The earliest phase of relativistic heavy-ion collisions

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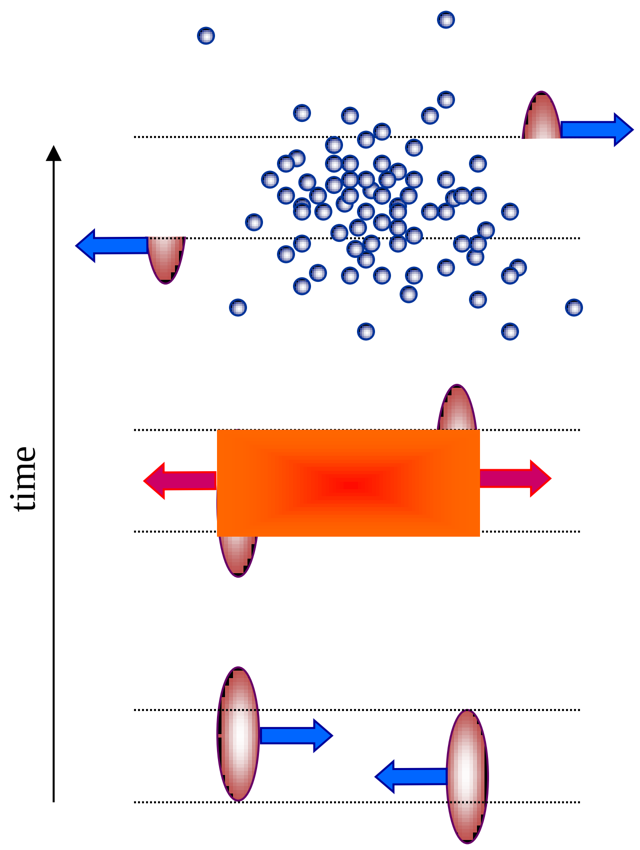
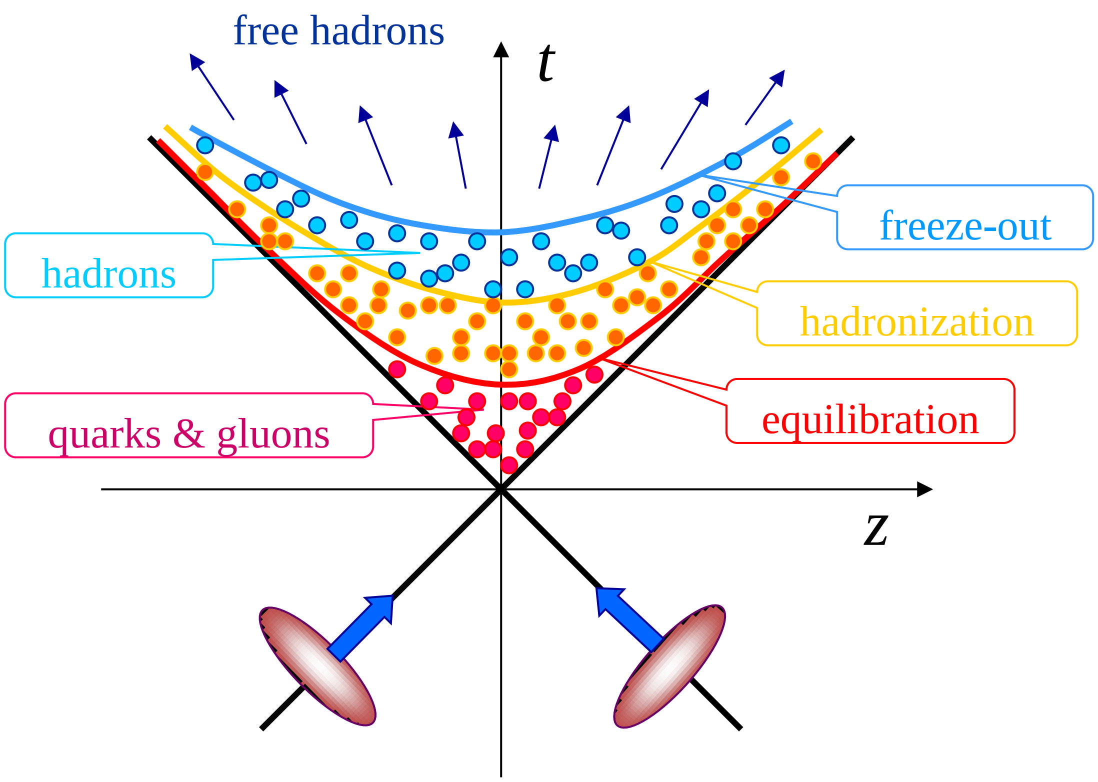
Alina Czajka, Wade Cowie, Bryce Friesen,
Jean-Yves Ollitrault and Doug Pickering

Scenario of relativistic heavy-ion collisions



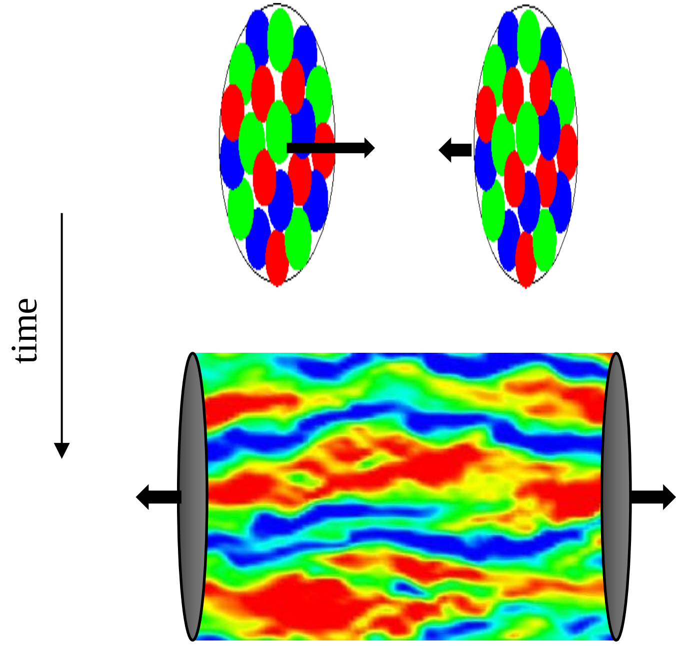


Leitmotif of this talk

The earliest phase of relativistic heavy-ion collisions, when the produced matter is maximally anisotropic, has record high energy density, and its dynamics is strongly nonlinear, does not merely provide the initial condition for the further hydrodynamic evolution of the system, but many phenomena observed in the collision final state have their origin in this phase.

Color Glass Condensate

Color charges confined in the colliding nuclei generate **glasma** – the system of strong mostly classical chromodynamic fields which evolves towards equilibrium.



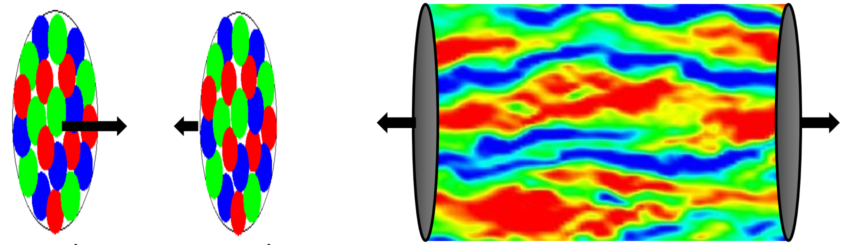
Color Glass Condensate

Classical Yang-Mills equation

$$D_\mu F^{\mu\nu}(x) = j^\nu(x)$$

$$j^\mu(x) = j_1^\mu(x) + j_2^\mu(x)$$

$$j_{1,2}^\mu(x) = \pm\delta^{\mu\mp}\delta(x^\pm)\rho_{1,2}(\mathbf{x}_\perp)$$

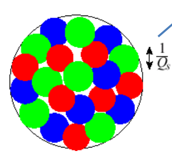


β_1^i β_2^i pre-collision potentials

$\alpha, \alpha_\perp^i$ post-collision potentials

Ansatz of gauge potentials

$$\begin{cases} A^+(x) = \Theta(x^+)\Theta(x^-)x^+\alpha(\tau,\mathbf{x}_\perp) \\ A^-(x) = -\Theta(x^+)\Theta(x^-)x^-\alpha(\tau,\mathbf{x}_\perp) \\ A^i(x) = \Theta(x^+)\Theta(x^-)\alpha_\perp^i(\tau,\mathbf{x}_\perp) \\ \qquad + \Theta(-x^+)\Theta(x^-)\beta_1^i(\mathbf{x}_\perp) + \Theta(x^+)\Theta(-x^-)\beta_2^i(\mathbf{x}_\perp) \end{cases}$$



