Making a fluid atom by atom

Emergent hydrodynamic behavior of few strongly-interacting fermions

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CERN

QTI-TH Forum
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Based on work within the ISOQUANT Collaborative Research Centre

https://www.isoquant-heidelberg.de

IsoQuant SFB1225

New!!

ABC

Origins of collectivity in few-body systems

Heavy-Ion Collisions, Ultracold Atoms

Principal Investigators

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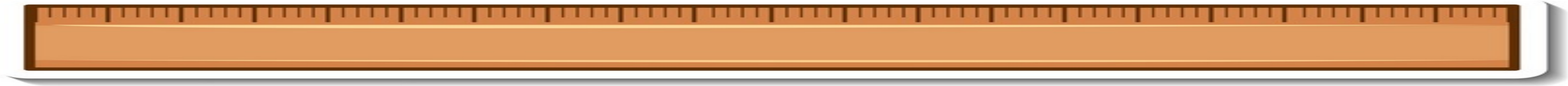
OUTLINE

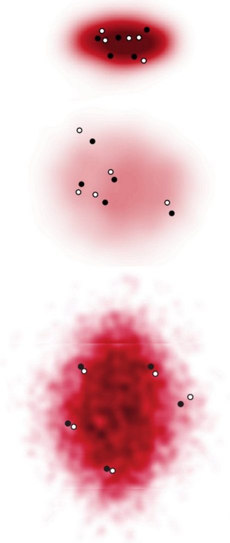
- Hydrodynamics, heavy-ion collisions, elliptic flow
- The small system question in high-energy collisions
- Why ultra-cold atoms? Full control of a quantum system
- Elliptic flow in a few-fermion system: hydrodynamic interpretation
- Some prospects

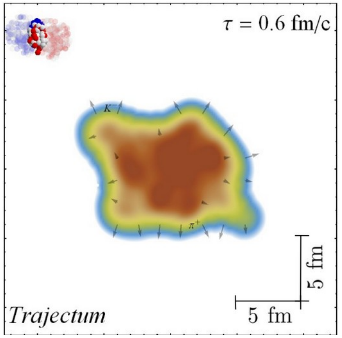
THE PHYSICAL WORLD AS AN EMERGENT PHENOMENON

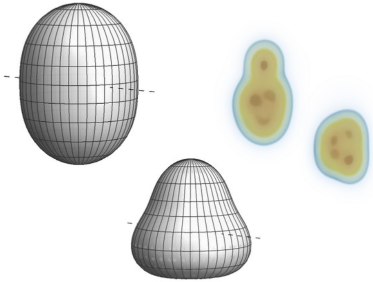
10^{15} m

10^{-15} m

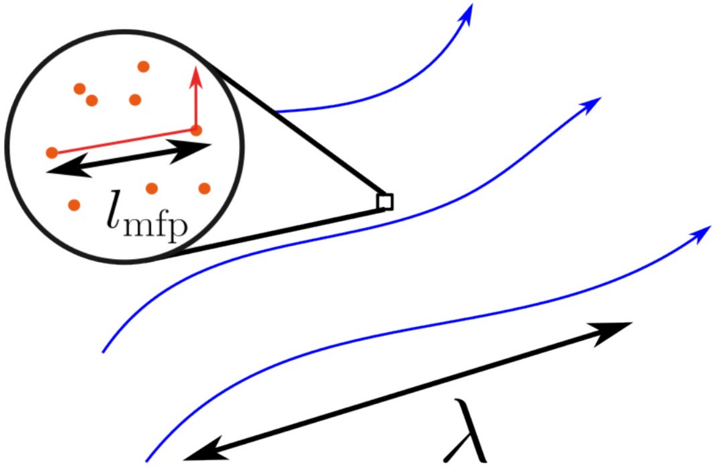








(COLLISIONAL) HYDRODYNAMICS



$$l_{\mathrm{mfp}} \ll \lambda$$

… from kinetic theory

The *pressure tensor* is defined as the fluctuation of the velocities of the ensemble from the mean velocity, i.e. as the 2-nd order moment:

$$\mathbf{P} = m \int (\mathbf{v} - \mathbf{v}_b)(\mathbf{v} - \mathbf{v}_b) f(\mathbf{v}) d^3 v$$

$$\mathbf{u}_\alpha = \langle \mathbf{v}_\alpha \rangle$$

Motion from conservation laws

$$\rho \left(\partial_t + \vec{u} \cdot \vec{\nabla} \right) \vec{u} = -\vec{\nabla} p + \eta \nabla^2 \vec{u}$$

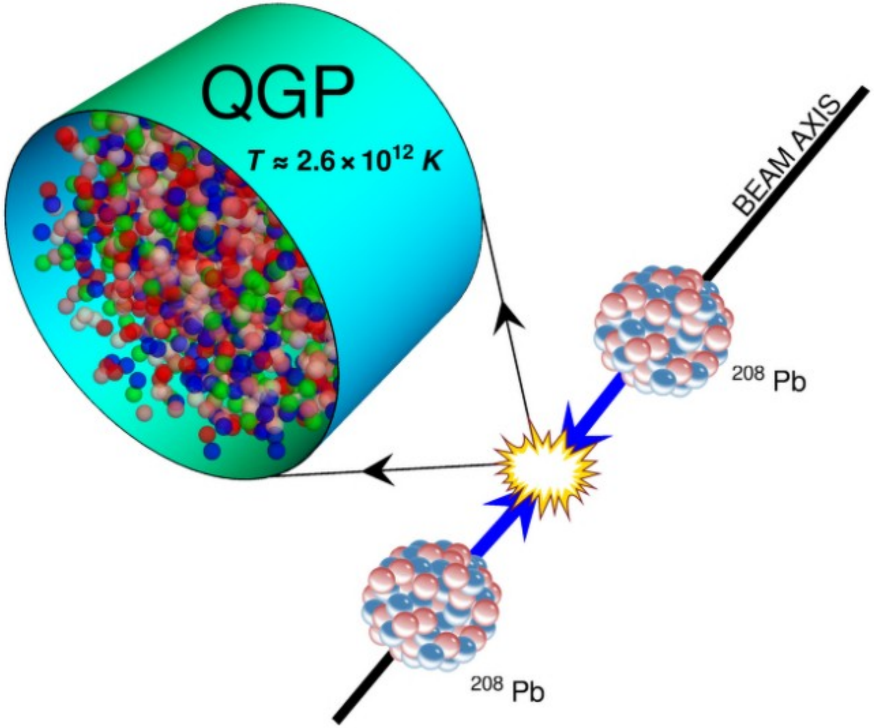
density pressure shear

Large N, separation of scales (micro vs. macro), equilibrium, …

Heavy Ion Collisions – Somehow adhering to the“More is Different” paradigm in the context of high-energy physics **[P.W. Anderson, 1972]**

turn to a different direction; we should investigate some "bulk" phenomena by distributing high energy or high nucleon density over a relatively large volume. *The fact*

[T-D. Lee, 1974 link]



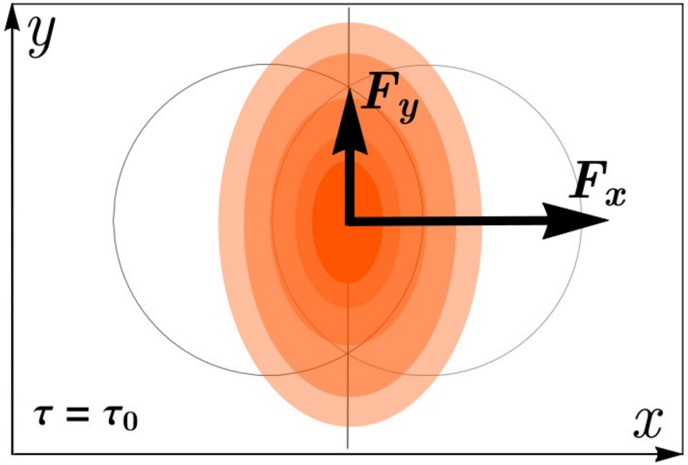
Key probe of hydrodynamic behavior – Elliptic flow

[Ollitrault, PRD **46** (1992) 229-245]

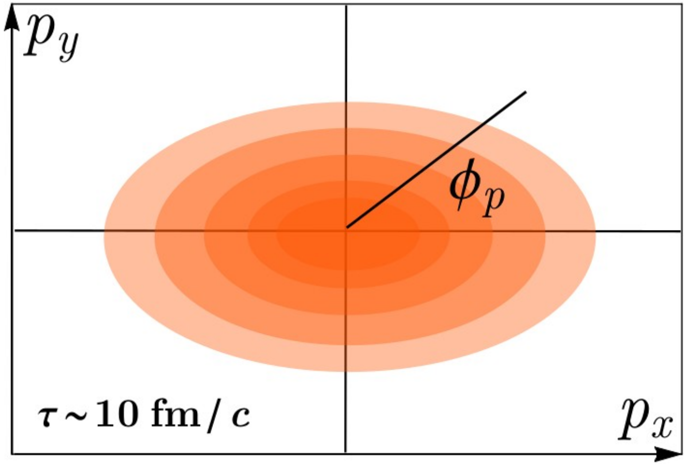
$$F = -\nabla P \quad \longrightarrow \quad V_2 = \frac{1}{N}\int_{\mathbf{p}_t} \frac{dN}{d^2\mathbf{p}_t} e^{-i2\phi_p}$$

1/R

Coordinate space



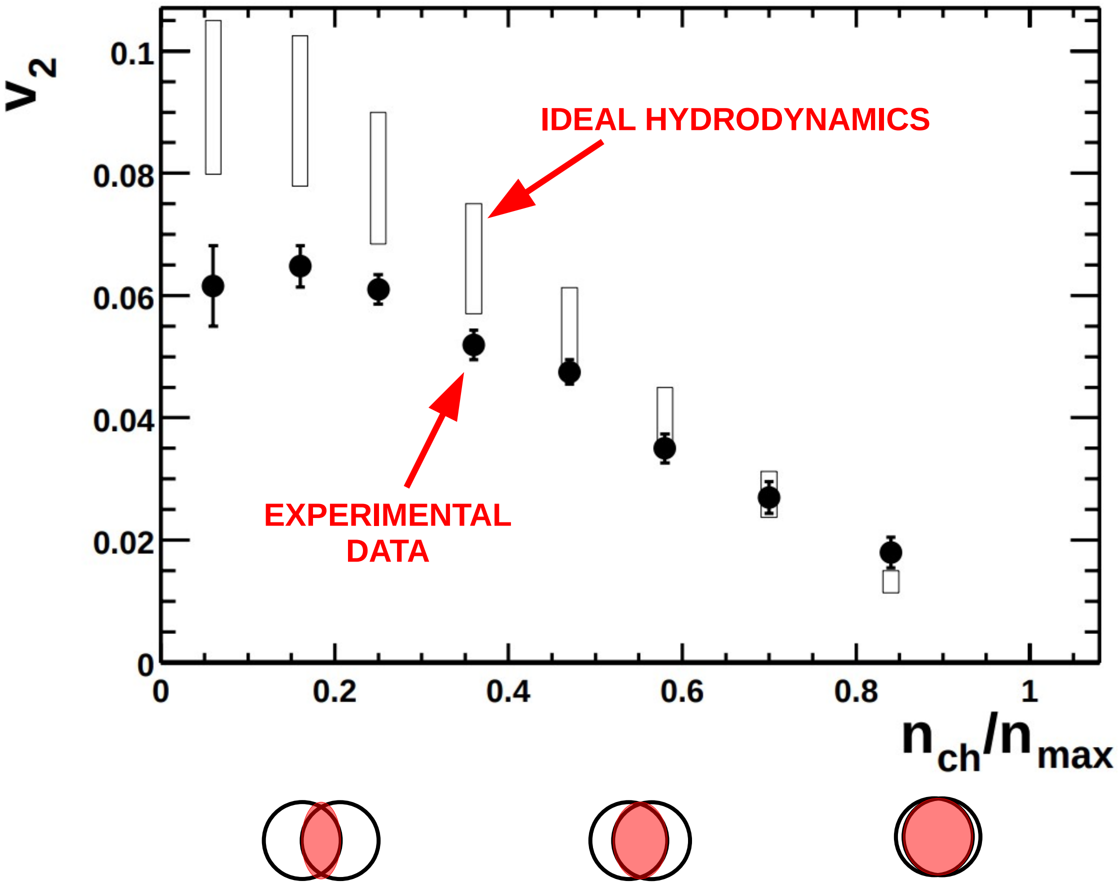
Momentum space



2000 – Evidence of a strongly-coupled quark-gluon plasma

[STAR collaboration, PRL **86** (2001) 402-407]

