Heavy Quark Polarization in a QCD Plasma

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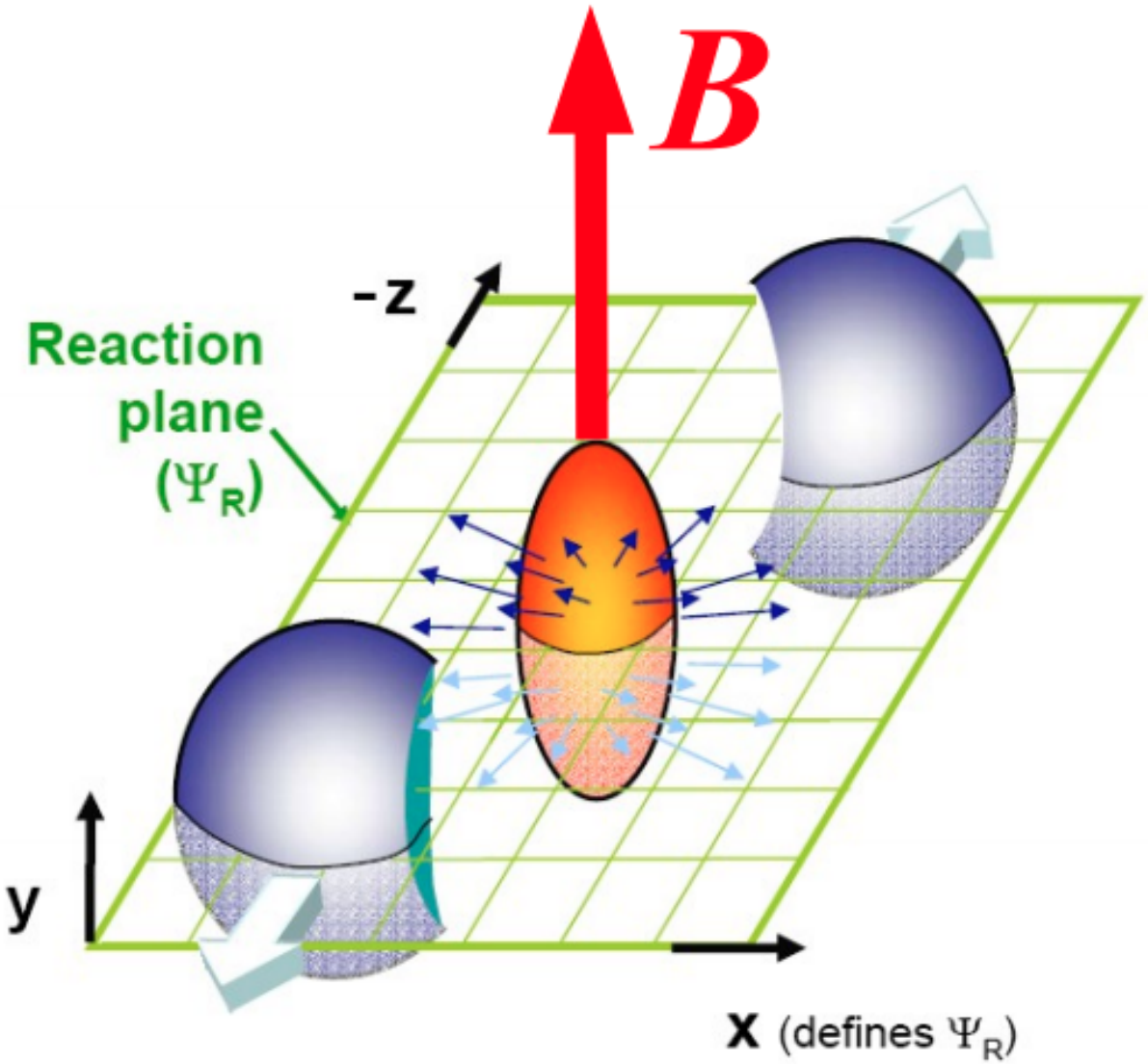
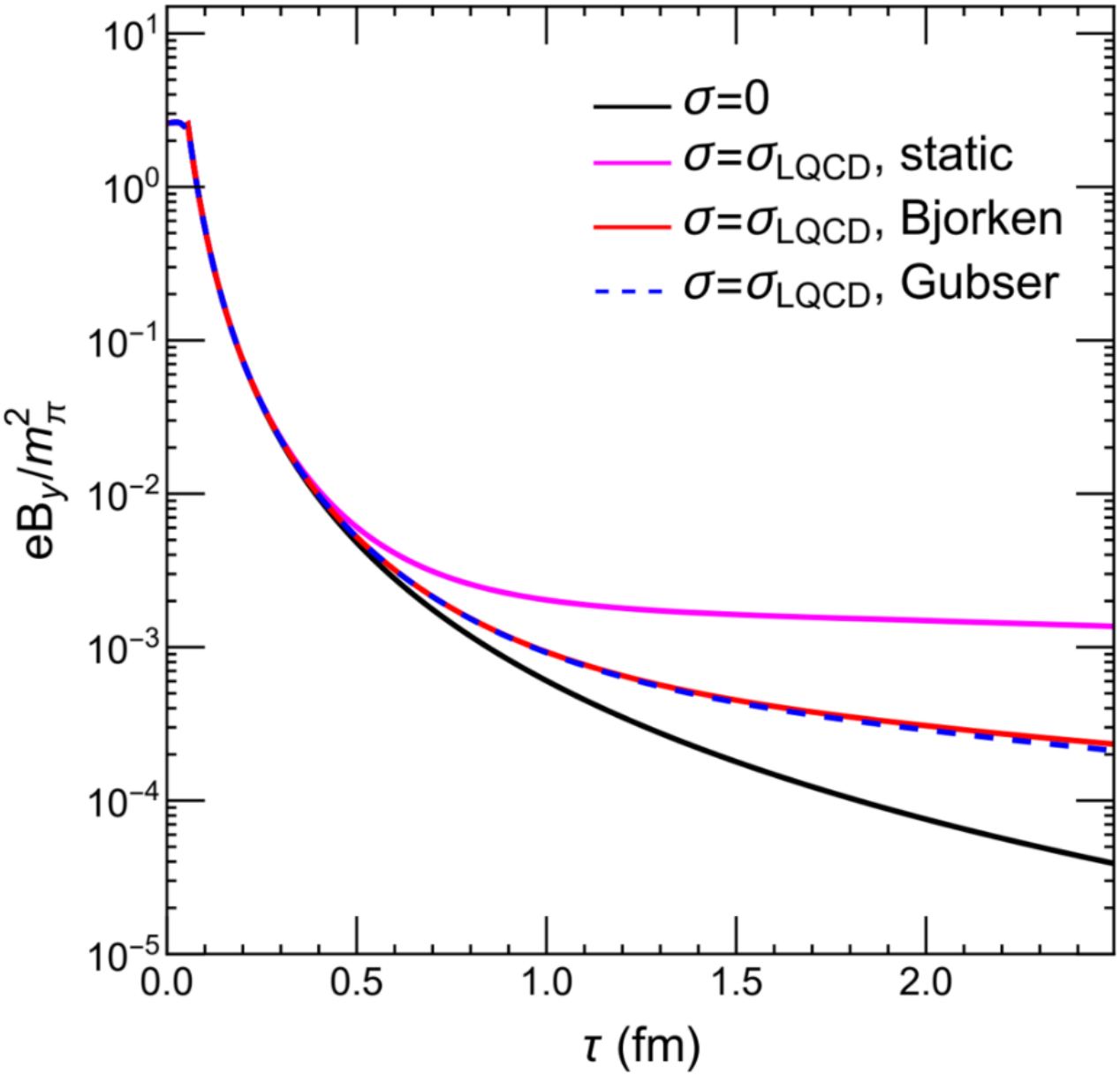
[S. Dey and A. Jaiswal, arXiv:2502.20352]

