Fragmentation Function Fallacies

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Introduction

Prompted by previous discussions on fragmentation functions, which assume clean separation of perturbative and nonperturbative physics:

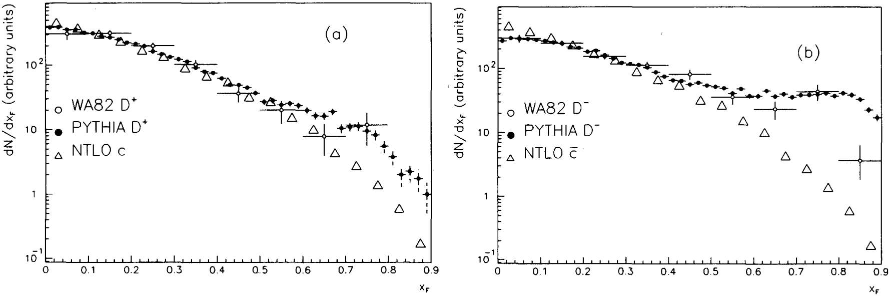
$$D_{\mathrm{H/Q}}(x_{\mathrm{H}} = x_{\mathrm{Q}} z, \mu^2) = D_{\mathrm{Q/Q}}(x_{\mathrm{Q}}, \mu^2, \mu_0^2) \otimes f_Q(z) \times P_{\mathrm{H/Q}}^{\mathrm{flavour}}$$

where $D_{\mathrm{Q/Q}}$ evolves with μ^2 according to DGLAP,
usually from $D_{\mathrm{Q/Q}}(x_{\mathrm{Q}}, \mu^2 = \mu_0^2 \approx m_{\mathrm{Q}}^2) = \delta(x_{\mathrm{Q}} - 1)$;
and the μ^2-independent $f_{\mathrm{Q}}(z)$ has $0 < z < 1$.

Old knowledge seems lost to people of today, so time to remind that

- fragmentation functions fail in hadronic collisions;
- they are based on the concept of independent fragmentation, which has been disproven in $\mathrm{e^+e^-}$ collisions; and
- string and cluster fragmentation of a Q introduce a dependence on the colour connections and momenta of nearby partons.

Factorization breakdown in fixed-target $\pi^- p$



($x_{\mathrm{F}} = p^*_L / p^*_{L,max}$, L = longitudinal, * = in CM)

WA82, Phys.Lett. B305 (1993) 402

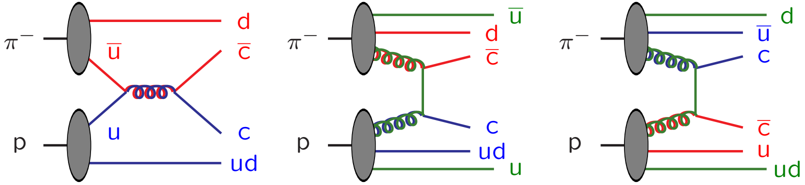
Fragmentation function factorization

$$\frac{\mathrm{d}N_{\mathrm{D}}}{\mathrm{d}x_{\mathrm{F}}} = \frac{\mathrm{d}N_{\mathrm{c}}}{\mathrm{d}x_{\mathrm{F}}} \otimes f(z) \quad , \quad 0 < z < 1 \quad , \quad z \approx \frac{x_{\mathrm{F,D}}}{x_{\mathrm{F,c}}} \approx \frac{E_{\mathrm{D}}}{E_{\mathrm{c}}} \approx \frac{p^+_{\mathrm{D}}}{p^+_{\mathrm{c}}}$$

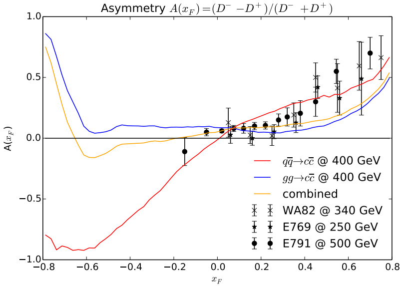
does not work!

Production asymmetries in fixed-target $\pi^- \mathrm{p}$

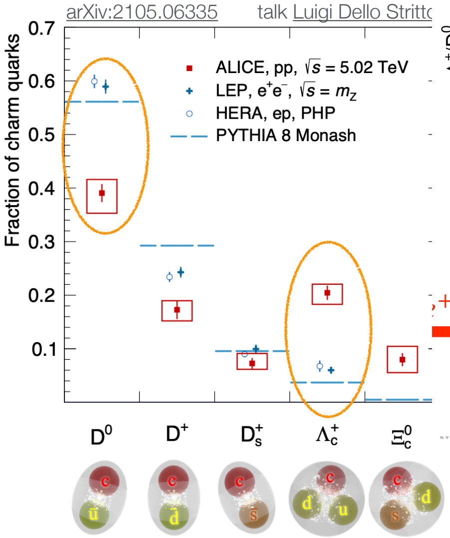
$\mathrm{u\overline{u}} \to \mathrm{c\overline{c}}$ pulls $\overline{\mathrm{D}}$ forwards, while $\mathrm{gg} \to \mathrm{c\overline{c}}$ can pull either D or $\overline{\mathrm{D}}$:

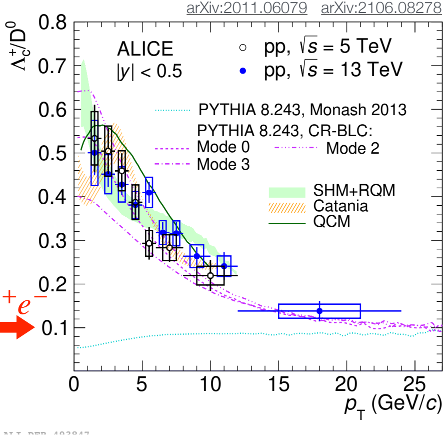


Asymmetry $A(x_{\mathrm{F}}) = (\sigma(\mathrm{D}^-) - \sigma(\mathrm{D}^+))/(\sigma(\mathrm{D}^-) + \sigma(\mathrm{D}^+))$:



Charm hadron composition at the LHC





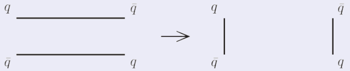
Would require $P_{\mathrm{H/Q}}^{\mathrm{flavour}}$
to depend on $p_{\perp}$ and process
(+ multiplicity, more?).

K. Reygers, EPS-HEP 2021

QCD-based Colour Reconnection

Christiansen & Skands(2015): QCD-inspired CR (QCDCR, CR-BLC)

Ordinary string reconnection

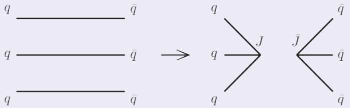


($q\bar{q}$: 1/9, gg: 1/8, model: 1/9)

Double junction reconnection

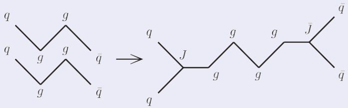
(qq: 1/3, gg: 10/64, model: 2/9)

Triple junction reconnection

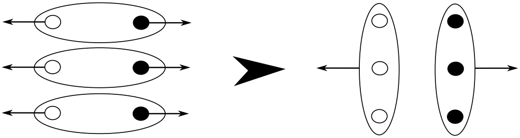


($q\bar{q}$: 1/27, gg: 5/256, model: 2/81)

Zipping reconnection



(Depends on number of gluons)

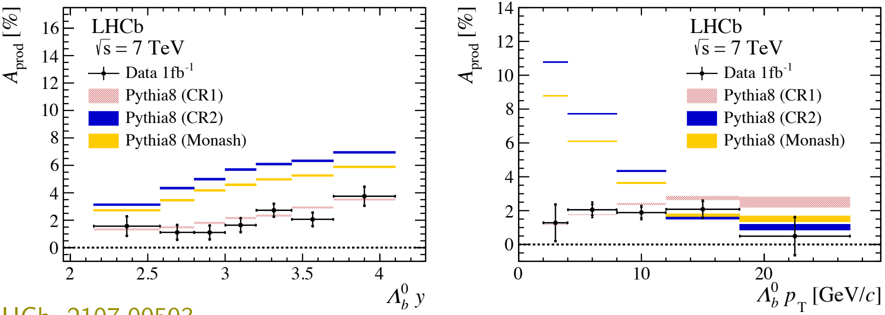


Triple-junction also in HERWIG cluster model (2017).

Bottom production asymmetries

Asymmetries predicted and observed also for charm and bottom hadrons at the LHC, but full picture not yet clear.

$$A = (\sigma(\Lambda_b^0) - \sigma(\overline{\Lambda}_b^0))/(\sigma(\Lambda_b^0) + \sigma(\overline{\Lambda}_b^0))$$



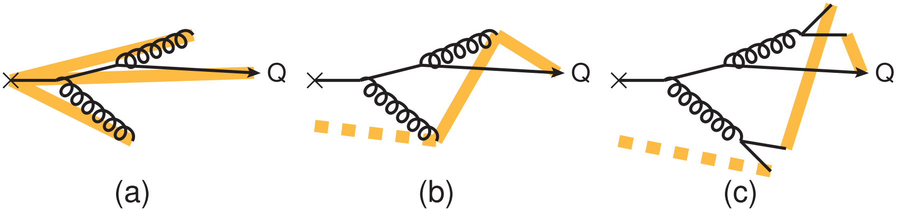
LHCb, 2107.09593

Enhanced Λ_b production at low $p_\perp$, like for Λ_c, dilutes asymmetry?

Little/no support for fragmentation function approach in hadron colliders.

