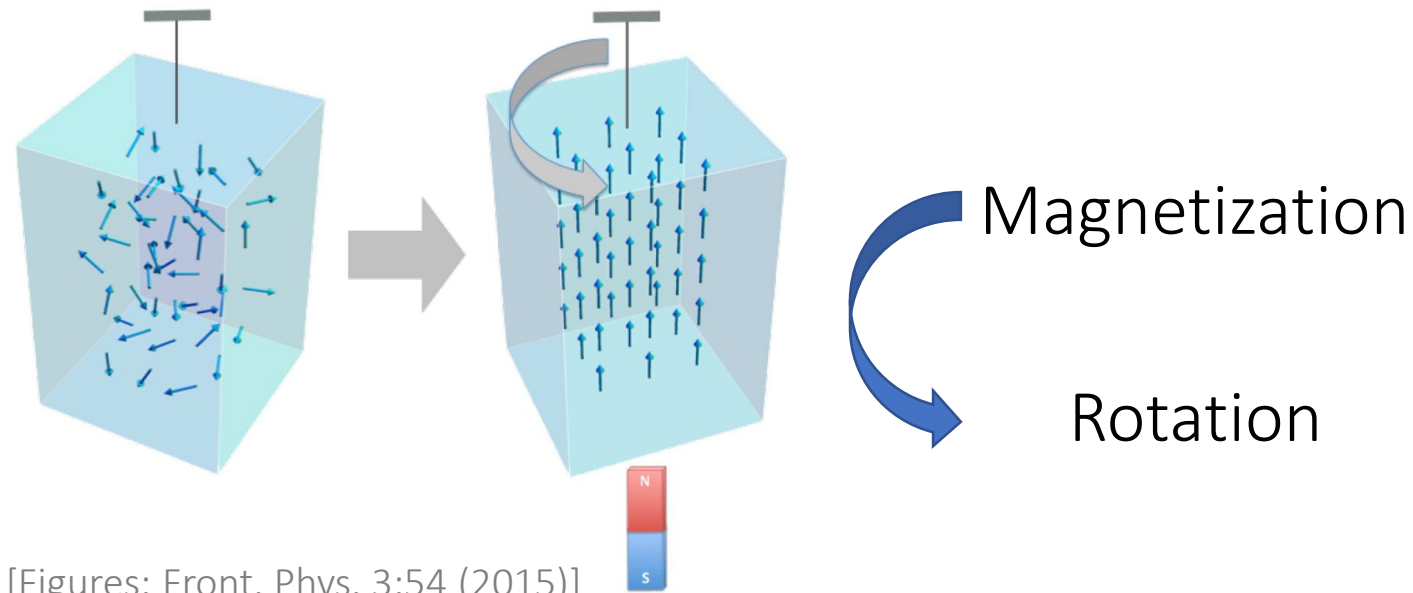
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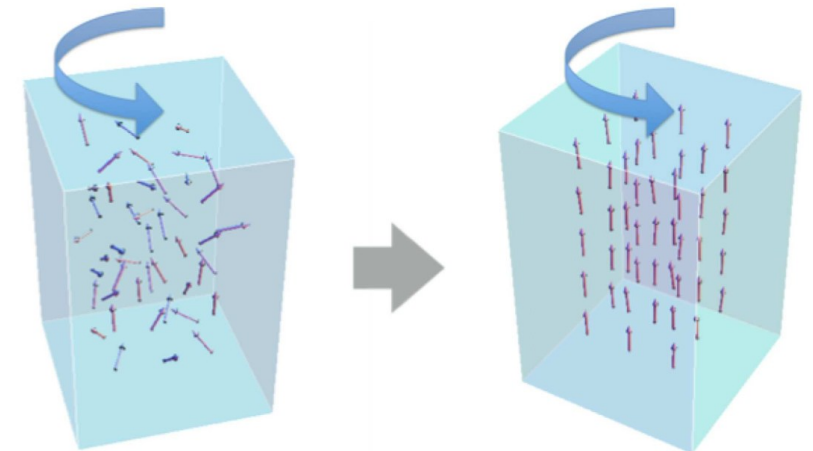
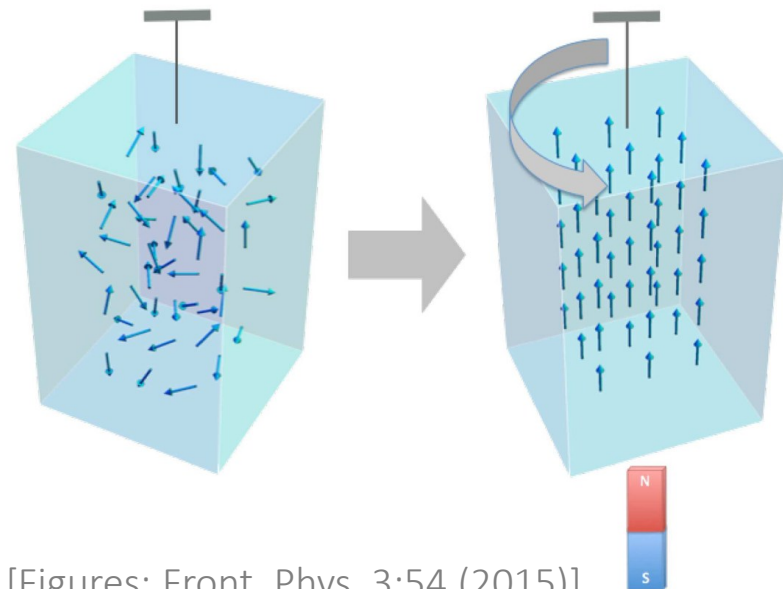
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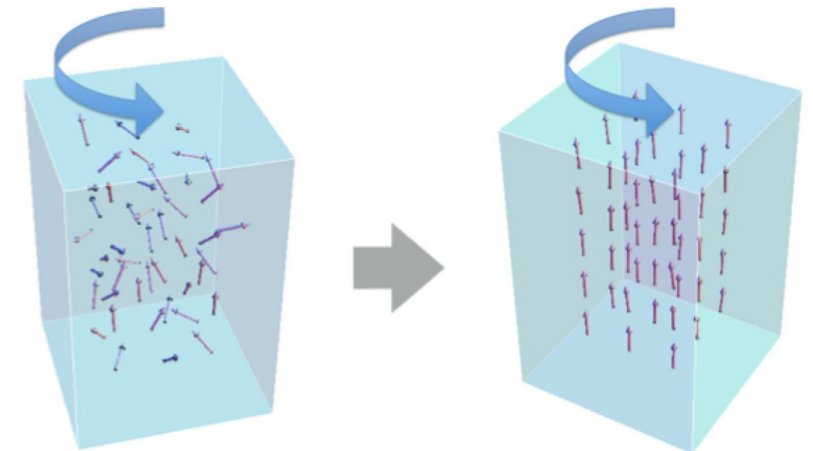
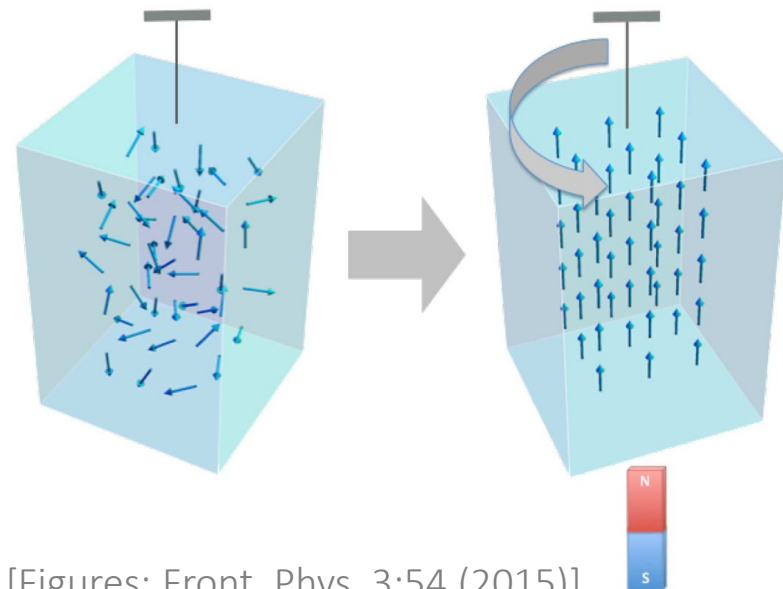
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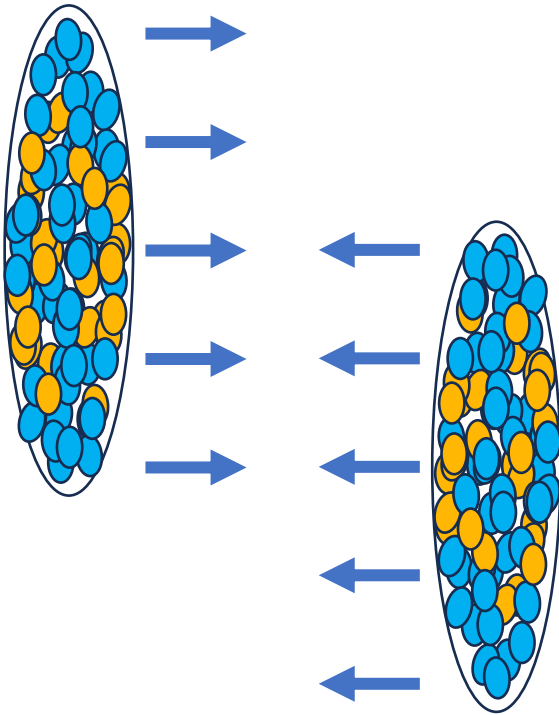
