Using net quark number gain to probe the phases of QCD

Based on: 2504.06459

Tomas Mari Surkau and **Urko Reinosa**

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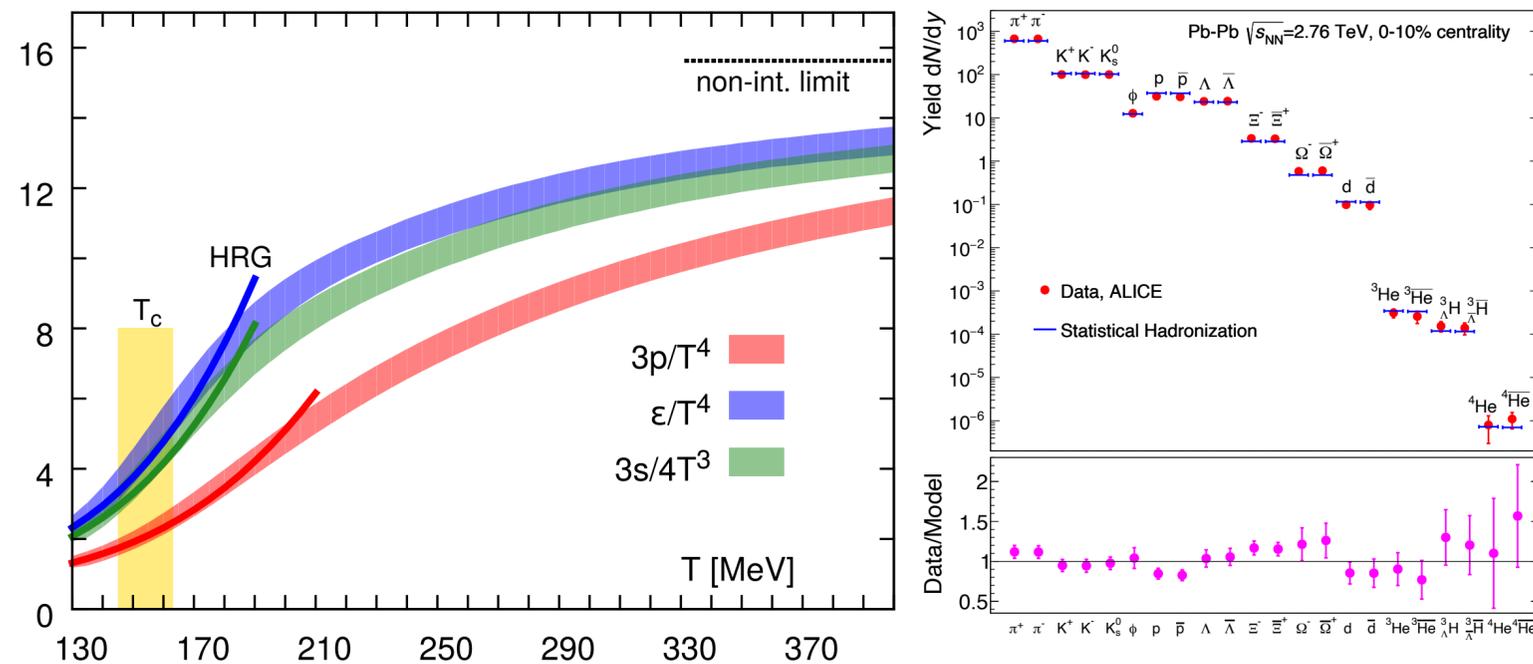
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THE 21ST INTERNATIONAL CONFERENCE ON QCD IN EXTREME CONDITIONS

Our rough understanding

Low $T \to$ high T: rapid change in degrees of freedom

- **Low T:** Hadron resonance gas.

  - Agrees w. Lattice QCD up to T_c,

  - Predicts freeze-out yields of HIC.

HotQCD, PRD 90, 094503 (2014), [1407.6387]

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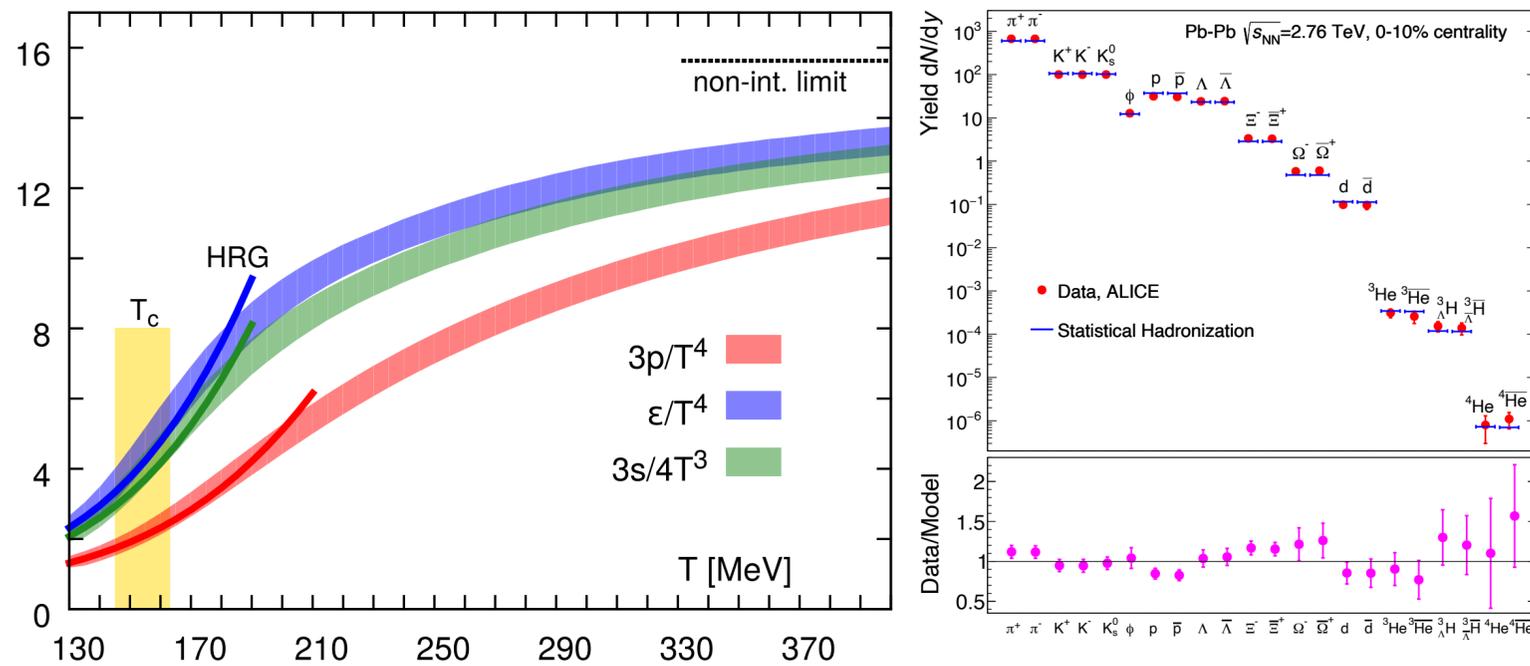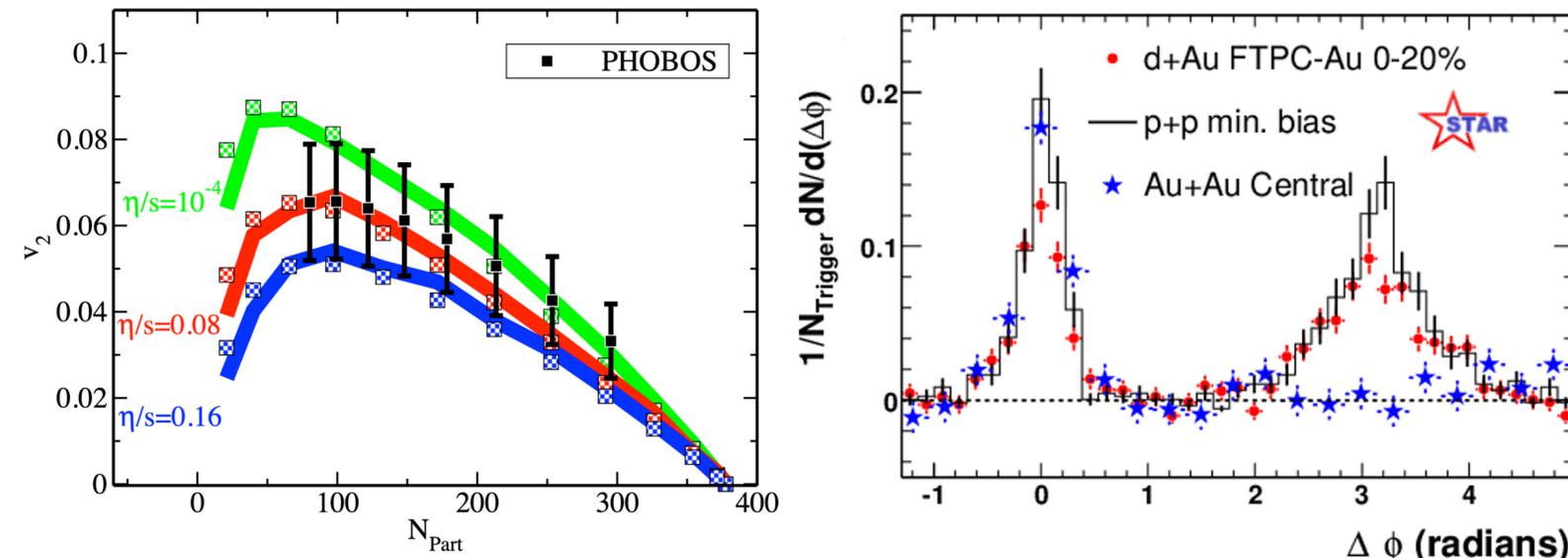 - Agrees w. Lattice QCD up to T_c,

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- **High T:** Strongly coupled fluid (~QGP),

  - Analytic crossover from the HRG,

  - HIC: anisotropic flow & jet quenching $\to$ low viscosity, thermalized medium created.

