Effect of anomalous HHH couplings on the decay width of $H \to Z^*Z^* \to 4l$

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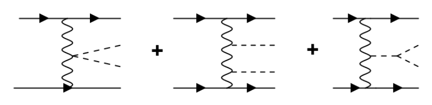
Overview

Motivation : $H \to Z^* Z^* \to 4l$

- Higgs sector in SM is not well explored, in particular HHH, $HHHH$ and $VVHH$ couplings are still not well measured.
- Few processes can probe these coupling.
  - VBF mechanism for HH production

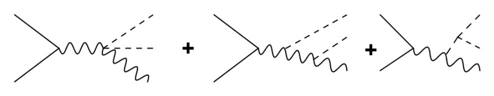


The obtained bound are $-5.4 < \kappa_{HHH} < 11.4$ and $-0.1 < \kappa_{V_2H_2} < 2.1$ at 95% confidence level [1]. The bound comes from both the couplings $WWHH$ and $ZZHH$.

[1] ATLAS Col.,PRD 108, 052003(23)

Motivation : $H \to Z^* Z^* \to 4l$

- Higgs-strahlung : HHV (V=W, Z) production



At the HL-LHC the bound will be quite weak $-9 < \kappa_{V_2 H_2} < 11$ [2].

- VVH (V=W, Z) production

We can probe two $VVHH$ couplings separately.

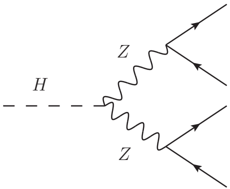
[2] Eur. Phys. J. Plus (2019) 134: 288

Motivation : $H \to Z^* Z^* \to 4l$

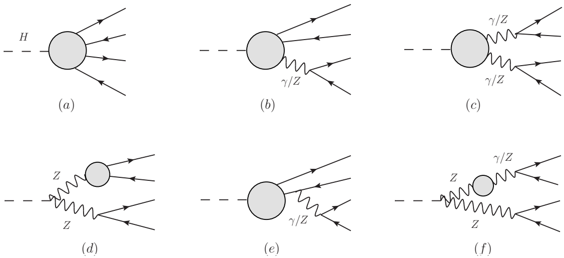
- One loop EW correction to Higgs partial decay widths.
- These processes are sensitive to HHH and $VVHH$ couplings beyond LO.
- Effect of anomalous HHH and $ZZHH$ coupling on the Higgs decay width.
- Effect of scaling of $ZZWW$ coupling on Higgs decay width.

LO and NLO diagrams

Tree level diagram :



One loop level generic diagrams :



Feynman Diagrams

- Total number of diagrams :
  - LO : 1 diagram
  - NLO virtual : Pentagon + Box + Triangle + Self Energy diagrams.
    - Total 118 diagrams for $H \to \nu_e \bar{\nu}_e \nu_\mu \bar{\nu}_\mu$.
    - Total 256 diagrams for $H \to e^+ e^- \mu^+ \mu^-$.
  - NLO real emission :
    - No diagrams for $H \to \nu_e \bar{\nu}_e \nu_\mu \bar{\nu}_\mu$.
    - 4 diagrams for $H \to e^+ e^- \mu^+ \mu^-$.

Helicity Techniques

This formalism has been done for massless fermions[3].
Dirac equation for massless fermion

$$\not{p}U(p) = 0$$

Representation :

$$\overline{U}_L(p) = \langle p\,,\;\; \overline{U}_R(p) = [p$$

$$U_L(p) = p]\,,\;\; U_R(p) = p\rangle$$

Then the Lorentz-invariant spinor product are

$$\overline{U}_L(p)U_R(q) = \langle pq\rangle \text{ and } \overline{U}_R(p)U_L(q) = [pq]$$

Vector currents are

$$\overline{U}_L(p)\gamma^\mu U_L(q) = \langle p\gamma^\mu q]$$

and

$$\overline{U}_R(p)\gamma^\mu U_R(q) = [p\gamma^\mu q\rangle$$

[3] **arXiv:1101.2414v1[hep-ph]12 Jan 2011**.

