

# Out-of-equilibrium CME from Lattice QCD

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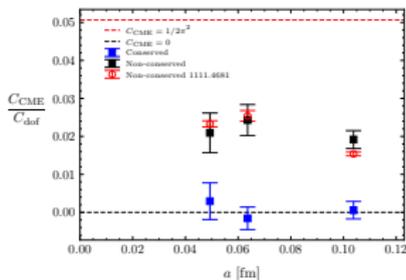
# Anomalous transport

- ▶ Axial anomaly +  $\begin{matrix} \text{EM fields} \\ \text{Vorticity} \end{matrix} \rightarrow$  **Anomalous transport phenomena**
- ▶ Examples:
- Chiral Magnetic Effect (CME) *✍ Fukushima, Kharzeev, Warringa '08*
  - Chiral Separation Effect (CSE)
  - Chiral Vortical Effect (CVE)
  - Chiral Magnetic Wave (CMW)
  - ...
- ▶ Probe of the topological nature of the QCD vacuum
- ▶ Experimental detection in heavy-ion collisions and condensed matter systems, review *✍ Kharzeev, Liao, Tribedy '24*
- ▶ Objective: use Lattice QCD to understand CME properties in QCD

# Previous works

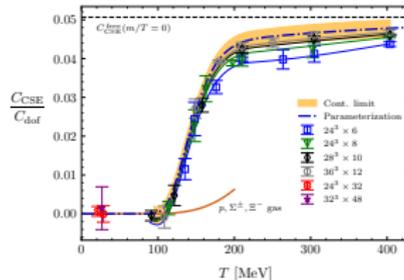
## CSE conductivity in physical QCD

✍ Brandt, Endrődi, EGV, Markó '23



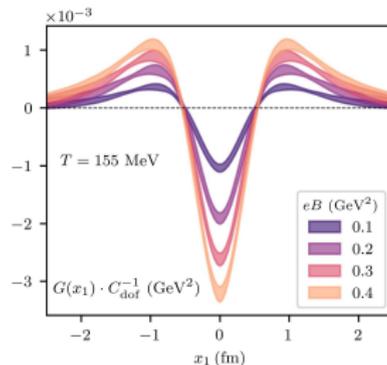
## Local CME with inhomogeneous $B$

✍ Brandt, Endrődi, EGV, Markó, Valois '24



## Absence of CME in equilibrium QCD

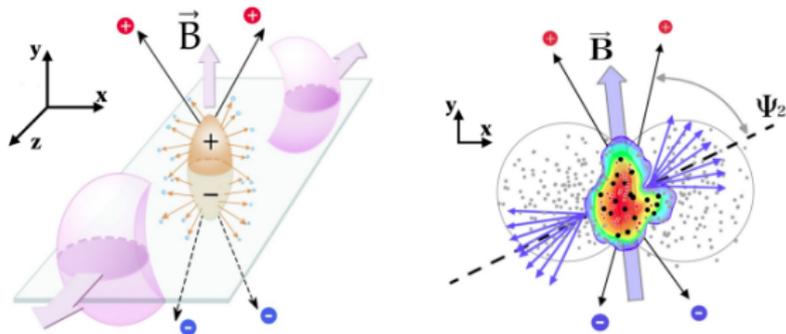
✍ Brandt, Endrődi, EGV, Markó '24



# Out-of-equilibrium CME

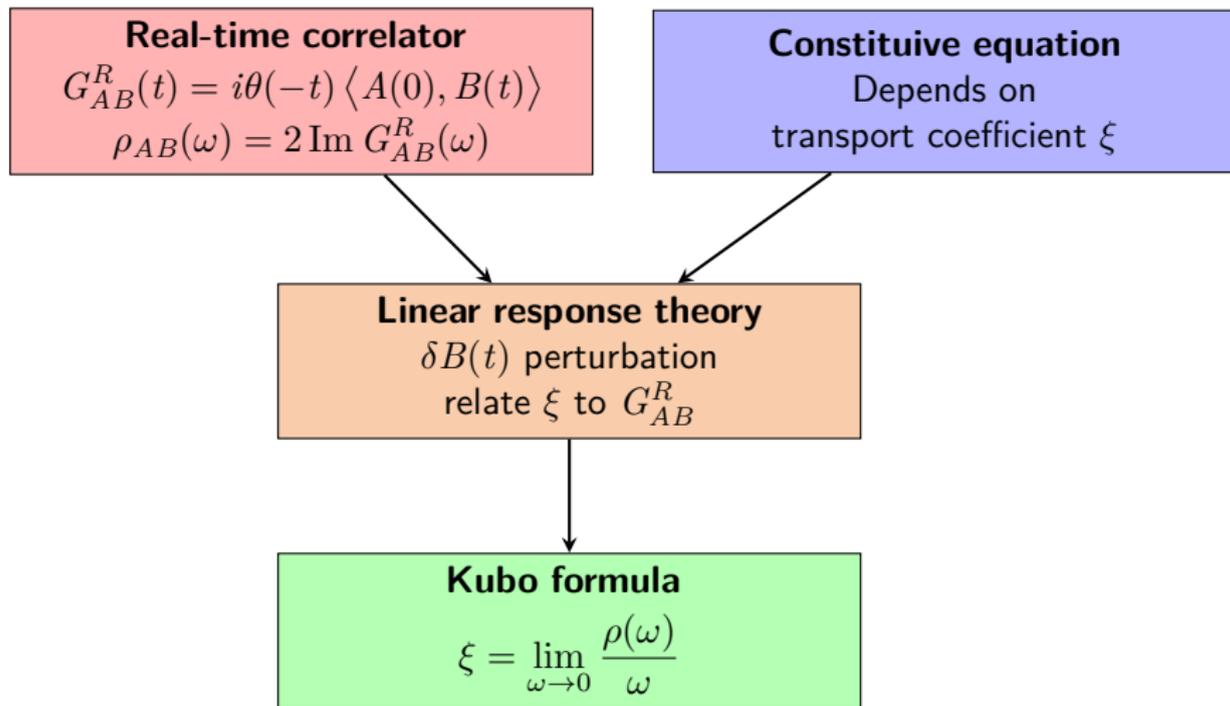
- ▶ So far we concentrated on a *global thermal equilibrium* system.
- ▶ In experiments out-of-equilibrium effects can be very relevant

✍ Kharzeev, Liao, Tribedy '24



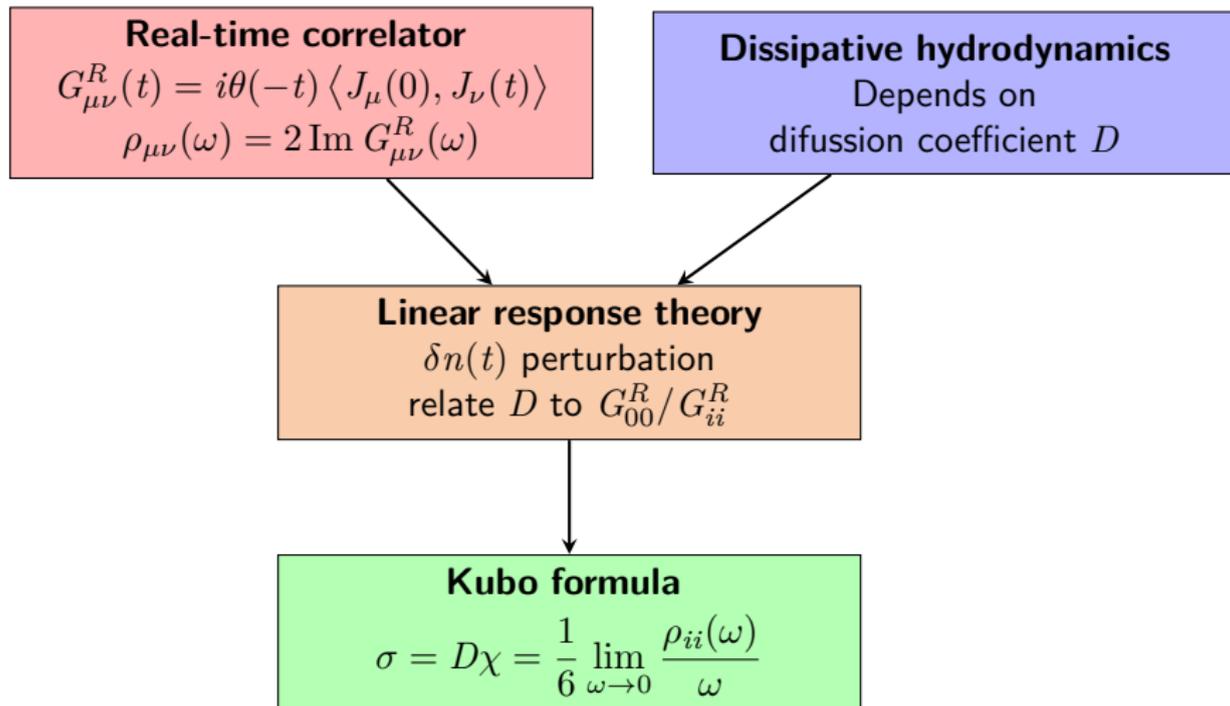
- ▶ What can we say from Euclidean lattice QCD about non-equilibrium effects?