Generation of magnetic field in heavy ion collisions



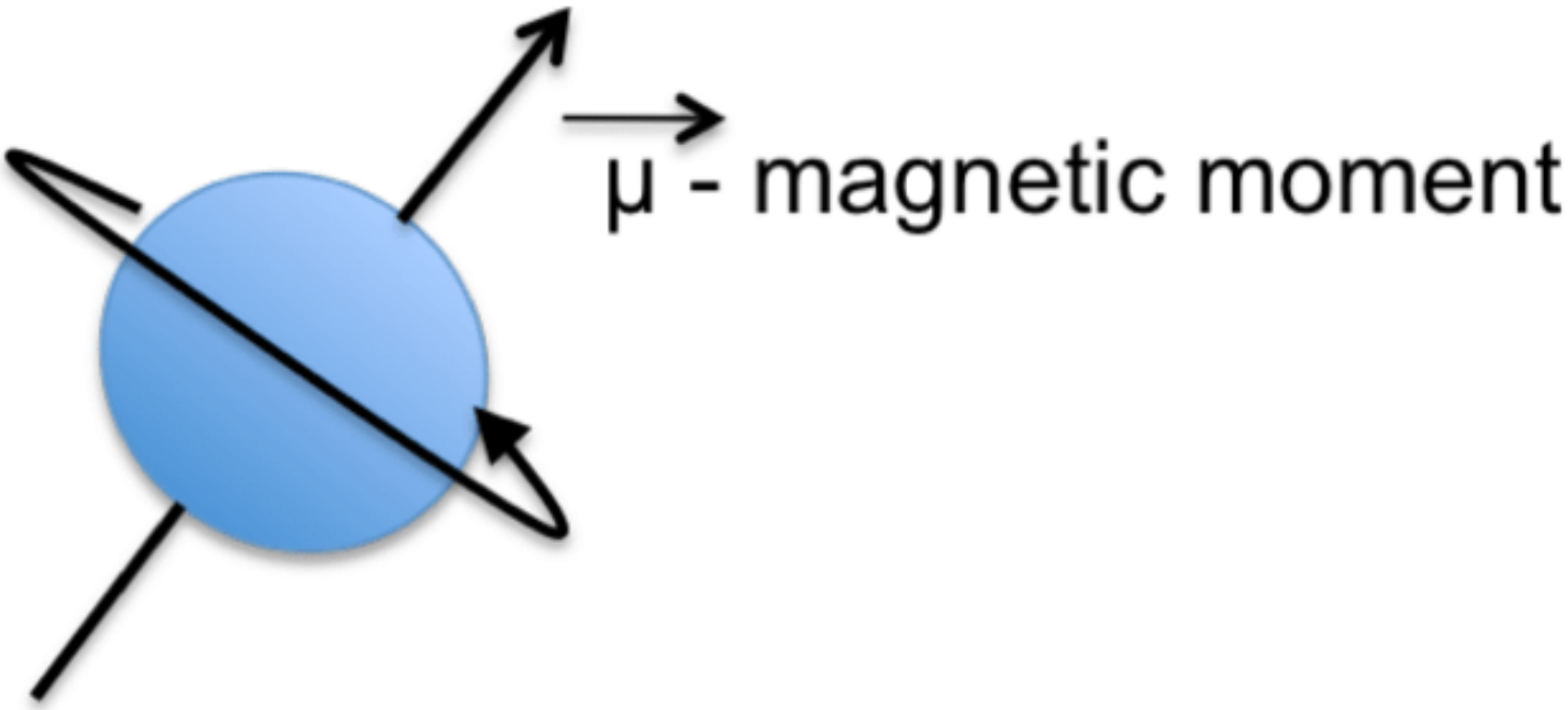
[Adapted from D. Kharzeev @ CPOD 2013.]

Magnetic field time evolution



[A. Huang, D. She, S. Shi, M. Huang and J. Liao, Phys. Rev. C 107, 034901 (2023).]

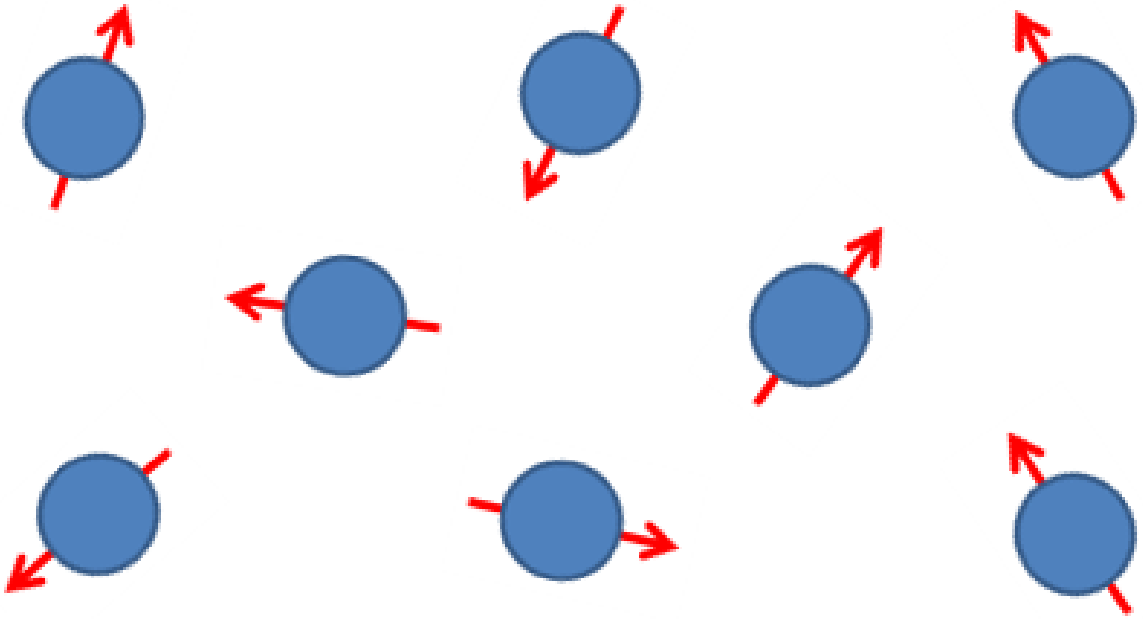
Heavy quarks: charged, spin-1/2 particles



