Transverse Beam Optics V

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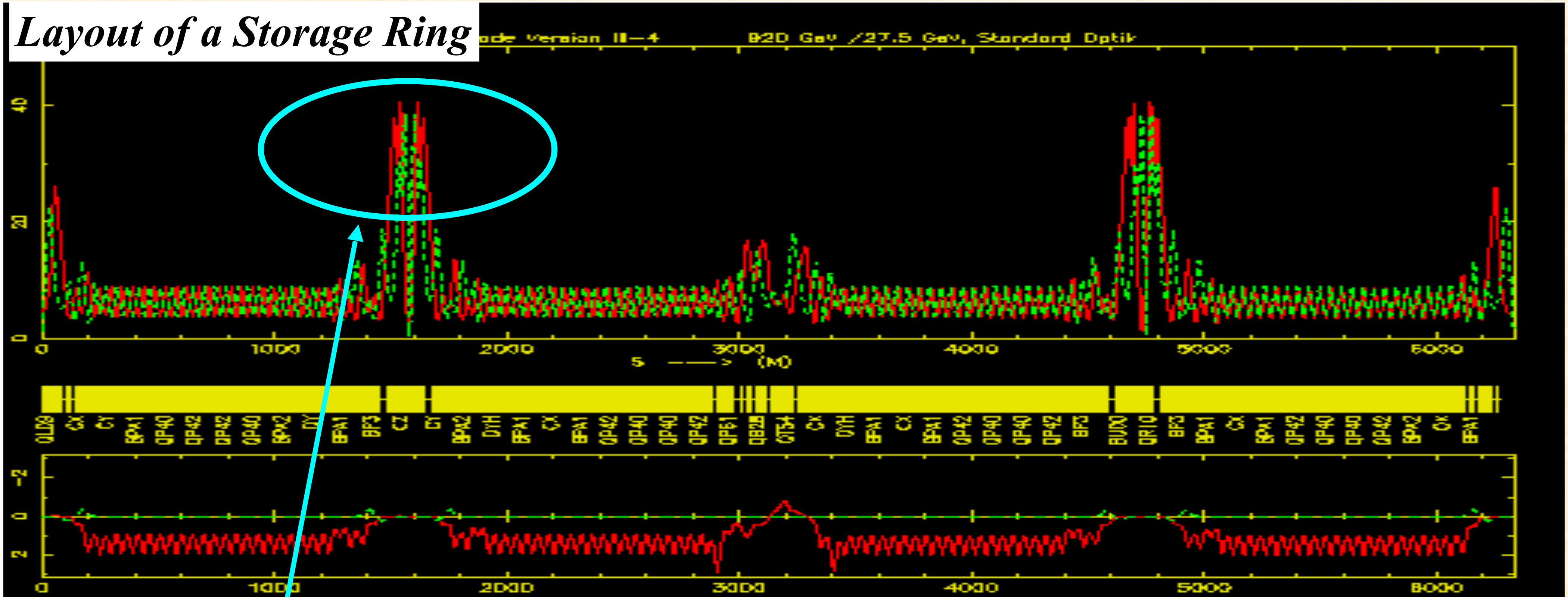
Particle Colliders

28.) Insertions

Layout of a Storage Ring



Arc: regular (periodic) magnet structure:

bending magnets —> define the energy of the ring
main focusing & tune control, chromaticity correction,
multipoles for higher order corrections

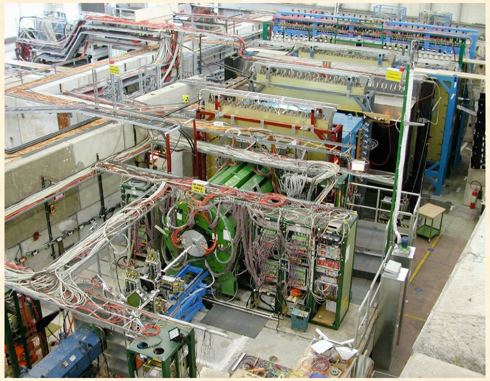
Straight sections:

drift spaces for injection, dispersion suppressors,
low beta insertions, RF cavities, etc....
… and the high energy experiments if they cannot be avoided

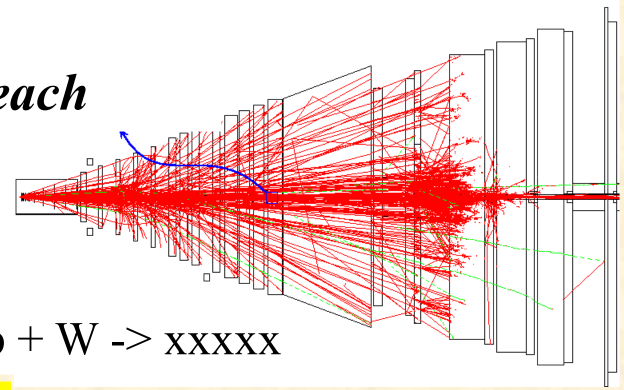
General Aspects:

Fixed target < — > beam-beam collisions

Fixed Target



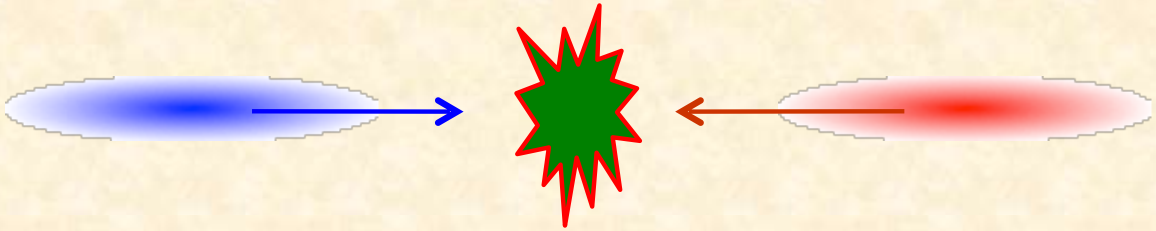
high event rate,
limited energy reach

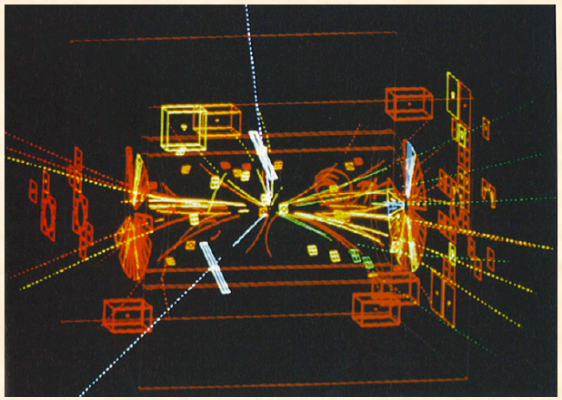


fixed target event p + W -> xxxxx

$$E_{lab} \propto \sqrt{E_{beam}}$$

Collider experiments:
E=mc²





Z_0 —> e+e− pair
(white dashed lines)

low event rate (luminosity)
high energy reach

$$E_{lab} = E_{beam\ 1} + E_{beam\ 2}$$

Insertions for Particle Colliders

... the most complicated one: the drift space

Question to the audience: ***what will happen to the beam parameters α, β, γ if we stop focusing for a while ...?***

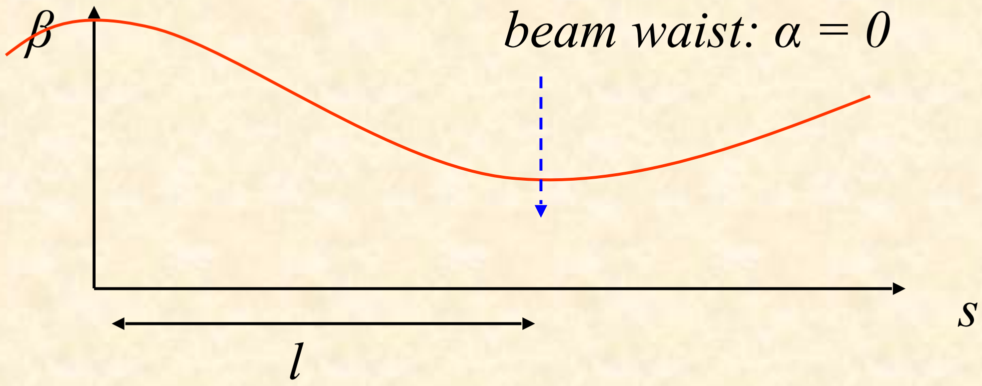
$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_S = \begin{pmatrix} C^2 & -2SC & S^2 \\ -CC' & SC'+S'C & -SS' \\ C'^2 & -2S'C' & S'^2 \end{pmatrix} * \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_0$$

transfer matrix for a drift:

$$M = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \longrightarrow$$

$$\beta(s) = \beta_0 - 2\alpha_0 s + \gamma_0 s^2$$
$$\alpha(s) = \alpha_0 - \gamma_0 s$$
$$\gamma(s) = \gamma_0$$

location of the waist:



given the initial conditions $\alpha_0, \beta_0, \gamma_0$: where is the point of smallest beam dimension in the drift … or at which location occurs the beam waist ?