^{197}Au+^{197}Au collisions @ RHIC

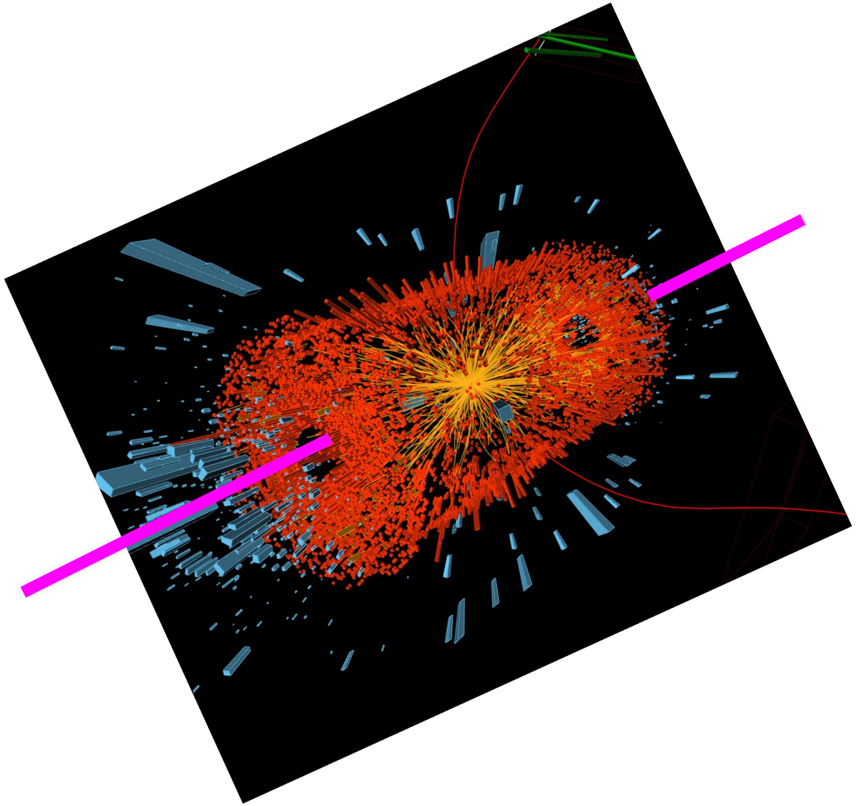


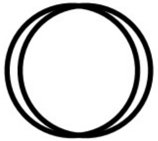
≈ 1000 final-state hadrons

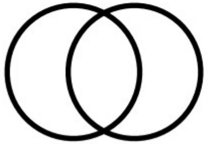
Viscous corrections to fix the visible discrepancies

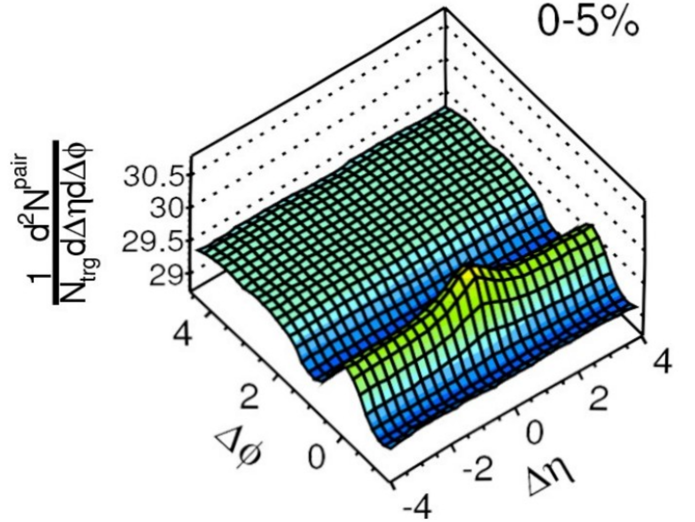
[Teaney, PRC **68** (2003) 034913]
[Romatschke & Romatschke, PRL **99** (2007) 172301]

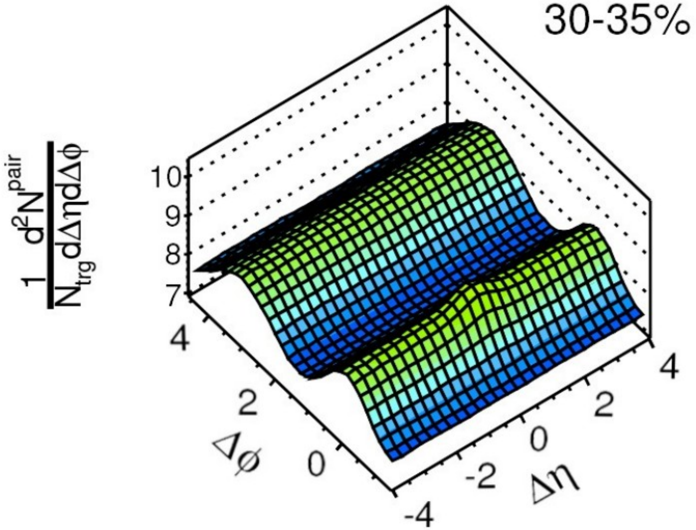
^{208}Pb+^{208}Pb collisions @ LHC











[CMS collaboration, EPJC **72** (2012) 2012]

Big discovery at the LHC – Small system collectivity

[Wiedemann, Grosse-Oetringhaus, arXiv:2407.7484]

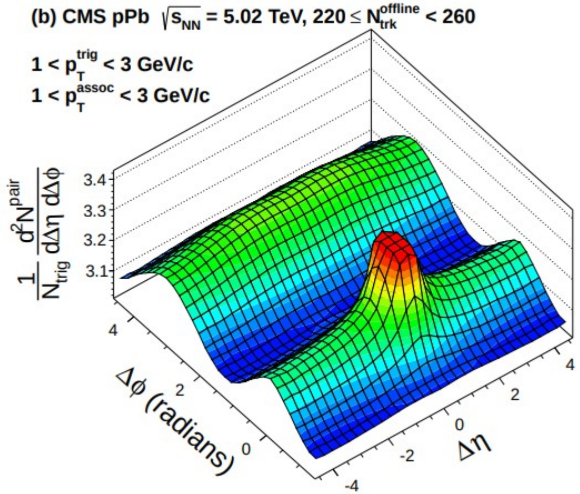
A hydrodynamic description is not justified based on “standard criteria”

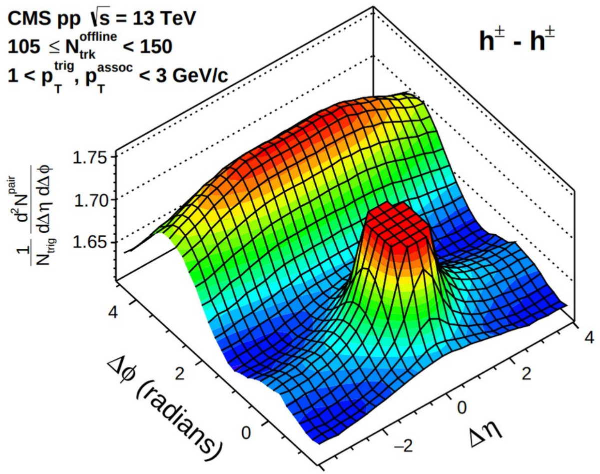
[Ambrus, Schlichting, Werthmann, PRL **130** (2023) 15, 152301]
[Kurkela, Wiedemann, Wu, EPJC **79** (2019) 11, 965]

Triggered vast program on thermalization and out-of-equilibrium dynamics

[Berges, Heller, Mazeliauskas, Venugopalan, RMP **93** (2021) 3, 035003]

[CMS collaboration, PLB **724** (2013) 213-240]
[CMS collaboration, PLB **765** (2017) 193-220]





“More is different” ?

Can we quantify what “more” means?

Are p-p collisions “different”?

Why ultracold atoms?

Exquisite experimental control

particle number

$$\mathcal{H} = -\sum_{i=1}^{N} \frac{\hbar^2}{2m} \boldsymbol{\nabla}_i^2 + \sum_{i<j} \frac{\hbar^2}{2m} g_0 \, \delta^{(d)} \left(\boldsymbol{r}_i - \boldsymbol{r}_j \right) + \sum_{i=1}^{N} \mathcal{V}_{\mathrm{ext}} \left(\boldsymbol{r}_i \right)$$

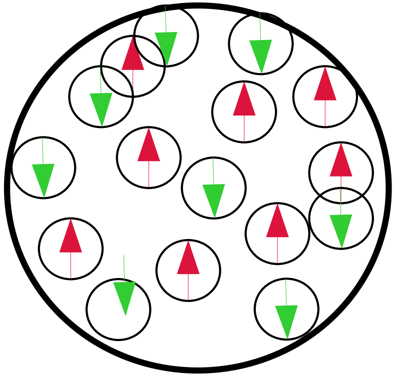
interaction strength

geometry

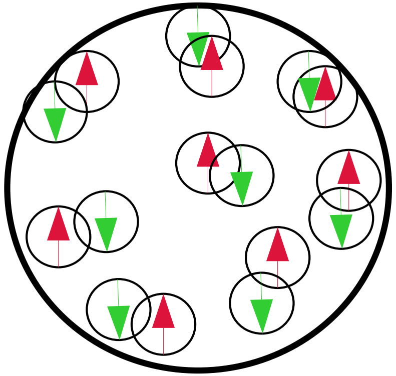
Parameters can be tuned in a table-top experiment

