$U_A(1)$ symmetry restoration from PHENIX measurements

in $\sqrt{s_{NN}}$ = 200 GeV Au+Au collisions

T. Csörgő [1,2] for the PHENIX Collaboration

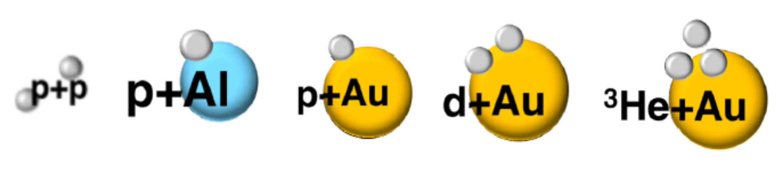
[1] HUN-REN Wigner FK, Budapest, Hungary
[2] MATE Institute of Technology, Gyöngyös, Hungary

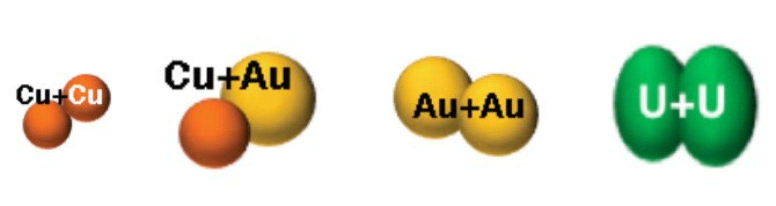
Introduction
Bose-Einstein correlations
Levy stable source distributions, Levy expansions
Centrality and m_T dependence of α, λ, λ / λ_{max}, R and $\hat{R}$
Centrality dependence of α_0, A, B, $\hat{A}$, $\hat{B}$, H and σ
Comparisons to Monte-Carlo simulations
Indirect observation of $U_A(1)$ symmetry restoration by PHENIX

Based on: arXiv:2407.08586, Phys. Rev. C to appear
See also T. Novák's talk at ICNFP 2024 and Phys. Rev. C 97 (2018) 064911, and Phys. Rev. C 108 (2023) 049905

PHENIX DETECTOR @ RHIC – RUN HISTORY

Completed 16 years of operation with **versatility**.
9 collision species and 9 collision energies.
Both **geometry and beam energy scan.**



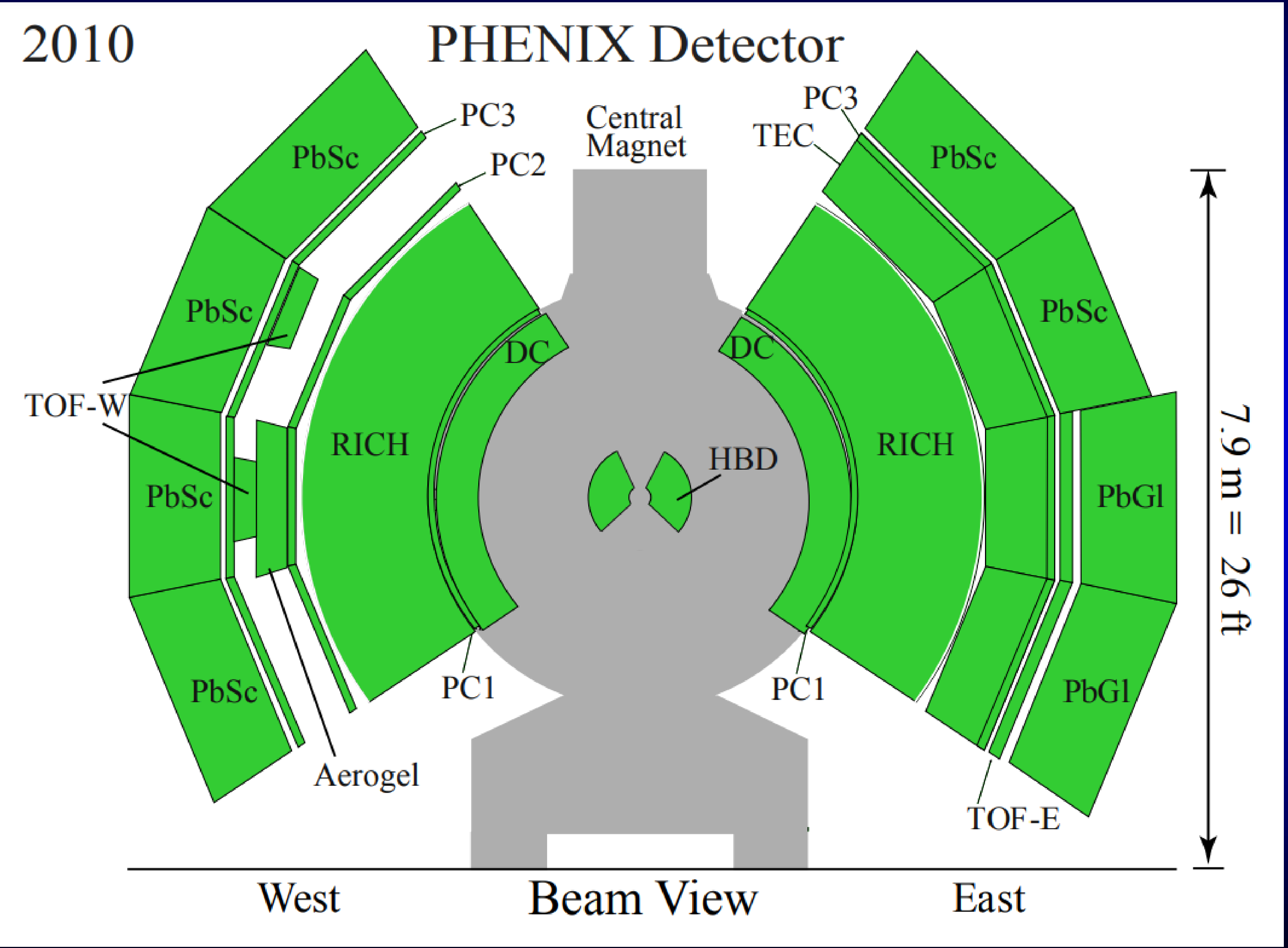


PHENIX has completed its datataking and has been replaced by sPHENIX. But PHENIX data analysis continues, exploiting the **discovery potential** of PHENIX.

In this talk: centrality dependence of Bose-Einstein correlations in a **special 2010 run** for Au+Au at $\sqrt{s_{NN}}$ = 200 GeV, that allows for **charged pions identification at low m_T** .

Species	Run Year
Au+Au	2001, 2002, 2004, 2007, 2008, **2010**, 2011, 2014, 2016
d+Au	2003, 2008, 2016
Cu+Cu	2005
U+U	2012
Cu+Au	2012
^{3}He+Au	2014
p+Au	2015
p+Al	2015

THE PHENIX DETECTOR - 2010



For PHENIX Heavy Ion overview,
T. Novák's talk at ICNFP'24

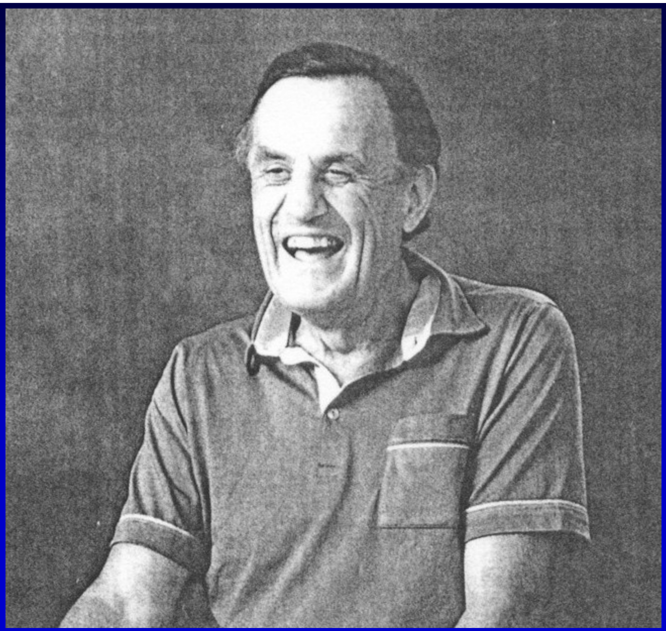
Central Arm detectors

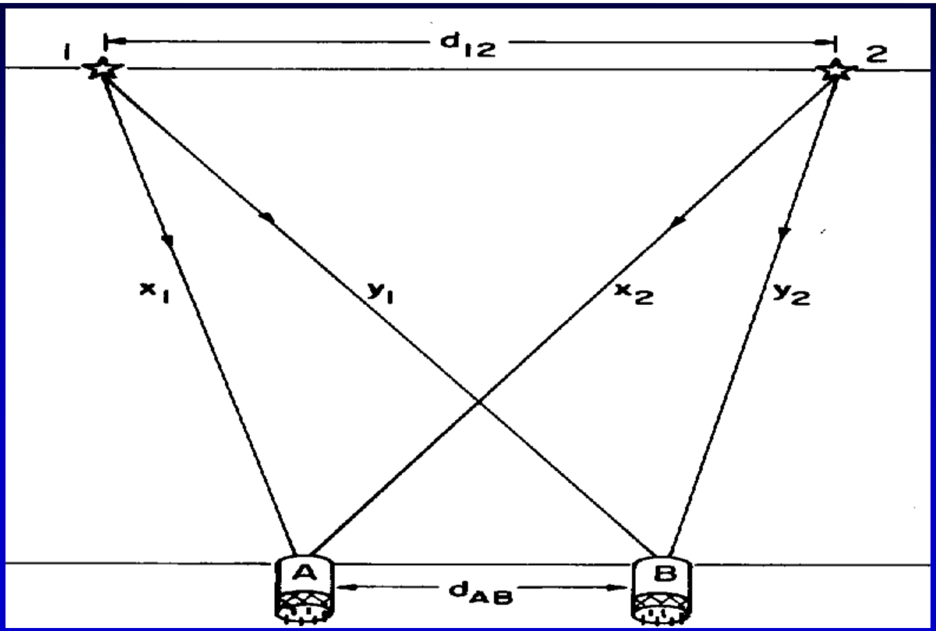
DC: Drift Chamber
PC1 – PC3: PAD Chamber
PbSc: EM Cal
PbGl: EM Cal

HBD: Hadron Blind Det
(half magnetic field in CM)

Not shown:
BBC, ZDC: centrality

HBT: Robert HANBURY BROWN – Richard Quentin TWISS





Two people: Robert Hanbury Brown and Richard Quentin Twiss

– Robert, Hanbury as well as Richard and Quentin: can be given names, but…

– Sir Robert Hanbury Brown had a *compound family* name…

R. Hanbury Brown and R. Q. Twiss: Engineers, who worked in radio and optical astronomy

„Interference between two different photons can never occur."

P. A. M. Dirac, The Principles of Quantum Mechanics, Oxford, 1930

„As an engineer my education in physics had stopped far short of the quantum theory.
Perhaps just as well … ignorance is sometimes a bliss in science."

R. H. Brown: Boffin: A Personal Story … ISBN 0-7503-0130-9

In particle physics: GGLP effect (Goldhaber, Goldhaber, Lee, Pais) discovered independently,

Explanation: Bose-Einstein statistics of pions

Two particle Bose-Einstein/HBT:

$C_2(q)$ = 1 + positive-definite term

1+ |Fourier–transform of the source|^2,

Usually evaluated in Gaussian approximation

Dubna school: use it as a tool

Kopylov, Podgoretskii, Lednicky:

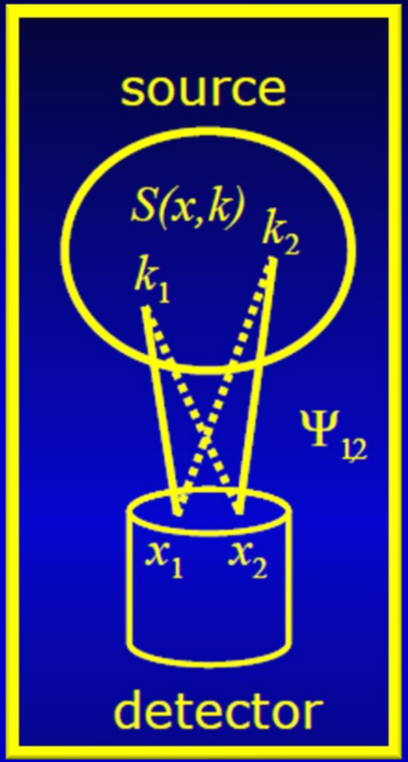
x <-> k

C_2 = 1 + |Fourier–transform source|^2

Introduction to Bose-Einstein or HBT correlations

Two plane waves
Symmetrized, + for bosons, - for fermions
Expansion dynamics, final state interactions,
multiparticle symmetrization effects: negligible

$$\Psi_1 = \mathrm{e}^{-ik_1x_1}$$
$$\Psi_2 = \mathrm{e}^{-ik_2x_2}$$



$$A_{12} \propto \frac{1}{\sqrt{2}} \left[\mathrm{e}^{ik_1x_1+ik_2x_2} \pm \mathrm{e}^{ik_1x_2+ik_2x_1} \right],$$

$$N_2(k_1,k_2) \propto \int dx_1 \rho(x_1) \int dx_2 \rho(x_2) \; |A_{12}|^2$$

$$C_2(k_1,k_2) = \frac{N_2(k_1,k_2)}{N_1(k_1)N_2(k_2)} = 1 \pm |\tilde{\rho}(k_1-k_2)|^2$$

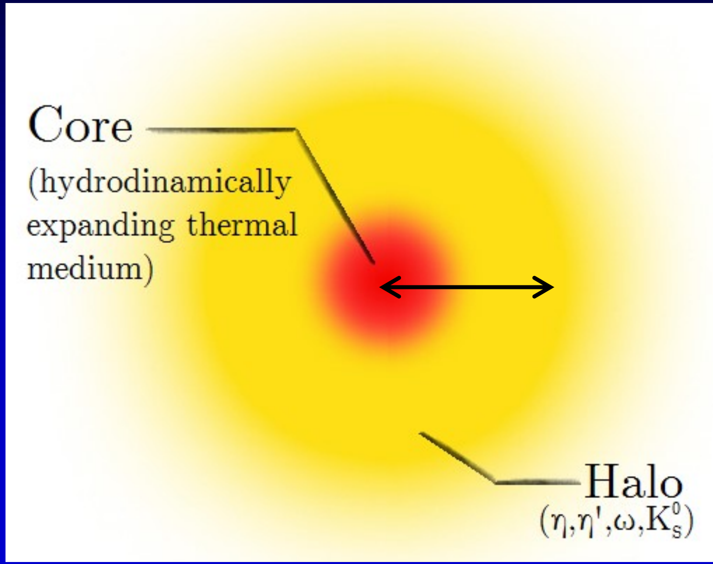
Two particle HBT correlations, typically, but needs cross-checks:
C(q) = 1 + positive-definite term
C(q) = 1+ |Fourier–transform of the source|^2,

$$\tilde{\rho}(q) = \int dx \; \mathrm{e}^{iqx} \; \rho(x)$$

Earlier: evaluated in Gaussian approximation
Dependence on mean momentum:
expansion dynamics ρ(x) → S(x,k)

Dubna school: <u>use it as a tool</u>
Kopylov, Podgoretskii, Lednicky: x <-> k
1+ |Fourier–transform of the source|^2

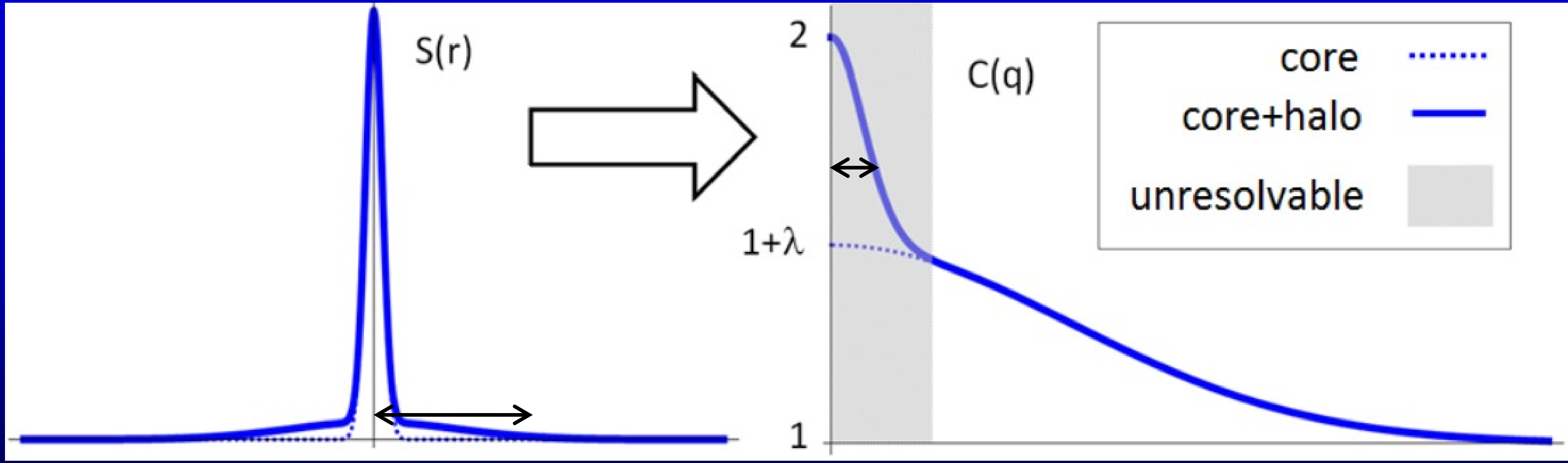
Core/halo model, long-lived resonances



Resonance pions reduce the corr. strength [1, 2]
Core-Halo model: $S = S_C + S_H$
Primordial pions - Core $\lesssim$ 10 fm
Resonance pions - from very far regions - Halo
Corr. strength→C-H ratio: $\lambda = \left(\frac{N_C}{N_C+N_H}\right)^2$

$R = \sqrt{\langle x^2\rangle - \langle x\rangle^2}.$ **Variance: halo dominated!**

Precise measurement of λ important !

