# Cosmological gravitational particle production: Starobinsky vs Bogolyubov

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Supervisor: Prof. Oleg Lebedev

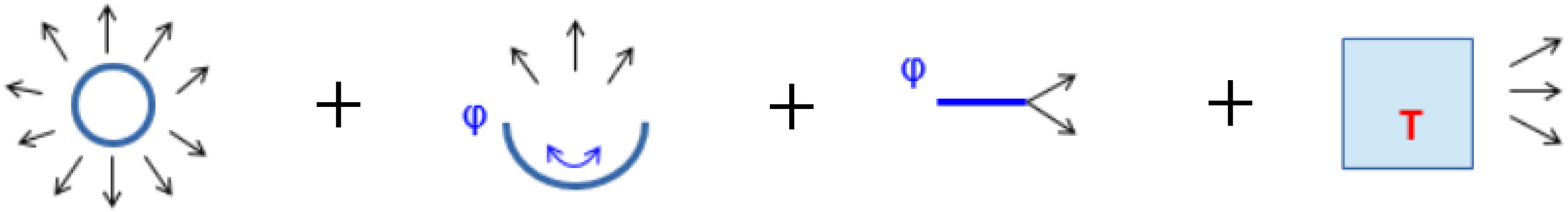
D. Feiteira and O. Lebedev; *Cosmological gravitational particle production: Starobinsky vs Bogolyubov, uncertainties, and issues;*
**arXiv:2503.14652**

# Motivation

Lebedev 2210.02293

- Gravitational particle production is crucial for **non-thermal dark matter** studies.

- **Add up** all production mechanisms:



**During inflation** + **Via inflaton oscillations** + **Inflaton decay** + **Thermal emission (freeze-in)**

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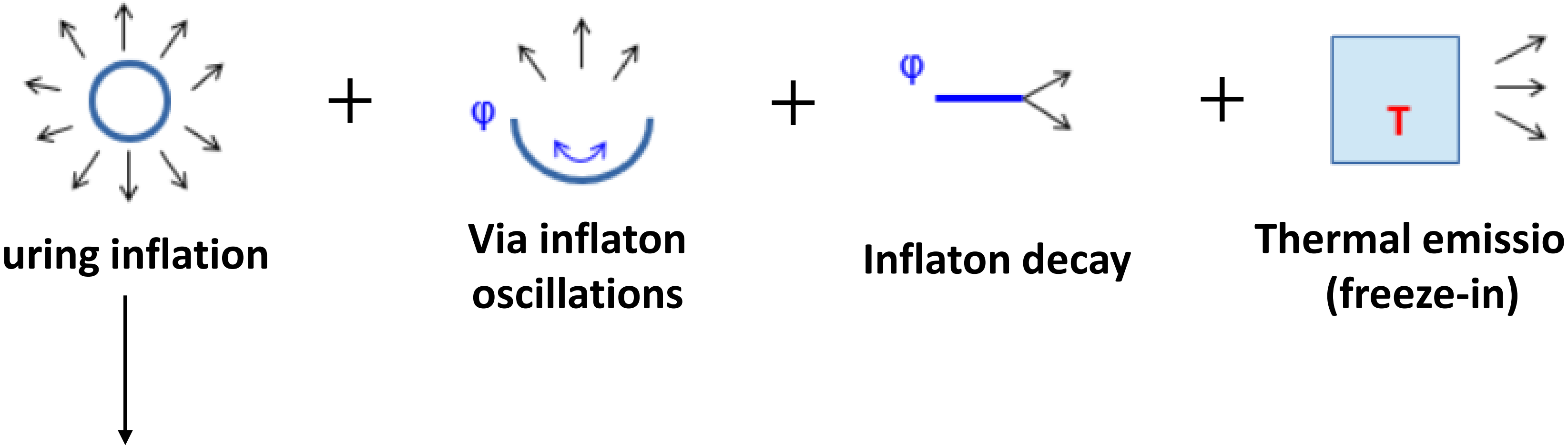

