Observing universal scaling properties of the chiral phase transition in (2+1)-flavor QCD

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MOTIVATION

Universal aspects of phase transition in Quantum Chromodynamics (QCD) control properties of strong-interaction matter in the vicinity of the chiral limit as well as close to the searched-for critical point at non-vanishing baryon chemical potential. We focus here on the former.

We aim at a controlled analysis of critical behavior in QCD in the (i) chiral limit, after taking (ii) the infinite volume and (iii) continuum limits.

The final goal is to reach an accuracy that allows to firmly establish the universality class of the chiral phase transition in (2+1)-flavor QCD, *i.e.* does it belong to the 3-d, $O(4)$ universality class or is it sensitive to a larger $U(2) \times U(2)$ symmetry group, which may arise, if the anomalous $U(1)_A$ is effectively restored at the chiral phase transition temperature.

SETUP:

Throughout this work we show results for (2+1)-flavor QCD using the Highly Improved Staggered Quark (HISQ) action and a Symanzik improved gauge action on lattices with temporal extent $N_\tau = 8$ and a strange quark mass (m_s) fixed to its physical value. We vary the spatial lattice size in the range $4 \leq N_\sigma/N_\tau \leq 11.5$ in order to keep the finite size scaling variable $\boldsymbol{z_L}$ at "small enough values" while changing the two degenerate light quark masses (m_l) towards the chiral limit.

Focus of this work:

- We focus here on a first analysis of the influence of finite volume effects on the extraction of universal parameters controlling the scaling near a critical point. We therefore do not yet aim at continuum extrapolated results.
- We assume here that the underlying chiral symmetry group is $O(2)$. However, eventually this ongoing work will lead to a determination of universal critical exponents in the chiral limit and fixed lattice cut-off.

IMPROVED CHIRAL ORDER PARAMETER

We work with an improved order parameter M defined as the difference of the multiplicatively renormalized light quark chiral condensate $M_\ell = m_s \langle \bar{\psi}\psi \rangle / f_K^4$ and its susceptibility $\chi_\ell\,(t,h,l) = \partial M_\ell/\partial m_\ell$

$$\boldsymbol{M = M_\ell - H\,\chi_\ell = h^{1/\delta}\, f_{G\chi}\,(z, z_L) + M_{reg}} \qquad (1)$$

with $H = m_\ell/m_s$ and $f_{G\chi}(z, z_L) = f_G(z, z_L) - f_\chi(z, z_L)$ denoting the difference of finite volume scaling functions [1,2] for M_ℓ and χ_ℓ, respectively.

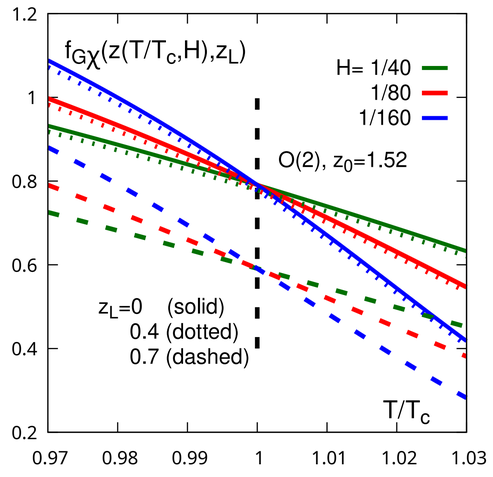
Scaling variables: $z = z_0 \dfrac{\tau}{H^{1/\beta\delta}}, z_L = z_{0,L} \dfrac{1}{LH^{\nu_c}}, \nu_c \equiv \dfrac{\nu}{\beta\delta} = (1+1/\delta)/3$

$$t \equiv \frac{\tau}{t_0} = \frac{1}{t_0}\left(\frac{T}{T_c} - 1\right)\ ,\ h \equiv \frac{H}{h_0} = \frac{1}{h_0}\frac{m_\ell}{m_s}\ ,\ l = \frac{l_0}{L} = l_0 \frac{N_\tau}{N_\sigma}$$

non-universal parameters: $T_c, t_0, h_0, l_0 \ \Rightarrow\ z_0 = h_0^{1/\beta\delta}\ ,\ z_{0,L} = l_0 h_0^{\nu_c}$