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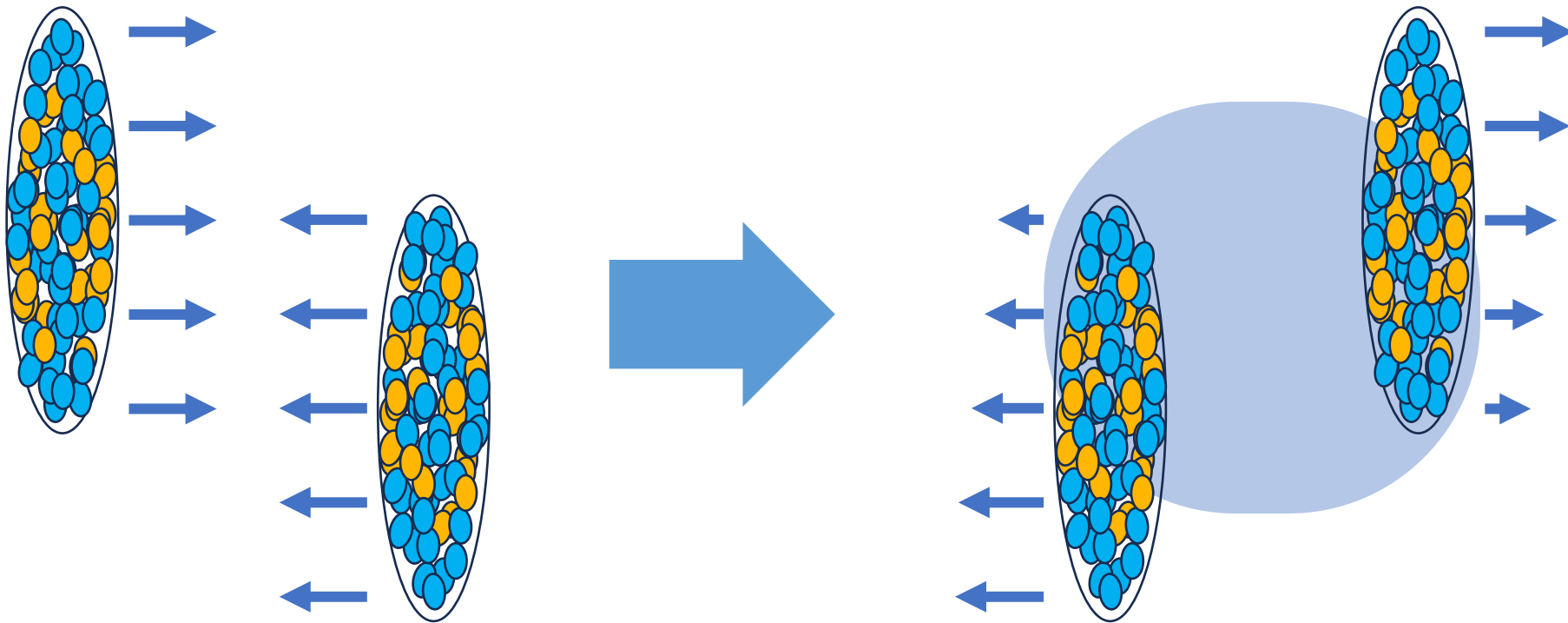
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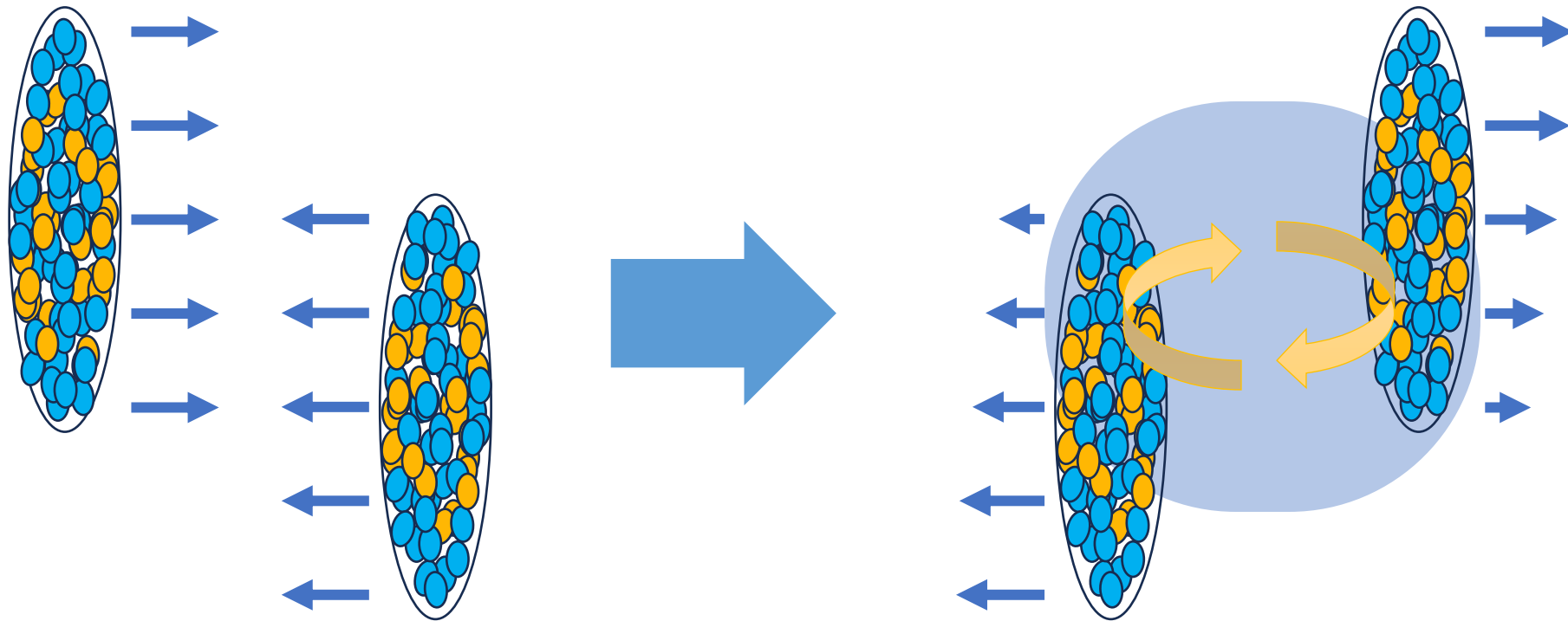
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$$\omega_y = (\nabla \times \vec{v})_y \approx -\frac{1}{2} \frac{\partial v_z}{\partial x} \Rightarrow \text{Guess: } \Delta x \sim 5 \text{ fm}, \Delta v \sim 0.2 \Rightarrow \frac{\omega}{T} \sim \text{up to few percent}$$