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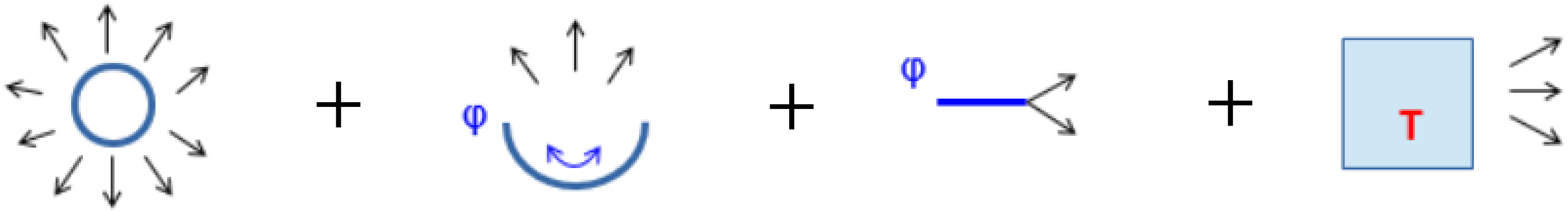


**During inflation**

**Via inflaton oscillations**

**Inflaton decay**

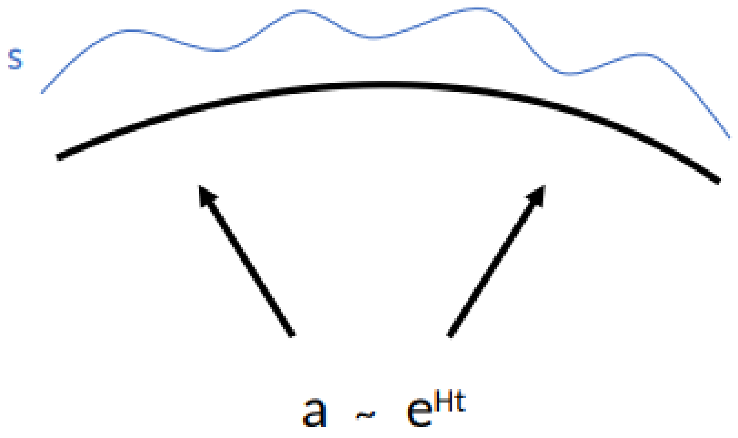
**Thermal emission (freeze-in)**

**Irreducible background for other production mechanisms.**

- **Dark relics:** if particle number is conserved (free or very weakly interacting particles), dark relics produced in inflation survive up to the present day.

# Bogolyubov coefficient approach

Parker, Grib,
Zeldovich,
Starobinsky.
60-70’s



- Condensate of a **scalar spectator field** with fluctuations.

$$\phi(\eta,\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[a_{\mathbf{k}}\,\chi_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}} + a_{\mathbf{k}}^{\dagger}\,\chi_k^*(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$

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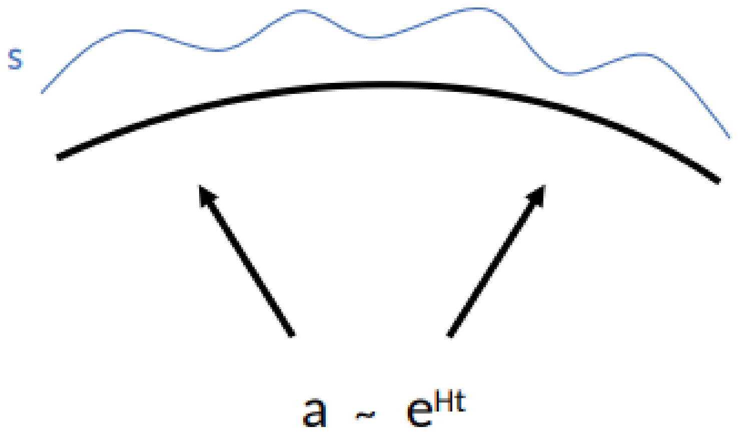