Fragmentation models

Consider hadronization in $e^+e^- \to \gamma^*/Z^0 \to Q\overline{Q}$:

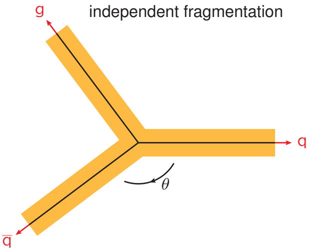


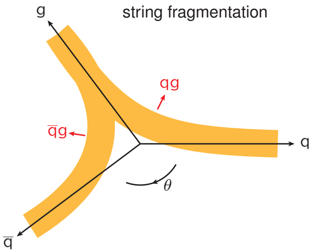
(a) Independent Fragmentation: each parton fragments separately along an axis stretching out from the CM origin; **ideological underpinning of fragmentation functions**.

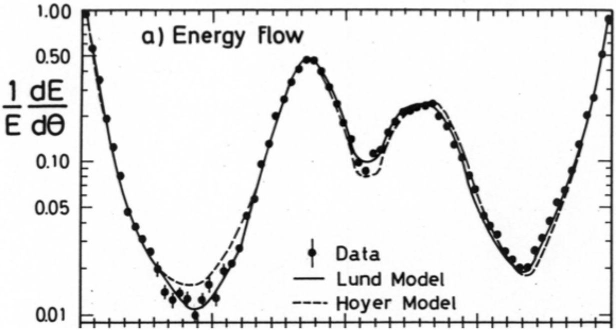
(b) String Fragmentation: string stretched from the Q via intermediate colour-ordered gluons to the $\overline{Q}$, with hadrons formed along its length (and an occasional $g \to q\overline{q}$ leads to the break of a string in two).

(c) Cluster Fragmentation: force all final gluons to split by $g \to q\overline{q}$ to give smaller and simpler clusters that decay to two hadrons (and massive clusters are split into smaller along "string" direction).

The string/JADE Effect







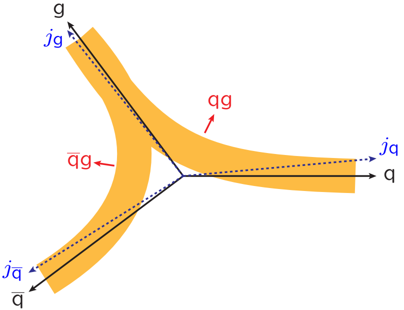
3 jets energy-ordered.

JADE (1980, 1983)

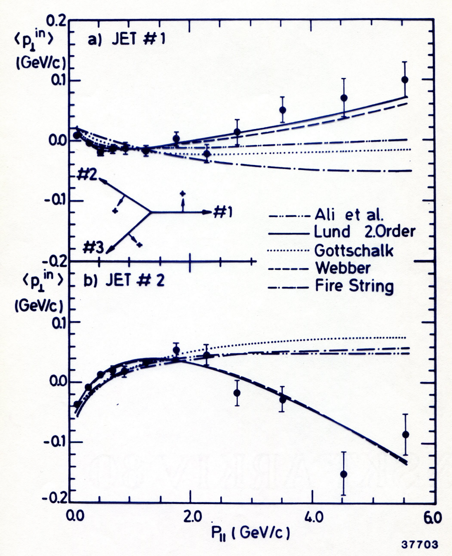
Jets are crooked

$(E, \mathbf{p})$ not preserved when massless partons become massive jets!

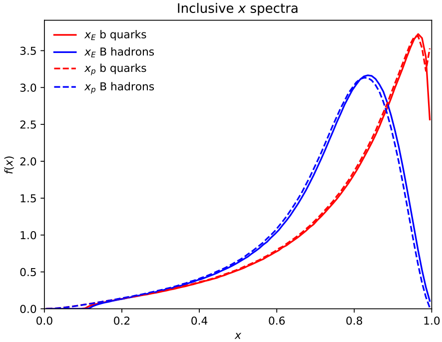
In the string model the reconstructed q and $\overline{\mathrm{q}}$ jet axes are shifted in the g direction:

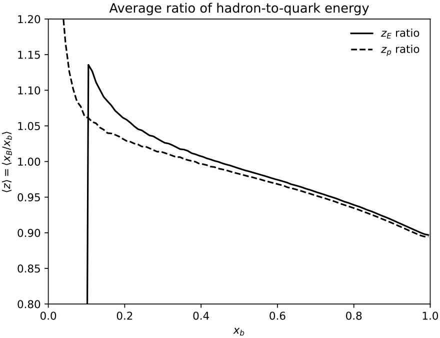


Clear PETRA/LEP evidence that independent fragmentation does not work (in $\mathrm{e^+e^-}$).