(SUPERFLUID) HYDRODYNAMICS

without interactions



with interactions



Fermions – BEC of molecules at T~0

molecule size << inter-molecule distance

Large system (N>>1), separation of scales ($a^3 n \ll 1$)

condensate fluctuations

$$\boxed{\hat{\Psi}(r,t) = \Psi(r,t) + \cancel{\delta\hat{\Psi}(r,t)}}$$

$$\Psi = \sqrt{n}\, e^{iS(r,t)} \qquad v_S = \frac{\hbar}{m}\nabla S$$

$\Rightarrow$

$$\frac{\partial}{\partial t} n + \nabla(v_S n) = 0$$

EOS

$$m\frac{\partial}{\partial t} v_S + \nabla\left(\frac{1}{2} m v_S^2 + \mu(n) + V_{ext}\right) = 0$$

Hydrodynamic equations of superfluids (T=0)
Closed equations for n and v_S

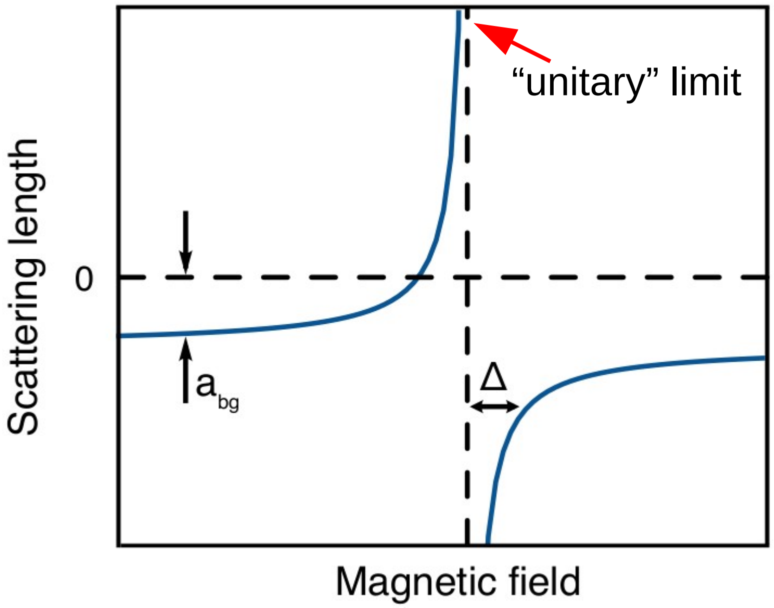
[from S. Stringari, **Lectures at Collège de France (2004/2005)**]

Control of interaction strength – From non-interacting to strongly-interacting systems

Interactions at low momenta described by an s-wave scattering length parameter

Tunable via a Feshbach resonance through an external magnetic field

[L. Bayha, PhD thesis, Heidelberg University (2020)]



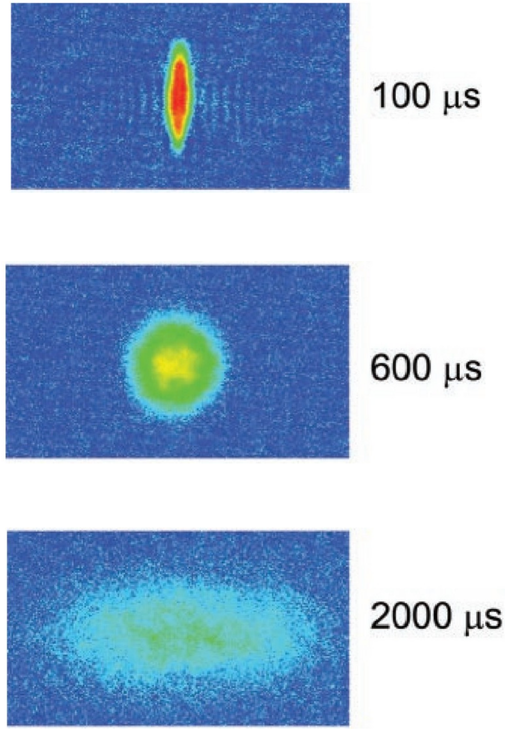
$$a_{3D} = a_{bg}\left(1 + \frac{\Delta}{B - B_0}\right)$$

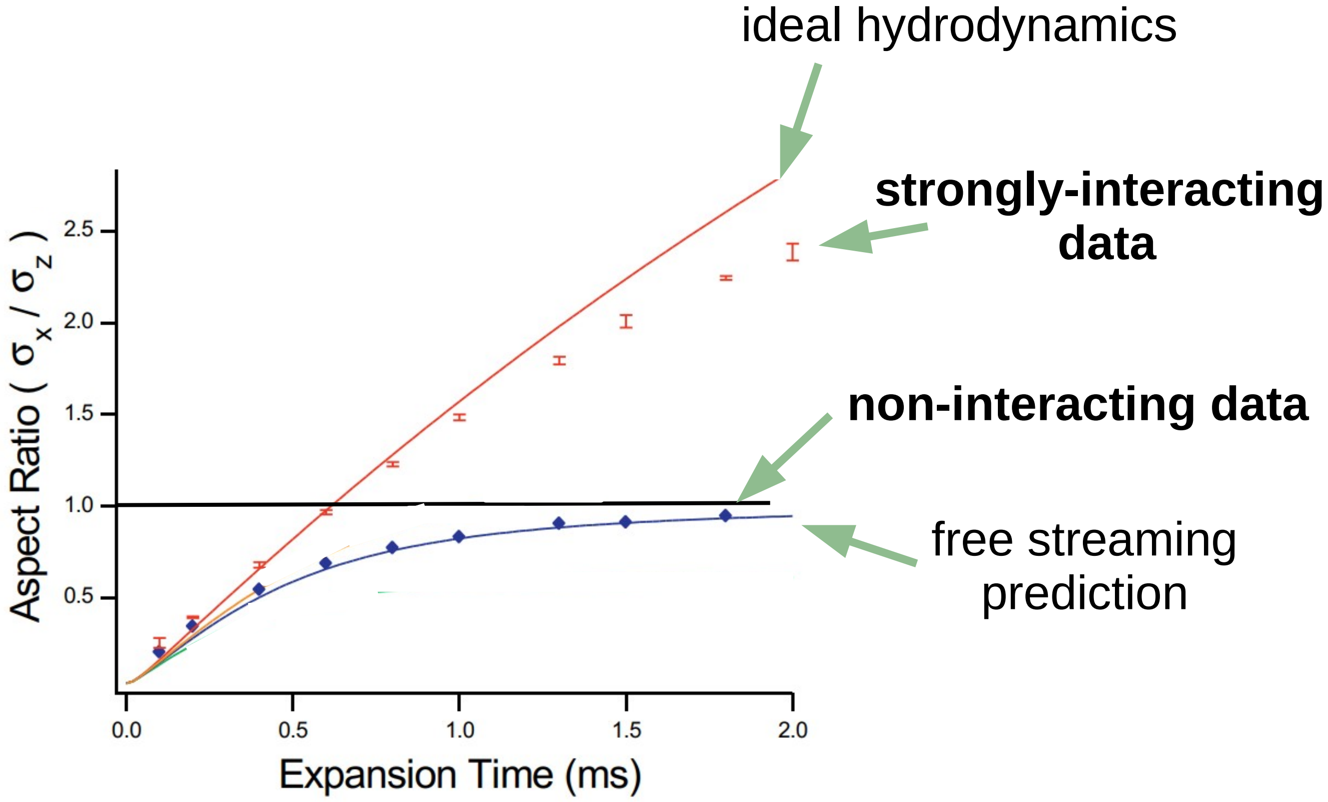
Values for lowest states of ^{6}Li:

$$a_{bg} = -2100\, a_{\mathrm{Bohr}}$$
$$B_0 = 690\,\mathrm{G}$$
$$\Delta = 200\,\mathrm{G}$$

Control of geometry and interactions – Elliptic flow to probe superfluid hydrodynamics

[O’Hara et al., Science **298** (2002) 2179-2182]
[Menotti, Pedri, Stringari, PRL **89**, 250402 (2002)]



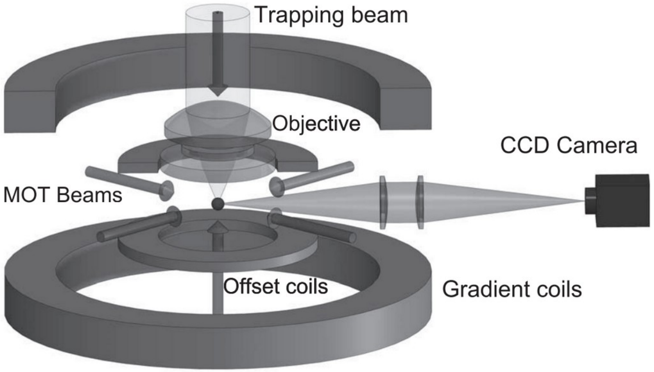


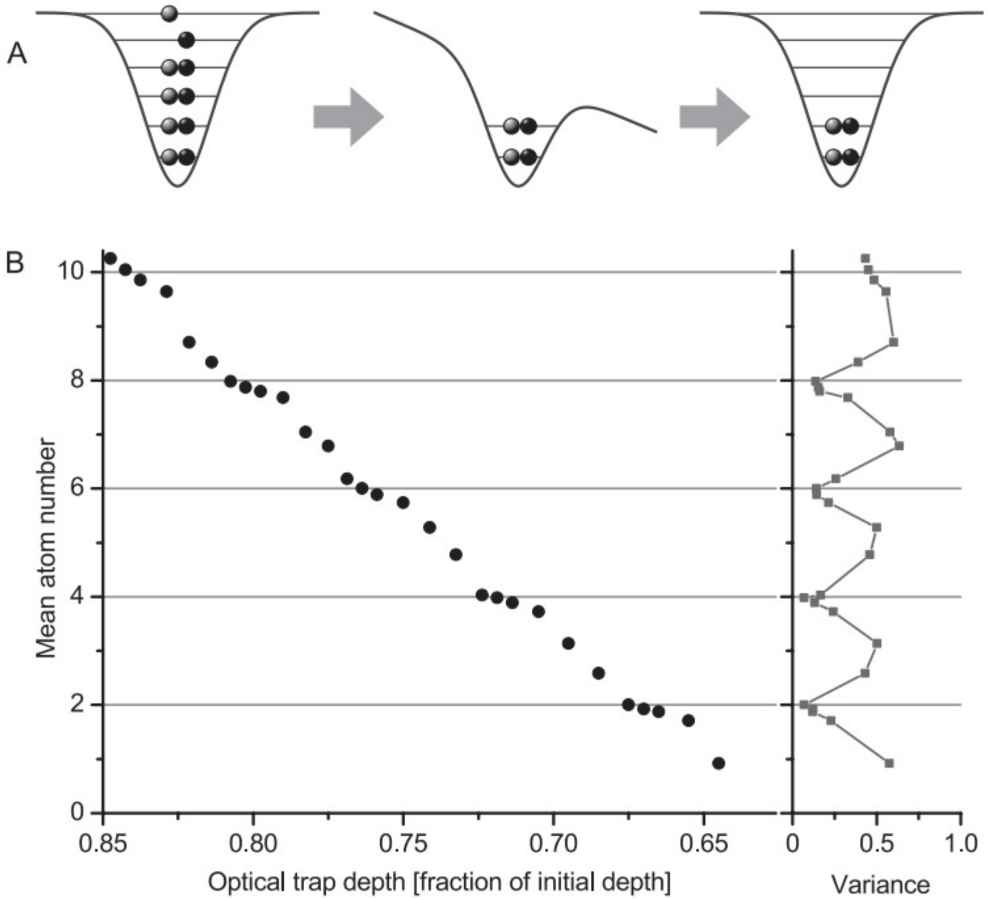
Recent new study in a thermal gas done @ Huzhou: Ke Li *et al.*,arXiv:2405.02847