beam waist:

$$\alpha(s) = 0 \quad \rightarrow \quad \alpha_0 = \gamma_0 * s$$

$$\ell = \frac{\alpha_0}{\gamma_0}$$

beam size at that position:

$$\left.\begin{matrix} \gamma(\ell) = \gamma_0 \\ \alpha(\ell) = 0 \end{matrix}\right\} \rightarrow \gamma(l) = \frac{1+\alpha^2(\ell)}{\beta(\ell)} = \frac{1}{\beta(\ell)}$$

$$\beta(\ell) = 1/\gamma_0$$

β-Function in a Drift:

let's assume we are at a symmetry point in the center of a drift.

$$\beta(s) = \beta_0 - 2\alpha_0 s + \gamma_0 s^2$$

as $\alpha_0 = 0, \quad \rightarrow \quad \gamma_0 = \frac{1+\alpha_0^2}{\beta_0} = \frac{1}{\beta_0}$

and we get for the β function in the neighbourhood of the symmetry point

$$\beta(s) = \beta_0 + \frac{s^2}{\beta_0}$$

! ! !

Nota bene:

1.) this is very bad !!!

2.) this is a direct consequence of the conservation of phase space density (... in our words: ε = const) … and there is no way out.

3.) Thank you, Mr. Liouville !!!

Joseph Liouville, 1809-1882

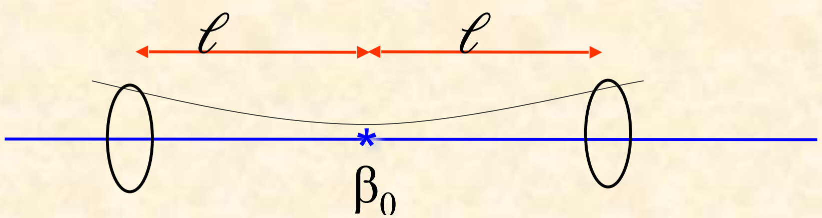
A bit more in detail: β-Function in a Drift

If we cannot fight against Liouville's theorem ... at least we can optimise

Optimisation of the beam dimension at position $s = \ell$:

$$\beta(\ell) = \beta_0 + \frac{\ell^2}{\beta_0}$$

Find the β at the center of the drift that leads to the lowest maximum β at the end:



$$\frac{d\hat{\beta}}{d\beta_0} = 1 - \frac{\ell^2}{\beta_0^2} = 0$$

$$\rightarrow \beta_0 = \ell$$

$$\rightarrow \hat{\beta} = 2\beta_0$$

If we choose $\beta_0 = \ell$ we get the smallest β at the end of the drift and the maximum β is just twice the distance ℓ

In any case: ***keep ℓ as SMALL as possible !!!***

Insertions:

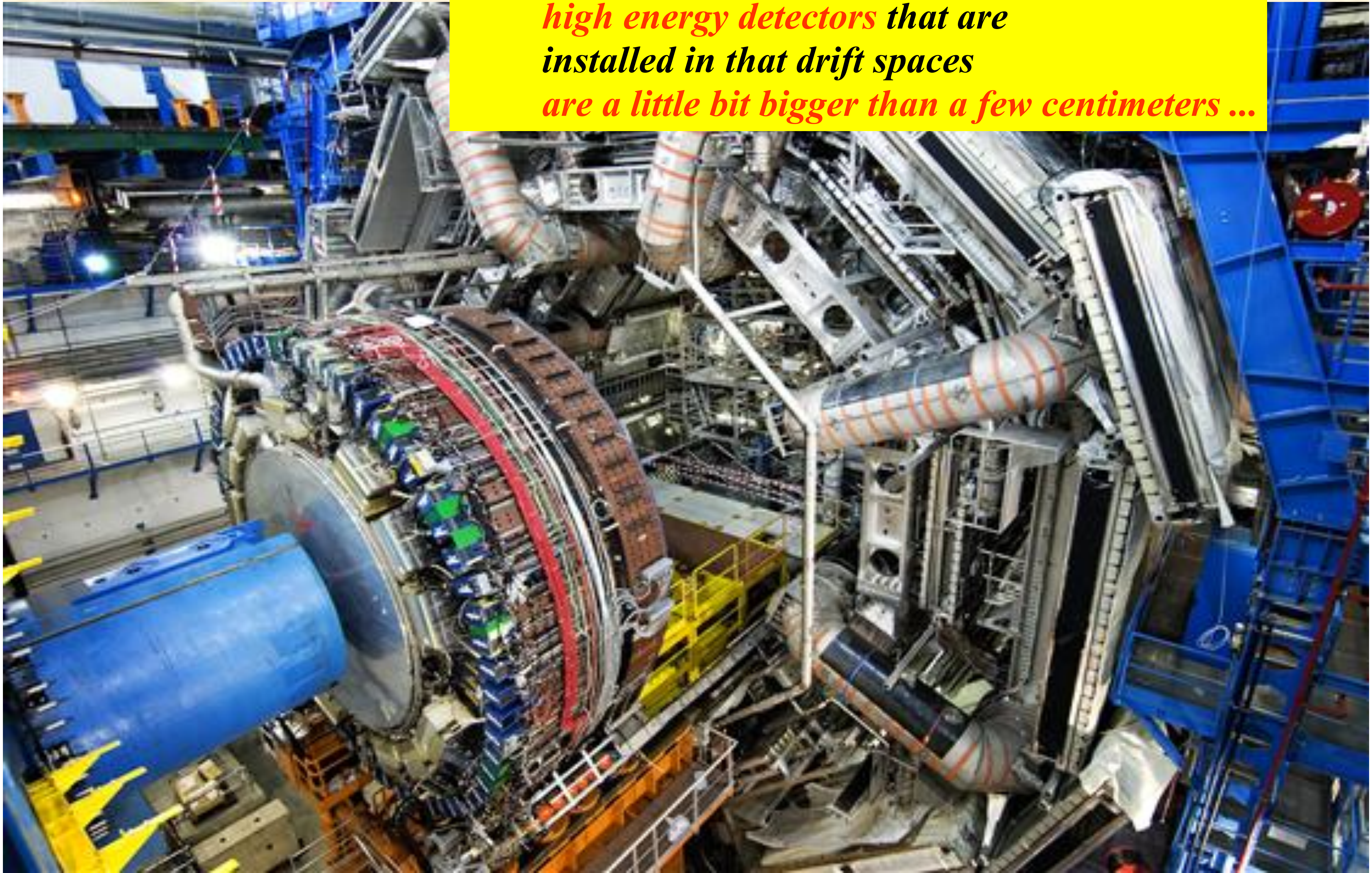
Prepare for Beam collisions

keep ℓ as SMALL as possible !!!