# $\Lambda$ polarization in spin-thermal approach

In the assumption of local thermal equilibrium one can find expression for spin 4-vector: [PRC 95, 054902 (2017)]

$$S^\mu(p, x) \approx -\frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\nu \varpi_{\rho\sigma}(x), \quad \varpi^{\mu\nu} = \frac{1}{2} \left( \partial^\nu \frac{u^\mu}{T} - \partial^\mu \frac{u^\nu}{T} \right)$$

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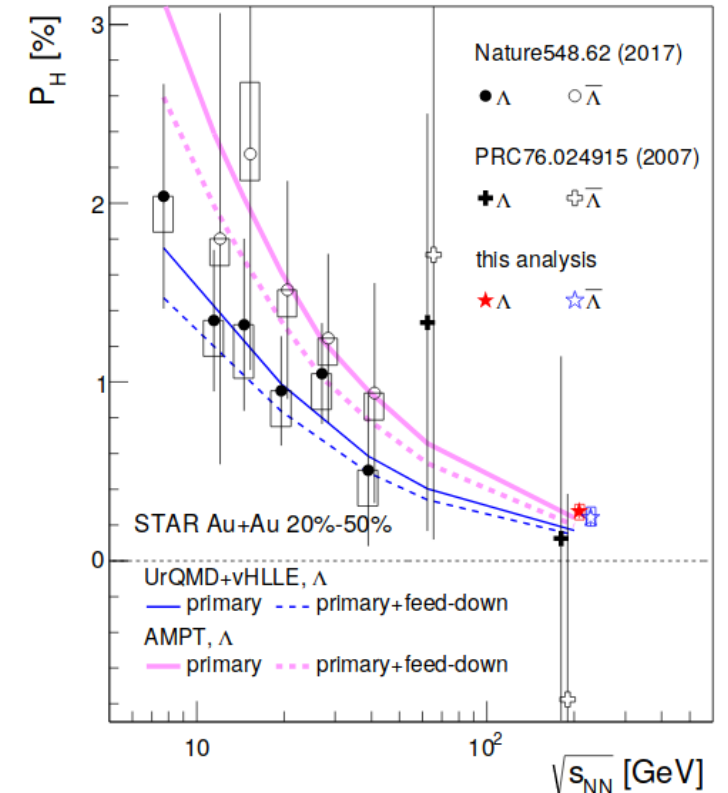
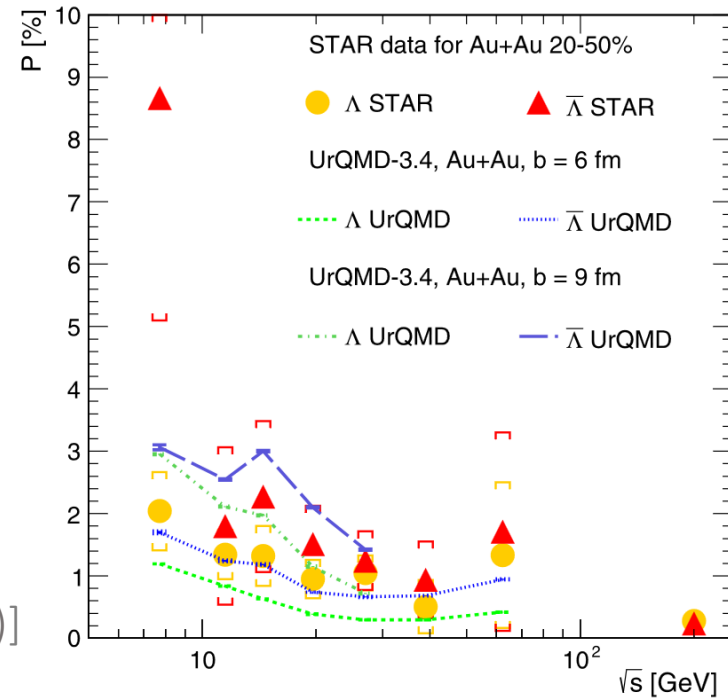
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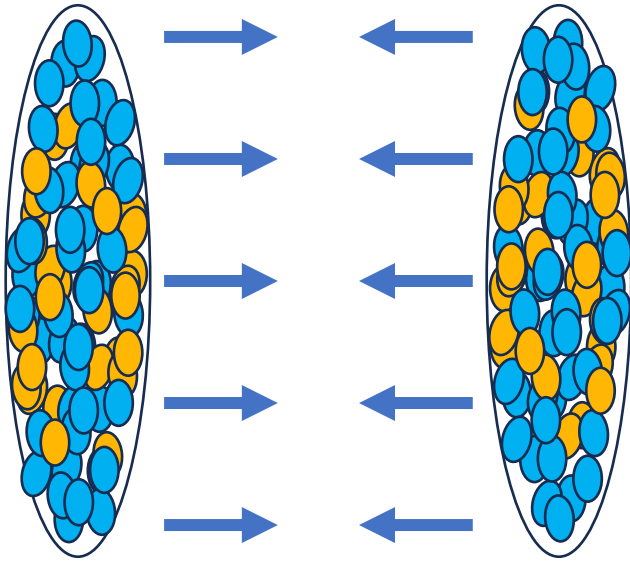
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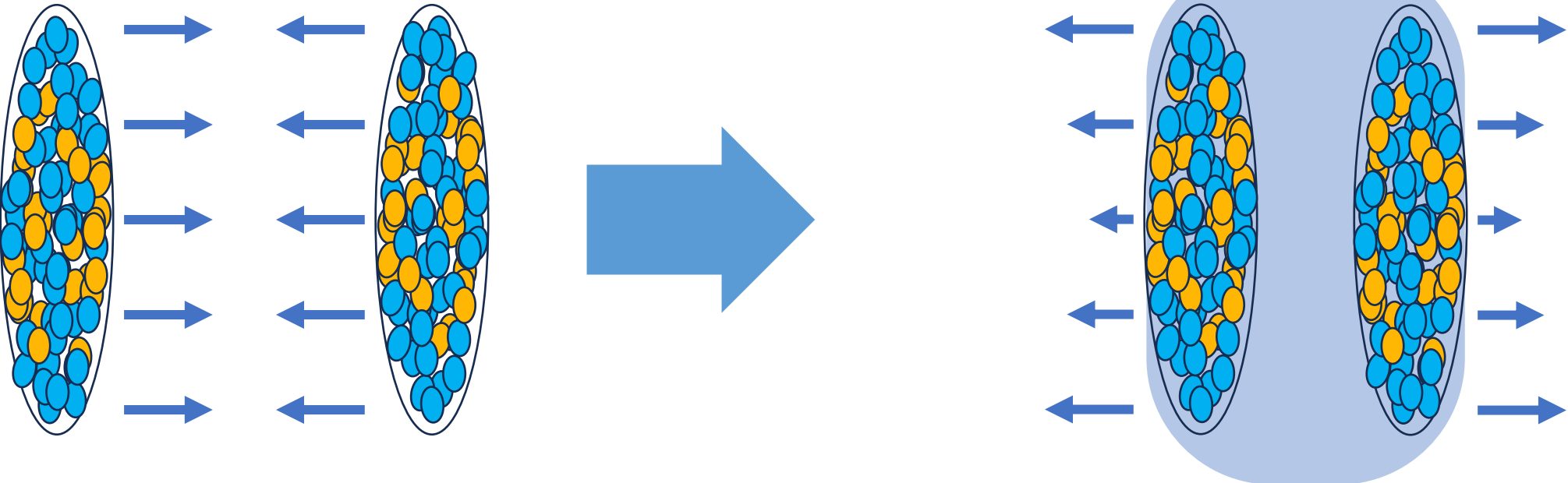
[PRC 98 (2018) 14910; PLB 803, 135298 (2020)]



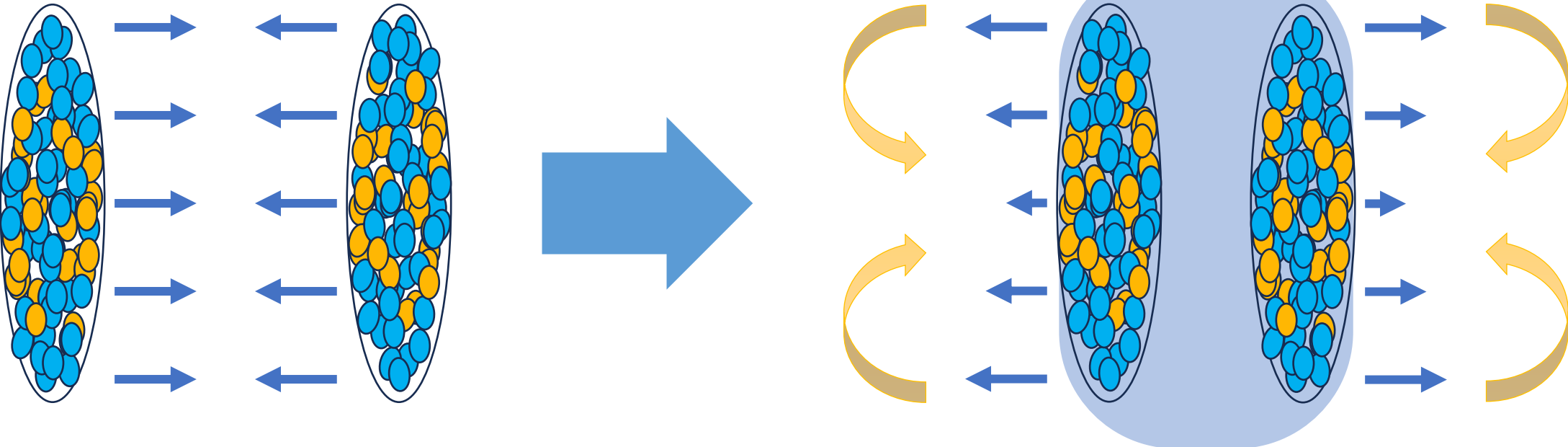
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# Ultrarelativistic Quantum Molecular Dynamics (UrQMD)

- Represents a Monte Carlo method for the time evolution of the various phase space densities of particle species
- Based on the covariant propagation of all hadrons on classical trajectories, stochastic binary scatterings, resonance and string formation with their subsequent decay
- Ideologically based on the Boltzmann equation
- The collision criterion (black disk approximation):  $d < d_0 = \sqrt{\sigma(\sqrt{s}, type)/\pi}$
- 55 baryons and 32 mesons are included. All antiparticles and isospin-projected states are implemented
- Cross sections are taken from PDG
- Resonances are implemented in Breit–Wigner form

[S. A. Bass et al, Prog. Part. Nucl. Phys. 41 (1998) 255-369,  
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**How to calculate it in the transport model?**

Solution: Coarse-graining approach + HRG Model

[PLB 803, 135298 (2020)]