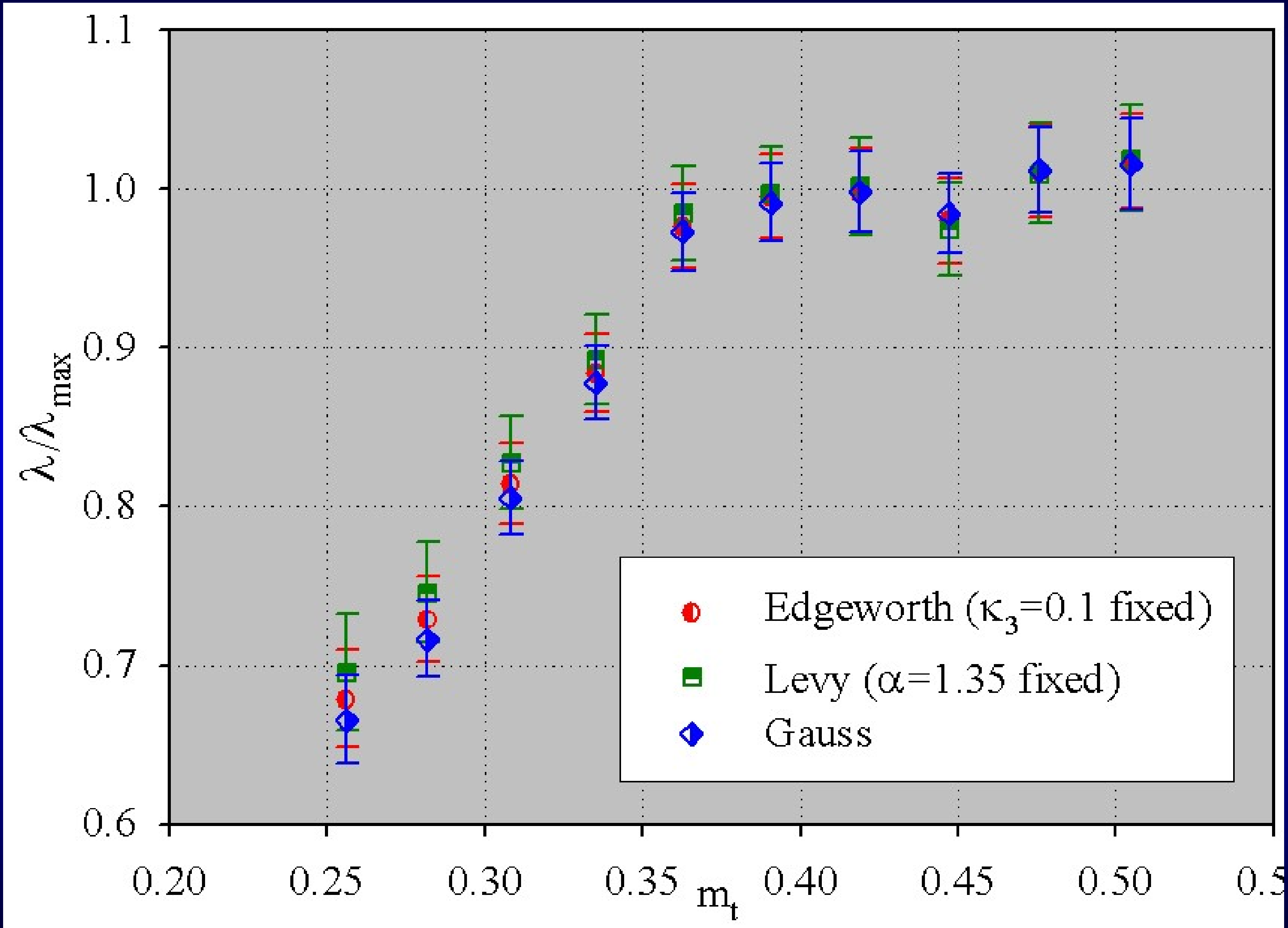


Precise measurement of λ is based on extrapolation to Q = 0, needs precise measurement of the shape of C(q):

→ Levy expansion

[1] J. Bolz et al: Phys.Rev. D47 (1993) 3860-3870
[2] T. Cs, B. Lörstad, J. Zimányi: hep-ph/9411307, Z.Phys. C71 (1996) 491-497

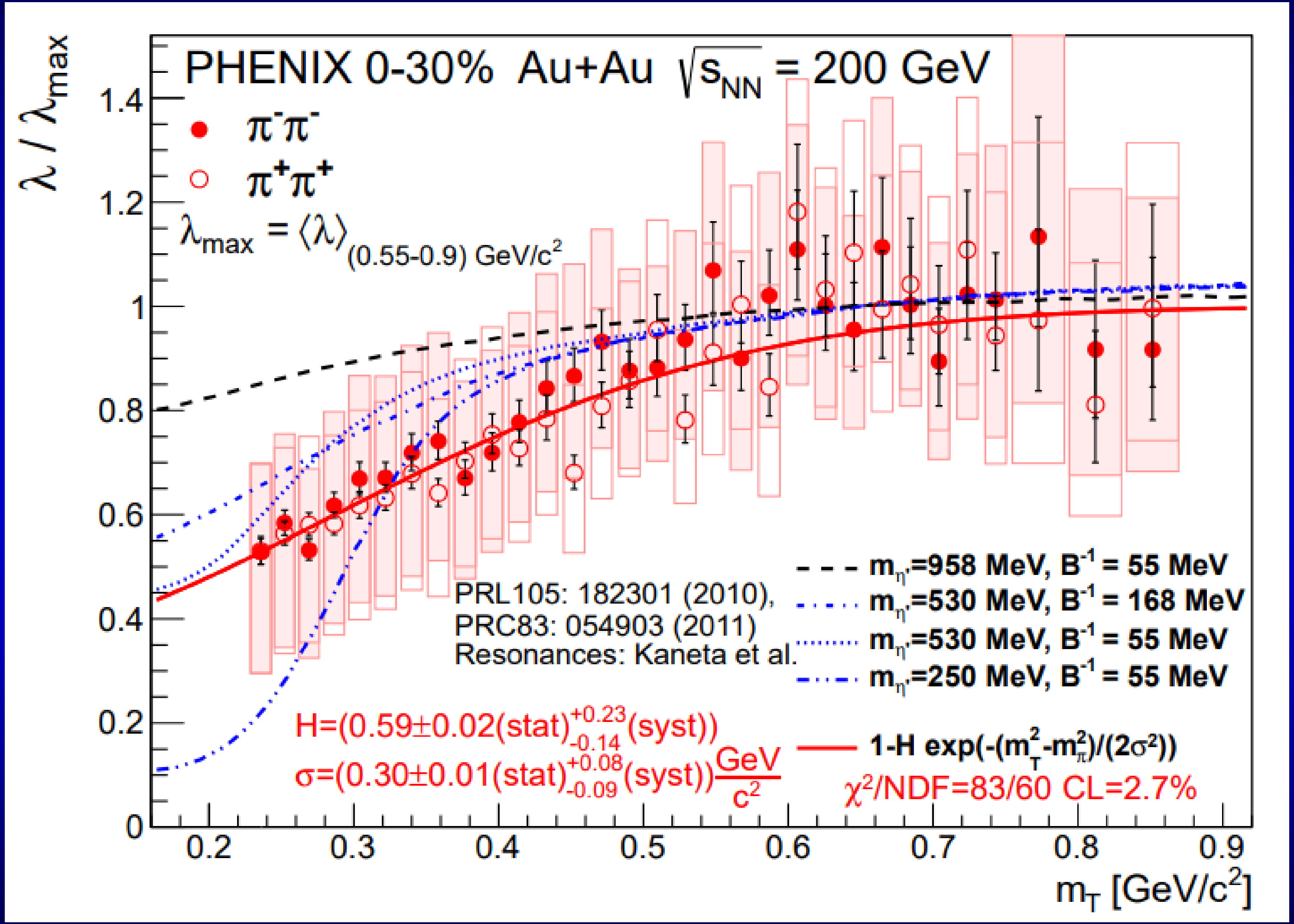
HBT: Interpretation of λ, α and R



M. Csanád for PHENIX Collaboration, arXiv:nucl-ex/0509042:
λ / λ_{max} is independent of the method of extrapolation of $C_2(q)$ to q = 0

$U_A(1)$ SYMMETRY RESTORATION: CAN WE TURN IT ON/OFF?

Is it centrality dependent?



PHENIX data + Monte Carlo simulations, PHENIX Phys. Rev. C 97 (2018) 064911:
0-30 % Au+Au @ 200 GeV Levy Bose-Einstein is sensitive to in-medium mass modification of η'

DATA SAMPLE

$\sqrt{s_{NN}}$ = 200 GeV Au+Au collisions, half field in PHENIX central magnet allows pion id down to transverse momentum $p_T > 0.16$ GeV.

Min. bias data sample ~ 7.3 billion events.

0 – 60 % centrality selection ~ 4.4 billion events.
Centrality vs N_{part} determination with PHENIX Glauber calculations.

Similar single track selections as in earlier 0 – 30 % central results, published in Phys. Rev. C 97 (2018) 064911.

Six centrality classes:
0-10%, 10-20%, 20-30%, 30-40%, 40-50% and 50-60%

In each centrality class:
23 bins in $m_T = \sqrt{m^2 + p_T^2}$, from 0.248 GeV to 0.876 GeV

Due to broader central range, more stringent pair cuts, as compared to our 0-30 % results published in Phys. Rev. C 97 (2018) 064911. Other details similar.

NEW PAIR CUTS AND SYSTEMATIC ERRORS

TABLE I. The values of the coordinates for the pair cuts and the alternative values used to determine systematic uncertainties.

Pair cuts	DC $\Delta\varphi_0$ [rad]	DC Δz_0 [cm]	DC $\Delta\varphi_1$ [rad]	TOF east $\Delta\varphi_0$ [rad]	TOF east Δz_0 [cm]	TOF west $\Delta\varphi_0$ [rad]	TOF west Δz_0 [cm]	EM Cal $\Delta\varphi_0$ [rad]	EM Cal Δz_0 [cm]	EM Cal $\Delta\varphi_1$ [rad]
Default cut settings	.12	8.	.017	.12	12	.075	14.0	.12	16	.015
Loose drift chamber cut	.11	7.	.016	.12	12	.075	14.0	.12	16	.015
Strict drift chamber cut	.13	9.	.018	.12	12	.075	14.0	.12	16	.015
Loose ID detector cuts	.12	8.	.017	.11	11	.070	13.0	.11	15	.013
Strict ID detector cuts	.12	8.	.017	.13	13	.080	15.0	.13	17	.017

Systematic errors fully propagated to the very end of this analysis chain:
Cross-checks with three alternative syst error calculation methods.
Most conservative estimate of the systematic errors is shown.
Correlated error propagation is taken into account.

Improvements in Coulomb corrections not detailed in this talk due to time limitations.

FITTING FUNCTION: LEVY SHAPE

$$C_2(Q) = 1 + \lambda \exp\left[-Q^\alpha R^\alpha\right],$$

Cs. T., S. Hegyi, W. A. Zajc, nucl-th/0310042

Approach: we do not know the shape a priori.
Precise measurement of the intercept λ needed:
λ has important physical meaning.

Is it Gaussian? Maybe, test if α = 2, or not.
Check also with Edgeworth and Gauss expansion.

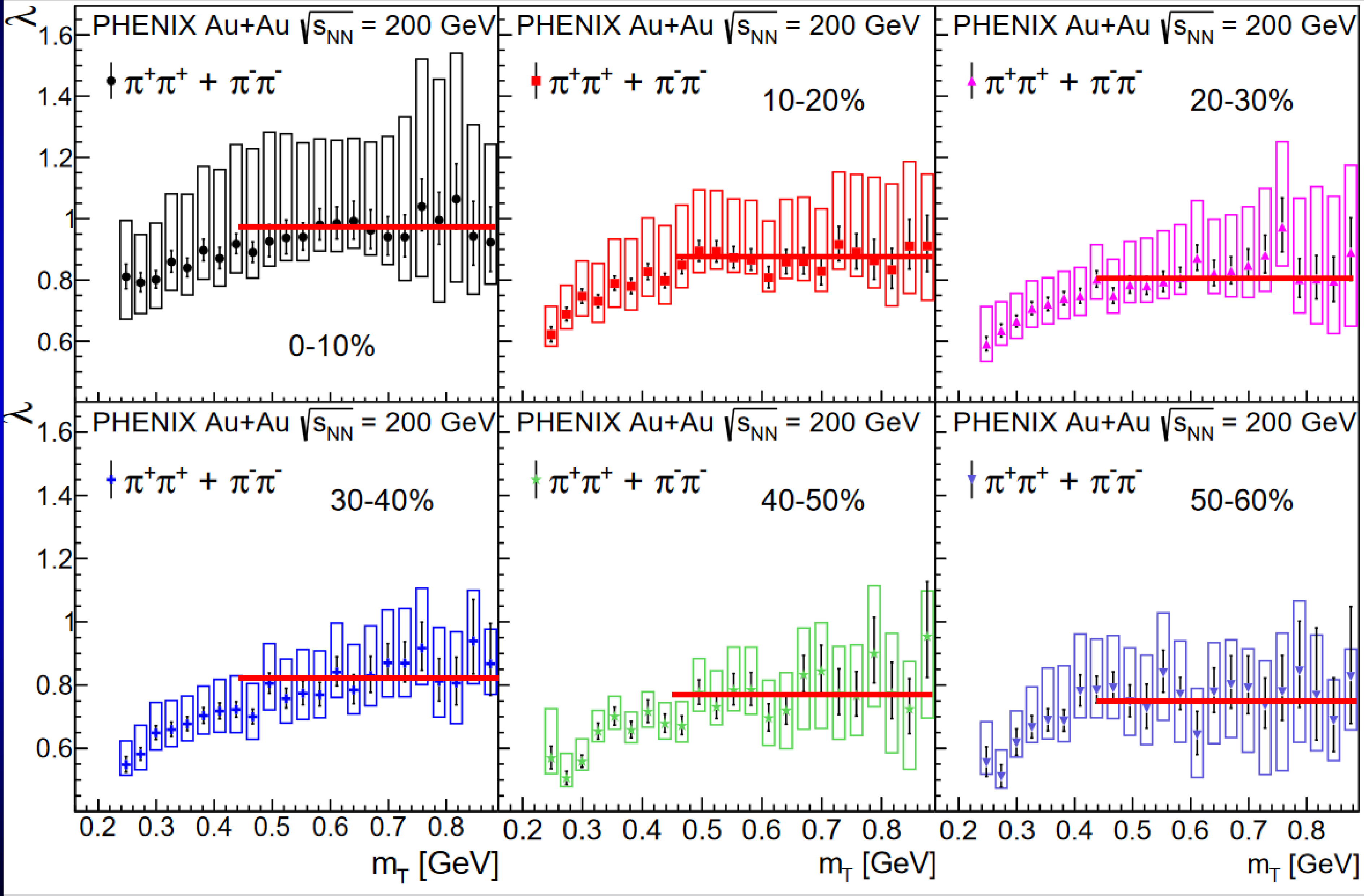
Is it exponential? Maybe, test if α = 1, or not.
Check also with Laguerre expansion.

Is it Levy? Maybe, test the fit quality.
We used Levy expansion. First order corrections are consistent with 0.

In every step of this analysis:
Fits represent data, p-value or confidence level (CL) > 0.1% required.

M_T AND CENTRALITY DEPENDENT RESULTS

M_T AND CENTRALITY DEPENDENCE OF LÉVY λ



Results for Levy λ

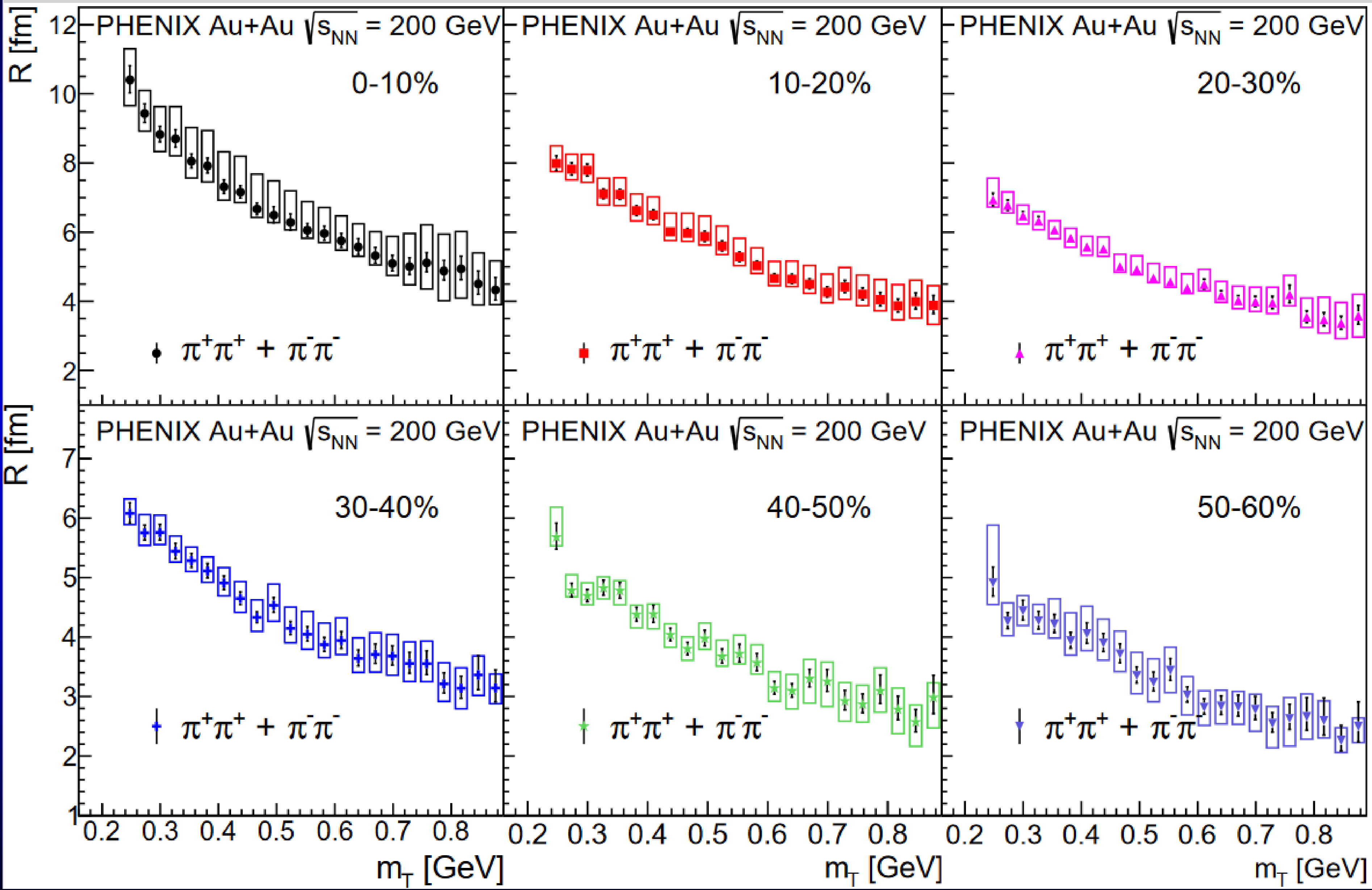
saturation at large m_T and suppression at low m_T