... there is just a little problem

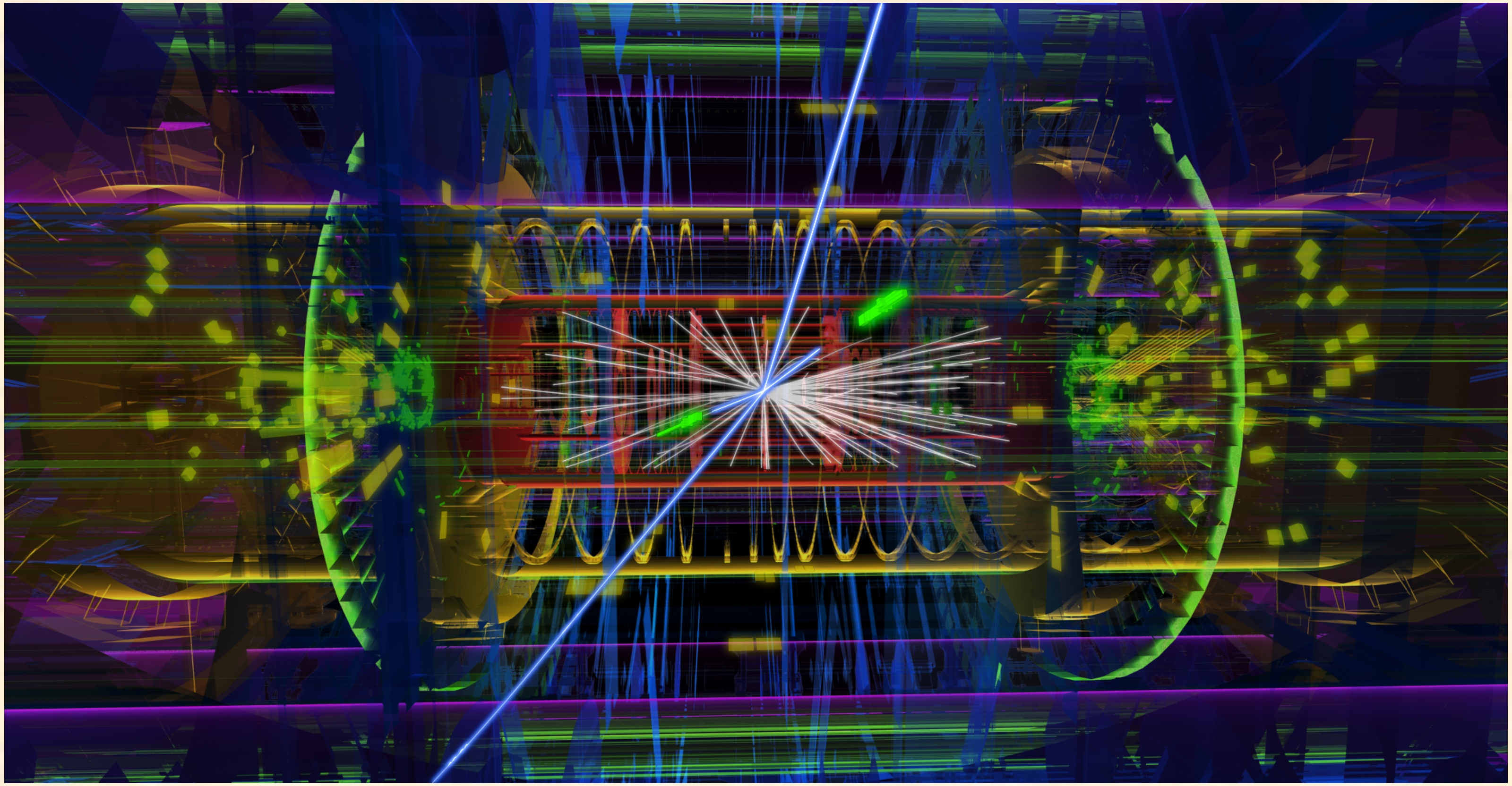
... clearly there is another problem !!!

... unfortunately ... in general
high energy detectors that are
installed in that drift spaces
are a little bit bigger than a few centimeters ...



... and why all that ??

High Light of the HEP-Year 2012 / 13 naturally the HIGGS



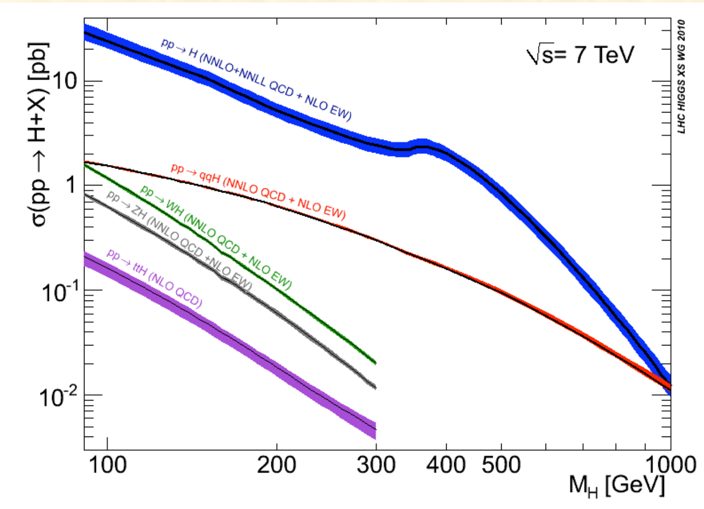
ATLAS event display: Higgs => two electrons & two muons

$$E = m_0 c^2 = m_{e1} + m_{e2} + m_{\mu 1} + m_{\mu 2} = 125.4\ GeV$$

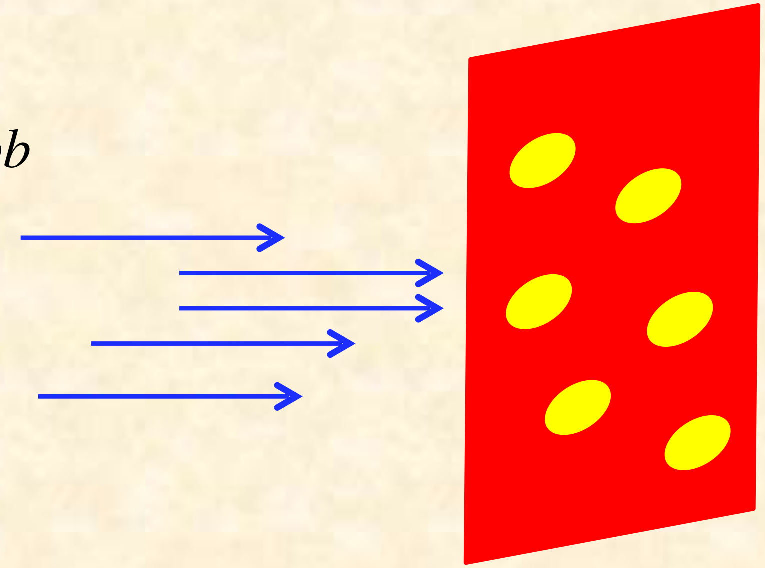
Problem: Our particles are VERY small !!

man trifft nicht so häufig.

Overall cross section of the Higgs:



$$\Sigma_{react} \approx 1pb$$



$$1b = 10^{-24} cm^2 = \frac{1}{mio} \cdot \frac{1}{mio} \cdot \frac{1}{mio} \cdot \frac{1}{10000} \quad mm^2 \longrightarrow \boxed{1pb = 10^{-12} b \approx ZERO}$$

The particles are "very small"

The only chance we have:
compress the transverse beam size … at the IP

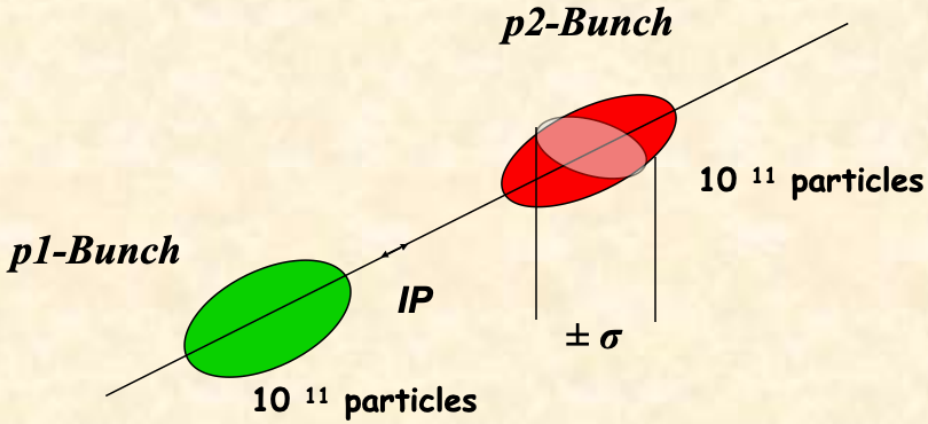
LHC typical → 16 µm

The Collider Performance: Luminosity

Event Rate: "Physics" per Second

$$R = L \cdot \Sigma_{react}$$

$$L = Luminosity$$

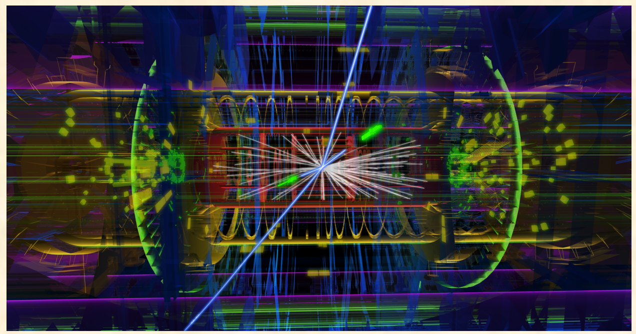


Example: Luminosity run at LHC

$\sigma_x = \sigma_y = 16\mu m$	*beam size at IP*
$f_0 = 11.245\ kHz$	*Revolution-Frequency*
$n_b = 2808$	*Number of Bunches*
$N_p = 1.2 \cdot 10^{11}$	*Particles per Bunch*
$I_p = 584\ mA$	*Beam Current*

