The curvature of the pseudo-critical line in the QCD phase diagram from mesonic lattice correlation functions

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Introduction & Overview

- The complete understanding of the QCD phase diagram remains an open question in particle physics.
- Of particular interest is the pseudo-critical line in the $T-\mu_q$ plane which describes the change of the pseudo-critical temperature as a function of the baryon chemical potential.
- The current understanding is a smooth crossover at $\mu_B = 0$ from the low-T phase, where chiral symmetry is spontaneously broken, to the high-T phase were it is restored [2].
- Several lattice studies have shown that T_{pc} decreases with μ_B according to

$$\frac{T_{pc}(\mu_B)}{T_{pc}(\mu_B=0)} = 1 - \kappa\left(\frac{\mu_B^2}{T_{pc}(\mu_B=0)^2}\right) + O(\mu_B\text{^}4)$$

Our contribution [1]

- We propose a new method to study the change of T_{pc} as a function of μ_B using meson correlation functions computed on the lattice.
- We expand the meson correlation functions in powers of $\frac{\mu_q}{T}$ around $\mu_q = 0$ [2].

$$G(\tau,\mu_q) = G(\tau,\mu_q)\Big|_{\mu_q=0} + \frac{\mu_q}{T}T\frac{\partial G(\tau,\mu_q)}{\partial \mu_q}\Big|_{\mu_q=0} + \frac{1}{2}\frac{\mu_q^2}{T^2}T^2\frac{\partial^2 G(\tau,\mu_q)}{\partial \mu_q^2}\Big|_{\mu_q=0} + O(\frac{\mu_q^3}{T^3})$$

- Upon chiral symmetry restoration we expect a degeneracy between the vector and axial-vector correlators which we use to determine $T_{pc}(\mu_q)$.

Methodology

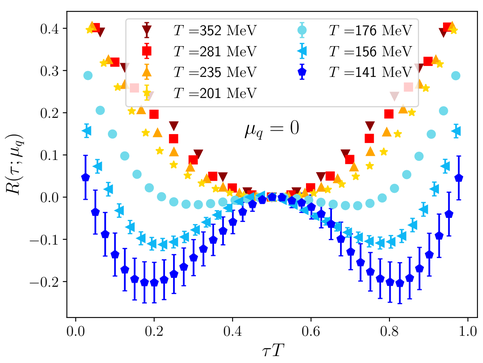
- We compute lattice correlation functions on two set of gauge ensembles generated by the FASTSUM collaboration. They are referred to as Generation 2 and Generation 2L with $M_\pi = 391(3)MeV$ and $M_\pi = 239(1)MeV$ respectively.
- To probe restoration of chiral symmetry we introduce the ratio:

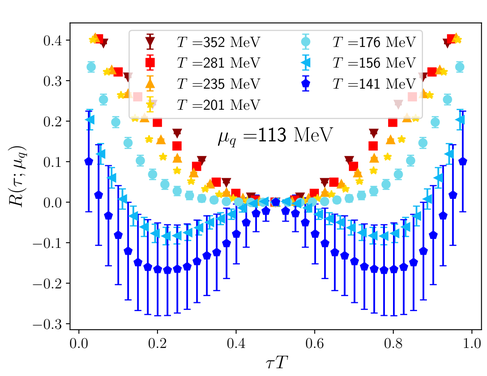
$$R(\tau,\mu_q) = \frac{\widetilde{G_V}(\tau,\mu_q) - \widetilde{G_A}(\tau,\mu_q)}{\widetilde{G_V}(\tau,\mu_q) + \widetilde{G_A}(\tau,\mu_q)}$$

With

$$\widetilde{G_V}(\tau,\mu_q) = \frac{G_V(\tau,\mu_q)}{G_V(N_\tau/2,\mu_q)}$$

And similarly for G_A. This ratio effectively removes any difference due to the wave-function renormalisations Z_V and Z_A.

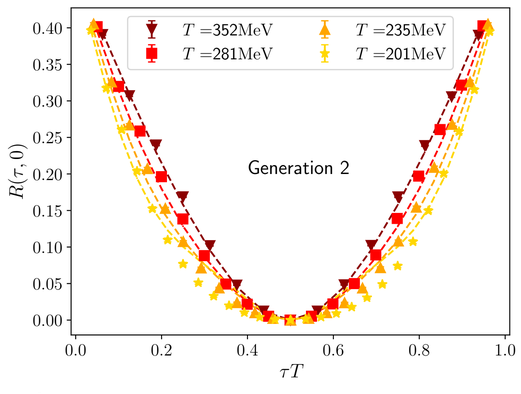




- At low temperatures, $R(\tau,\mu_q)$ presents a negative curvature near the middle of the lattice. This is understood as due to the higher mass of the axial-vector meson compared to the vector meson. Assuming a single pole ansatz, the ratio can be approximated as

$$R(\tau,0) = \frac{\cosh(m_V x) - \cosh(m_A x)}{\cosh(m_V x) + \cosh(m_A x)} \text{ with } x = \tau T - 1/2$$
$$\approx \frac{1}{4}(m_V^2 - m_A^2)x^2 + O(x\text{^}4)$$

- The negative curvature vanishes upon chiral symmetry restoration. However, due to the use of the Wilson fermion action, we do not expect a full degeneracy in the plot for $R(\tau,\mu_q)$ even when chiral symmetry is restored.
- At very high temperatures our results agree with the values of $R(\tau,0)$ computed using non-interacting free quarks, with the same fermion action and geometry [4]. The agreement shows that the non-degeneracy is explained at the level of the Wilson fermion action [4].