Jochim Lab @ Heidelberg University – Control of particle numbers

[Serwane *et al.*, Science **332** (2011) 6027]

Pure quantum states (ground states)





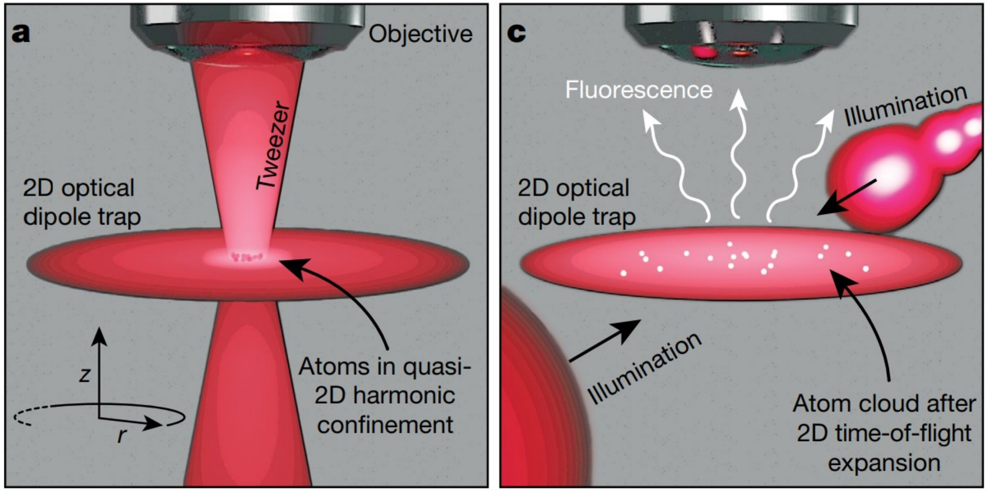
Jochim Lab @ Heidelberg University – Imaging of finite samples in “free space”

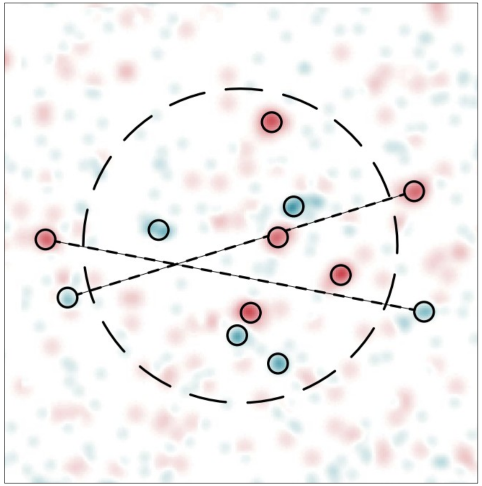
[Bergschneider *et al.*, PRA **97** 063613 (2018)]

For each atom one detects about 20 photons per 20µs of exposure

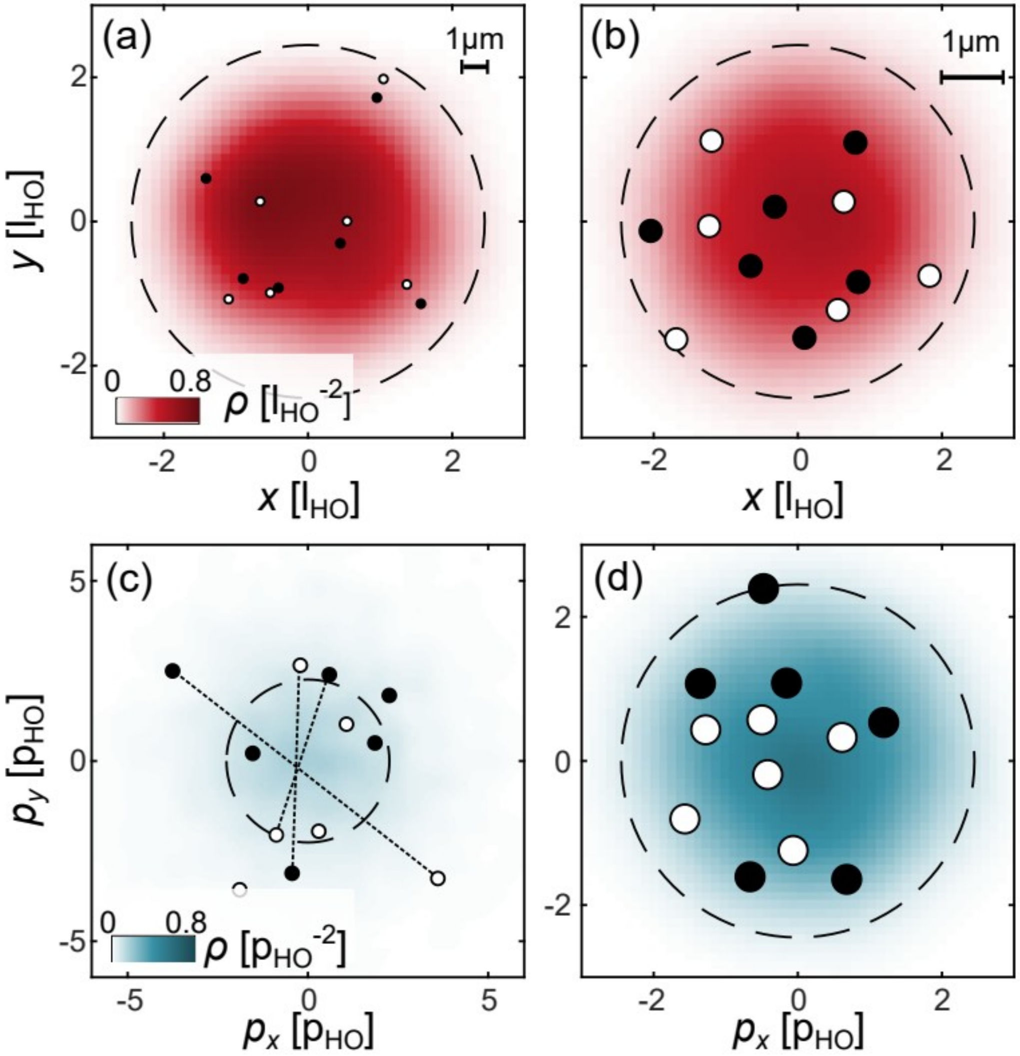
Localization fidelity: 99.4 ± 0.3%

[Holten *et al.*, Nature **606**, 287-291 (2022)]

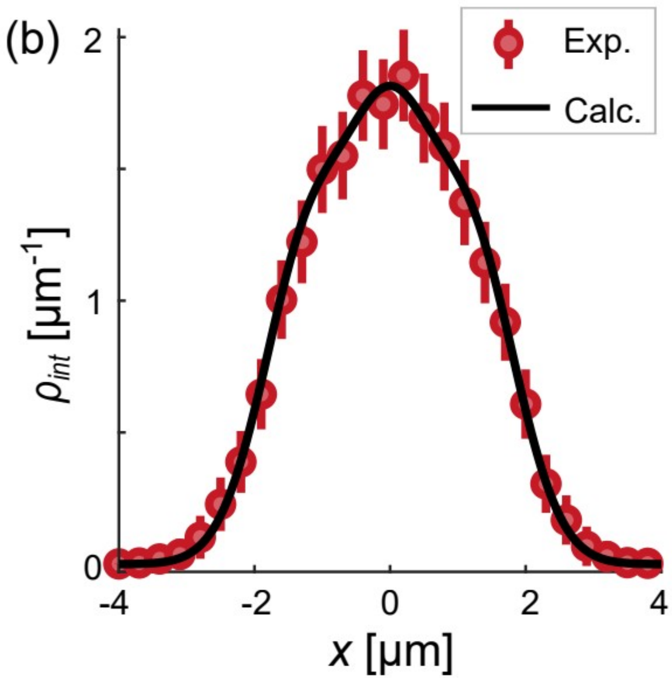




interacting non-interacting



non-interacting

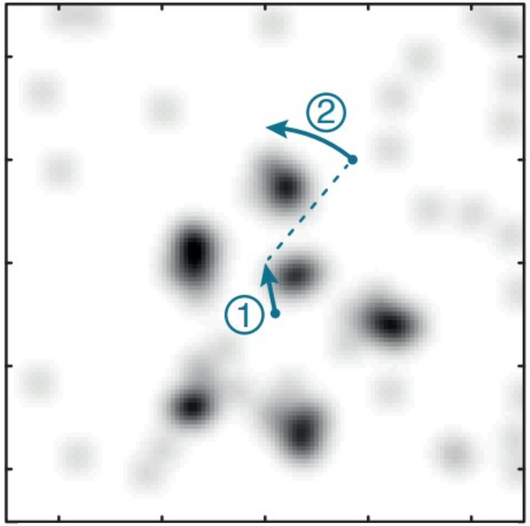


[Brandstetter *et al.*, arXiv:2409.18154]

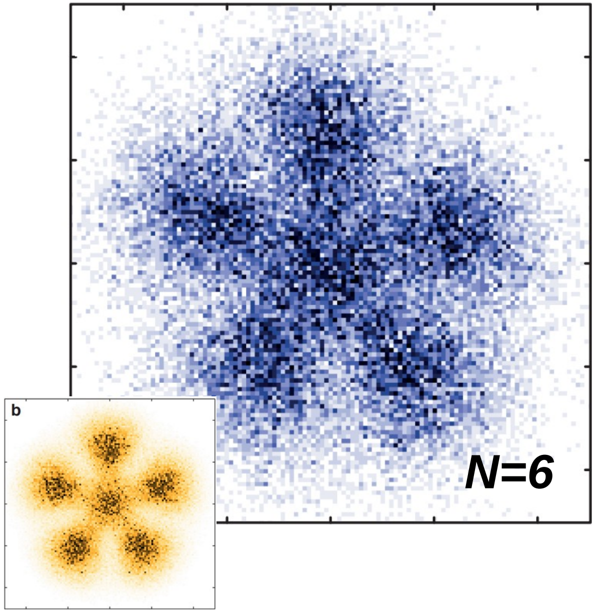
Demonstration – “seeing” the effect of Pauli exclusion