# Linear response theory

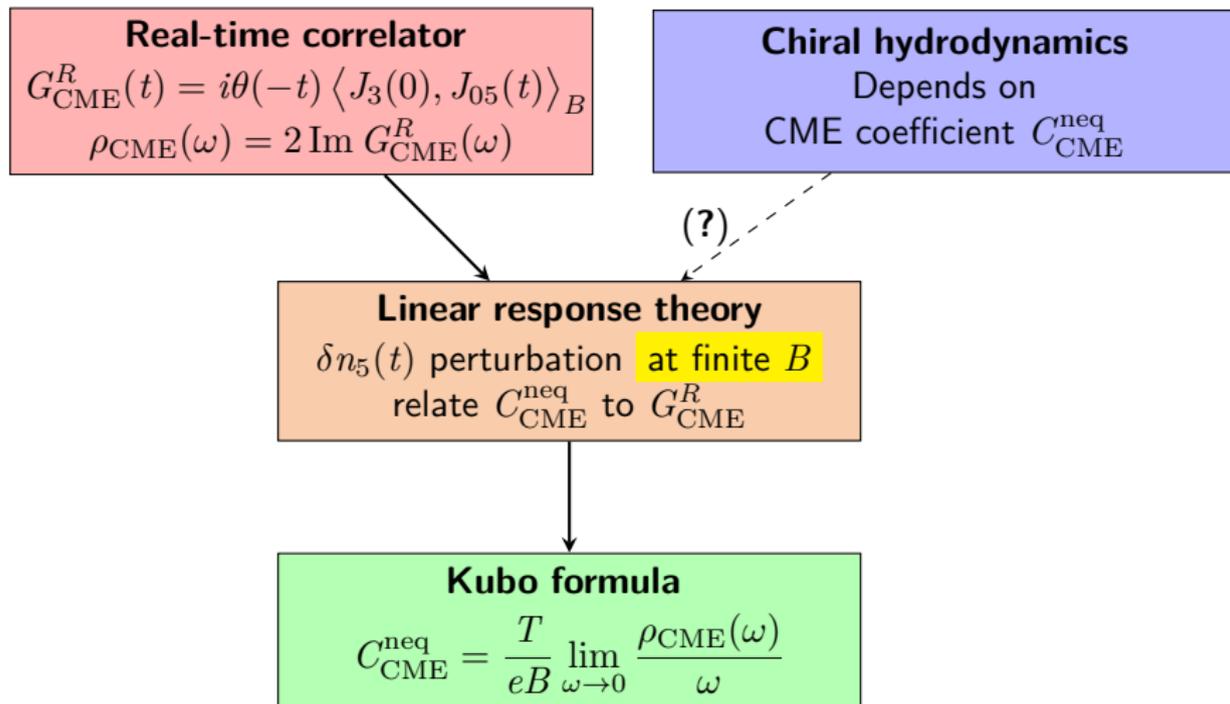


- ▶ Electric conductivity, bulk/shear viscosities,... in microscopic theory (QCD)
- ▶ Used in hydrodynamic simulations

# Electric conductivity



- ▶ Well-known derivation
- ▶ Used to calculate  $\sigma$  in lattice QCD



- ▶ Missing precise connection with chiral hydrodynamics!
- ▶ Correct normalization of Kubo formula?

