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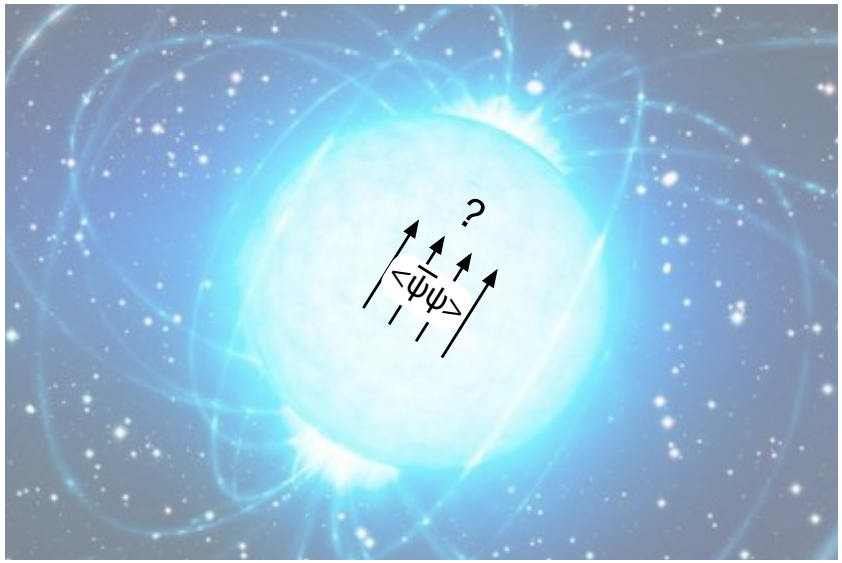
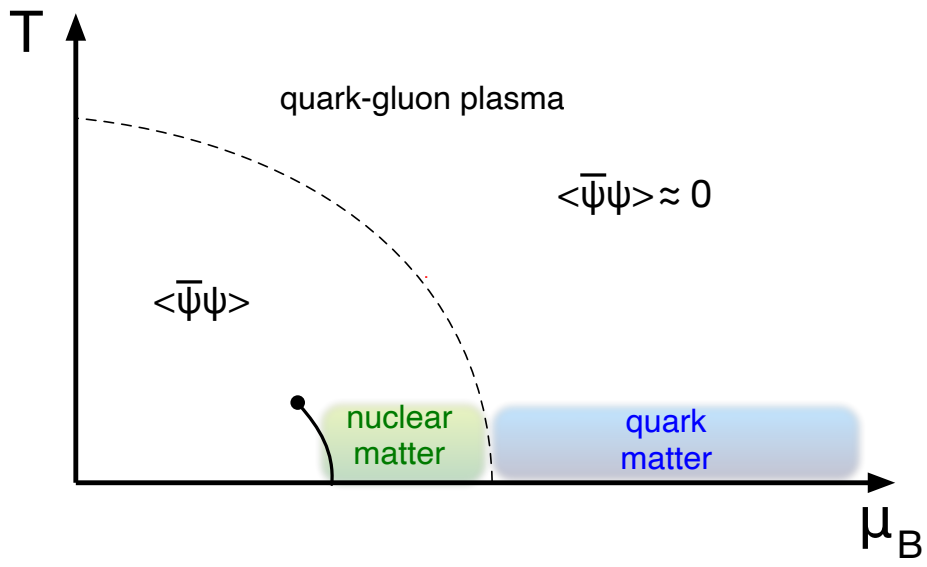
Nuclear chiral density wave in neutron stars?

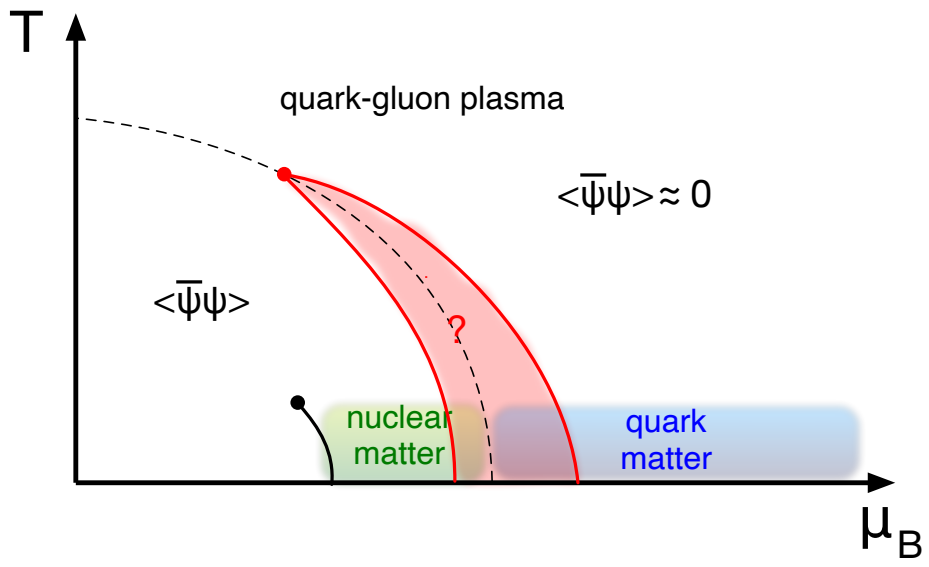
S. Pitsinigkos and A. Schmitt, PRD 109, 014024 (2024)
O. Papadopoulos and A. Schmitt, PRD 111, 034010 (2025)



Motivation


T
quark-gluon plasma
<$\bar{\psi}\psi$>≈ 0
<$\bar{\psi}\psi$>
nuclear
matter
quark
matter
μ_B

