Particle-theory input for neutron stars: Three applications of pQCD in NS physics

Aleksi Vuorinen

University of Helsinki & Helsinki Institute of Physics

XQCD 2025

Wrocław, 3 July 2025

Main references:

1) Cruz Rojas et al. (incl. AV), PRL 133 (2024), 2402.00621
2) Kärkkäinen, Navarrete, Nurmela, Paatelainen, Seppänen, AV, Accepted in PRL, 2501.17921
3) Kiuru, Nättilä, Paatelainen, AV, In preparation

Lecture notes and reviews:

1) Ghiglieri et al. (incl. AV), Phys. Rept. 880, 2002.10188
2) AV, Acta Phys. Polon. B 55 (2024), 2405.01141

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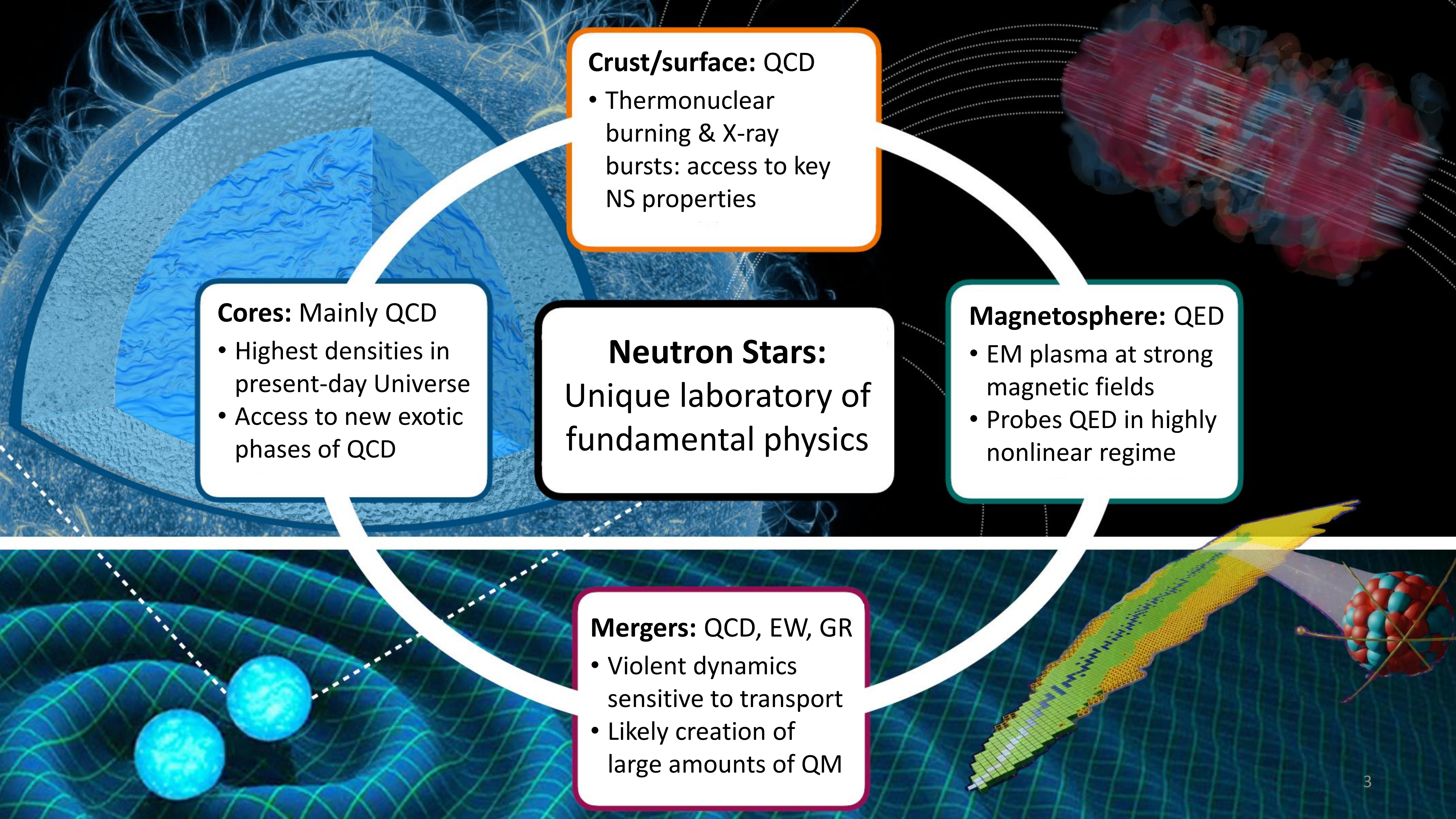

Crust/surface: QCD
• Thermonuclear burning & X-ray bursts: access to key NS properties
Cores: Mainly QCD
• Highest densities in present-day Universe
• Access to new exotic phases of QCD
Neutron Stars:
Unique laboratory of fundamental physics
Magnetosphere: QED
• EM plasma at strong magnetic fields
• Probes QED in highly nonlinear regime
Mergers: QCD, EW, GR
• Violent dynamics sensitive to transport
• Likely creation of large amounts of QM

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Unique laboratory of fundamental physics