Interaction with magnetic field: $\mathcal{H} = -\vec{\mu} \cdot \vec{B}$

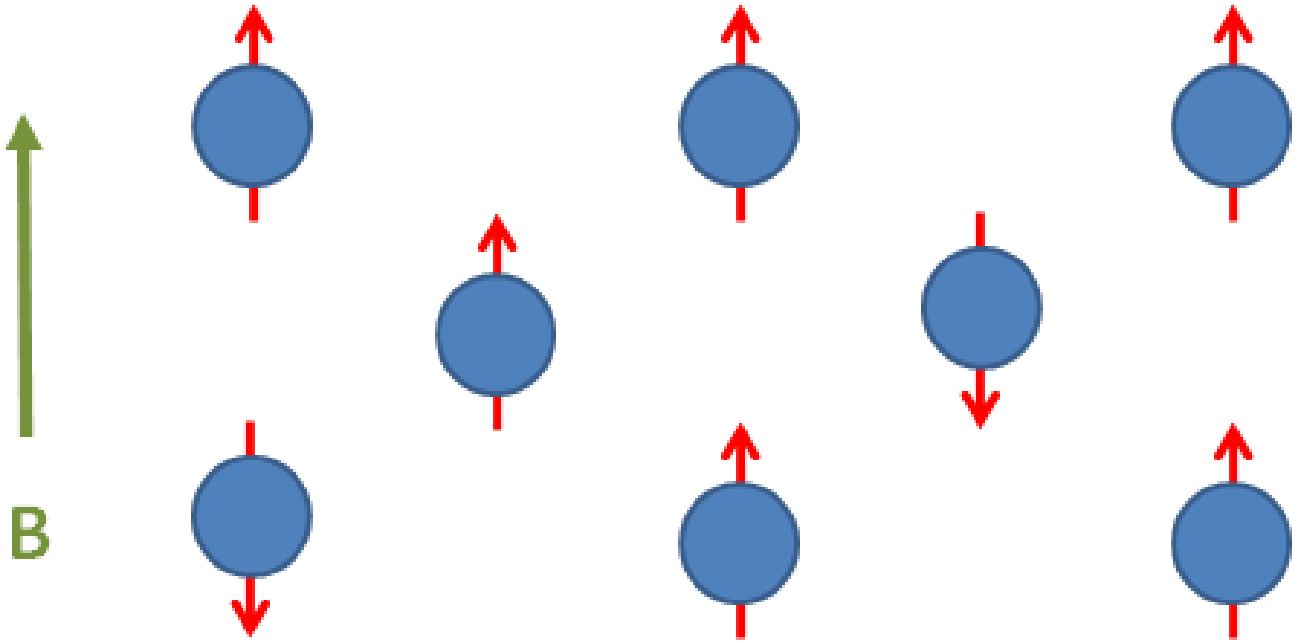
Magnetic moment and spin: $\vec{\mu} = \gamma \vec{s}$

No external magnetic field



Un-aligned spins of heavy quarks.