IN EACH CENTRALITY CLASS

Saturated region:

$0.45 \leq m_T \leq 0.9$ GeV
average value: λ_{max}

M_T AND CENTRALITY DEPENDENCE OF LEVY R

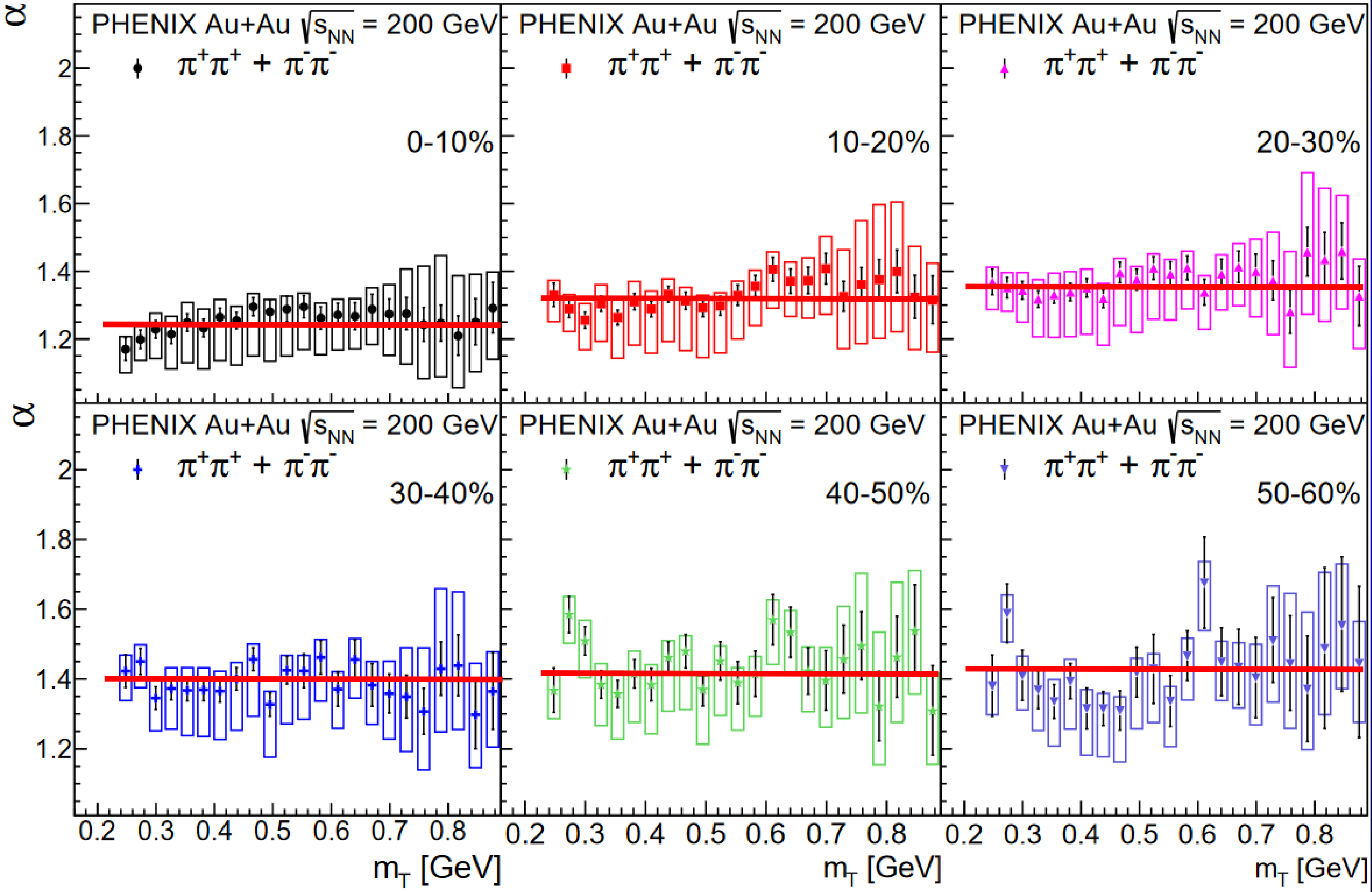


Results for Levy R

monotonic decrease with increasing m_T

IN EACH CENTRALITY CLASS

M_T AND CENTRALITY DEPENDENCE OF LÉVY α



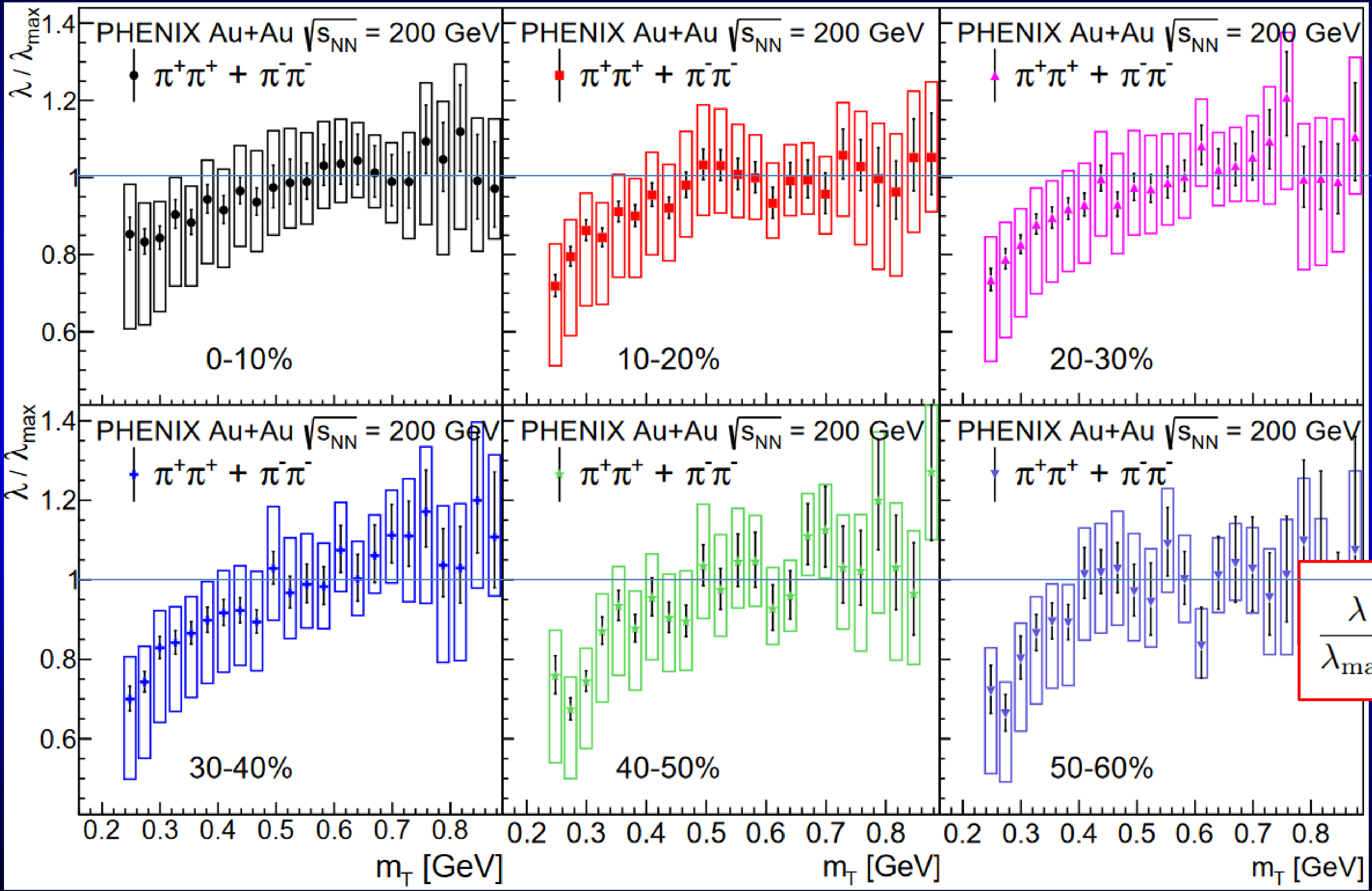
Results for Levy α

m_T independent constant α_0 value

IN EACH CENTRALITY CLASS

PARAMETERIZATION OF M_T DEPENDENCE

M_T DEPENDENCE OF LÉVY λ / λ_{max}



$0.45 \le m_T \le 0.9$ GeV saturated value: λ_{max}

Scale it out!

Results for λ / λ_{max}

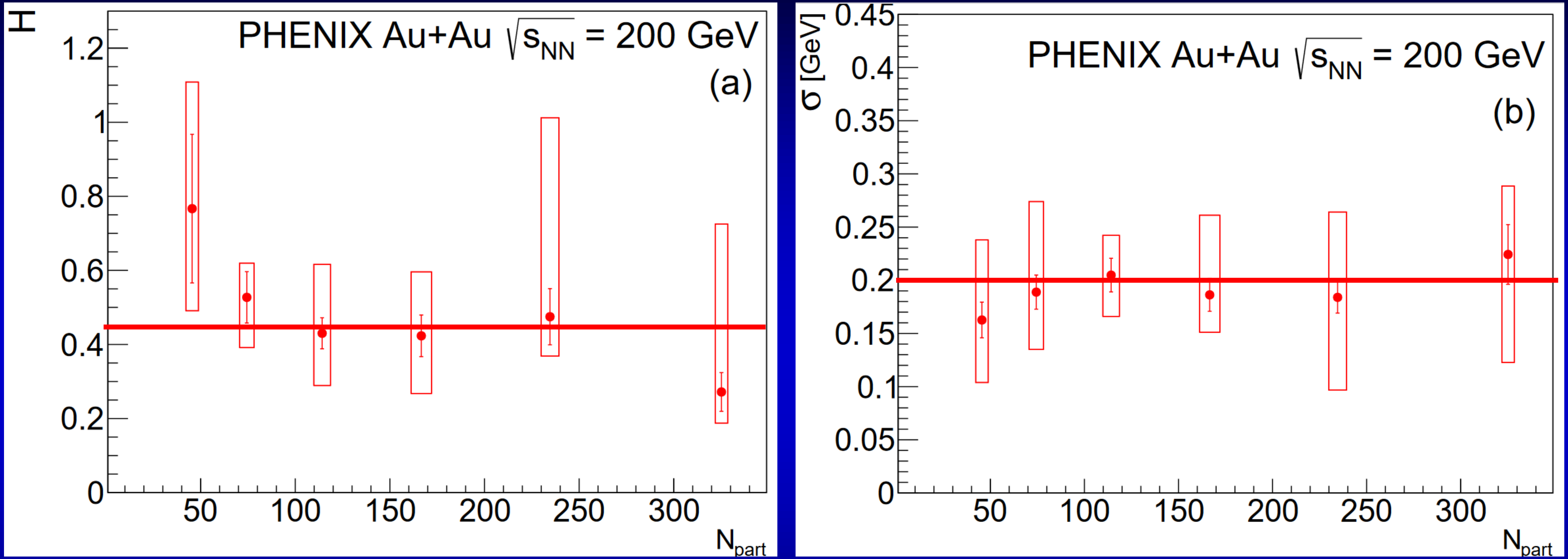
UNEXPECTED, CENTRALITY INDEPENDENT SCALING

IN EACH CENTRALITY CLASS

$$\frac{\lambda}{\lambda_{max}} = 1 - H \exp\left(-\frac{m_T^2 - m_\pi^2}{2\sigma^2}\right)$$

Values of σ and H are expected to be independent of centrality

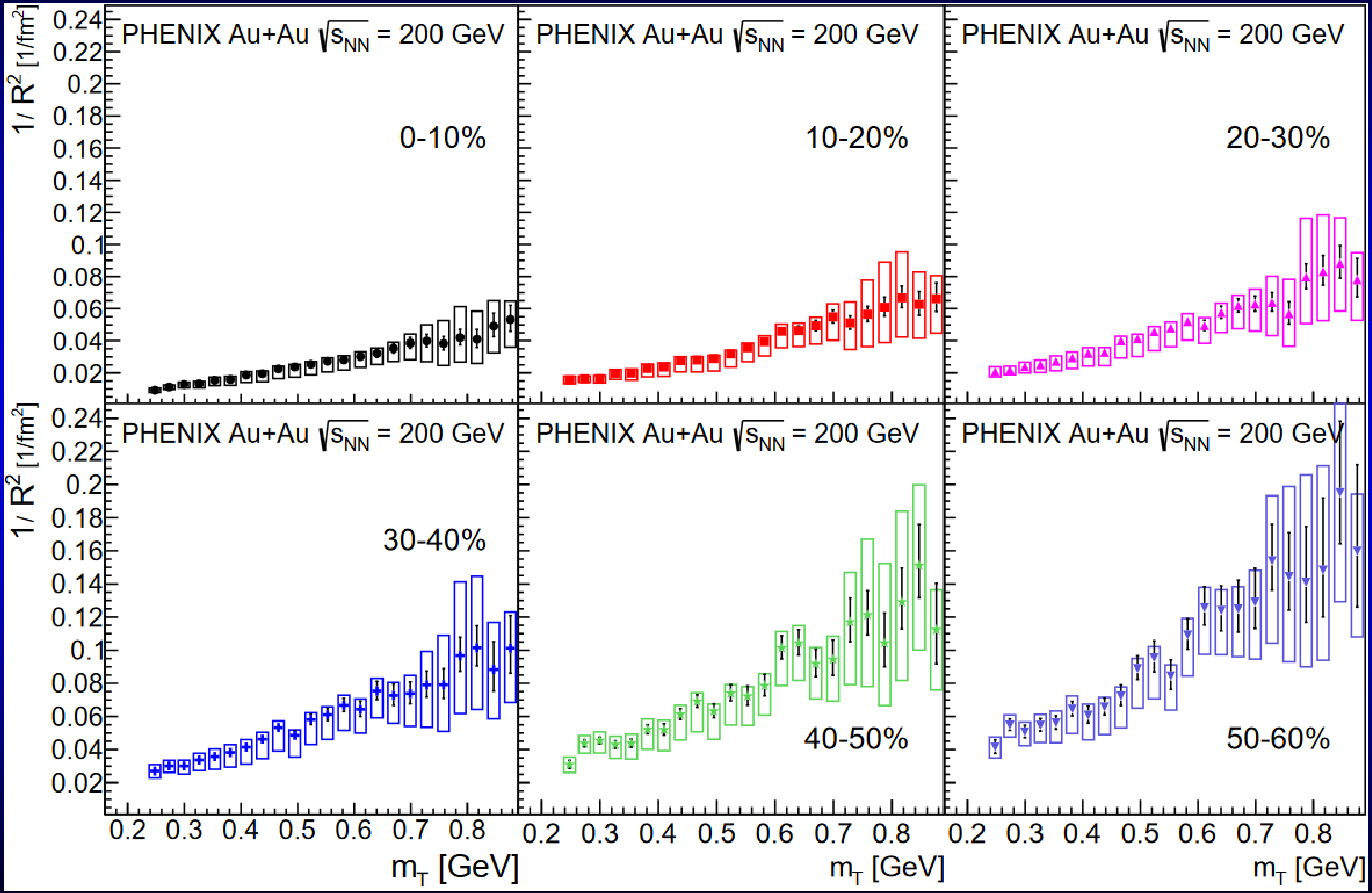
CENTRALITY DEPENDENCE OF σ AND H



$$\frac{\lambda}{\lambda_{\max}} = 1 - H \exp\left(-\frac{m_T^2 - m_\pi^2}{2\sigma^2}\right)$$

Values of σ and H are (within errors) independent of centrality, with a CL > 0.1 %

M_T DEPENDENCE OF LÉVY $1/R^2$



Analytic hydro predicts for $\alpha = 2$:

$1/R^2 = A\, m_T + B$

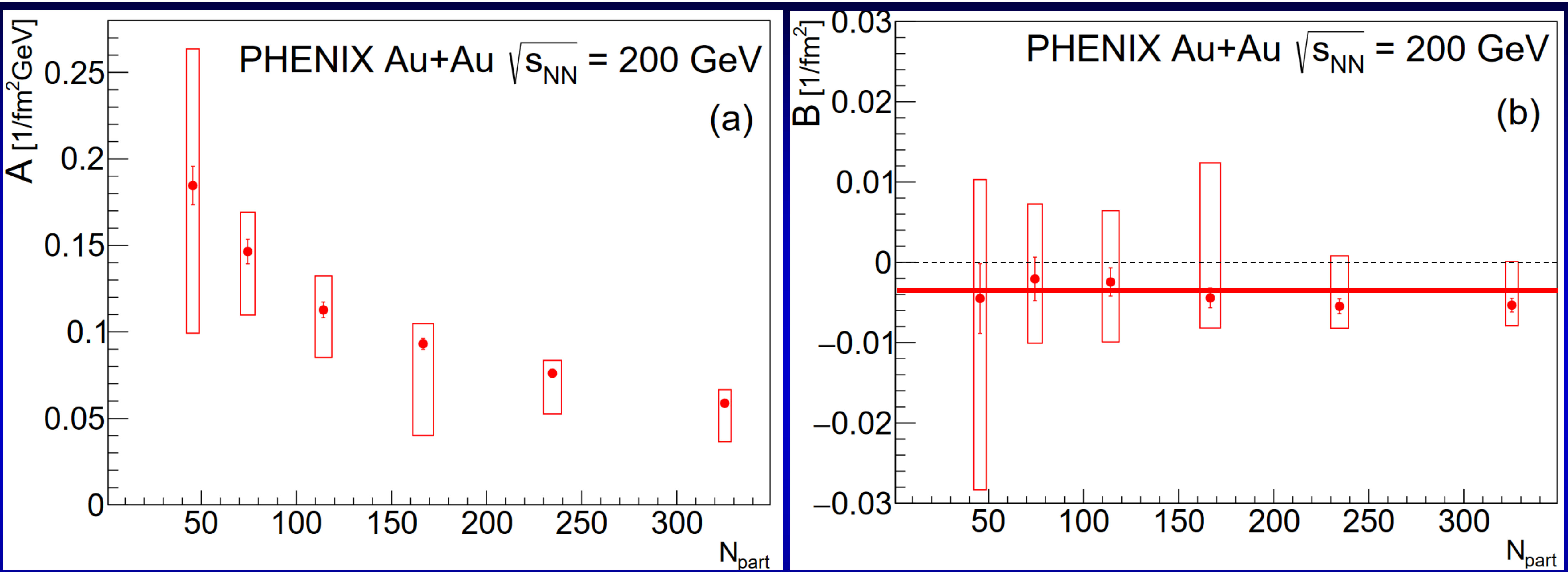
UNEXPECTEDLY, $1/R^2$ SCALING HOLDS ALSO FOR $\alpha < 2$

