Neutrality constraints and the CFL phase in the quark meson diquark model

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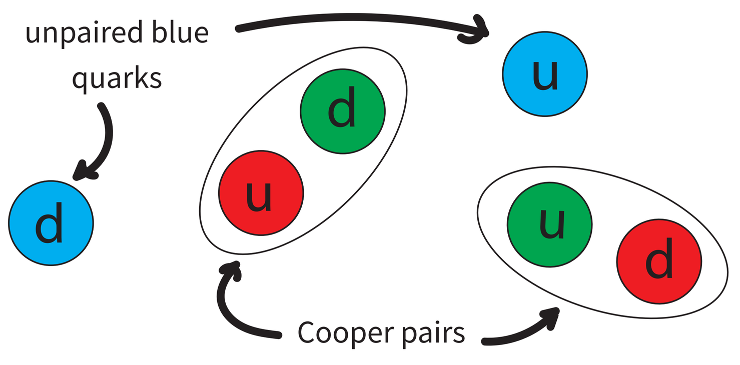
Abstract



Pairing in the 2SC phase

We employ the two-flavor quark-meson diquark model as a low energy renormalizable model for QCD at nonzero quark chemical potentials [1, 2]. Parameters originating from the quark-meson sector are determined from vacuum physics, while we keep the new parameters as free parameters. Our model is calculated to one loop in the large N_c apprixmation. To study the effects of enforcing charge neutrality, we choose two main parameter sets for the two-flavor superconducting sector inspired by their quark-meson counterparts. Several interesting analytical results can be obtained for large quark chemical potential. We also discuss the three flavor extension of this model.

Model and theory

The two-flavor QMD model possesses the global symmetries $SU(2)_L \times SU(2)_R \times U(1)_B \times SU(3)_C$. The Lagrangian density is [1]

$$
\begin{aligned}
\mathcal{L} \;=\;& \frac{1}{2}(\partial_\mu\sigma)(\partial^\mu\sigma) + \frac{1}{2}(\partial_\mu\pi_0)(\partial^\mu\pi_0) + (\partial_\mu + i\mu_I\delta_{\mu 0})\,\pi^+\left(\partial^\mu - i\mu_I\delta^{\mu 0}\right)\pi^- - \frac{1}{2}m^2(\sigma^2+\vec{\pi}^2) - \frac{\lambda}{24}(\sigma^2+\vec{\pi}^2)^2 \\
&+h\sigma + (\partial_\mu + 2i\mu_a\delta_{\mu 0})\,\Delta_a^\dagger\left(\partial^\mu - 2i\mu_a\delta^{\mu 0}\right)\Delta_a - m_\Delta^2\Delta_a^\dagger\Delta_a - \frac{\lambda_3}{12}(\sigma^2+\vec{\pi}^2)\Delta_a^\dagger\Delta_a - \frac{\lambda_\Delta}{6}\left(\Delta_a^\dagger\Delta_a\right)^2 \\
&+\bar{\psi}(i\,\partial\!\!\!/ + \gamma^0\hat{\mu})\psi - g\bar{\psi}[\sigma + i\gamma^5\vec{\pi}\cdot\vec{\tau}]\psi + \frac{1}{2}g_\Delta\bar{\psi}_b^C\Delta_a\gamma^5\tau_2\epsilon_{abc}\psi_c + \frac{1}{2}g_\Delta\bar{\psi}_b\Delta_a^\dagger\gamma^5\tau_2\epsilon_{abc}\psi_c^C\;,
\end{aligned}
$$

Our goal is to determine the four free parameters through neutron star observables. One expects such dense matter to be charge- and color neutral. In this work we have investigated the effects of neutrality with a two-flavor color-superconducting system. For two flavors the only possibility is the 2SC phase. A diquark condensate breaks the $SU(3)_C$ global symmetry down to $SU(2)_C$. We apply this model to non-strange quark stars as a first step towards more realistic hybrid constructions. Through a renormalization group improvement we calculate the thermodynamic potential Ω, which is independent of renormalization scale. To determine the quark condensate, superconducting gap and chemical potentials we solve the gap equations and neutrality conditions,

$$
\frac{d\Omega}{d\phi}=0,\quad \frac{d\Omega}{d\Delta}=0,\quad \frac{d\Omega}{d\mu_e}=0,\quad \frac{d\Omega}{d\mu_8}=0
$$

Condensates

Dashed = Asymptotic values!