Motivation



inhomogeneous chiral condensate

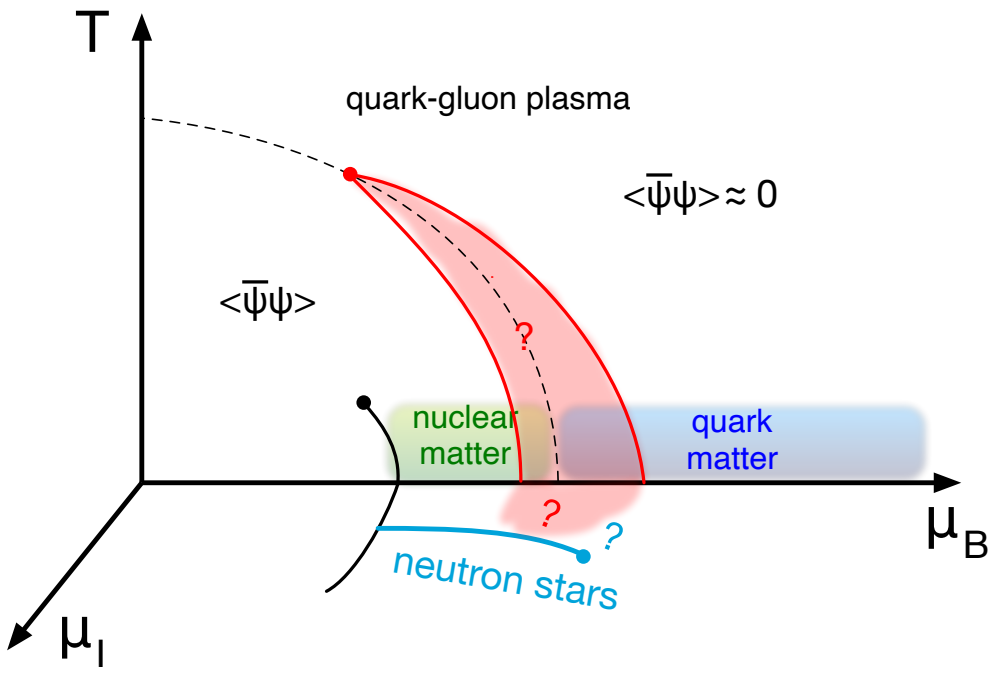
D. Nickel, PRD 80, 074025 (2009)

chiral density wave

$$\langle\bar{\psi}\psi\rangle = \phi\cos(2\vec{q}\cdot\vec{x})$$

$$\langle\bar{\psi}\gamma^5\tau_3\psi\rangle = \phi\sin(2\vec{q}\cdot\vec{x})$$

Motivation



inhomogeneous chiral condensate

D. Nickel, PRD 80, 074025 (2009)

chiral density wave (CDW)

$$\langle\bar{\psi}\psi\rangle = \phi\cos(2\vec{q}\cdot\vec{x})$$

$$\langle\bar{\psi}\gamma^5\tau_3\psi\rangle = \phi\sin(2\vec{q}\cdot\vec{x})$$

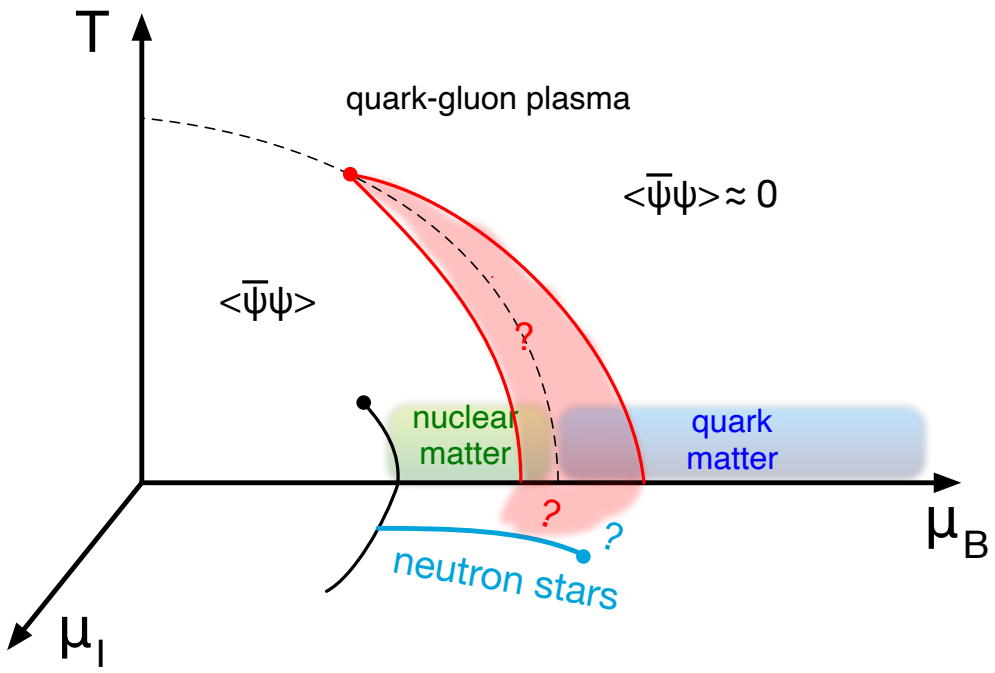
neutron stars: β-equilibrium

$$\mu_p + \mu_e = \mu_n$$

and electric charge neutrality

$$n_p = n_e$$

Motivation



most literature on CDW: quark matter (NJL, quark-meson model)

E. Nakano and T. Tatsumi, PRD 71, 114006 (2005)

S. Carignano, M. Buballa, and B.-J. Schaefer, PRD 90, 014033 (2014)

CDW in quark star: M. Buballa and S. Carignano, EPJA 52, 57 (2016)

very few works on nuclear CDW

F. Dautry and E.M. Nyman, NPA 319, 323 (1979)

A. Heinz, F. Giacosa, and D.H. Rischke, NPA 933, 34 (2015)

Nucleon-meson model

M. Drews and W. Weise, PRC 91, 035802 (2015)

E. S. Fraga, M. Hippert, A. Schmitt, PRD 99, 014046 (2019)

E. S. Fraga, R. da Mata, S. Pitsinigkos and A. Schmitt, PRD 106, 074018 (2022)

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu + \gamma^0\hat{\mu})\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi_a\partial^\mu\pi^a - U(\sigma,\pi) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} - \frac{1}{8}\mathrm{Tr}[\rho_{\mu\nu}\rho^{\mu\nu}]$$

$$+ \frac{m_\omega^2}{2}\omega_\mu\omega^\mu + \frac{m_\rho^2}{2}\rho_\mu\rho^\mu + \frac{d}{2}\omega_\mu\omega^\mu\rho_\nu\rho^\nu - \bar{\psi}\Big[g_\sigma(\sigma + i\gamma^5\pi) + \gamma^\mu(g_\omega\omega_\mu + g_\rho\rho_\mu)\Big]\psi$$

- no nucleonic mass parameter → chiral symmetry [explicit breaking in $U(\sigma,\pi)$]
- neglect meson fluctuations: condensates $\langle\omega_0\rangle$, $\langle\rho_0^3\rangle$, and

$$\langle\sigma\rangle = \phi\cos(2\vec{q}\cdot\vec{x})\,, \qquad \langle\pi_3\rangle = \phi\sin(2\vec{q}\cdot\vec{x})$$

