



Chiral Properties of (2+1)-Flavor QCD in Magnetic Fields at Zero Temperature

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Introduction

- At zero temperature, there is a phenomenon called magnetic catalysis (MC), where the order parameter of the chiral phase transition, the chiral condensate, increases with the magnetic field strength [1].
- The behavior of meson masses and decay constants, especially for the neutral pion, in strong magnetic fields provides valuable insights into chiral symmetry dynamics. As a Goldstone boson, the neutral pion's mass and decay constant are significantly affected by the magnetic field, shedding light on the impact of magnetic fields on QCD.
- Charged mesons, such as pions and kaons, exhibit complex mass behaviors. At the lowest Landau level approximation, their mass typically increases with the magnetic field strength, but recent studies have shown different trends at larger magnetic field strengths, where the masses decrease [2].
- The sum of the up and down quark contributions to neutral pion correlators provides a lower bound for the mass of the charged ρ meson [3]. The condensation of ρ mesons is of particular interest, as it could indicate a transition of the QCD vacuum to a superconducting state under a strong magnetic field [4].
- Earlier studies were conducted using a larger-than-physical pion mass, $M_\pi \approx 220$ MeV (cf. Ref. [2]). In this work, we perform lattice QCD simulations with the physical pion mass.

Lattice setup

In this study, $N_f = 2 + 1$ lattice QCD simulations are based on

♦ **Action:** Highly improved staggered quarks and tree-level improved Symanzik action

♦ **Quark mass:** Physical quark masses, $m_u = m_s^{\text{phy}}/27$

♦ **Lattice spacing:** $a = 0.112$ fm, 0.084 fm, 0.067 fm, 0.056 fm

♦ **eB ranges:** $0 \leq eB \leq 1.22$ GeV² ($N_b = 0, 2, 4, 6, 8, 10, 12$), $eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$

where $N_x(N_y)$ is the number of points in the $x(y)$ direction on the lattice, $N_b \in Z$ is the number of magnetic fluxes through a unit area in the $x-y$ plane.

TABLE I: Lattice configurations and statistics.

Lattice size	a [fm]	# of conf. at different eB [GeV ²]						
		0.0	0.2	0.41	0.61	0.81	1.02	1.22
$24^3 \times 48$	0.112	600	701	1401	1350	1350	1350	1350
$32^3 \times 64$	0.084	600	800	800	800	800	800	800
$40^3 \times 80$	0.067	475	555	555	555	555	321	316
$48^3 \times 96$	0.056	150	266	250	250	220	306	215

Light quark chiral condensates

To examine the eB dependence of the chiral condensate, we investigate the renormalized condensate $\Delta\Sigma_q(B)$ [5], defined as:

$$\Delta\Sigma_q(B) = \frac{2m_q}{m_\pi^2 F_\pi^2} \left(\langle \bar{\psi}\psi \rangle_q(B, T=0) - \langle \bar{\psi}\psi \rangle_q(B=0, T=0) \right).$$

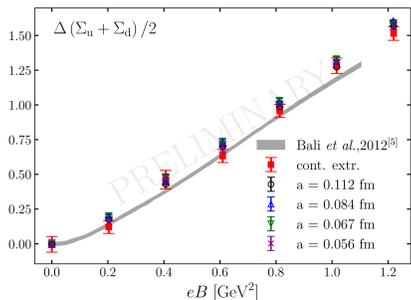


FIG. 1. The average light renormalized condensates.

As expected, we observed the phenomenon of magnetic catalysis at zero temperature, where a magnetic field enhances chiral symmetry breaking. However, our continuum extrapolated chiral condensate is slightly larger compared to the results of Bali *et al.*, who employed $N_f = 2 + 1$ flavors of stout-smearred staggered quarks with physical pion masses.

Methodology

Two-point temporal correlation function in the Euclidean space,

$$G(\tau) = \int d^3\vec{x} \left\langle \mathcal{M}(\vec{x}, \tau) \left(\mathcal{M}(\vec{0}, 0) \right)^\dagger \right\rangle,$$

where $\mathcal{M} = \bar{\psi}(\tau, \vec{x})\Gamma\psi(\tau, \vec{x})$ is a meson operator that projects to a certain quantum channel $\Gamma = \Gamma_D \otimes t^a$ with Dirac matrices Γ_D and a flavor matrix t^a . The connected part of the correlation function of the staggered bilinear,

$$G(\tau) = - \sum_{x,y,z} \zeta(\vec{n}) \text{Tr} \left[\left(M^{-1}(\vec{x}, \tau; \vec{0}, 0) \right)^\dagger M^{-1}(\vec{x}, \tau; \vec{0}, 0) \right],$$

where $M^{-1}(\vec{x}, \tau; \vec{0}, 0)$ is the staggered propagator from $(\vec{0}, 0)$ to (\vec{x}, τ) , and the phase factor $\zeta(\vec{n}) = 1$ for the pseudoscalar channel.

The temporal correlator decays exponentially at large distances τ , $\lim_{\tau \rightarrow \infty} G(\tau) \sim A e^{-m_\Gamma \tau}$,

where A is the amplitude of the decay, and it is related to the meson's decay constant f_Γ by $A \propto f_\Gamma$.