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- Equation of motion for $\chi_k(\eta)$:

$$\chi_k''(\eta) + \omega_k^2(\eta)\, \chi_k(\eta) = 0, \qquad \omega_k^2(\eta) = k^2 + a^2(\eta)\, m^2 + \left( \frac{1}{6} - \xi \right) a^2(\eta) R(\eta)$$

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- **Comoving particle density**:

$$a^3 n = \int\frac{dk}{k}\,a^3 n_k, \qquad a^3 n_k \equiv \frac{k^3}{2\pi^2}\,|\beta_k|^2$$

# Bogolyubov coefficient approach

- **Analytical results:**

Inflation followed by **radiation dominated** epoch:

$$a^3 n = \frac{3\kappa^2 a_e^3}{4\pi^2}\boxed{\frac{H_e^{11/2}}{m^{5/2}}}$$

**Very large number**

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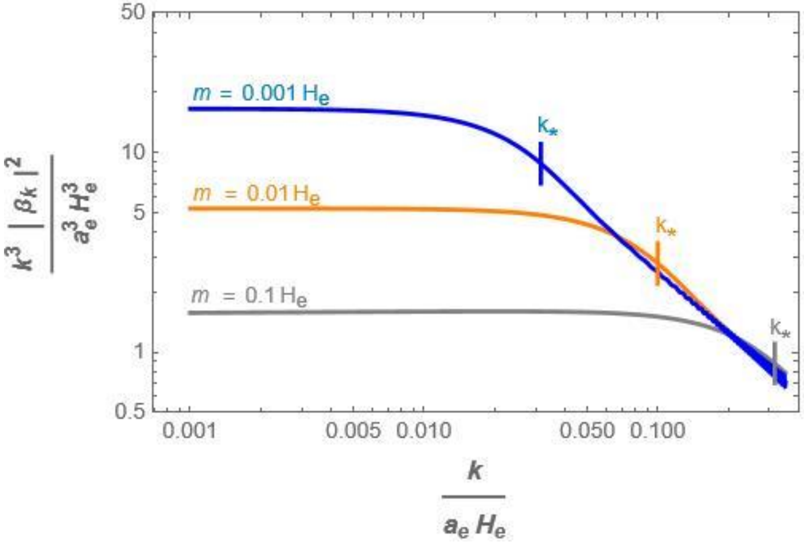

*Radiation dominated*

# Starobinsky approach

Starobinsky ‘86

$$\Phi(t,\mathbf{x}) = \underset{\substack{\Big\downarrow \\ \text{Long wavelength}}}{\bar{\Phi}(t,\mathbf{x})} + \underbrace{\int \frac{d^3\mathbf{k}}{(2\pi)^3}\theta(k - \epsilon a(t)H)\left[a_{\mathbf{k}}\,\chi_k(\eta)e^{i\mathbf{k}\cdot\mathbf{r}} + a_{\mathbf{k}}^{\dagger}\,\chi_k^*(\eta)e^{-i\mathbf{k}\cdot\mathbf{r}}\right]}_{\text{Short wavelength}}$$

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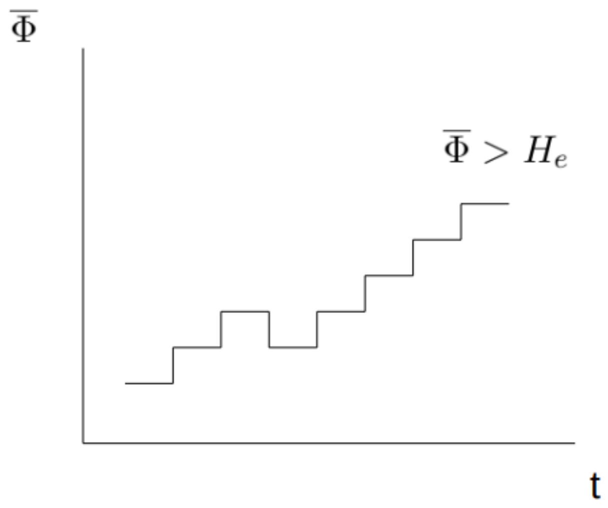
- Random walk:



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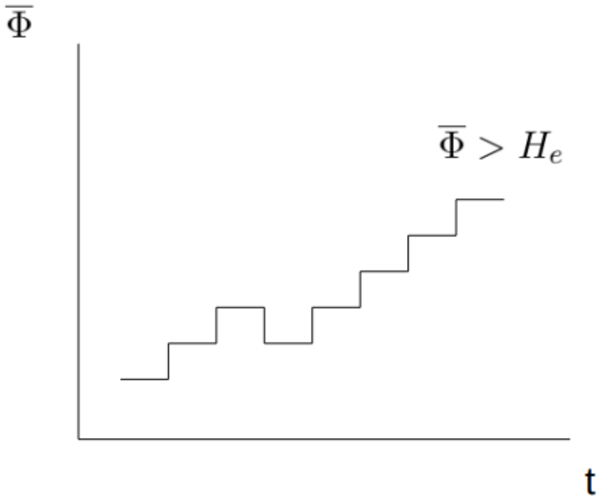

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$$\langle\bar{\Phi}^2\rangle = \frac{3}{8\pi^2}\frac{H^4}{m^2} + \left(\underset{\substack{\Big\downarrow \\ \text{Pre-inflationary} \\ \text{initial conditions}}}{\langle\bar{\Phi}^2\rangle_0} - \frac{3}{8\pi^2}\frac{H^4}{m^2}\right)\underset{\substack{\Big\downarrow \\ \text{Duration of} \\ \text{inflation}}}{e^{-\frac{2m^2}{3H}(t-t_0)}}$$

- **Infinitely long inflation:** agrees with Bogolyubov approach.
- **Finite inflation:** strong dependence on the initial conditions.

# Starobinsky approach

- When $H \sim m$: condensate $\overline{\Phi}$ is **converted into particles**.
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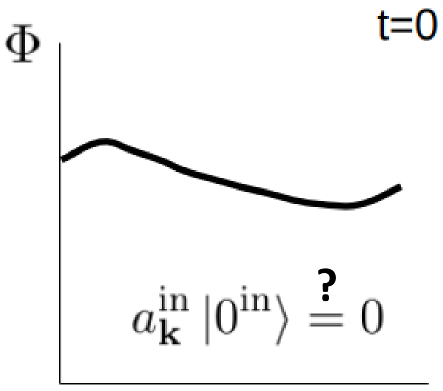
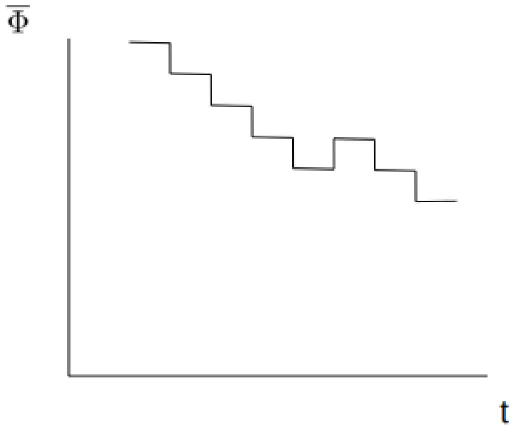
⇒ **Consistent with Bogolyubov approach**

- **Issues**:

Inflation has a finite duration;

Scalars can have non-trivial initial conditions.





⇒ **Starobinsky approach provides a more realistic description**

# Constraints on dark relics

- Abundance of stable particles produced by inflation cannot exceed that of dark matter.