$$L = 1.0 * 10^{34}\ \frac{1}{cm^2 s}$$

$$L = \frac{1}{4\pi} \cdot N_{p1} \cdot \frac{N_{p2}}{\sigma_x \sigma_y} \cdot (n_b \cdot f_0)$$



Mini-β Insertions: practical guide lines

** calculate the periodic solution in the arc*

** introduce the drift space needed for the insertion device (detector ...)*

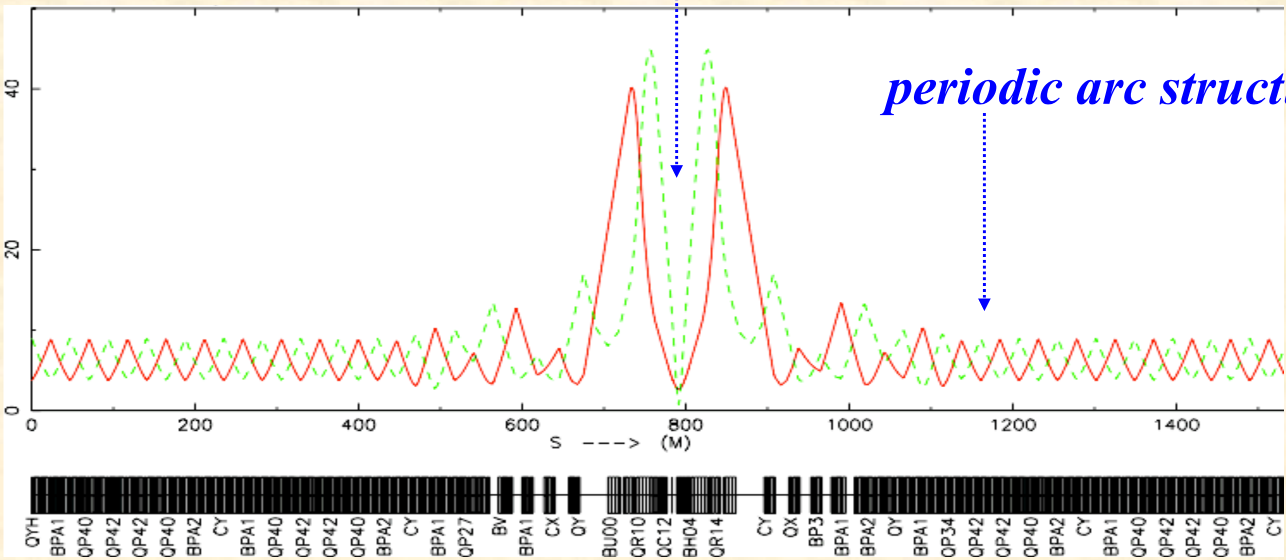
** put a quadrupole doublet (triplet ?) as close as possible*

** introduce additional quadrupole lenses to match the beam parameters to the values at the beginning of the arc structure*

β^*
$\alpha^* = 0$

8 individually powered quad magnets are needed to match the insertion (... at least)

periodic arc structure



30.) Lattice Design

Whenever we combine two different lattice structures we need a "matching section" in between to adapt the optics functions between the two lattices.

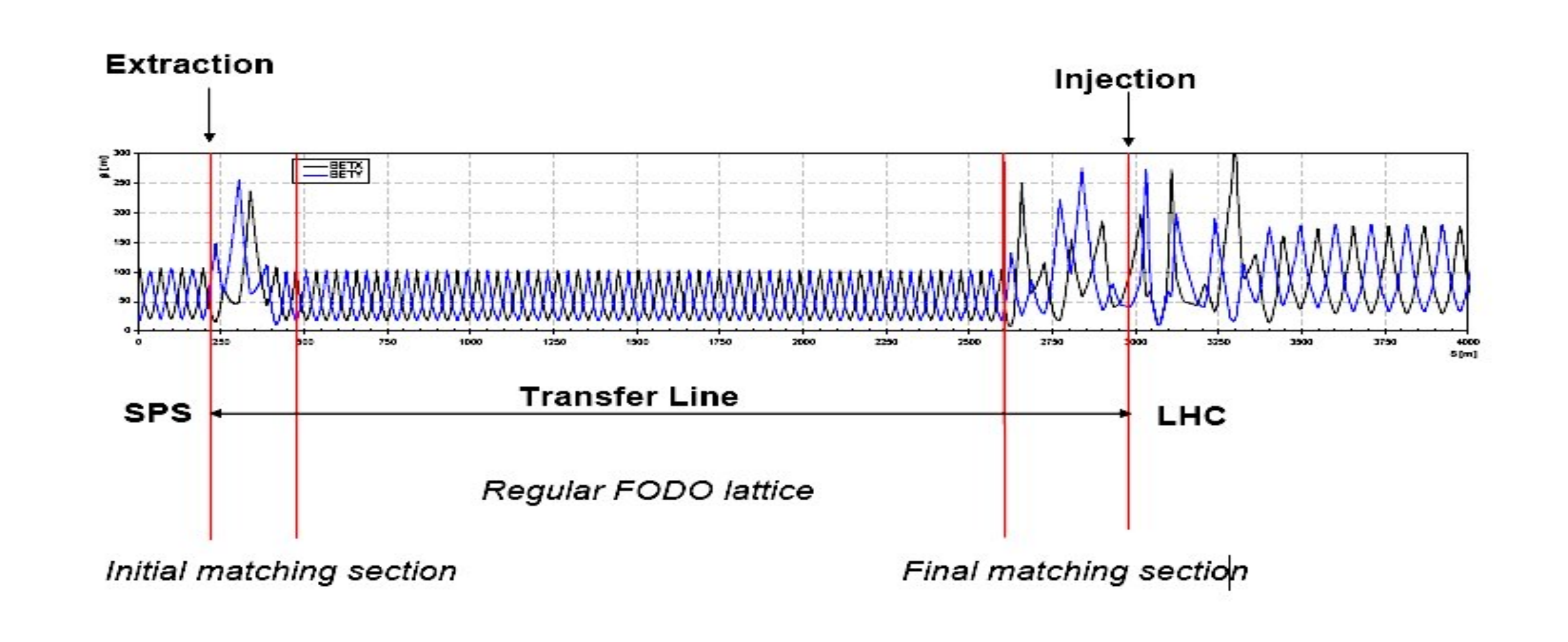
In this matching section we need as many independent quadrupoles as parameters that we have to match.

Usual parameters to be optimised & matched:

$$\alpha_x,\ \beta_x \qquad D_x,\ D_x'$$

$$\alpha_y,\ \beta_y \qquad Q_x,\ Q_y \quad \text{(…or phase advance)}$$

Example:
Transferline SPS-LHC



One word about Mini-Beta Insertions:

Mini Beta Insertions must be installed in

… straight sections (no dipoles that drive dispersion)

… that are dispersion free

… that are connected to the arc lattice by dispersion suppressors

if not, the dispersion dilutes the particle density and increases the effective transverse beam size.