IN EACH CENTRALITY CLASS

$$\frac{1}{R^2} = A m_T + B$$

Values of A and B are expected to be centrality dependent

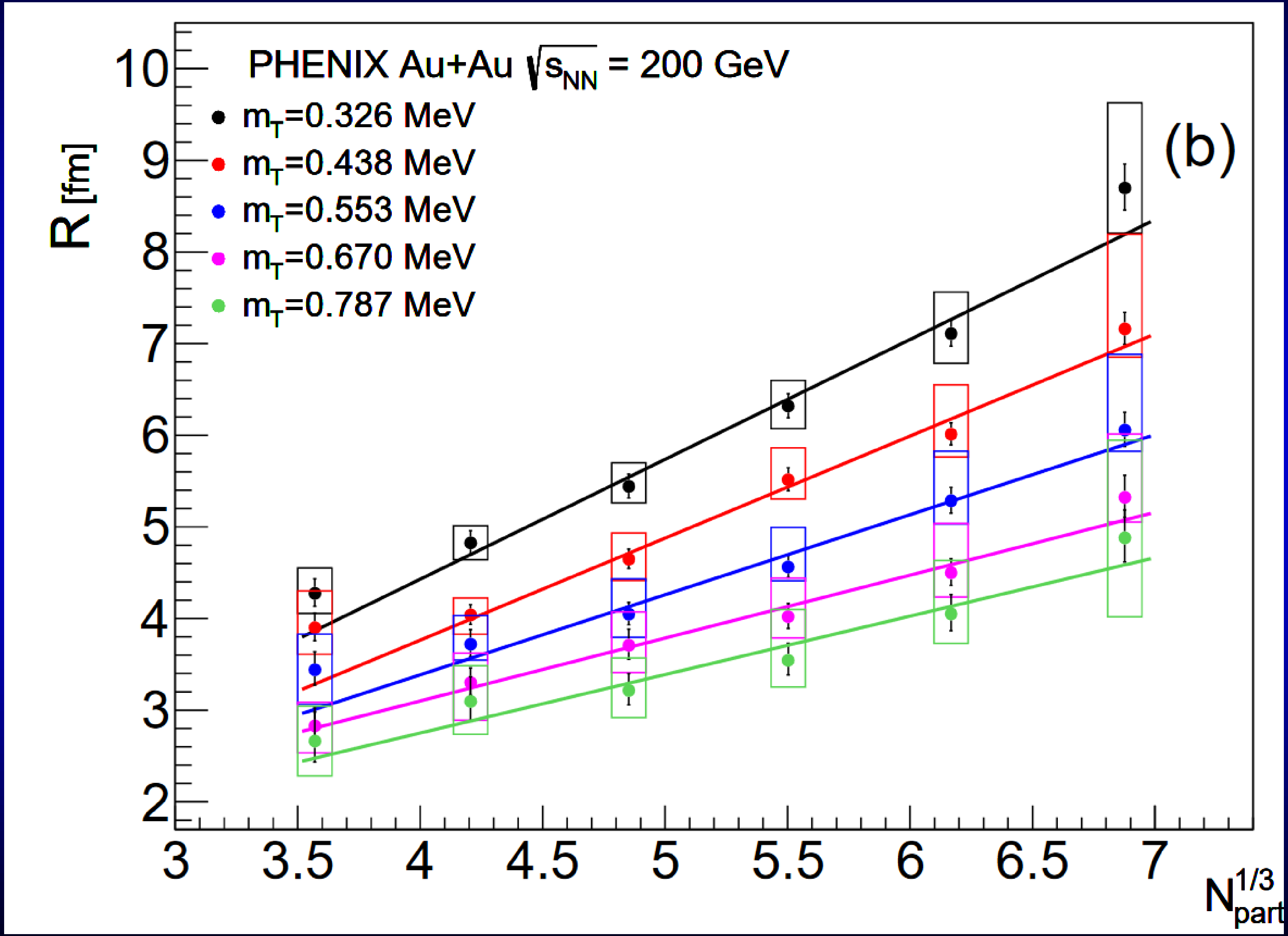
CENTRALITY DEPENDENCE OF A AND B



$$\frac{1}{R^2} = Am_T + B$$

Values of A decrease for more central collisions: R increases with centrality
B are (within errors) nearly vanishing, suggesting large geometrical size and a possible Cooper-Frye effect

N_{part} DEPENDENCE OF LEVY R



Levy scale R in selected m_T bins

Affine linear in $N_{part}^{1/3}$

$$p_0 + p_1 * N_{\mathrm{part}}^{1/3}$$

Volume of the Levy source

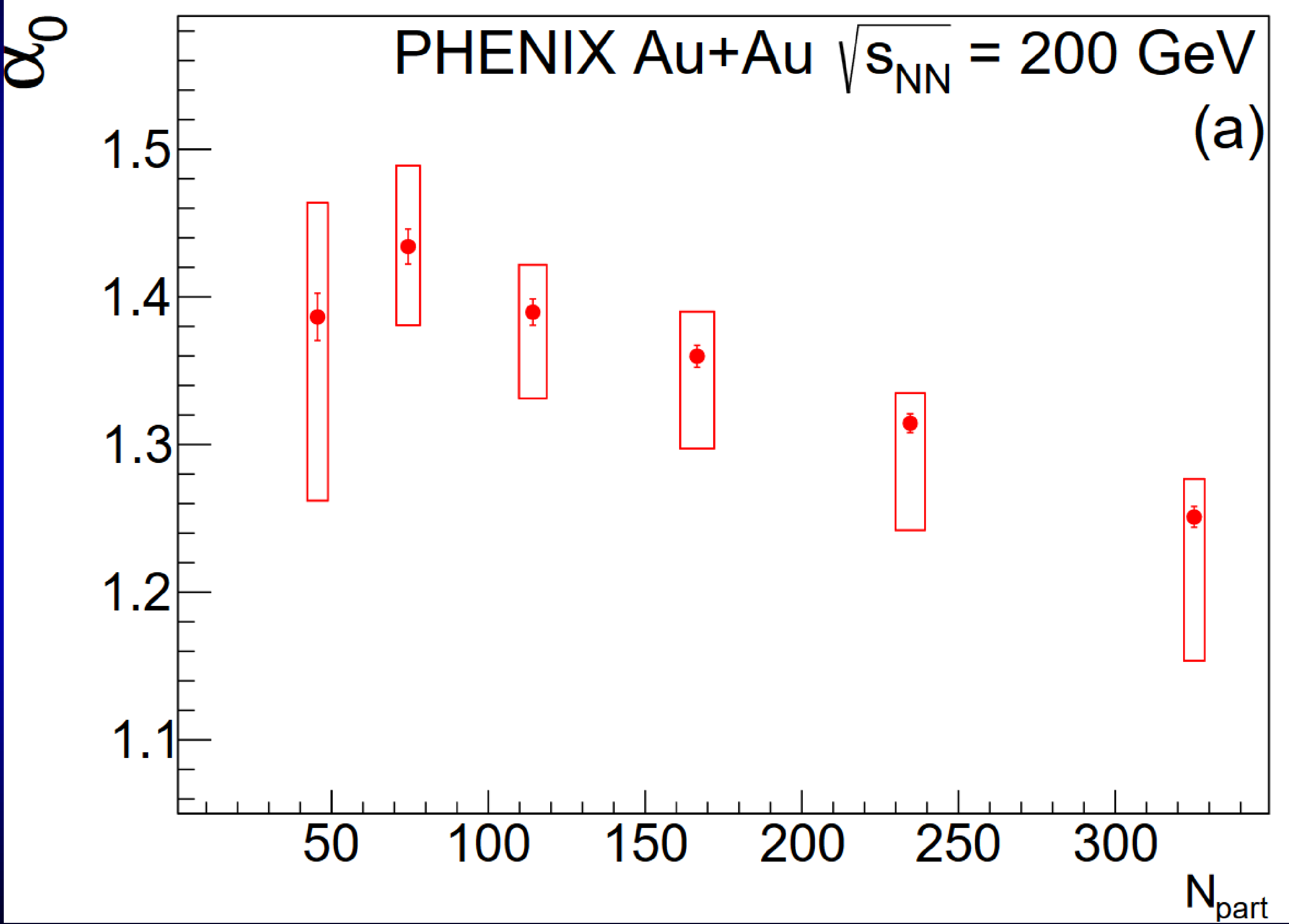
$V \sim R^3 \sim N_{part}$

Similarly, volume of a Gaussian source $V \sim R^3_G \sim N_{part}$

in each centrality class

PHENIX,
Phys.Rev.Lett. 93 (2004), 152302

N_{part} DEPENDENCE OF LEVY α_0



Levy fits

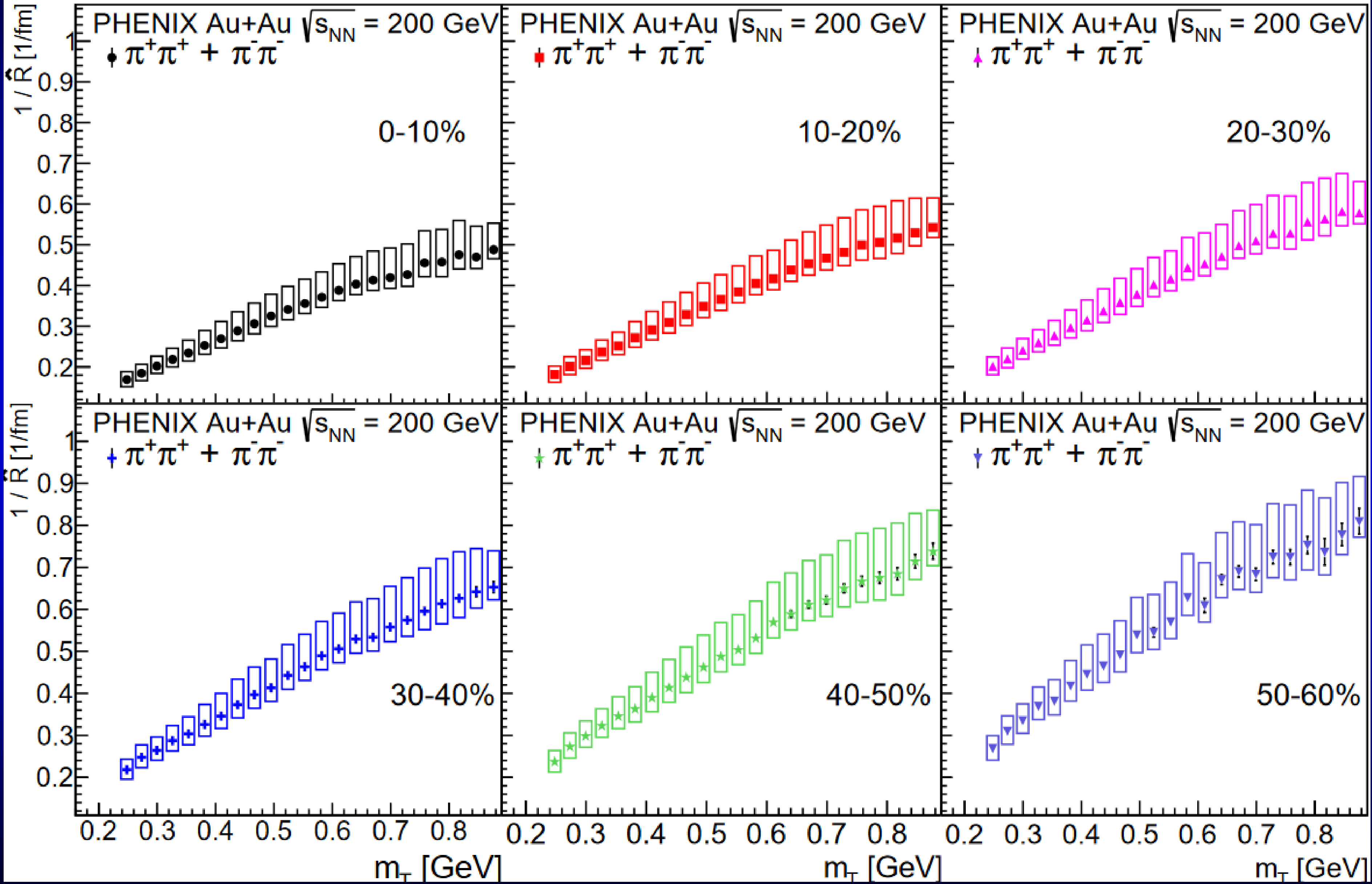
$1 < \alpha_0 < 2$

Far from 1: not exponential
Far from 2: not Gaussian
in any centrality class

α_0 decreases with increasing N_{part}

Comparison with kaons and protons needed

M_T AND CENTRALITY DEPENDENCE OF $\hat{R}$



An unexpected scaling law was found by PHENIX in Phys. Rev. C 97 (2018) 064911, $\sqrt{s_{NN}}$ = 200 GeV Au+Au, in 0-30 % centrality class:

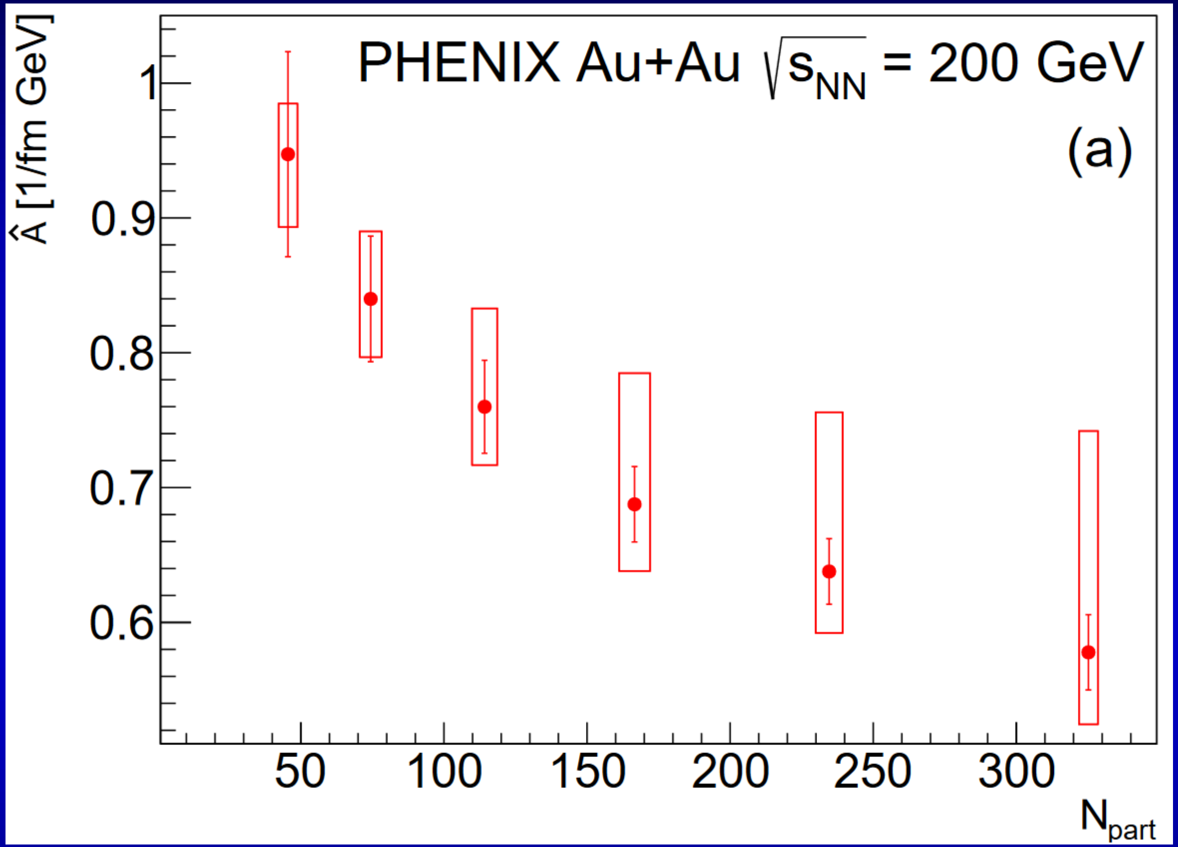
$$\frac{1}{\hat{R}} = \frac{\lambda(1+\alpha)}{R},$$