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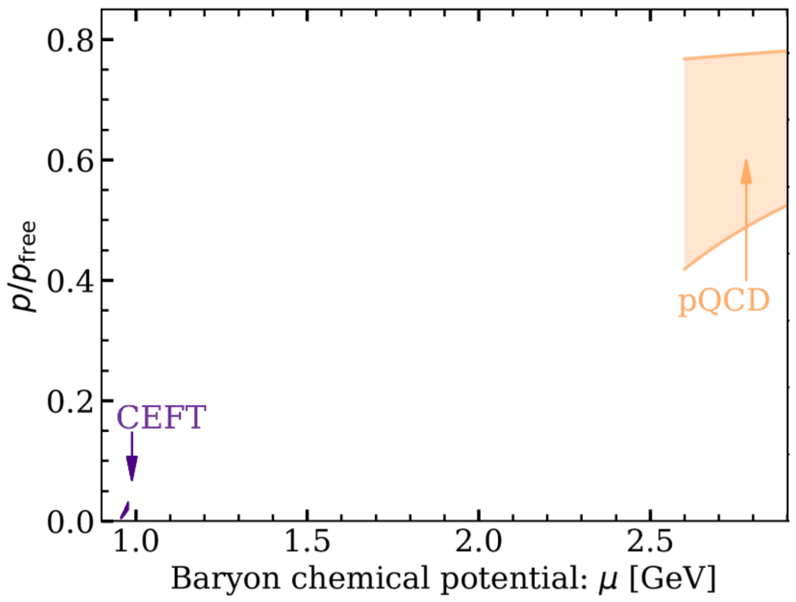
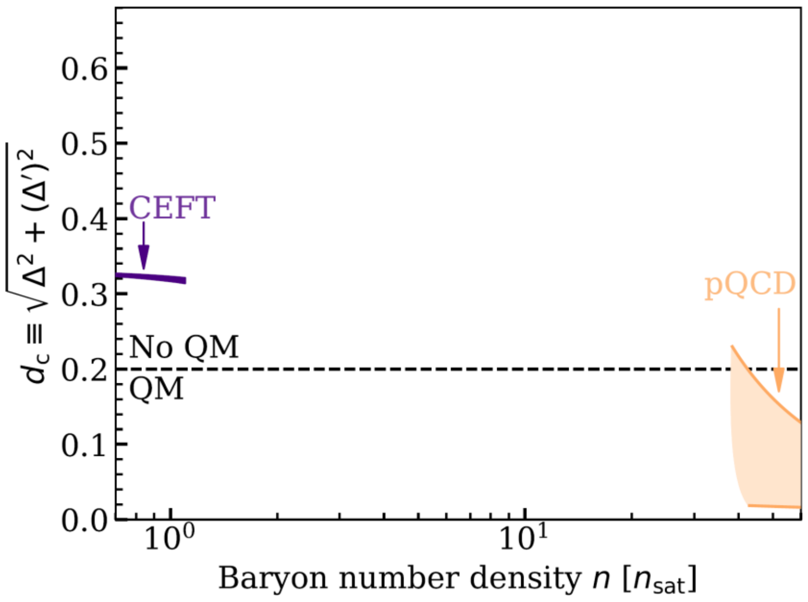
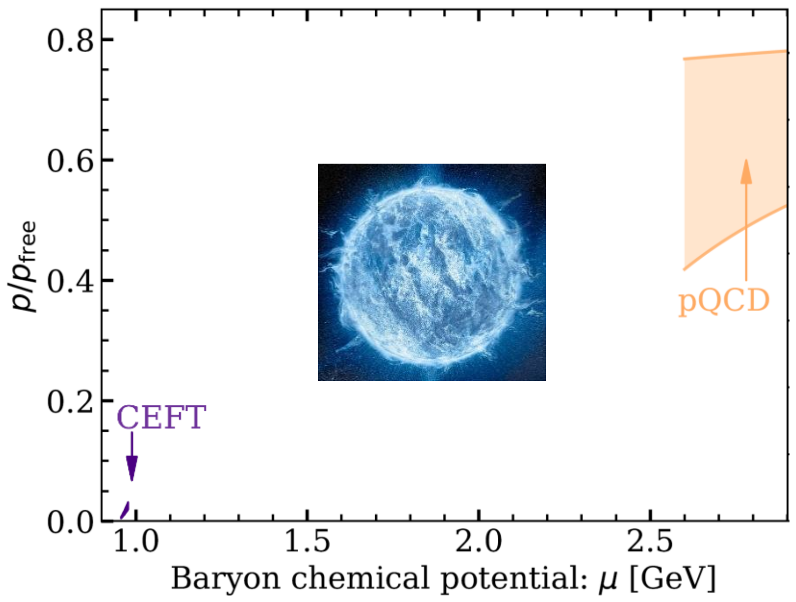
Topic 1: From QCD thermodynamics to composition of NS cores

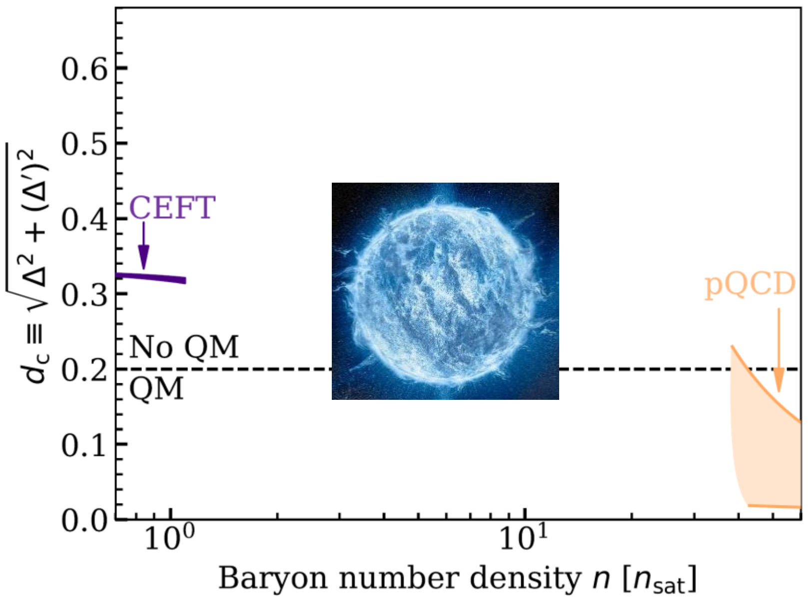




For the properties of NS matter – controlling stellar structure and composition – only low- and high-density limits under control via Chiral EFT & pQCD

Topic 1: From QCD thermodynamics to composition of NS cores





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Model-independent approach: Use observational data to constrain NS matter everywhere