HotQCD, PRD 90, 094503 (2014), [1407.6387]

A. Andronic et al., Nature 561, 321–330 (2018), [1710.09425]

M. Luzum et al., PRC 78, 034915 (2008), [0804.4015]

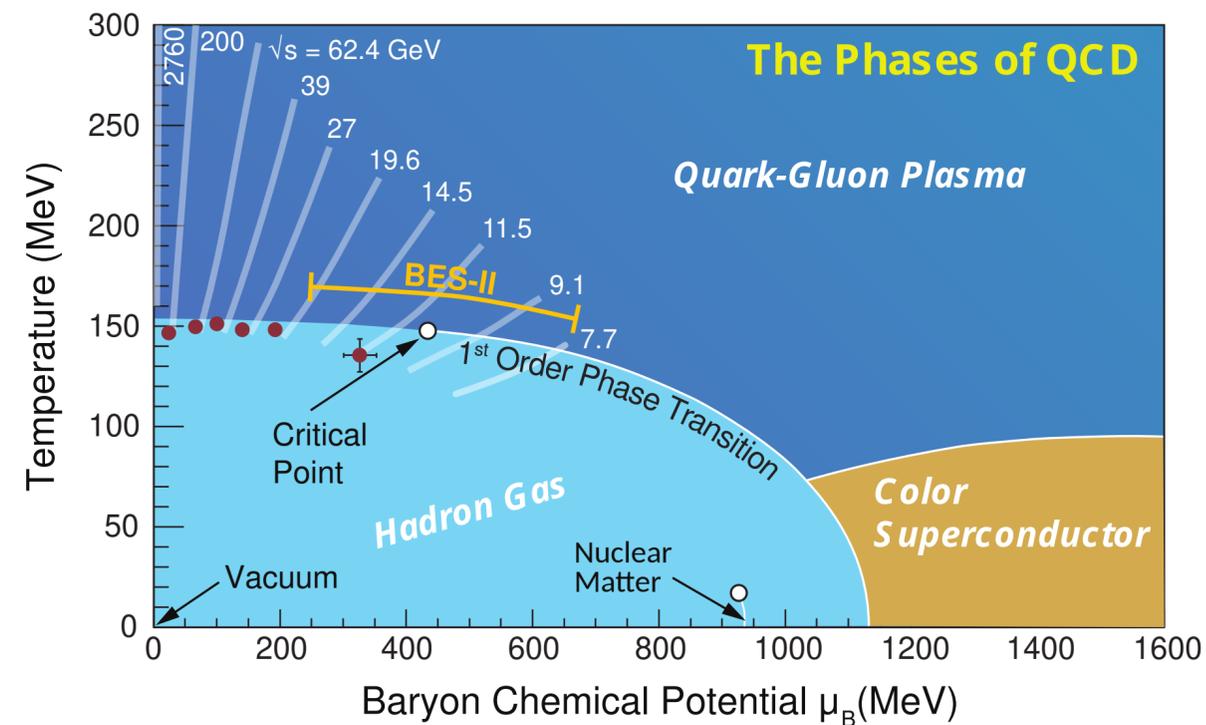STAR, PRL 91, 072304 (2003), [nucl-ex/0306024]

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$(T \to \infty:$ asymptotically free $(\alpha_s(2\pi T) \to 0)$ parton gas$)$

The desired understanding

Full QCD phase diagram

- Need to study: QCD at $\mu \neq 0$.

- Expectations(funct.QCD, models):

A. Bzdak et al., Phys. Rep. 853, 1-87 (2020), [1906.00936]

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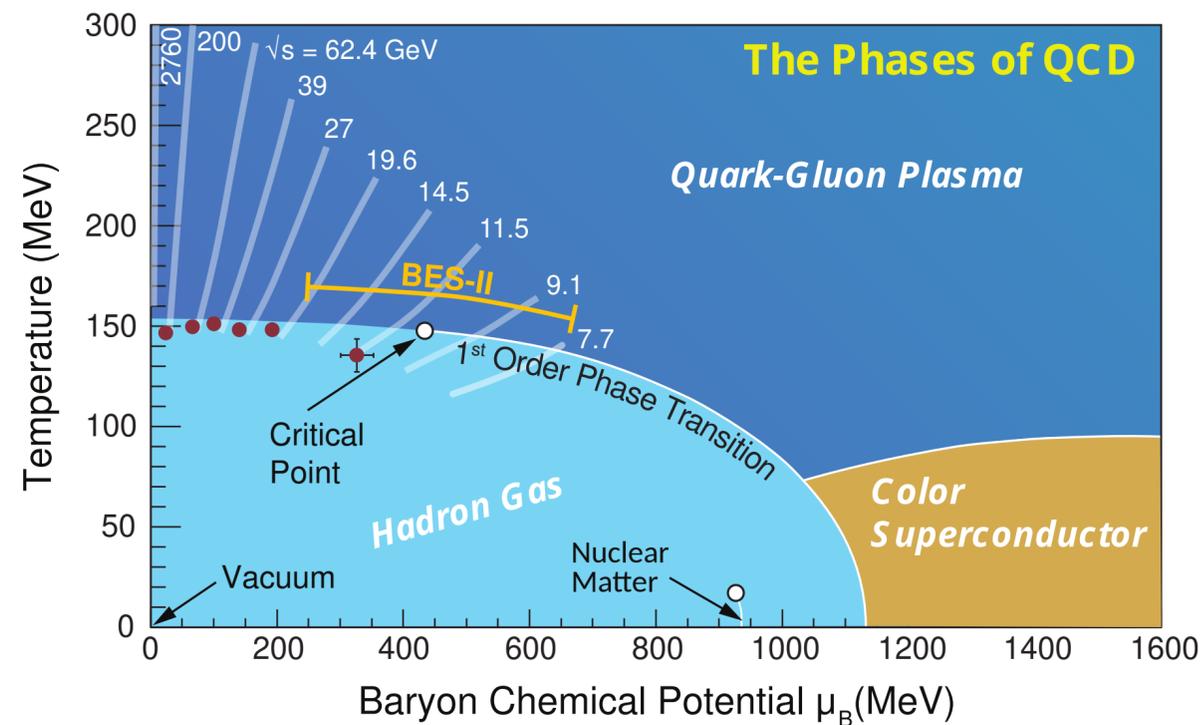
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- Crossover $\Rightarrow T_c$ depends on observable,

- Approximate QCD order parameters:

  - Chiral condensates $\langle \bar{q}q \rangle$ (chiral sym. $m_q \rightarrow 0$),

  - Polyakov loops $\ell, \bar{\ell}$ (center sym. $m_q \rightarrow \infty$). $\ell \equiv \langle \Phi \rangle$

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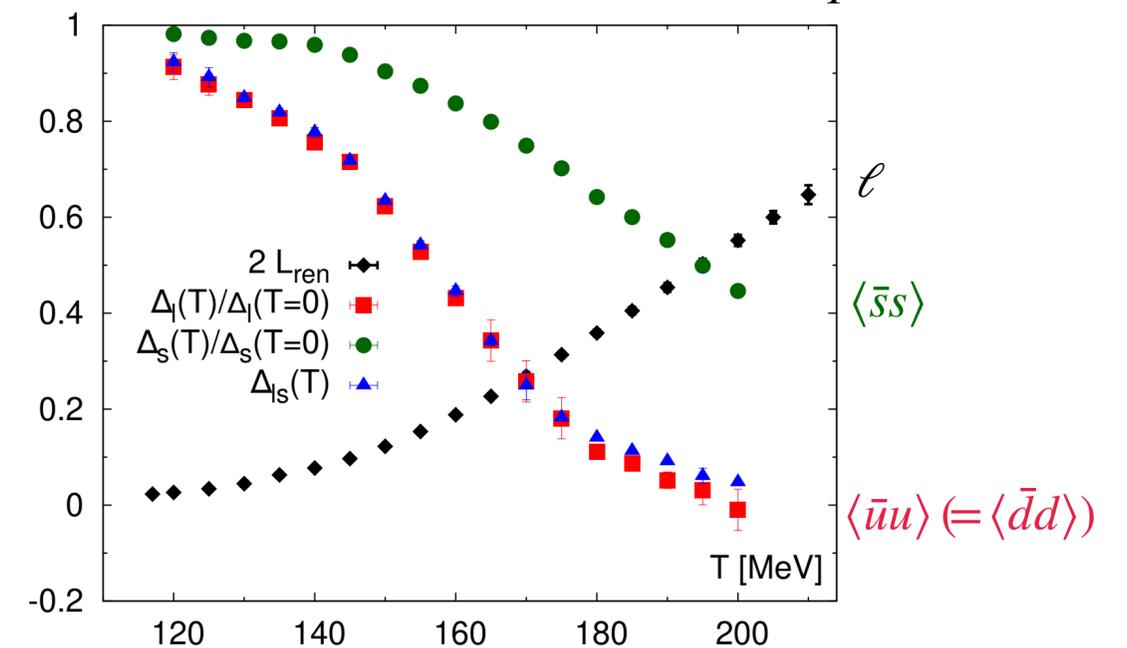- Expectations(funct.QCD, models):