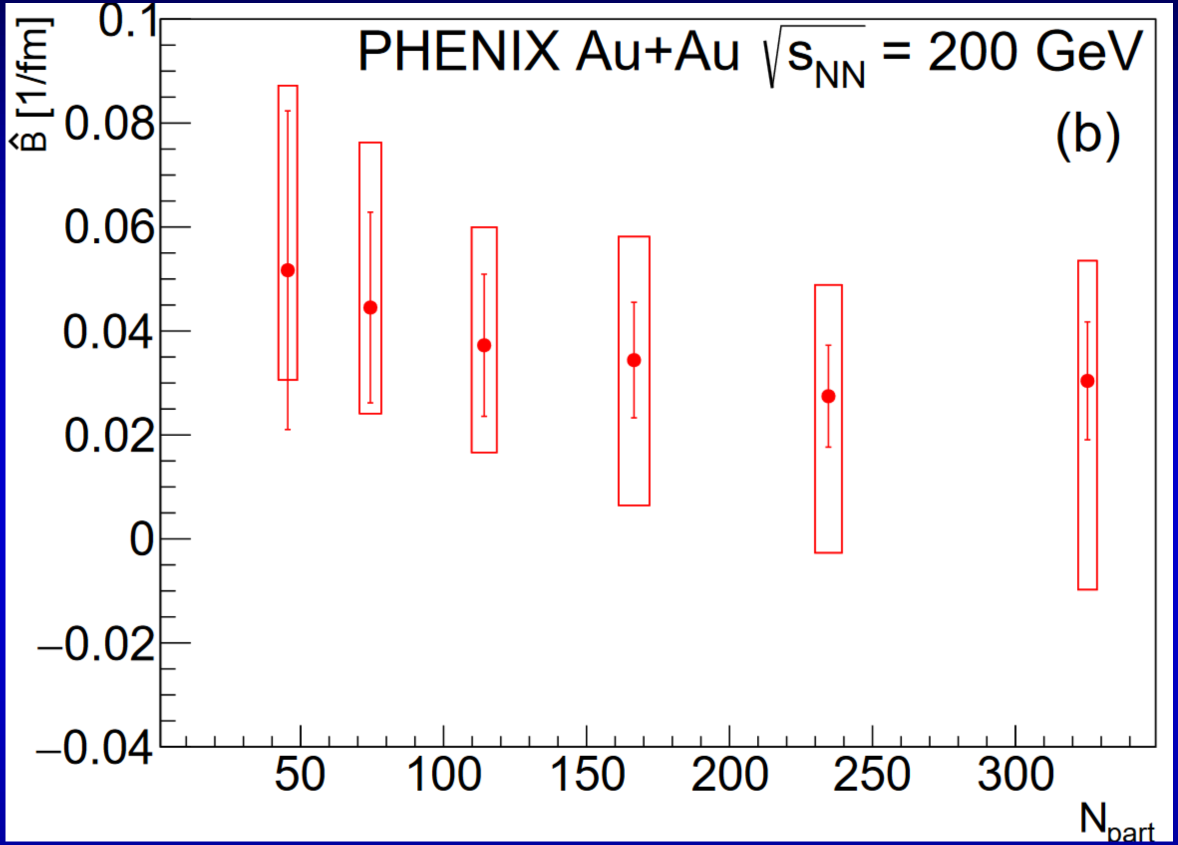
$$\frac{1}{\hat{R}} = \hat{A} m_T + \hat{B}.$$

NOW IT IS SEEN IN EACH CENTRALITY CLASS - CHALLENGE FOR THEORY

Part of systematics cancel, less correlated as λ, R and α

N_{part} DEPENDENCE OF $\hat{A}$ AND $\hat{B}$



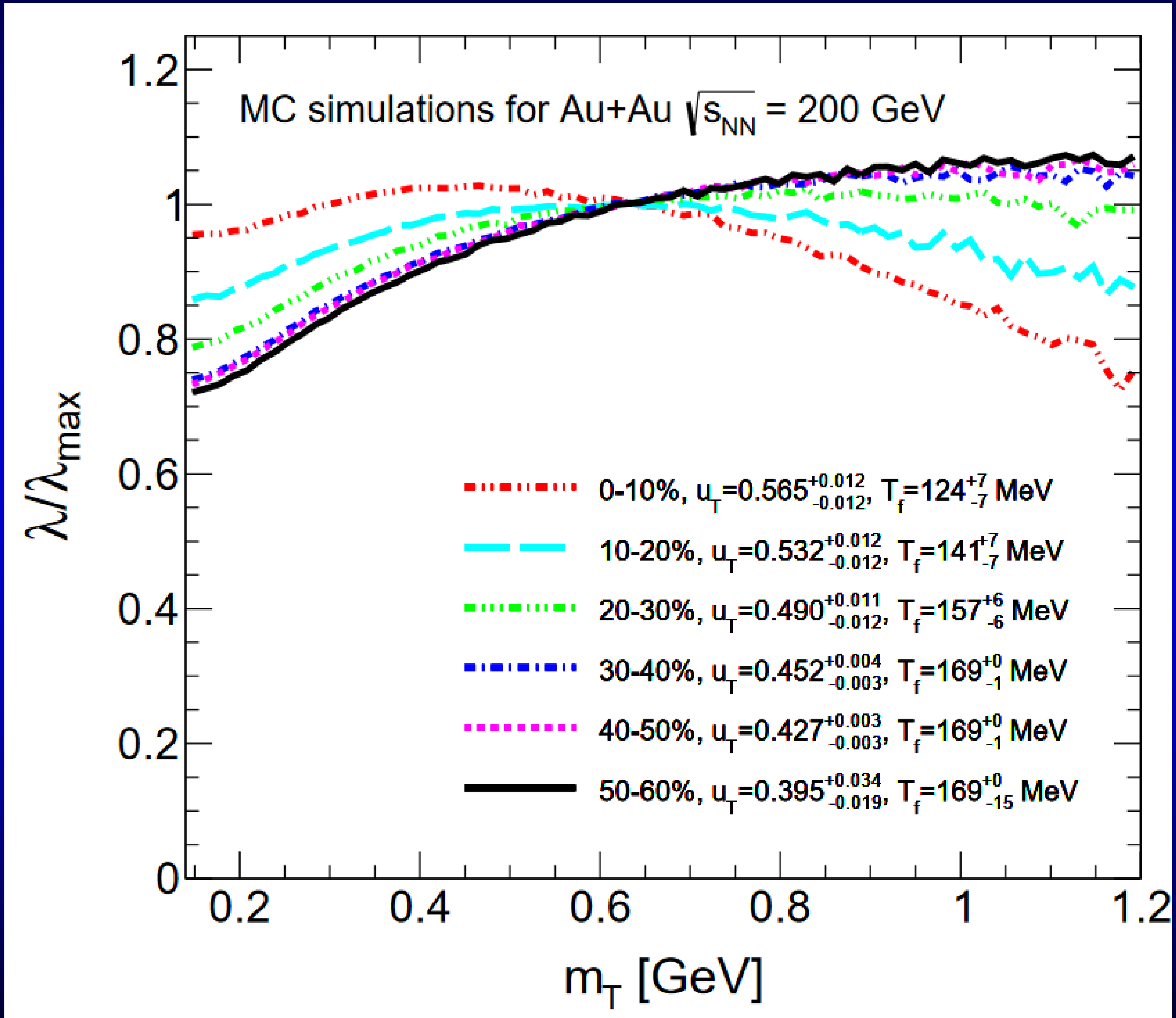


$\hat{A}$ decreases with increasing N_{part}, similarly to A

$\hat{B}$ is independent of N_{part}, similarly to B, but its average is positive.

COMPARISON WITH MONTE-CARLO SIMULATIONS: SEARCH FOR $U_A(1)$ SYMMETRY RESTORATION

MONTE-CARLO SIMULATIONS FOR LÉVY λ / λ_{MAX}



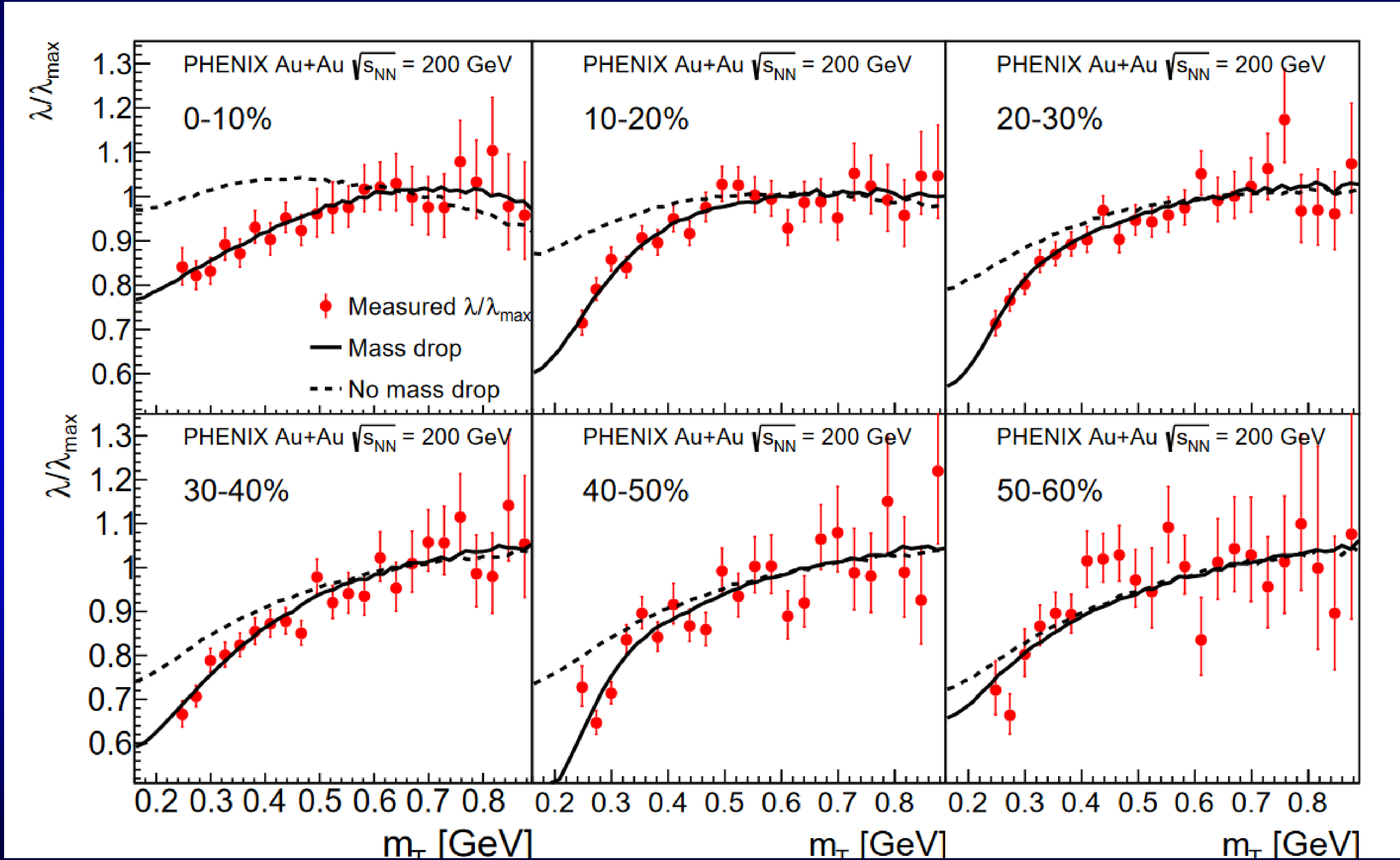
Simulations WITHOUT in-medium η' mass modification

MC results indicate EXPECTED and monotonic centrality dependence

An interplay of radial flow and resonance chain decay effects.

$\eta' \rightarrow$
$\eta + \pi^+ + \pi^- \rightarrow$
$(\pi^+ + \pi^- + \pi^0) + \pi^+ + \pi^-$

MONTE-CARLO SIMULATIONS FOR LÉVY λ / λ_{MAX}

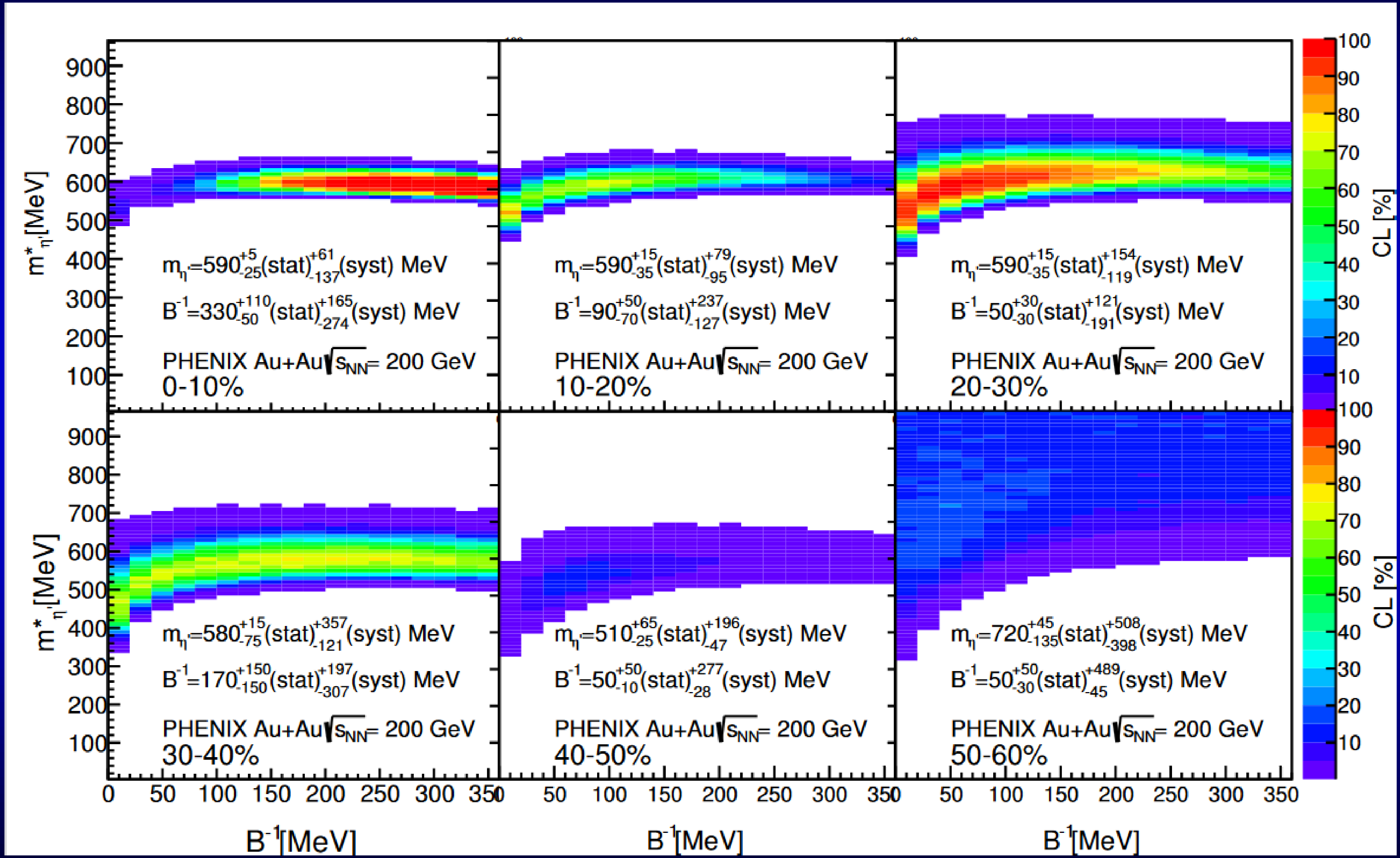


Scale out saturated value of λ at large m_T
Results for λ / λ_{max}

Simulations WITH AND WITHOUT IN-MEDIUM η' MASS MODIFICATION in each centrality CLASS

THERMAL model (SHARE) for resonance production, T_{chem} and μ_B values from STAR. K^+, K^-, p and anti-p spectra fitted. Resonance chain decays included.

χ^2/NDF AND CL (OR P-VALUE) MAPS FOR LÉVY λ/λ_{MAX}



Maps out allowed regions and best values of

IN-MEDIUM η' MASS MODIFICATION in each centrality CLASS

Colored region: allowed with CL > 0.1 %

Selective, except in 50-60 % centrality class

CENTRALITY DEPENDENCE OF IN-MEDIUM MASS OF η′

Centrality	$m^*_{\eta'}$	(stat)	(syst)
0%–10%	590	$^{+5}_{-25}$	$^{+61}_{-137}$
10%–20%	590	$^{+15}_{-35}$	$^{+79}_{-95}$
20%–30%	590	$^{+15}_{-35}$	$^{+154}_{-119}$
30%–40%	580	$^{+15}_{-75}$	$^{+357}_{-121}$
40%–50%	510	$^{+65}_{-25}$	$^{+196}_{-47}$
50%–60%	720	$^{+45}_{-135}$	$^{+508}_{-398}$

In-medium mass of η′ determined indirectly from Levy Bose-Einstein correlations.

Similar to the vacuum mass of η (548 MeV) in each centrality class!

Lower, than the vacuum mass of η′ (958 MeV) except the 50-60% centrality class!

- The Kapusta-Kharzeev-McLerran prediction [43] is in agreement with our measurements in each investigated centrality class.
- The lower limit of Kwon, Lee, Morita, and Wolf [58] is also consistent with our measurement in each investigated centrality class.
- Our measured centrality-average value of $m^*_{\eta'}$ is slightly below, but consistent with, the lower limit predicted by Pisarski and Wilczek [42].
- However, the upper limit of Weinberg [55] is several standard deviations below the central values obtained in each investigated centrality class.
- The lower limit predictions of Horvatić, Kekez and Klabučar [56] and of Huang and Wang [57] are excluded except in the 50%–60% centrality class.
- Our results also suggest that the prediction of Ref. [64] slightly underestimates the in-medium mass change of the η'.

CENTRALITY DEPENDENCE OF IN-MEDIUM MASS OF η′

R. D. Pisarski and F. Wilczek, Remarks on the Chiral Phase Transition in Chromodynamics, Phys. Rev. D **29**, 338 (1984).
J. I. Kapusta, D. Kharzeev, and L. D. McLerran, The Return of the prodigal Goldstone boson, Phys. Rev. D **53**, 5028 (1996), arXiv:hep-ph/9507343.

Y. Kwon, S. H. Lee, K. Morita, and G. Wolf, Renewed look at η' in medium, Phys. Rev. D **86**, 034014 (2012), arXiv:1203.6740 [nucl-th].

Z. Huang and X.-N. Wang, Partial U(1)A restoration and eta enhancement in high-energy heavy ion collisions, Phys. Rev. D **53**, 5034 (1996), arXiv:hep-ph/9507395.

S. Weinberg, The U(1) Problem, Phys. Rev. D **11**, 3583 (1975).
D. Horvatić, D. Kekez, and D. Klabučar, η' and η mesons at high T when the $U_A(1)$ and chiral symmetry breaking are tied, Phys. Rev. D **99**, 014007 (2019), arXiv:1809.00379 [hep-ph].

G. Kovács, P. Kovács, and Z. Szép, One-loop constituent quark contributions to the vector and axial-vector meson curvature mass, Phys. Rev. D **104**, 056013 (2021), arXiv:2105.12689 [hep-ph].