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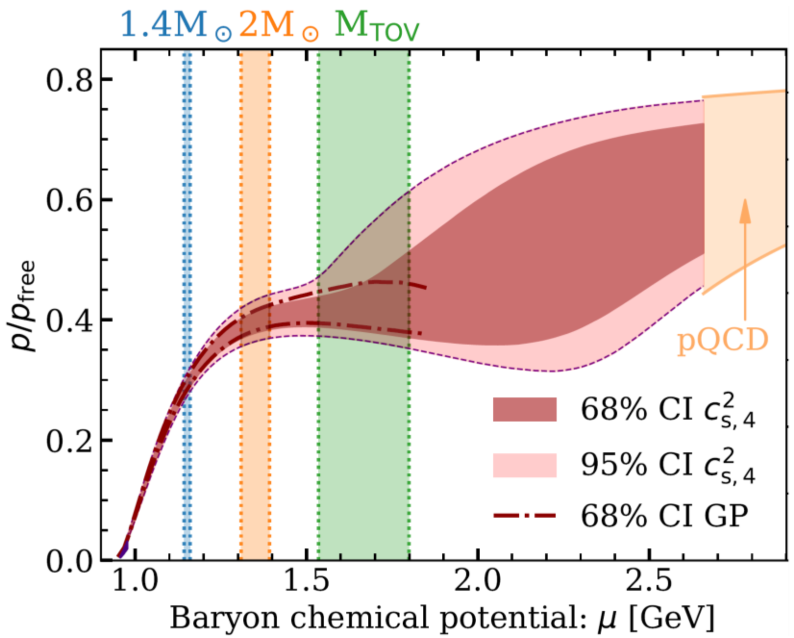
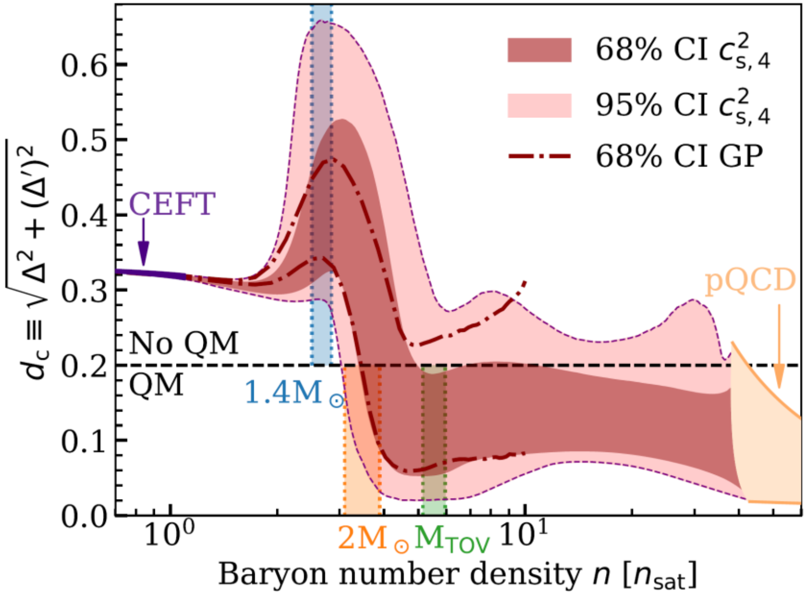


Annala et al. (incl. AV), Nature Comm. 14 (2023)



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- Evidence for quark-matter cores in massive NSs

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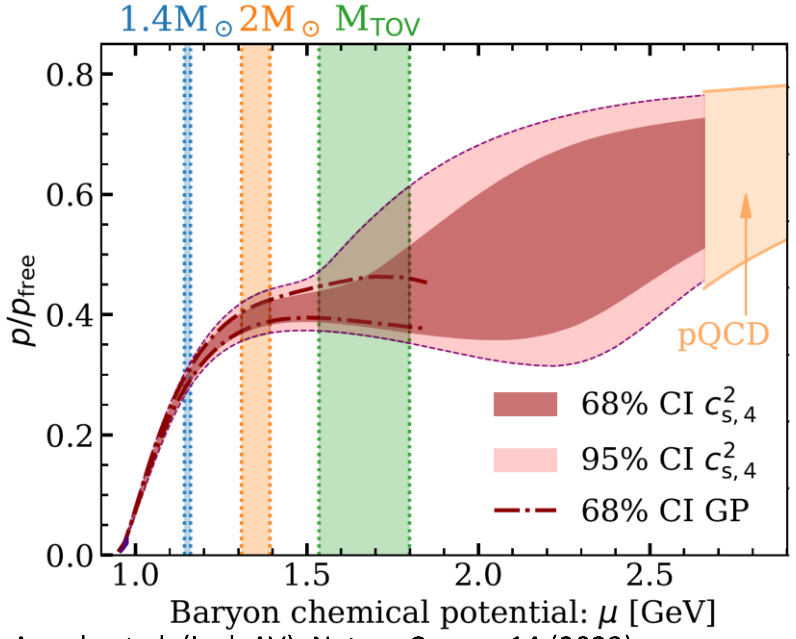

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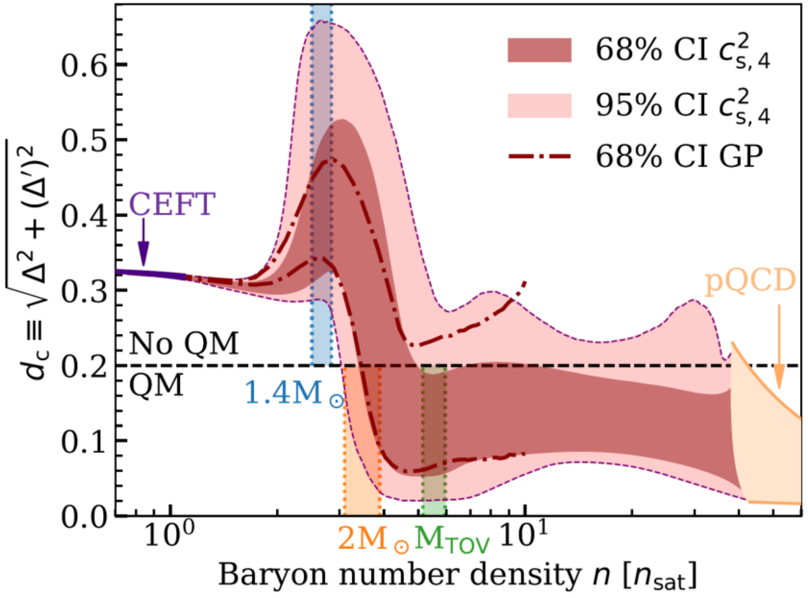

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Challenge: Is it possible to qualitatively improve the situation with theoretical calculations?