Mini-β Insertions: Phase advance

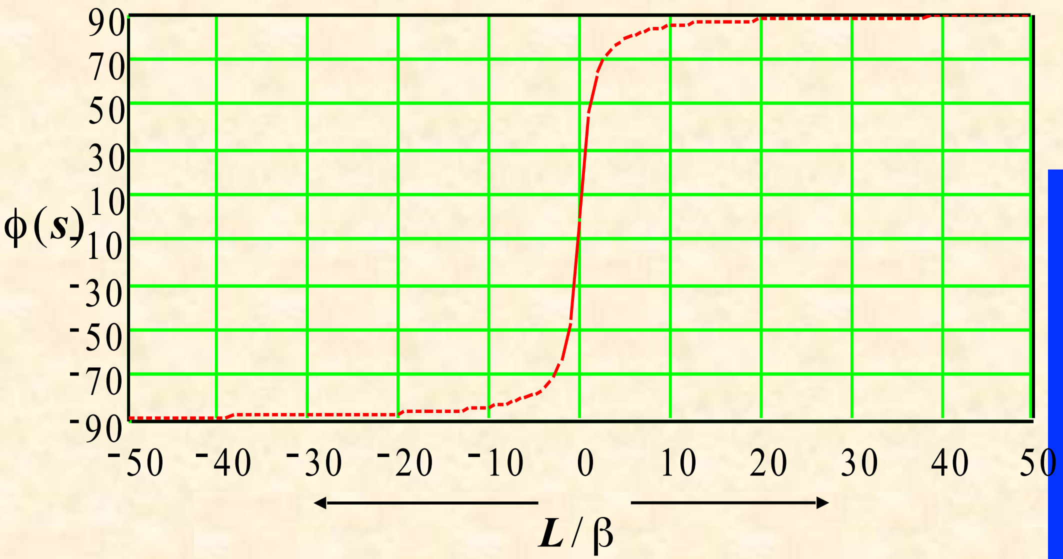
By definition the phase advance is given by:

$$\Phi(s) = \int \frac{1}{\beta(s)}\, ds$$

Now in a mini β insertion:

$$\beta(s) = \beta_0 \left(1 + \frac{s^2}{\beta_0^2}\right)$$

$$\rightarrow \Phi(s) = \frac{1}{\beta_0} \int_0^L \frac{1}{1 + s^2/\beta_0^2}\, ds = \arctan \frac{L}{\beta_0}$$



Consider the drift spaces on both sides of the IP:

the phase advance of a mini β insertion is approximately π,

in other words: the tune will increase by half an integer.

Mini-β Insertions: A look into Phase Space

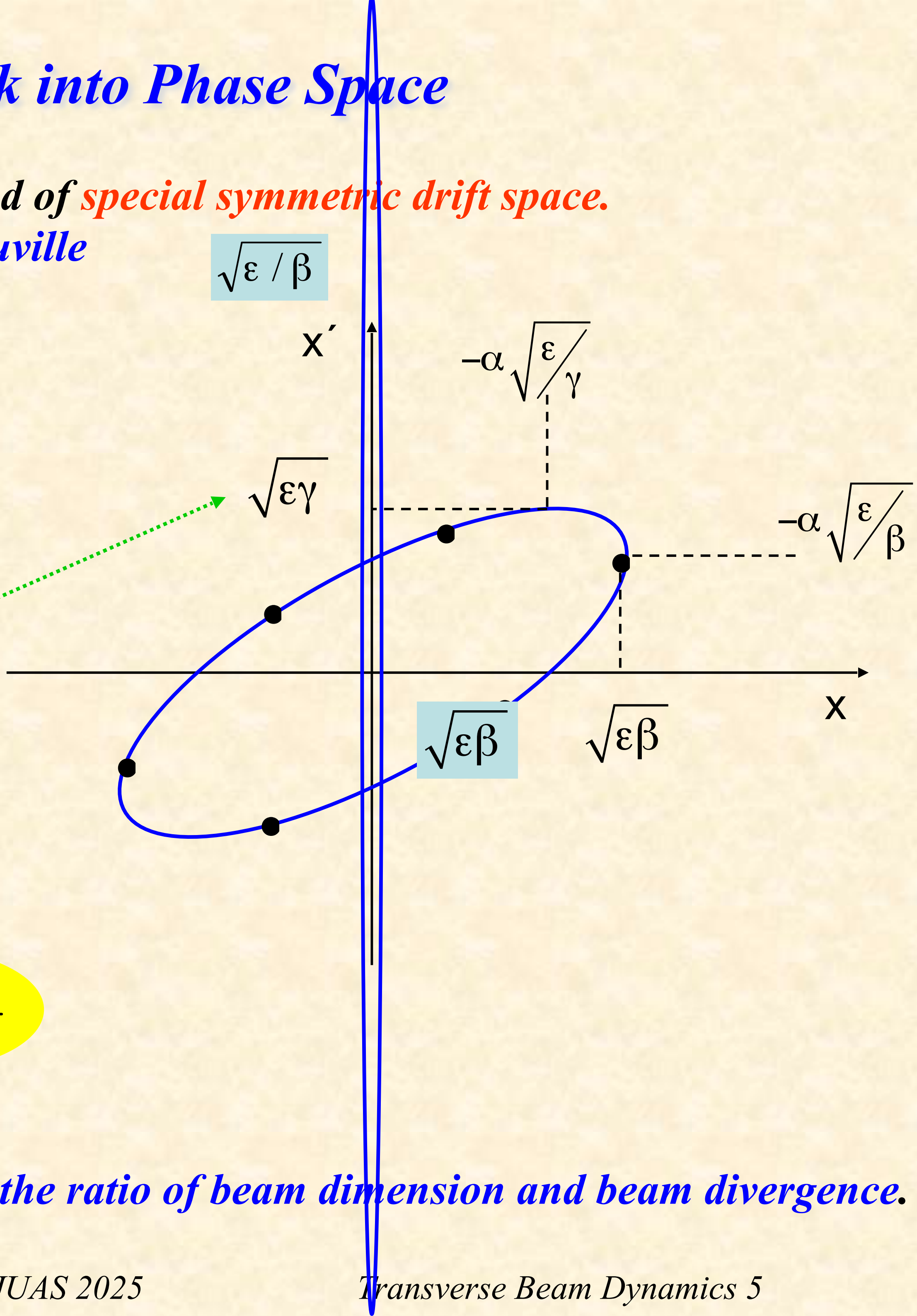
A mini-β insertion is always a kind of special symmetric drift space.

—> greetings from Liouville

$$\alpha^* = 0$$

$$\gamma^* = \frac{1+\alpha^2}{\beta} = \frac{1}{\beta^*}$$

$$\sigma'^* = \sqrt{\frac{\varepsilon}{\beta^*}}$$



$$\beta^* = \frac{\sigma^*}{\sigma'^*}$$

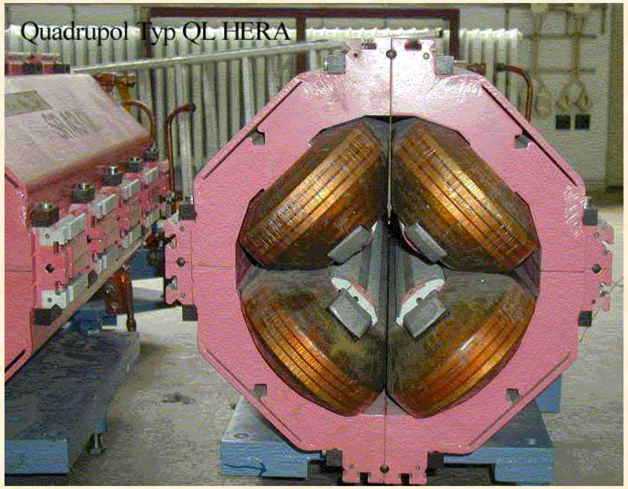
at a symmetry point β is just the ratio of beam dimension and beam divergence.

Are there any problems ??