Study $\mathrm{e}^+\mathrm{e}^- \to \mathrm{Z}^0 \to \mathrm{b}\overline{\mathrm{b}}$ at $E_{\mathrm{cm}} = m_{\mathrm{Z}}$ with shower and hadronization; exclude events with additional $\mathrm{g} \to \mathrm{b}\overline{\mathrm{b}}$ branchings

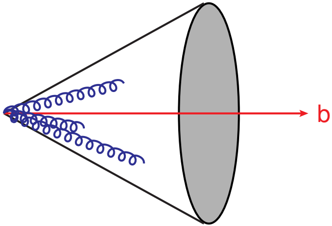




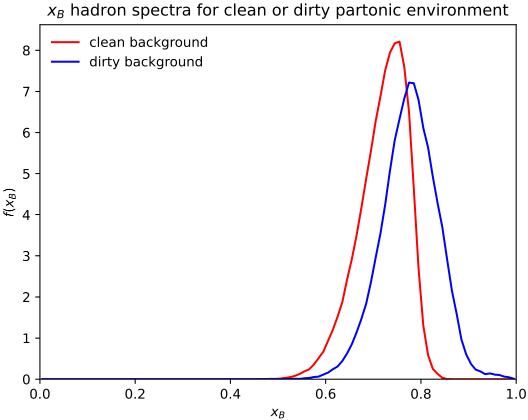
$x_E = 2E/m_{\mathrm{Z}}$ and $x_p = 2|\mathbf{p}|/m_{\mathrm{Z}}$ similar, except at small x values.
Red: b quarks after shower.
Blue: B hadrons after hadronization.

Here $\langle z \rangle(x_{\mathrm{b}})$ with $z = x_{\mathrm{B}}/x_{\mathrm{b}}$.
Large x_{b}: "deceleration" in $\mathrm{b} \to \mathrm{B}$.
Small x_{b}: "acceleration" in $\mathrm{b} \to \mathrm{B}$.

Environmental dependence



Given a b quark after the shower, draw a $R = 0.5$ cone on the unit sphere around it, and find other partons inside the cone.



Study bin $0.78 < x_{E,\mathrm{b}} < 0.80$ and define $x_{\mathrm{cone}} = 2 \sum_{i \in \mathrm{cone}} E_i / m_{\mathrm{Z}}$

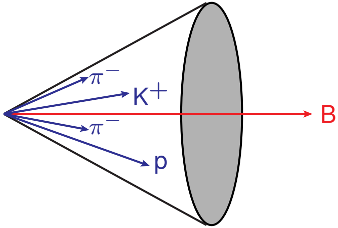
Red: $x_{\mathrm{cone}} < 0.05$, "clean" background

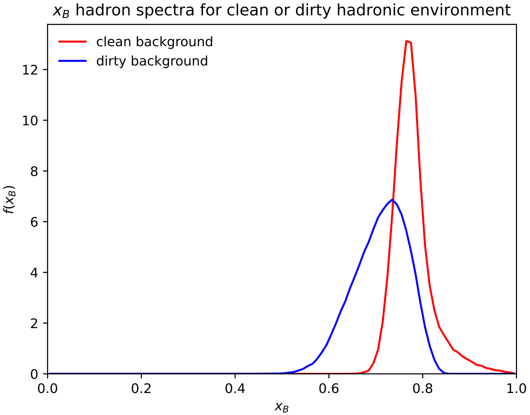
Blue: $x_{\mathrm{cone}} > 0.15$, "dirty" background

Harder x_{B} in dirty case! "Reabsorption" of emitted energy.

Environmental dependence — caveat

Results not translated to hadron level:





Same procedure as before, but now a B hadron at center and other hadrons in a cone around it.

Still study parton-level bin $0.78 < x_{E,\mathrm{b}} < 0.80$. (Unphysical!)
Reversed order "clean" $\leftrightarrow$ "dirty", since now energy lost in $\mathrm{b} \to \mathrm{B}$ contributes to background.

Issues and conclusion

- Naively $\mathrm{d}\sigma(\mathrm{D}) = \mathrm{d}\sigma(\mathrm{c}) \otimes f(z = E_{\mathrm{B}}/E_{\mathrm{b}}) \times P_{\mathrm{flavour}}(\mathrm{c} \to \mathrm{D})$.
  Such factorization is strongly broken in hadron collisions,
  as manifested by beam drag "speedup", D (and B) asymmetries,
  and an environment-dependent charm hadron composition.
- PETRA and LEP data disprove Independent Fragmentation.
- Fragmentation functions are static. They may work
  for some simple tasks, but do not offer a full picture.
- Event generators with strings/clusters are not perfect,
  but they offer a more realistic and dynamic approach.
- Generator uncertainty from many issues: $\mathrm{N}^n\mathrm{LO}$, PDFs,
  $m_{\mathrm{c,b}}$, α_{s}, shower, match&merge, colour reconnection, ...