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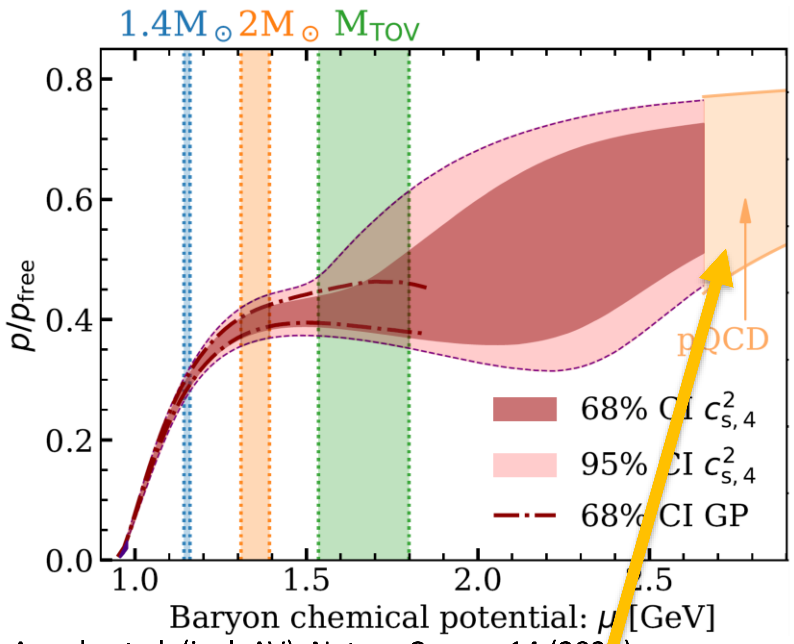

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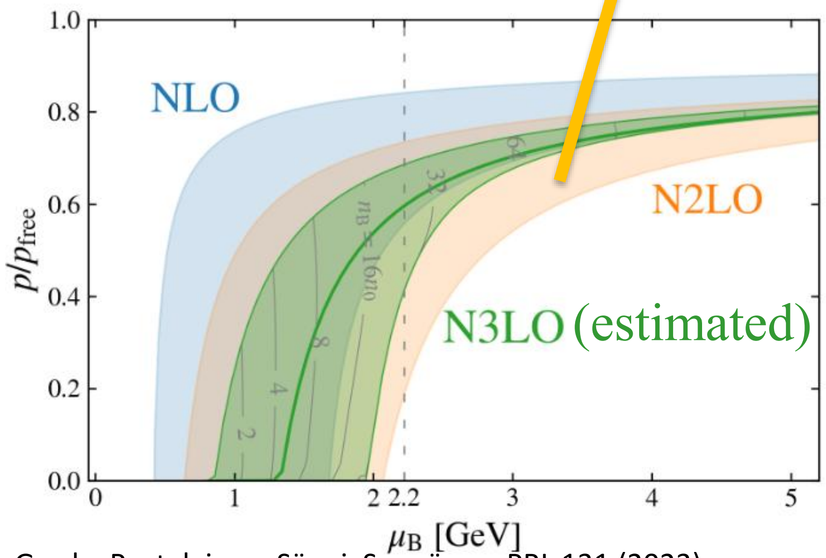

Gorda, Paatelainen, Säppi, Seppänen, PRL 131 (2023)

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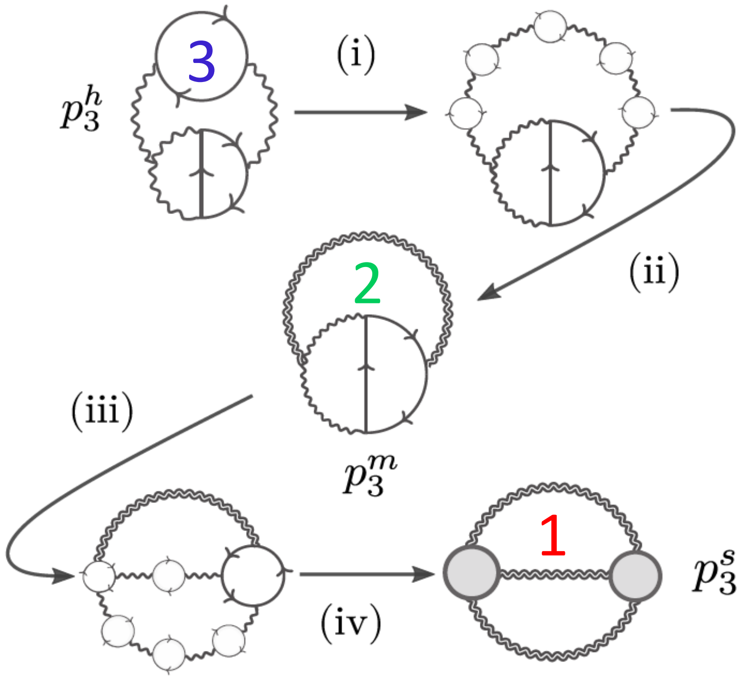
Opportunity: Completion of N3LO QM pressure has potential for qualitative leap (3 N's may suffice)

- Easier said than done: determination of missing contribution an open problem since late 1970s!