Helicity Identities

Identities :

$$\not{p} = p\rangle[p + p]\langle p$$

$$[p_1\, p_2] = \langle p_2\, p_1\rangle^*$$

$$\langle p_1 \gamma^\mu p_2]\langle p_3 \gamma_\mu p_4] = 2\langle p_1\, p_3\rangle[p_4 p_2]$$

$$1 = \frac{\not{l}_1 \not{l}_2 + \not{l}_2 \not{l}_1}{2(l_1.l_2)} \qquad \text{!useful}$$

Reference momenta l_1 and l_2 should be unequal and massless, and can be chosen from the process to get compact amplitudes.

Functional form of spinor product[4] and vector current

Functional form :

$$[p_1\, p_2] = (p_1^y + ip_1^z)\left[\frac{p_2^0 - p_2^x}{p_1^0 - p_1^x}\right]^{\frac{1}{2}} - (p_2^y + ip_2^z)\left[\frac{p_1^0 - p_1^x}{p_2^0 - p_2^x}\right]^{\frac{1}{2}}$$

$$\langle p_1 \gamma^0 p_2] = \frac{(p_1^y - ip_1^z)(p_2^y + ip_2^z) + (p_1^0 - p_1^x)(p_2^0 - p_2^x)}{\sqrt{(p_1^0 - p_1^x)(p_2^0 - p_2^x)}}$$

$$\langle p_1 \gamma^x p_2] = \frac{(p_1^y - ip_1^z)(p_2^y + ip_2^z) - (p_1^0 - p_1^x)(p_2^0 - p_2^x)}{\sqrt{(p_1^0 - p_1^x)(p_2^0 - p_2^x)}}$$

$$\langle p_1 \gamma^y p_2] = (p_1^y - ip_1^z)\left[\frac{p_2^0 - p_2^x}{p_1^0 - p_1^x}\right]^{\frac{1}{2}} + (p_2^y + ip_2^z)\left[\frac{p_1^0 - p_1^x}{p_2^0 - p_2^x}\right]^{\frac{1}{2}}$$

$$\langle p_1 \gamma^z p_2] = (p_1^z + ip_1^y)\left[\frac{p_2^0 - p_2^x}{p_1^0 - p_1^x}\right]^{\frac{1}{2}} + (p_2^z - ip_2^y)\left[\frac{p_1^0 - p_1^x}{p_2^0 - p_2^x}\right]^{\frac{1}{2}}$$

[4] **Kleiss and Stirling, Nucl.Phys.B262, 235(1985).**

Techniques to compute amplitudes :

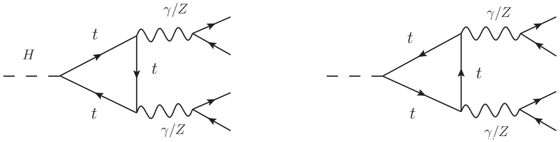
- We compute helicity amplitudes by using spinor helicity formalism at the matrix element level.
- We use `FeynArts` to generate diagrams and then we use `FormCalc` to generate raw Feynman amplitudes.
- The helicity amplitudes have been computed using our generic `FORM` routines.
- The final amplitudes are written in terms of spinor product and dot products among the vector currents, momenta and polarizations.

- We adopt t'Hooft-Veltman (HV) dimensional scheme to compute the amplitudes.
- In this scheme, the loop part has been computed in d-dimension and the rest has been computed in 4-dimension.

$$\bar{\gamma}^{\mu}\bar{\gamma}_{\mu} = d \qquad \bar{\not{q}} = \not{q} + \tilde{\not{q}}$$

- We use the package 'OneLOop' for scalar integrals computation.
- We use an in-house routine *OVReduce*, based on Oldenborgh-Vermaseren reduction techniques to reduce tensor integrals in terms of scalar integrals.

γ^5-anomaly



- The trace is inconsistent, as one can get different results depending on the different starting points of the trace.
- The formal treatment of $\gamma^5 = \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$ is not consistent in d-dimension as the anti-symmetric tensor $\epsilon_{\mu\nu\rho\sigma}$ lives in 4-dimension.
- We use KKS scheme (Korner-Kreimer-Schilcher) in which all γ^5-matrices has to be taken to a particular vertex ('reading point') by anti-commuting with other γ-matrices. Then one can do d-dimensional algebra and compute trace.