- Particle number remains constant after reheating and is bounded by the dark matter abundance:

$$Y \leq 4.4 \times 10^{-10} \frac{\mathrm{GeV}}{m}, \qquad Y = \frac{n}{s_{SM}}, \qquad s_{SM} = \frac{2\pi^2 g_*}{45} T^3$$

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- **Matter domination:** $\boldsymbol{Y \simeq 0.07 \times \frac{1}{\Delta}\frac{H_e^{1/2}\bar{\Phi}^2}{m\, M_{PL}^{3/2}} \Rightarrow \bar{\Phi} < \Delta^{1/2}\frac{5\times10^9}{(H_e/\mathrm{GeV})^{1/4}}\ \mathrm{GeV}}$ ➡ $\boxed{\boldsymbol{T_R \lesssim \mathrm{GeV}}}$

$$\Delta \equiv \sqrt{\frac{H_e}{H_R}} \simeq \frac{T_{inst}}{T_R} \gg 1$$

# Overproduction of particles

- Huge production of dark relics during and after inflation:

**Gravitationally produced relics may be the end of the story**

**OR**

**Dilution of the relics + Non thermal production**

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Production of the right amount of DM
through **freeze-in at stronger coupling**

- Dilution factor: $\Delta \equiv \sqrt{\frac{H_e}{H_R}} \simeq \frac{T_{inst}}{T_R}$ ⇨ $\boxed{\textbf{Low } \boldsymbol{T_R}}$

- All we know: $\boldsymbol{T_R > 4\ \mathrm{MeV}}$ (Hannestad 2004)

# Low temperature freeze-in

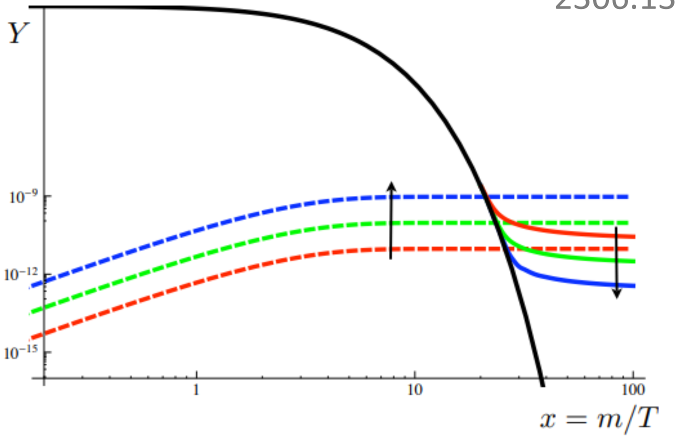
- **Freeze-in:**

  Initially negligible number of DM particles;

  DM is non-thermal;

  DM produced from SM through scattering;

Cosme, Costa, Lebedev
2306.13061



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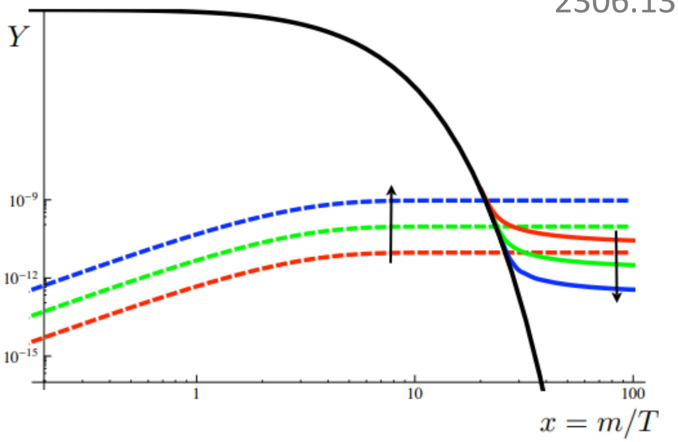
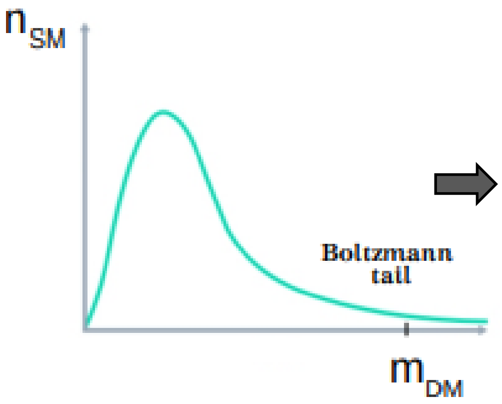