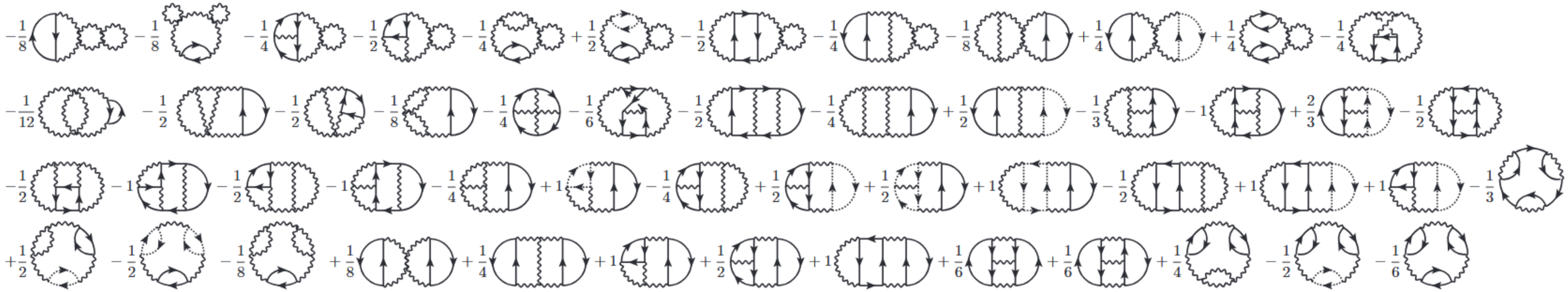
Freedman, McLerran, PRD 16 (1977)

Topic 1: From QCD thermodynamics to composition of NS cores



$$p = p_{\mathrm{FD}} + p_1^h \alpha_s + p_2^h \alpha_s^2 + \boxed{p_3^h \alpha_s^3}$$
$$+ p_2^s \alpha_s^2 + p_3^s \alpha_s^3$$
$$+ p_3^m \alpha_s^3$$

1. Soft ($g\mu_{\mathrm{B}}$) contribution: Gorda et al. (incl. AV), PRL 121 (2018), PRL 127 (2021)
2. Mixed contribution: Gorda, Paatelainen, Säppi, Seppänen, PRL 131 (2023)
3. Incomplete hard (μ_{B}): Kärkkäinen et al. (incl. AV), Accepted to PRL, 2501.17921



Topic 1: From QCD thermodynamics to composition of NS cores

Resolve IR issues by matching to HTL ✓

↙

Perform temporal integrals analytically ✓

↘

Systematic removal of UV divs. w/ vacuum renormalization (✓)

↙

IR- and UV-safe 3d integrals with Monte-Carlo methods

Secret weapon: dense Loop Tree Duality (dLTD)* – details in **poster by Pablo Navarrete**:

1) Use combination of dense Loop Tree Duality* and alorithmic tools to extract IR and UV poles:

$G = G - G_{\mathrm{CT}} + G_{\mathrm{CT}}$, where

- $G - G_{\mathrm{CT}}$ finite and computed numerically
- $G_{\mathrm{CT}} = G_{\mathrm{CT}}^{\mathrm{IR}} + G_{\mathrm{CT}}^{\mathrm{UV}}$ with
  - $G_{\mathrm{CT}}^{\mathrm{IR}}$ matches with HTL effective theory
  - $G_{\mathrm{CT}}^{\mathrm{UV}}$ using Bogoliubov R theorem

2) Perform remaining fully finite integrals numerically using Monte Carlo methods

*Navarrete, Paatelainen, Seppänen, PRD 110 (2024)
Kärkkäinen et al. (incl. AV), Accepted to PRL, 2501.17921

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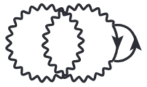
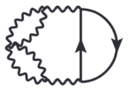
IR- and UV-safe 3d integrals with Monte-Carlo methods

Diagram	ε^{-2}	ε^{-1}	$\varepsilon^{0}_{\text{traditional}}$	$\varepsilon^{0}_{\text{dLTD}}$	$N\,[10^6]$	$[\mu\text{s}]$
$-\frac{1}{12}$ 	0	$\frac{3}{4}$	12.375	12.36(4)	110	7.1
$-\frac{1}{8}$ 	$\frac{27}{4}$	$\frac{189}{2}$	716.38	716.32(7)	120	6.5

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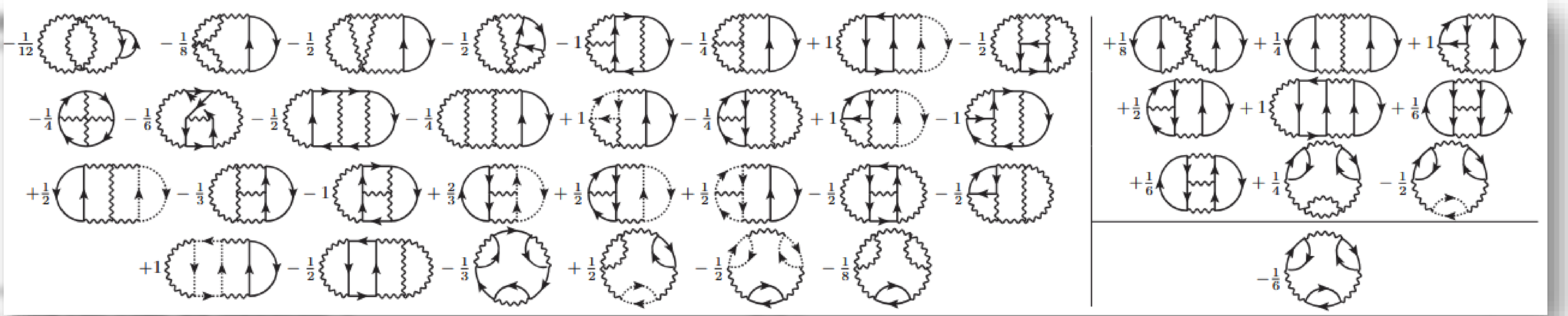
1) Use combination of dense Loop Tree Duality*



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○ G_{CT}^{UV} using Bogoliubov R theorem