- Removing γ^5 from the trace

$$1 = \frac{\not{l}_1 \not{l}_2 + \not{l}_2 \not{l}_1}{2(l_1.l_2)} \qquad \text{!useful}$$

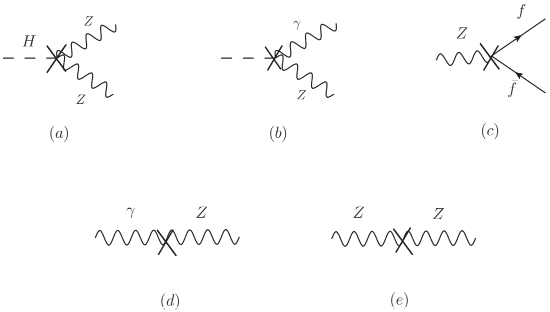
$$\gamma^5 |p\rangle = 0 \quad \text{and} \quad \gamma^5 |p] = -|p]$$

$$\text{Trace}(p\rangle\langle q) = \langle qp\rangle$$

- Same prescription has been followed to calculate $Zf\bar{f}$-vertex correction where the current is with the d-dimensional γ-matrices and γ^5-matrices.

- For final state phase space integral, we use a Monte Carlo integration package called `AMCI`.
- We adopt random numbers from `AMCI` and use `RAMBO` routine for phase-space point generation.
- The `AMCI` has been implemented via a parallel virtual machine called `PVM` for parallel computation across the CPUs.

Renormalization and CT diagrams



- We do on-shell renormalization for the EW correction for these processes.

OL EW Renormalization

We use on-shell renormalization scheme to calculate the required CTs for this process. The SM parameters are renormalized as

$$e_0 = (1 + \delta Z_e)e, \quad M^2_{W,0} = M^2_W + \delta M^2_W, \quad \text{and} \quad M^2_{Z,0} = M^2_Z + \delta M^2_Z.$$

We also need wave function(w.f.) renormalization of involve fields in this process to remove the UV divergences. The required w.f. renormalization are defined as

$$Z_0 = (1 + \frac{1}{2}\delta Z_{ZZ})Z + \frac{1}{2}\delta Z_{ZA}A, \quad A_0 = (1 + \frac{1}{2}\delta Z_{AA})A + \frac{1}{2}\delta Z_{AZ}Z,$$
$$W^{\pm}_0 = (1 + \frac{1}{2}\delta Z_W)W^{\pm}, \quad H_0 = (1 + \frac{1}{2}\delta Z_H)H, \quad f^L_{i,0} = (\delta_{ij} + \frac{1}{2}\delta Z^{f,L}_{ij})f^L_j.$$

CMS and OL Renormalization

- The complex mass scheme (CMS) has been used to treat unstable particle in one loop electroweak correction. The unstable masses are defined with a complex part as

$$m_V^2 \to \mu_V^2 = m_V^2 - i m_V \Gamma_V ,$$

  where $V = W, Z$ and Γ_V is the corresponding decay width. This treatment also makes Weinberg angle complex as $\cos^2\theta_W = \mu_W^2/\mu_Z^2$.

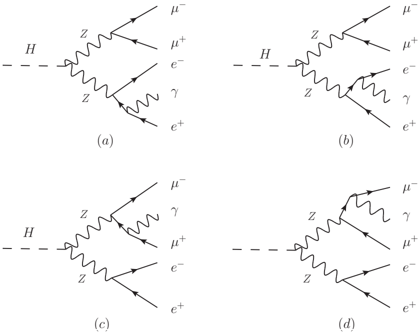
- The renormalization in CMS has been done in a modified version of the on-shell scheme where the renormalized mass is the pole of the corresponding propagator in the complex plane. When renormalized conditions being imposed, one need to perform the self energy computation with complex momenta.

- This computation can be done with Taylor expansion of self energies about the real mass and maintaining the one loop accuracy.

Input parameter scheme

- Different choice of α leads to different input parameter scheme.
- We calculate this process in $\alpha(M_Z)$ and G_F input parameter scheme.
- In $\alpha(M_Z)$ scheme, $\alpha(0)$ is evolved via renormalization-group equations from zero-momentum transfer to Z pole.
- In G_F scheme, the effective value of α is derived from the Fermi constant G_F in muon decay process.
- The charge renormalization constant contains mass singular terms like $\alpha \log m_f$ from each light fermion f which remain uncancelled in the EW corrections.
- Running of α absorbs these mass singular terms from the charge renormalization.