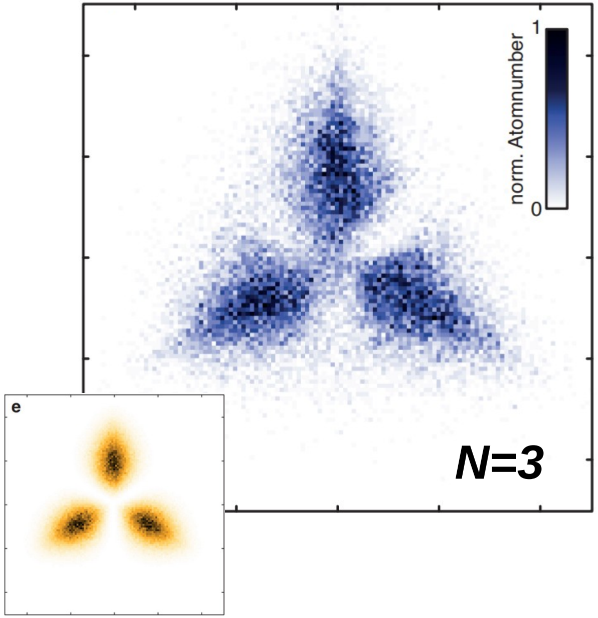
[Holten *et al.*, PRL **126**, 020401 (2021)]

single image



average image





High-fidelity preparation and imaging of zero-temperature states

Proposal – Elliptic flow with few trapped fermions

Flörchinger *et al.*, PRC **105** (2022) 4, 044908

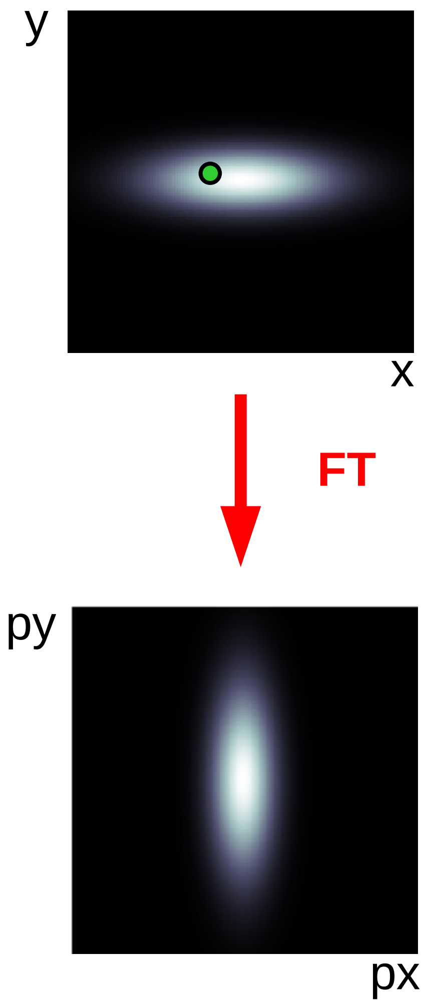
Studying emergence of “hydrodynamics” particle by particle

Beware of Heisenberg relations!

One atom in a harmonic potential

$$\boxed{\lambda = \frac{\omega_y}{\omega_x}}$$

$$\begin{aligned}\langle \cos(2\phi_p) \rangle_{\psi_{0,0}} &= \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle_{\psi_{0,0}} \\ &= \int \mathrm{d}p_x \mathrm{d}p_y \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} |\psi_{0,0}(p_x, p_y)|^2 \\ &= \frac{1-\sqrt{\lambda}}{1+\sqrt{\lambda}} = v_2\end{aligned}$$



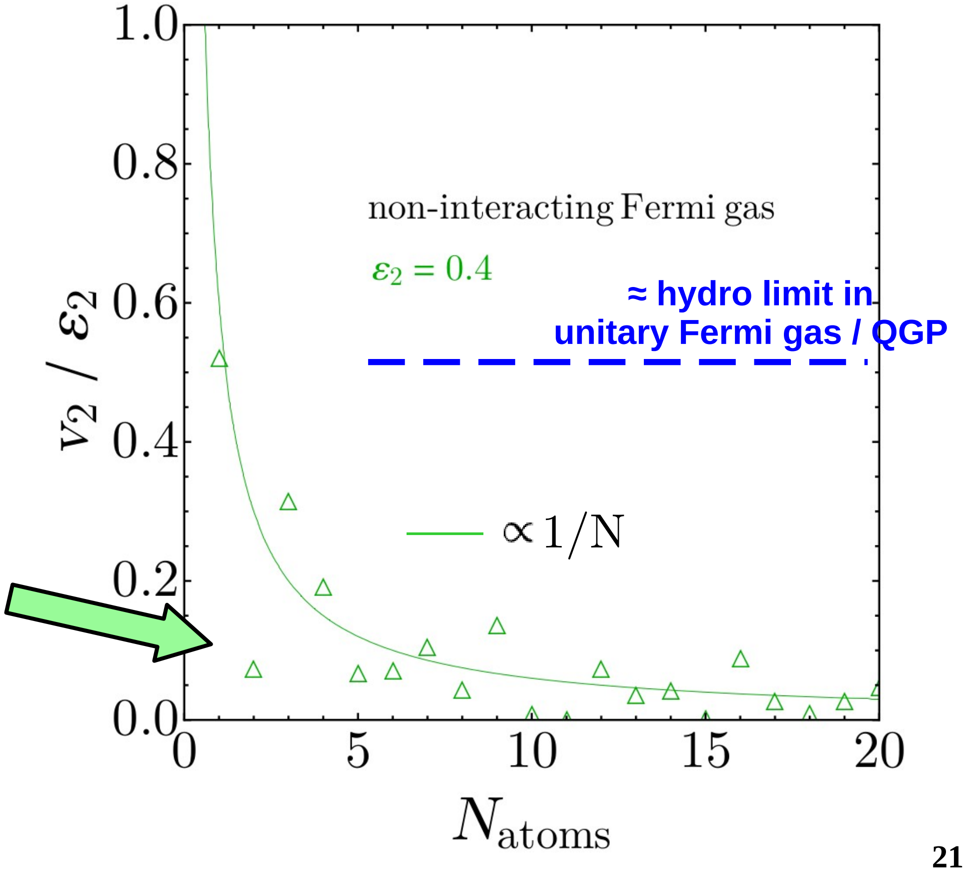
“Background” elliptic flow from position-momentum relation

Dependence of background elliptic flow on particle number

$$\varepsilon_2 = (1-\lambda)/(1+\lambda) \qquad \boxed{\lambda = \frac{\omega_y}{\omega_x}}$$

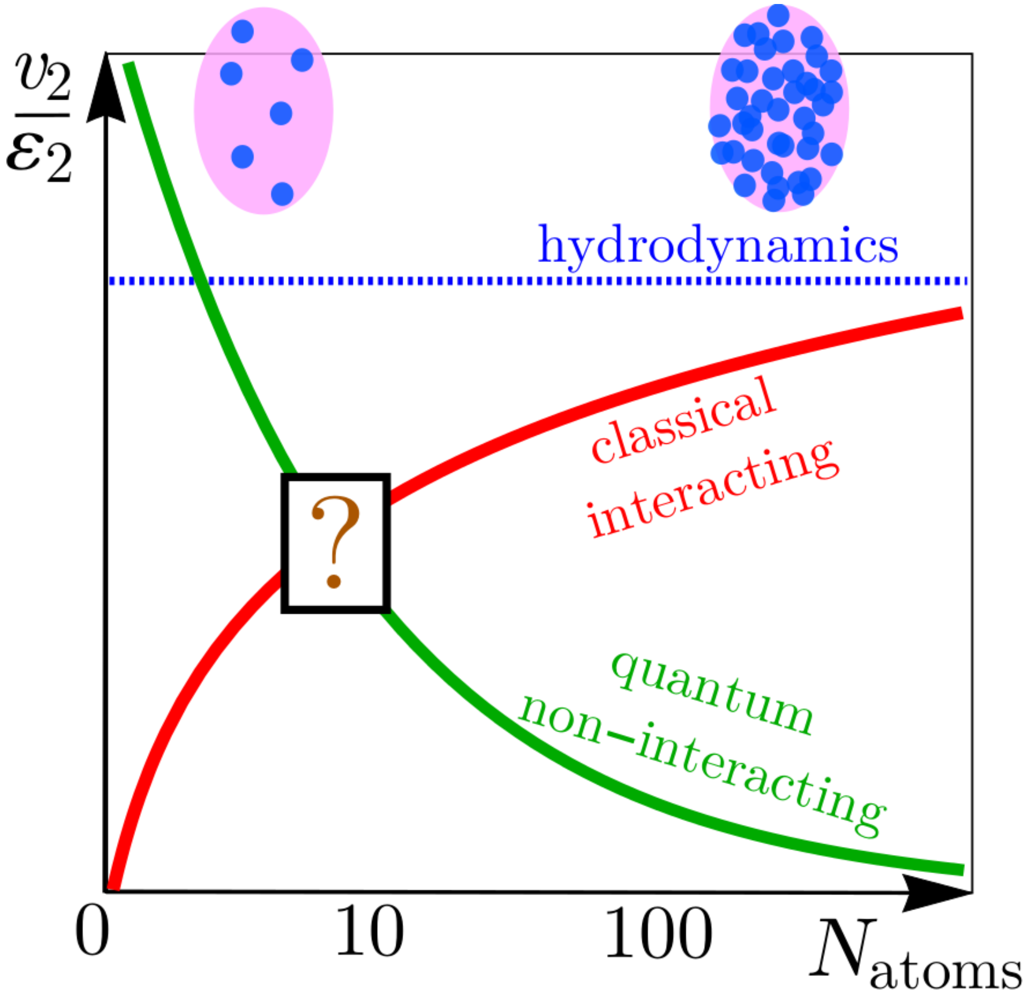
$$v_2 = \langle \cos(2\phi_p) \rangle_\Psi$$