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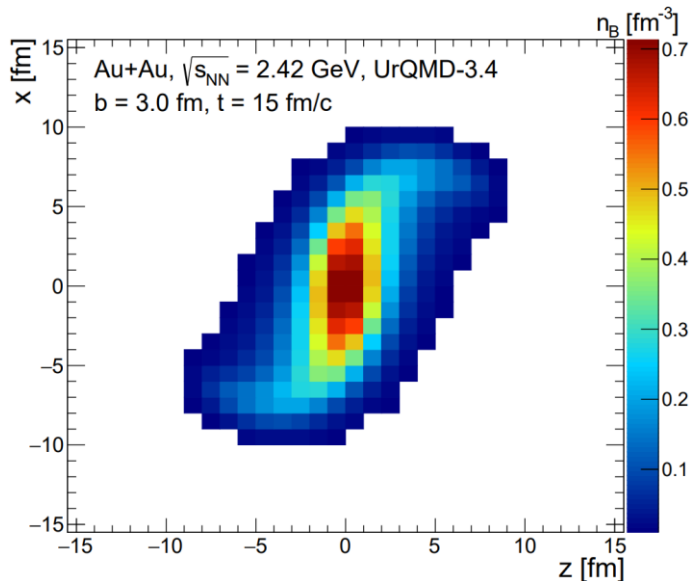
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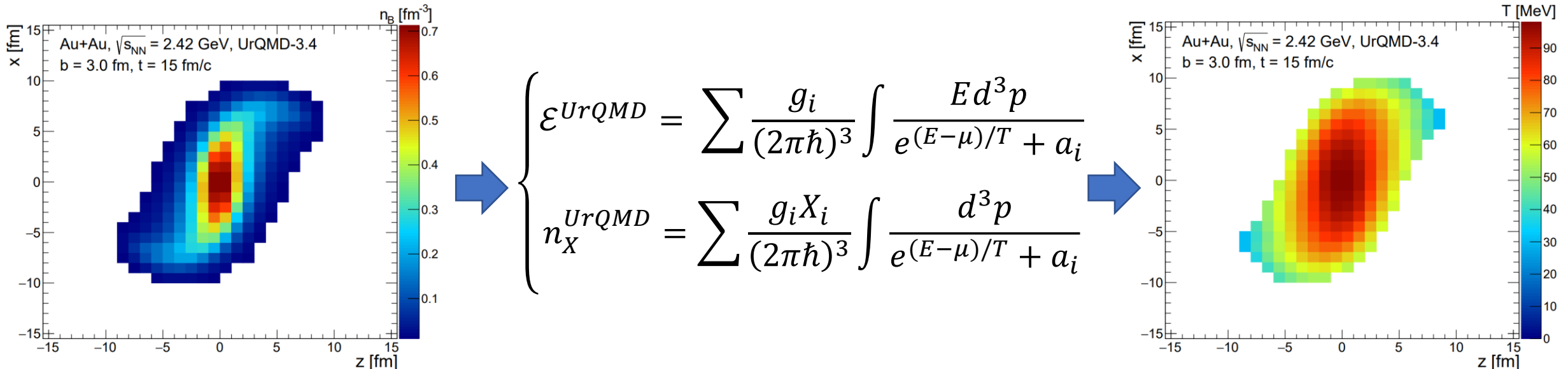
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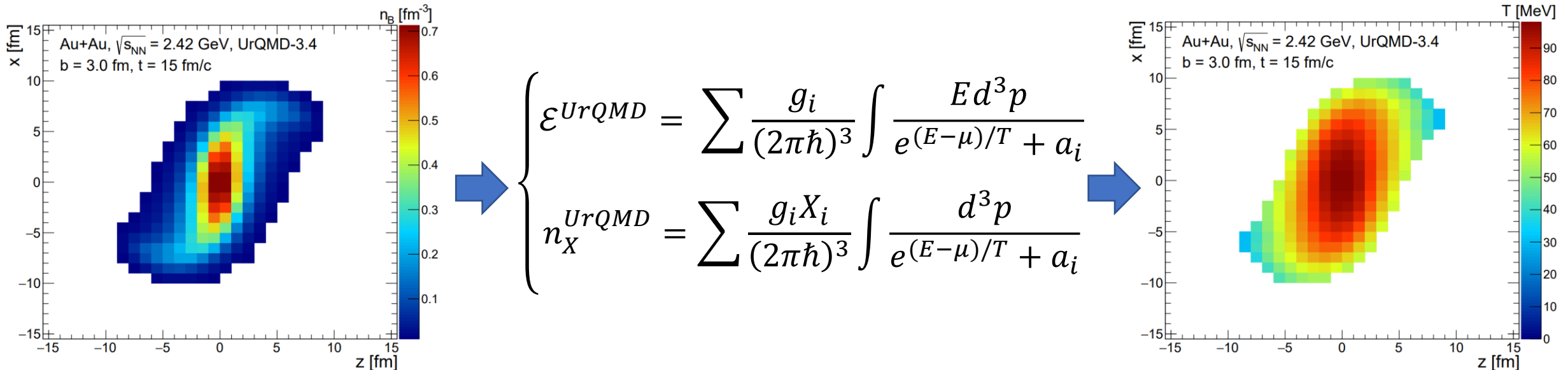
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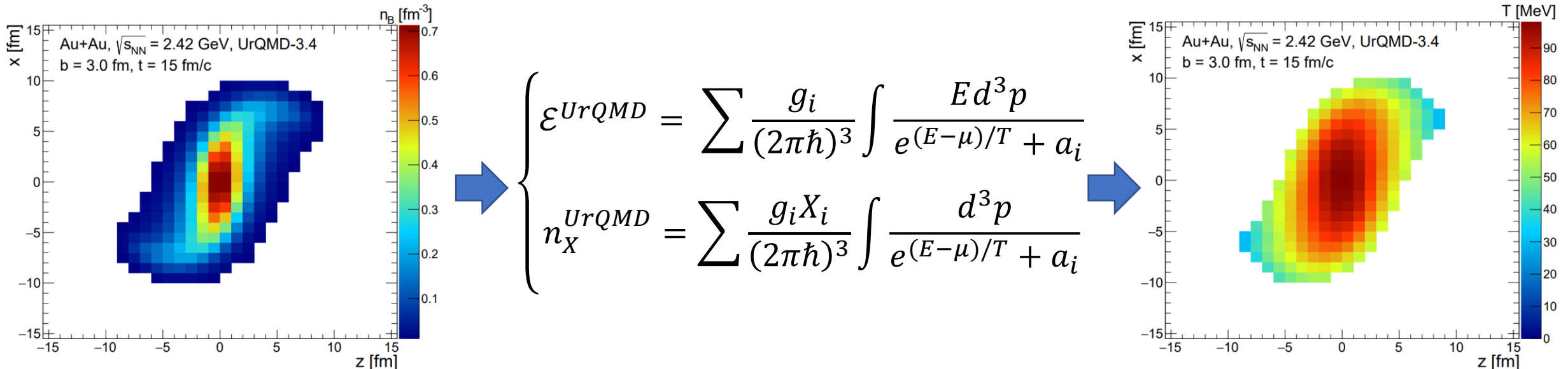
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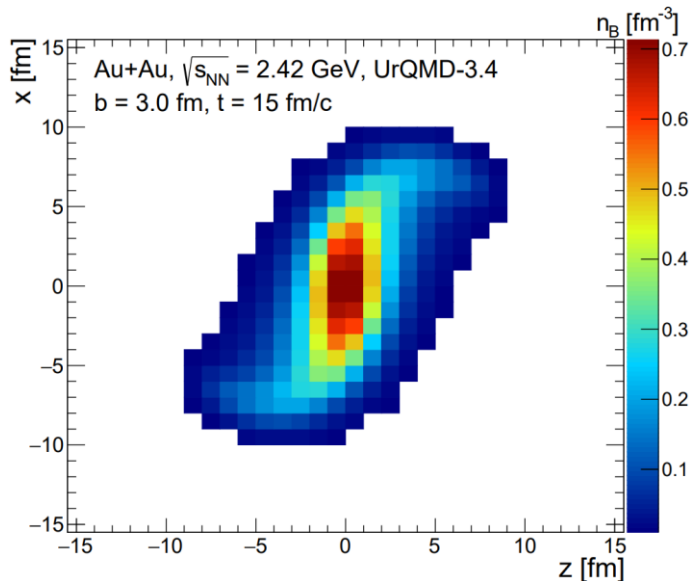
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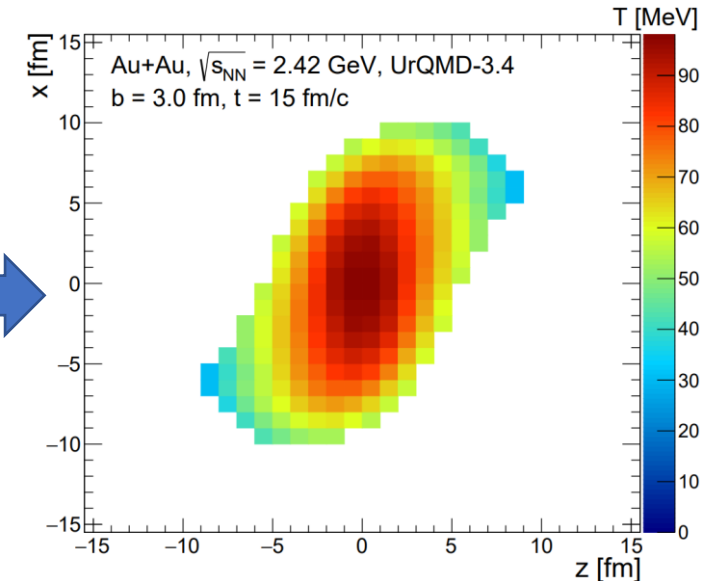