IR singularities and dipole subtraction [5].



- Virtual diagrams are IR singular and there are $\frac{1}{\epsilon}$ and $\frac{1}{\epsilon^2}$ poles.
- Photon is emitted from charged leptons from the final states and emission diagrams are IR singular in soft and collinear regions.
- We adopt Catani-Seymour dipole subtraction to get the IR safe observables.

[5] arXiv:1712.07975, M. Schonherr

- The ***I*** term removes all IR singularities from virtual diagrams.
- Fermion split in to photon and fermion. Here both emitter and spectator are in the final state.
- Total 12 dipole terms in this process.
- The dipole terms $D_{ij,k}$ have similar behaviour as real emission diagrams in collinear and soft regime.

SM Results

Input parameter scheme	Γ^{LO} (10^{-9} GeV)	Γ^{NLO} (10^{-9} GeV)	RE
$H \to \nu_e\bar{\nu}_e\nu_\mu\bar{\nu}_\mu$			
G_F	930.7	959.7	3.11%
$\alpha(M_Z)$	1007.7	948.0	-5.92%
$H \to e^+e^-\mu^+\mu^-$			
G_F	238.04	241.03	1.26%
$\alpha(M_Z)$	256.82	237.69	-7.45%

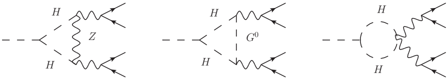
We define relative enhancement as RE $= \frac{\Gamma^{NLO}-\Gamma^{LO}}{\Gamma^{LO}} \times 100$.

- Depending on the input parameter scheme Γ^{LO} differ by $\sim 8\%$ and Γ^{NLO} differ by $\sim 1\%$.
- Our results differ by only $\sim 0.1\%$ with the package `Prophecy4f`[6].

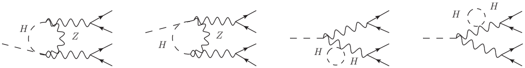
[6] Nucl. Phys. B 160, 131 (2006)

Anomalous coupling effect

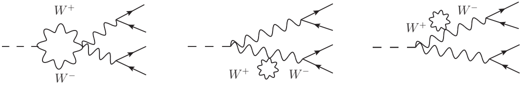
- NLO EW virtual diagrams with HHH and $ZZHH$ couplings.



- NLO EW virtual diagrams with $ZZHH$ couplings.



- NLO EW virtual diagrams with $ZZWW$ couplings.



- HHH, $ZZHH$ and $ZZWW$ couplings also appear in CT diagrams.

Anomalous HHH coupling effects

κ_{HHH}	RI (%)	
	G_F scheme	$\alpha(M_Z)$ scheme
	$H \to \nu_e \bar{\nu}_e \nu_\mu \bar{\nu}_\mu$	
10	-7.54	-8.48
6	-1.22	-1.36
2	0.34	0.40
-2	-2.84	-3.20
-6	-10.6	-12.15
-10	-23.53	-26.48
	$H \to e^+ e^- \mu^+ \mu^-$	
10	-7.65	-8.62
6	-1.23	-1.39
2	0.36	0.40
-2	-3.17	-3.25
-6	-10.95	-12.41
-10	-23.91	-26.72

We define relative increment (RI) as RI $= \frac{\Gamma_\kappa^{NLO} - \Gamma_{SM}^{NLO}}{\Gamma_{SM}^{NLO}} \times 100.$

Anomalous $ZZHH$ coupling effects

Effect of anomalous $ZZHH$ coupling :

κ_{ZZHH}	RI (%)	
	G_F scheme	$\alpha(M_Z)$ scheme
$H \to \nu_e \bar{\nu}_e \nu_\mu \bar{\nu}_\mu$		
10	5.73	0.28
6	3.17	0.16
2	0.64	0.03
−2	−1.92	−0.09
−6	−4.47	−0.22
−10	−7.01	−0.35
$H \to e^+ e^- \mu^+ \mu^-$		
10	5.13	−0.50
6	2.85	−0.28
2	0.57	−0.06
−2	−1.71	0.16
−6	−3.91	0.38
−10	−6.26	0.88