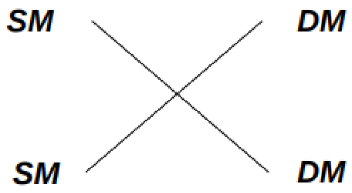
- **High temperature freeze-in – very small couplings.**

- **Low temperature freeze-in – large couplings provided that $T_R \ll m$.**



**Boltzmann supression of the DM production reaction**

**Only at the tail:**



# Higgs portal to DM

- Simplest model: scalar DM with small coupling to Higgs

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$$\dot{n} + 3Hn = \Gamma(h_i h_i \rightarrow ss) - \Gamma(ss \rightarrow h_i h_i)$$

- For sufficiently small coupling and when DM abundance is negligible $\Gamma(ss \rightarrow h_i h_i) \simeq 0.$

$$\Gamma(h_i h_i \rightarrow ss) = \langle \sigma(h_i h_i \rightarrow ss)\, v_r \rangle\, n_h^2 \simeq \frac{\lambda_{hs}^2\, T^3 m}{2^7\, \pi^4} e^{-2m/T}$$

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- To reproduce the correct relic abundance:

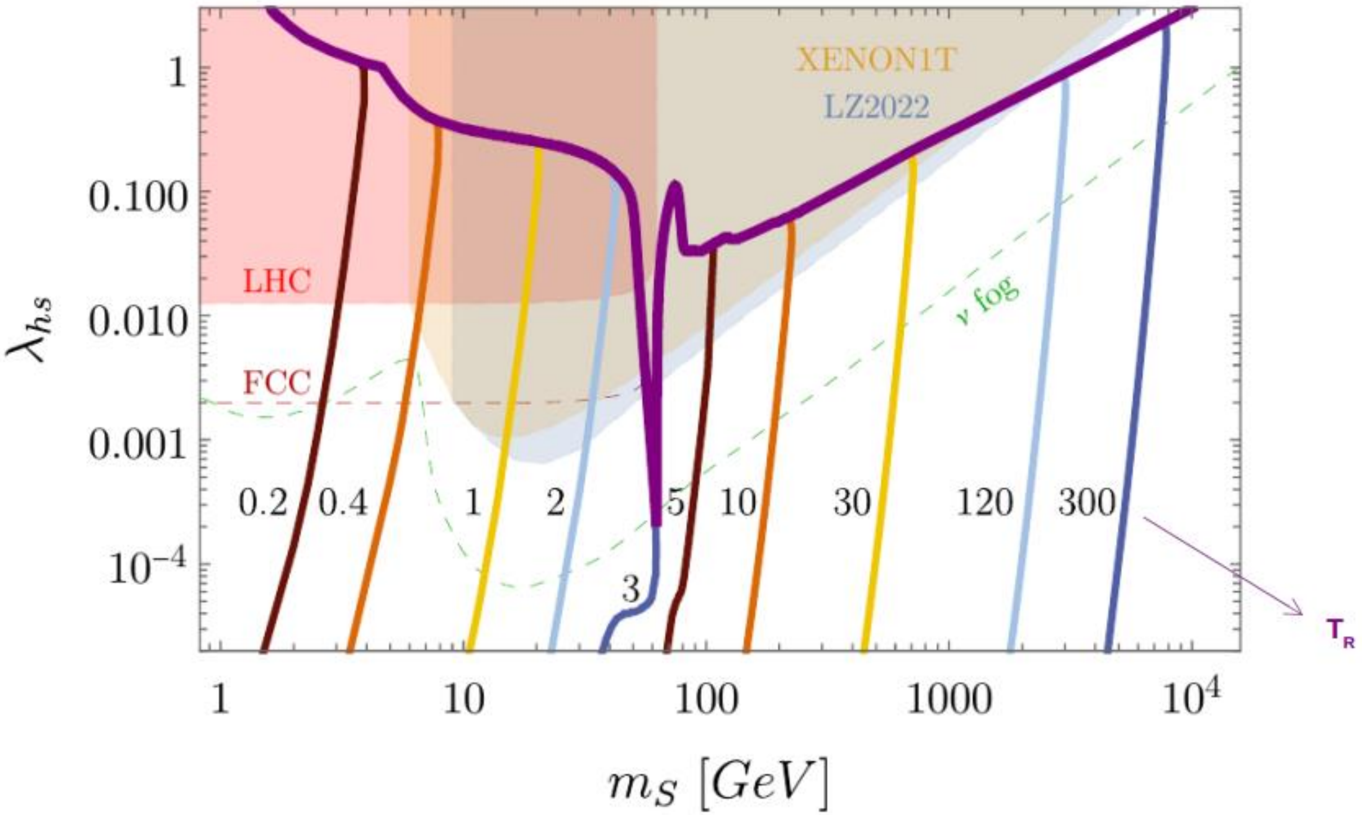
$$\lambda_{hs} \simeq 3 \times 10^{-11}\, \boldsymbol{e^{m/T_R}} \sqrt{\frac{T_R}{m}}$$