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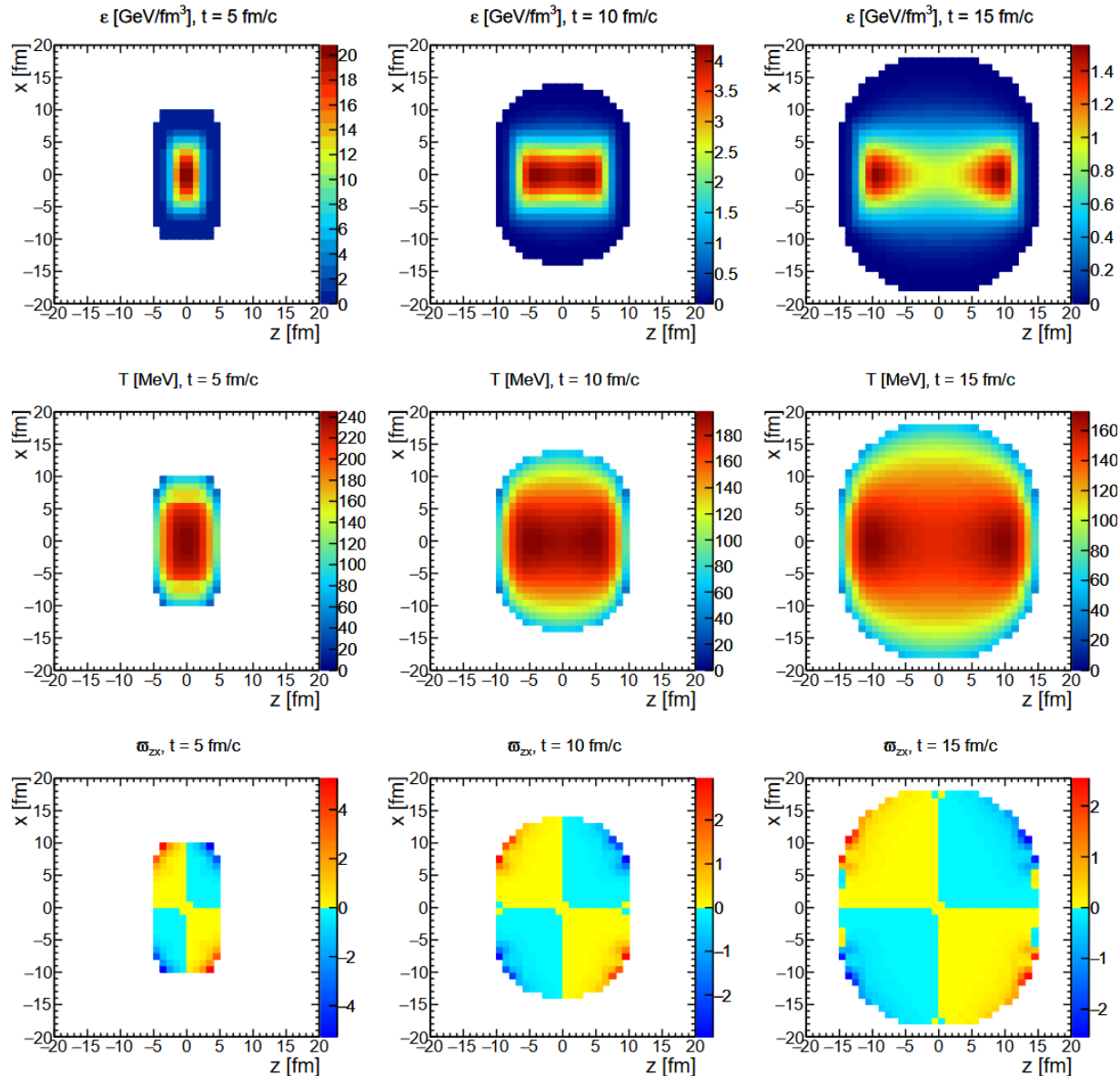
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7. Finally, polarization and other observables can be calculated



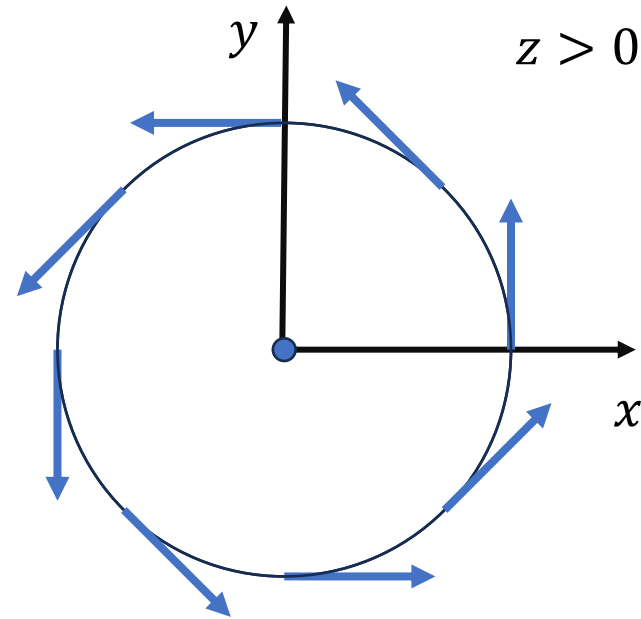
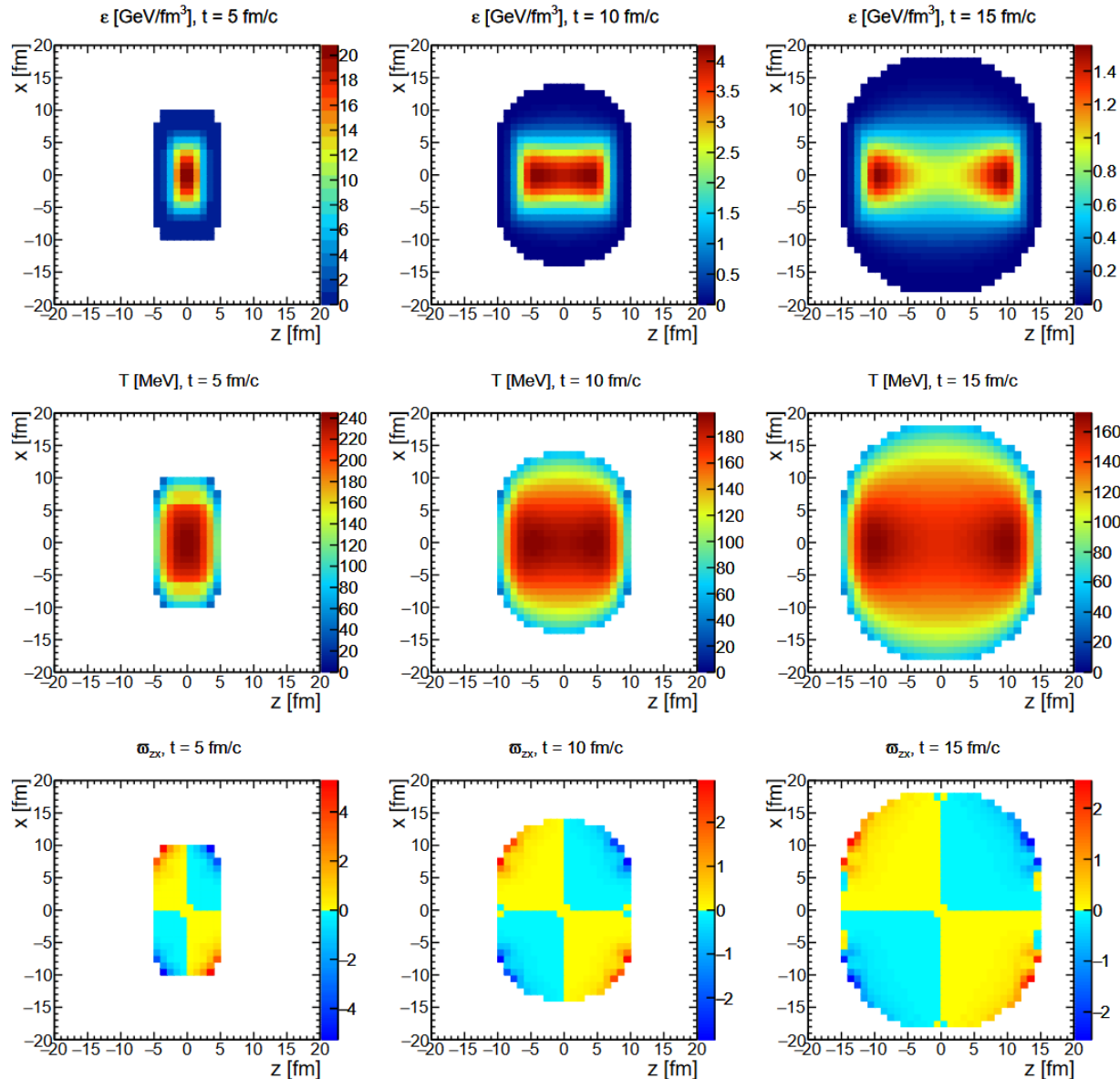
$$\left\{ \begin{array}{l} \varepsilon^{UrQMD} = \sum \frac{g_i}{(2\pi\hbar)^3} \int \frac{E d^3p}{e^{(E-\mu)/T} + a_i} \\ n_X^{UrQMD} = \sum \frac{g_i X_i}{(2\pi\hbar)^3} \int \frac{d^3p}{e^{(E-\mu)/T} + a_i} \end{array} \right.$$