N trapped
non-interacting
fermions



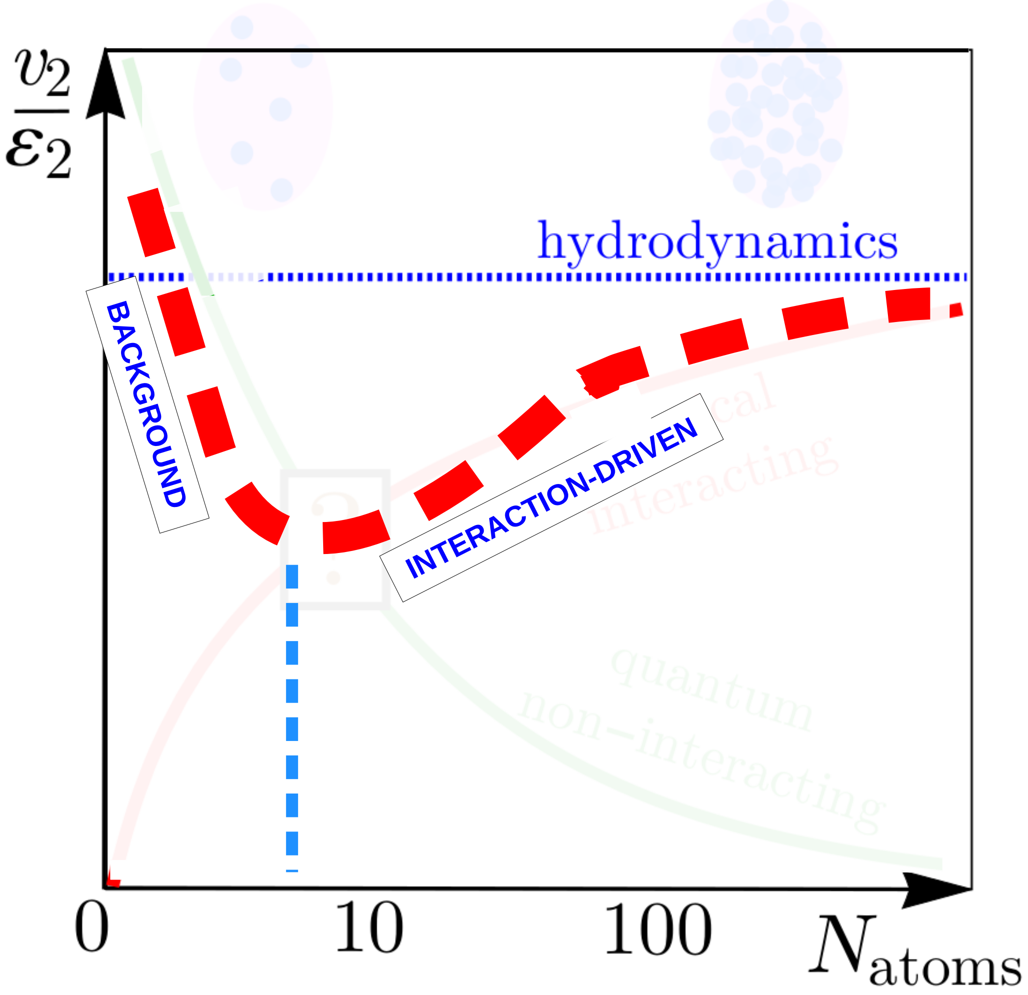
It disappears quickly, like 1/N

Qualitative expectations



Combining the curves…

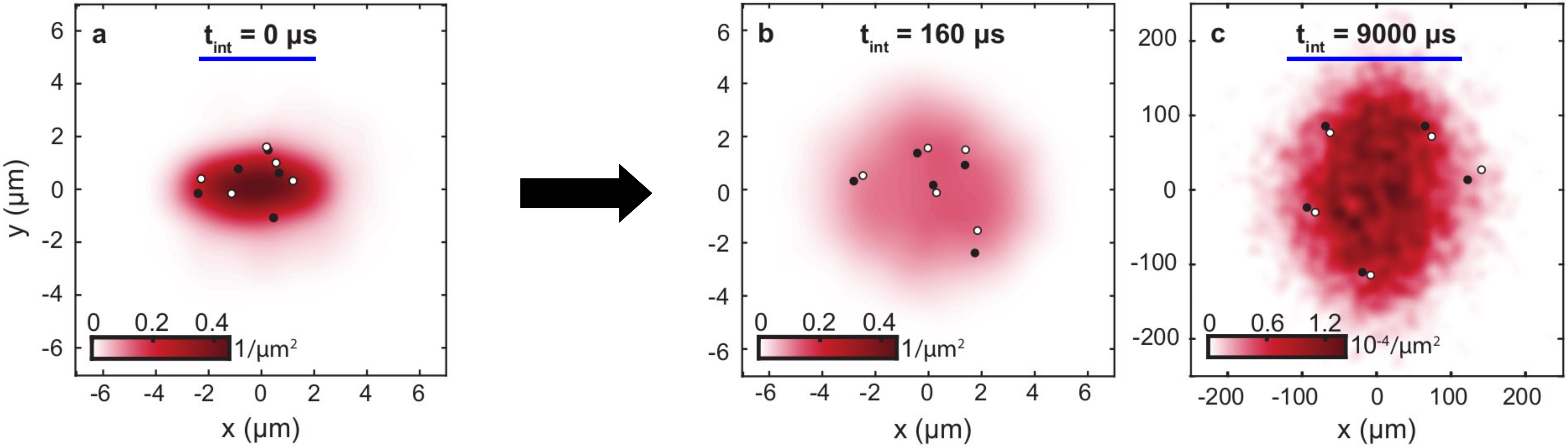
We predict non-monotonic behavior



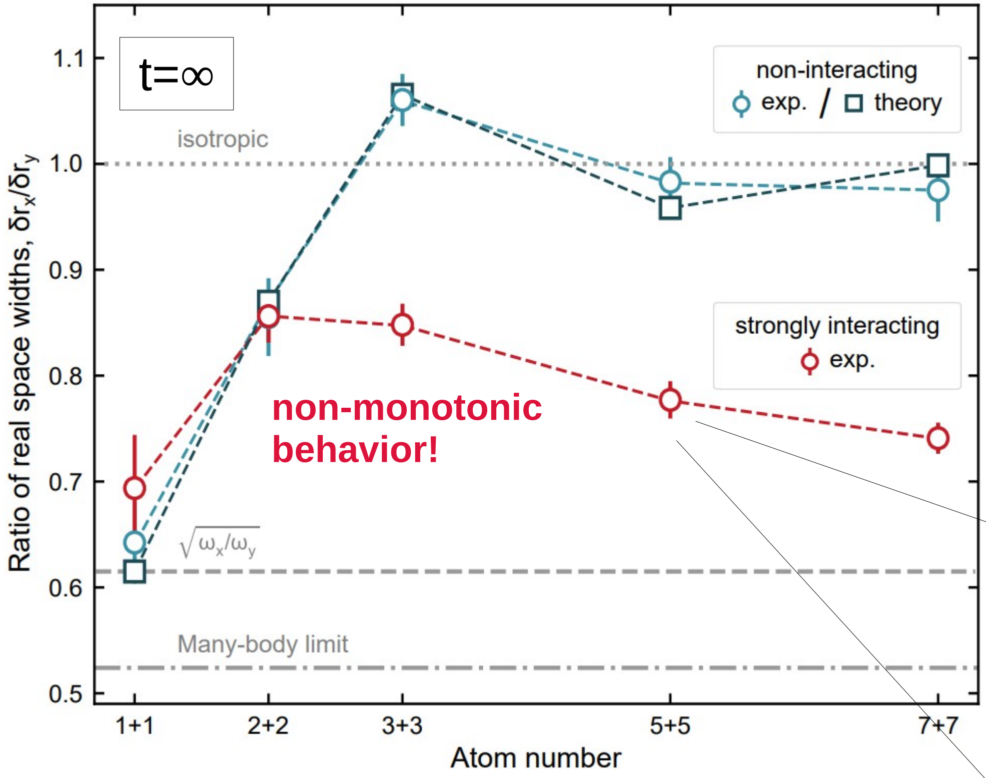
Experimental realization – Elliptic flow of few fermions

[Brandstetter *et al.*, arXiv:2308.09699, to appear in Nature Physics]

Phenomenon observed with just 10 atoms – small system puzzle?

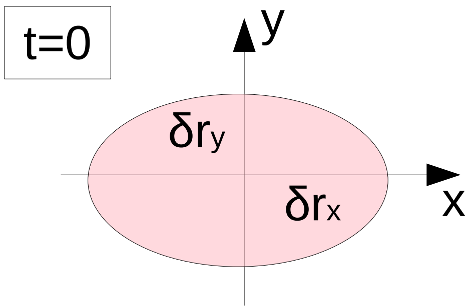


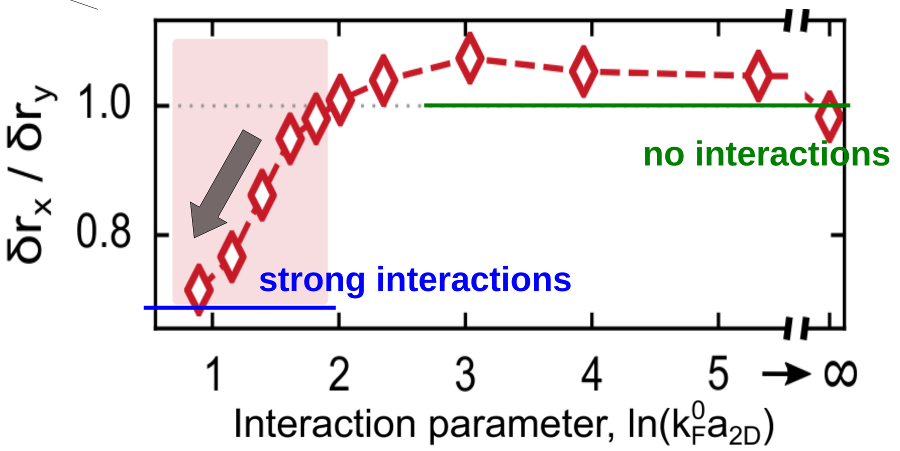
Emergence of elliptic flow as a function of particle number and interaction strength



Final state is isotropic

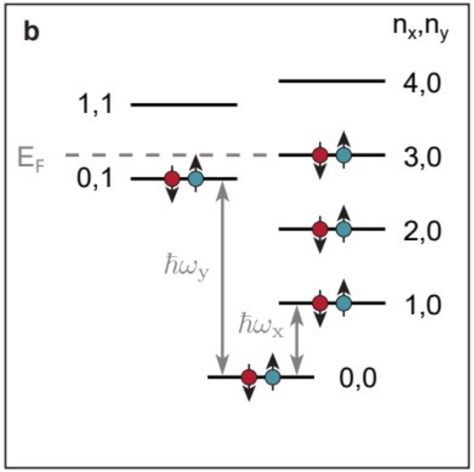
Final state is elliptical

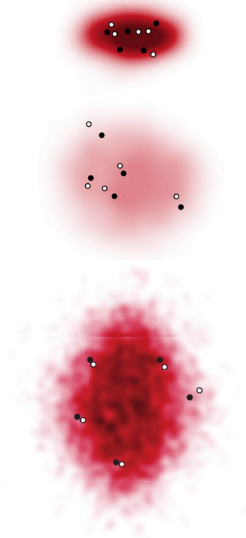




“More is different” …

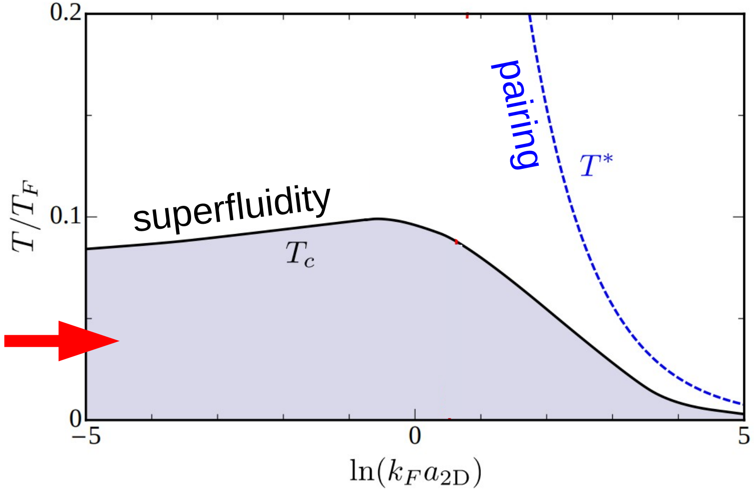
… 10 is different?!