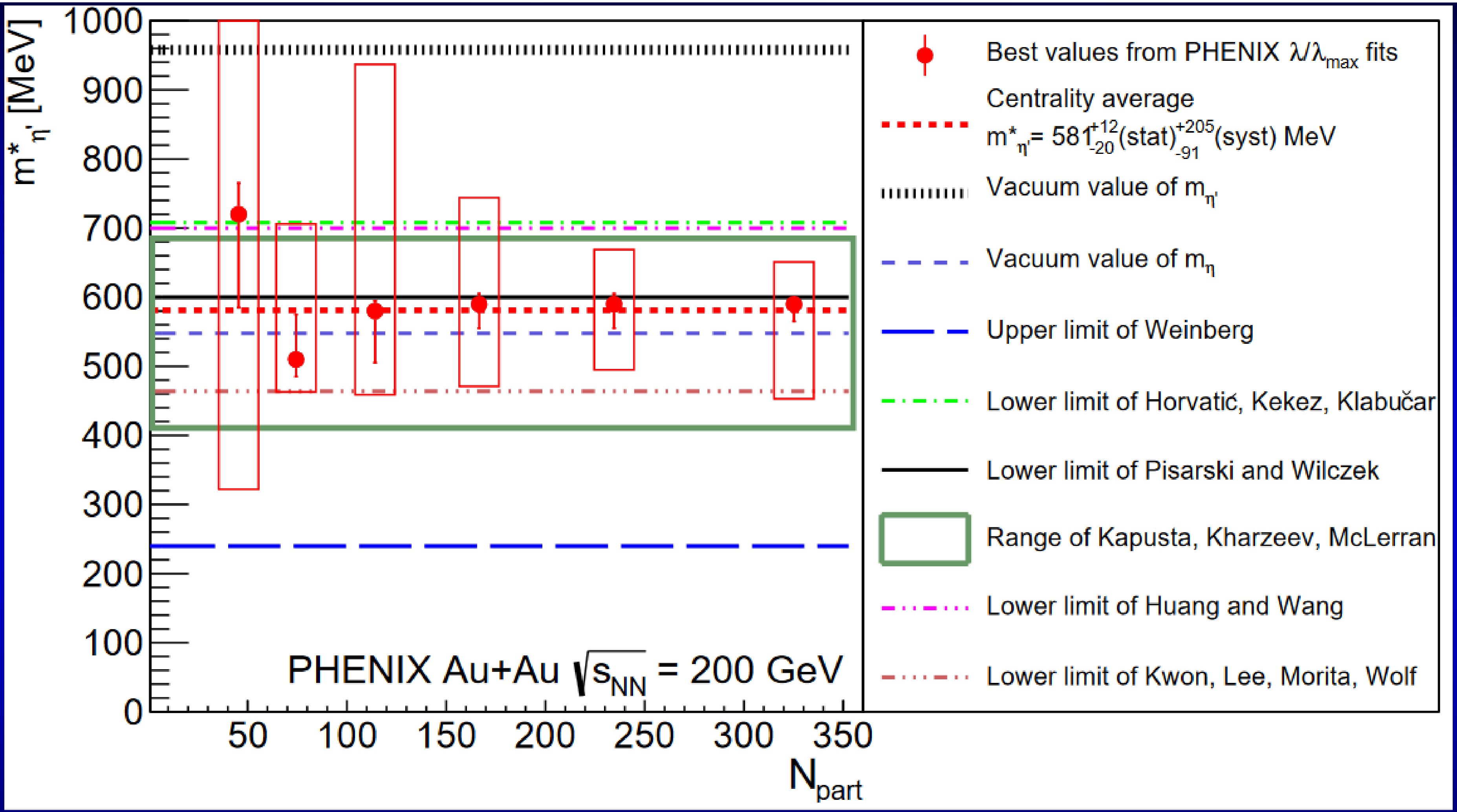
In-medium mass of η′ determined indirectly from Levy Bose-Einstein correlations.

Similar to the vacuum mass of η (548 MeV) in each centrality class!

Lower, than the vacuum mass of η′ (958 MeV) except the 50-60% centrality class!

- The Kapusta-Kharzeev-McLerran prediction [43] is in agreement with our measurements in each investigated centrality class.
- The lower limit of Kwon, Lee, Morita, and Wolf [58] is also consistent with our measurement in each investigated centrality class.
- Our measured centrality-average value of $m^*_{\eta'}$ is slightly below, but consistent with, the lower limit predicted by Pisarski and Wilczek [42].
- However, the upper limit of Weinberg [55] is several standard deviations below the central values obtained in each investigated centrality class.
- The lower limit predictions of Horvatić, Kekez and Klabučar [56] and of Huang and Wang [57] are excluded except in the 50%–60% centrality class.
- Our results also suggest that the prediction of Ref. [64] slightly underestimates the in-medium mass change of the η'.

N_{part} DEPENDENCE OF IN-MEDIUM MASS OF η′



In-medium mass of η′ is determined with the help of Levy Bose-Einstein correlation measurements and Monte-Carlo simulations to be similar to the vacuum mass of η in each centrality class: indirectly, return of the prodigal Goldstone boson η′
Centrality dependent selection power, successful: KHM, KLMW, PW: m*(η′) ~ m(η)

SUMMARY AND CONCLUSIONS

Centrality dependent Levy stable Bose-Einstein correlations in $\sqrt{s_{NN}}$ = 200 GeV Au+Au collisions by PHENIX

$1 < \alpha < 2$ singificantly, decreasing with increasing N_{part}

Unexpected scaling laws found

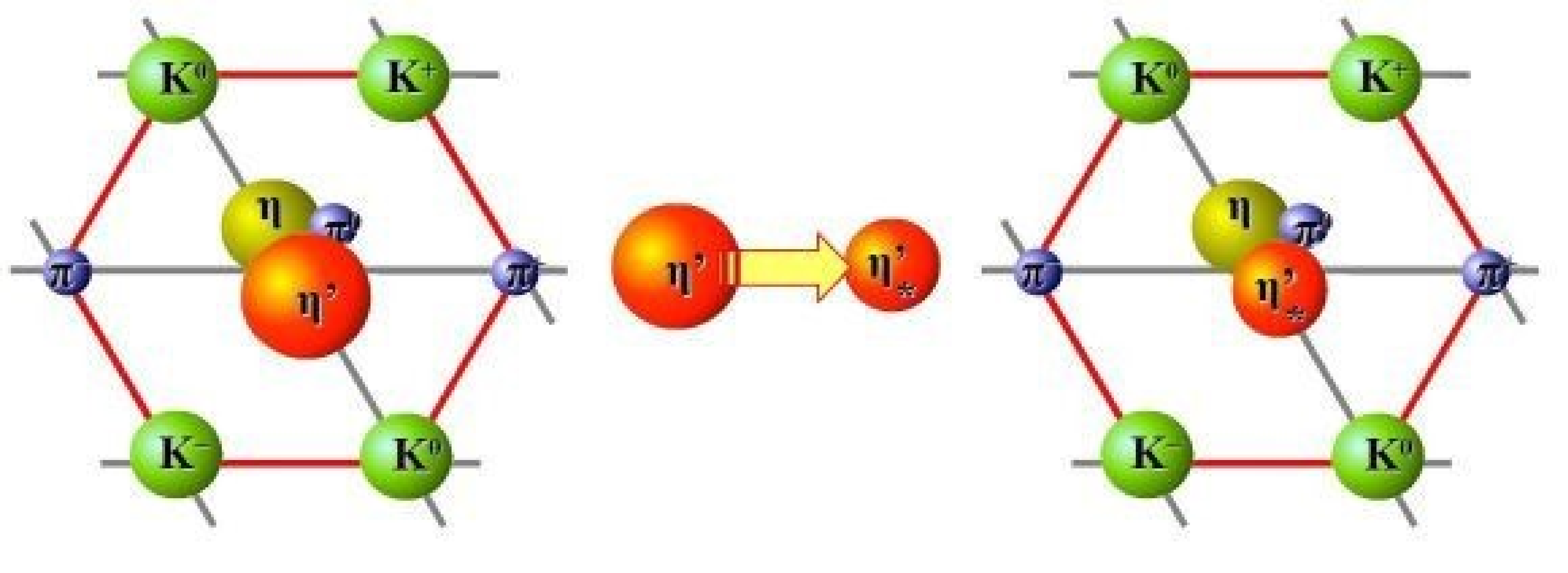
Data not inconsistent with $U_A(1)$ symmetry restoration: In-medium mass modification of η' with indirect method

Direct observation e.g. $\eta' \rightarrow \gamma + \gamma$ is particularly challenging but also particularly rewarding:

Challenge for sPHENIX?

Thank you for your attention!

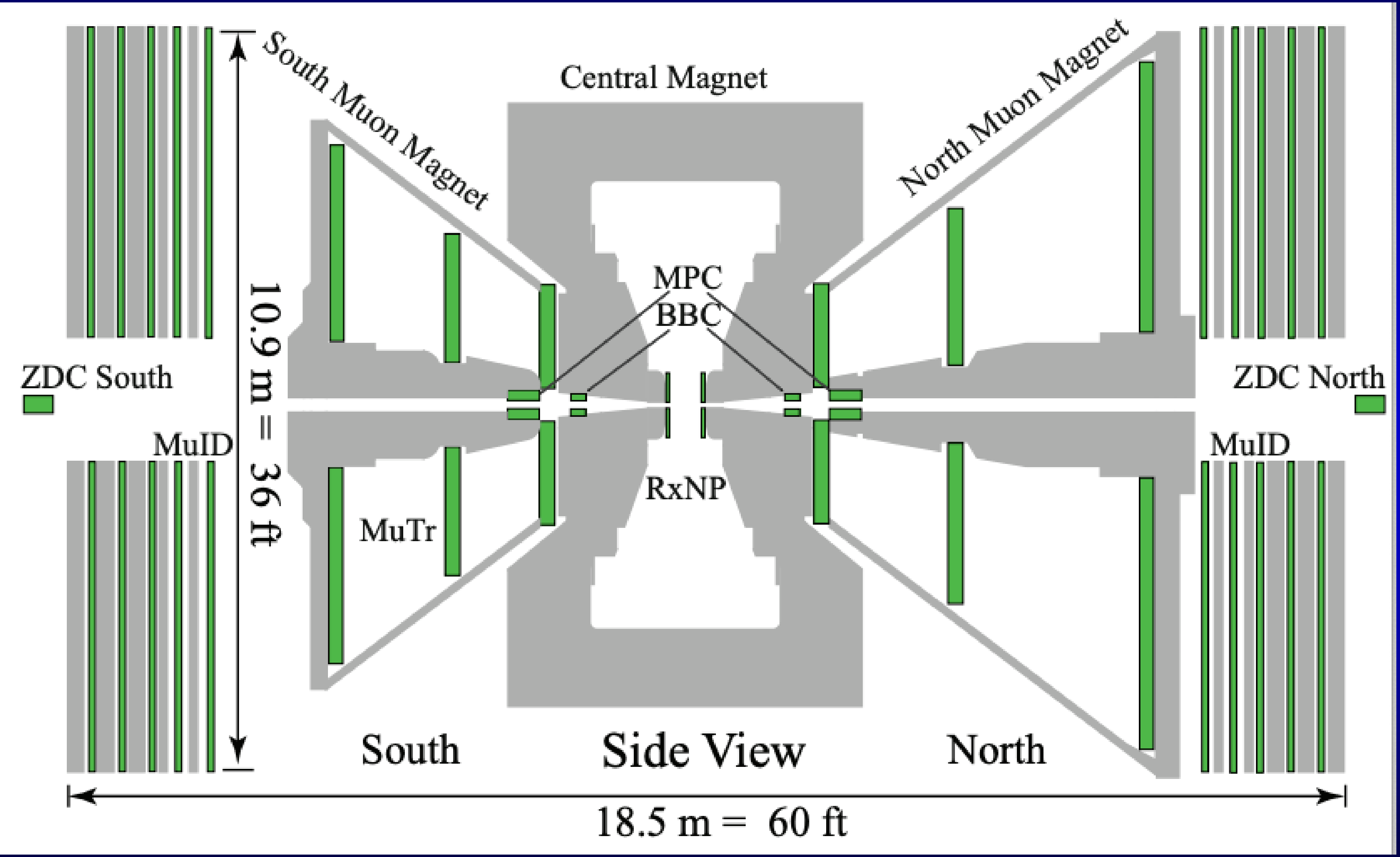
Questions?



Partially supported by NKFIH and MATE KKP FRG, Hungary
and by the PHENIX funding agencies and organizations listed at https://www.bnl.gov/rhic/phenix.php

BACKUP SLIDES

THE PHENIX DETECTOR @ RHIC – SIDE VIEW



Muon Magnets followed by Muon Arm Detectors

MuTr: Muon Tracker
MuID: Muon Identification

For centrality in Au+Au:
BBC: Beam to Beam Counter
ZDC: Zero Degree Calorimeter

Hanbury Brown: a *family* name

1971MNRAS.151..161H

Mon. Not. R. astr. Soc. (1971) **151**, 161–176.

A STUDY OF α VIRGINIS WITH AN INTENSITY INTERFEROMETER

D. Herbison-Evans, R. Hanbury Brown, J. Davis and L. R. Allen

(Received 28 August 1970)

Grandfather: Sir Robert Hanbury Brown, K.C.M.G., a notable irrigation engineer (Wiki link)

Father: Basil Hanbury Brown

Twin sons:
- **Robert Hanbury Brown**
- **Jordan Hanbury Brown**

Daughter:
- **Marion Hanbury Brown**

„It is not all that unusual that an English last name is a compound one, with or without a hyphen.”
Wes Metzger

Thank you Wes!
For private communications on the family tree of Sir Robert *Hanbury Brown*

R. Q. Twiss: Richard *Quentin* TWISS

OBITUARIES

Richard Quentin Twiss 1920–2005

Fellow and Eddington Medallist of the RAS, pioneer of radio astronomy and interferometry.

Richard Twiss was born in Simla in India in 1920. He was educated at Rugby School and completed the Mathematical Tripos at Cambridge with distinction in 1941. He spent the war years in the Admiralty working on radar, and after the war was appointed British Liaison Officer to the Research Laboratory for Electronics (RLE) at MIT in the USA, where he assisted in editing the 27-volume RLE

and the non-classical Bose–Einstein statistics of the photons must be taken into account. The debate surrounding the HBT effect led to a much deeper understanding of the nature of light and marks the beginning of modern quantum optics. In 1968 Hanbury Brown and Twiss were jointly awarded the Eddington Medal of the RAS for their work.

In 1955 Richard moved to Sydney, Australia, where he took up a research position in the CSIRO Division of Radiophysics. As well as doing more work on the HBT effect, his work on electromagnetic-wave propagation laid the theoretical foundation for both astrophysical masers

ferometer in 1972. Although the Mark II did not produce significant astronomical results, it was a major step in the development of modern optical interferometry. The Monteporzio station was closed in 1976 and Richard effectively retired from active scientific research to pursue his interests in art and music.

In 1998 Richard came to Sydney for the summer opera season and visited the Sydney University Stellar Interferometer, the modern Michelson successor to the Narrabri Stellar Intensity Interferometer, also at Narrabri. Of course, much had changed since the 1950s. He visited regularly thereafter, and shortly before his death in Sydney he applied for Australian permanent residence.
Bill Tango

Reference:

Bill Tango: Richard Quentin TWISS (1920-2005), A&G vol 47, p. 4.38 (2006)

Apologies:

For my earlier mistaken communications on resolving „Q." in Richard *Quentin* TWISS

HBT: „Has to be a Gaussian", IF …

Model-independent but Gaussian IF we assume:

- 1 + positive definite forms
- Plane wave approximation
- Two-particle symmetrization (only)
- IF f(q) is analytic at q = 0 and
- IF means and variances are finite
- Follows an approximate Gaussian(α = 2)

$$C_2(k_1,k_2) = 1 + |\tilde{f}(q_{12})|^2,$$

$$\tilde{f}(q_{12}) = \int dx \, \exp(iq_{12}x) \, f(x),$$

$$q_{12} = k_1 - k_2.$$

$$\tilde{f}(q) \approx 1 + iq\langle x\rangle - q^2\langle x^2\rangle/2 + \ldots,$$

$$C(q) = 1+|\tilde{f}(q)|^2 \approx 2-q^2(\langle x^2\rangle - \langle x\rangle^2) \approx 1+\exp(-q^2R^2),$$

$$R = \sqrt{\langle x^2\rangle - \langle x\rangle^2}.$$

Model-independent but non-Gaussian IF we assume:

- 1 + positive definite form (same as above)
- Plane wave approximation (same)
- Two-particle symmetrization only (same)
- IF f(q) is NOT analytic at q = 0 and
- IF means and variances are NOT finite
- IF Generalized Central Limit theorems are valid
- Follows a Levy shape(0 < α ≤ 2)
- Earlier Gaussian recovered for α = 2

$$f(x) = \int \Pi_{i=1}^{n} dx_i \, \Pi_{j=1}^{n} f_j(x_j) \, \delta(x - \sum_{k=1}^{n} x_k).$$

$$\tilde{f}(q) = \prod_{i=1}^{n} \tilde{f}_i(q)$$

$$C(q;\alpha) = 1 + \lambda \exp\left(-|qR|^\alpha\right).$$

Cs. T, S. Hegyi, W. A. Zajc, nucl-th/0310042

Edgeworth expansion method

Gaussian w(t), -∞ < t < ∞

$$t = \sqrt{2} Q R_E,$$
$$w(t) = \exp(-t^2/2),$$
$$\int_{-\infty}^{\infty} dt\, \exp(-t^2/2) H_n(t) H_m(t) \propto \delta_{n,m},$$

$$H_n(t) = \exp(t^2/2) \left(-\frac{d}{dt}\right)^n \exp(-t^2/2).$$

$$H_1(t) = t,$$
$$H_2(t) = t^2 - 1,$$
$$H_3(t) = t^3 - 3t,$$
$$H_4(t) = t^4 - 6t^2 + 3, \; \ldots$$

$$C_2(Q) = \mathcal{N} \left\{ 1 + \lambda_E \exp(-Q^2 R_E^2) \times \left[1 + \frac{\kappa_3}{3!} H_3(\sqrt{2} Q R_E) + \frac{\kappa_4}{4!} H_4(\sqrt{2} Q R_E) + \ldots \right] \right\}.$$

3d generalization straightforward

- **Applied by NA22, L3, STAR, PHENIX, ALICE, CMS**

Laguerre expansion method

Model-independent but experimentally tested:

w(t): Exponential

0 < t < ∞

Laguerre polynomials

$$t = QR_L,$$
$$w(t) = \exp(-t)$$
$$\int_0^\infty dt\,\exp(-t)L_n(t)L_m(t) \propto \delta_{n,m},$$

$$L_n(t) = \exp(t)\frac{d^n}{dt^n}(-t)^n\exp(-t).$$

$$L_0(t) = 1,$$
$$L_1(t) = t - 1,$$

$$C_2(Q) = \mathcal{N}\left\{1 + \lambda_L \exp(-QR_L)\left[1 + c_1 L_1(QR_L) + \frac{c_2}{2!}L_2(QR_L) + \ldots\right]\right\}$$

First successful tests

on NA22, UA1 data , convergence criteria satisfied

$$\int_0^\infty dt\, R_2^2(t)\exp(+t) < \infty,$$

Intercept: $\lambda_* \sim 1$

$$\lambda_* = \lambda_L[1 - c_1 + c_2 - \ldots],$$
$$\delta^2\lambda_* = \delta^2\lambda_L\left[1 + c_1^2 + c_2^2 + \ldots\right] + \lambda_L^2\left[\delta^2 c_1 + \delta^2 c_2 + \ldots\right]$$

Gauss expansion method

Gaussian w(t), $0 \le t < \infty$

$$L_0(t\,|\,\alpha=2) = \frac{\sqrt{\pi}}{2},$$
$$L_1(t\,|\,\alpha=2) = \frac{1}{2}\left\{\sqrt{\pi}t - 1\right\},$$
$$L_2(t\,|\,\alpha=2) = \frac{1}{32}\left\{(\pi-2)t^2 - \sqrt{\pi}t + 2 - \frac{\pi}{2}\right\}.$$

Provides a new expansion around a Gaussian shape that is defined for the non-negative values of *t* only.