Boundary condition

$$\begin{cases} \alpha(0,\mathbf{x}_\perp) = \beta_1^i(\mathbf{x}_\perp) + \beta_2^i(\mathbf{x}_\perp) \\ \alpha_\perp^i(0,\mathbf{x}_\perp) = -\dfrac{ig}{2}[\beta_1^i(\mathbf{x}_\perp), \beta_2^i(\mathbf{x}_\perp)] \end{cases}$$

Gauge condition

$$x^+A^- + x^-A^+ = 0$$

E. Iancu, R. Venugopalan, in *Quark-Gluon Plasma 3*,
ed. by R.C. Hwa, X.-N. Wang (World Scientific, Singapore, 2004

Proper time expansion

$$\alpha(\tau,\mathbf{x}_\perp)=\sum_{n=0}^{\infty}\tau^n\alpha_{(n)}(\mathbf{x}_\perp), \qquad \alpha_\perp^i(\tau,\mathbf{x}_\perp)=\sum_{n=0}^{\infty}\tau^n\alpha_{\perp(n)}^i(\mathbf{x}_\perp)$$

Proper time τ is treated as a small parameter $\tau << Q_s^{-1}$

Yang-Mills equations for the expanded potentials are solved recursively

$$\alpha_{(n)}=\alpha_{\perp(n)}^i=0 \quad \text{for} \quad n=1,3,5,\ldots$$

0th order - oboundary conditions

$$\begin{cases}\alpha_{(0)}=-\dfrac{ig}{2}[\beta_1^i,\beta_2^i]\\ \alpha_{\perp(0)}^i=\beta_1^i+\beta_2^i\end{cases}$$

Post-collision potentials are expressed through pre-collision potentials

2nd order

$$\begin{cases}\alpha_{(2)}=-\dfrac{ig}{16}[D^j,[D^j,[\beta_1^i,\beta_2^i]]]\\ \alpha_{\perp(2)}^i=\dfrac{ig}{4}\varepsilon^{zij}\varepsilon^{zkl}[D^j,[\beta_1^k,\beta_2^l]]\end{cases}$$

$$D^i\equiv\partial^i-ig(\beta_1^i+\beta_2^i)$$

Fully analytic approach!

R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:nucl-th/0604054
G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Energy-momentum tensor

$$T^{\mu\nu} = 2\mathrm{Tr}[F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{1}{4}g^{\mu\nu}F^{\rho\sigma}F_{\rho\sigma}]$$

$$F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu} - ig[A^{\mu}, A^{\nu}]$$

The energy-momentum tensor is symmetric, gauge invariant and obeys

$$\partial_{\mu}T^{\mu\nu} = 0$$

T^{00} - energy density

T^{0i} - energy flux, Poynting vector

T^{xx}, T^{yy}, T^{zz} - pressures

T^{ij} - momentum flux

Averaging over collisions

$$T^{\mu\nu} \sim \sum \partial^i \partial^j \beta^k \beta^l \ldots \beta^m \quad \Rightarrow \quad \left\langle T^{\mu\nu} \right\rangle \sim \sum \partial^i \partial^j \left\langle \beta^k \beta^l \ldots \beta^m \right\rangle$$

The pre-collision potentials in covariant gauge $\partial_\mu \beta^\mu = 0$ obey

$$-\nabla^2 \beta^+(\mathbf{x}_\perp) = \rho(\mathbf{x}_\perp) \quad \Rightarrow \quad \beta^+(\mathbf{x}_\perp) = \frac{1}{2\pi} \int d^2 x'_\perp K_0(m\,|\,\mathbf{x}_\perp - \mathbf{x}'_\perp\,|)\rho(\mathbf{x}'_\perp)$$

IR regulator $m = \Lambda_{\text{QCD}}$

The potentials are transformed from the covariant to light-cone gauge $\beta_1^+ = \beta_2^- = 0$

Wick theorem

$$\left\langle \rho_a^k(\mathbf{x}_\perp)\rho_b^l(\mathbf{y}_\perp)\ldots\rho_c^m(\mathbf{z}_\perp)\right\rangle = \sum \prod \left\langle \rho_a^i(\mathbf{x}_\perp)\rho_b^j(\mathbf{y}_\perp)\right\rangle$$

Glasma graph approximation

$$\left\langle \beta_a^k(\mathbf{x}_\perp)\beta_b^l(\mathbf{y}_\perp)\ldots\beta_c^m(\mathbf{z}_\perp)\right\rangle = \sum \prod \left\langle \beta_a^i(\mathbf{x}_\perp)\beta_b^j(\mathbf{y}_\perp)\right\rangle = \sum \prod B_{ab}^{ij}(\mathbf{x}_\perp, \mathbf{y}_\perp)$$

Basic correlator

$$B^{ij}_{ab}(\mathbf{x}_\perp, \mathbf{y}_\perp) \equiv \left\langle \beta^i_a(\mathbf{x}_\perp)\beta^j_b(\mathbf{y}_\perp) \right\rangle = \int d^2x'_\perp \, d^2y'_\perp \cdots\cdots \left\langle \rho^i_a(\mathbf{x}'_\perp)\rho^j_b(\mathbf{y}'_\perp) \right\rangle$$

$$\left\langle \rho^i_a(\mathbf{x}_\perp)\rho^j_b(\mathbf{y}_\perp) \right\rangle = g^2 \mu(\mathbf{x}_\perp)\delta^{ab}\delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

color charge surface density

$$\mu = g^{-4} Q_s^2$$

Projected Woods-Saxon distribution

$$\mu(\mathbf{x}_\perp) = \frac{\bar{\mu}}{\ln(1+e^{R_A/a})} \int_{-\infty}^{\infty} \frac{dz}{1+\exp\left[\left(\sqrt{\mathbf{x}_\perp{}^2 + z^2} - R_A\right)/a\right]}$$

System uniform in the transverse plane

$$B^{ij}_{ab}(\mathbf{x}_\perp, \mathbf{y}_\perp) = \delta^{ab} f^{ij}(\mathbf{x}_\perp - \mathbf{y}_\perp) = \delta^{ab} f^{ij}(\mathbf{r})$$

$$\begin{cases} \mathbf{R} = \frac{1}{2}(\mathbf{x}_\perp + \mathbf{y}_\perp) \\ \mathbf{r} = \mathbf{x}_\perp - \mathbf{y}_\perp \end{cases}$$

System weakly nonuniform in the transverse plane

$$B^{ij}_{ab}(\mathbf{x}_\perp, \mathbf{y}_\perp) = \delta^{ab} f^{ij}(\mathbf{R}, \mathbf{r}) \approx \text{``}\textit{gradient expansion in } \mathbf{R}\text{''}$$

