# Guiding center hydrodynamics with spin

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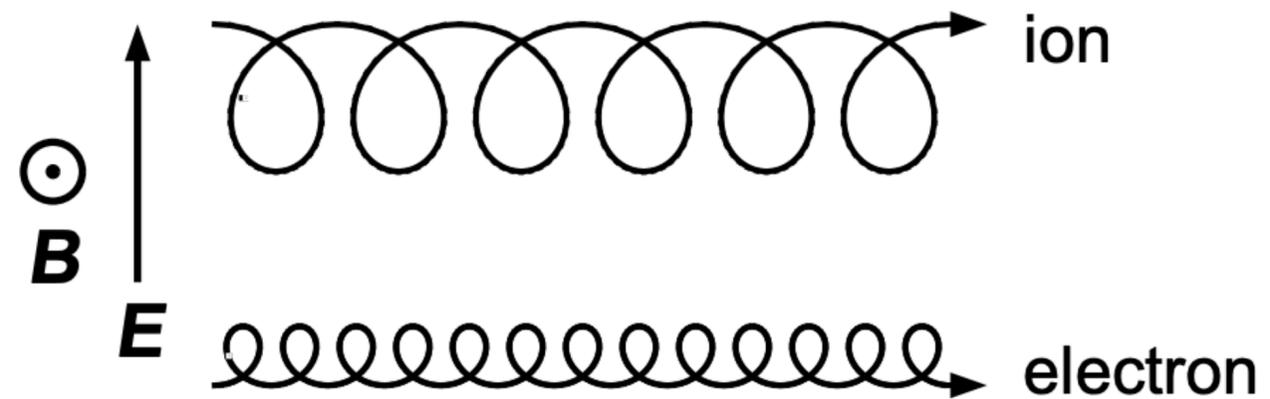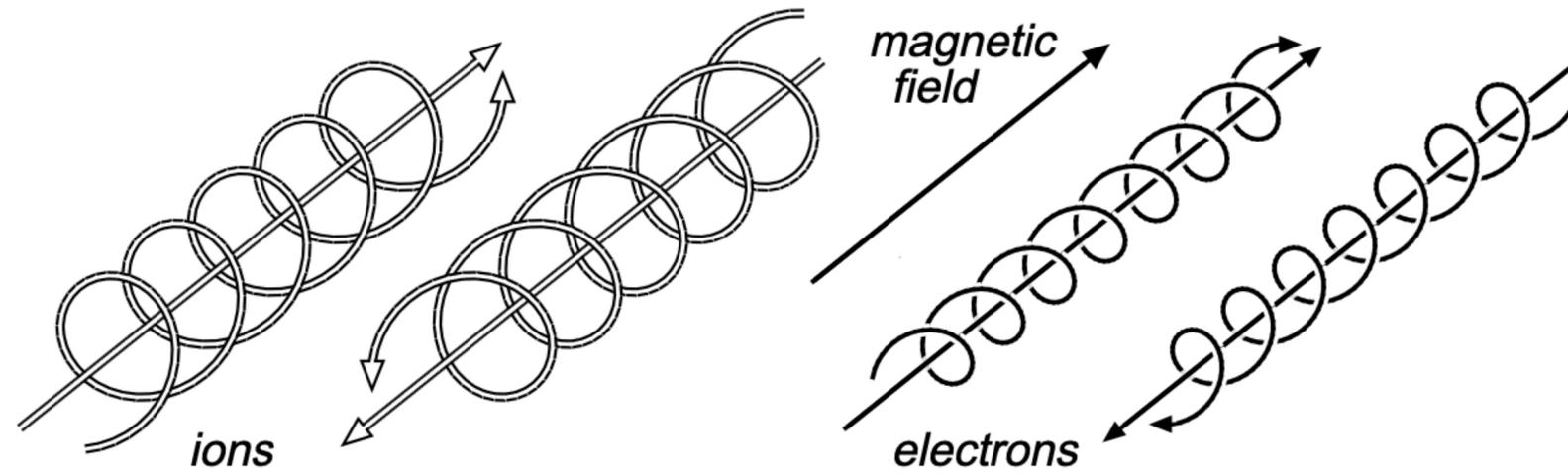2-4 Jul, 2025

# Outline

Guiding-center

Guiding-center hydrodynamics

D. T. Son and M. Stephanov, PRL 133 no. 14, (2024) 145201

Guiding-center hydrodynamics with spin

# Guiding-center



magnetic field

ions

electrons



$\odot$

$B$

$E$

ion

electron

3

# Assumptions

A plasma of particles immersed in a strong magnetic field can be envisioned as a collection of guiding centers. To effectively substitute the motion of particles with that of their guiding centers, the cyclotron radius should be smaller than the mean free path.