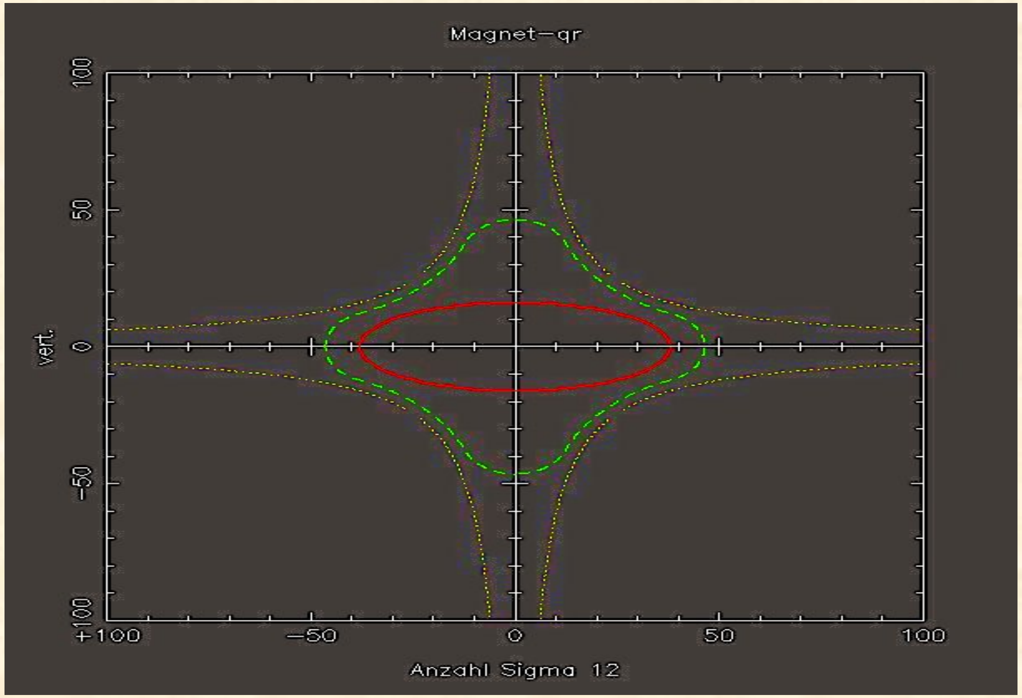
sure there are ...

* *aperture of mini β quadrupoles limit the luminosity*

* *remember: large apertures make weak quads*



gradient of a quadrupole magnet: $g = \frac{2\mu_0 n I}{r^2}$



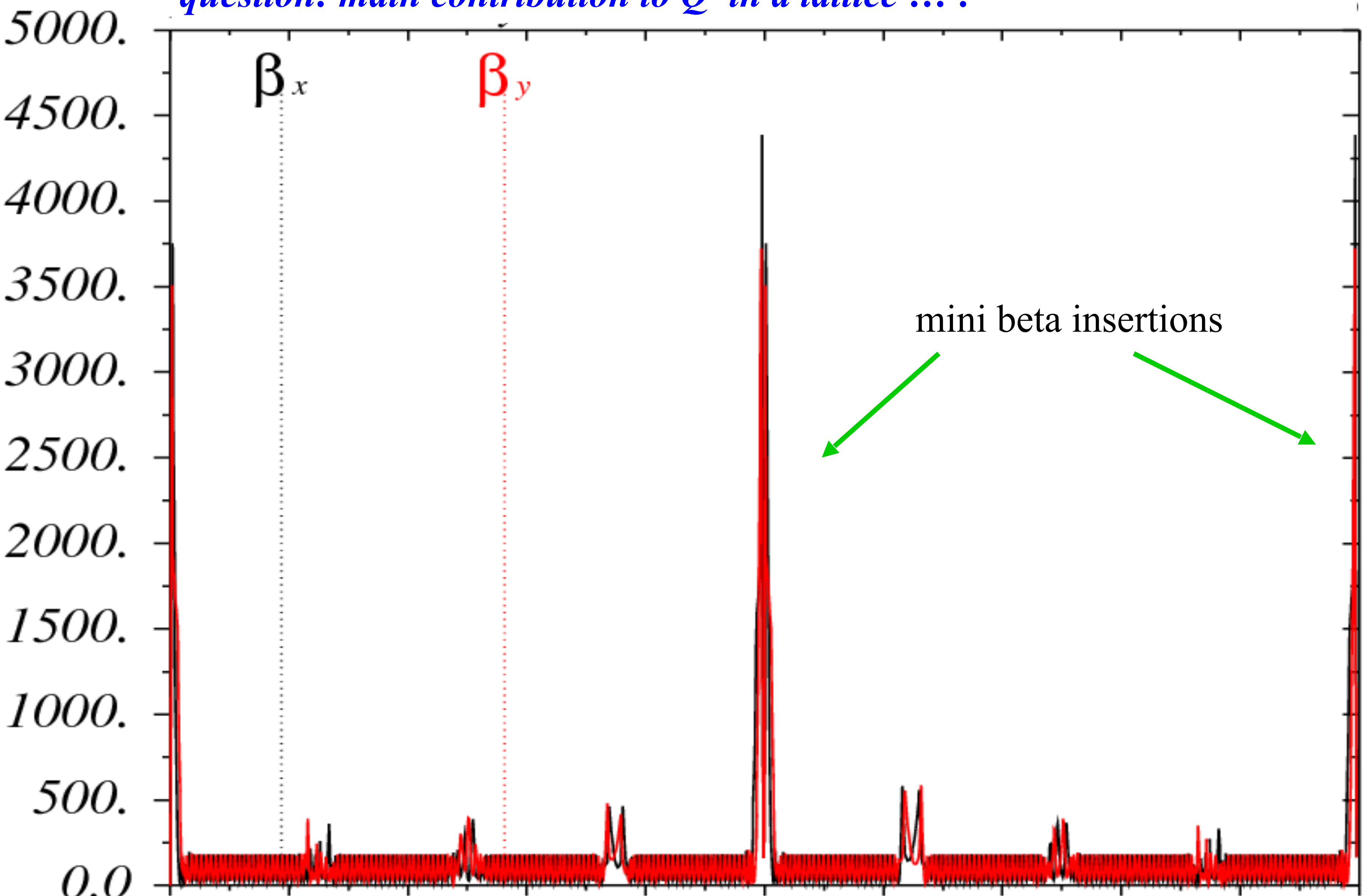
beam envelope at the first mini β quadrupole lens in the HERA proton storage ring

keep distance „s“ to the first mini β quadrupole as small as possible to limit $\hat{\beta}$

… and now back to the Chromaticity

$$Q' = -\frac{1}{4\pi}\oint k(s)\beta(s)ds$$

question: main contribution to Q' in a lattice … ?



Mini-Beta Insertion

Whenever we collide the beams the beam size has to be small to get highest luminosity.

This adds a large contribution to the chromaticity.

Which needs to be corrected by (non-linear) sextupole fields.

Non-linear fields however disturb the harmonic motion and can lead to particle losses.

Correction of Q':

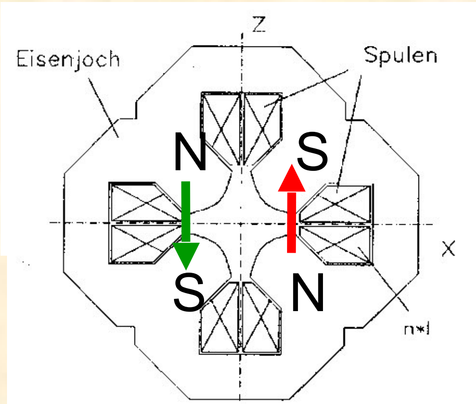
k_1 normalised quadrupole strength

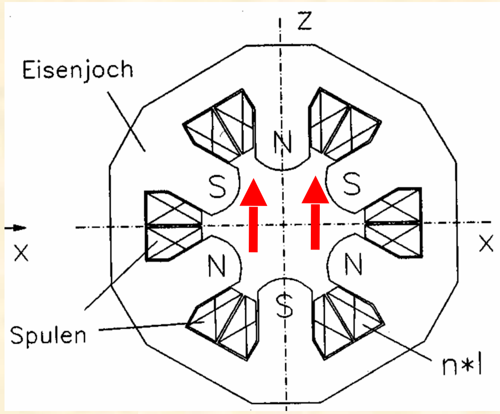
k_2 normalised sextupole strength

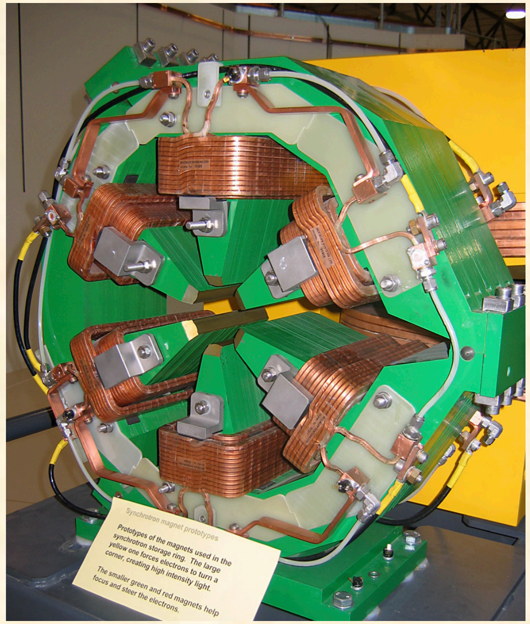
Sextupole Magnets:

$$k_1(sext) = \frac{\tilde{g}\,x}{p/e} = k_2 * x$$

$$k_1(sext) = k_2 * D * \frac{\Delta p}{p}$$







corrected chromaticity

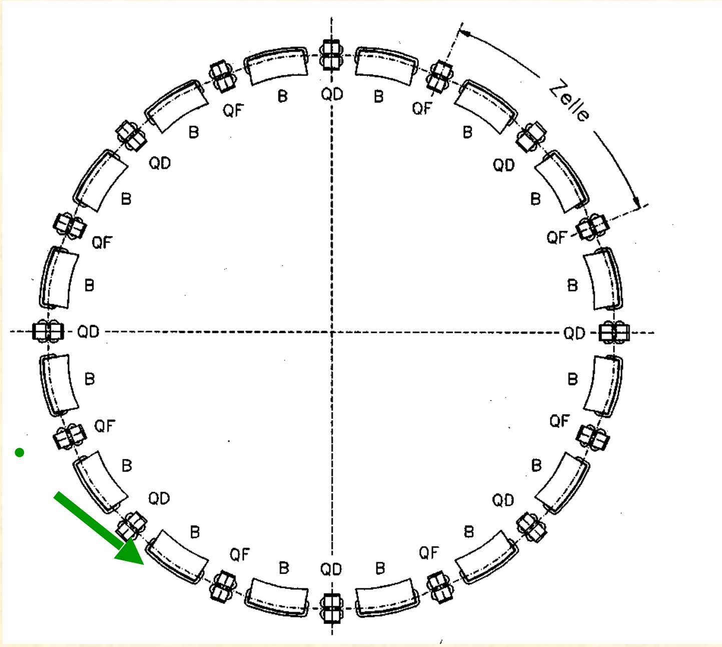
counter acting effect in the two planes

$$Q'_x = -\frac{1}{4\pi}\oint \beta_x(s)\ [+k_q(s) - S_F D_x(s) + S_D D_x(s)]\ ds$$

$$Q'_y = -\frac{1}{4\pi}\oint \beta_y(s)\ [-k_q(s) + S_F D_x(s) - S_D D_x(s)]\ ds\ ,\qquad \text{with}\qquad S_F = k_2^F \cdot l_{sext}\ ,\ \ S_D = k_2^D \cdot l_{sext}$$

"natural" chromaticity — *sextupole correction of chromaticity*

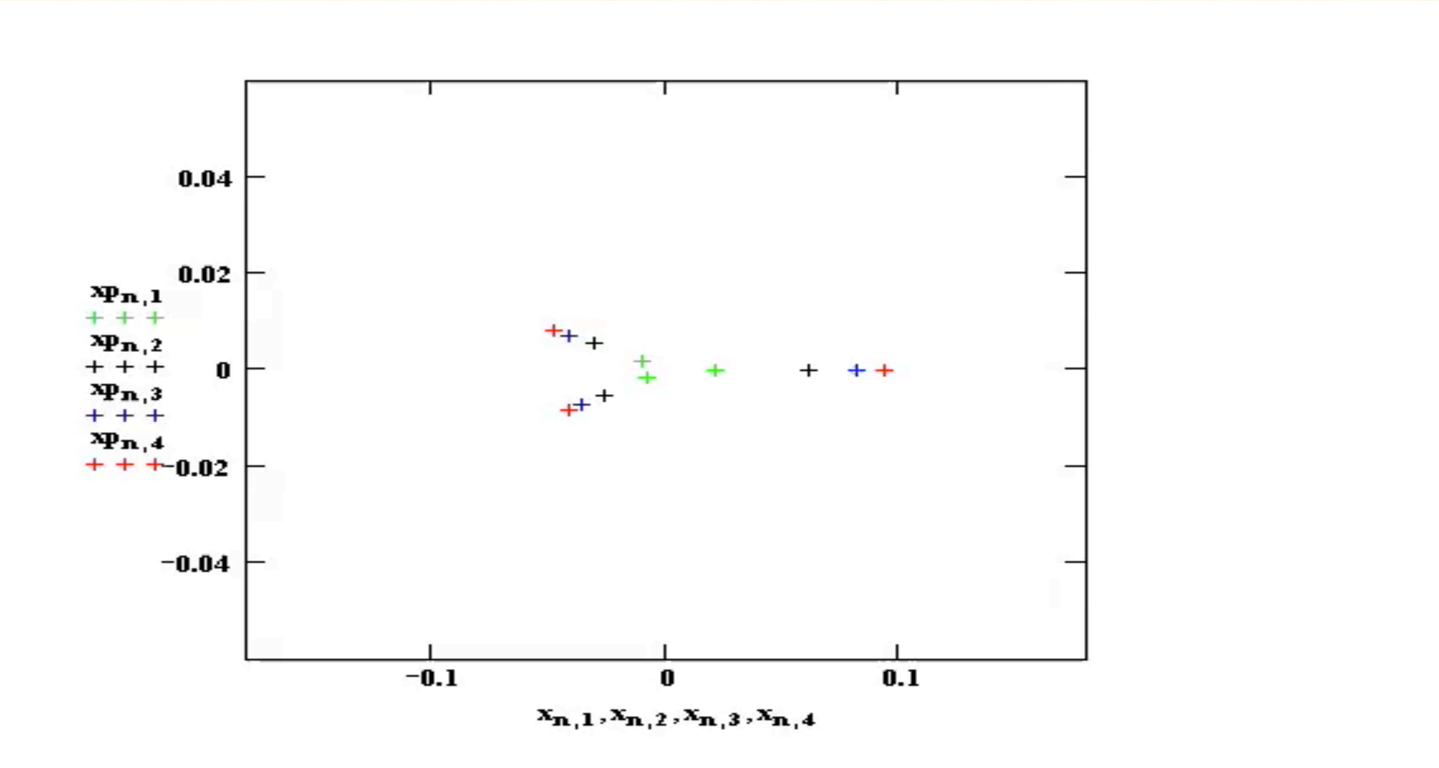
31.) Particle Tracking Calculations



Again: the phase space ellipse

for each turn write down – at a given position „s" in the ring – the single particle amplitude x and the angle x′... and plot it.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{turn} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



4 particles,
each having a slightly different emittance:
observed in phase space,
in a linear lattice

Particle Tracking Calculations

particle vector: $\begin{pmatrix} x \\ x' \end{pmatrix}$ ***field:*** $B = \begin{pmatrix} g'xz \\ \frac{1}{2} g'(x^2 - z^2) \end{pmatrix}$

Idea:

And if you encounter a nonlinear element (e.g. sextupole): stop and calculate explicitly the magnetic field at the particles coordinate.
—> determine the Lorentz-force and thus the new x'.