G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Numerical results

Pb-Pb collisions at LHC

$N_c = 3$

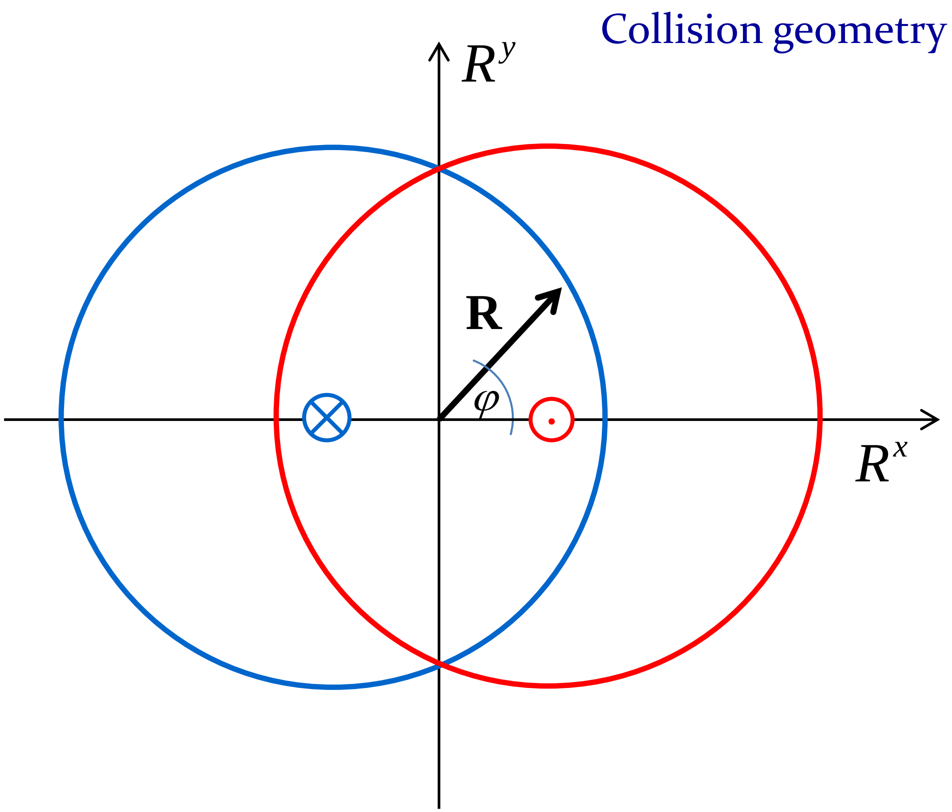
$g = 1$

$Q_s = 2\ \text{GeV}$

$m = 0.2\ \ \text{GeV}$

$R_A = 7.4\ \text{fm}$

$a = 0.5\ \text{fm}$

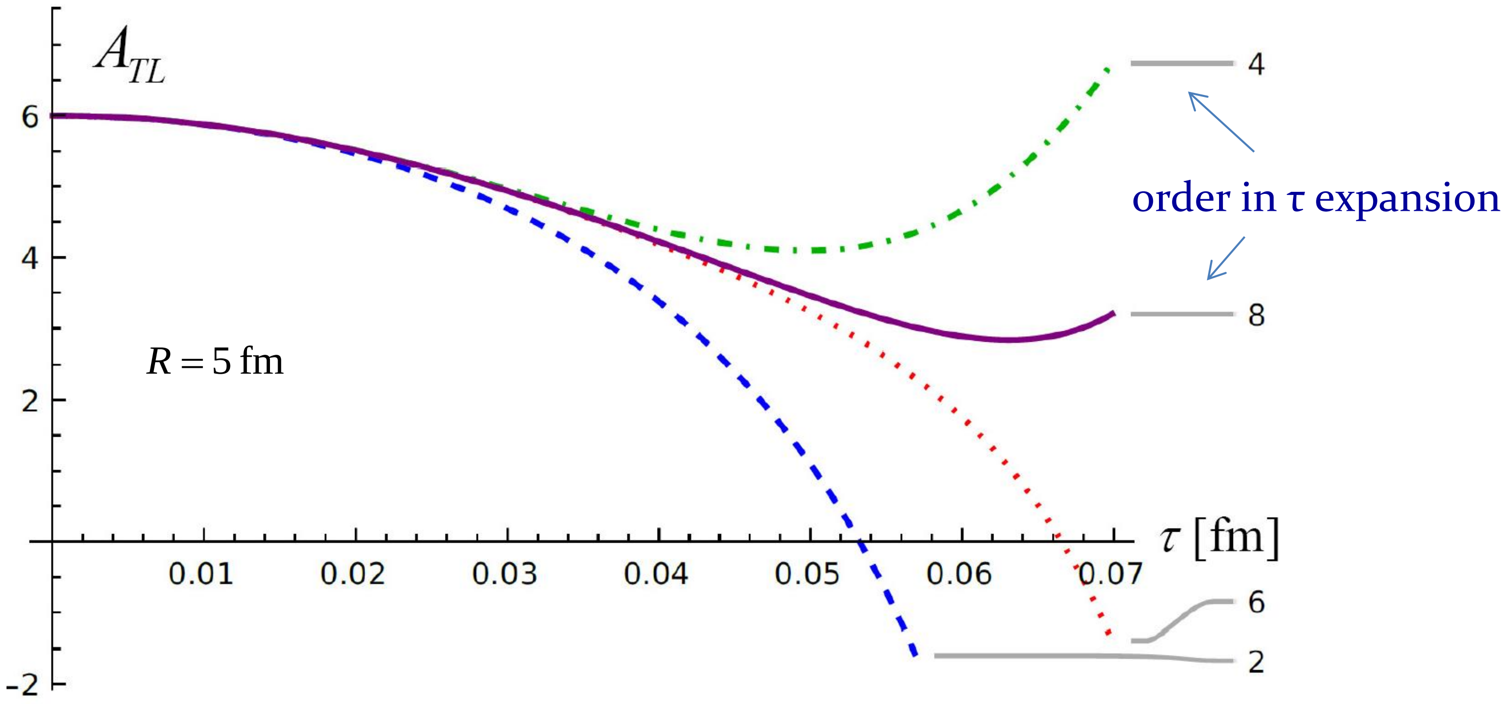


Anisotropy

Central Pb-Pb collsions

$$A_{TL} \equiv \frac{3(p_T - p_L)}{2p_T + p_L} \qquad p_T \equiv \langle T^{xx} \rangle, \quad p_L \equiv \langle T^{zz} \rangle$$

$$\tau = 0 \quad \Rightarrow \quad p_T = -p_L = \varepsilon \quad \Rightarrow \quad A_{TL} = 6$$

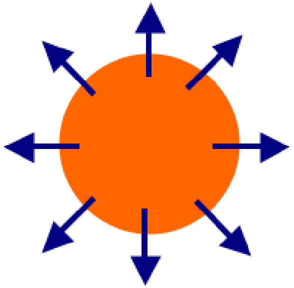


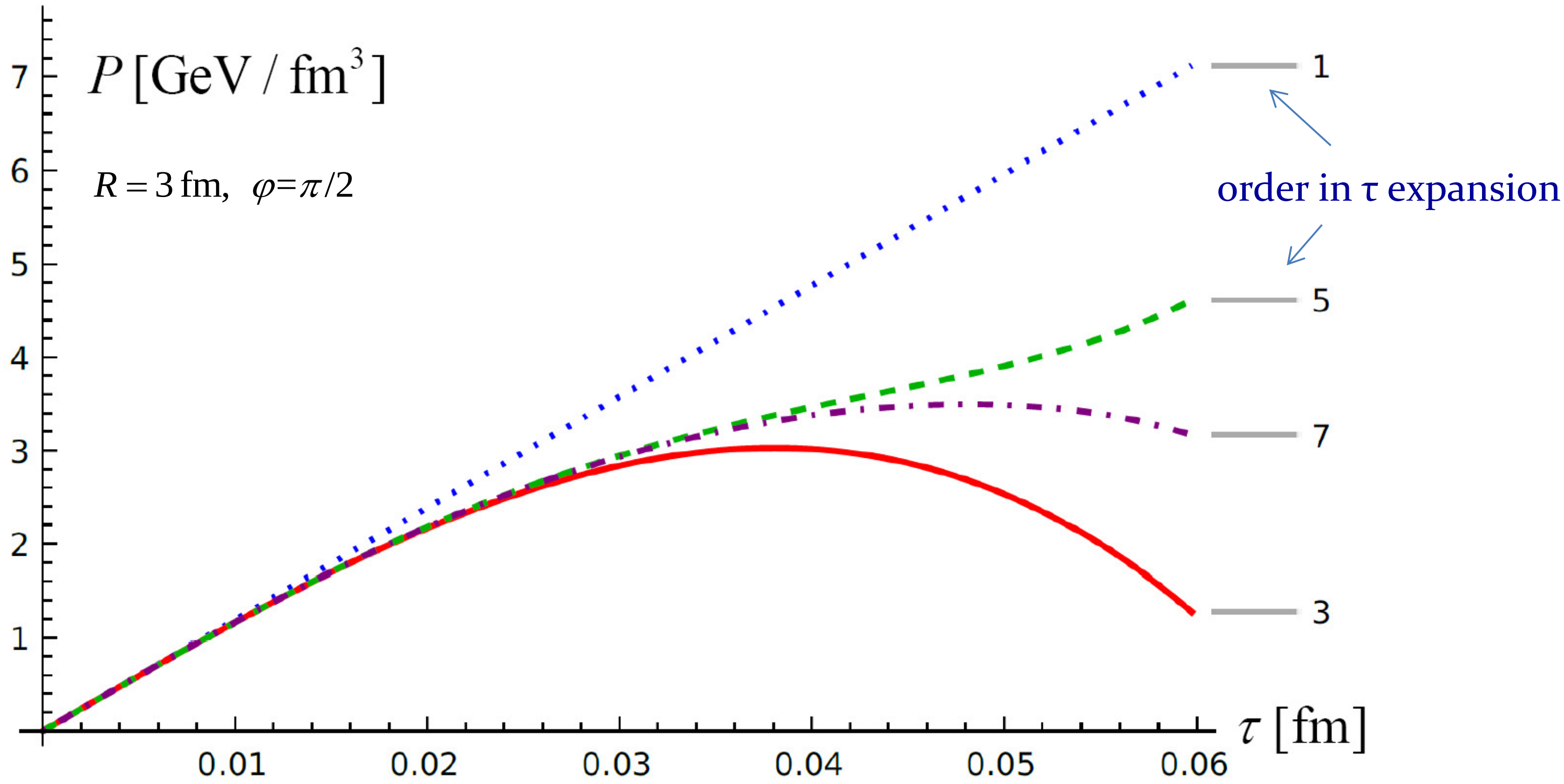
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Radial flow