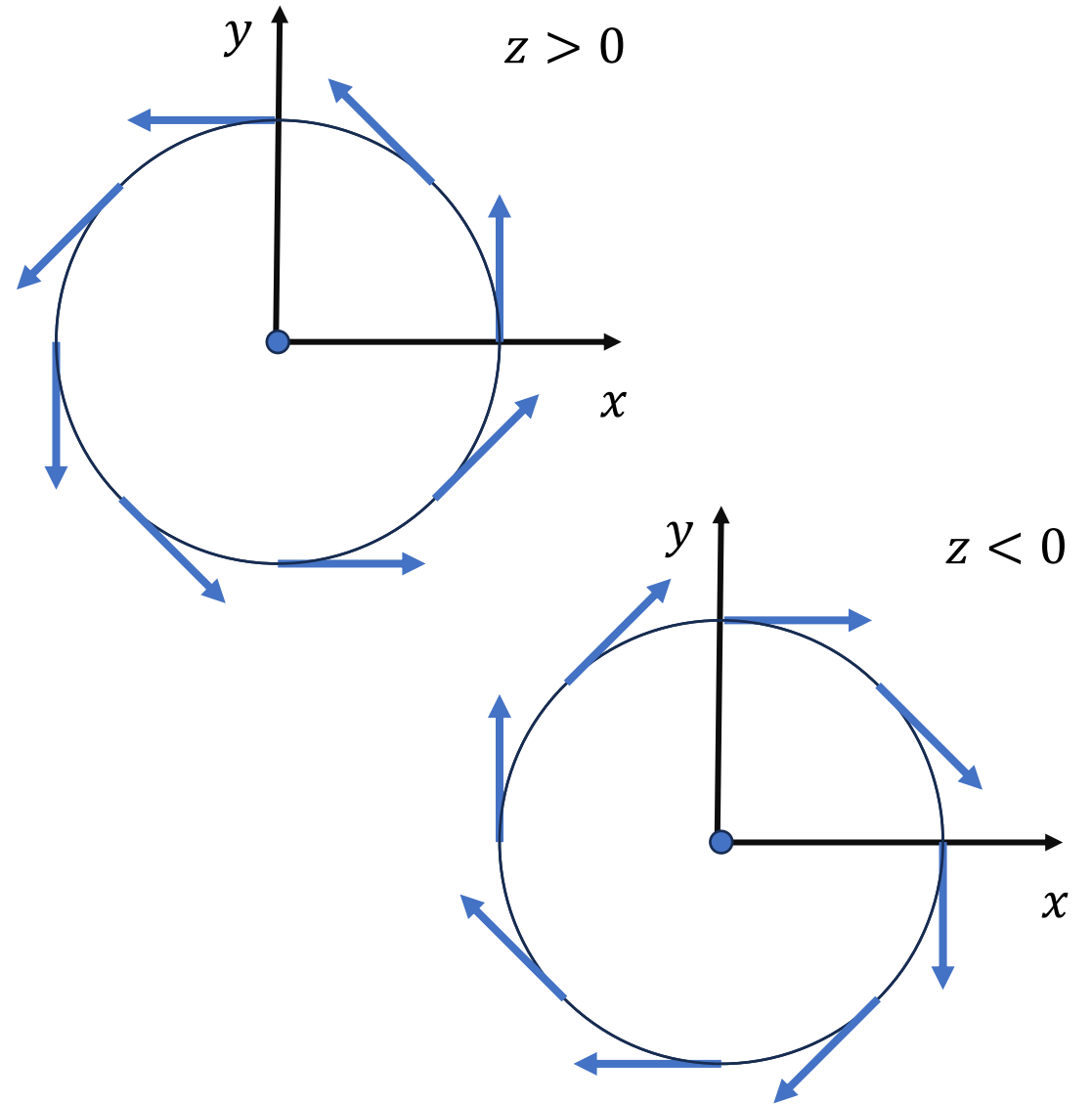
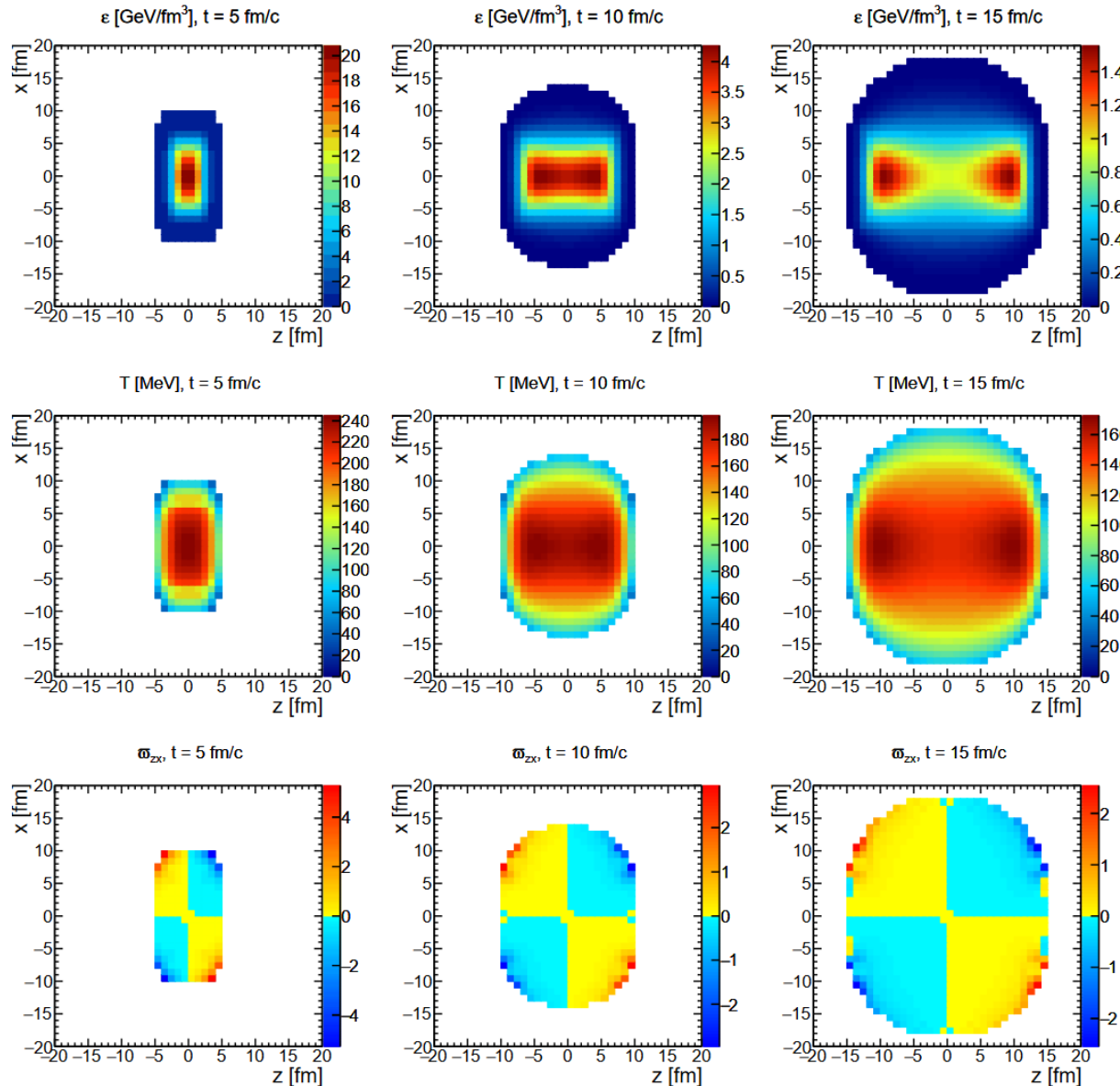
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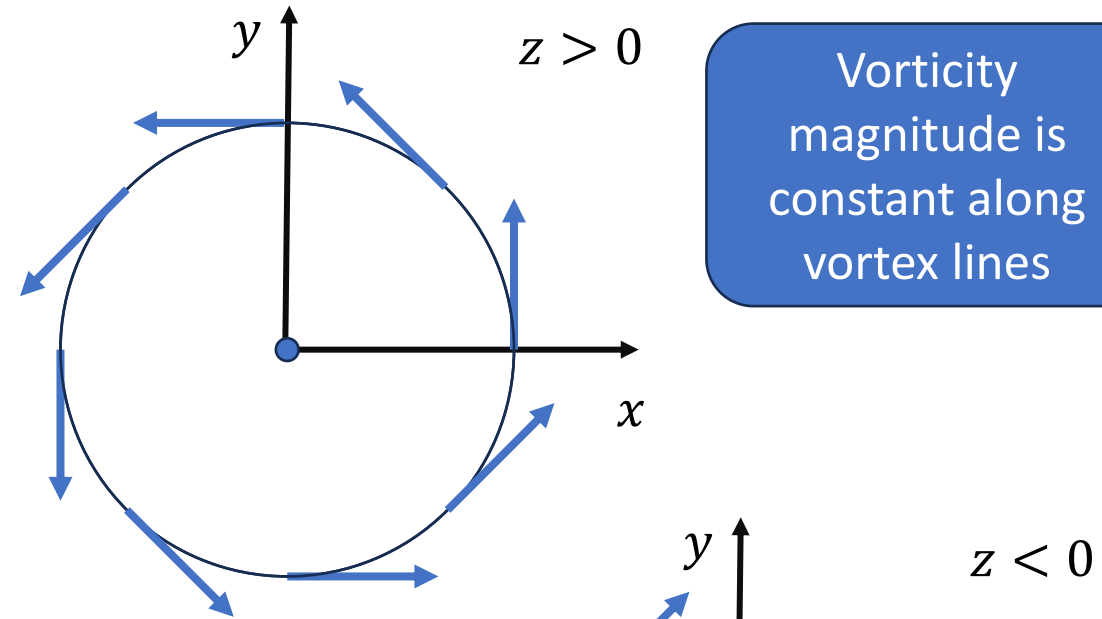
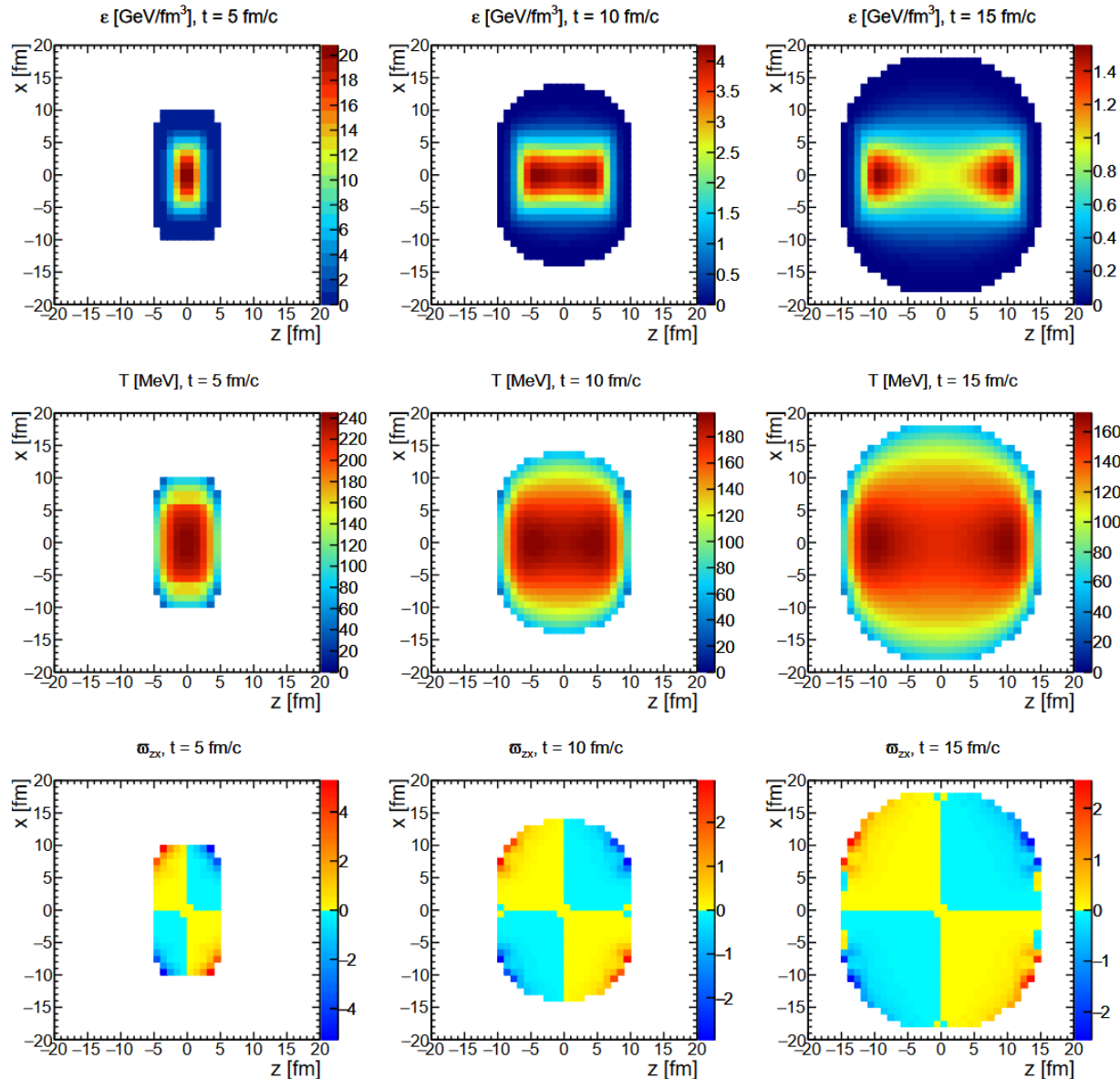
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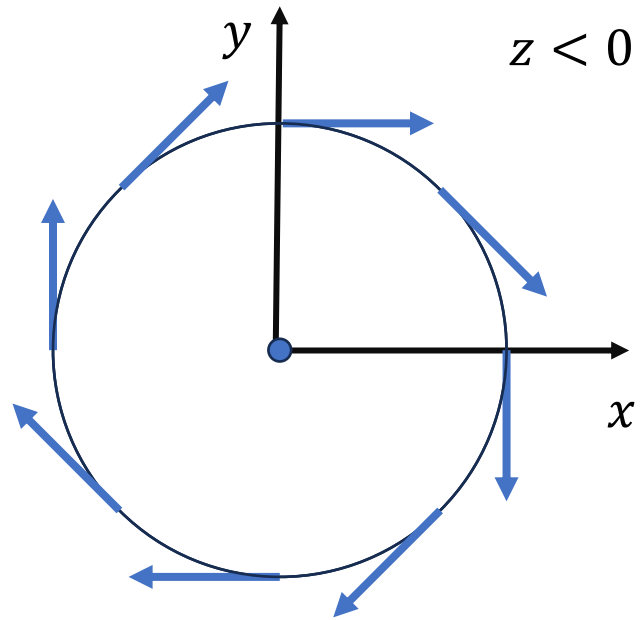