Anomalous $ZZWW$ coupling effects

Effect of anomalous $ZZWW$ coupling :

κ_{ZZWW}	RI (%)	
	G_F scheme	$\alpha(M_Z)$ scheme
$H \to \nu_e\bar{\nu}_e\nu_\mu\bar{\nu}_\mu$		
10	10.45	-19.29
6	5.80	-10.72
2	1.16	-2.14
-2	-3.49	6.43
-6	-8.14	15.00
-10	-12.79	23.58
$H \to e^+e^-\mu^+\mu^-$		
10	6.56	-23.68
6	3.95	-13.15
2	0.78	-2.79
-2	-2.35	7.85
-6	-5.48	18.38
-10	-8.59	28.90

Summary

- We have studied one loop EW correction to $H \to \nu_e \bar{\nu}_e \nu_\mu \bar{\nu}_\mu$ and $H \to e^+ e^- \mu^+ \mu^-$ processes.
- These processes have significant dependency on HHH coupling.
- The relative increment goes from $\sim 0.5\%$ to $\sim -27\%$ depending upon the allowed scaling of HHH coupling.
- The dependencies of $ZZHH$ coupling on partial decay width is marginal in $\alpha(M_Z)$ scheme but in G_F scheme, the RI goes from -7% to 5% depending on κ_{ZZHH}.
- The partial decay width also depends on $ZZWW$ coupling strongly.
- Gauge invariance is maintained with the scaling of HHH in this process, whereas the same is not maintained with the scaling of $ZZHH$ and $ZZWW$ couplings.
- A better theory is needed to vary $VVHH$ and $VVVV$ couplings with out disturbing the gauge invariance.

Thank You

OL EW Renormalization

The counterterms can be calculated by imposing suitable renormalization conditions. The required counterterms read as

$$\delta M_W^2 = \text{Re}\,\Sigma_T^W(M_W^2), \quad \delta Z_W = -\text{Re}\frac{\partial}{\partial k^2}\Sigma_T^W(k^2)\Big|_{k^2=M_W^2},$$

$$\delta M_Z^2 = \text{Re}\,\Sigma_T^{ZZ}(M_Z^2), \quad \delta Z_{ZZ} = -\text{Re}\frac{\partial}{\partial k^2}\Sigma_T^{ZZ}(k^2)\Big|_{k^2=M_Z^2},$$

$$\delta Z_{ZA} = 2\frac{\Sigma_T^{AZ}(0)}{M_Z^2}, \quad \delta Z_{AA} = -\frac{\partial}{\partial k^2}\Sigma_T^{AA}(k^2)\Big|_{k^2=0},$$

$$\delta Z_e = -\frac{1}{2}\delta Z_{AA} - \frac{s_W}{c_w}\frac{1}{2}\delta Z_{ZA}, \quad \delta Z_H = -\text{Re}\frac{\partial}{\partial k^2}\Sigma^H(k^2)\Big|_{k^2=M_H^2},$$

$$\delta Z_{ii}^{f,L} = -\text{Re}\,\Sigma_{ii}^{f,L}(m_{f,i}^2) - m_{f,i}^2\frac{\partial}{\partial k^2}\text{Re}[\Sigma_{ii}^{f,L}(k^2) + \Sigma_{ii}^{f,R}(k^2) + \Sigma_{ii}^{f,L}(k^2)].$$

The self energies Σ, can be calculated from self energy of W-boson, Z-boson, photon(also $Z\gamma$) and H-boson propagators.

CMS and OL Renormalization

- The complex mass scheme(CMS) has been used to treat unstable particle in one loop electroweak correction. The unstable masses are defined with a complex part as

$$m_V^2 \to \mu_V^2 = m_V^2 - i m_V \Gamma_V,$$

where $V = W, Z$ and Γ_V is the corresponding decay width. This treatment also makes Weinberg angle complex as $\cos^2\theta_W = \mu_W^2/\mu_Z^2$.
- The renormalization in CMS has been done in a modified version of the on-shell scheme where the renormalized mass is the pole of the corresponding propagator in the complex plane. When renormalized conditions being imposed, one need to perform the self energy computation with complex momenta.
- This computation can be done with Taylor expansion of self energies about the real mass and maintaining the one loop accuracy.