Edgeworth expansion is different from this: Edgeworth is around two-sided Gaussian, includes negative values of *t* also.

arXiv:1604.05513 [physics.data-an]

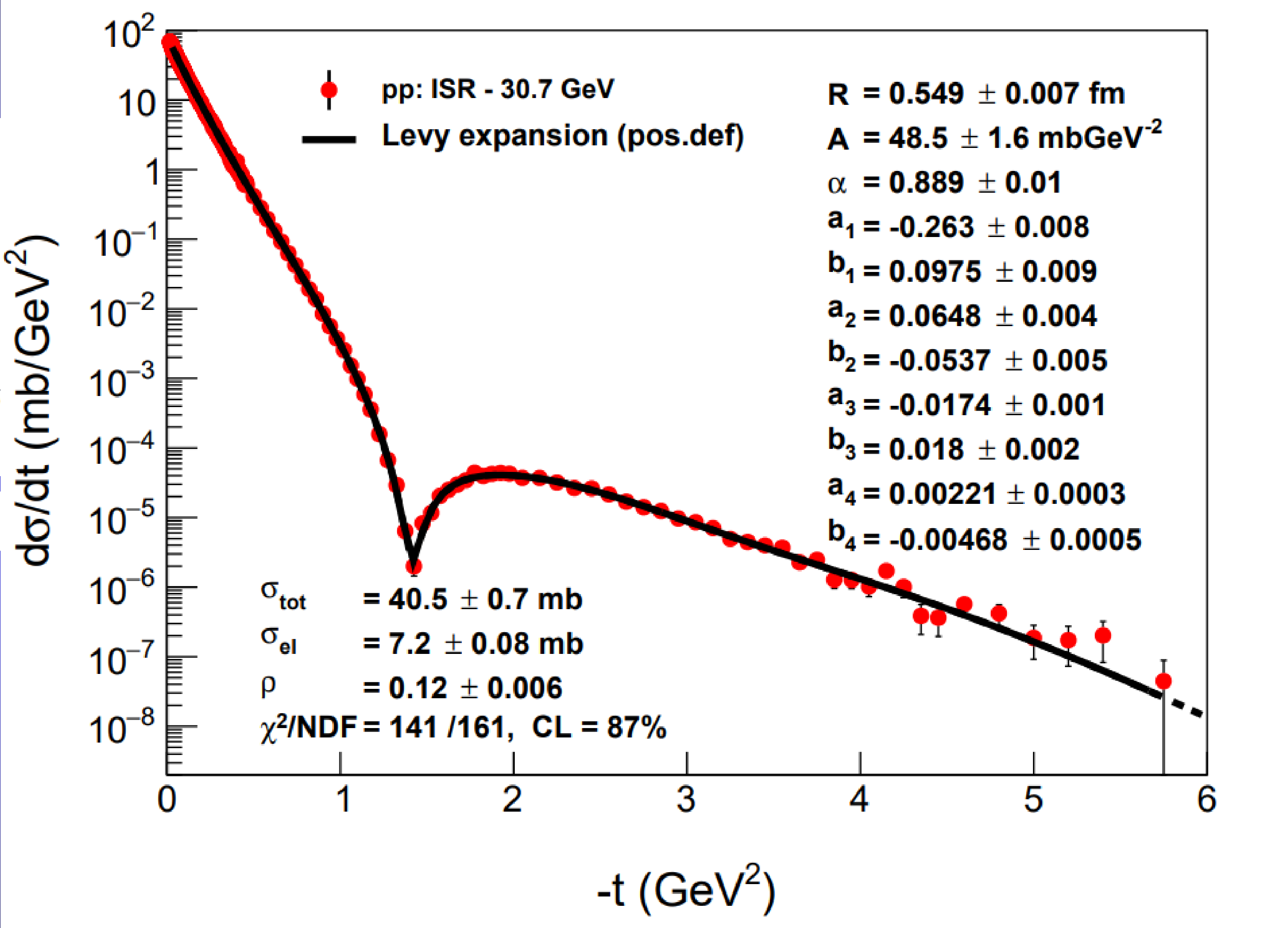
Levy expansions for 1+ positive definite forms

$$C_2(k_1, k_2) = 1 + |\tilde{f}(q_{12})|^2,$$

$$t = \left(\sum_{i,j=\text{side,out,long}} R^2_{i,j} q_i q_j \right)^{1/2},$$

$$C_2(t) = N \left\{ 1 + \lambda \exp(-t^\alpha) \left| 1 + \sum_{n=1}^{\infty} (a_n + ib_n \right. \right.$$

$\{c_n = a_n + ib_n\}_{n=1}^{\infty}$ are now complex valued



Model-independent but:

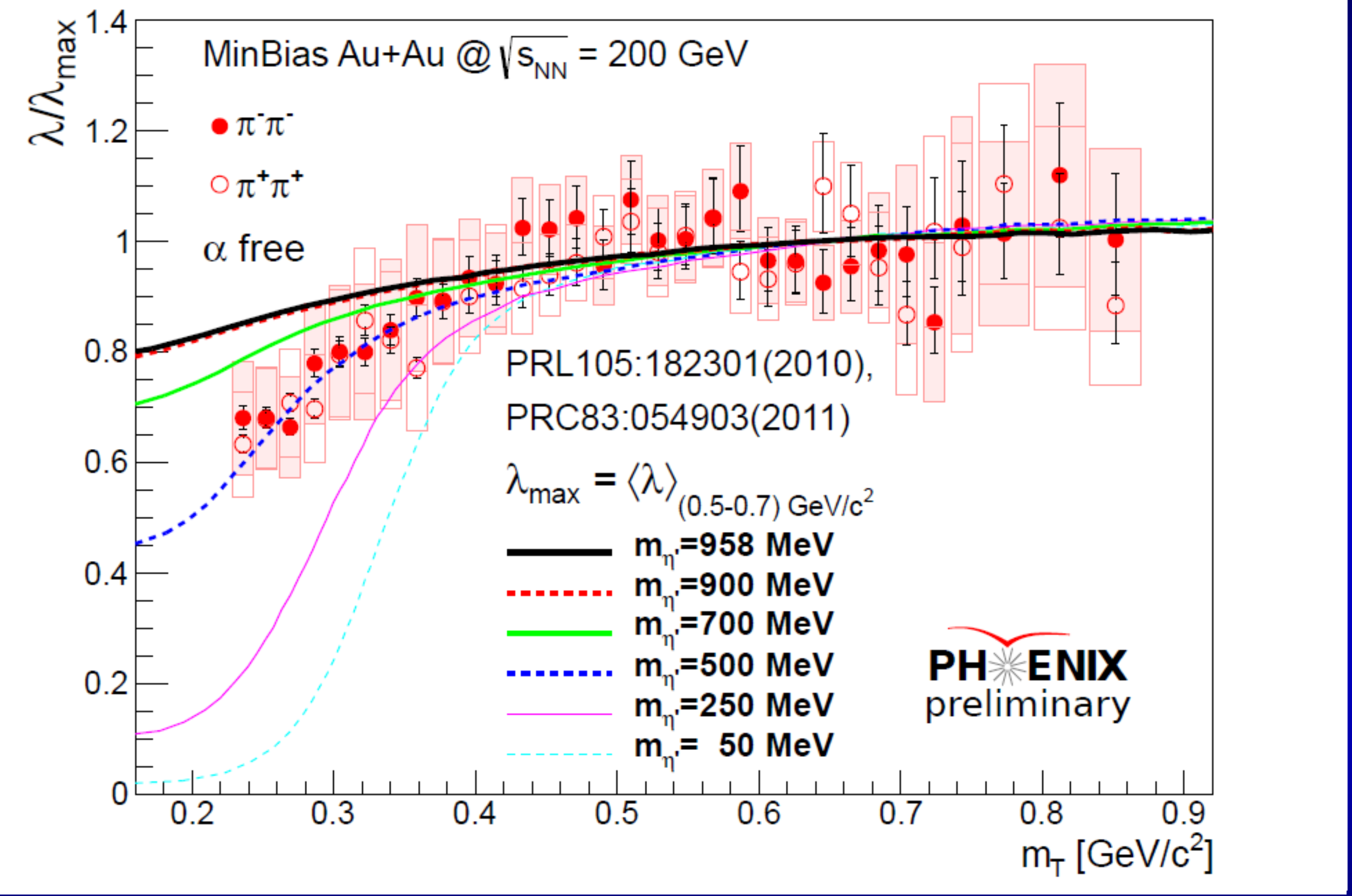
- Generalizes exponential (α =1) and Gaussia
- In this case, for 1+ positive definite forms
- ubiquoutous in nature
- How far from a Levy?
- Works **also for in elastic pp scattering**

In 200 GeV Au+Au, 1st order corrections vanish

T. Novák, T. Cs., H. C. Eggers, M. de Kock: arXiv:1604.05513
T. Cs., R. Pasechnik, A. Ster: arXiv:1807.02897 [hep-ph]

Interpretation of λ

$$C(q;\alpha) = 1 + \lambda \exp\left(-|qR|^{\alpha}\right).$$



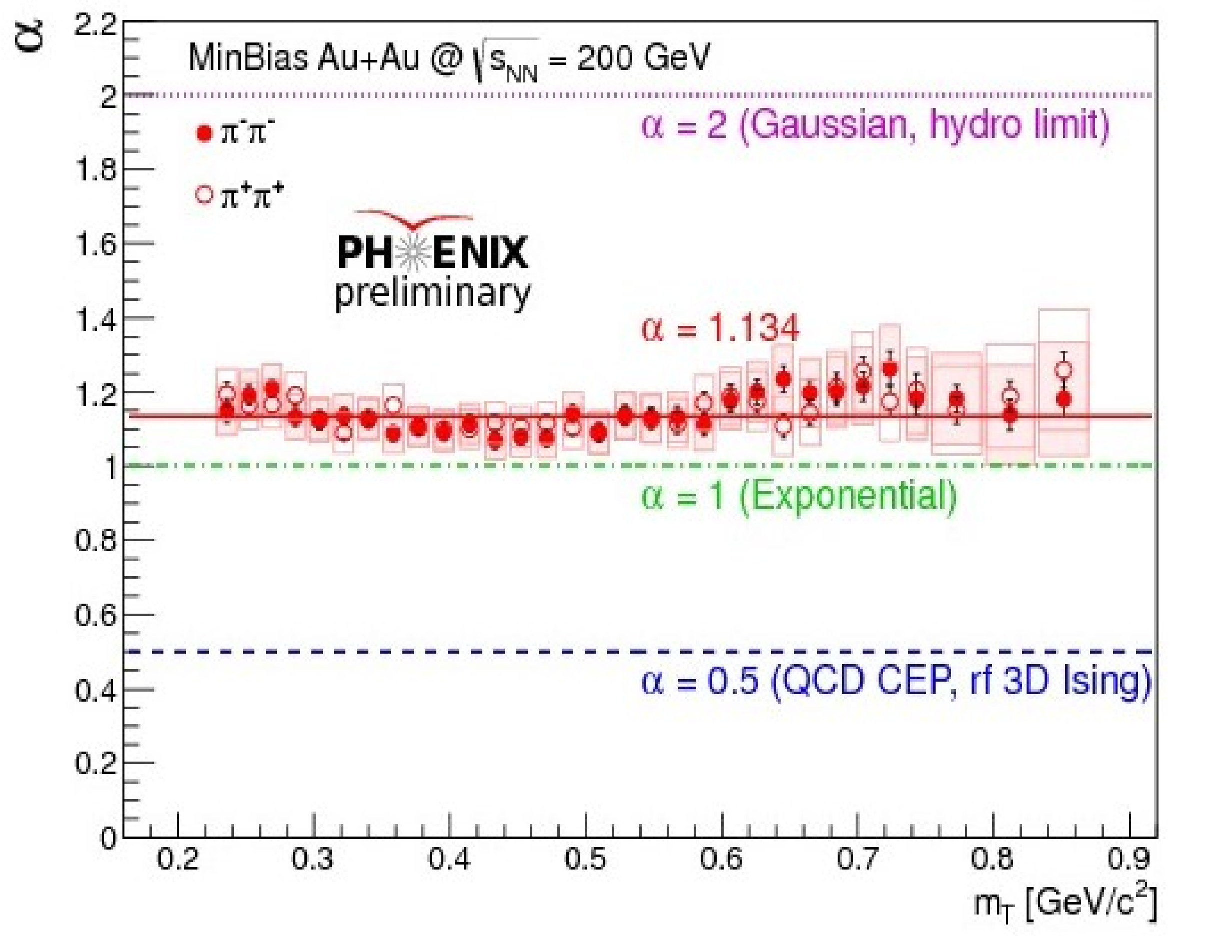
PHENIX preliminary data from arXiv:1610.05025

Method: S. Vance, T. Cs., D. Kharzeev: PRL 81 (1998) 2205-2208 , nucl-th/9802074

Predictions: Cs. T., R. Vértesi, J. Sziklai, arXiv:0912.5526 [nucl-ex] arXiv:0912.0258 [nucl-ex]

Interpretation of α

$$C(q;\alpha) = 1 + \lambda \exp\left(-|qR|^{\alpha}\right).$$

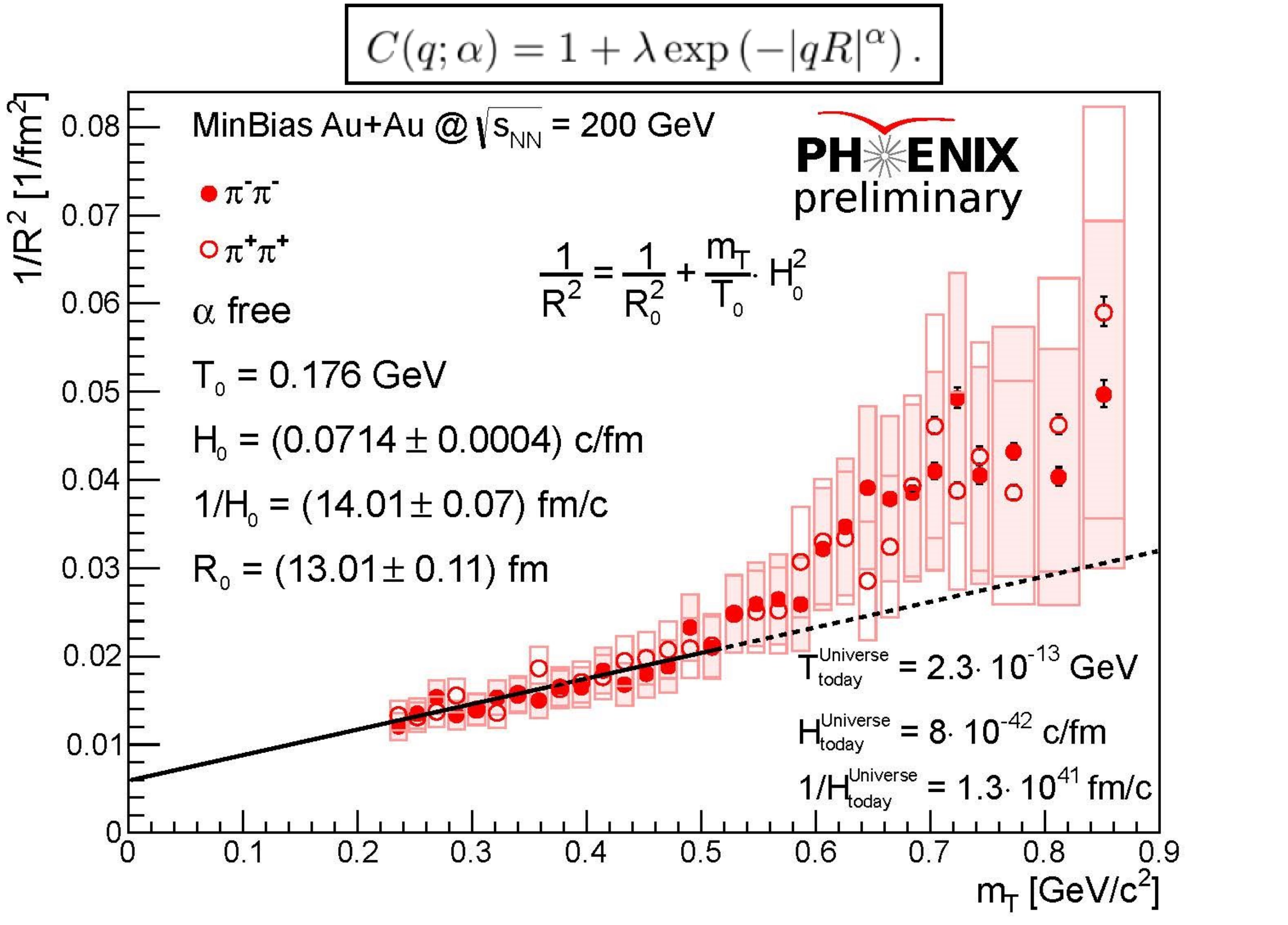


Prediction: at QCD CEP, $\alpha = \eta_c \leq 0.5$ (critical exponent of the correlation function)

T. Cs, S.Hegyi, T. Novák, W.A. Zajc, nucl-th/0512060 T. Cs, arXiv.org:0903.0669

Search for the QCD critical point with α (m_T, $\sqrt{s}$, %, …)

HBT: Interpretation of R



Possibility: hydro scaling behaviour of R at low m_T

Hubble ratio of Big Bang and Little Bangs ~ 10^{40} (α = 2, centrality dependence, …)

M. Csanád, T. Cs, B. Lörstad, A. Ster, nucl-th/0403074

NEEDS generalization for α < 2 !