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The Phases of QCD

$\sqrt{s} = 62.4$ GeV

Quark-Gluon Plasma

BES-II

1st Order Phase Transition

Critical Point

Hadron Gas

Color Superconductor

Vacuum

Nuclear Matter

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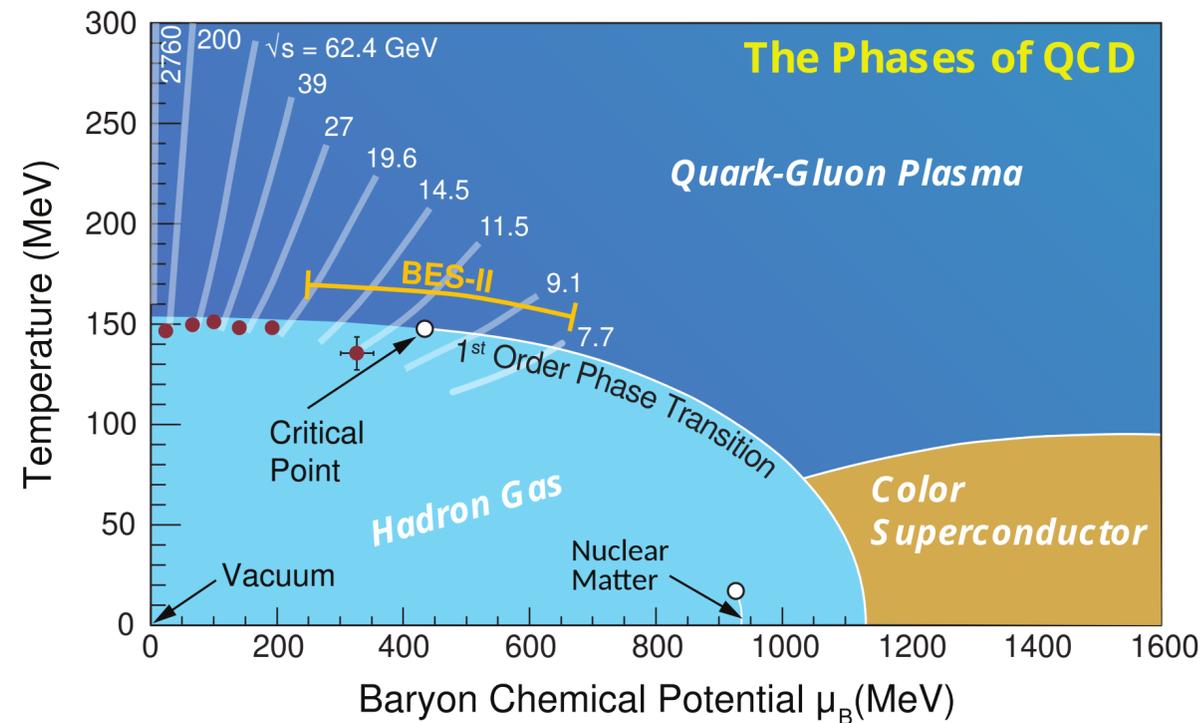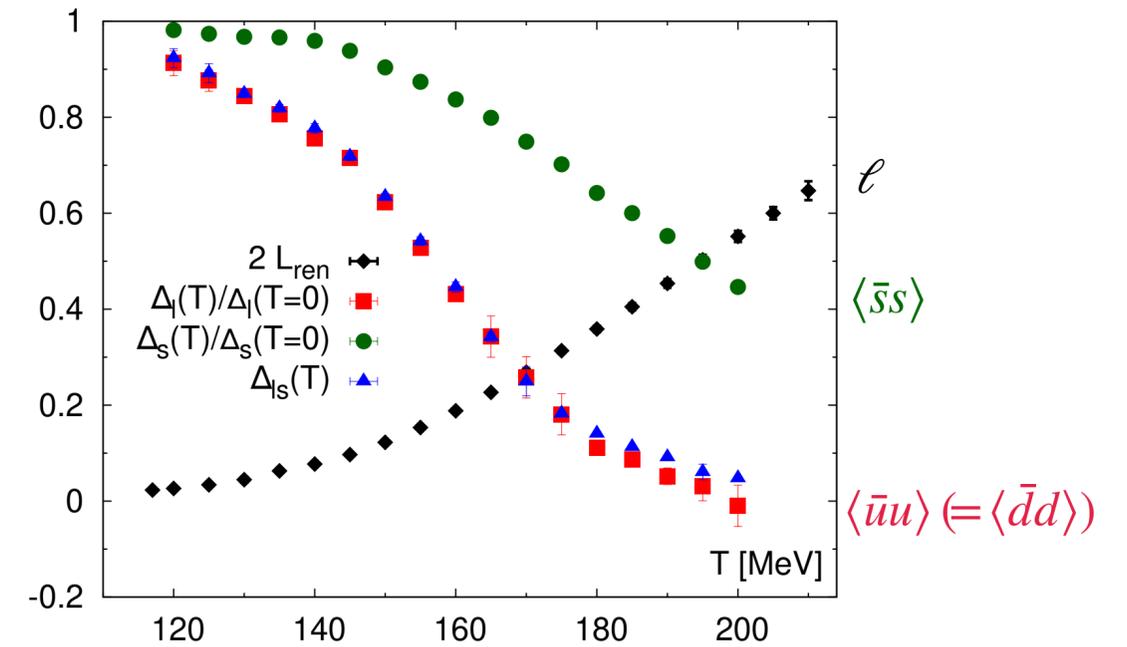Oversimplified... likely various different CSC phases

Use $\ell, \bar{\ell}$ in this talk!

ℓ

$\langle \bar{s}s \rangle$

$\langle \bar{u}u \rangle (= \langle \bar{d}d \rangle)$

2 L_{ren}
$\Delta_l(T)/\Delta_l(T=0)$
$\Delta_s(T)/\Delta_s(T=0)$
$\Delta_{ls}(T)$

T [MeV]

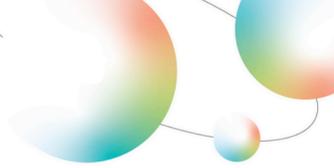A. Bazavov et al., PRD 87, 094505 (2013), [1301.3943]

Polyakov loops and confinement

Meaning of the Polyakov loops

- QCD medium with a static quark q:

  Part. fct.: $Z_q = \mathrm{Tr}_q \, e^{-\beta(H-\mu Q)}$,

  Quark number

- $\ell = Z_q/Z, \quad \bar{\ell} = Z_{\bar{q}}/Z \quad \Rightarrow$

  $-T \ln(\ell) = \Delta F_q, \; -T \ln(\bar{\ell}) = \Delta F_{\bar{q}},$

- $\ell, \bar{\ell}$ measure **free energy increase** related to adding a static quark/anti-quark source to a QCD medium.

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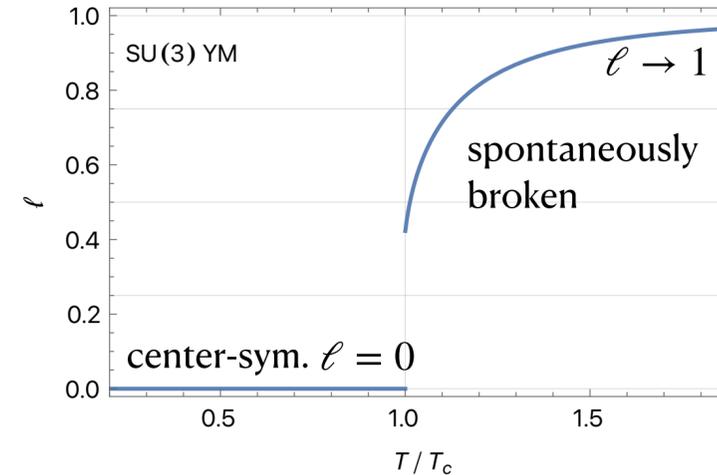
- YM: if $\ell = 0 \Leftrightarrow \Delta F_q = \infty \rightarrow$ confined q,

SU(3) YM

$\ell \to 1$

spontaneously broken

center-sym. $\ell = 0$

Adding a quark is forbidden at low T.

ℓ is order parameter!

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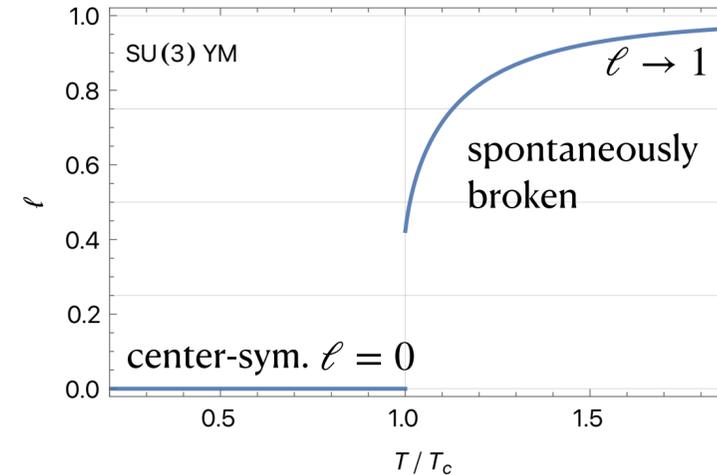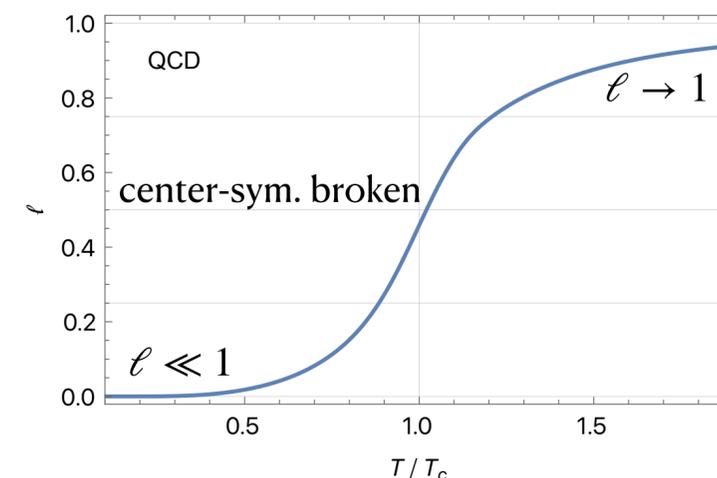Adding a quark is forbidden at low T.

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- QCD: $M < \infty$ breaks center-sym.:

Adding a quark is never forbidden?!

ℓ not real order parameter!

But, very high ΔF_q at low T.

Still, how can it be compatible with hadronic dof's?

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Thermodynamics with Polyakov loops

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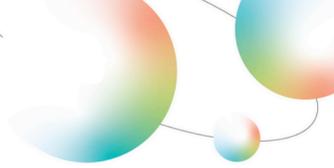
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upon bringing q : $\Delta Q_q + 1$,

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Important distinction:

- This is not the same as the net quark number **density**, $n_q = -\dfrac{\partial \Omega}{\partial \mu}$.
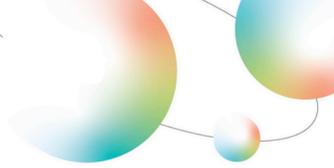
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- Order parameters can feed back into n_q, $\Omega = \Omega(T, \mu, \ell(T,\mu), \langle\bar{q}q\rangle(T,\mu), \dots),$ but n_q corresponds to Q-density of the system **without** an added quark.

- $\Delta Q_{q,\bar{q}} \pm 1$, the net quark number gain, is a "global" observable, not a density.

Thermodynamic potential at low T

Polyakov loop potential

heavy quark or chirally
broken constituent mass

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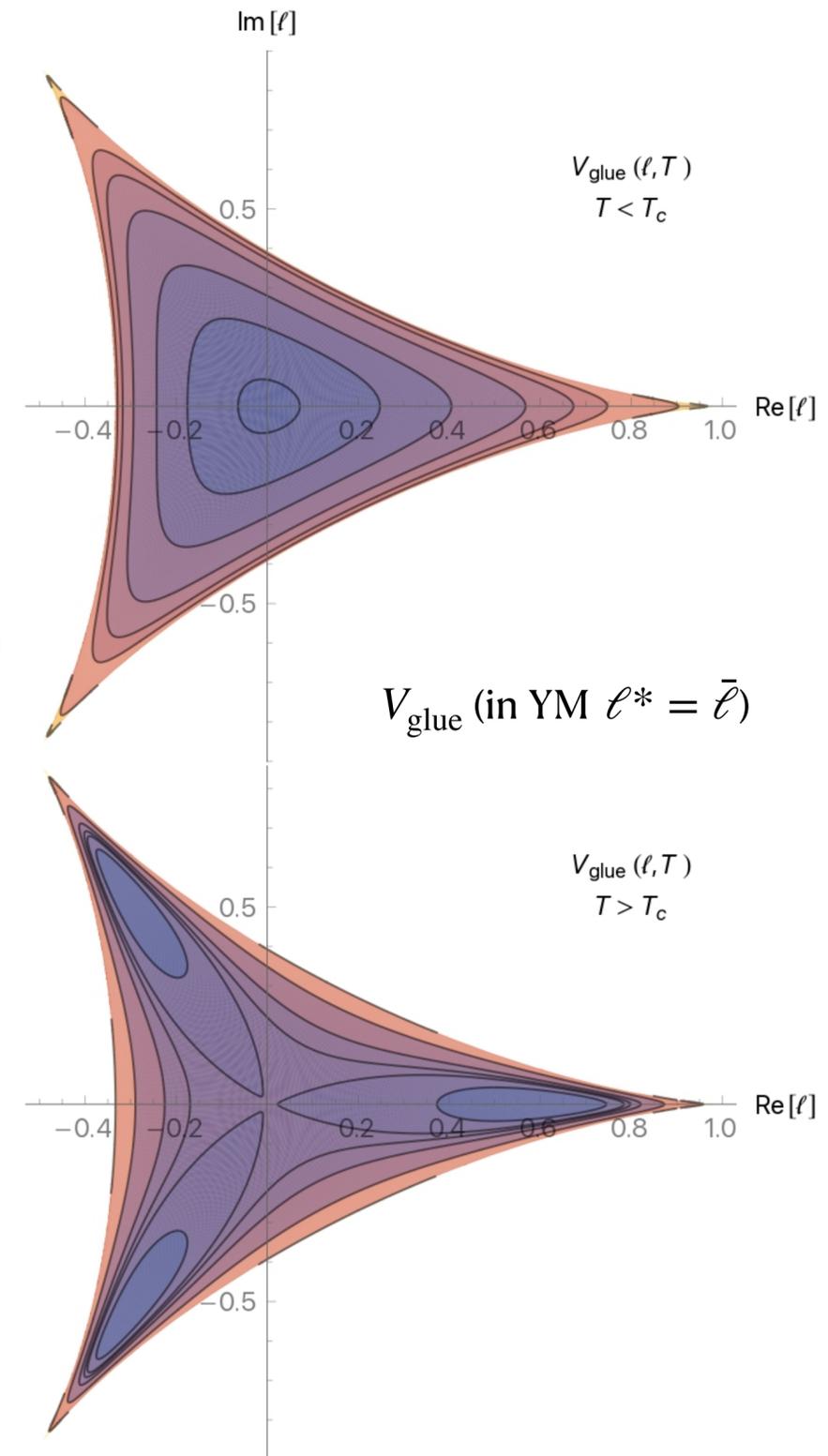
- $V_{\text{glue}}(\ell, \bar{\ell}, T)$ satisfying:

  - **Center-symmetric**: $V_{\text{glue}}(\ell, \bar{\ell}) = V_{\text{glue}}(e^{i2\pi/3}\ell, e^{-i2\pi/3}\bar{\ell})$
    (as given by YM),

  - **Confining** at low T: $\ell, \bar{\ell} \xrightarrow{T \to 0} 0$ (as seen on lattice),

  - T-**power-law** behaviour at low T: $V_{\text{glue}} \sim T^{\#}$,

$\text{Im}[\ell]$

$V_{\text{glue}}(\ell, T)$
$T < T_c$

$\text{Re}[\ell]$

V_{glue} (in YM $\ell* = \bar{\ell}$)

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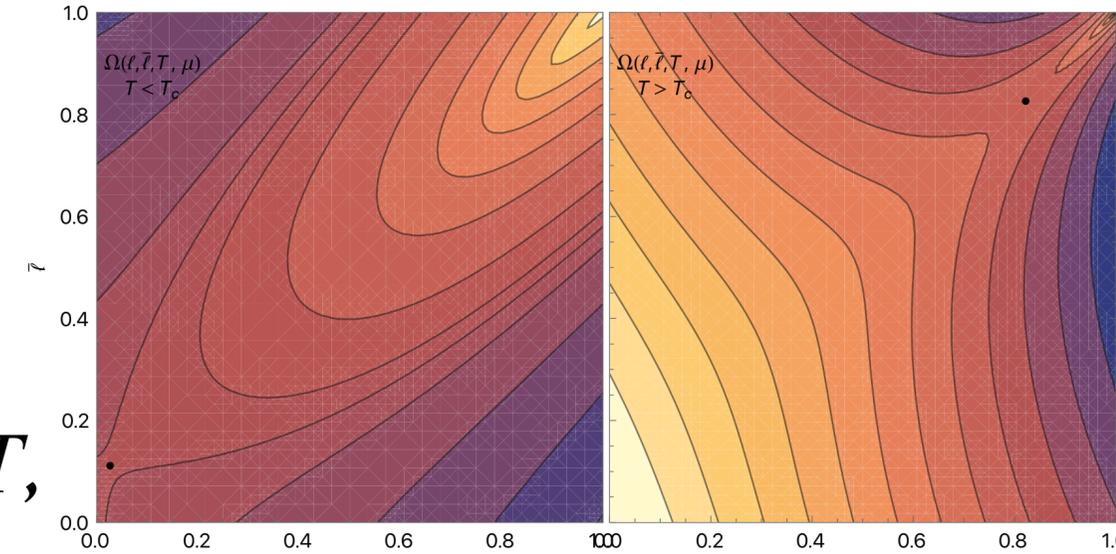- $V_{\text{glue}}(\ell, \bar{\ell}, T)$: confining, center-sym., T-power-law at low T,

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$$\left. + \ln\left[1 + 3\bar{\ell} e^{-\beta(\varepsilon_q + \mu)} + 3\ell e^{-2\beta(\varepsilon_q + \mu)} + e^{-3\beta(\varepsilon_q + \mu)} \right] \right\},$$

see e.g. K. Fukushima, Phys. Lett. B 591, 277 (2004), [hep-ph/0310121]

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$$\left. + \ln\left[1+3\bar{\ell} e^{-\beta(\varepsilon_q+\mu)}+3\ell e^{-2\beta(\varepsilon_q+\mu)}+e^{-3\beta(\varepsilon_q+\mu)} \right] \right\},$$

$$\underset{\substack{\beta M \gg 1,\\ \ell,\bar{\ell} \ll 1\\ |\mu| < M}}{\simeq} - \left\{ \ell(e^{\beta\mu}f_{\beta M} + e^{-2\beta\mu}f_{2\beta M}) + \bar{\ell}(e^{2\beta\mu}f_{2\beta M} + e^{-\beta\mu}f_{\beta M}) + (e^{3\beta\mu} + e^{-3\beta\mu})f_{3\beta M}/3 \right\}$$

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- With $f_y \pi^2 / 3TN_f = \displaystyle\int_0^\infty dx\, x^2 e^{-y\sqrt{1+x^2}} \sim 3\sqrt{2\pi} e^{-y} y^{-3/2}$ for $y \gg 1$.

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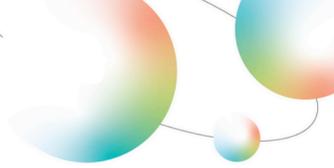# Calculating ℓ, $\bar{\ell}$ at low T

$$\Omega \simeq V_{\text{glue}} - \left\{ \ell(e^{\beta\mu}f_{\beta M} + e^{-2\beta\mu}f_{2\beta M}) + \bar{\ell}(e^{2\beta\mu}f_{2\beta M} + e^{-\beta\mu}f_{\beta M}) + (e^{3\beta\mu} + e^{-3\beta\mu})f_{3\beta M}/3 \right\}$$

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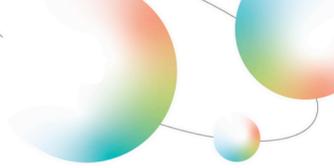
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- $\left. \begin{pmatrix} \partial_\ell^2 V_{\text{glue}}^{\ =\ 0} & \partial_\ell \partial_{\bar{\ell}} V_{\text{glue}} \\ \partial_\ell \partial_{\bar{\ell}} V_{\text{glue}} & \partial_{\bar{\ell}}^2 V_{\text{glue}}^{\ =\ 0} \end{pmatrix} \right|_{\ell,\bar{\ell}=0} \begin{pmatrix} \ell \\ \bar{\ell} \end{pmatrix} \simeq \begin{pmatrix} e^{\beta\mu}f_{\beta M} + e^{-2\beta\mu}f_{2\beta M} \\ e^{-\beta\mu}f_{\beta M} + e^{2\beta\mu}f_{2\beta M} \end{pmatrix}.$

8

Calculating $\ell, \bar{\ell}$ at low T

Chemical potential μ dependence

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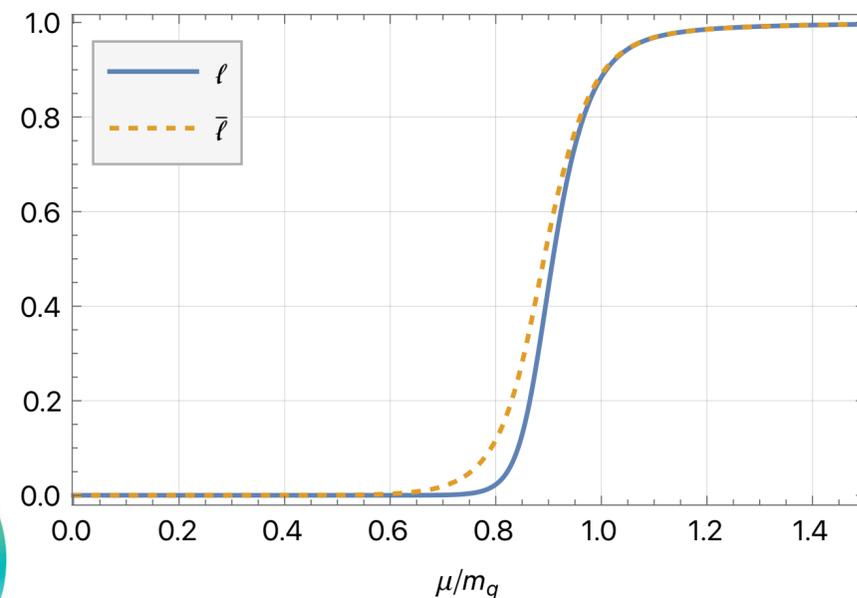# Calculating ℓ, $\bar{\ell}$ at low T

Chemical potential μ dependence

- $\left. \begin{pmatrix} 0 & \partial_\ell \partial_{\bar{\ell}} V_{\text{glue}} \\ \partial_\ell \partial_{\bar{\ell}} V_{\text{glue}} & 0 \end{pmatrix} \right|_{\ell, \bar{\ell}=0} \begin{pmatrix} \ell \\ \bar{\ell} \end{pmatrix} \simeq \begin{pmatrix} e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M} \\ e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M} \end{pmatrix},$