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- As $R(\tau,\mu_q)$ switches sign going from the low-temperature phase to the high-temperature phase, by necessity there is a temperature where the curvature of $R(\tau,\mu_q)$ vanishes near the centre of the lattice. We use this point to define $T_{pc}(\mu_q)$ as the temperature at which the two correlators are degenerate.
- To quantitatively estimate the μ_q dependence of T_{pc} we define the time-averaged quantity

$$\overline{R}(\mu_q,T) = \frac{\sum_{\tau=\tau_{min}}^{\frac{N_\tau}{2}} \frac{R(\tau;\mu_q,T)}{\sigma^2(\tau;\mu_q,T)}}{\sum_{\tau=\tau_{min}}^{\frac{N_\tau}{2}} \frac{1}{\sigma^2(\tau;\mu_q,T)}}$$

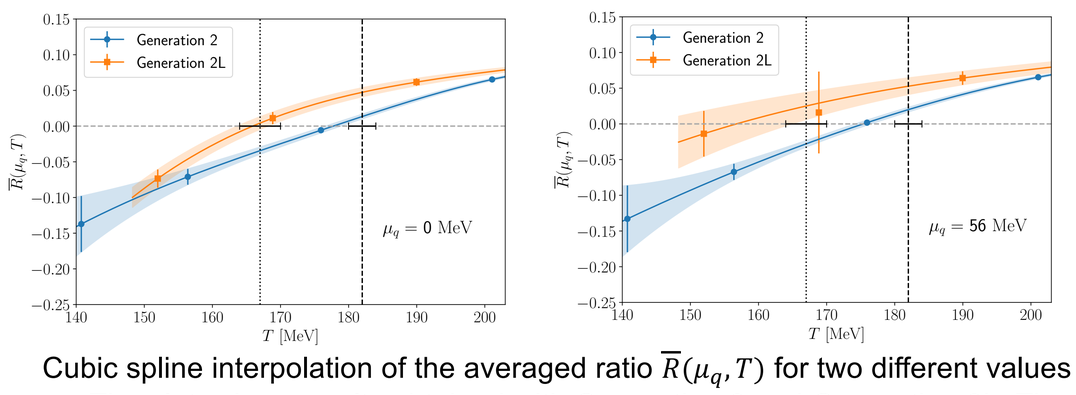
where σ^2 is the variance of the correlator at time τ

- At fixed μ_q we interpolate $\overline{R}(\mu_q,T)$ with cubic spines and find T_{pc} from the x-intercept.
- We fit the results of $T_{pc}(\mu_q)$ obtained from the interpolation to the equation:

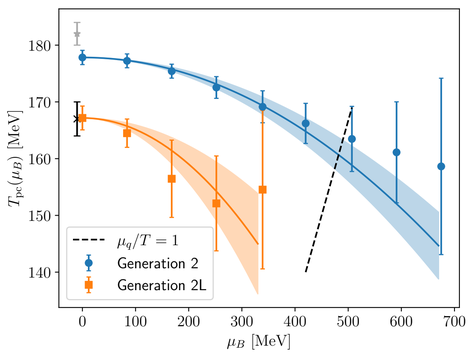
$$\frac{T_{pc}(\mu_B)}{T_{pc}(\mu_B=0)} = 1 - \kappa\left(\frac{\mu_B^2}{T_{pc}(\mu_B=0)^2}\right) + O(\mu_B\text{^}4)$$

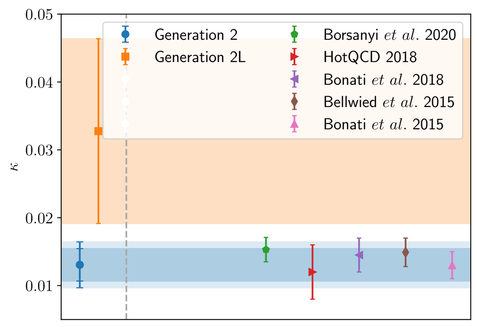
To extract the parameter κ

Results



Cubic spline interpolation of the averaged ratio $\overline{R}(\mu_q,T)$ for two different values of μ_q. The plots show results obtained with Generation 2 and Generation 2L. The dashed (dotted) vertical lines correspond to $T_{pc}(\mu_q=0)$ obtained from the renormalised chiral condensate.





(Left) The pseudo-critical temperature as a function of μ_B for the two set of ensembles shown by blue circles and orange squares. The blue and orange curves are the fit to the data. The $T_{pc}(0)$ values obtained from the renormalised chiral condensate are shown as gray and black crosses. The dashed black line correspond to $\frac{\mu_q}{T} = 1$. (Right) Final results for κ obtained using Generation 2 (blue circle) and Generation 2L (orange square) ensembles compared with results from the literature.

		Generation 2	Generation 2L
μ_q [MeV]	μ_B [MeV]	$T_{pc}(\mu_q)$ [MeV]	$T_{pc}(\mu_q)$ [MeV]
0	0	177.8(1.2)(5.8)	167.1(2.1)(3.1)
28	84	177.2(1.2)(5.7)	164.5(2.5)(3.1)
56	168	175.4(1.2)(5.5)	156.5(6.8)(0.8)
84	252	172.5(1.9)(3.9)	152.1(8.4)(0.9)
113	339	169.1(2.8)(1.5)	155(14)(2.0)
140	420	166.2(3.5)(0.6)	
	κ	0.0131(23)(23)	0.034(14)

Results of $T_{pc}(\mu_q)$ determined using the interpolation of $\overline{R}(\mu_q,T)$, including the values of κ obtained from the fits. The results of T_{pc} obtained with the renornalised chiral condensate are $T_{pc}(0) = 182(2)MeV$ and $T_{pc}(0) = 167(3)MeV$ for Generation 2 and Generation 2L respectively.

Outlook

- Use better noise vector estimators (Z2, Z4) combined with different methods of noise reduction.
- Repeat this study using spectral functions extracted from the lattice correlators.

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