We are in a very large magnetic field and in a reference frame where $\mathbf{E} \parallel \mathbf{B}$.

We focus on a plasma composed of a single species of charged spin-half particles, with a stationary background charge density & static EM field (for simplicity)

# Guiding-center hydrodynamics

⟶ We use classical single-particle distribution function in a phase space extended to spin

"+" means particle contribution

"−" means antiparticle contribution

⟶ Identifying the collisional invariants of the local Boltzmann equation

$$f^{\pm}(x, p, s) = \exp\left(-\beta(x) \cdot p \pm \xi(x)\right) \left[1 + \frac{1}{2}\omega_{\mu\nu}(x)s^{\mu\nu}\right]$$

Spin potential is assumed small (for simplicity)

$$s^{\mu\nu} = (1/m)\,\epsilon^{\mu\nu\alpha\beta}p_{\alpha}s_{\beta}, \quad \xi(x) = \mu_B/T, \quad \beta_{\mu}(x) = U^{\mu}/T$$

$$N^\mu = \int dP\, dS\, \left(p^\mu + v^\mu\right)\left(f^+ - f^-\right) = N^\mu_\parallel + N^\mu_\perp$$

Net current

$$T^{\mu\nu} = \int dP\, dS\, \left(p^\mu p^\nu + \overline{p^\mu_\perp p^\nu_\perp}\right)\left(f^+ + f^-\right) = \tilde{T}^{\mu\nu} + T^{\mu\nu}_\perp$$

Energy-momentum tensor

$$S^{\lambda,\mu\nu} = \int dP\, dS\, p^\lambda\, s^{\mu\nu}\left(f^+ + f^-\right)$$

Spin tensor

# Guiding-center hydrodynamics

$$\rightarrow \quad N^\mu = \int dP\, dS\, \left(p^\mu + v^\mu\right)\left(f^+ - f^-\right) = N^\mu_\parallel + \boxed{N^\mu_\perp}$$

drift current

$$v^\mu = B_*^{-2}\left(p_\nu p^\lambda \partial_\lambda B^{\mu\nu} + \frac{\tilde{m}^2 - m^2}{2B^*}B^{\mu\nu}\partial_\nu B_*\right)$$

$$dP = \frac{d^4 p}{2\pi^2}\theta(p_0)\,\delta(p^2 - \tilde{m}^2)\,\delta^2(\Delta^\mu{}_\nu p^\nu)\left(B_* + \frac{p_\mu \partial_\nu B^{\mu\nu}}{B_*}\right)$$

takes care of the mass-shell condition

$p_0^2 = p_\parallel^2 + \tilde{m}^2$

$\tilde{m}^2 = \overline{p_\perp^2} + m^2$

forces the phase space to have only one momentum direction, **p ∥ B**

$\Delta^\mu{}_\nu = (1/B_*^2)B^{\mu\alpha}B_{\nu\alpha}$

$$\int dS = \frac{2\,m}{\sqrt{3}\,\pi}\int d^4 s\,\delta\left(s\cdot s + \frac{3}{4}\right)\delta(p\cdot s) = 2$$

projector onto the transverse plane (x,y)
perpendicular to **B** plane in the **E ∥ B** frame

normalization of s          orthogonality condition

$$N^\mu_\parallel = n\,u^\mu = 4\,\sinh(\xi)\,n_0\,u^\mu = 4\,\sinh(\xi)\,u^\mu\frac{B_*}{(2\pi)^2}\,\tilde{m}\,K_1(\tilde{z})$$

$\tilde{z} = \tilde{m}/T$

$$N^\mu_\perp = B_*^{-2}\left(\tilde{T}^\lambda{}_\nu \partial_\lambda B^{\mu\nu} - MB^{\mu\nu}\partial_\nu B_*\right)$$

$$\boxed{\partial_\mu N^\mu = 0}$$

$\tilde{\Delta}^\mu{}_\nu = \delta^\mu_\nu - \Delta^\mu{}_\nu$ projector onto the 1+1 **E ∥ B** space, i.e., (t,z) hyperplane