# 1-loop spectral function

▶ Free fermions  $\rightarrow$  1-loop exact  $\rho$

▶ Electric conductivity: e.g.  $\nearrow$  Aarts, Nikolaev 21'

$$\rho_{ii}(\omega) = \chi\left(\frac{m}{T}\right)\omega\delta(\omega) + \Theta(\omega^2 - 4m^2)\frac{m^2}{4\pi^2}\sqrt{\frac{\omega^2 - 4m^2}{\omega^2}}(\omega^2 + 2m^2)\left[1 - 2n_F\left(\frac{\omega}{2}\right)\right]$$

▶  $\sigma \rightarrow \infty \Rightarrow$  resummation required  $\nearrow$  Arnold, Moore, Yaffe '00

# 1-loop spectral function

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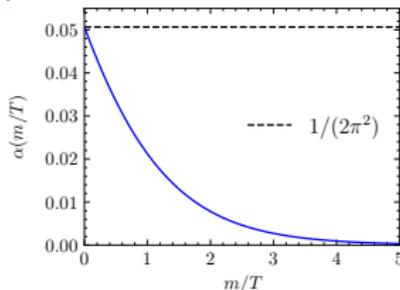
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▶ CME:  $\nearrow$  Brandt, Endrődi, EGV, Markó, Valois, Lattice '24

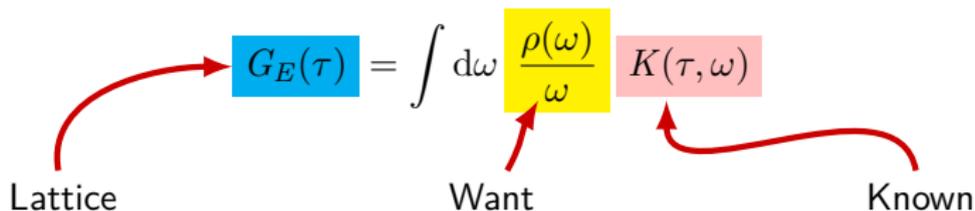
$$\frac{1}{eB}\rho_{\text{CME}}(\omega) = \alpha\left(\frac{m}{T}\right)\omega\delta(\omega) + \Theta(\omega^2 - 4m^2)\frac{m^2}{\pi^2}\frac{\tanh[|\omega|/(4T)]}{\omega\sqrt{\omega^2 - 4m^2}}$$



▶  $C_{\text{CME}}^{\text{neq}} \rightarrow \infty \Rightarrow$  resummation required  $\leftrightarrow$  relation to the anomaly?

# Spectral reconstruction

- ▶  $G_{R,\rho}$  not directly accessible from lattice QCD
- ▶ Spectral representation of Euclidean correlators


$$G_E(\tau) = \int d\omega \frac{\rho(\omega)}{\omega} K(\tau, \omega)$$

Lattice                      Want                      Known

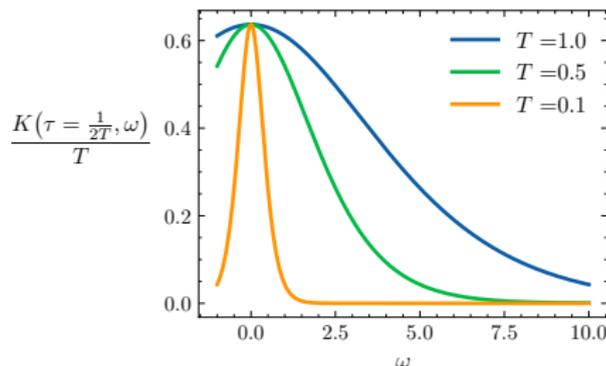
- ▶ On the lattice:  $N_t \sim \mathcal{O}(10)$  ill-posed inverse problem
- ▶ Many methods on the market  $\rightarrow$  applied to get other transport coefficients
- ▶ Examples: Maximum entropy method, Backus-Gilbert, machine learning, ...