- $\Rightarrow \ell \simeq \dfrac{1}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}} \big|_{\ell, \bar{\ell}=0}} (e^{-\beta\mu} f_{\beta m_q} + e^{2\beta\mu} f_{2\beta m_q}), \quad \bar{\ell} \simeq \dfrac{1}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}} \big|_{\ell, \bar{\ell}=0}} (e^{\beta\mu} f_{\beta m_q} + e^{-2\beta\mu} f_{2\beta m_q})$
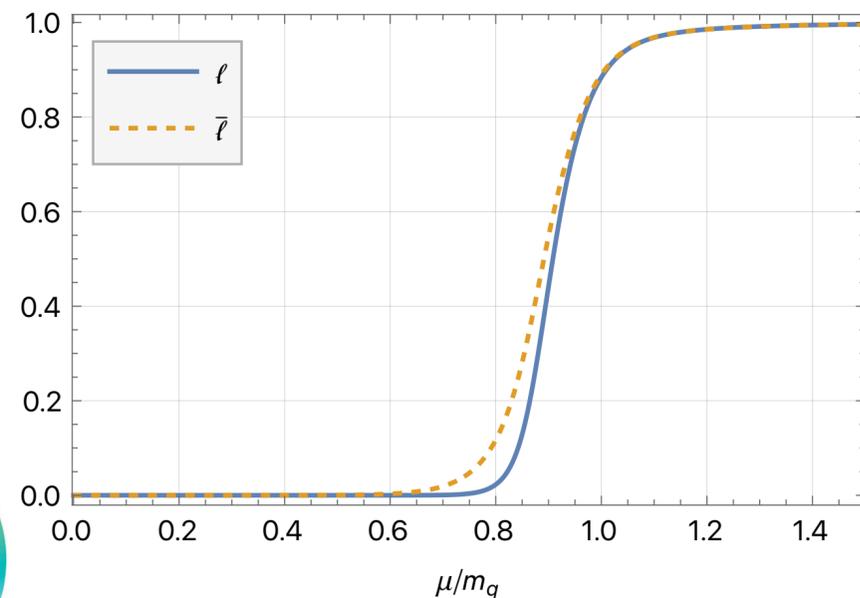
Calculating ℓ, $\bar{\ell}$ at low T

Chemical potential μ dependence

- $$\begin{pmatrix} 0 & \partial_\ell \partial_{\bar{\ell}} V_{\text{glue}} \\ \partial_\ell \partial_{\bar{\ell}} V_{\text{glue}} & 0 \end{pmatrix}\Bigg|_{\ell,\bar{\ell}=0} \begin{pmatrix} \ell \\ \bar{\ell} \end{pmatrix} \simeq \begin{pmatrix} e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M} \\ e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M} \end{pmatrix},$$

- $$\Rightarrow \ell \simeq \frac{1}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}}|_{\ell,\bar{\ell}=0}} (e^{-\beta\mu} f_{\beta m_q} + e^{2\beta\mu} f_{2\beta m_q}), \quad \bar{\ell} \simeq \frac{1}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}}|_{\ell,\bar{\ell}=0}} (e^{\beta\mu} f_{\beta m_q} + e^{-2\beta\mu} f_{2\beta m_q})$$

$|\mu| > M$: deconfined (see backup)
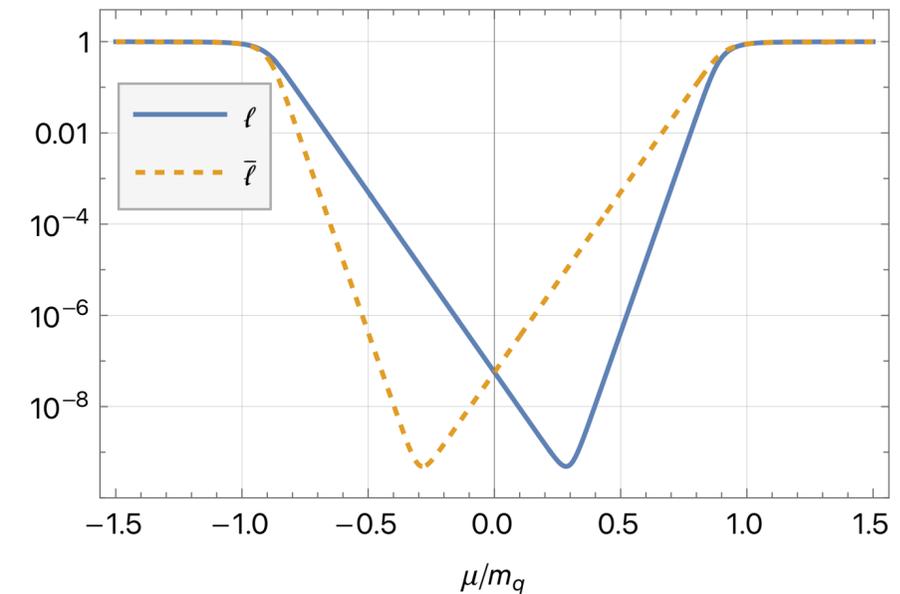
9

Calculating ℓ, $\bar{\ell}$ at low T

Chemical potential μ dependence

- $$\begin{pmatrix} 0 & \partial_\ell \partial_{\bar{\ell}} V_{\text{glue}} \\ \partial_\ell \partial_{\bar{\ell}} V_{\text{glue}} & 0 \end{pmatrix}\bigg|_{\ell,\bar{\ell}=0} \begin{pmatrix} \ell \\ \bar{\ell} \end{pmatrix} \simeq \begin{pmatrix} e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M} \\ e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M} \end{pmatrix},$$

- $$\Rightarrow \ell \simeq \frac{1}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}}\big|_{\ell,\bar{\ell}=0}} (e^{-\beta\mu} f_{\beta m_q} + e^{2\beta\mu} f_{2\beta m_q}), \quad \bar{\ell} \simeq \frac{1}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}}\big|_{\ell,\bar{\ell}=0}} (e^{\beta\mu} f_{\beta m_q} + e^{-2\beta\mu} f_{2\beta m_q})$$
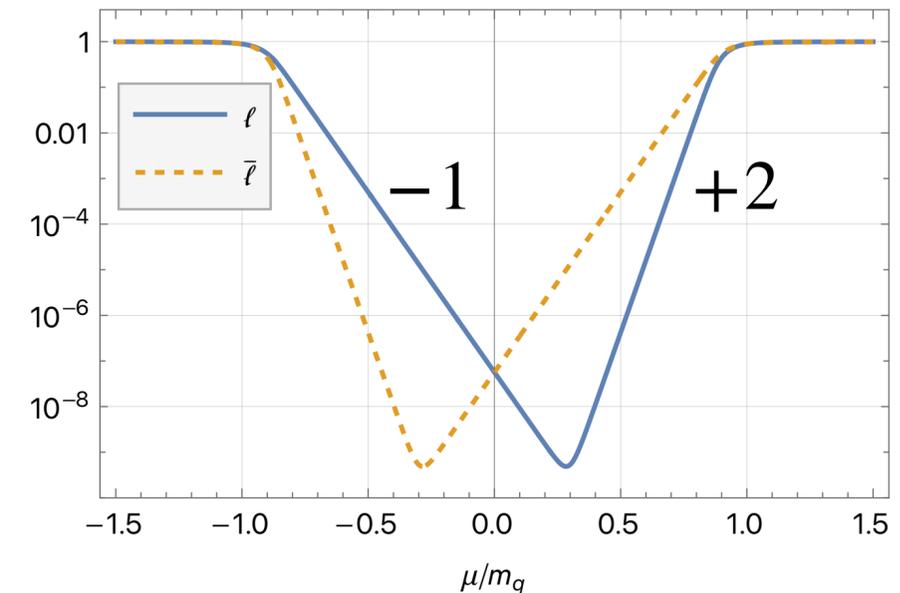

Log-plot shows more structure!

9

Calculating ℓ, $\bar{\ell}$ at low T

Chemical potential μ dependence

- $$\begin{pmatrix} 0 & \partial_\ell \partial_{\bar{\ell}} V_{\text{glue}} \\ \partial_\ell \partial_{\bar{\ell}} V_{\text{glue}} & 0 \end{pmatrix}\Bigg|_{\ell,\bar{\ell}=0} \begin{pmatrix} \ell \\ \bar{\ell} \end{pmatrix} \simeq \begin{pmatrix} e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M} \\ e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M} \end{pmatrix},$$

- $\Rightarrow \ell \simeq \dfrac{1}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}}\big|_{\ell,\bar{\ell}=0}} \boxed{(e^{-\beta\mu} f_{\beta m_q} + e^{2\beta\mu} f_{2\beta m_q})}, \quad \bar{\ell} \simeq \dfrac{1}{\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}}\big|_{\ell,\bar{\ell}=0}} (e^{\beta\mu} f_{\beta m_q} + e^{-2\beta\mu} f_{2\beta m_q})$