- CDW wave vector $\vec{q}$ induces anisotropic nucleon dispersions

$$E_k^\pm = \sqrt{\left(\sqrt{k_\parallel^2 + M^2} \pm q\right)^2 + k_\perp^2}$$

Nucleonic fluctuations and renormalization (page 1/2)

nucleonic contribution to free energy: vacuum ("Dirac sea") + medium

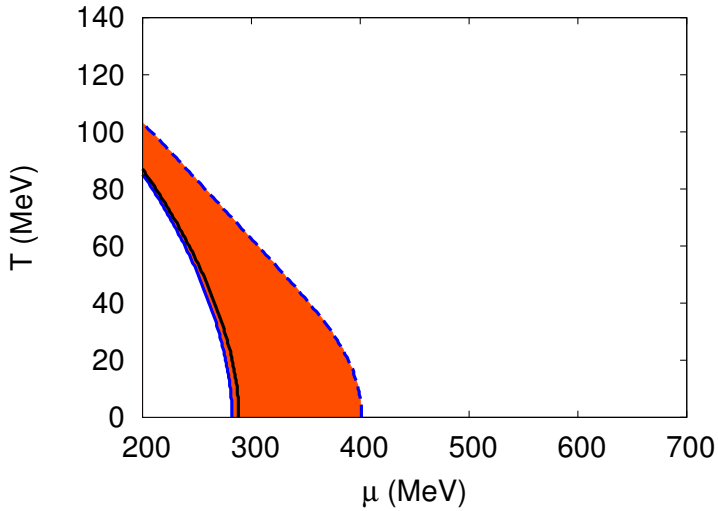
$$\Omega_{\rm nucl} = -\sum_{N=n,p}\sum_{e=\pm}\sum_{s=\pm}\int\frac{d^3\vec{k}}{(2\pi)^3}\left[\frac{E_k^s}{2}+(e\mu_N^*-E_k^s)\Theta(e\mu_N^*-E_k^s)\right]$$

Dirac sea often neglected ("no-sea approximation") for simplicity
A. Heinz, F. Giacosa, and D.H. Rischke, NPA 933, 34 (2015)

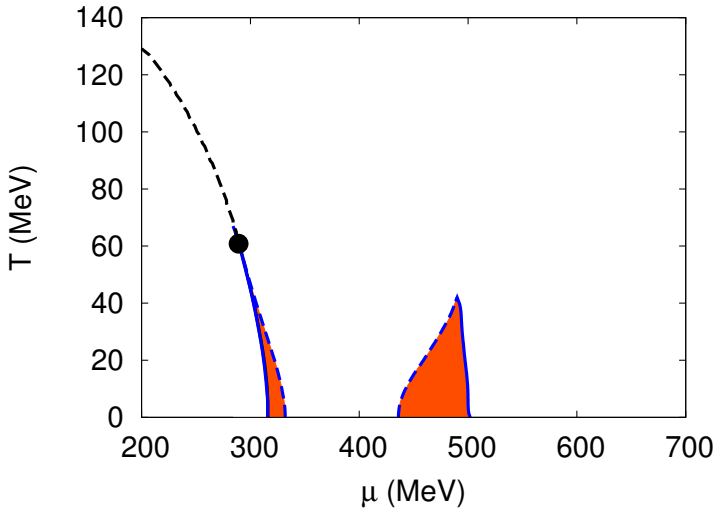
may contain important qualitative effects, e.g., magnetic catalysis in nucleons
A. Haber, F. Preis and A. Schmitt, PRD 90, 125036 (2014)

artifacts from Dirac sea in quark-meson model: re-entrance of CDW
S. Carignano, M. Buballa, and B.-J. Schaefer, PRD 90, 014033 (2014)

without Dirac sea:



with Dirac sea:



Nucleonic fluctuations and renormalization (page 2/2)

proper time regularization → cutoff Λ and renormalization scale ℓ

renormalization: counterterms to cancel Λ divergences → renormalized couplings (fitted to vacuum and saturation properties)

Dirac sea effectively modifies vacuum potential

$$U = \ldots + \frac{m_N^4}{96\pi^2}\left(1 - 8\frac{\phi^2}{f_\pi^2} - 12\frac{\phi^4}{f_\pi^4}\ln\frac{\phi^2}{f_\pi^2} + 8\frac{\phi^6}{f_\pi^6} - \frac{\phi^8}{f_\pi^8}\right) - \frac{q^2M^2}{2\pi^2}\ln\frac{M^2}{\ell^2} - \frac{q^4}{2\pi^2}F\left(\frac{M}{q}\right)$$

suitable choice of ℓ gives $\Omega = 2f_\pi^2 q^2$ (small q) and bounded potential (large q)

$$\ell = \sqrt{m_N^2 + (2q)^2}$$

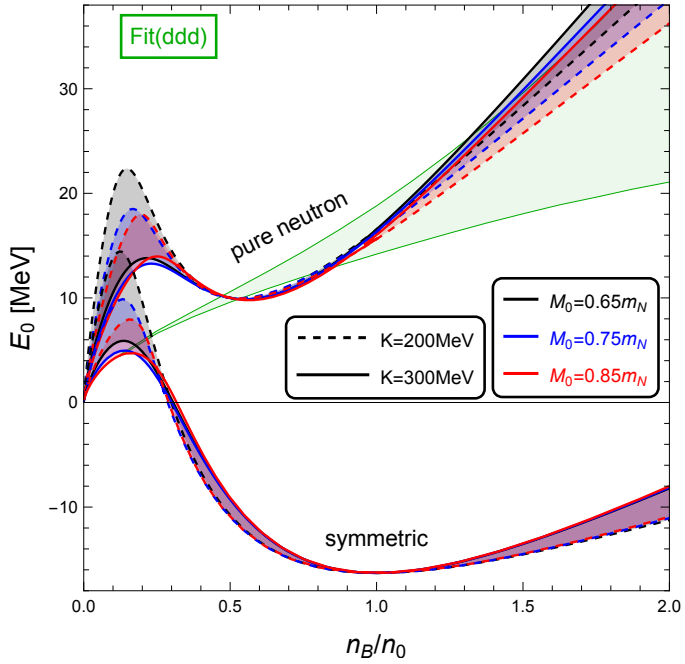
→ artificial re-entrance of CDW is eliminated!