Pb-Pb collisions at b = 6 fm

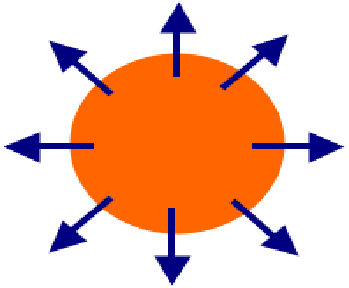
$$P \equiv R^i T^{0i}$$





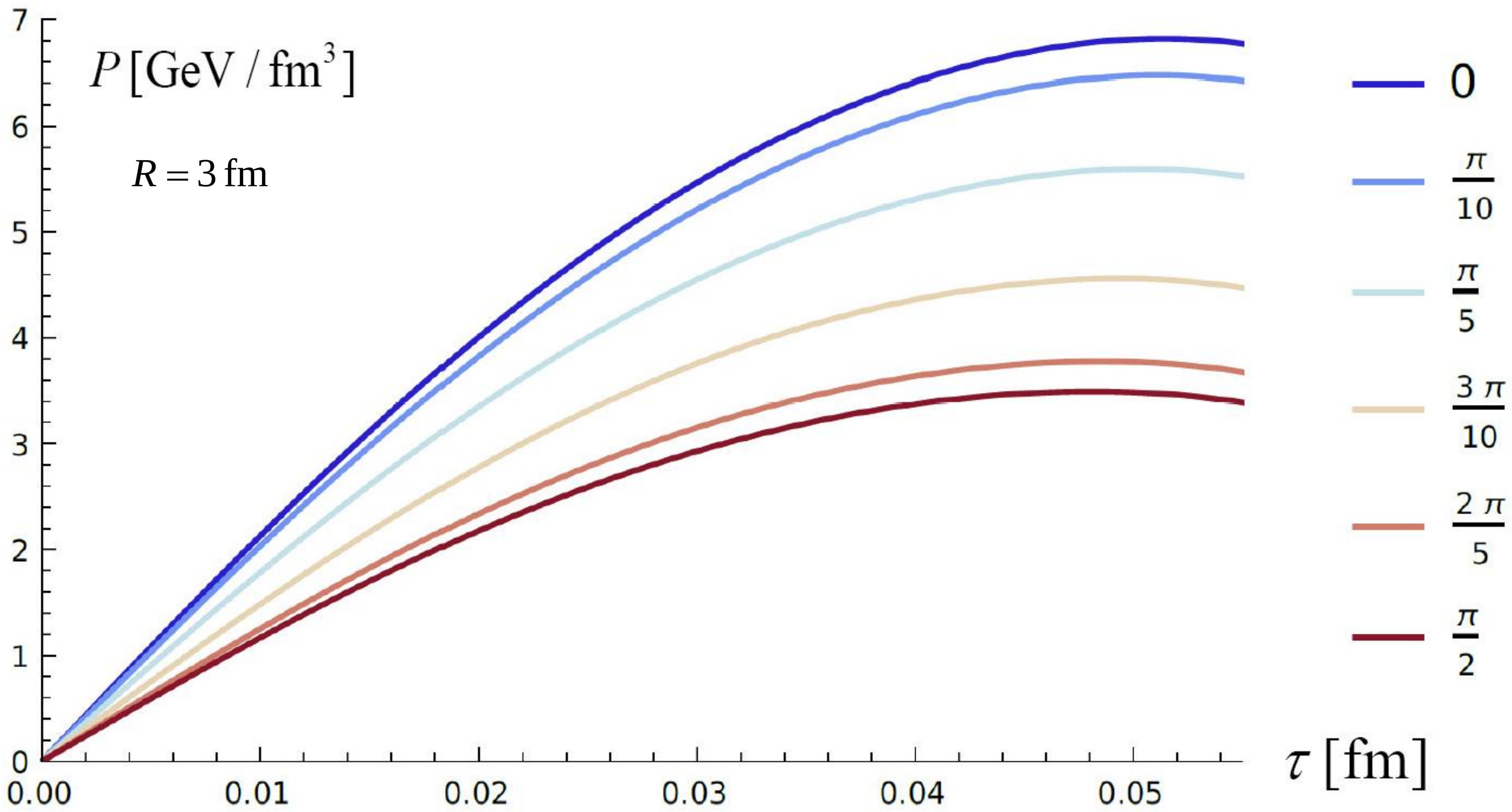
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Radial flow cont.



Pb-Pb collisions at b = 6 fm

$$P \equiv R^i T^{0i}$$

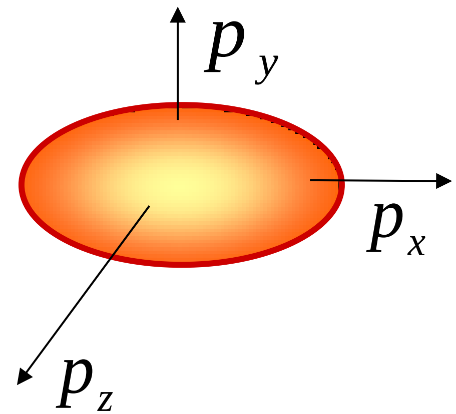


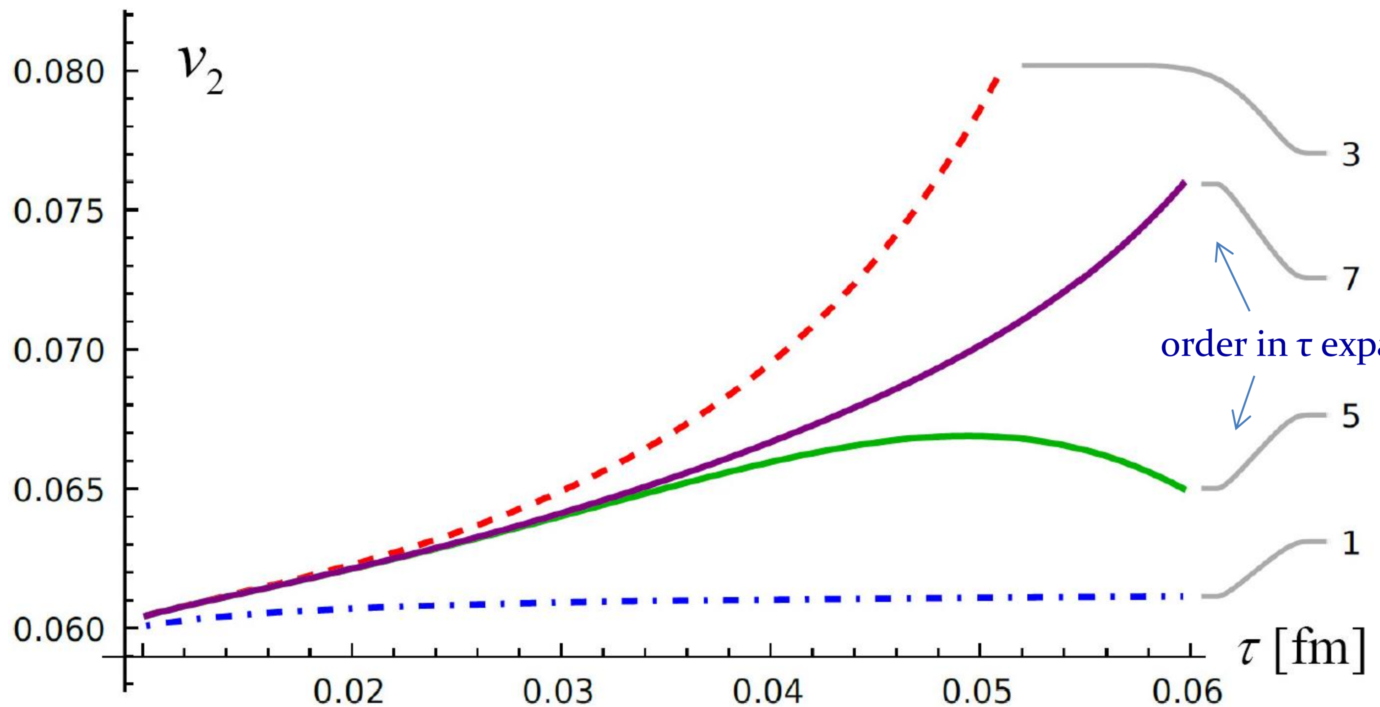
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Elliptic flow

Pb-Pb collisions at b = 2 fm

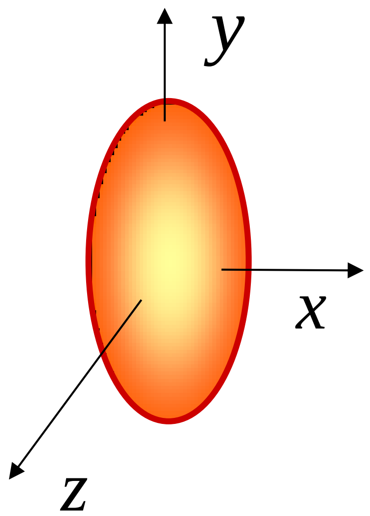
$$v_2 = \frac{\int d^2R \frac{T_{0x}^2 - T_{0y}^2}{\sqrt{T_{0x}^2 + T_{0y}^2}}}{\int d^2R \sqrt{T_{0x}^2 + T_{0y}^2}}$$