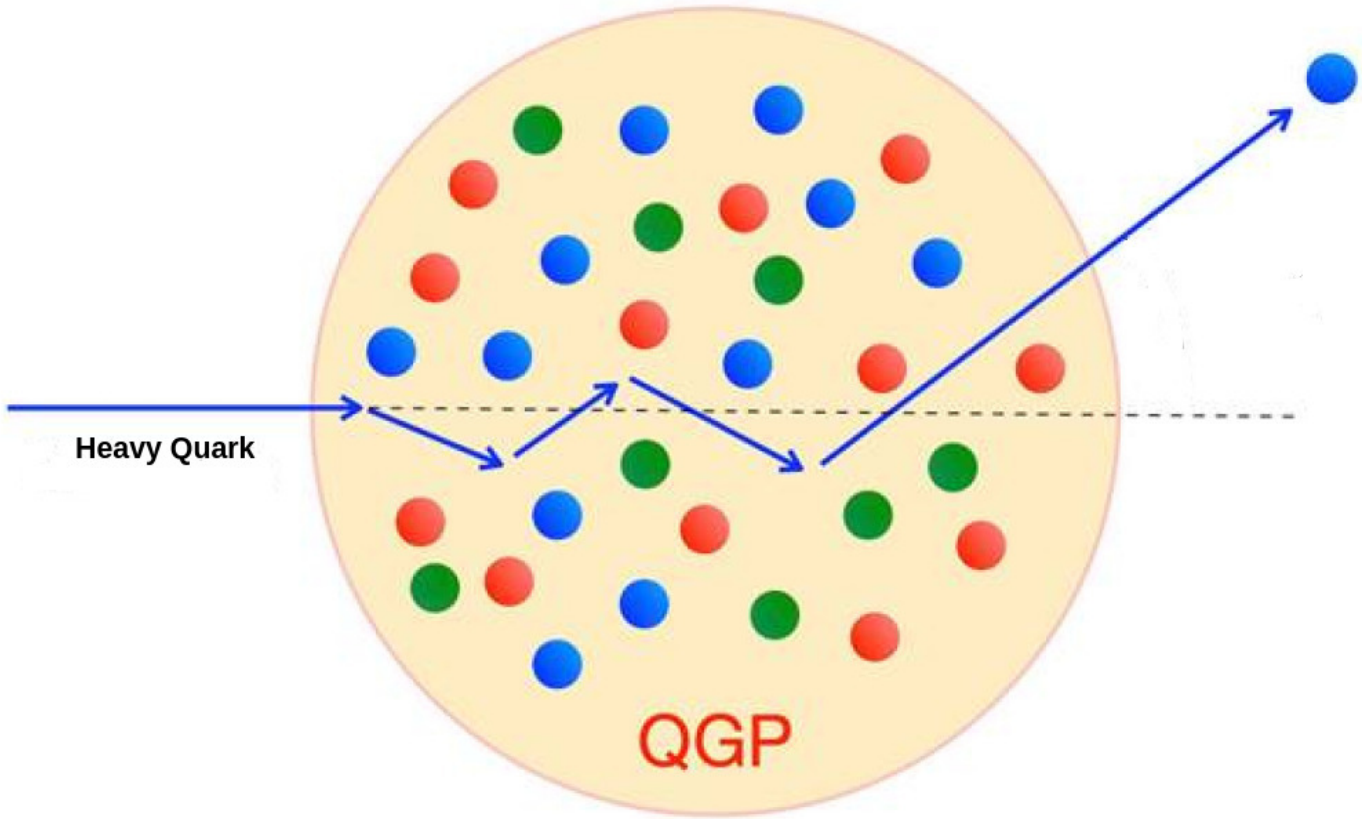
Heavy quarks in magnetic field



Aligned spins in presence of magnetic field.

Spin-polarization of heavy quarks.

Heavy quarks in QGP



Polarized heavy quarks propagates through QGP.

Rotational Brownian motion

- Random rotational motion (orientation and angular velocity) of a microscopic particle due to thermal fluctuations caused by collisions with surrounding medium particles.
- Rotational Brownian motion problem: first considered by Debye.
- For classical spins, the Langevin equation corresponds to the stochastic Landau–Lifshitz-Gilbert equation

$$\frac{d\mathbf{s}}{d\tau} = \mathbf{s} \times \left[\tilde{\mathbf{B}} + \boldsymbol{\xi}(\tau)\right] - \lambda\, \mathbf{s} \times \left(\mathbf{s} \times \tilde{\mathbf{B}}\right)$$

- Here $\tilde{\mathbf{B}} \equiv \gamma \mathbf{B} = -\frac{\partial \mathcal{H}}{\partial \mathbf{s}}$ and γ is the gyromagnetic ratio $\boldsymbol{\mu} = \gamma\, \mathbf{s}$.
- $\mathbf{s} \times \tilde{\mathbf{B}}$ represents precession dynamics of the system.
- $\boldsymbol{\xi}(\tau)$ is the random torque on the particle by the medium.
- λ is the damping coefficient.