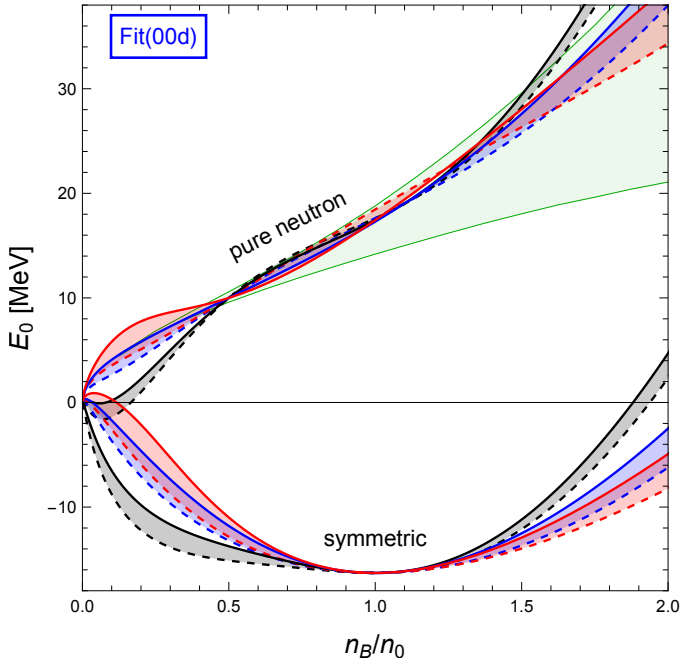
S. Pitsinigkos and A. Schmitt, PRD 109, 014024 (2024)

Parameter fit

Model parameters fitted to

1. vacuum properties (m_π, m_N, ...)
2. saturation properties of symmetric nuclear matter (n_0, E_B, ...)
   → explore dependence on (Dirac) mass M_0 at saturation
   [empirically: $M_0 \simeq (0.7 - 0.8)\, m_N$]
3. binding energy of pure neutron matter
   chiral effective theory: I. Tews, J. Carlson, S. Gandolfi, S. Reddy, Astrophys.J. 860, 149 (2018)





Fit(ddd): meson couplings ω^4, ρ^4, $\omega^2\rho^2$

Fit(00d): meson coupling $\omega^2\rho^2$

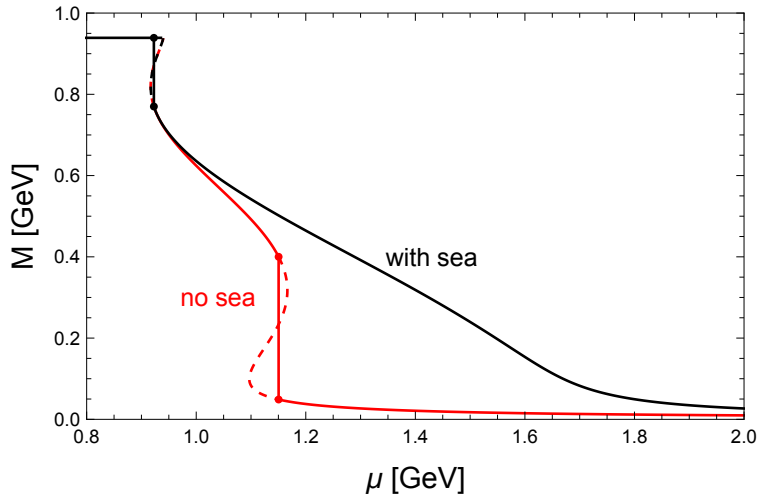
Stationarity equations and chiral crossover

isospin-symmetric nuclear matter:
stationarity of Ω with respect to condensates ϕ, ω, CDW wavenumber q

$$\frac{\partial\Omega}{\partial\phi} = \frac{\partial\Omega}{\partial\omega} = \frac{\partial\Omega}{\partial q} = 0$$

without CDW: chiral transition is
a crossover (for physical pion mass)

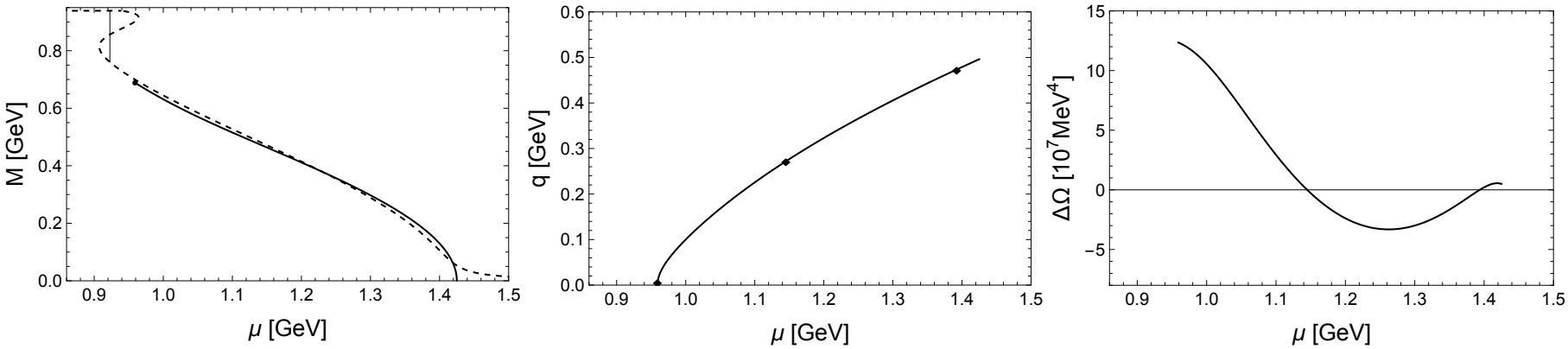
effective nucleon mass $M = g_\sigma\phi$



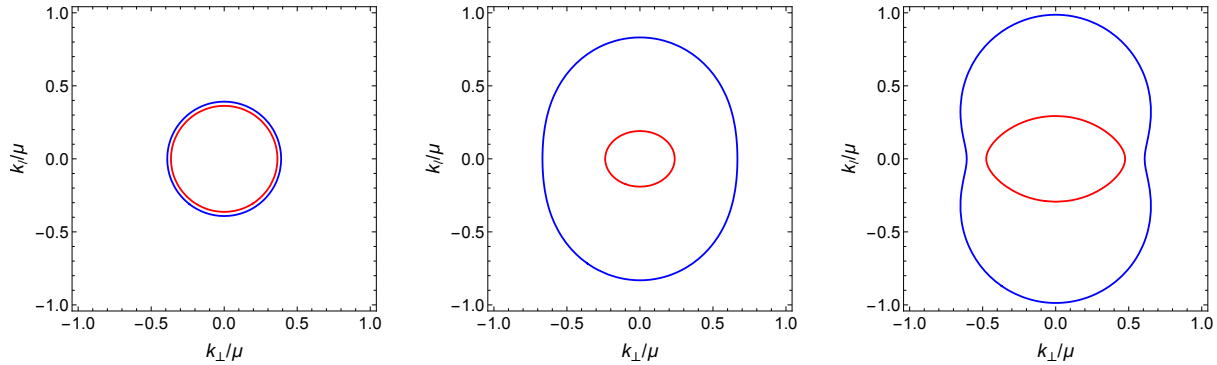
neutron star conditions: add ρ, leptons, neutrality, β-equilibrium

$$\frac{\partial\Omega}{\partial\rho} = \frac{\partial\Omega}{\partial\mu_e} = 0$$