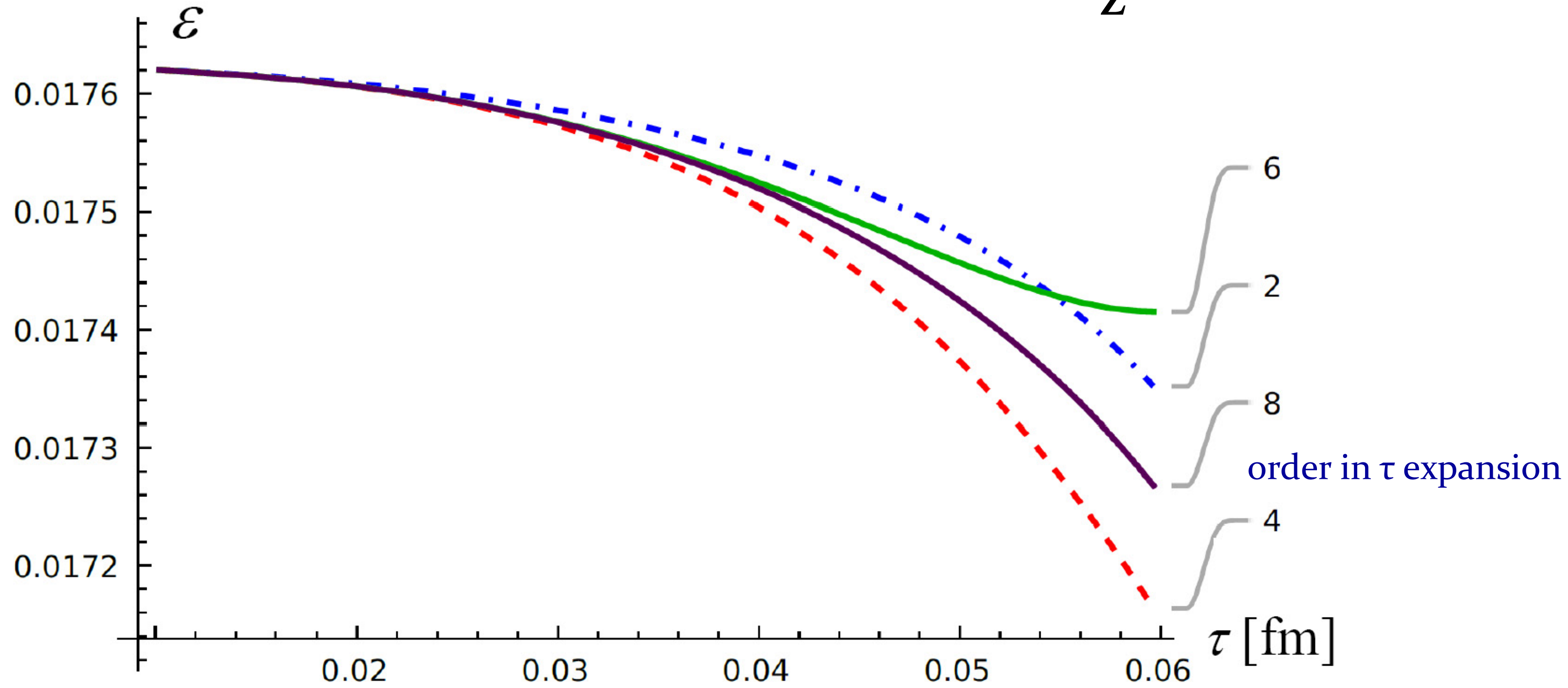
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Eccentricity



Pb-Pb collisions at b = 2 fm

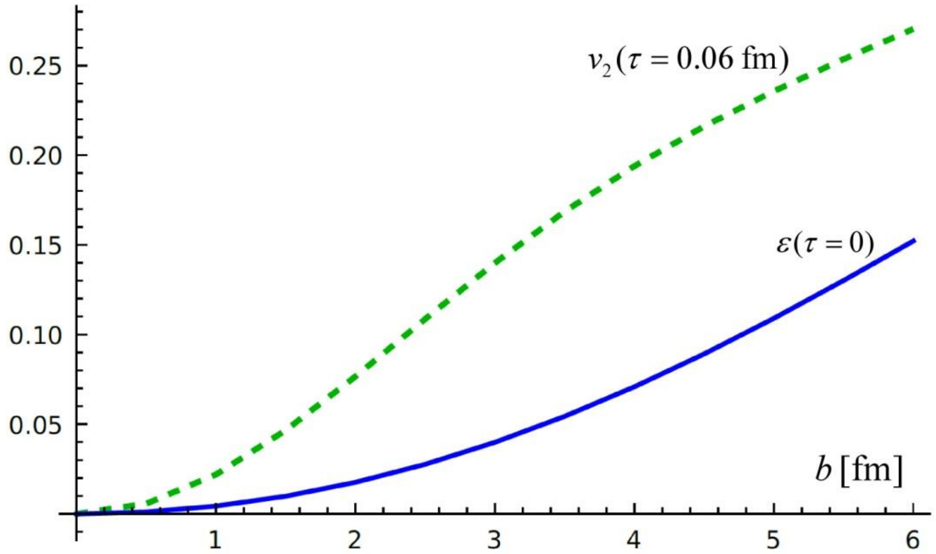
$$\varepsilon = \frac{\int d^2R \frac{R_x^2 - R_y^2}{\sqrt{R_x^2 + R_y^2}} T^{00}}{\int d^2R \sqrt{R_x^2 + R_y^2} T^{00}}$$

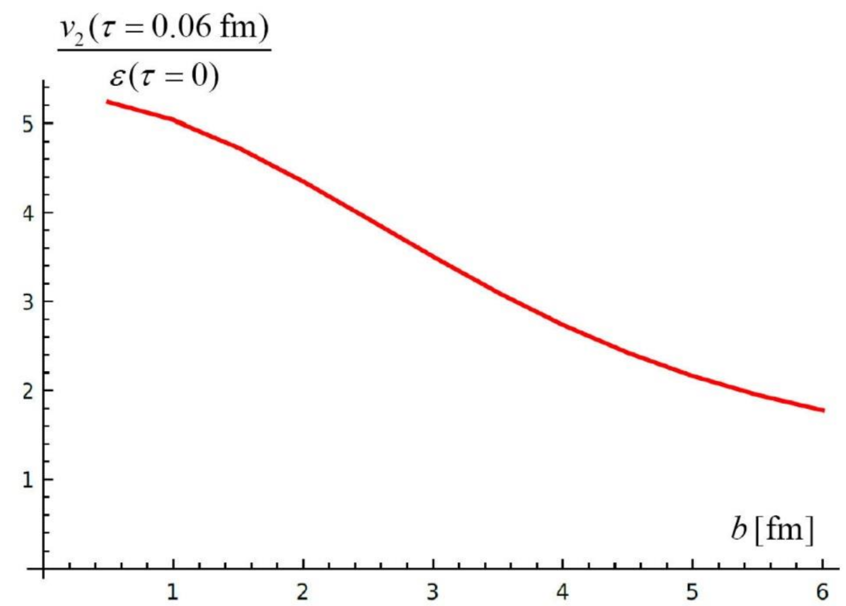


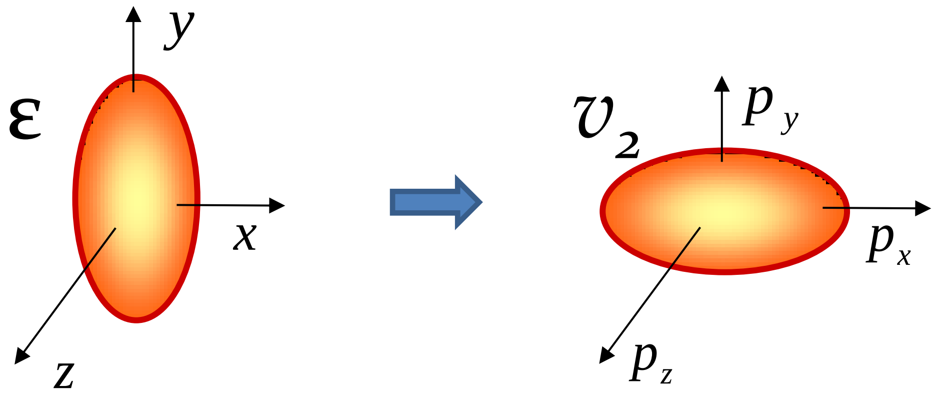
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Hydrodynamic-like behavior