Langevin and Fokker-Planck equations

- Generalized multivariate Langevin equation

$$\frac{dy_i}{d\tau} = A_i(y,\tau) + C_{ik}(y,\tau)\,\xi_k(\tau).$$

- Reduces to Landau–Lifshitz-Gilbert equation with $s_i = y_i$ and

$$A_i = \epsilon_{ijk} s_j \tilde{B}_k + \lambda\,(s^2\delta_{ik} - s_i s_k)\tilde{B}_k, \qquad C_{ik} = \epsilon_{ijk} s_j.$$

- Assume statistical properties of white noise for the random torque

$$\langle \xi_k(\tau) \rangle = 0, \qquad \langle \xi_k(\tau_1)\,\xi_l(\tau_2) \rangle = 2\,D\,\delta_{kl}\,\delta(\tau_1 - \tau_2).$$

- Using the Kramers–Moyal expansion, one arrives at the Fokker–Planck equation

$$\frac{\partial \mathcal{P}}{\partial \tau} = -\frac{\partial}{\partial y_i}\left[A_i(y,\tau) + D C_{jk}(y,\tau)\frac{\partial C_{ik}(y,\tau)}{\partial y_j}\right]\mathcal{P} + D\frac{\partial^2}{\partial y_i \partial y_j}[C_{ik}(y,\tau)C_{jk}(y,\tau)\mathcal{P}]$$

Fokker-Planck equation for spin

- Fokker–Planck equation corresponding to the stochastic Landau–Lifshitz-Gilbert equation

$$\frac{\partial \mathcal{P}}{\partial \tau} = -\frac{\partial}{\partial s_i}\Big[\epsilon_{ijk}\, s_j \tilde{B}_k + \lambda(s^2\delta_{ik} - s_i\, s_k)\tilde{B}_k - 2D s_i\Big]\mathcal{P} + D\,\frac{\partial^2}{\partial s_i\, \partial s_j}\big[s^2\delta_{ij} - s_i s_j\big]\mathcal{P}$$

- Assuming that the field $\tilde{\mathbf{B}}$ is independent of particle spin $\mathbf{s}$

$$\frac{\partial \mathcal{P}}{\partial \tau} = D\,\frac{\partial}{\partial \mathbf{s}} \cdot \left[\mathbf{s} \times \left(\mathbf{s} \times \left(\frac{\lambda}{D}\tilde{\mathbf{B}} - \frac{\partial}{\partial \mathbf{s}}\right)\right)\right]\mathcal{P}$$

- To find: Probability of a spin-polarized particle having an instantaneous orientation in the direction (θ, ϕ).
- Consider a sphere in spin-space of fixed radius s, i.e., $\mathbf{s} = (s, \theta, \phi)$: each point on the sphere represents a different spin orientation.
- Choose z-axis to be along $\tilde{\mathbf{B}}$.

Fokker-Planck equation for spin contd...

- Since we have an axially symmetric Hamiltonian

$$\tau_s \frac{\partial \mathcal{P}}{\partial \tau} = \frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \left[\sin\theta \left(\frac{\lambda}{D} \frac{\partial \mathcal{H}}{\partial \theta} \mathcal{P} + \frac{\partial \mathcal{P}}{\partial \theta} \right) \right], \qquad \tau_s \equiv \frac{1}{D}.$$

- Here, $\mathcal{H} = -\boldsymbol{\mu} \cdot \boldsymbol{B} = -\boldsymbol{s} \cdot \tilde{\boldsymbol{B}}$.
- For spatially homogeneous and time varying magnetic field $\mathbf{B}$,

$$\tau_s \frac{\partial \mathcal{P}}{\partial \tau} = \frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \left[\sin\theta \left(\frac{\lambda}{D} \frac{\partial \mathcal{H}}{\partial \theta} \mathcal{P} + \frac{\partial \mathcal{P}}{\partial \theta} \right) \right].$$