$\Rightarrow$ **Large coupling for $\boldsymbol{T_R \ll m}$**

# Constraints

Arcadi, Costa, Goudelis, Lebedev
2405.03760

- Solving Boltzmann equation numerical, we can plot the curves reproducing the correct relic abundance in parameter space:



- New imput: treat unknown $T_R$ as a free parameter.

# Conclusions

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- For a stable dark relic with $m \ll H$ to exist and be compatible with observations, a **very low reheating temperature** is required (GeV range or below).

- Gravitationally produced particles can be diluted, provided that $\boldsymbol{T_R}$ **is low**. Then DM can be produced non-thermally through **freeze-in at stronger coupling**.

# Constraints on dark relics: general result

- General result for inflationary scalar production:

$$\boxed{\boldsymbol{Y \propto \frac{\bar{\Phi}^2}{m_{\text{eff}}^{1/2}}\left(\frac{H_R}{m_{\text{eff}}}\right)^{\gamma}}}$$

- Radiation domination: $\gamma = 0$;

- Matter domination: $\gamma = 1/2$.

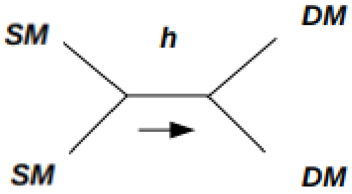
- This general expression includes:

Weak self-coupling $\lambda$;
Small non-minimal coupling to gravity $\xi$;
General initial conditions (included in $\bar{\Phi}$).

# Probes of freeze-in

Lebedev, Morais, Oliveira, Pasechnik
2410.21874

- **Invisible Higgs decay:**



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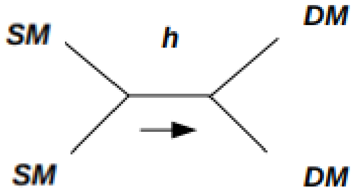

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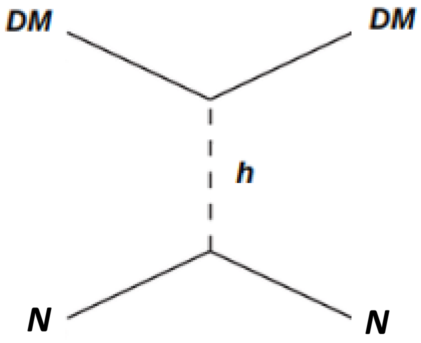

- XENON and Darwin can probe freeze-in at stronger coupling down to $\nu$-fog.