Pb-Pb collisions







M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Universal flow

Continuity equation: $\partial_\mu T^{\mu\nu} = 0$

- short-time evolution
- $T^{\mu\nu}(\tau = 0) = \mathrm{diag}(\varepsilon, \varepsilon, \varepsilon, -\varepsilon)$
- boost invariance

$$T^{tx} \approx -\frac{1}{2}\tau \frac{\partial T^{tt}}{\partial x}$$

Universal flow of glasma

$$T^{\mu\nu} = \sum_{n=0}^{\infty} \tau^n T_n^{\mu\nu}$$

$$T_{n+1}^{tx} = -\frac{1}{2}\tau \frac{\partial T_n^{tt}}{\partial x}$$

$$n = 1, 2, \ldots 7$$

J. Vredevoogd and S. Pratt,
Phys. Rev. C **79**, 044915 (2009)

M. Carrington, St. Mrówczyński and
J.-Y. Ollitrault, Phys. Rev. C 110, 054903 (2024)

Hydrodynamic-like behavior

Mapping of glasma $T^{\mu\nu}_{\rm glasma}(\tau,\mathbf{x}_T)$ on hydrodynamic $T^{\mu\nu}_{\rm hydro}(\tau,\mathbf{x}_T)$

Eigenvalue problem: $$T^{\mu\nu}_{\rm glasma} w_\nu = \lambda w^\mu$$

Ideal hydrodynamics

$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

$$T^\mu{}_\mu = 0 \quad \Rightarrow \quad p = \frac{1}{3}\varepsilon$$

$$T^{\mu\nu} u_\nu = \varepsilon u^\mu$$

Anisotropic hydrodynamics

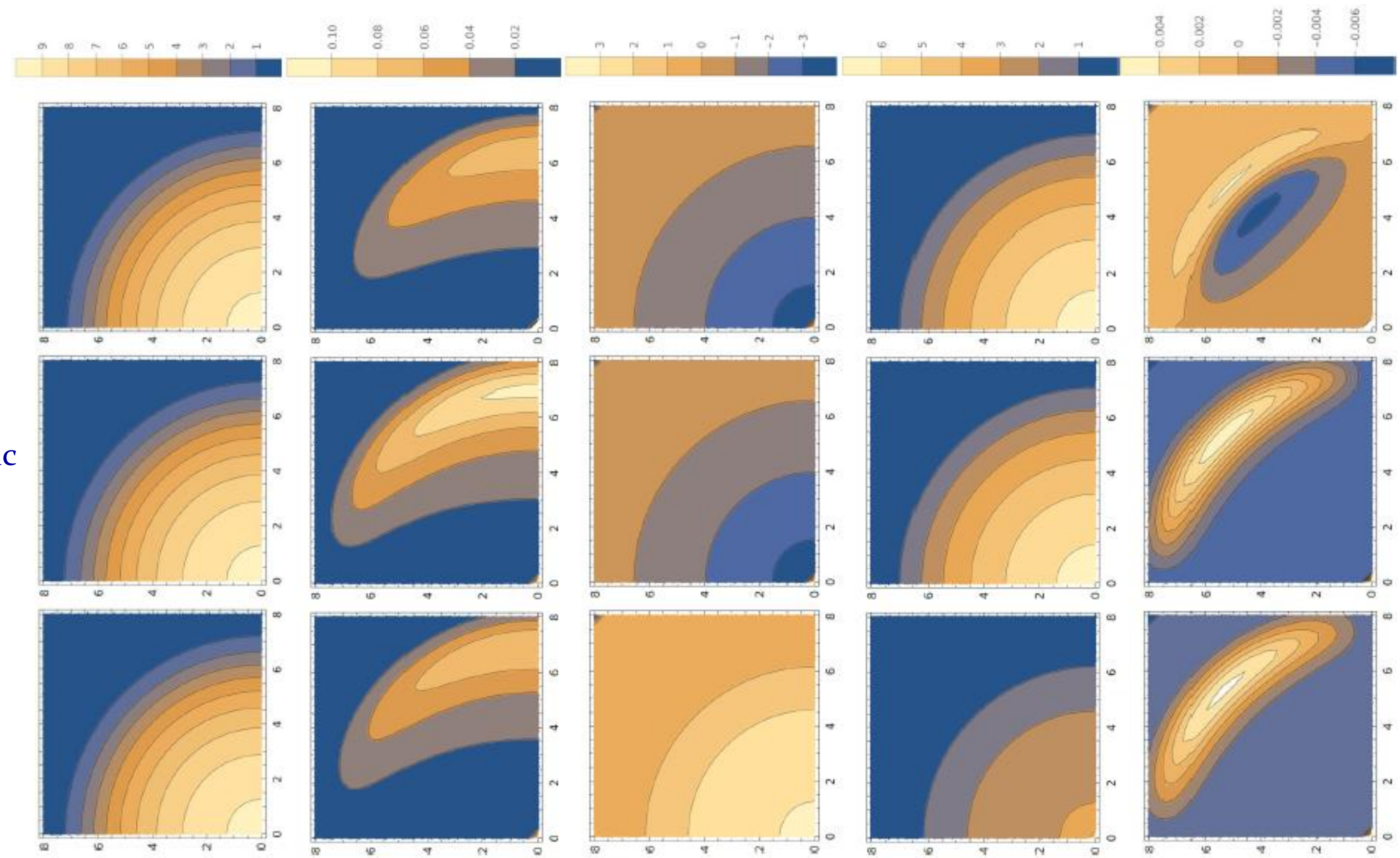
$$T^{\mu\nu} = (\varepsilon + p_T) u^\mu u^\nu - p_T g^{\mu\nu} - (p_T - p_L) z^\mu z^\nu$$

$$T^\mu{}_\mu = 0 \quad \Rightarrow \quad p_L = \varepsilon - 2 p_T$$

$$T^{\mu\nu} u_\nu = \varepsilon u^\mu, \qquad T^{\mu\nu} z_\nu = -p_L z^\mu$$

W. Florkowski & R. Ryblewski, Phys. Rev. C **83**, 034907 (2011)
M. Martinez & M. Strickland, Nucl. Phys. A **848**, 183 (2010)