CDW solutions (isospin-symmetric matter)



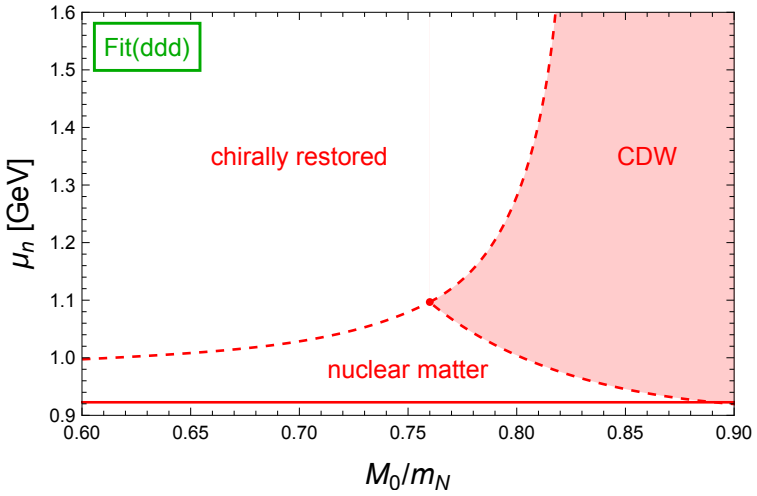
CDW "disrupts" crossover with two first-order transitions (physical m_π)



different contributions from $s = +$ and $s = -$ states to axial current:

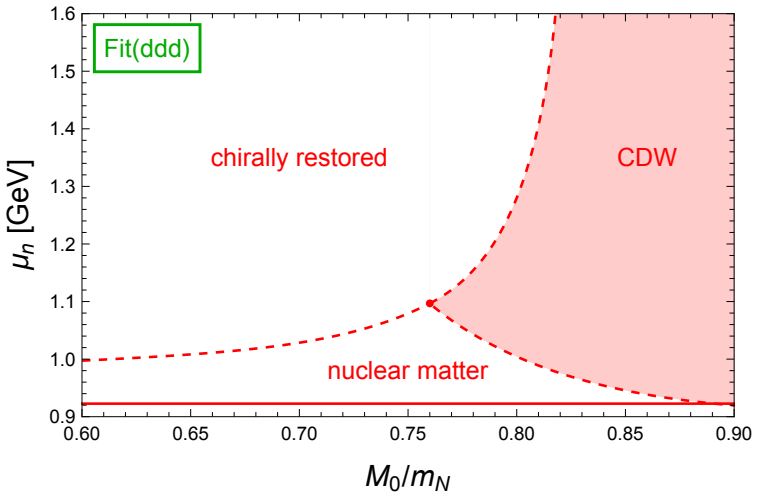
net axial baryon current to cancel axial mesonic current