$-1 \qquad +2$

μ-derivative

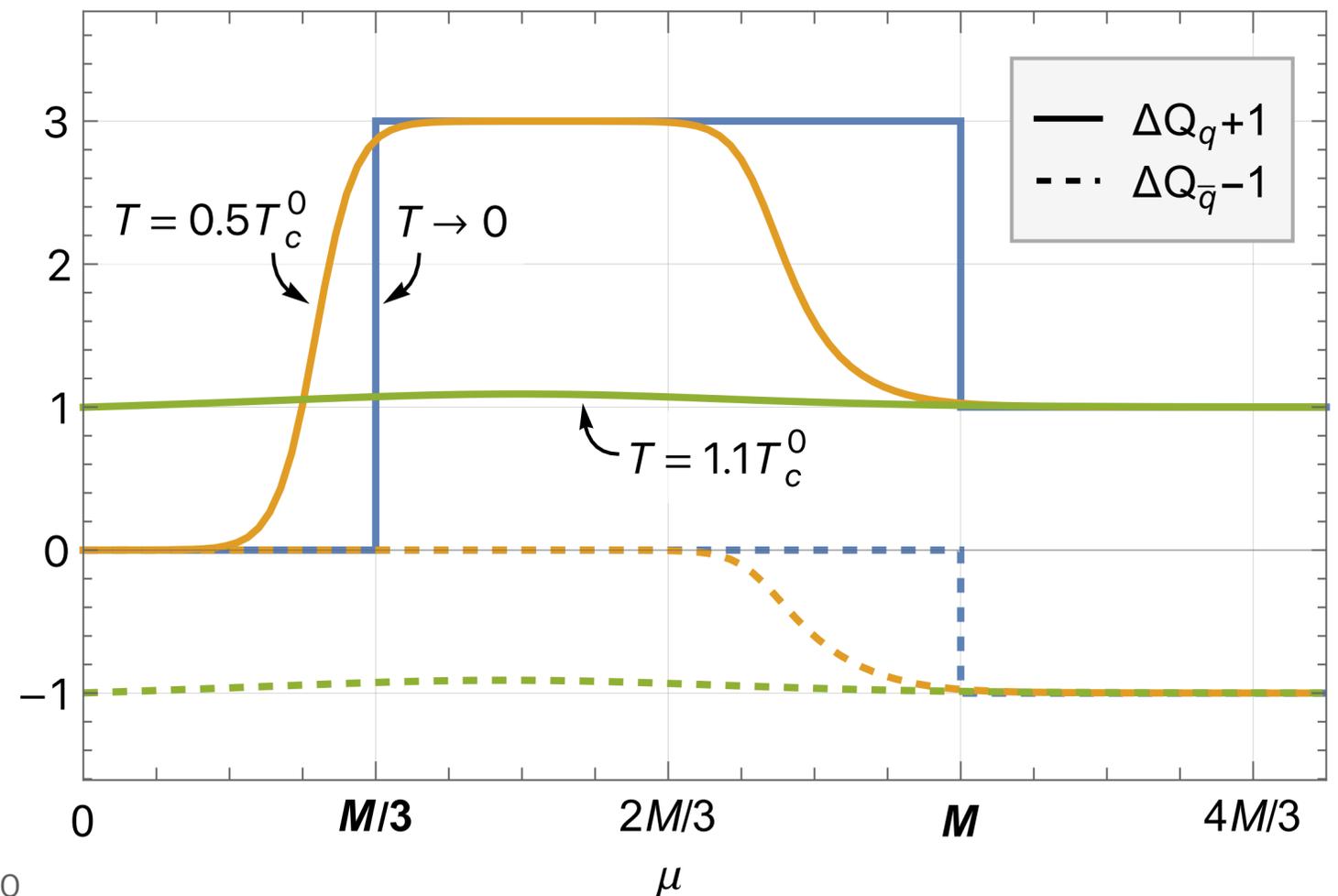

9

Net quark number gains $\Delta Q_{q,\bar{q}}$
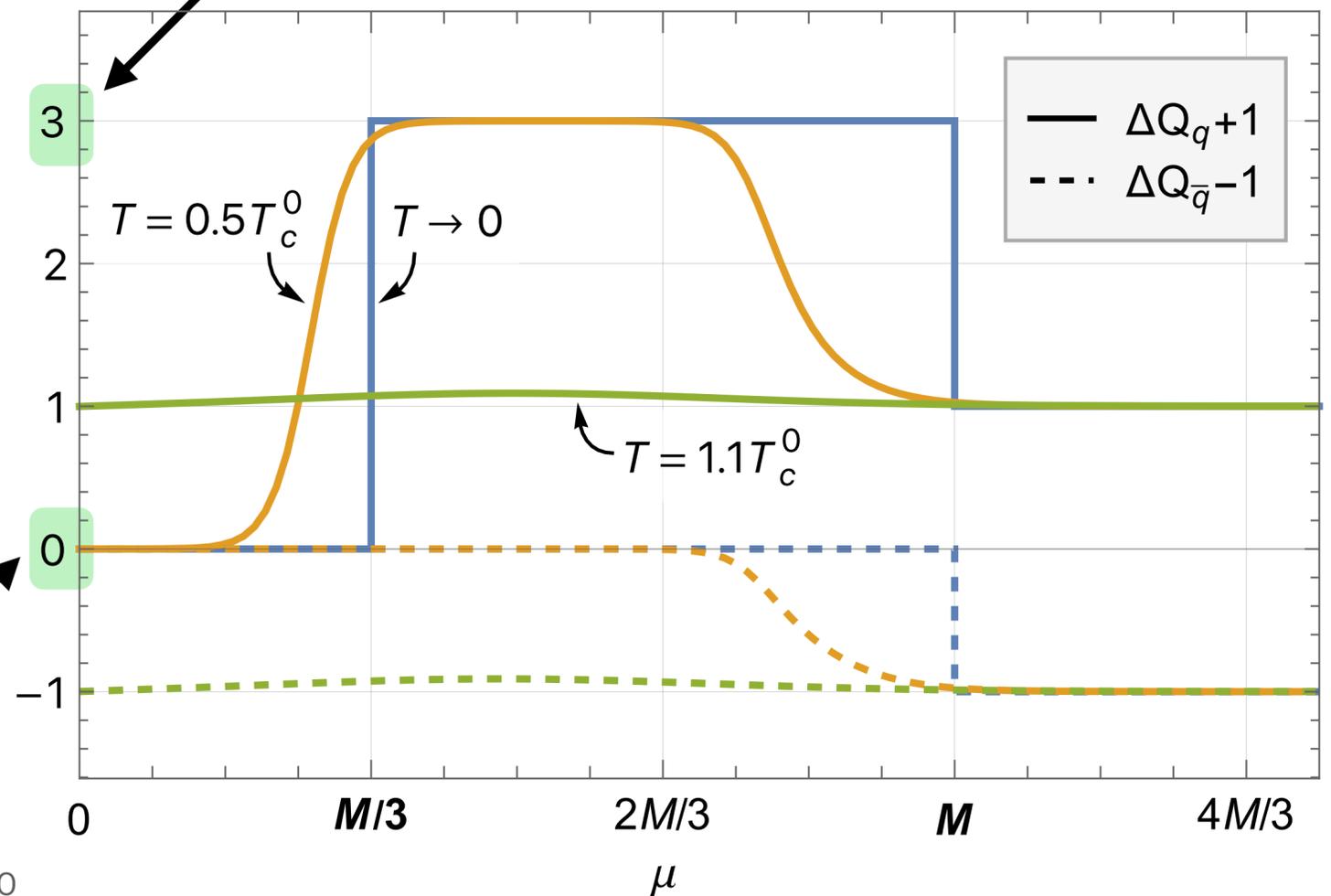
From Polyakov loop potential

- Then the **net quark number gain** is:

$$\Delta Q_q + 1 \simeq \frac{3}{1 + e^{-3\beta\mu} f_{\beta M}/f_{2\beta M}},$$

$$\Delta Q_{\bar{q}} - 1 \simeq \frac{-3}{1 + e^{3\beta\mu} f_{\beta M}/f_{2\beta M}},$$
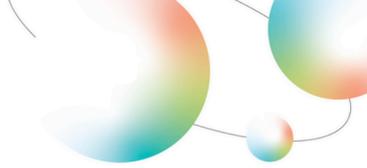
- Independent of V_{glue}.

"-like" since don't know color representation

Net quark number gains $\Delta Q_{q,\bar{q}}$

From Polyakov loop potential
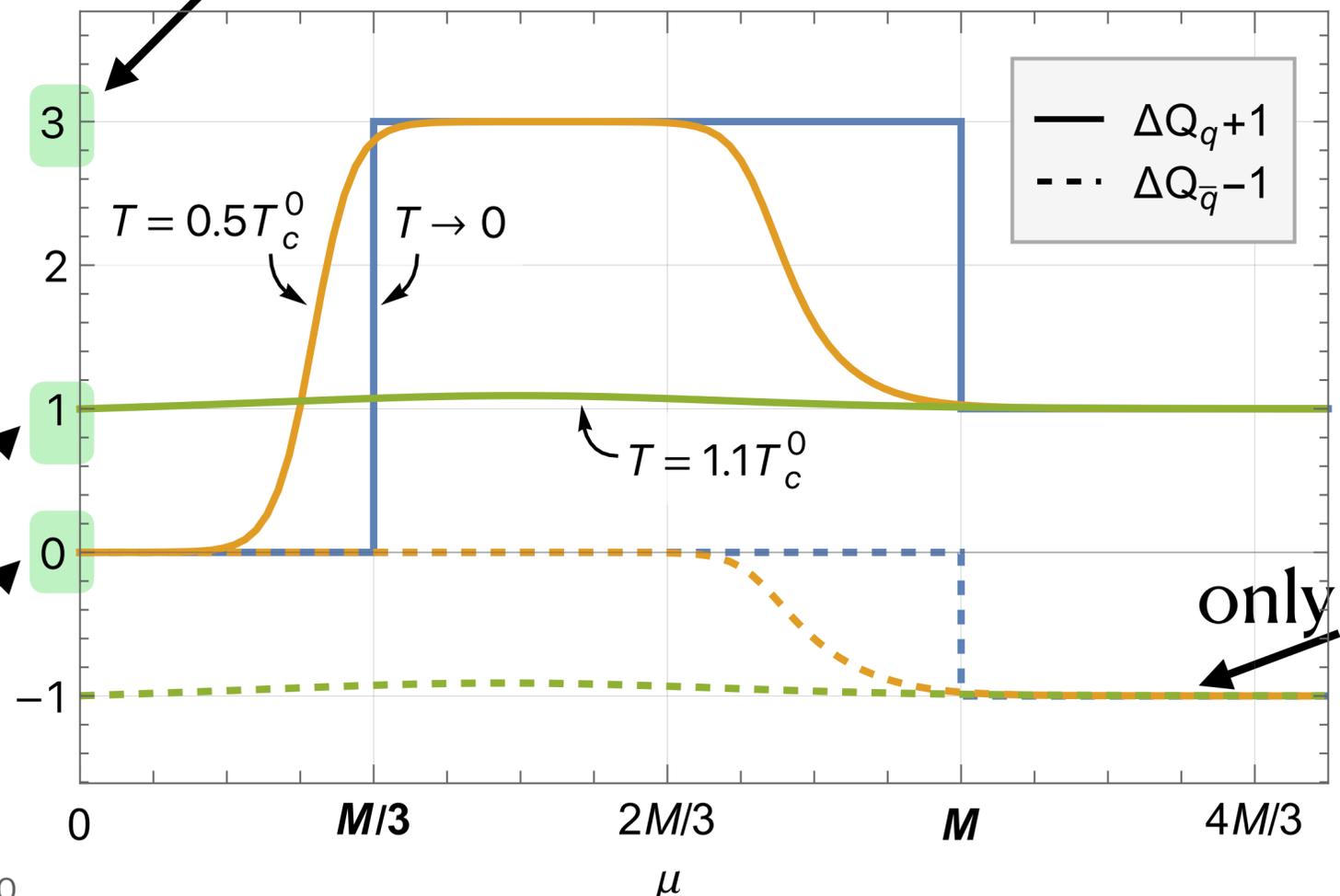
- Then the **net quark number gain** is:

- $\Delta Q_q + 1 \simeq \dfrac{3}{1 + e^{-3\beta\mu} f_{\beta M}/f_{2\beta M}},$

- $\Delta Q_{\bar{q}} - 1 \simeq \dfrac{-3}{1 + e^{3\beta\mu} f_{\beta M}/f_{2\beta M}},$

- Independent of V_{glue}.

baryon-like configurations

meson-like configurations

"-like" since don't know color representation

Net quark number gains $\Delta Q_{q,\bar{q}}$

From Polyakov loop potential

- Then the **net quark number gain** is:

- $$\Delta Q_q + 1 \simeq \frac{3}{1 + e^{-3\beta\mu} f_{\beta M}/f_{2\beta M}},$$

- $$\Delta Q_{\bar{q}} - 1 \simeq \frac{-3}{1 + e^{3\beta\mu} f_{\beta M}/f_{2\beta M}},$$

- Independent of V_{glue}.