# Midpoint method

- ▶ Kernel evaluated at the midpoint

$$K(\tau, \omega) = \frac{\omega \cosh[\omega(\tau - 1/(2T))]}{\pi \sinh[\omega/(2T)]} \rightarrow K(\tau = 1/(2T), \omega) = \frac{\omega}{\pi \sinh[\omega/(2T)]}$$

- ▶ Behaves as a smeared  $\delta(\omega)$  when  $T \rightarrow 0$

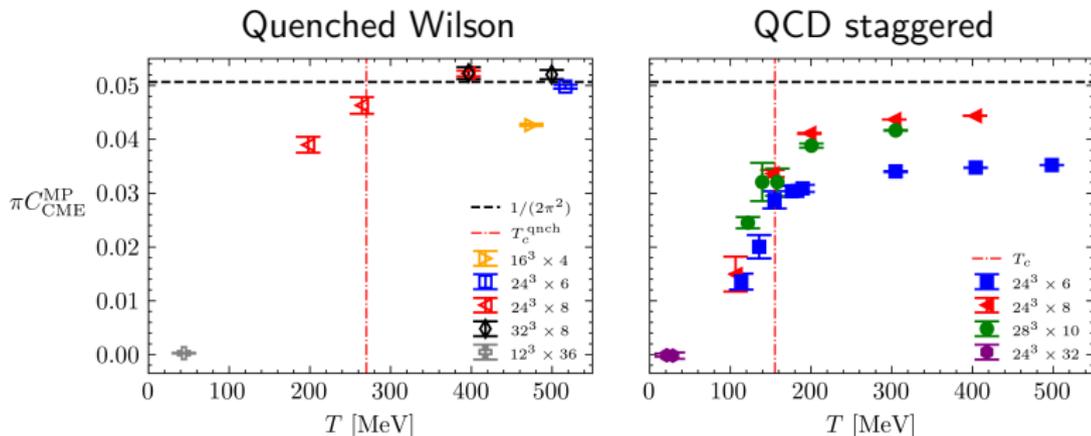


- ▶  $G_E(a\tau = N_t/2)$  carries first estimate! ⓘ Buividovich '24 ⓘ poster

$$C_{\text{CME}}^{\text{MP}} = \frac{G_E(N_t/2)}{eB T \pi}$$

# Midpoint in QCD

- ▶ Estimation of  $C_{\text{CME}}^{\text{neq}}$  in QCD: [Brandt, Endrödi, EGV, Markó, Valois, Lattice '24](#)



- ▶ Estimation reliable when  $T \rightarrow 0$
- ▶ Suppression at low temperatures
- ▶ Intuition at high  $T \rightarrow$  at 1-loop:

$$\frac{G_E(N_t/2)}{eB T} = \pi C_{\text{CME}}^{\text{MP}}(m/T) = C_{\text{CSE}}^{\text{free}}(m/T) \xrightarrow{m/T \rightarrow 0} \frac{1}{2\pi^2}$$

# Multipoint estimator

- ▶ Improve midpoint estimator by using several points
- ▶ Expanding  $K(\tau, \omega)$  and  $\rho(\omega) = \sum_{n=1}^{\infty} \frac{c_n}{n!} \omega^n$  to obtain

$$G(N_t/2) = \pi T^2 c_1 + \frac{14}{\pi} T^3 \zeta(3) c_2 + \frac{30}{\pi} T^4 \zeta(4) c_3 + \dots$$