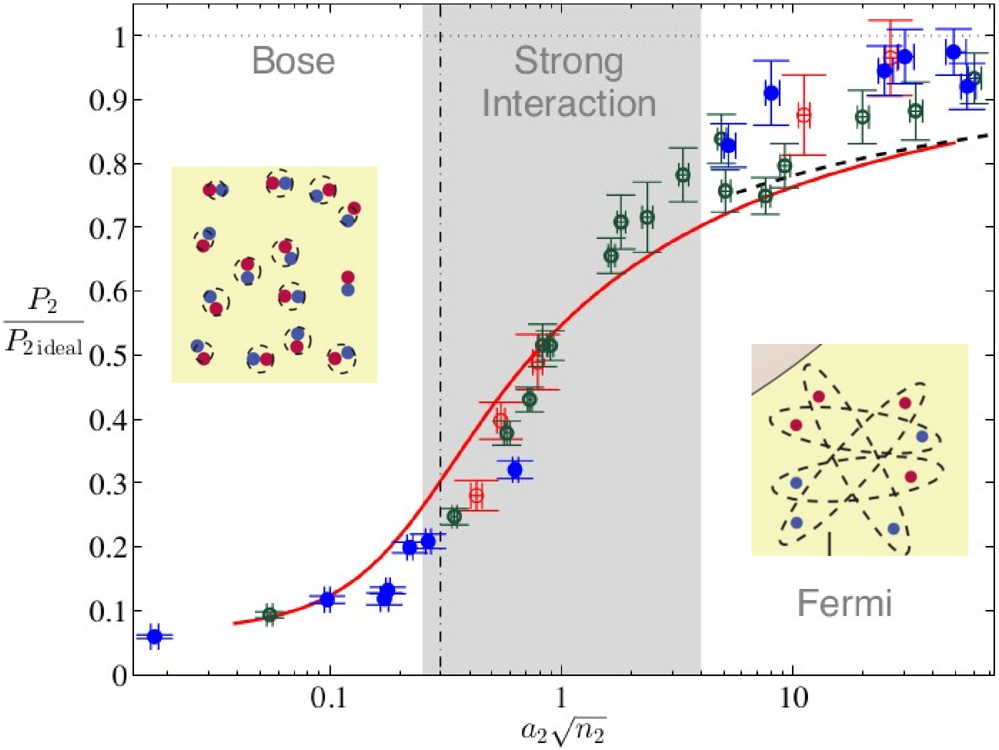




Hydrodynamic interpretation – superfluid in 2D (many-body limit)

[Levinsen, Parish, arXiv:1408.2737]





Continuity and Euler equations (ideal fluid)

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$$

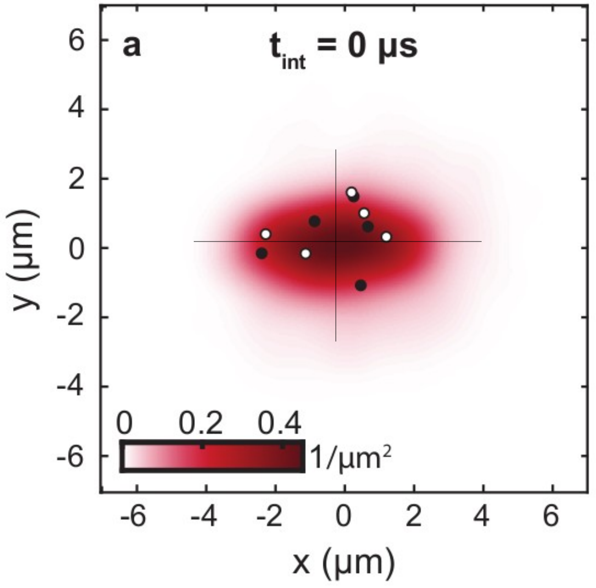
$$\rho(\partial_t + \mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla P$$

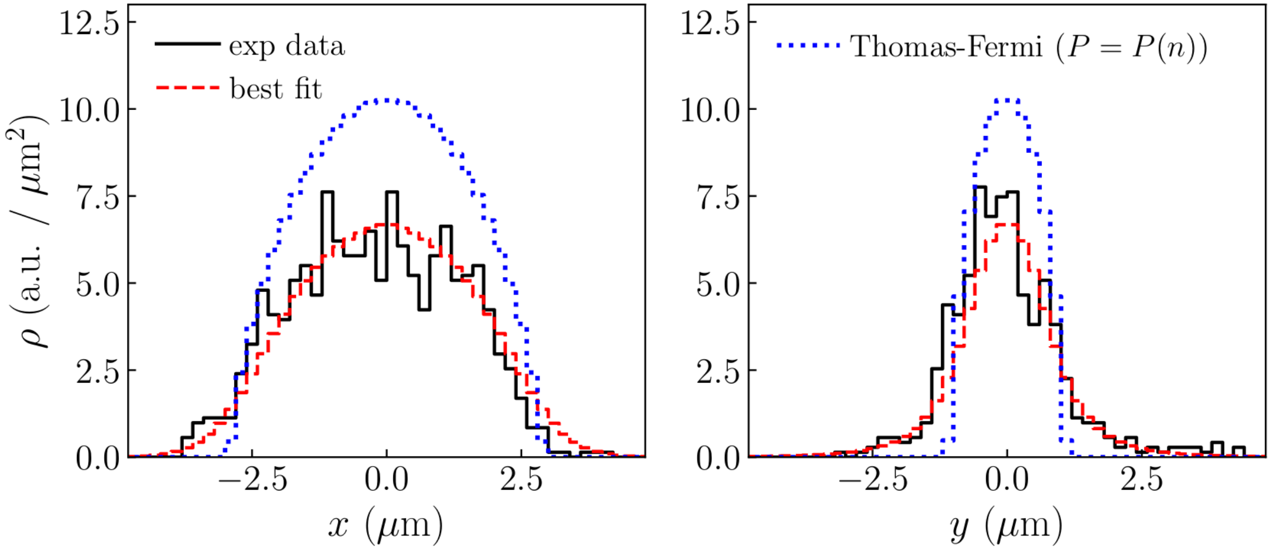
$$P_{\mathrm{ideal}} = \frac{\pi \hbar^2}{2M} n^2$$

Mass of ^{6}Li

Matching to mass density at t=0 to the measured initial condition

Thomas-Fermi = "ideal hydrostatics" $\nabla p + n\nabla V = 0$

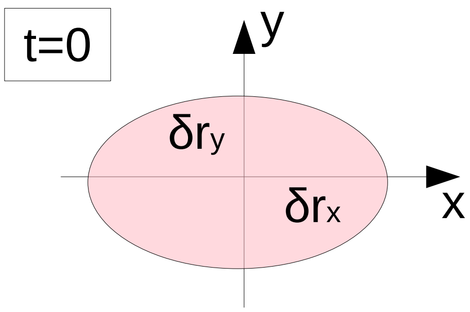




This is something we can not do in heavy-ion collisions!

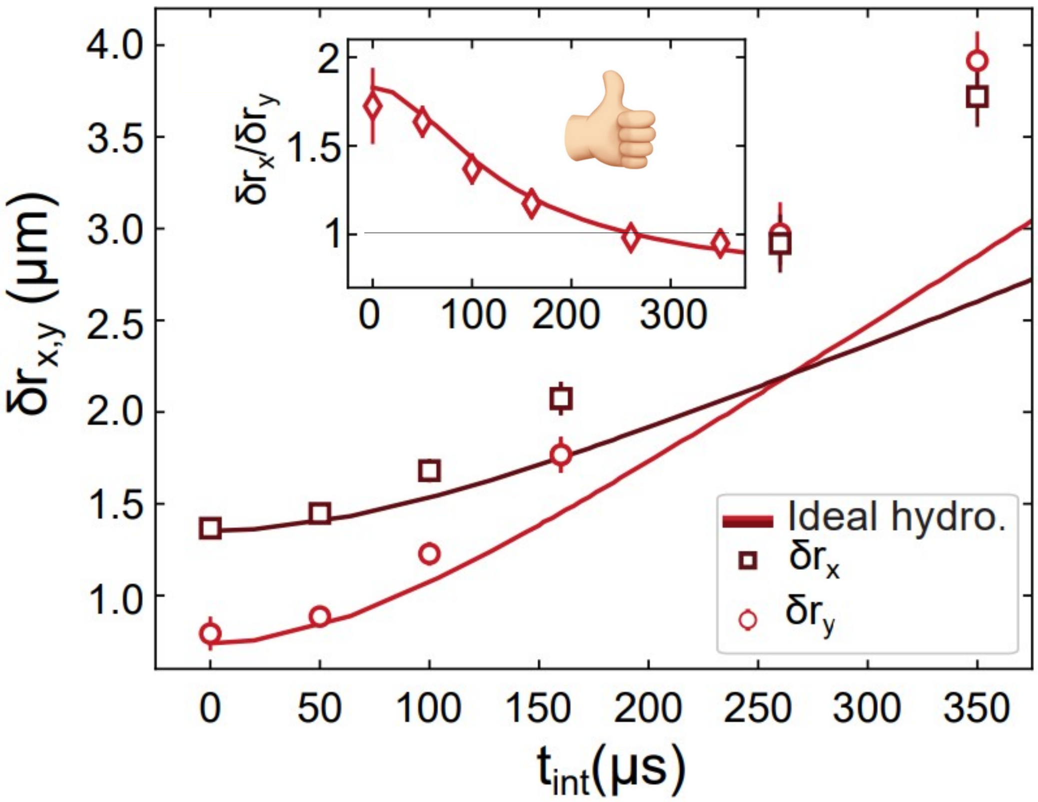
Mesoscopic **physics (quantum corrections) … tails of the distributions?**

Direct comparison in real space – Aspect ratio of system is perfectly reproduced



Brilliant hydro solver from Stony Brook

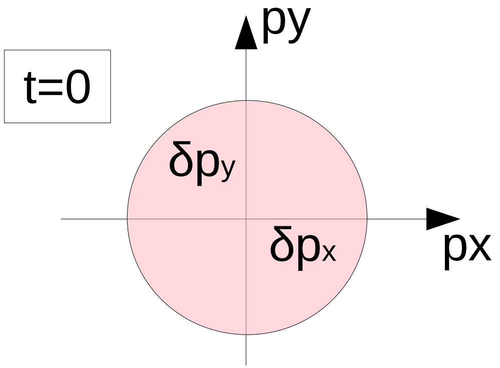
https://python-hydro.github.io/pyro2/index.html



Absolute sizes off by fractions of μm – Better understanding of systematics?