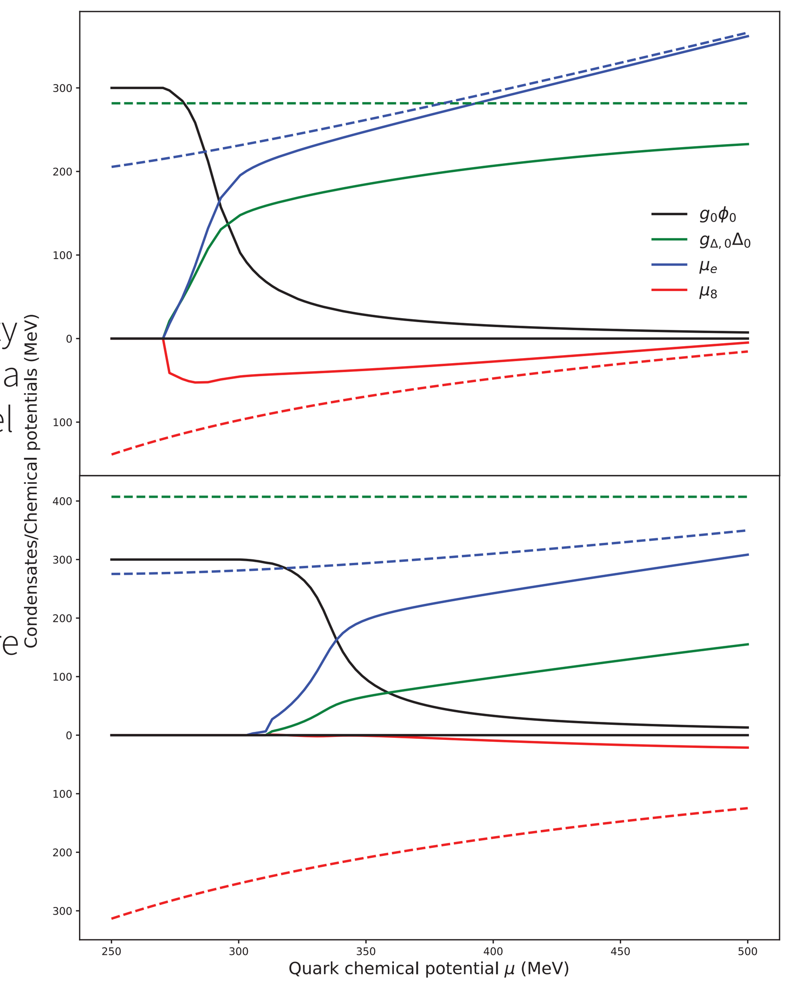
We observe strong dependence on g_Δ of the magnitude of the gap and transition point
Quark condensate decreases to zero for large μ
Too low value of g_Δ leads to very large asymptotic value of the gap

- μ_8 cross the x-axis for large μ
For large μ we can solve all gap and neutrality conditions analytically. The gap approaches a constant - an effect not observed at tree-level

$$
\bar{\Delta}_0^2 \simeq \frac{m_q^2}{g_{\Delta,0}^2}\exp\left(\frac{(4\pi)^2}{4g_{\Delta,0}^2} - 1\right)
$$

- The electric and color chemical potentials are

$$
\begin{aligned}
\mu_e &\simeq 0.57\mu\left(1 + 0.36\frac{g_{\Delta,0}^2\bar{\Delta}_0^2}{\mu^2}\right)\\
\mu_8 &\simeq 0.09\mu\left(1 - 5.91\frac{g_{\Delta,0}^2\bar{\Delta}_0^2}{\mu^2}\right)
\end{aligned}
$$