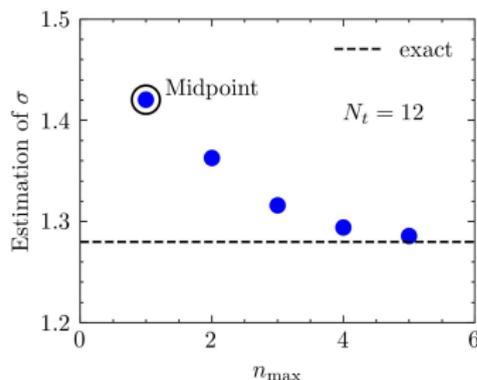
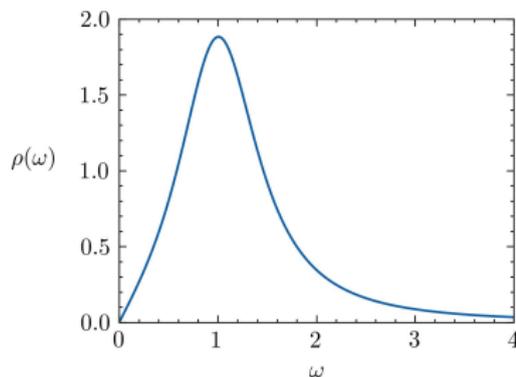
- ▶ Similar expression for other  $\tau$  values  $\rightarrow$  system of equations

$$\begin{pmatrix} A_{11} & A_{12} & \dots & A_{1n_{\max}} \\ A_{21} & A_{22} & \dots & A_{2n_{\max}} \\ \vdots & \ddots & \ddots & \vdots \\ A_{n_{\max}1} & A_{n_{\max}2} & \dots & A_{n_{\max}n_{\max}} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_{n_{\max}} \end{pmatrix} = \begin{pmatrix} G(N_t/2) \\ G(N_t/2 - 1) \\ \vdots \\ G(N_t/2 - (n_{\max} - 1)) \end{pmatrix}$$

- ▶  $c_1$  ( $\sim$  conductivity) improved estimation!

# Multipoint Breit-Wigner

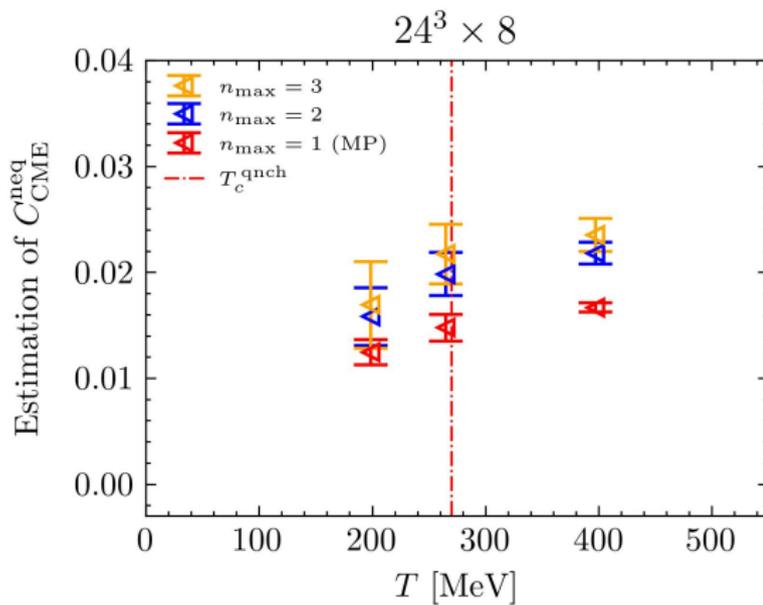
- ▶ Toy model: Breit-Wigner spectral function
- ▶ Conductivity  $\sigma \equiv \lim_{\omega \rightarrow 0} \rho(\omega)/\omega$