… calculate kick on the particle …

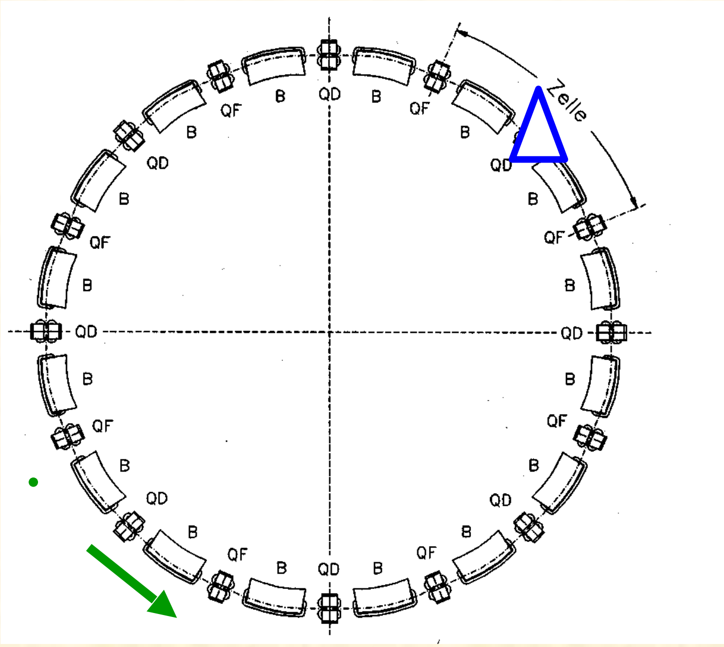
$$\Delta x_1' = \frac{B_z l}{p/e} = \frac{1}{2}\frac{g'}{p/e} l(x_1^2 - z_1^2) \qquad \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} \rightarrow \begin{pmatrix} x_1 \\ x_1' + \Delta x' \end{pmatrix}$$

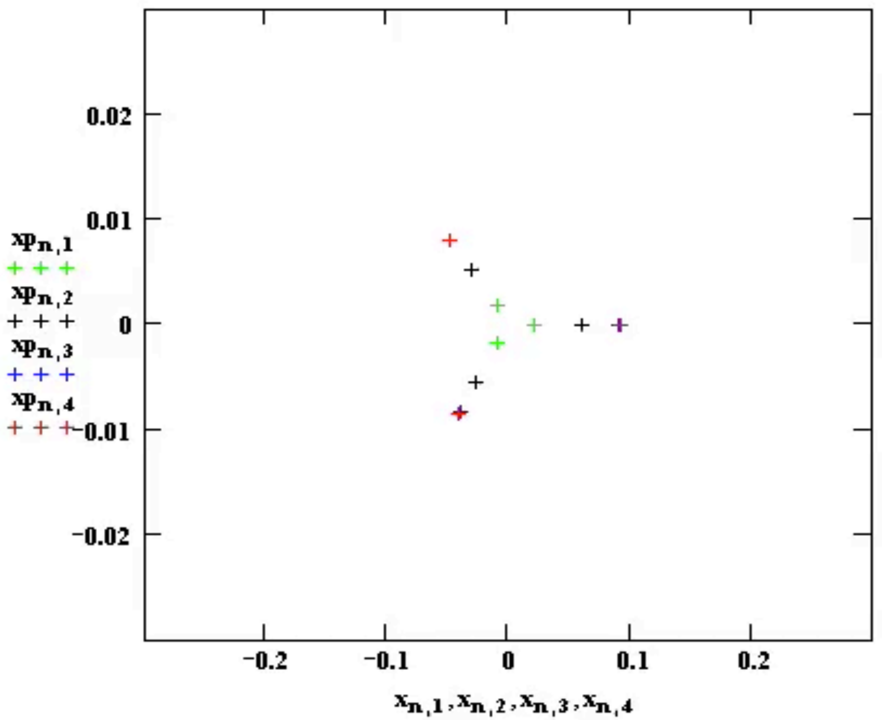
$$\Delta z_1' = \frac{B_x l}{p/e} = \frac{g' x_1 z_1}{p/e} l \qquad \begin{pmatrix} z_1 \\ z_1' \end{pmatrix} \rightarrow \begin{pmatrix} z_1 \\ z_1' + \Delta z_1' \end{pmatrix}$$

… and continue with the linear matrix transformations

Installation of a weak (!!!) sextupole magnet

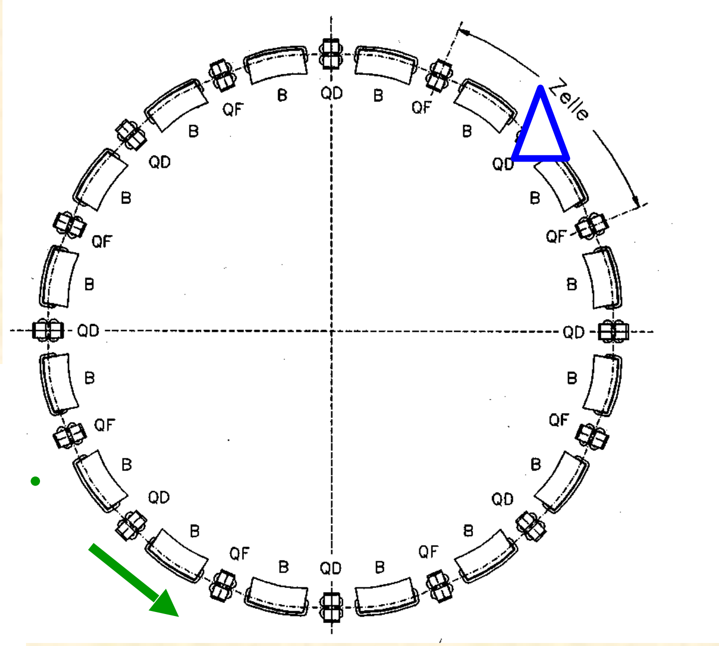
The good news: sextupole fields in accelerators cannot be treated analytically anymore.
—> no equations; instead: Computer simulation
„ particle tracking “

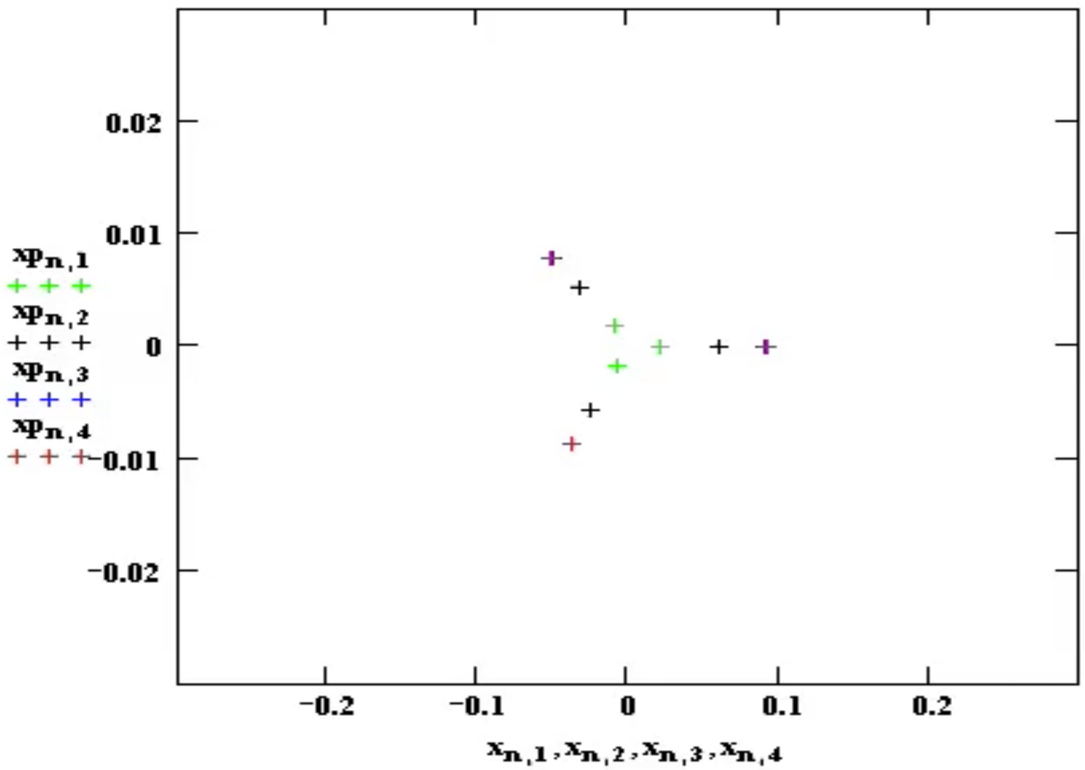




Installation of a strong (!!!) sextupole magnet

The non-linear field effects are so strong, that
- from a certain amplitude x - they destroy the stability of the motion and the particles are lost.





"Dynamic Aperture"

Than'x for your attention