only the added q

meson-like configurations

baryon-like configurations

$T = 0.5T_c^0$ $T \to 0$

$T = 1.1T_c^0$

only the $\bar{q}$

Legend:
—— $\Delta Q_q + 1$
- - - $\Delta Q_{\bar{q}} - 1$

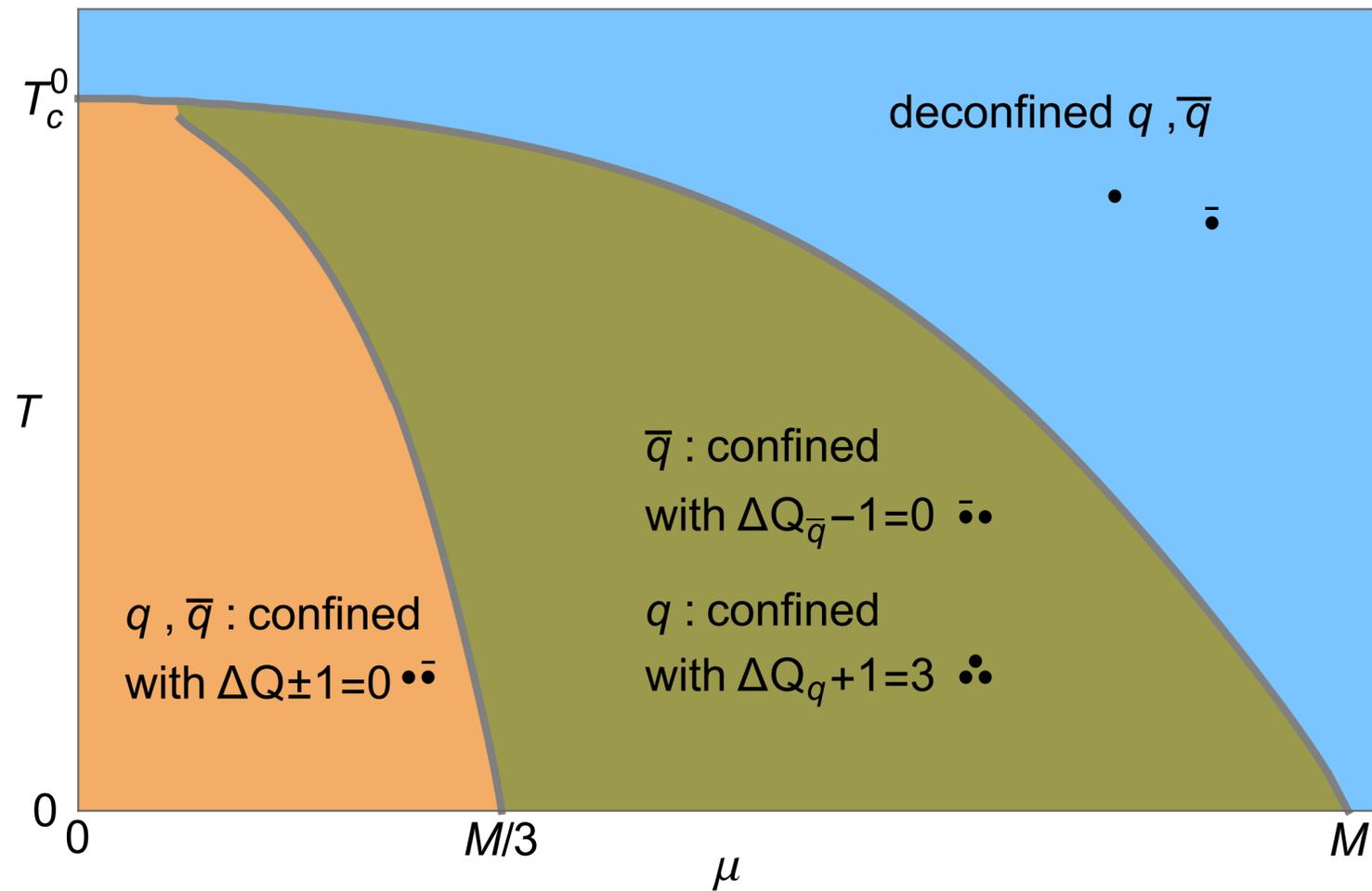x-axis: 0, $M/3$, $2M/3$, M, $4M/3$, μ

"-like" since don't know color representation

QCD phase diagram
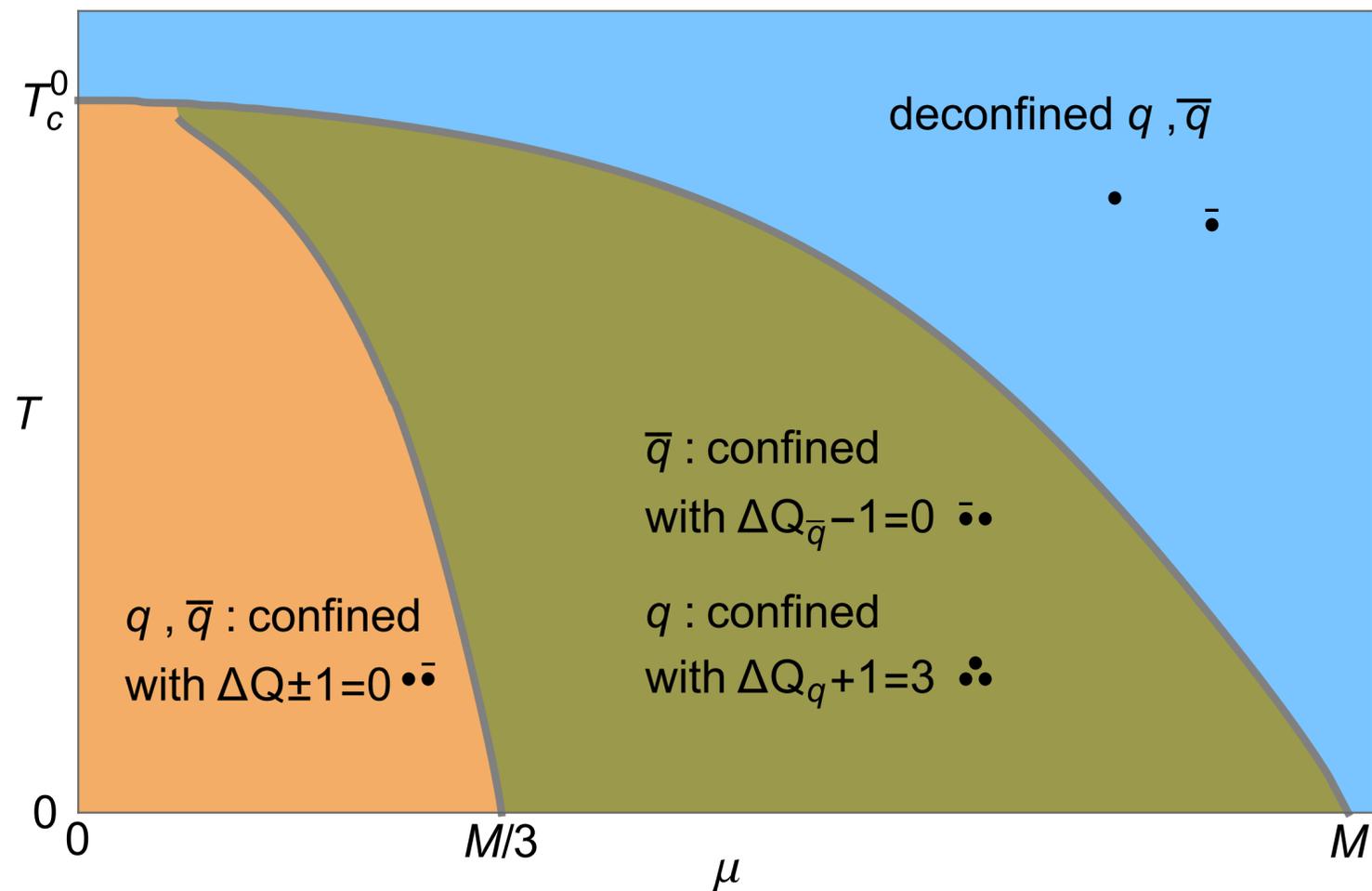## Net quark number probes degrees of freedom

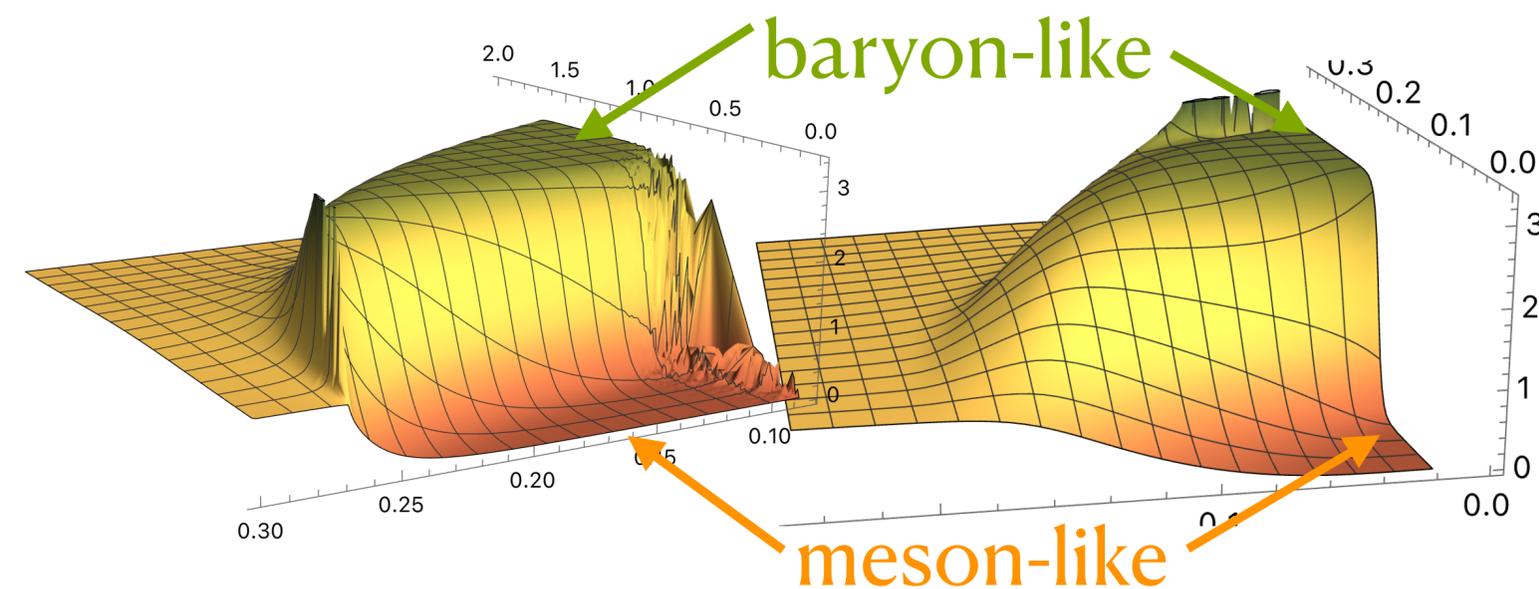- **Net quark number gain** for heavy quark QCD:

QCD phase diagram
Net quark number probes degrees of freedom

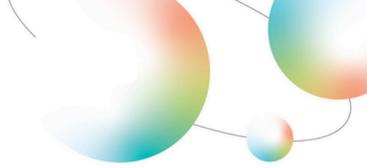- **Net quark number gain** for heavy quark QCD:

Heavy-quark QCD vs PNJL model:

Plateau of $\Delta Q_{q,\bar{q}} \pm 1$ for:

whole confined phase vs only at low T

Outlook

Possible applications of the net quark number gain

- Theoretical observable, sensitive to net quark number content of active dof's,

- Sensitive to a critical point $\rightarrow \Delta Q_q + 1 \gg 3$,

- Probe dof's in other phases, e.g. diquarks $\rightarrow \Delta Q_q + 1 \overset{?}{=} 2$,

- The net quark number gain is essentially $\langle \Phi Q \rangle$,
  other combinations of Polyakov loops and conserved charges could be interesting:
  e.g. $\langle \Phi Q^a Q^a \rangle$ color Casimir, or $\langle \Phi S \rangle$ strangeness, ...