The renormalized counterterms in CMS are

$$\delta\mu_W^2 = \Sigma_T^W(M_W^2) + (\mu_W^2 - M_W^2)\Sigma_T'^W(M_W^2) + c_T^W + \mathcal{O}(\alpha^3),$$
$$\delta\mu_Z^2 = \Sigma_T^{ZZ}(M_Z^2) + (\mu_Z^2 - M_Z^2)\Sigma_T'^Z(M_Z^2) + \mathcal{O}(\alpha^3),$$
$$\delta\mathcal{Z}_W = -\Sigma_T'^W(M_W^2), \quad \delta\mathcal{Z}_{ZZ} = -\Sigma_T'^{ZZ}(M_Z^2),$$
$$\delta\mathcal{Z}_H = -\Sigma'^H(M_H^2), \quad \delta\mathcal{Z}_{ZA} = \frac{2}{\mu_Z^2}\Sigma_T^{AZ}(0), \quad \delta\mathcal{Z}_{AA} = -\Sigma_T'^{AA}(0).$$

The charge renormalization constant δZ_e is calculated from the photon self energy renormalization constant δZ_{AA} as it can be seen from Equ. 31. The renormalization constant δZ_{AA} contains mass singular terms $\alpha \log m_f$ where m_f is the mass of the fermion. These contribution comes from every light fermion loop in δZ_{AA} and remain uncancelled in EW corrections. To renormalize the electric charge, the standard QED on-shell renormalization condition is being imposed in the Thomson limit , where the photon momentum transfer is zero. This renormalize the QED coupling $\alpha = \alpha(0)$ at $Q^2 = 0$. To have the weak coupling α at desire scale ($Q^2 \sim M_Z^2$) one need running of α from $Q^2 = 0$ to $Q^2 = M_Z^2$. The running of α remove the mass singular terms from the charge renormalization.

The choice of the running of the coupling α leads to the notion of the input parameter scheme. In $\alpha(M_Z)$ input parameter scheme, the $\Delta\alpha(M_Z)$ is given by

$$\Delta\alpha(M_Z) = \frac{\alpha(0)}{3\pi} \sum_{f \neq t} N_f^c Q_f^2 \left[\ln\left(\frac{M_Z^2}{m_f^2}\right) - \frac{5}{3} \right]. \tag{1}$$

The shift in charge renormalization $\delta Z_e|_{\alpha(M_Z^2)} \to \delta Z_e|_{\alpha(0)} - \frac{1}{2}\Delta\alpha(M_Z^2)$, will remove all mass singularities in δZ_e. The numerical value of $\alpha(M_Z)$ has been extracted from an experimental analysis of e^+e^- annihilation to hadrons.

In α_{G_F} scheme, the electromagnetic coupling is derived from the Fermi constant as

$$\alpha_{G_F} = \frac{\sqrt{2} G_F M_W^2 (M_Z^2 - M_W^2)}{\pi M_Z^2} = \alpha(0)(1 + \Delta r) + \mathcal{O}(\alpha^3) \qquad (2)$$

$$\Delta r = \Delta\alpha(M_Z) - \Delta\rho \frac{c_W^2}{s_W^2} + \Delta r_{rem}. \qquad (3)$$

In this scheme, the shift in charge renormalization is given by $\delta Z_e|_{G_F} \to \delta Z_e|_{\alpha(0)} - \frac{1}{2}\Delta r$, where the Δr is the radiative corrections to muon decay. The Δr is given by

$$\begin{aligned}\Delta r \quad = \quad & \Sigma_T^{AA}(0) - \frac{c_W^2}{s_W^2}\left(\frac{\Sigma_T^{ZZ}(M_Z^2)}{M_Z^2} - \Sigma_T^W(M_W^2) M_W^2\right) + \frac{\Sigma_T^W(0) - \Sigma_T^W(M_W^2)}{M_W^2} \\ & +2\,\frac{c_W}{s_W}\frac{\Sigma_T^{AZ}(0)}{M_Z^2} + \frac{\alpha}{4\pi s_W^2}\left(6 + \frac{7 - 4s_W^2}{2s_W^2}\log c_W^2\right).\end{aligned} \qquad (4)$$