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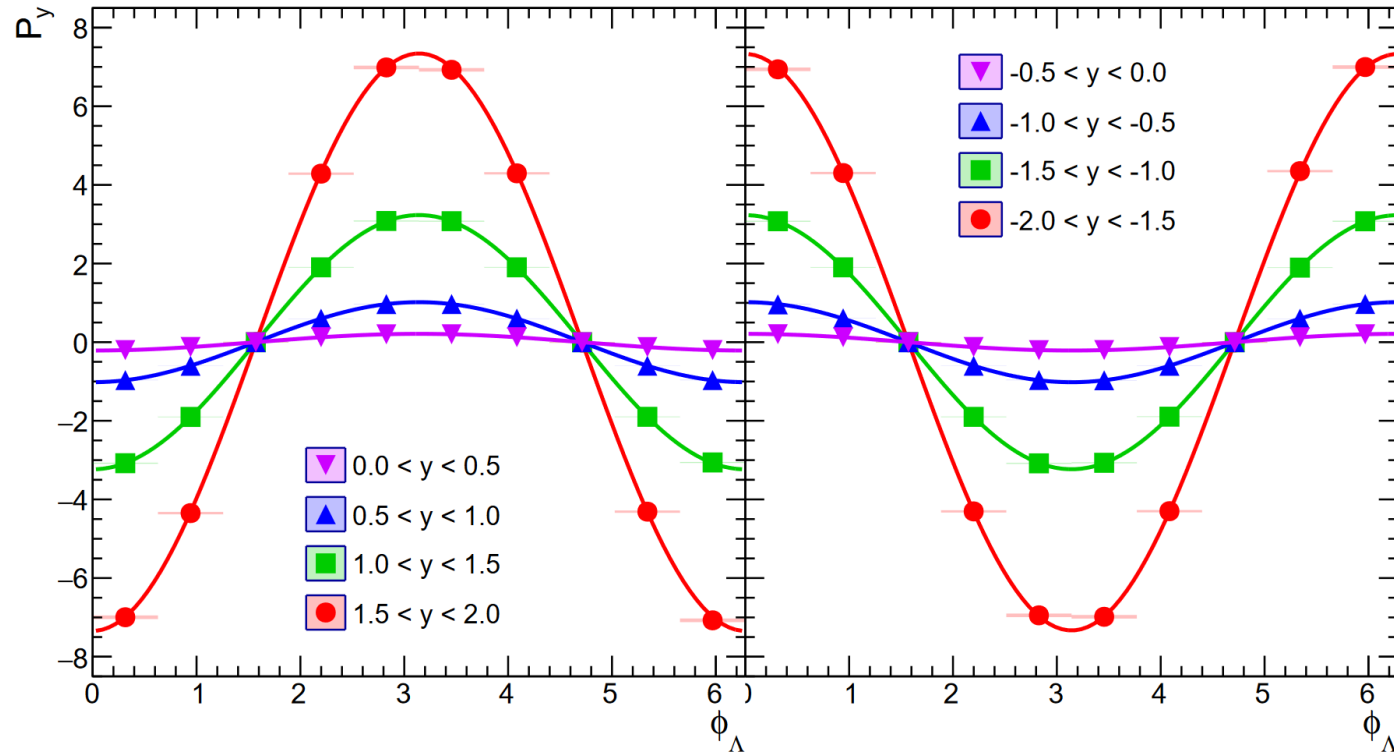