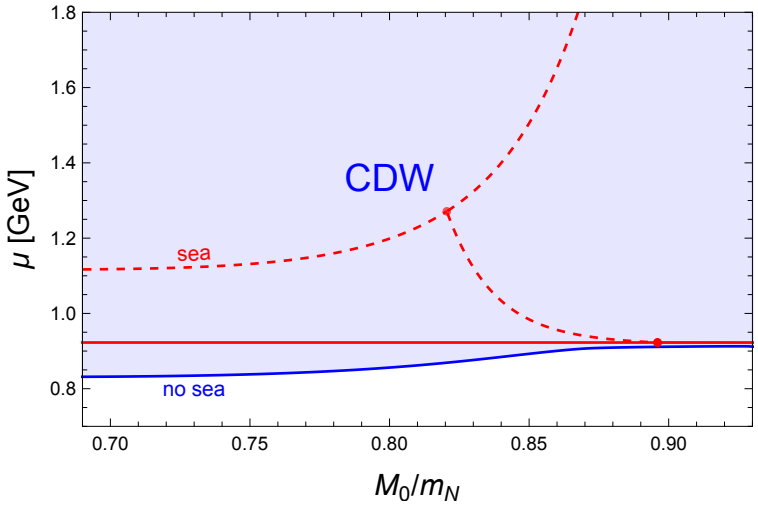
Locating the CDW in parameter space



chiral limit: 2nd-order chiral transition and 2nd-order CDW transition

Locating the CDW in parameter space

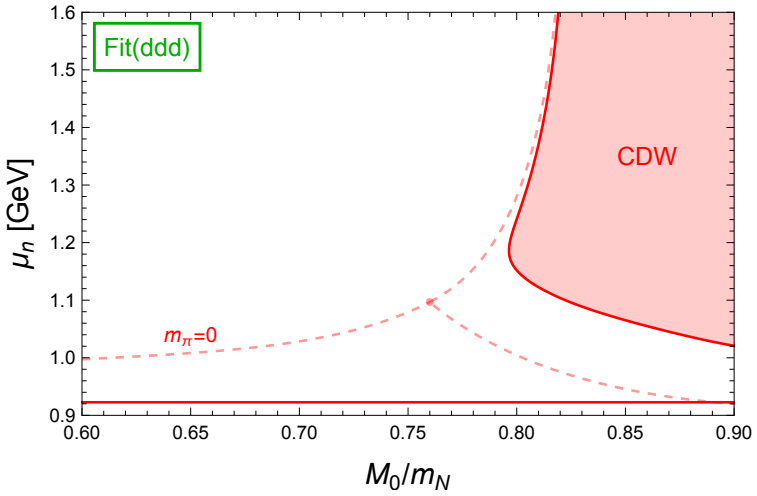




chiral limit: 2nd-order chiral transition and 2nd-order CDW transition

no-sea approximation favors CDW unphysically

Locating the CDW in parameter space

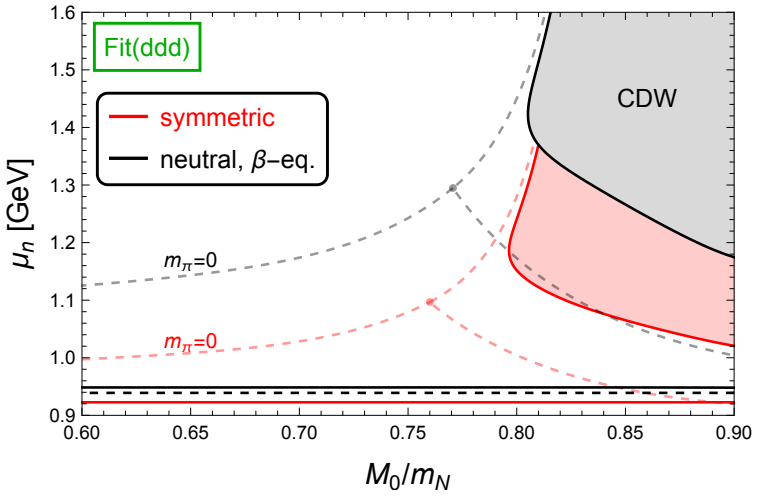


chiral limit: 2nd-order chiral transition and 2nd-order CDW transition

no-sea approximation favors CDW unphysically

physical m_π: chiral crossover, 1st-order CDW transition

Locating the CDW in parameter space



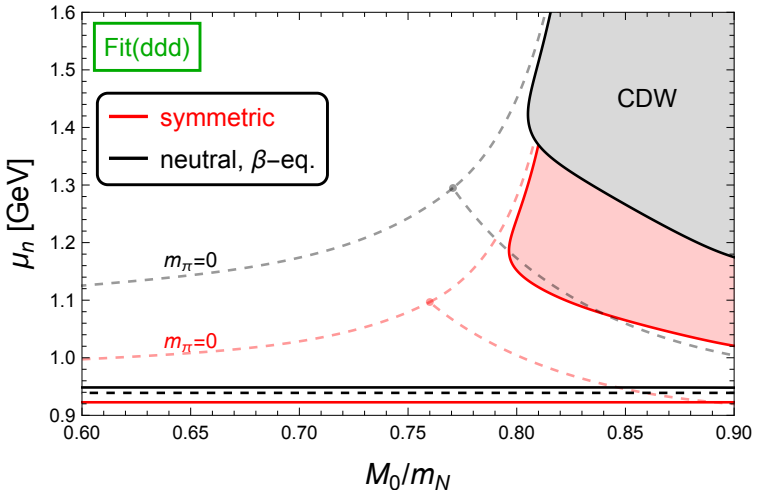
chiral limit: 2nd-order chiral transition and 2nd-order CDW transition

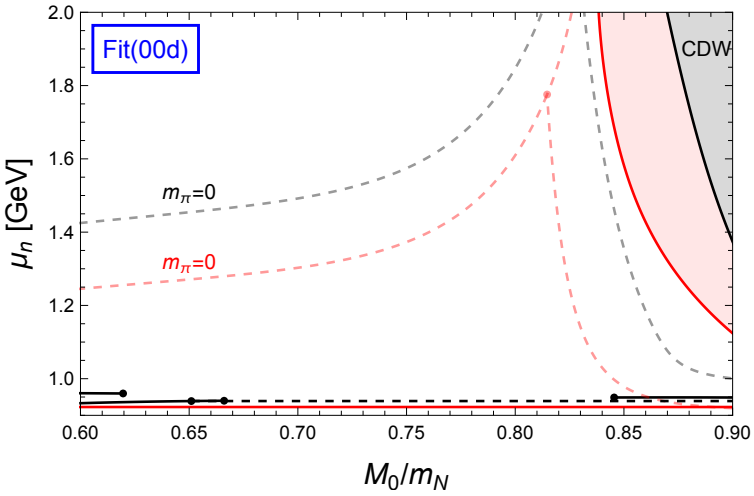
no-sea approximation favors CDW unphysically

physical m_π: chiral crossover, 1st-order CDW transition

neutron star conditions: CDW less favored

Locating the CDW in parameter space





chiral limit: 2nd-order chiral transition and 2nd-order CDW transition

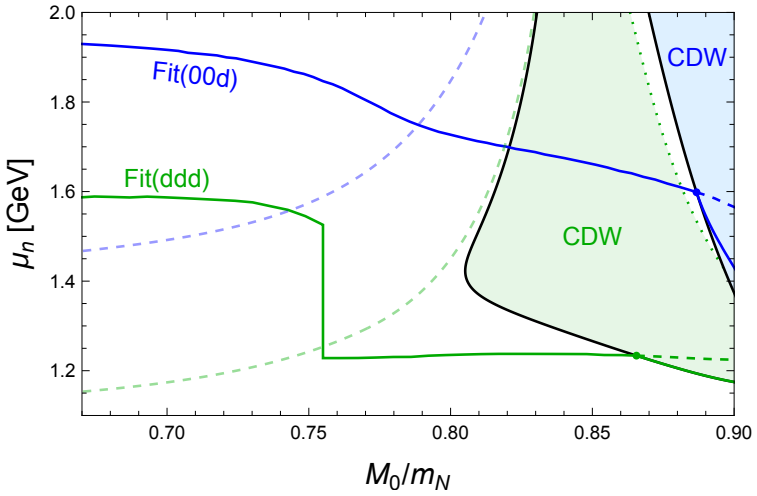
no-sea approximation favors CDW unphysically

physical m_π: chiral crossover, 1st-order CDW transition

neutron star conditions: CDW less favored

more realistic Fit(00d): CDW even less favored

Neutron stars with CDW cores?