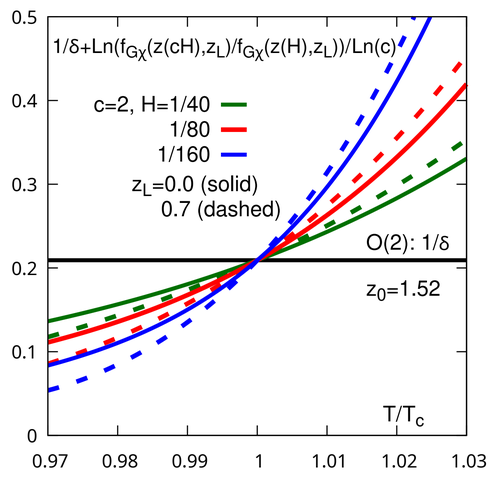
Infinite volume limit, $T = T_c \Leftrightarrow z_L \equiv 0, z \equiv 0 : f_{G\chi}(0,0) = 1 - 1/\delta.$

UNIVERSAL PROPERTIES OF ORDER PARAMETER



- The scaling function $f_{G\chi}(z, z_L)$ has a unique crossing point at T_c
- The location of the crossing point is volume independent in the finite volume scaling regime
- leads to a unique crossing point for the scaled order parameter $M/H^{1/\delta}$ in the (finite size, quark mass) scaling region

Construct a new observable $B(T, H_1, H_2)$ as the infinite volume limit of $B(T, H_1, H_2, L_1, L_2)$ that eliminates z_L-dependence and overall amplitudes at T_c

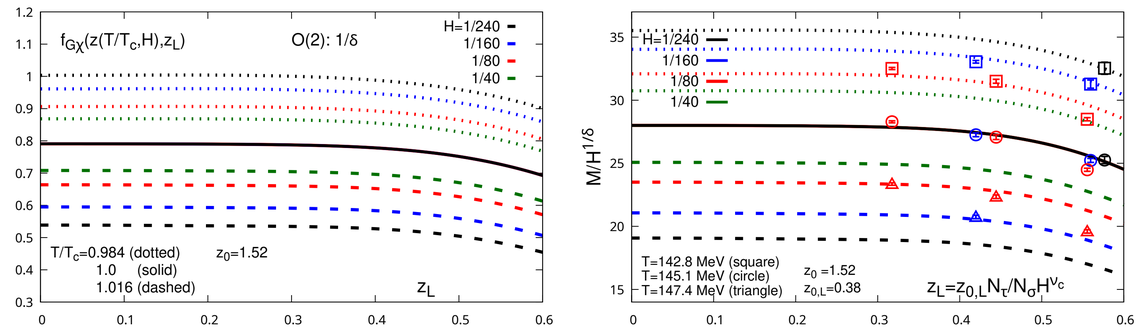


$$B(T, H_1, H_2, L_1, L_2) = \ln\left[\frac{M(T, H_1, L_1)}{M(T, H_2, L_2)}\right] \Big/ \ln\left[\frac{H_1}{H_2}\right]$$

Unique intersection points at $T = T_c$ in infinite ($z_L = 0$) and finite volumes for $z_{L_1} = z_{L_2} \neq 0$