- ▶ Multipoint improves the conductivity estimation

# Multipoint quenched QCD wilson

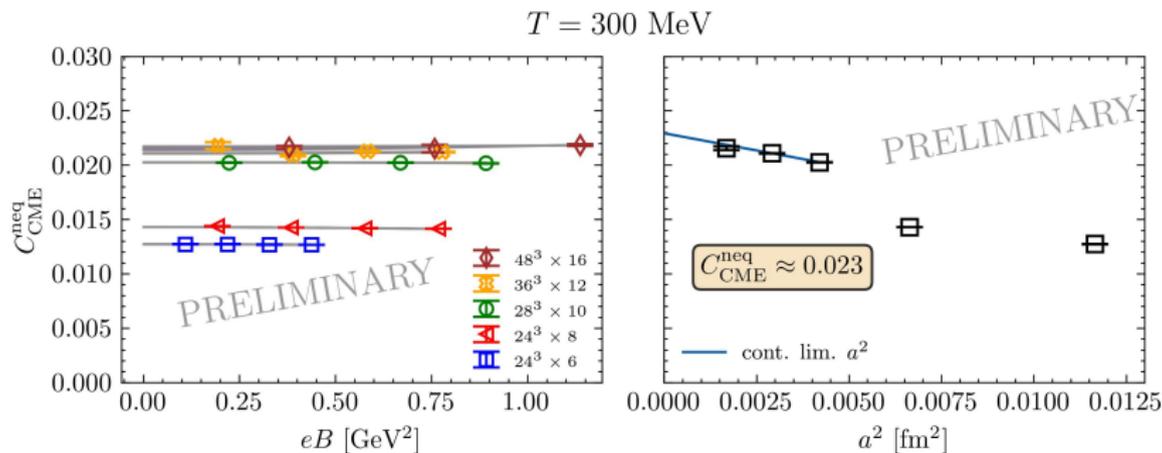
- Use multipoint estimation with quenched QCD Wilson data



- Estimation of  $C_{\text{CME}}^{\text{neq}} \approx 0.022 - 0.025$  at high  $T$

# Conductivity in QCD

- ▶ Reconstructing staggered QCD data using Gaussian processes [Horak et al. '21](#)



- ▶ Precise normalization of Kubo formula missing!
- ▶ Detailed systematic error analysis required
- ▶ Consistent with multipoint quenched Wilson estimation

## Summary

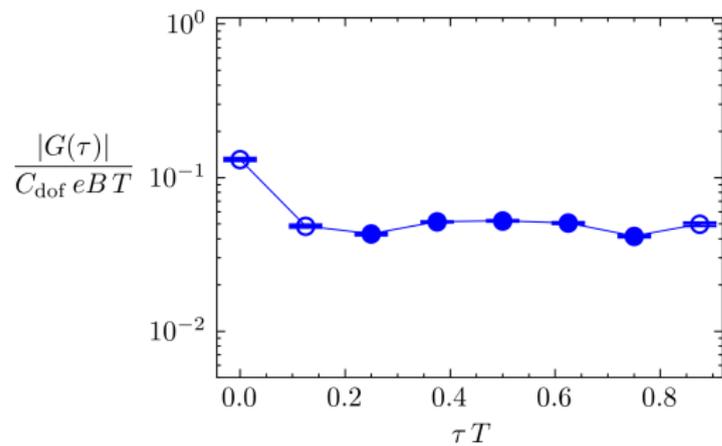
- ▶ Study of out-of-equilibrium CME in QCD using lattice simulations
- ▶ Spectral function calculated at 1-loop in perturbation theory
- ▶ First estimation yields  $C_{\text{CME}}^{\text{neq}}$  suppressed at low  $T$
- ▶ Preliminary results towards continuum limit at  $T = 300$  MeV

## Outlook

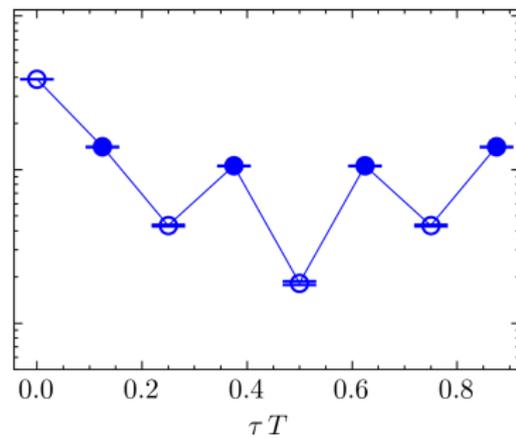
- ▶ How to obtain Kubo formula from chiral hydrodynamics?
- ▶ Relation between required resummation and axial anomaly?
- ▶ Apply several different reconstruction methods
- ▶ Systematic scan in  $T$

Backup slides

# Correlators



Wilson



Staggered