Hydrodynamic-like behavior



glasma

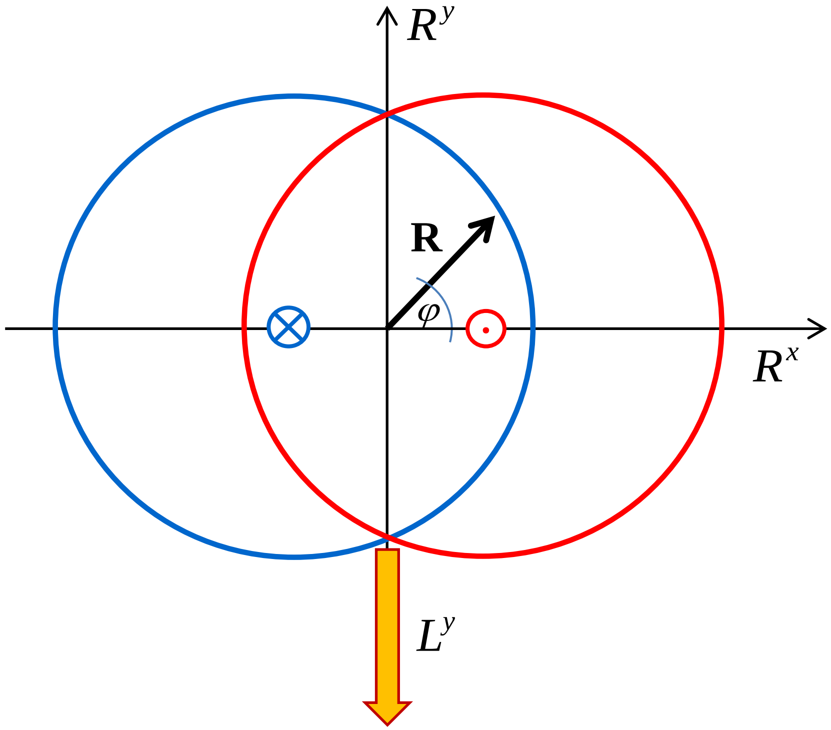
anisotropic hydro

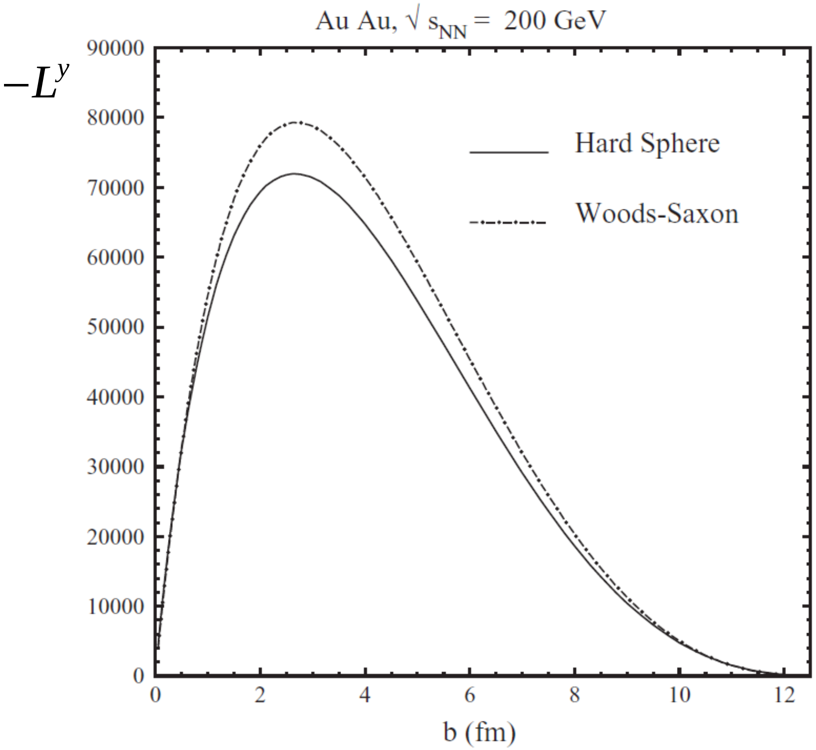
ideal hydro

τ = 0.06 fm $T^{tt} = \varepsilon$ T^{tx} T^{zz} T^{xx} T^{xy}

M. Carrington, St. Mrówczyński & J.-Y. Ollitrault, Phys. Rev. C 110, 054903 (2024)

Angular momentum





$$L^y \sim 10^7 \hbar \text{ @ LHC } ?$$

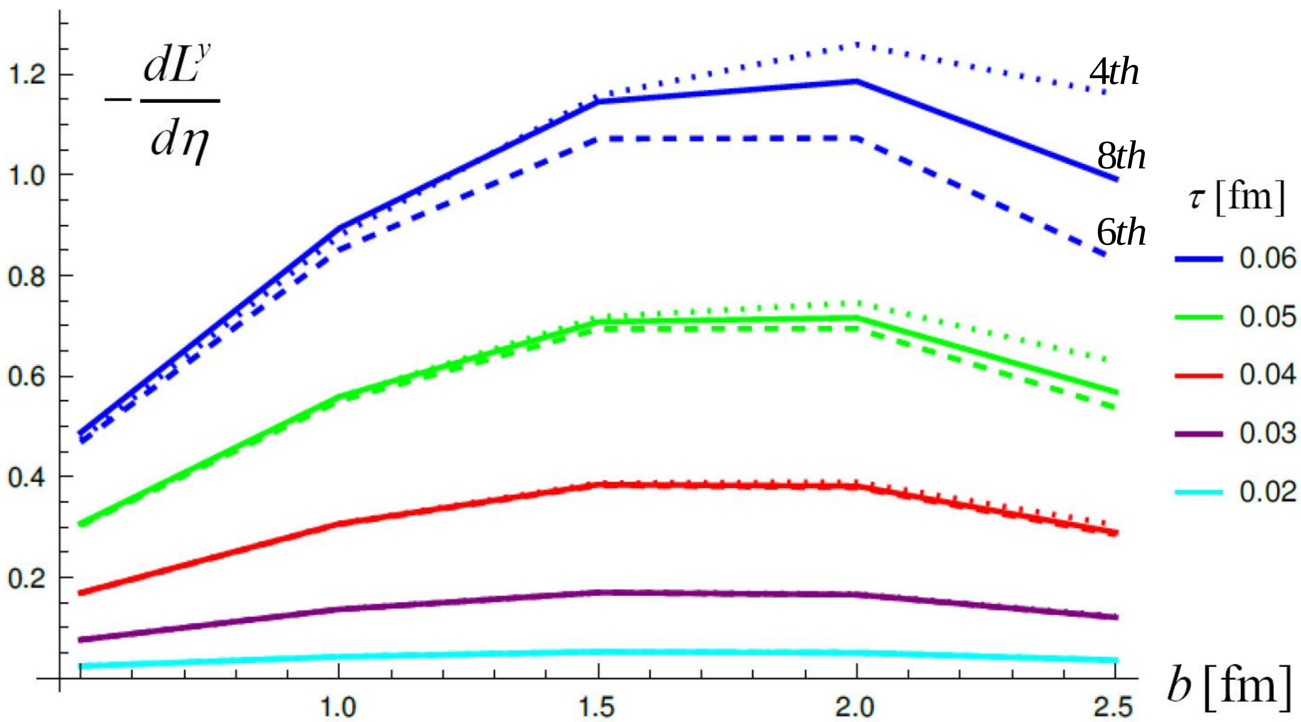
F. Becattini, F.Piccini, J. Rizzo, Physical Review C **77**, 024906 (2008)

Angular momentum cont.

Pb-Pb collisions at b = 2 fm

$$\frac{dL^\eta}{d\eta} = -\tau^2 \int d^2R\, R^x\, T^{\tau\eta}$$

Milne coordinates

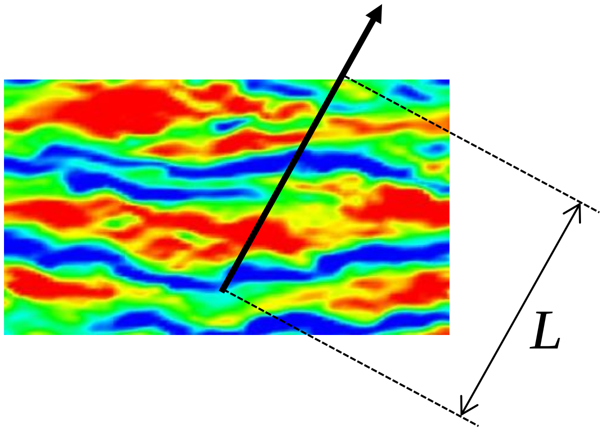


Glasma does not rotate!

M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Jet quenching in glasma

How hard probes propagate through the glasma?



$$\frac{dE}{dx},\ \hat{q}\ ?$$

$\dfrac{dE}{dx}$ - collisional energy loss

$\hat{q}$ - transverse momentum broadening

$\dfrac{dE^{\mathrm{rad}}}{dx} = -\dfrac{1}{8}\alpha_s N_c \hat{q} L$ - radiative energy loss

Fokker-Planck equation

- Transport of hard probes can be described using the Fokker-Planck equation.

drift / collisions

$$\underbrace{\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)}_{\text{drift}}n(t,\mathbf{r},\mathbf{p})=\underbrace{\left(\nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j+\nabla_p^i Y^i(\mathbf{v})\right)}_{\text{collisions}}n(t,\mathbf{r},\mathbf{p})$$

$n(t,\mathbf{r},\mathbf{p})$ - distribution function of hard probes

$$\mathbf{v}\equiv\frac{\mathbf{p}}{E_{\mathbf{p}}},\quad \nabla_p^i\equiv\frac{\partial}{\partial p_i}$$

$$X^{ij}(\mathbf{v}),\ \ Y^i(\mathbf{v})\quad\Rightarrow\quad\begin{cases}\dfrac{dE}{dx}=-\dfrac{\mathrm{v}^i}{\mathrm{v}}Y^i(\mathbf{v}) & \text{collisional energy loss}\\[2ex] \hat{q}=\dfrac{2}{\mathrm{v}}\left(\delta^{ij}-\dfrac{\mathrm{v}^i\mathrm{v}^j}{\mathrm{v}^2}\right)X^{ji}(\mathbf{v}) & \text{momentum broadening}\end{cases}$$

$$n(t,\mathbf{r},\mathbf{p})=n_{\mathrm{eq}}(\mathbf{p})\sim e^{-\frac{E_{\mathbf{p}}}{T}}\quad\Leftrightarrow\quad Y^j(\mathbf{v})=\frac{\mathrm{v}^i}{T}X^{ij}(\mathbf{v})$$

solves FK equation

Fokker-Planck equation of a hard probe in glasma

- Lorentz force $\mathbf{F} \equiv g\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$

- $$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right) n(t,\mathbf{r},\mathbf{p}) = \left(\nabla_p^i X^{ij}(\mathbf{v}) \nabla_p^j + \nabla_p^i Y^i(\mathbf{v})\right) n(t,\mathbf{r},\mathbf{p})$$

- $$X^{ij}(\mathbf{v}) = \frac{1}{N_c} \int_0^t dt' \left\langle F^i(t,\mathbf{r}) F^j(t',\mathbf{r} - \mathbf{v}(t-t'))\right\rangle, \qquad Y^j(\mathbf{v}) = \frac{\mathrm{v}^i}{T} X^{ij}(\mathbf{v})$$

- The collision term is given by field correlators $\left\langle E^i E^j \right\rangle, \left\langle B^i E^j \right\rangle, \left\langle B^i B^j \right\rangle$

- Gauge covariance requires: $\left\langle E_a^i(t,\mathbf{r}) E_a^j(t',\mathbf{r}')\right\rangle \rightarrow \left\langle E_a^i(t,\mathbf{r})\, \Omega_{ab}(t,\mathbf{r}\,|\,t',\mathbf{r}')\, E_b^j(t',\mathbf{r}')\right\rangle$

$$\Omega(t,\mathbf{r}\,|\,t',\mathbf{r}') \equiv P\exp\left[ig \int_{(t',\mathbf{r}')}^{(t,\mathbf{r})} ds_\mu A^\mu(s)\right]$$