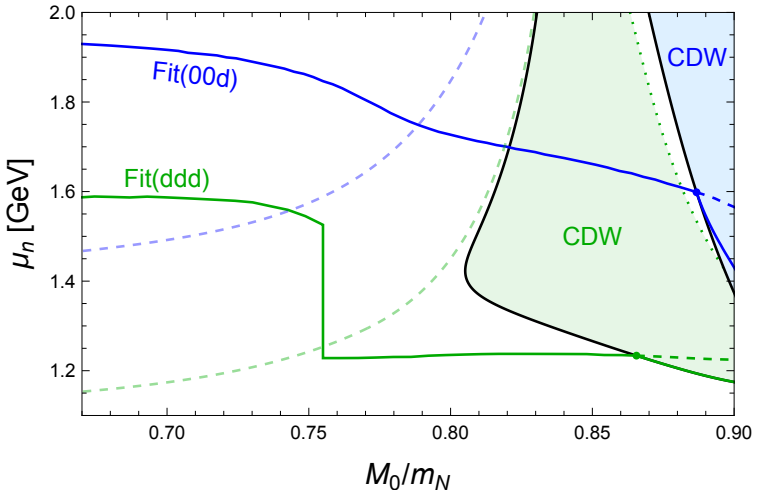


solve TOV equation:

- neglect anisotropy
- homogeneous matter instead of crust (does not affect maximal mass)

→ maximal μ_n in the center of maximally massive star

Neutron stars with CDW cores?



solve TOV equation:

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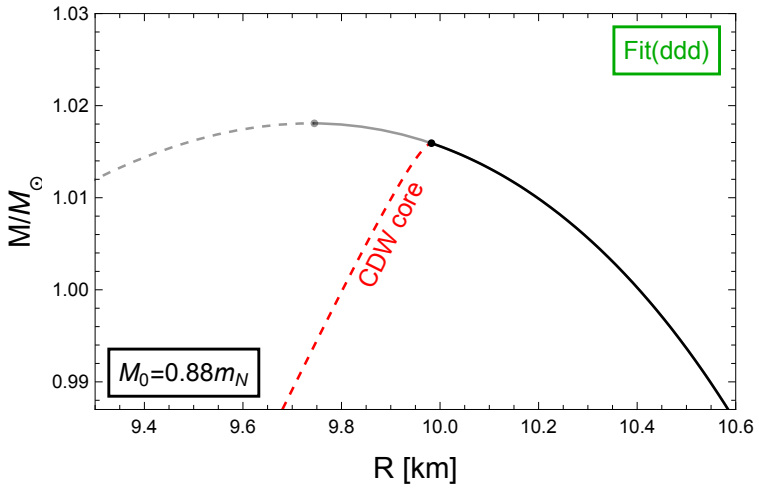
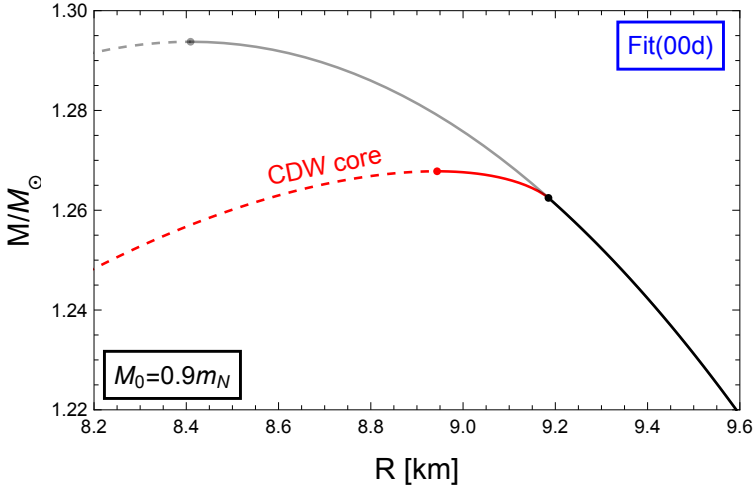
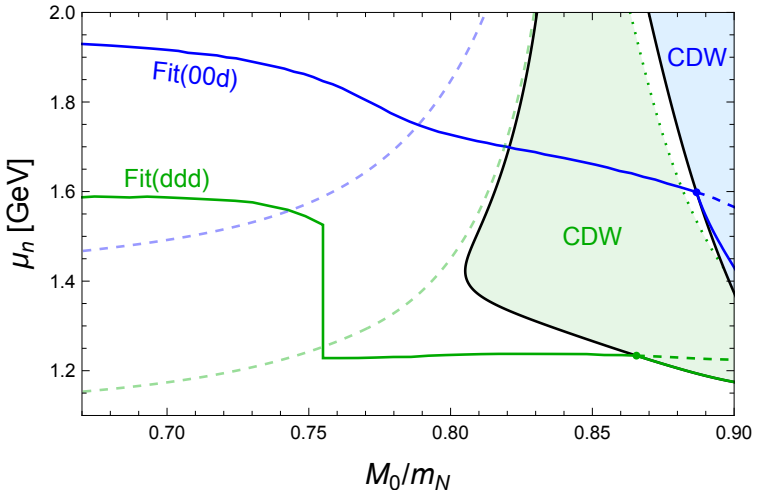
→ maximal μ_n in the center of maximally massive star

CDW may or may not destabilize the star





Neutron stars with CDW cores?



solve TOV equation:

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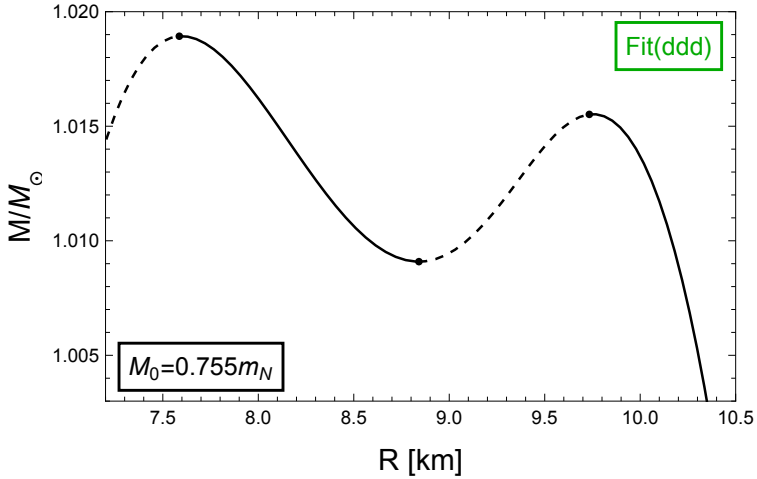
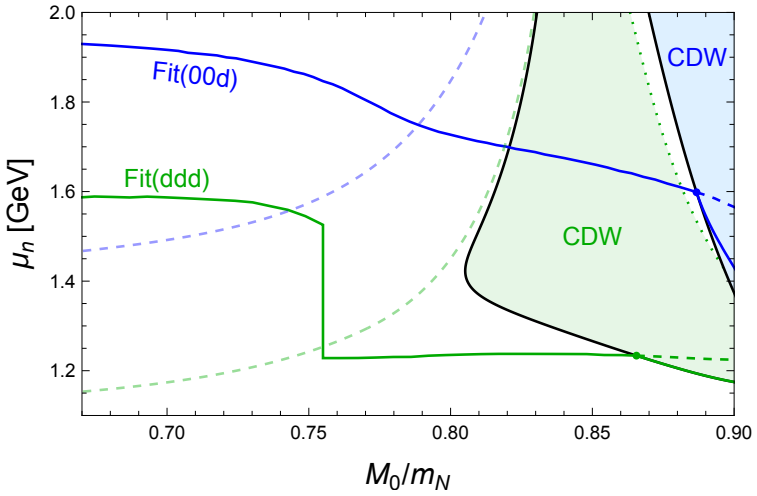
→ maximal μ_n in the center of maximally massive star

twin stars (despite crossover)



Neutron stars with CDW cores?