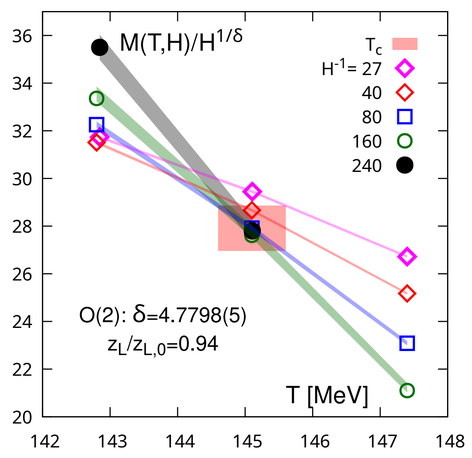
FINITE SIZE SCALING ANALYSIS

- masses: $m_\ell = m_s/27$ (phys.), $m_s/40$, $m_s/80$, $m_s/160$, $m_s/240$ (new)
- temperatures: 142 MeV $< T <$ 148 MeV
- spatial volumes: several sets of N_σ, chosen such that z_L is approximately constant for different values of H



$f_{G\chi}$ (left) and scaled order parameter $M/H^{1/\delta}$ (right) vs z_L for fixed z

need $z_{L,b} \equiv N_\tau/N_\sigma H^{\nu_c} \lesssim 1$ to be close to the infinite volume limit

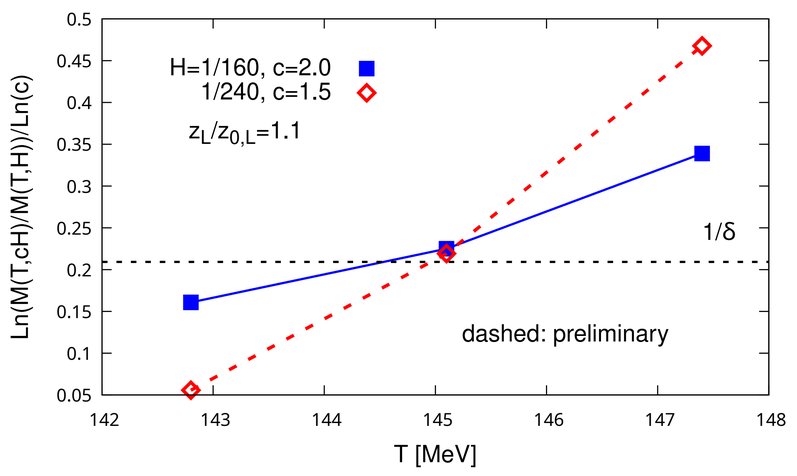


- δ of $O(2)$ is chosen, since HISQ preserves $O(2) \subset O(4) \cong SU(2)_L \times SU(2)_R$
- Finite volume analysis $\Rightarrow$ $m_s/40$ out of the scaling region.
- Smaller $m_\ell \leq m_s/80$ manifest a unique intersection point : $M_{reg} \sim \mathcal{O}(H^3)$ small
- This indicates $T_c \sim 145$ MeV for $N_\tau = 8$
- Right now, we approach closer towards $z_L = 0$

Next steps towards a direct determination of the universality class:

Aiming to derive δ in a parameter-independent manner from Lattice-QCD

- Perform calculations with $H = 1/160, 1/240$ on lattices of size $N_\sigma = 78, 92$: corresponds to $z_{L,b} \sim 0.8$, *i.e.* $z_L \sim 0.3$
- Calculate B including the data of the new $m_s/240$, hoping to get a more precise value of T_c and δ (resolution between $U(2) \times U(2)$ and $O(4)$)
- Perform similar set of runs for $N_\tau = 12$ lattices too, and thereby approach continuum limit.



- Showing lattice results (solid blue line) for $m_s/160$ on $N_\sigma = 56$, yielding $z_{L,b} = z_L/z_{0,L} \sim 1.1$
- $m_s/240$ to be done on $N_\sigma = 64$ for same $z_{L,b}$ (red dotted line). **THIS NEEDS TO BE DONE.**

REFERENCES

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[2] Heng Tong Ding, Olaf Kaczmarek, Frithjof Karsch, Peter Petreczky, Mugdha Sarkar, Christian Schmidt, and Sipaz Sharma, "Curvature of the chiral phase transition line from the magnetic equation of state of (2+1)-flavor QCD", Phys.Rev.D 109, 114516, 2024.

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