Momentum space – Hydrodynamics does not make any prediction there

$$\mathscr{P}_{jk}(t,\mathbf{x}) = \rho(t,\mathbf{x})v_j(t,\mathbf{x})v_k(t,\mathbf{x}) + P(t,\mathbf{x})\delta_{jk}$$



Insights from kinetic theory:
Match hydro momentum flux density to f(t,**x**,**p**) of the non-interacting system after the quench

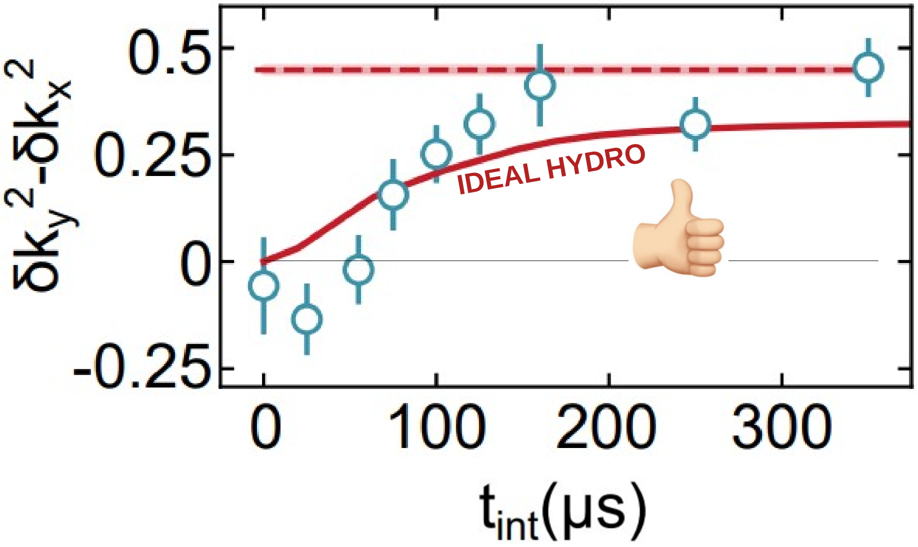
MOMENT OF DISTRIBUTION FUNCTION

STRESS-ENERGY TENSOR

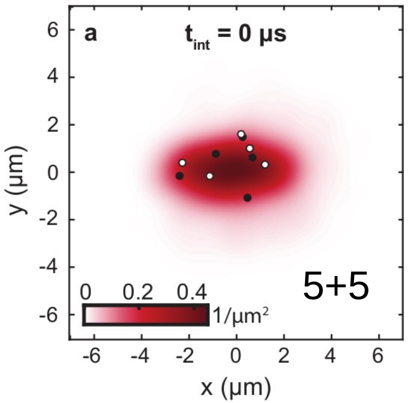
$$\mathscr{P}_{jk}(t,\mathbf{x}) = \int d^2p \left\{ \frac{p_j p_k}{m} f(t,\mathbf{x},\mathbf{p}) \right\}$$

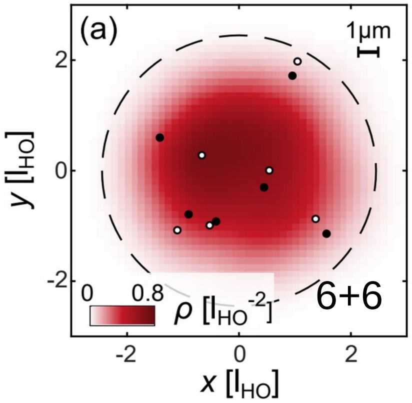
$$(\delta p_y)^2 - (\delta p_x)^2 =$$

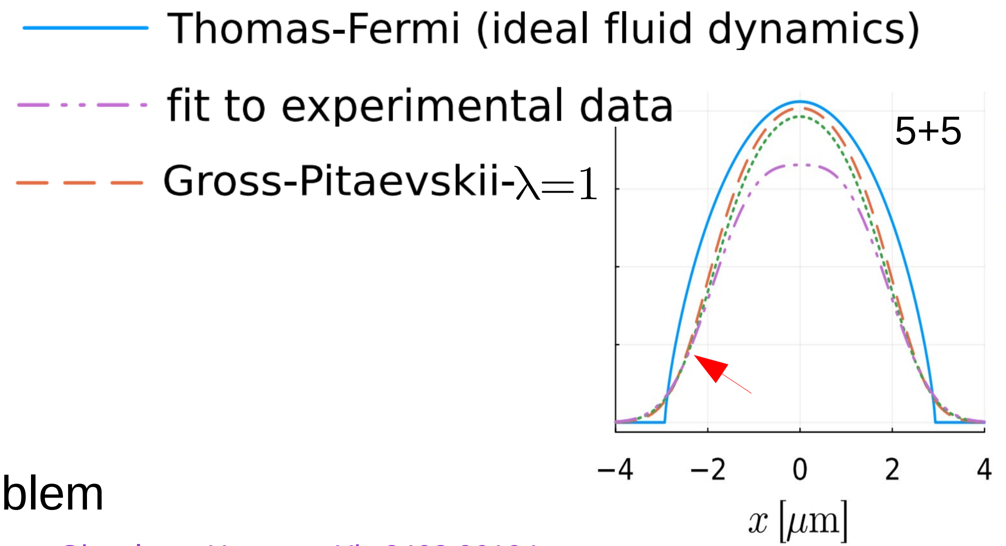
$$\frac{m}{2N}\int_{\mathbf{x}} \rho(t,\mathbf{x}) \left[v_y^2(t,\mathbf{x}) - v_x^2(t,\mathbf{x}) \right]$$



Prospects – Nature of the trapped density







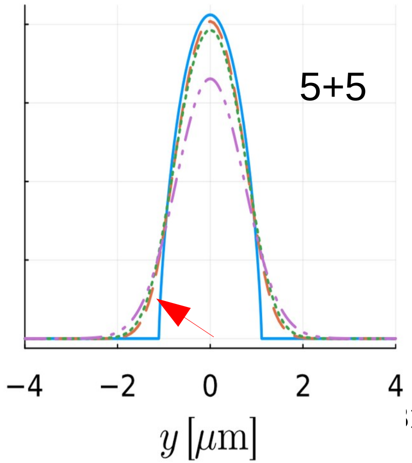
Second-order hydrostatic problem

[Floerchinger, Giacalone, Heyen, arXiv:2408.06104

quantum pressure

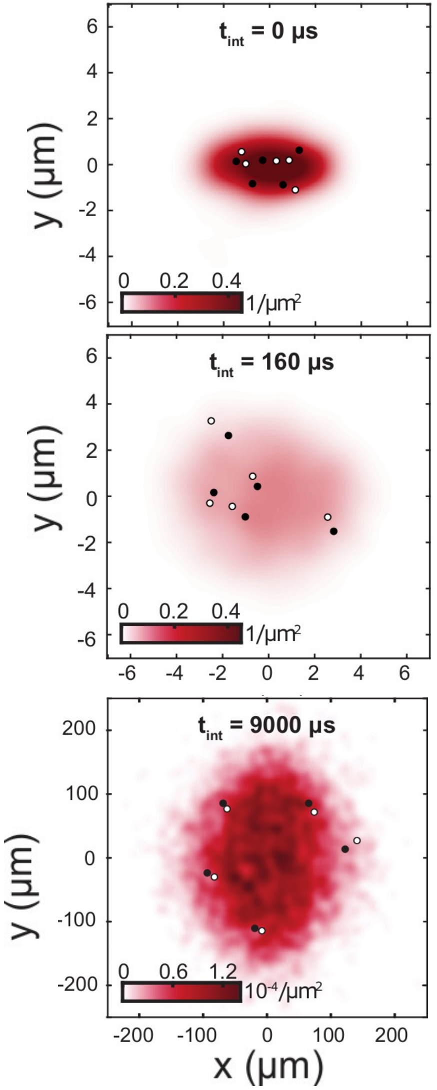
$$\underline{-\lambda \frac{\hbar^2}{2m^2}\rho \boldsymbol{\nabla}\left(\frac{\boldsymbol{\nabla}^2\sqrt{\rho}}{\sqrt{\rho}}\right)} + \boldsymbol{\nabla}p + \frac{1}{m}\rho\boldsymbol{\nabla}V = 0$$

Gross-Pitaevskii theory applied to 6 pairs of fermions?
"Condensate" of 6 bosons?

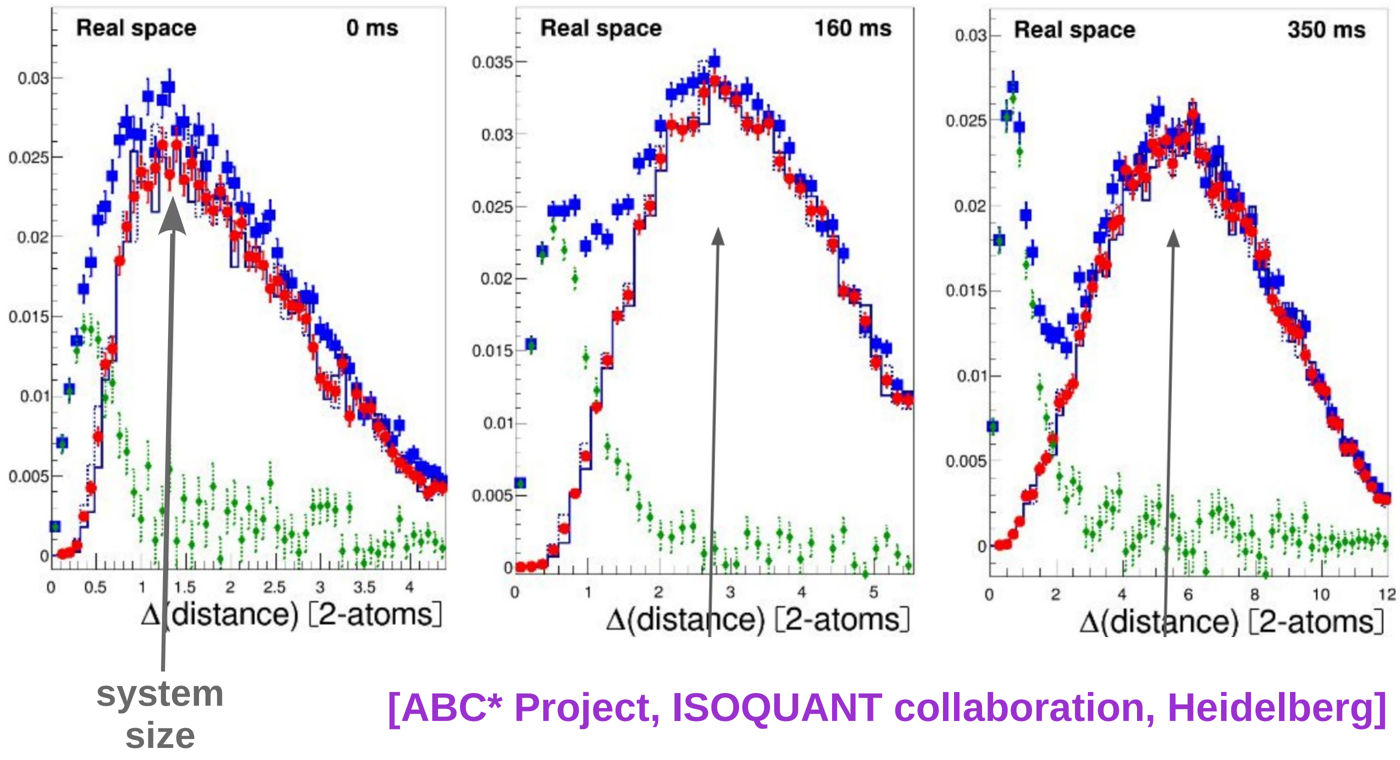


Prospects – Impact of correlations on dynamics

Role of two-body correlations for collectivity



Red = mixed events, **Blue** = up-down correlations



[ABC* Project, ISOQUANT collaboration, Heidelberg]

SUMMARY

- Major discovery by the LHC: collectivity in small collision systems
- Exploit exquisite degree of control of cold atomic gases to address the issue from a new angle
- **“Small system question” emerges with ultra-cold atoms**
- Hydro model based on many-body Fermi gas gives compelling description of the measurements
- Unique platforms to explore microscopic origins of emergent collective behavior