- This equation can be written in shorthand notation as

$$\tau_s \, \partial_\tau \mathcal{P}(\theta, \tau) = \mathcal{L}_\theta(\tau) \, \mathcal{P}(\theta, \tau) \qquad \Longrightarrow \qquad \tau_s \, \partial_\tau \left| \mathcal{P}, \tau \right\rangle = \hat{\mathcal{L}}(\tau) \left| P, \tau \right\rangle .$$

- The generic solution has the structure

$$\left| \mathcal{P}, \tau \right\rangle = \exp \left[\frac{1}{\tau_s} \int_0^\tau d\tau' \, \hat{\mathcal{L}}(\tau') \right] \left| \mathcal{P}, 0 \right\rangle .$$

Heavy quark polarization

- Considering the time dependence of the magnetic field to be of the form $B(t) = B_0\,\phi(\tau)$

$$\hat{\mathcal{L}}(\tau) = \hat{\mathcal{L}}^0 + \alpha\,\hat{\mathcal{L}}'(\tau), \quad \hat{\mathcal{L}}^0 = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right), \quad \mathcal{L}'(\tau) = \frac{\phi(\tau)}{\sin\theta}\frac{\partial}{\partial\theta}\,\sin^2\theta\,.$$

- Here $\alpha \equiv \mu B_0 \lambda/D$. Solve perturbatively assuming $\alpha \ll 1$.

$$\alpha \equiv \frac{\mu\,\gamma_v B_{\rm Lab}\lambda}{D} = \frac{{\rm g}\,s\,q\,\gamma_v B_{\rm Lab}}{2\,m_Q T} = \frac{(f\,e)\gamma_v B_{\rm Lab}}{2\,m_Q T}$$

- ${\rm g} \approx 2$ is the g-factor, $\gamma_v \equiv E/m_Q$, the charge $q = f\,e$, where f is $2/3$ and $-1/3$ for charm and bottom quarks, respectively.

- For $eB_{\rm Lab} \lesssim 0.1 m_\pi^2$ and $T = 300$ MeV, we obtain $\alpha = 0.034$ for charm and $\alpha = -0.007$ for bottom quarks with $E \simeq 50$ GeV .

Heavy baryon and meson polarization

- For baryons, the angular distribution of one of the decay daughter

$$\frac{dN}{d\cos\theta} = \frac{1}{2}\left(1 + \alpha_B |\vec{P}_B| \cos\theta\right)$$

- α_B is decay parameter. Using this distribution, one gets

$$\langle\cos\theta\rangle = \int \cos\theta \frac{dN}{d\cos\theta} d\cos\theta \implies |\vec{P}_B| = \frac{3}{\alpha_B}\langle\cos\theta\rangle$$

- Similarly, for mesons, the angular distribution is

$$\frac{dN}{d\cos\theta} = \frac{3}{4}\left[1 - \rho_{00} + \left(3\rho_{00} - 1\right)\cos^2\theta\right]$$

- ρ_{00} is element of spin density matrix; unpolarized $\implies \rho_{00} = 1/3$.
- Using this distribution, one gets

$$\langle\cos^2\theta\rangle = \int \cos^2\theta \frac{dN}{d\cos\theta} d\cos\theta \implies \Delta\rho_{00} = \frac{5}{2}\left[\langle\cos^2\theta\rangle - \frac{1}{3}\right]$$