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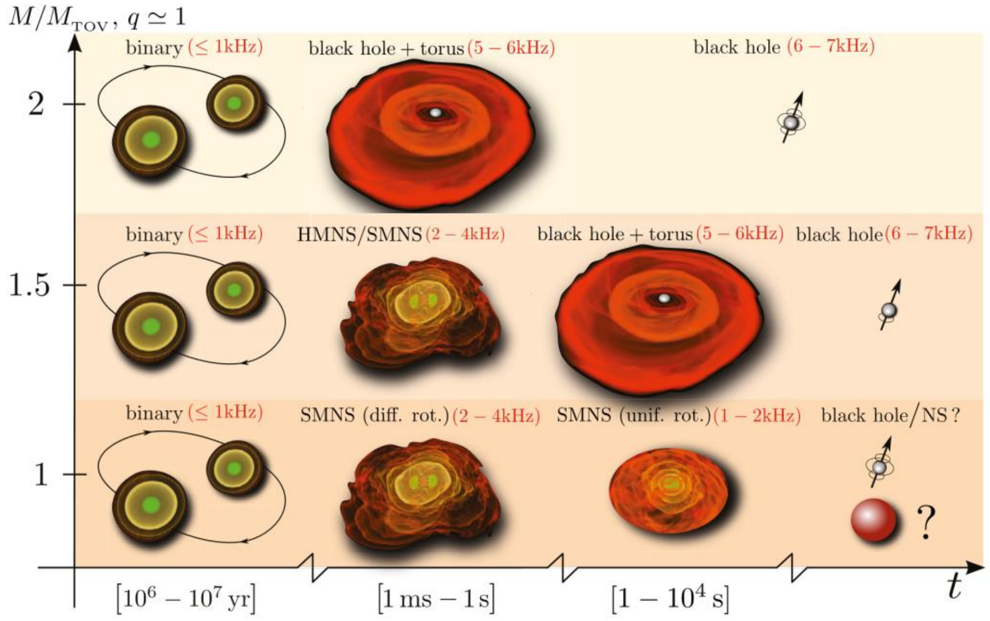
Topic 2: Transport coefficients for binary merger simulations

NS mergers feature rich dynamics imprinted in the postmerger GW waveform

To enable accurate simulations, need:

- EoS at nonzero T and charge fraction Y_Q
- Transport properties of both phases



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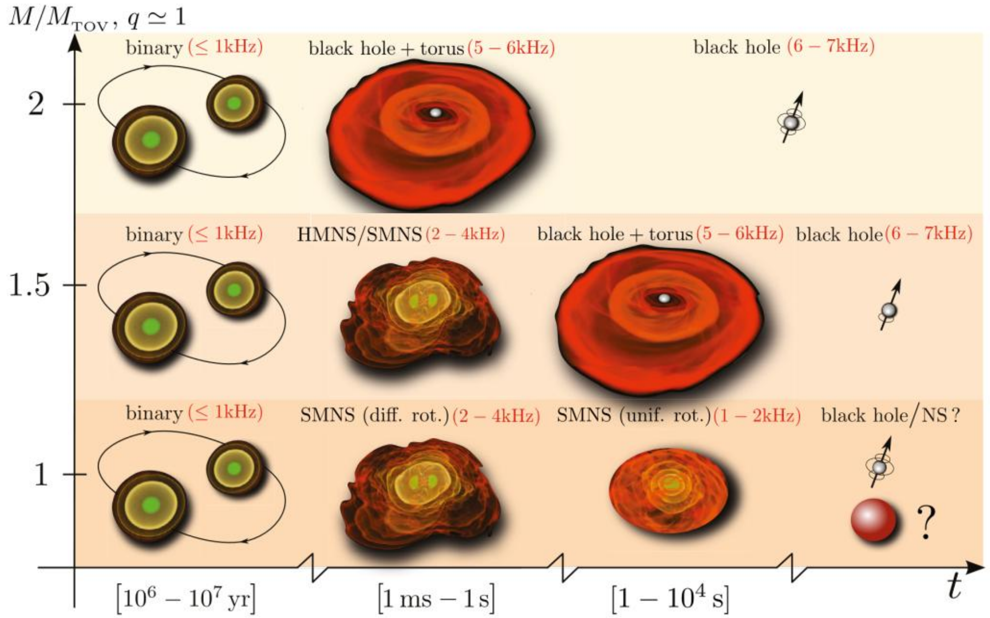

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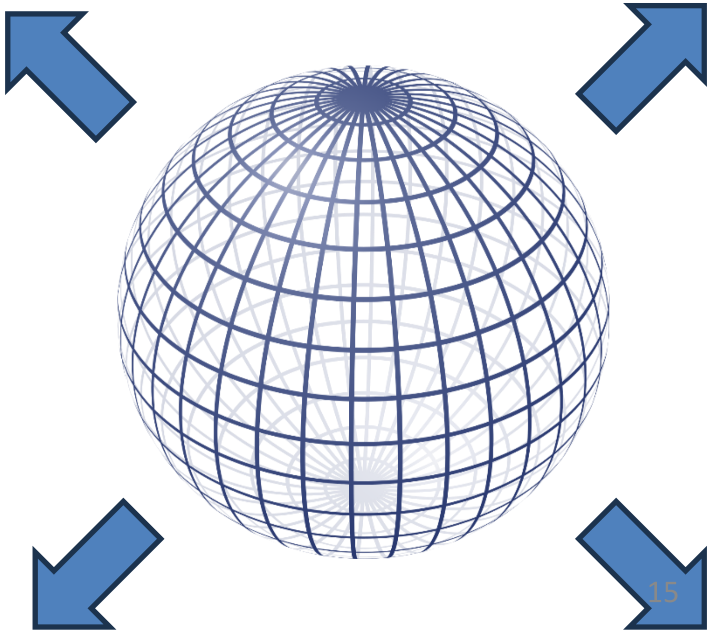
Alford, Bovard, Hanauske, Rezzolla, Schwenzer, PRL 120 (2018):

Inferring the properties of dense matter is one of the most exciting prospects from the measurement of gravitational waves from neutron star mergers. However, it will require reliable numerical simulations that incorporate viscous dissipation and energy transport if these can play a significant role within the survival time of the post-merger object. We calculate timescales for typical forms of dissipation and find that thermal transport and shear viscosity will not be important unless neutrino trapping occurs, which requires temperatures above about 10 MeV and gradients over lengthscales of 0.1 km or less. On the other hand, if direct-Urca processes remain suppressed, leaving modified-Urca processes to establish flavor equilibrium, then bulk viscous dissipation could provide significant damping to density oscillations observed right after the merger. When comparing with data from a state-of-the-art merger simulation we find that the bulk viscosity takes values close to its resonant maximum in a typical neutron-star merger, motivating a more careful assessment of the role of bulk viscous dissipation in the gravitational-wave signal from merging neutron stars.