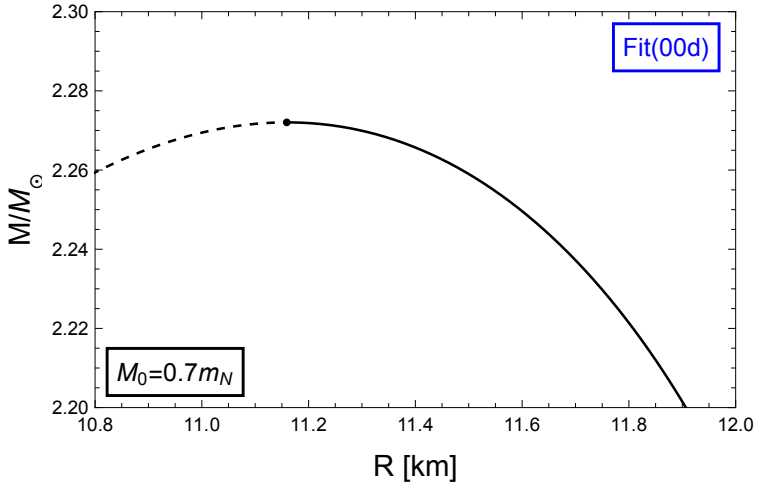


solve TOV equation:

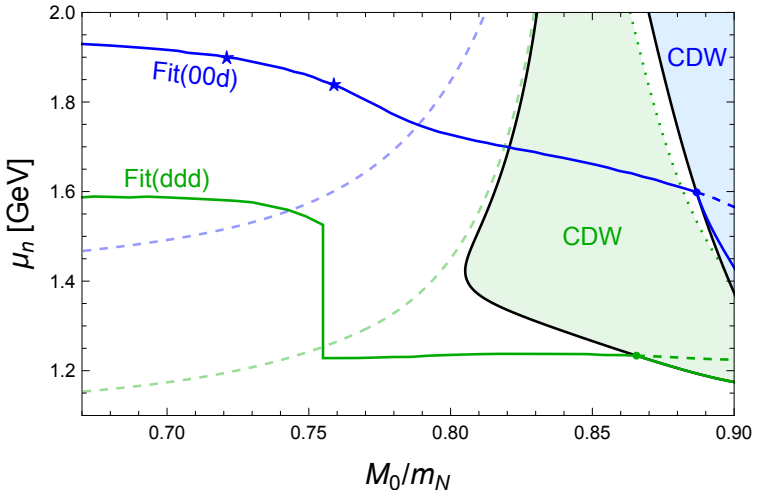
- neglect anisotropy
- homogeneous matter instead of crust (does not affect maximal mass)

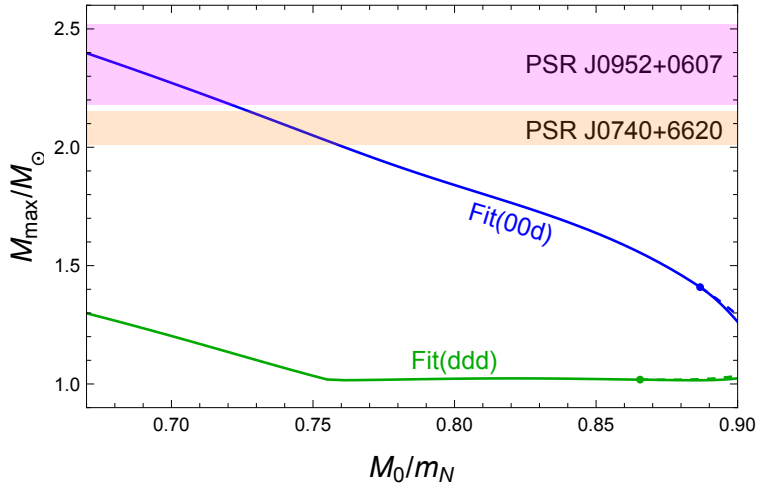
→ maximal μ_n in the center of maximally massive star

realistic (hybrid) stars exist with Fit(00d)



Neutron stars with CDW cores?





CDW is preferred in a corner of the parameter space where matter is (much) too soft to generate stars with realistic masses

Summary

- nucleon-meson model describes realistic (low-density) nuclear matter and chiral phase transition (crossover in presence of nucleonic fluctuations)
- suitable renormalization scheme cures unphysical CDW re-entrance (applicable to large class of models)
- CDW is disfavored by neutron star conditions (neutrality, electroweak equilibrium)
- CDW is favored in a region of parameter space where EoS is too soft (even in the absence of CDW)

Outlook

- stabilization of nuclear CDW through magnetic field
  A. Schmitt, work in progress
  S. Carignano, E.J. Ferrer, V. de la Incera, and L. Paulucci, PRD 92, 105018 (2015)
  W. Gyory and V. de la Incera, PRD 106, 016011 (2022)

- more complicated spatial structures
  "shifted" CDW Y. Takeda, H. Abuki, and M. Harada, PRD 97, 094032 (2018)
  2D chiral crystals S. Carignano and M. Buballa, PRD 86, 074018 (2012)
  quantum spin liquid R. D. Pisarski, A. M. Tsvelik, and S. Valgushev, PRD 102, 016015 (2020)

- model dependence? large effect of Dirac sea?
  parity doubling E. S. Fraga, R. da Mata and J. Schaffner-Bielich, PRD 108, 116003 (2023)
  holographic nuclear matter N. Kovensky, A. Poole, and A. Schmitt, PRD 105, 034022 (2022)

- include strangeness
  E. S. Fraga, R. da Mata, S. Pitsinigkos and A. Schmitt, PRD 106, 074018 (2022)

- interplay with (inhomogeneous) superconductivity