# Guiding-center hydrodynamics

$$T^{\mu\nu} = \int dP \, dS \left( p^\mu p^\nu + \overline{p_\perp^\mu p_\perp^\nu} \right) (f^+ + f^-) = \tilde{T}^{\mu\nu} + T_\perp^{\mu\nu}$$

$$\tilde{T}^{\mu\nu} = 4 \cosh(\xi) \int dP \, p^\mu p^\nu e^{-\beta \cdot p} = (\epsilon + P) u^\mu u^\nu - P \tilde{\Delta}^{\mu\nu}$$

$$\epsilon = 4 \cosh(\xi) \int dP \, (u \cdot p)^2 e^{-\beta \cdot p} = 4 \cosh(\xi) \frac{B_*}{(2\pi)^2} \tilde{m} \, T \left( \tilde{z} K_0(\tilde{z}) + K_1(\tilde{z}) \right)$$

$$T_\perp^{\mu\nu} = -2 \cosh(\xi) \int dP \, \Delta^{\mu\nu} p_\perp^2 \, e^{-\beta \cdot p} = \boxed{- P_\perp \Delta^{\mu\nu}}$$

$$P = 4 \cosh(\xi) \int dP \, \frac{1}{3} \left( (u \cdot p)^2 - p \cdot p \right) e^{-\beta \cdot p} = 4 \cosh(\xi) \frac{B_*}{(2\pi)^2} \tilde{m} \, T K_1(\tilde{z})$$

$$P_\perp = 2 \cosh(\xi) \frac{B_*}{(2\pi)^2} (\tilde{m}^2 - m^2) K_0(\tilde{z}) = - M B_*$$

spin potential
contribution
appears in higher-
order

$$\partial_\mu T^{\mu\nu} = F^{\nu\mu} J_\mu \implies \tilde{\Delta}^\sigma_{\ \nu} \partial_\mu T^\mu_{\ \sigma} = E_{\nu\lambda} N_\parallel^\lambda$$

$\tilde{\Delta}^\mu_{\ \nu} = \delta^\mu_\nu - \Delta^\mu_{\ \nu}$ projector onto the 1+1 $\mathbf{E} \parallel \mathbf{B}$ space, i.e., (t,z) hyperplane

$$\longrightarrow S^{\lambda,\mu\nu} = \int dP\, dS\, p^\lambda\, s^{\mu\nu}\left(f^+ + f^-\right)$$

$$= 2\cosh(\xi)\int dP\, p^\lambda e^{-\beta\cdot p}\int dS\, s^{\mu\nu}\left(1 + \frac{1}{2}\omega_{\alpha\beta}\, s^{\alpha\beta}\right)$$

$$= \frac{P}{4}\beta^\lambda\omega^{\mu\nu} + T^2\left((\epsilon + P)\frac{3\,T^2}{\tilde{m}^2} + \frac{P}{2}\right)\beta^\lambda\beta^\delta\beta^{[\mu}\omega^{\nu]}{}_\delta - \frac{T^2\,(\epsilon + P)}{2\tilde{m}^2}\left\{\tilde{\Delta}^{\lambda[\mu}\omega^{\nu]}{}_\delta\beta^\delta + \tilde{\Delta}^{\delta[\mu}\omega^{\nu]}{}_\delta\beta^\lambda + \tilde{\Delta}^{\lambda\delta}\beta^{[\mu}\omega^{\nu]}{}_\delta\right\}$$

$$\boxed{\partial_\lambda S^{\lambda,\mu\nu} = 0}$$

# Summary & Outlook

Incorporated spin degrees of freedom into the guiding center (ideal) hydrodynamics

In this restrictive scenario, number of EoMs are less than the usual hydro formalism

Easiest method to include EM field contribution in the hydrodynamics

Ongoing: include dissipation and dynamical EM using Maxwell equations

Ongoing: coupling between spin and background EoMs

*Thank you for listening!*

# Entropy current

$$\rightarrow \mathfrak{S}^\mu = -\int dP\, dS\, p^\mu \left( f^+ \log(f^+) + f^- \log(f^-) \right) + \int dP\, dS\, p^\mu \left( f^+ + f^- \right)$$

$$\partial_\mu \mathfrak{S}^\mu = T^{\mu\alpha} \partial_\mu \beta_\alpha + \beta_\alpha \left( \tilde{\Delta}^{\lambda\alpha} \partial_\mu \tilde{T}^\mu{}_\lambda - E^\alpha_\lambda N^\lambda_\parallel \right) - \frac{1}{2} S^{\mu,\alpha\beta} \partial_\mu \omega_{\alpha\beta} - N^\mu (\coth(\xi))^2 \partial_\mu \xi = 0$$

$$n = \frac{\partial P}{\partial \mu}, \qquad \epsilon = T\frac{\partial P}{\partial T} + \mu\frac{\partial P}{\partial \mu} - P, \qquad P_\perp = P - B_*\frac{\partial P}{\partial B_*}$$