- Here $\Delta\rho_{00} \equiv \rho_{00} - 1/3$

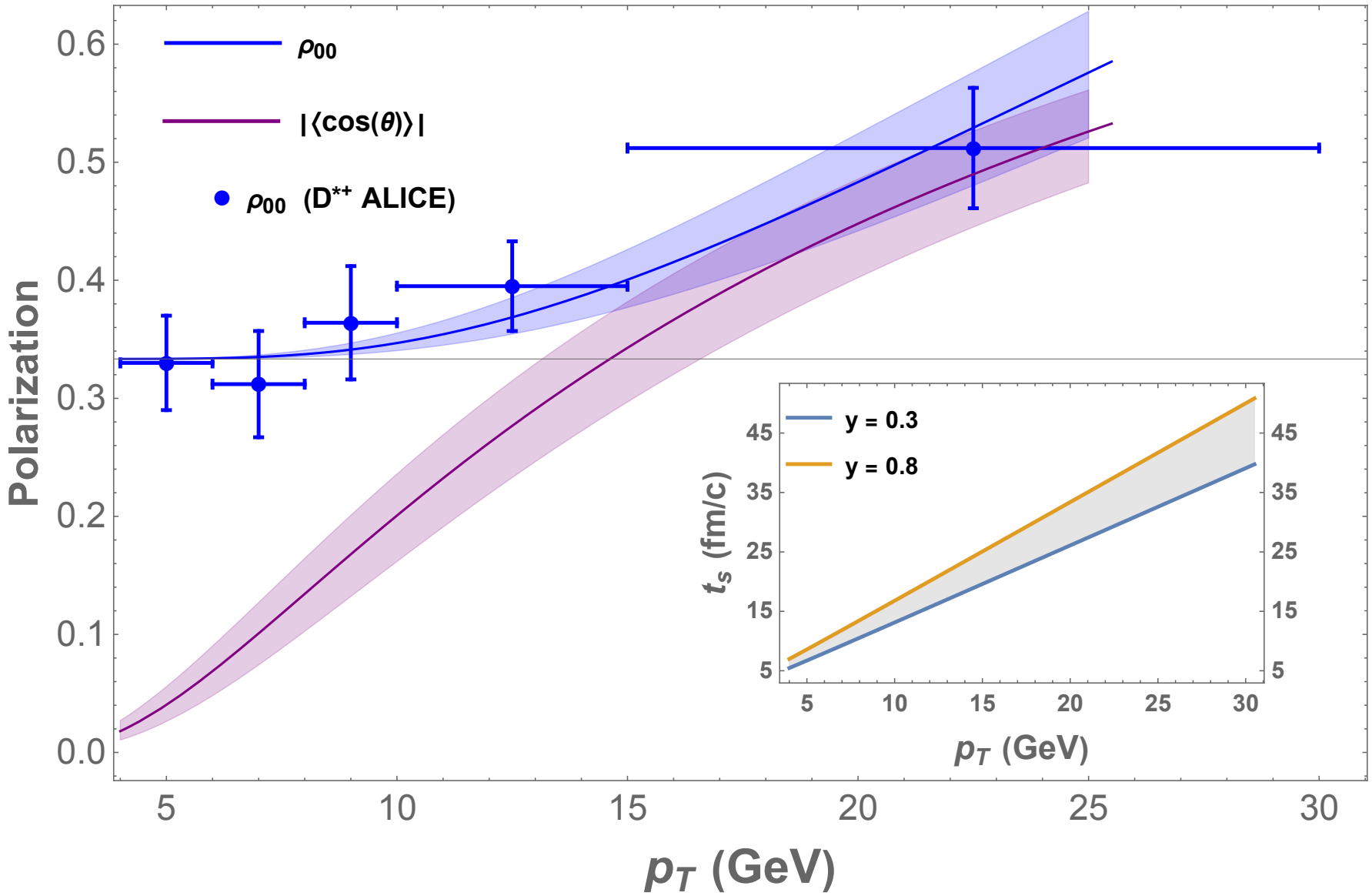
Results

- Since α is small, we find dominant contributions to be

$$\langle \cos\theta \rangle = \cos\theta_0 \, e^{-2\tau/\tau_s}, \qquad \langle \cos^2\theta \rangle = \frac{1}{3} + \frac{2}{3} \, e^{-6\tau/\tau_s}.$$

- Here τ_s is spin relaxation time. Supposedly large but not well constrained. Could be treated as a fit parameter.
- Initially $\theta_0 = 0, \pi$ depending on charge of the quark.
- Open heavy baryon and anti-baryon have opposite polarization.
- Open heavy vector mesons and its anti-particle, same sign.
- Quarkonium pol. small: heavy quark and anti-quark opp. pol.
- Static uniform fireball of constant temperature, $\tau = \frac{(R/\gamma_v)}{v_T}$.
- In the mid-rapidity region, $v_T = \frac{p_T}{E}$ and $\gamma_v = \frac{E}{m_Q}$.

Open heavy hadron polarization



Heavy quark polarization along initial magnetic field direction.

Ongoing and future works in this direction

- Fireball assumed to be static with constant average temperature.
- More realistic space-time evolution of the fireball and external magnetic field necessary.
- Predictions at forward rapidities.
- Calculation of spin relaxation time τ_s for heavy quarks.
- Derivation of an Einstein-Stokes-like relation between the spin diffusion coefficient and the dissipative parameters in spin hydrodynamics.
- Derivation of rotational Fokker-Planck equation from Kinetic theory with non-local collision terms.