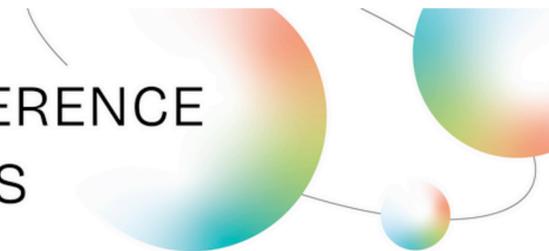- SU(2) and Heavy-quark (& imaginary μ?) lattice data exists $\rightarrow$ tests possible.

Thank you! :)

Backup

For questions also: victor-tomas.mari-surkau@polytechnique.edu

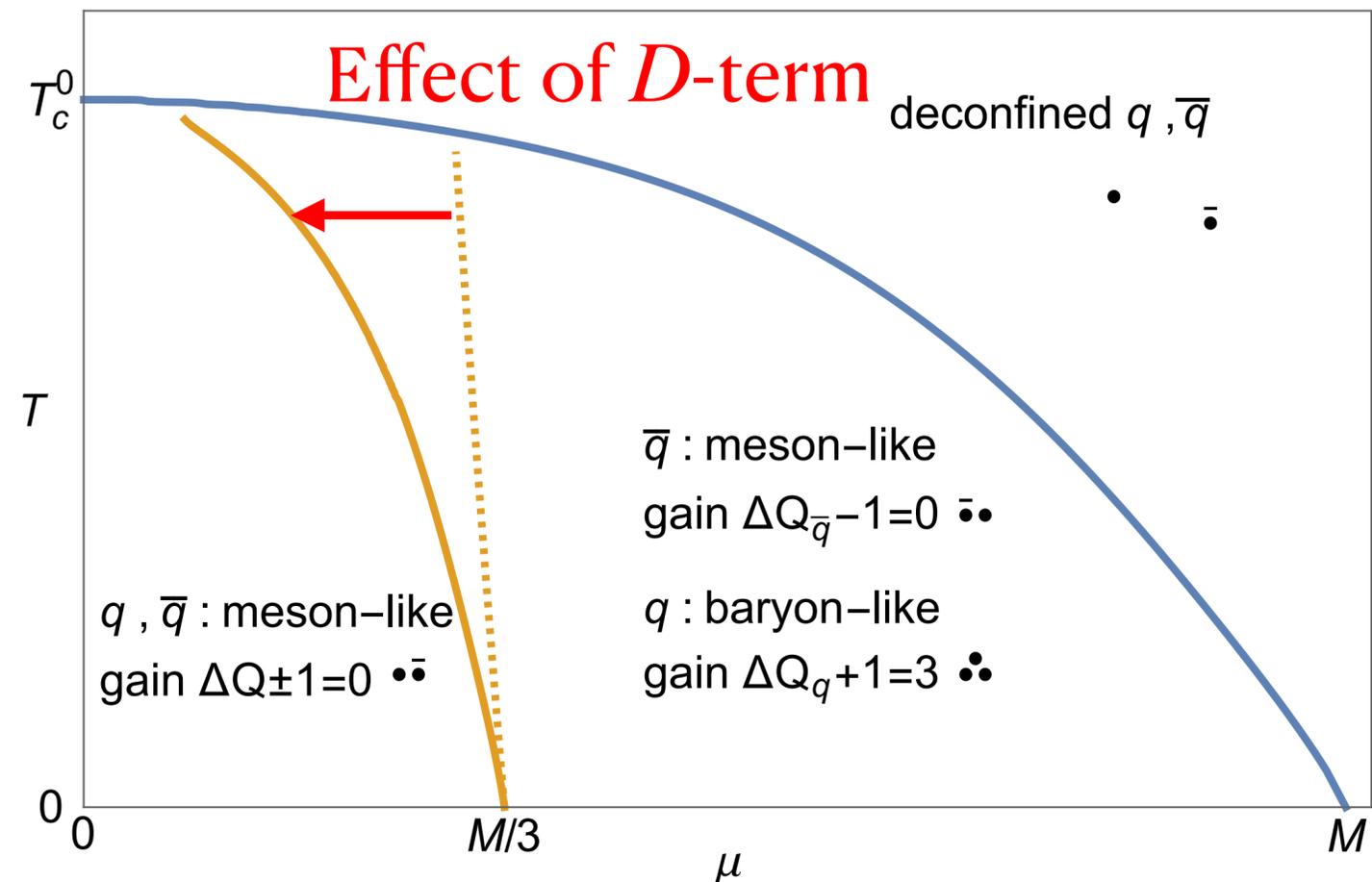# Beyond the linear order in ℓ, $\bar{\ell}$

Modifications at finite temperatures

- At non-zero $\mu > 0$: ℓ more suppressed than $\bar{\ell}$, and vice-versa at $\mu < 0$

- In equation with ℓ take into account the order $\dfrac{1}{2}\bar{\ell}^2\partial_{\bar{\ell}}^3 V_{\text{glue}}$,

- $\Delta Q_q + 1 \simeq \dfrac{3}{1 + D\, e^{-3\beta\mu} f_{\beta M}/f_{2\beta M}}$, with

$$D = \frac{1 - C\, \dfrac{\partial_{\bar{\ell}}^3 V_{\text{glue}}}{(\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}})^2}\, f_{2\beta M}}{1 - \dfrac{1}{2} C\, \dfrac{\partial_{\bar{\ell}}^3 V_{\text{glue}}}{(\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}})^2}\, f_{\beta M}^2/f_{2\beta M}}$$

$C \equiv 3 N_f T M^3$

Effect of D-term

deconfined q, $\bar{q}$

$\bar{q}$: meson–like
gain $\Delta Q_{\bar{q}} - 1 = 0$

q, $\bar{q}$: meson–like
gain $\Delta Q \pm 1 = 0$

q : baryon–like
gain $\Delta Q_q + 1 = 3$

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Thermodynamics with confinement

Expectations for $\Delta Q_q + 1$

- Consider **heavy-quark QCD** at low T: what states dominate Z_q?

- Heavy-quark, non-relativistic system:

$$H - \mu Q \simeq (N_q + N_{\bar{q}})m_q - \mu(N_q - N_{\bar{q}}),$$

- If only hadron dof's: $N_q - N_{\bar{q}} + 1 = 3k, \; k \in \mathbb{Z} \Rightarrow (N_q, N_{\bar{q}}) = (0,1), (2,0) \, (3,1), \dots$

$$H - \mu Q \simeq \underset{(0,1)}{m_q + \mu}, \; \underset{(2,0)}{2m_q - 2\mu}, \; \underset{(3,1)}{4m_q - 2\mu}, \dots$$
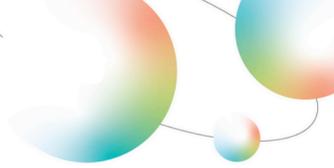
- If $\mu < m_q/3$ then (0,1) dominates, if $\mu > m_q/3$ then (2,0), never (3,1) and higher.

$$\to \Delta Q_q + 1 = 0 \qquad\qquad\qquad \to \Delta Q_q + 1 = 3$$

Meson   Baryon
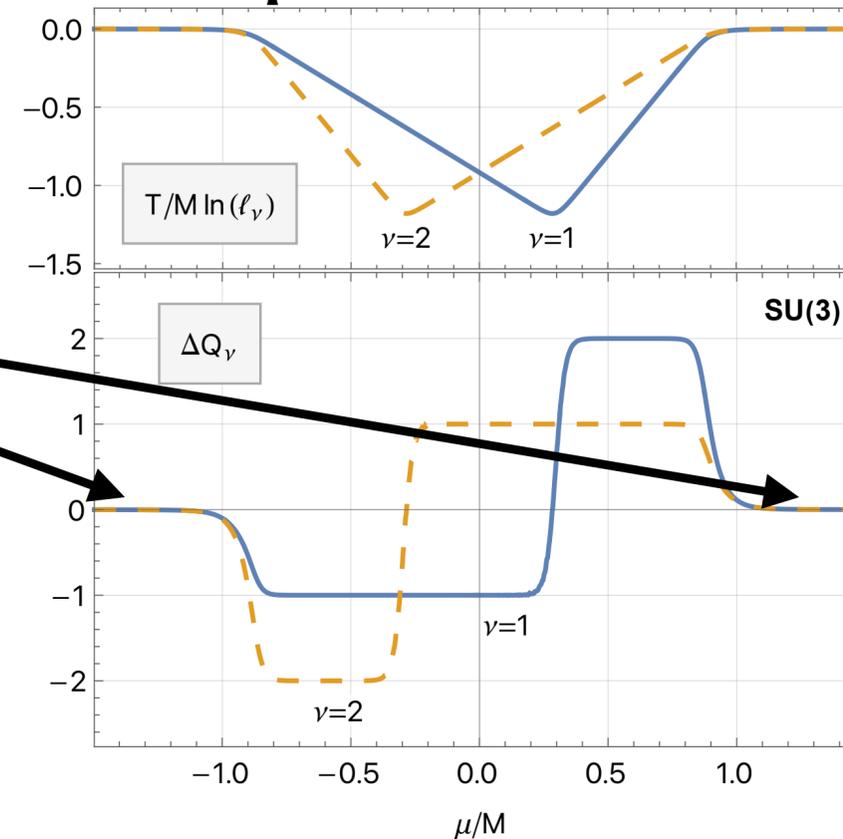
2025
XQCD

The case of $|\mu| > m_q$

Heavy quarks deconfine

- The quark potential is only power-law suppressed at low T, no longer exponentially,

- Polyakov loops either $\rightarrow 1$, or $\xrightarrow{\text{power−law}} 0$, depending on if V_q or V_{glue} dominate,

- For heavy quarks and any V_{glue}: see as deconfined phase, (nuclear liquid in QCD),

- μ-derivatives of $\ln(\ell)$ and $\ln(\bar{\ell})$ vanish,

- $\Delta Q_{q,\bar{q}} \pm 1 \simeq \pm 1, \Delta Q_{q,\bar{q}} = 0$

Generalization to SU(N$_c$)

$(N_c - 1)$ **fundamental Polyakov loops** ℓ_ν ↔ **adding** ν **quarks**

- $\Delta Q_\nu + \nu$: with the ν added q's get a ν-mesons- or one-baryon-like configuration, $\xrightarrow{}$ N$_c$ quarks

- Change from meson- to baryon-like being favorable at $\mu = m_q(1 - 2\nu/N_c)$,

- Access to other non-fundamental and product representations $\mathcal{R}$:

adjoint Polyakov loops not confined