Topic 2: Transport coefficients for binary merger simulations

In the NS setting, matter close to β equilibrium and local charge neutrality:

- $u + d \leftrightarrow u + s,\ u + e \leftrightarrow d/s + \nu_e \Longrightarrow \mu_s = \mu_d, \mu_u = \mu_d - \mu_e$
- $\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s = n_e \Longrightarrow \mu_e = \mu_e(\mu_d)$

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When system taken out of chemical equilibrium at low T, leading restoring process (perturbatively) is the non-leptonic $u + d \leftrightarrow u + s$ with rate

$$\frac{dn_d}{dt} \approx \lambda_1(\mu_s - \mu_d), \qquad \lambda_1 \approx \left(1 + \frac{4\alpha_s}{9\pi}\ln\frac{\Lambda}{T}\right)\frac{64}{5\pi^3}G_F^2 \sin^2\theta_c \cos^2\theta_c\, \mu_d^5 T^2$$

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The bulk viscosity describes energy dissipation in radially pulsating system and can be computed from $\langle \dot{E}_{\rm diss} \rangle \approx \frac{n_0}{\tau}\int_0^\tau \mathrm{d}t\, p(t)\frac{d\left(1/n(t)\right)}{dt}$.

Topic 2: Transport coefficients for binary merger simulations

The result for QM:

$$\zeta = \frac{\lambda_1 A_1^2}{\omega^2 + (\lambda_1 C_1)^2}, \text{ with}$$

$$\begin{aligned}
A_1 &\equiv H_p^{-1}\Big\{ n_s \left[\chi_{uu}\left(\chi_{dd}+\chi_{ds}\right)-\chi_{du}\left(\chi_{ud}+\chi_{us}\right)\right] \\
&\quad -n_d\left[\chi_{uu}\left(\chi_{ss}+\chi_{sd}\right)-\chi_{su}\left(\chi_{ud}+\chi_{us}\right)\right] \\
&\quad +n_u\left[\chi_{ud}\left(\chi_{ss}+\chi_{sd}\right)-\chi_{us}\left(\chi_{dd}+\chi_{ds}\right)\right]\Big\} \\
C_1 &\equiv H_p^{-1}\left[\left(\chi_{ud}+\chi_{us}\right)^2-\chi_{uu}\left(\chi_{dd}+2\chi_{ds}+\chi_{ss}\right)\right]
\end{aligned}$$

and $H_p \equiv \det \frac{\partial^2 p}{\partial \mu_i \partial \mu_j}$

Topic 2: Transport coefficients for binary merger simulations

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and $H_p \equiv \det \frac{\partial^2 p}{\partial \mu_i \partial \mu_j}$

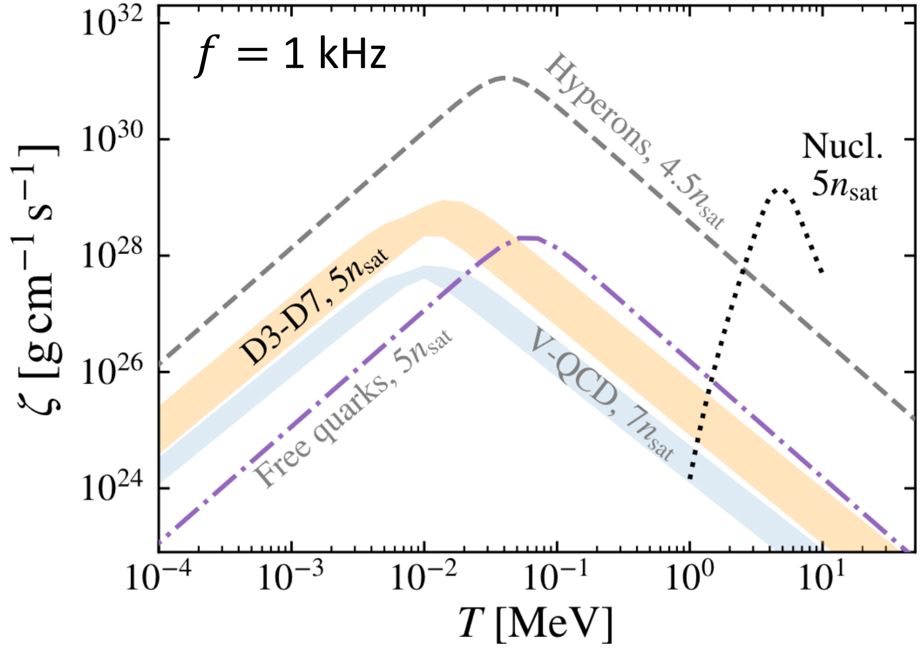
∴ The evaluation of ζ reduces to that of quark number densities and susceptibilities, i.e. the bread and butter of thermal perturbation theory. Catch: dependence on quark masses crucial – **talk by Loïc Fernandez!**