St. Mrówczyński, European Physical Journal A **54**, 43 (2018)

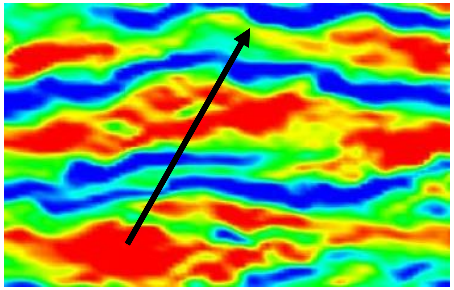
Transport of hard probes in glasma

$$X^{ij}(\mathbf{v}) = \frac{g}{N_c}\int_0^t dt' \Big\{ \big\langle E^i(t,\mathbf{r}) E^j(t',\mathbf{r}')) \big\rangle + \varepsilon^{jkl} \mathrm{v}^k \big\langle E^i(t,\mathbf{r}) B^l(t',\mathbf{r}')) \big\rangle$$

$$+ \varepsilon^{ikl} \mathrm{v}^k \big\langle B^l(t,\mathbf{r}) E^j(t',\mathbf{r}')) \big\rangle + \varepsilon^{ikl} \varepsilon^{jmn} \mathrm{v}^k \mathrm{v}^m \big\langle B^l(t,\mathbf{r}) B^n(t',\mathbf{r}')) \big\rangle \Big\}$$

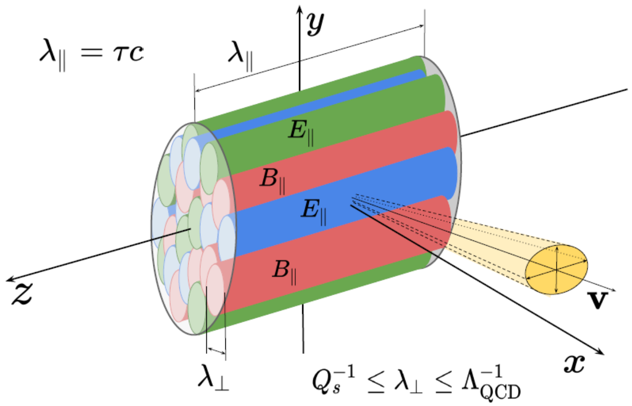
$$\mathbf{r}' \equiv \mathbf{r} - \mathbf{v}(t-t')$$

$$\begin{cases} \hat{q} = \dfrac{2}{\mathrm{v}} \left(\delta^{ij} - \dfrac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}^2} \right) X^{ji}(\mathbf{v}) \\[2ex] \dfrac{dE}{dx} = -\dfrac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}T} X^{ij}(\mathbf{v}) \end{cases}$$



St. Mrówczyński, European Physical Journal A **54**, 43 (2018)

Hard probes in glasma - $\hat{q}$



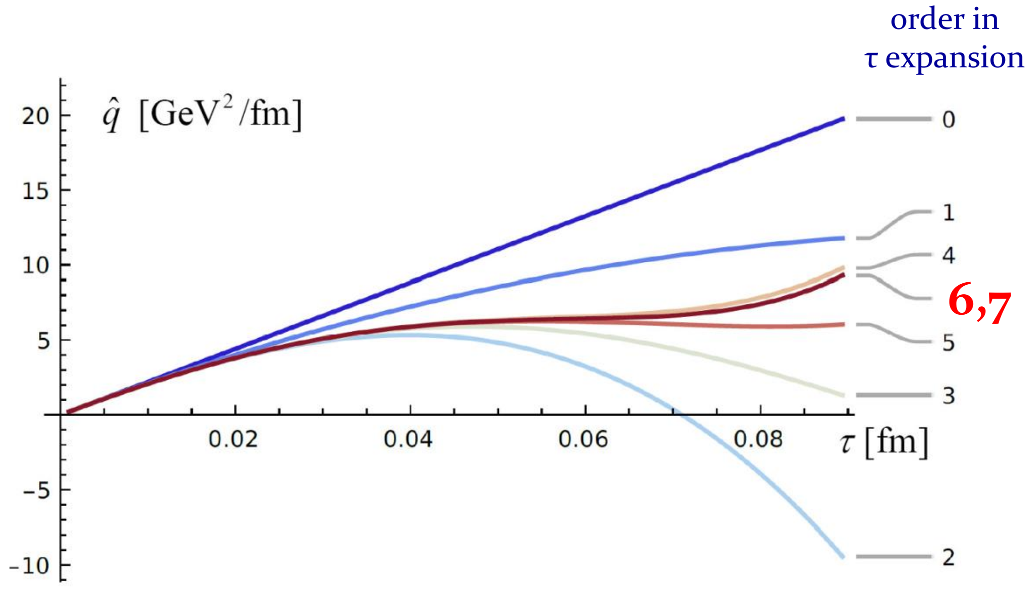
$$\hat{q} = \frac{2}{\mathrm{v}}\left(\delta^{ij} - \frac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}^2}\right) X^{ji}(\mathbf{v})$$

$N_c = 3, \quad g = 1$

$Q_s = 2\ \mathrm{GeV}$

$m = 0.2\ \mathrm{GeV}$

$\mathrm{v} = \mathrm{v}_{\perp} = 1$



M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Glasma impact on jet quenching

Glasma

$$\hat{q}_{\max} = 6 \ \text{GeV}^2 / \text{fm}$$

$$t_{\max} = 0.06 \ \text{fm}$$

Equilibrium QGP

$$\hat{q} = 3T^3$$

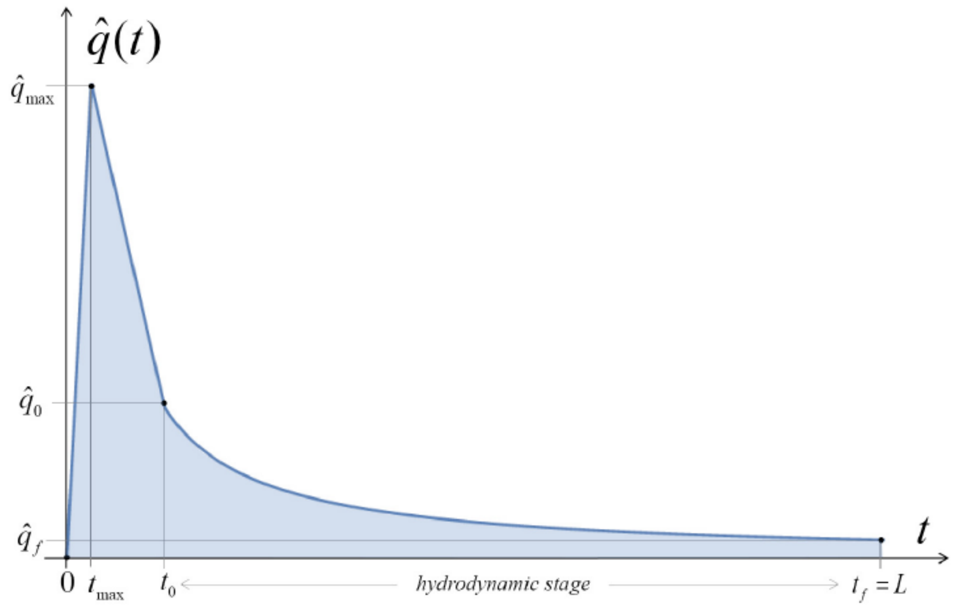
$$t_0 = 0.6 \text{ fm}$$

$$T_0 = 450 \text{ MeV}$$

$$\hat{q}_0 = 1.4 \ \text{GeV}^2 / \text{fm}$$

$$T = T_0 \left(\frac{t_0}{t} \right)^{1/3}$$

$$L = 10 \text{ fm}$$



$$\Delta p_T^2 \Big|^{\text{non-eq}} = \int_0^{t_0} dt \, \hat{q}(t)$$

$$\Delta p_T^2 \Big|^{\text{eq}} = \int_{t_0}^{L} dt \, \hat{q}(t)$$

$$\frac{\Delta p_T^2 \Big|^{\text{non-eq}}}{\Delta p_T^2 \Big|^{\text{eq}}} = 0.93$$

M. Carrington, A. Czajka & St. Mrówczyński, Physics Letters B **834**, 137464 (2022)

M. Carrington, A. Czajka & St. Mrówczyński, Physical Review C **105**, 064910 (2022)

Conclusions

- The glasma evolves in a hydrodynamic-like way.
- The glasma's orbital momentum is small,
  the system does not rotate.
- Momentum broadening and energy loss in the glasma
  are significantly bigger than in equilibrated QGP.
- In spite of its short lifetime the glasma provides
  a significant contribution to the jet quenching.