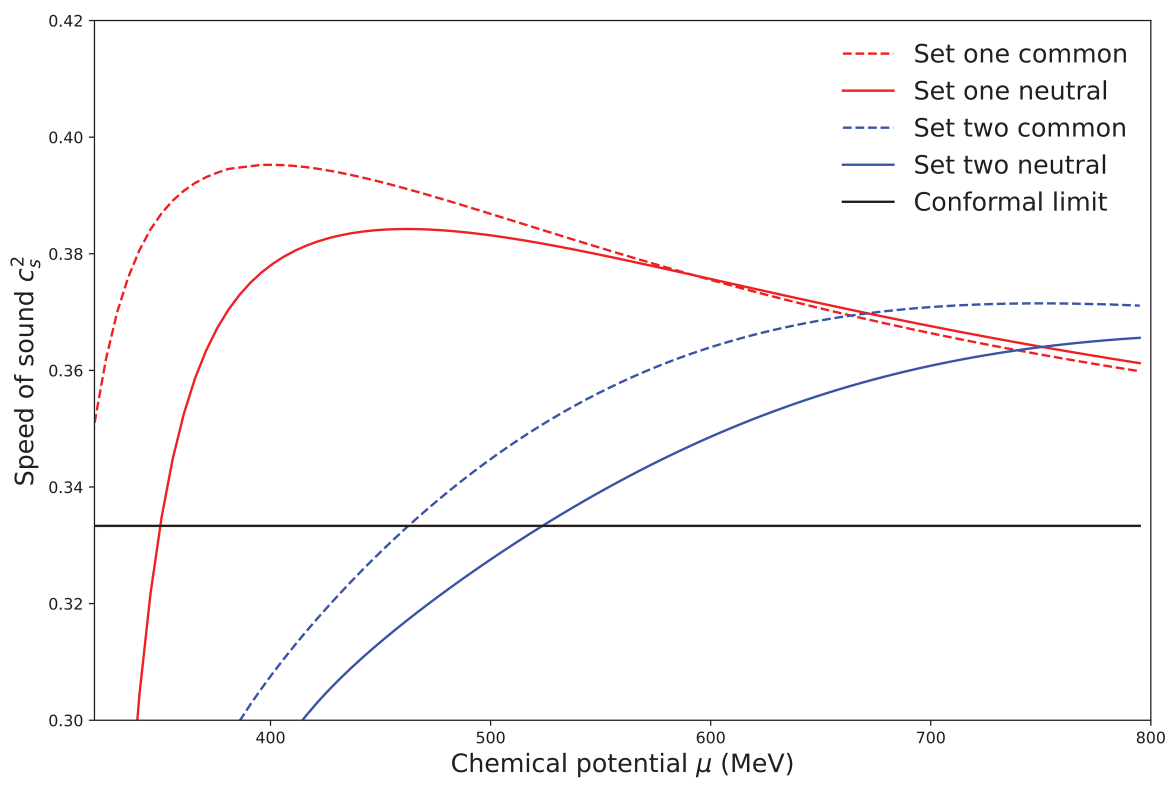


Speed of sound

- From the thermodynamic potential we can calculate then pressure P and energy density ε. The speed of sound is given by

$$
c_s^2 = \frac{dP}{d\epsilon}
$$

- The speed of sound is an interesting quantitiy to study when considering the application to neutron stars. Dashed lines are without electric and color charge neutrality, and solid lines are with.
- Imposing charge neutrality lowers the speed of sound for lower chemical potential and increases it for higher chemical potential.
- The speed of sound approaches the conformal limit from above - This is a generic behavior of the model

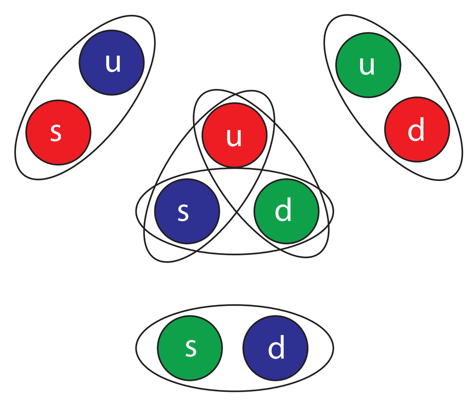


The three flavor quark meson diqark model

- The densities in the core of neutron stars are potentially high enough that strange quarks need to be included and we must extend the model to include three flavors of quarks
- We then have the color-flavor locked phase that breaks the global symmetry group

$$
SU(3)_{\text{color}} \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{\text{color}+L+R} \times Z_2
$$