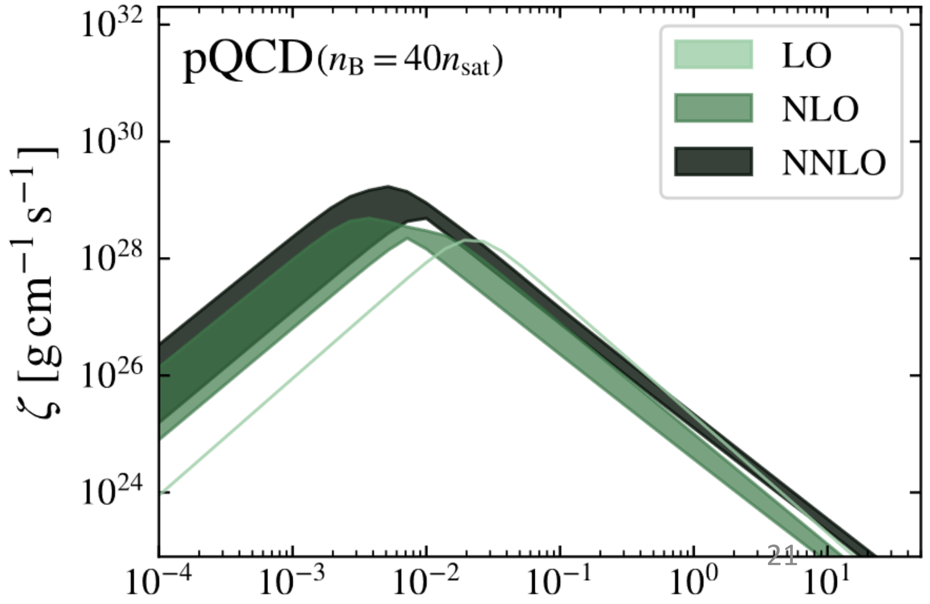
Topic 2: Transport coefficients for binary merger simulations

Observation 1: predictions from NNLO pQCD and holographic D3-D7 and V-QCD models qualitatively similar, far from hadronic results

Observation 2: Interactions important: even at $n_B = 40n_s$ differences between LO, NLO and NNLO (new work) results markedly different

Cruz Rojas et al., PRL 133 (2024), 2402.00621





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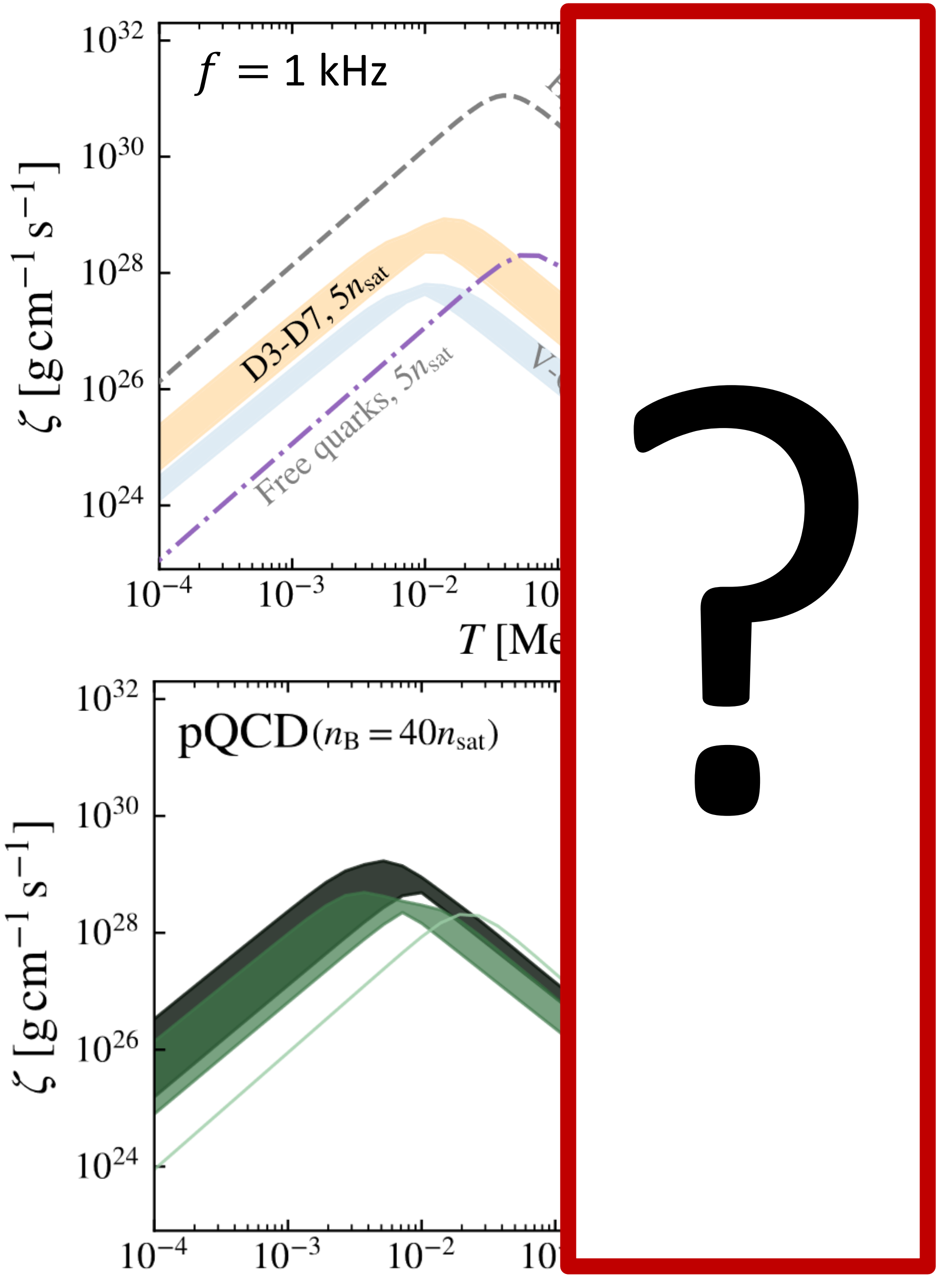
Cruz Rojas et al., PRL 133 (2024), 2402.00621

Important caveat: Very new results indicate nonleptonic processes becoming dominant at $T \gtrsim 0.1$ MeV, necessitating the use of a more involved description (**talk by Hernandez**)!

Hernández, Manuel, Säppi, Tolos, 2507.00794



Main takeaways:

- Neutron stars offers plenty of challenges for particle theorists – and not all of them lie in the core
- Within perturbative thermal QCD, the present main challenge in completing the N3LO pressure of cold and dense quark matter, promising a qualitative improvement in EoS inference
- Other interesting lines of future work include (at least):
  - EoS inference at nonzero T and Y_Q
  - Improving description of transport phenomena (in particular bulk viscous dissipation) in QM
  - Unraveling the nonlinearity of QED at extreme magnetic fields

Thank you!

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