Vorticity magnitude is constant along vortex lines

Vortex lines are circles in the transverse plane

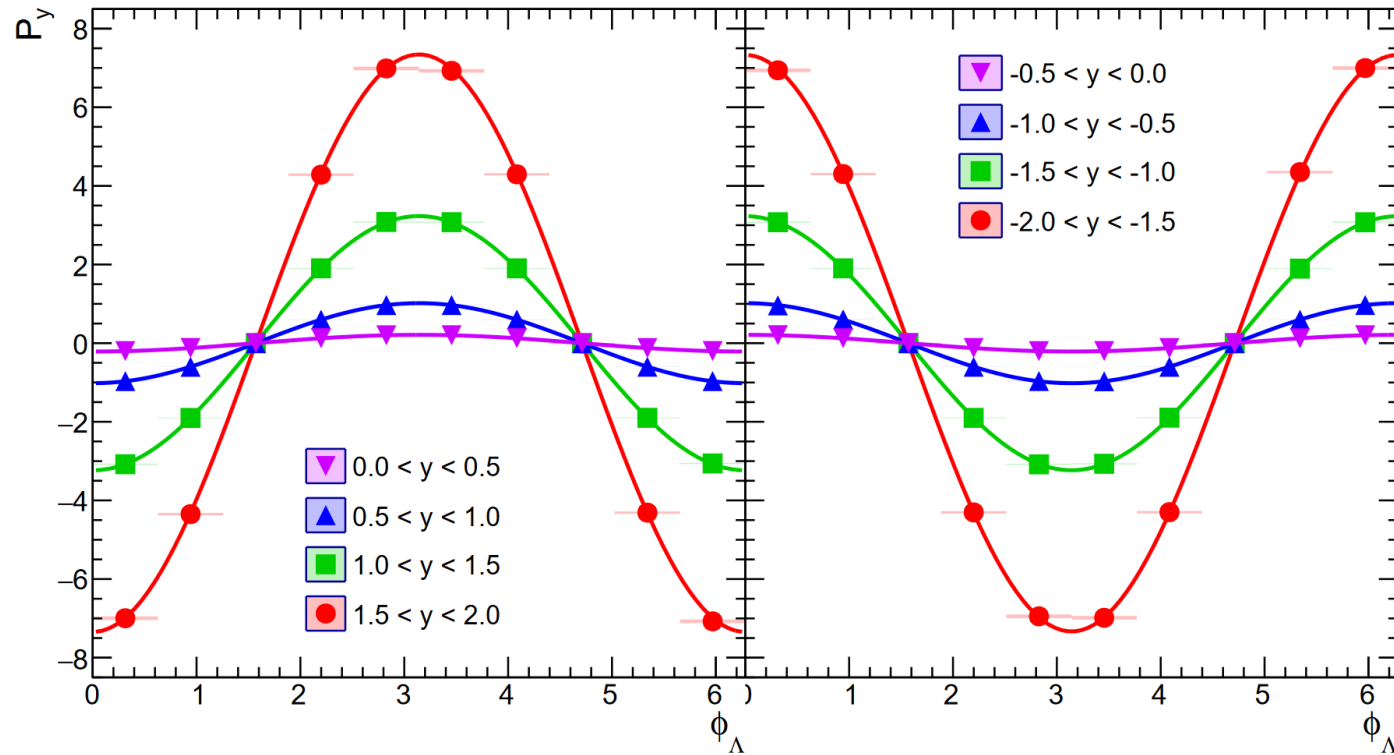


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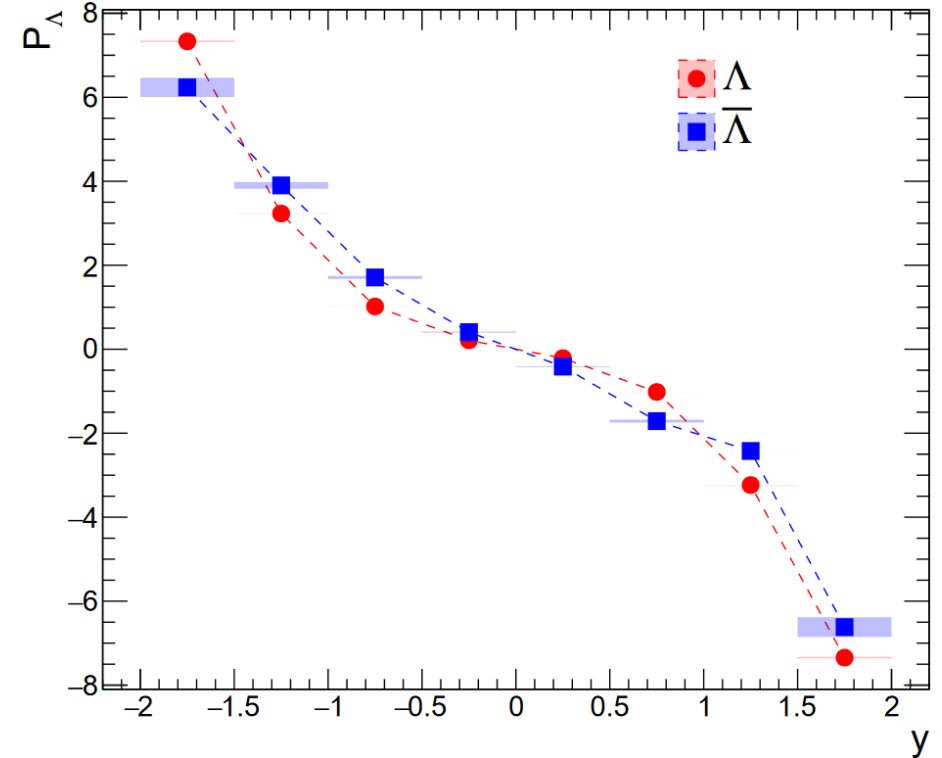
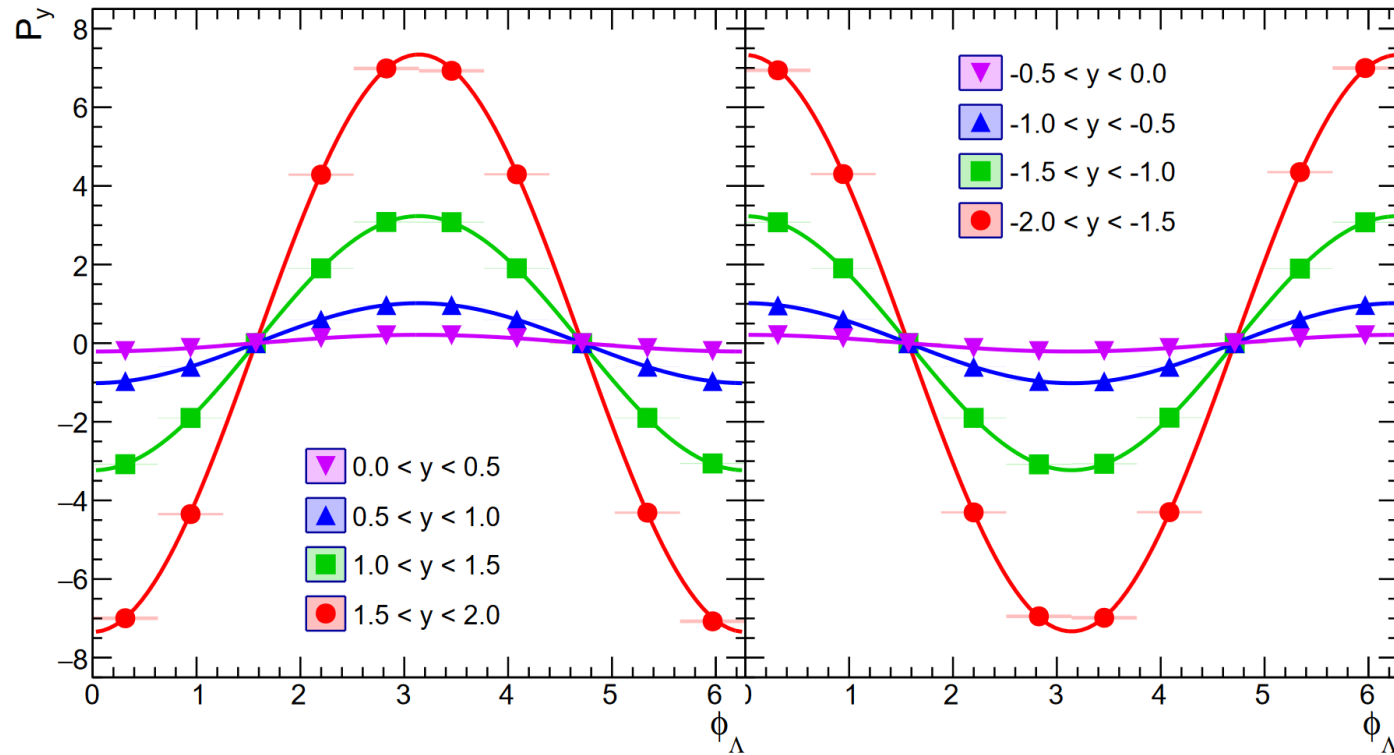


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- The  $\Lambda$  polarization clearly exhibits oscillatory behaviour as a function of the hyperon azimuthal angle.
- We fit the azimuthal angle dependence with a periodic function to extract magnitude of the local hyperon polarization  $P_\Lambda$  as a function of rapidity:

$$P_y = P_\Lambda \cos \phi_\Lambda$$

# Summary

- The thermal vorticity field has a structure which effectively resembles two vortex rings in the forward and backward hemispheres. The structure is stable in time, but the vorticity magnitude decreases due to system expansion.
- The polarization of  $\Lambda$ -hyperons exhibits oscillatory behaviour as a function of the hyperon azimuthal angle.
- The magnitude of the local  $\Lambda$  polarization is decreasing function of rapidity.
- The  $\Lambda$  and  $\bar{\Lambda}$  hyperons polarization are consistent with each other.
- The measurement of the azimuthal-angle dependence of local polarization can serve as a novel probe to investigate the internal structure and evolution of the fireball in central and semi-central heavy-ion collisions.