- Now have 6 gap equations and 8 free parameters that are coupled to the diquark section of the Lagrangian
- Goal is to constrain these parameters through NS-constraints on the equation of state

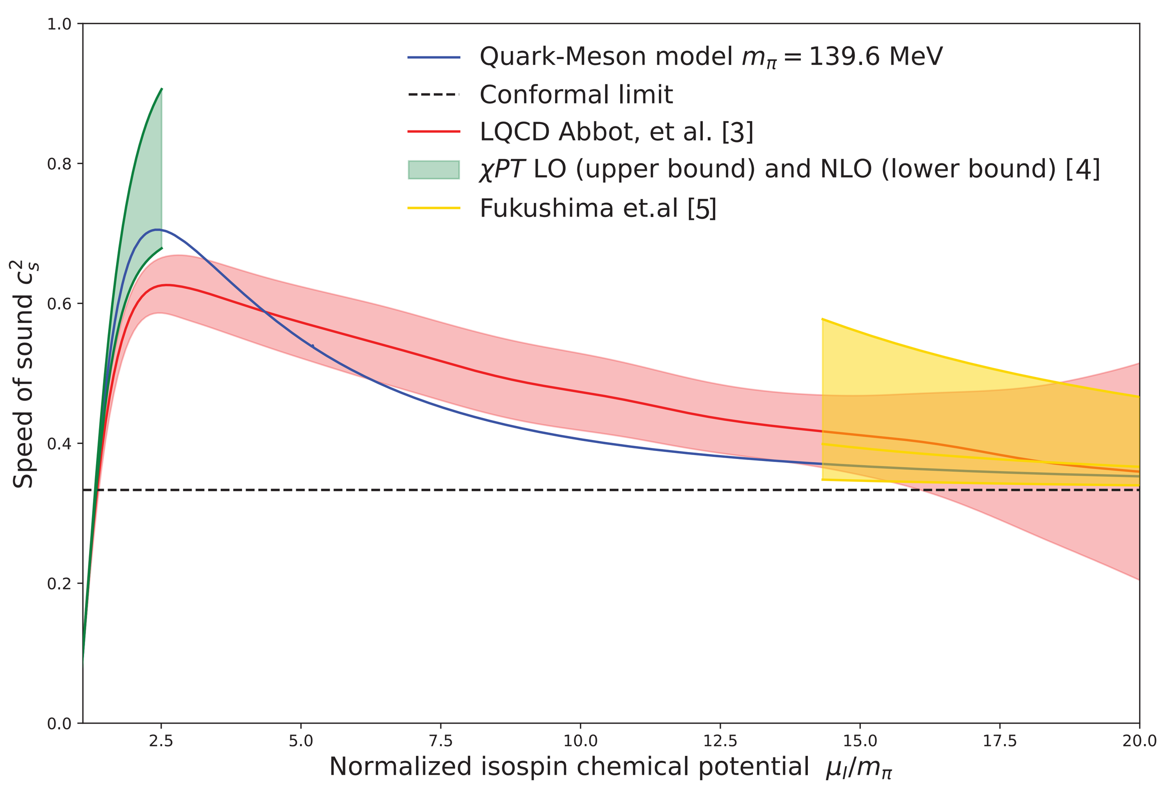


Pairing in the CFL phase

$$
\begin{aligned}
\mathcal{L}_{\text{scalar}} &= \langle D_\mu\Sigma^\dagger D^\mu\Sigma\rangle - m^2\langle\Sigma^\dagger\Sigma\rangle - \lambda_1\langle\Sigma^\dagger\Sigma\rangle^2 - \lambda_2\langle(\Sigma^\dagger\Sigma)^2\rangle + \langle H(\Sigma+\Sigma^\dagger)\rangle + c[\det\Sigma + \det\Sigma^\dagger]\;,\\
\mathcal{L}_{\text{quark}} &= \bar{\psi}(i\partial\!\!\!/ + \gamma^0\hat{\mu})\psi\;,\\
\mathcal{L}_{\text{diquark}} &= D_\mu(\Delta_{L,i}^a)^\dagger D^\mu\Delta_{L,i}^a - m_\Delta^2(\Delta_{L,i}^a)^\dagger\Delta_{L,i}^a - \lambda_1^\Delta[(\Delta_{L,i}^a)^\dagger\Delta_{L,i}^a]^2 - \lambda_2^\Delta(\Delta_{L,i}^a)^\dagger\Delta_{L,k}^a(\Delta_{L,k}^b)^\dagger\Delta_{L,i}^b\;,\\
&\quad +L\rightarrow R - \lambda_3^\Delta[(\Delta_{L,i}^a)^\dagger\Delta_{L,i}^a][(\Delta_{R,j}^b)^\dagger\Delta_{R,j}^b]\\
\mathcal{L}_{\text{scalar}-\text{quark}} &= -g\bar{\psi}T_a(\sigma_a + i\gamma^5\pi_a)\psi\;,\\
\mathcal{L}_{\text{scalar}-\text{diquark}} &= -\lambda_3\langle\Sigma^\dagger\Sigma\rangle(\Delta_{L,i}^a)^\dagger\Delta_{L,i}^a - \lambda_4\Sigma_{ij}^\dagger\Delta_{L,k}^a(\Delta_{L,j}^a)^\dagger\Sigma_{ki} + L\rightarrow R\;,\\
&\quad +\lambda_5\Sigma_{ij}(\epsilon_{klj}\Delta_{R,k}^a)^\dagger\Sigma_{ml}\epsilon_{nmi}\Delta_{L,n}^a + \lambda_5\Sigma_{ij}^\dagger(\epsilon_{klj}\Delta_{L,k}^a)^\dagger\Sigma_{lm}^\dagger\epsilon_{nmi}\Delta_{R,n}^a\\
\mathcal{L}_{\text{quark}-\text{diquark}} &= -\frac{1}{2\sqrt{2}}g_\Delta(\bar{\psi}_{L,j}^b)^C\Delta_{L,i}^a\gamma^5\epsilon_{abc}\epsilon_{ijk}\psi_{L,k}^c - \frac{1}{2\sqrt{2}}g_\Delta\bar{\psi}_{L,j}^b(\Delta_{L,i}^a)^\dagger\gamma^5\epsilon_{abc}\epsilon_{ijk}(\psi_{L,k}^c)^C - L\rightarrow R\;,