To determine pseudoscalar meson masses and decay constants, we perform correlated fits of temporal correlation functions, using a model that includes both non-oscillating and oscillating states. The optimal fit is selected via the corrected Akaike Information Criterion (AICc) to prevent overfitting. The correlation function is modeled as[7]:

$$G(\tau) = \sum_{i=1}^{N_{\text{nosc}}} A_{\text{nosc},i} e^{-M_{\text{nosc},i}\tau} - (-1)^\tau \sum_{i=0}^{N_{\text{osc}}} A_{\text{osc},i} e^{-M_{\text{osc},i}\tau}.$$

References

- [1] I. A. Shovkovy, *Lect. Notes Phys.* 871, 13 (2013)
[2] H.-T. Ding *et al.*, *Phys. Rev. D* 104, 014505 (2021)

- [3] Y. Hidaka and A. Yamamoto, *Phys. Rev. D* 87, 094502 (2013)
[4] M. N. Chernodub, *Phys. Rev. Lett.* 106, 142003 (2011)
[5] G. S. Bali *et al.*, *Phys. Rev. D* 86, 071502 (2012)

- [6] J. E. Cavanaugh, *Statistics & Probability Letters* 33, 201 (1997)
[7] A. Bazavov *et al.*, *Phys. Rev. D* 100, 094510 (2019)

Results

Neutral meson masses of $\pi^0, K^0, \eta_{s\bar{s}}^0$

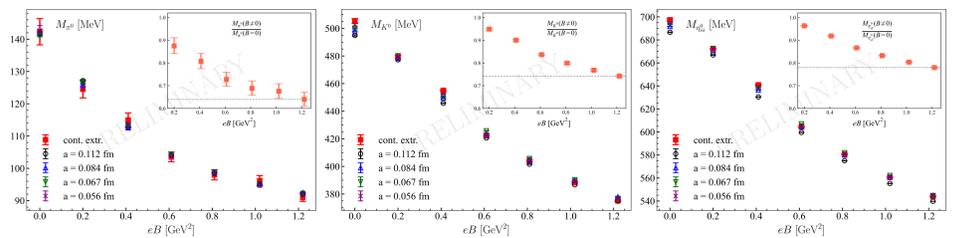


FIG. 2. M_{π^0} (left) using $G_{\pi^0} = (G_{\pi_u^0} + G_{\pi_d^0})/2$; $M_{K^0(s\bar{d})}$ (middle); $M_{\eta_{s\bar{s}}^0}$ (right).

As the magnetic field increases, the masses of all three neutral mesons decrease. At $eB \approx 1.2$ GeV², the mass of π^0 reduces to around 65% of its value at $eB = 0$, while the mass of K^0 decreases to around 74%, and the mass of $\eta_{s\bar{s}}^0$ decreases to around 78%.

Decay constants of $\pi^0, K^0, \eta_{s\bar{s}}^0$

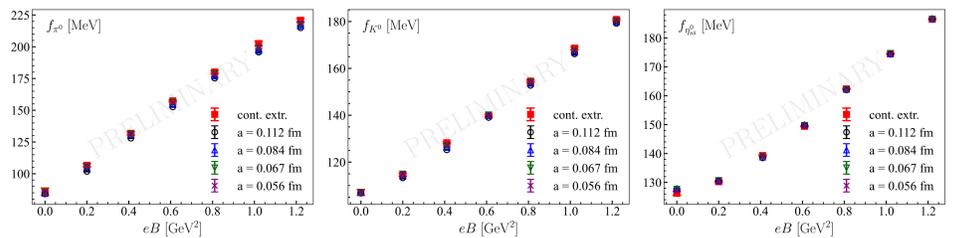


FIG. 3. f_{π^0} (left); $f_{K^0(s\bar{d})}$ (middle); $f_{\eta_{s\bar{s}}^0}$ (right).

All three decay constants increase with the magnetic field strength.

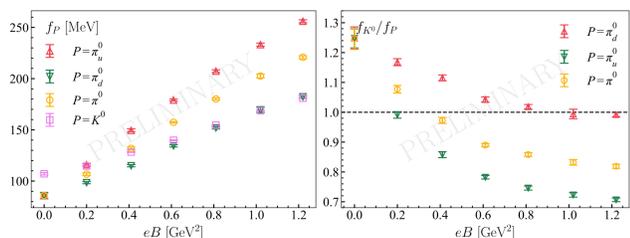


FIG. 4. Decay constants for the neutral pion and kaon, broken down by their respective flavor components (left), and the ratio of the kaon decay constant to the pion decay constants (right)."

The masses of charged meson π^\pm, K^\pm

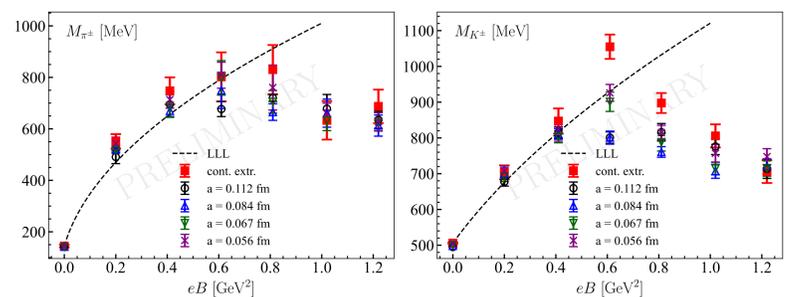


FIG. 5. M_{π^\pm} (left); M_{K^\pm} (right).

The Lowest Landau Level (LLL), $M_{ps}^\pm(B) = \sqrt{\left(M_{ps}^\pm(B=0) \right)^2 + |eB|}$.

For charged mesons, the masses of both pions and kaons initially increase with eB , but around $eB \approx 0.6$ GeV², they plateau or decrease slightly. The lowest Landau level line reveals a noticeable deviation in the masses at this point.

Conclusion

- ✓ Magnetic catalysis is observed at zero temperature up to $eB \sim 1.2$ GeV².
- ✓ The masses of neutral mesons decrease as eB increases, with heavier mesons being less affected by the magnetic field.
- ✓ The decay constants of all mesons increase as eB increases.
- ✓ The masses of charged mesons deviate from the lowest Landau level at $eB \approx 0.6$ GeV², with no further increase observed.



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