\end{aligned}
$$

Speed of sound at finite isospin

- At finite isospin chemical potential we can compare results to lattice QCD since there is no sign problem
- We observe good qualitiative agreement with results from χPT and lattice QCD
- Approaches the conformal limit from above in agreement with Ref. [3] and Ref. [5].
- Peak is at approximately $\mu_I \approx 320$ MeV



Asymptotic results for three flavors

- The gap equations needs to be evaluated numerically - but for asymptotic densities we find analytical results
- The CFL gap can be related to the 2SC gap

$$
\Delta_0^{\text{CFL}} = 2^{-\frac{1}{3}}\Delta_0^{\text{2SC}}
$$

- The speed of sound in the CFL phase relaxes slower to the conformal limit than in the 2SC phase, but approaches from above in both cases

$$
c_s^2 = \frac{1}{3}\left(1 + \frac{2}{3}\frac{g_\Delta^2\Delta_0^2}{\mu^2}\right) \qquad c_s^2 = \frac{1}{3}\left(1 + \frac{4}{3}\frac{g_\Delta^2\Delta_0^2}{\mu^2}\right)
$$

2SC CFL

Conclusions

- Neutrality conditions have noticable effect in the QMD model and will be relevant for neutron star calculations
- The speed of sound at finite isospin agrees well with lattice calculations
- We have formulated a three flavor extension to study the CFL phase - numerical work is underway

Check out our papers here for more information on neutrality:

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Check out our paper here for more information on three flavor QMD:

References:
[1] J. O. Andersen and M. P. Nødtvedt, Phys. Rev. D 111, 034020 (2025)
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[3] R. Abbott et al., NPLQCD Collaboration, Phys. Rev. D 108, 11, 114506 (2023)
[4] J. O. Andersen, M. K. Johnsrud, Q. Yu, and H. Zhou, Phys. Rev. D 111, 034017 (2025)
[5] K. Fukushima and S. Minato e-Print: 2411.03781 [hep-ph].