Variables and Coulomb corrections for Levy $C_2(Q)$

$$Q \equiv |\mathbf{q}_{\mathrm{LCMS}}| = \sqrt{q^2_{\mathrm{out,LCMS}} + q^2_{\mathrm{side,LCMS}} + q^2_{\mathrm{long,LCMS}}}.$$

$$\sum_{i=\mathrm{side,out,long}} R_i^2 q_i^2 \approx R^2 \left(\sum_{i=\mathrm{side,out,long}} q_i^2 \right) = R^2 Q^2,$$

$$Q \equiv |\mathbf{q}_{\mathrm{LCMS}}| = \sqrt{(p_{1,\mathrm{x}} - p_{m2,x})^2 + (p_{1,\mathrm{y}} - p_{2,\mathrm{y}})^2 + \frac{4(p_{1,\mathrm{z}}E_2 - p_{2,\mathrm{z}}E_1)^2}{(E_1+E_2)^2 - (p_{1,z}+p_{2,z})^2}},$$

$$C_2(Q;\lambda,R,\alpha,N,\varepsilon) = 1 - \lambda + \lambda C_2^{(0)}(Q;\lambda,R,\alpha,N,\varepsilon) \times \frac{\sum_j w_j K(q_{\mathrm{inv}})}{\sum_j w_j}$$

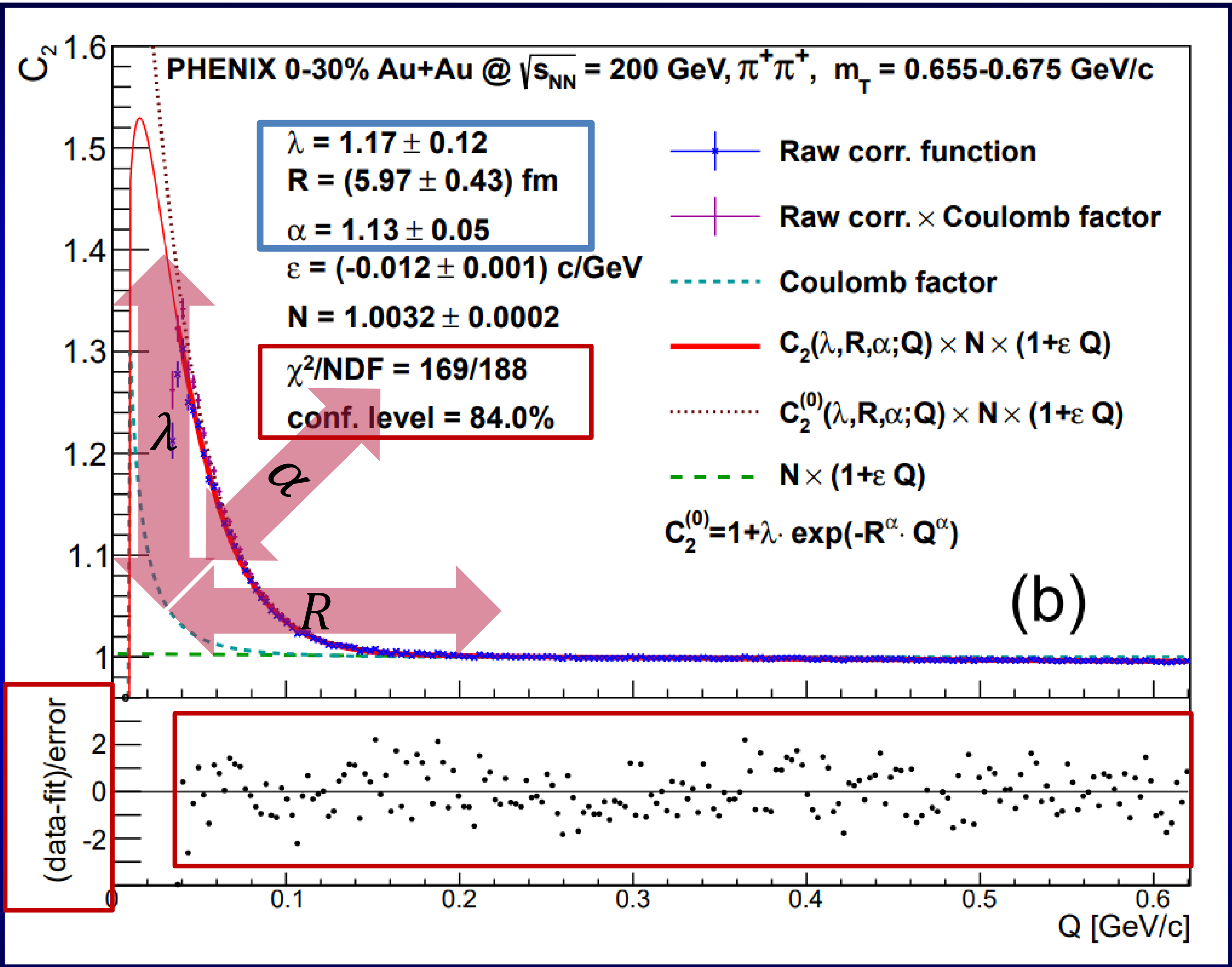
$$C_2^{(0)}(Q;R,\alpha,N,\varepsilon) = (1 + \exp(-R^\alpha Q^\alpha)) \times N \times (1 + \varepsilon Q),$$

From PHENIX, Phys.Rev.C 97 (2018) 6, 064911 and 2407.08586 [nucl-ex]

For recent results on Coulomb corrections for a Levy source, see:
M. Nagy, A. Purzsa et al, Eur.Phys.J.C 83 (2023) 11, 1015, arXiv:2308.10745 [nucl-th]

For a recent review on Levy Bose-Einstein correlations in heavy ions:
M. Csanád and D. Kincses, Universe 10 (2024) 2, 54, arXiv:2401.01249 [hep-ph]

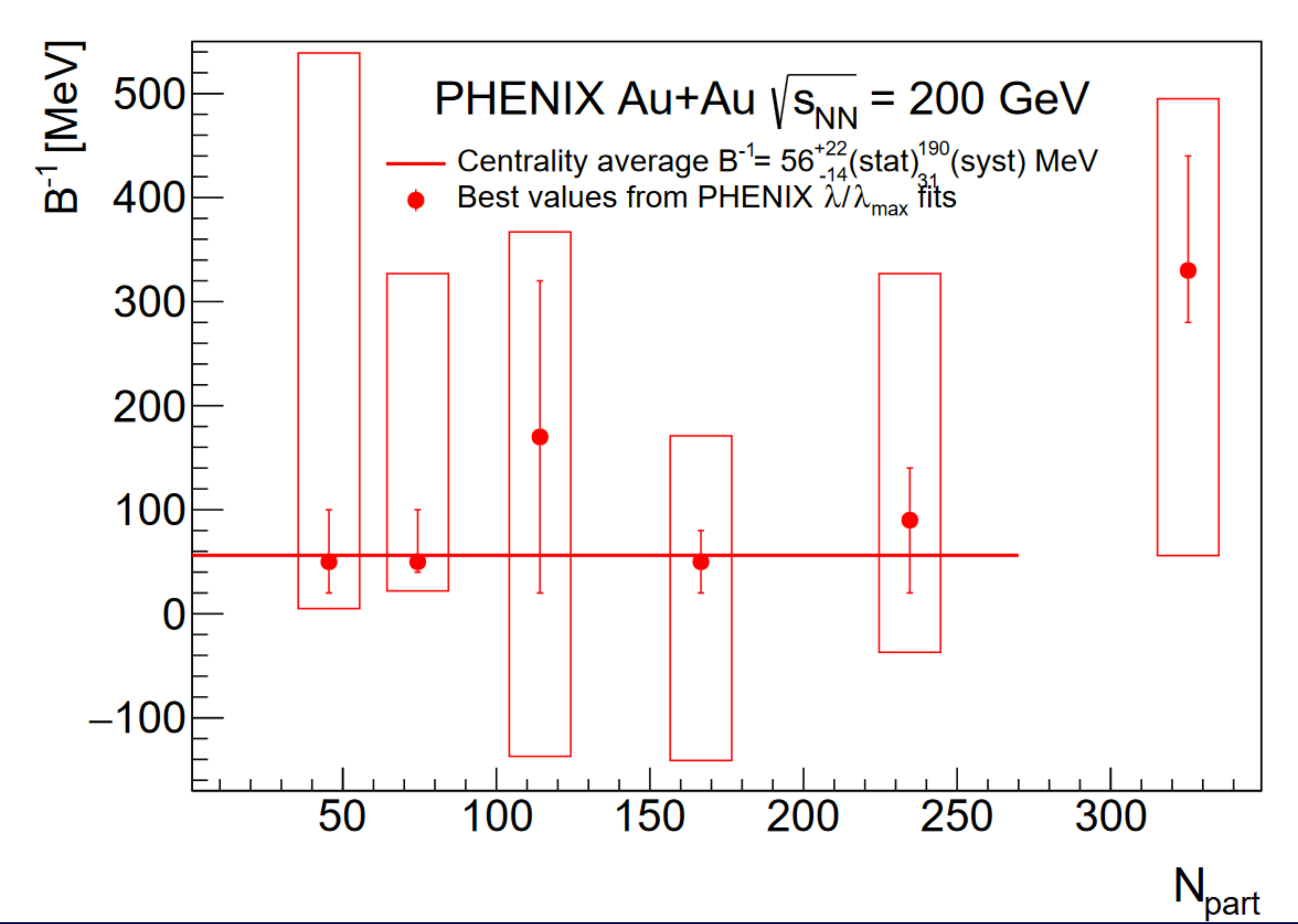
Quality plot for Levy fits of $C_2(Q)$



$\sqrt{s_{NN}}$ = 200 GeV 0-30 % Au+Au collisions, from Phys.Rev.C 97 (2018) 6, 064911

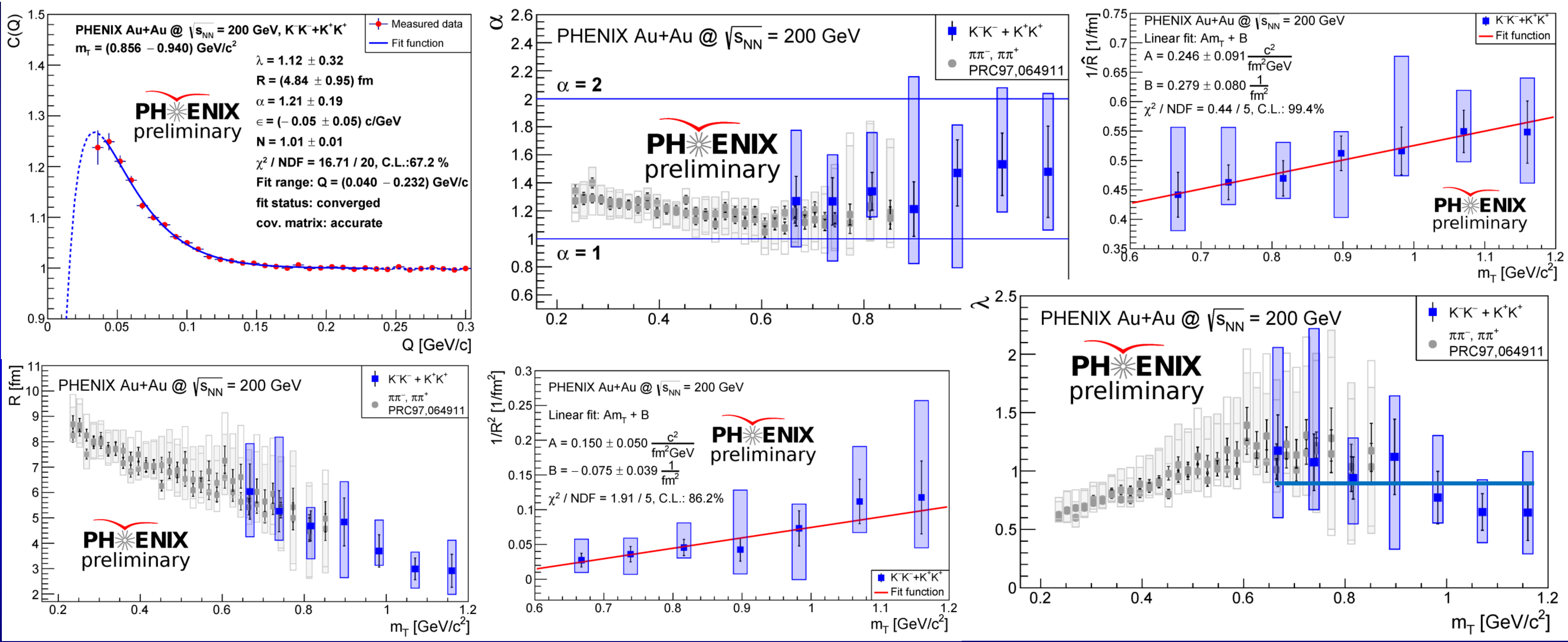
Note the good fit quality, p-value or CL > 0.1 %

CENTRALITY DEPENDENCE OF $B_{\eta'}^{-1}$



Cold source in 10-60 % centrality: very low $B_{\eta'}^{-1}$ or effective temperature of in-medium modified η' mesons, from the shape of suppression $\lambda(m_T)/\lambda_{max}$.

Levy C(Q) for kaons: no $U_A(1)$, but new m_T scalings



PHENIX preliminary, charged KK correlations, $\sqrt{s_{NN}}$ = 200 GeV min bias Au+Au:

λ(KK) ~ λ_{max}(ππ), no λ(KK)/λ_{max} signal for $U_A(1)$, as expected.

α(KK) ~ α(ππ): no anomalous diffusion??

L. Kovács for the PHENIX Collaboration, Universe 2023, 9(7), 336 , arXiv:2307.09573 [nucl-ex]

HBT: Two-particle symmetrization, chaotic source - or not ? Partial coherence: 3 vs 2 particle correlations

$$C_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \frac{N_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)}{N_1(\mathbf{k}_1)N_1(\mathbf{k}_2)N_1(\mathbf{k}_3)},$$

$$\lambda_2 = C_2(k_{12}\to 0) - 1,$$
$$\lambda_3 = C_3(k_{12}=k_{13}=k_{23}\to 0) - 1,$$

$$\kappa_3 = \frac{\lambda_3 - 3\lambda_2}{2\sqrt{\lambda_2^3}}.$$

$$p_c = \frac{N_{\text{coherent}}}{N_{\text{coherent}} + N_{\text{incoherent}}},$$

$$f_c = \frac{N_{\text{core}}}{N_{\text{core}} + N_{\text{halo}}}.$$

$$\lambda_2 = f_c^2\left[(1-p_c)^2 + 2p_c(1-p_c)\right]$$
$$\lambda_3 = 2f_c^3\left[(1-p_c)^3 + 3p_c(1-p_c)^2\right] + 3f_c^2\left[(1-p_c)^2 + 2p_c(1-p_c)\right].$$

arXiv:0604021: R. J. Glauber noted that partial coherence may be present, but swamped in a large background. Check it with three-particle vs two-particle Bose-Einstein correlations!

$$C_3(k_{12},k_{13},k_{23}) = K_3(k_{12},k_{13},k_{23})C_3^{(0)}(k_{12},k_{13},k_{23}).$$

$$C_3^{(0)}(k_{12},k_{13},k_{23}) = 1 + \ell_3 e^{-0.5(|2k_{12}R|^\alpha + |2k_{13}R|^\alpha + |2k_{23}R|^\alpha)} + \ell_2\left(e^{|2k_{12}R|^\alpha} + e^{|2k_{13}R|^\alpha} + e^{|2k_{23}R|^\alpha}\right).$$

$$K_3(k_{12},k_{13},k_{23}) = K_2(k_{12})K_2(k_{13})K_2(k_{23}),\ \text{where}$$

$$K_2(k) = \frac{\int d^4r S(\mathbf{r},\mathbf{k})|\Psi_{\mathbf{k}}^{(Coulomb)}(\mathbf{r})|^2}{\int d^4r S(\mathbf{r},\mathbf{k})|\Psi_{\mathbf{k}}^{(0)}(\mathbf{r})|^2}.$$

Three-body Coulomb correction in Riverside approximation, domain of validity checked.

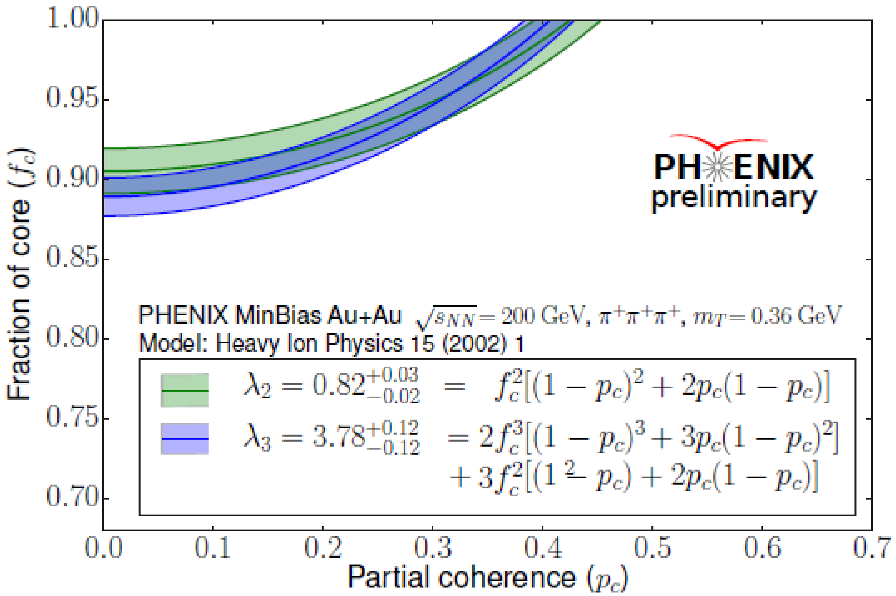
PHENIX preliminary data on three-pion Bose-Einstein: A.Bagoly, poster at QM17 Partial coherence measurement possible!

B. Kurgyis for the PHENIX Collaboration, *Phys.Part.Nucl.* 51 (2020) 3, 263-266, arXiv:1910.05019 [nucl-ex]

HBT: Two-particle symmetrization, chaotic source - or not ? Partial coherence: 3 vs 2 particle correlations

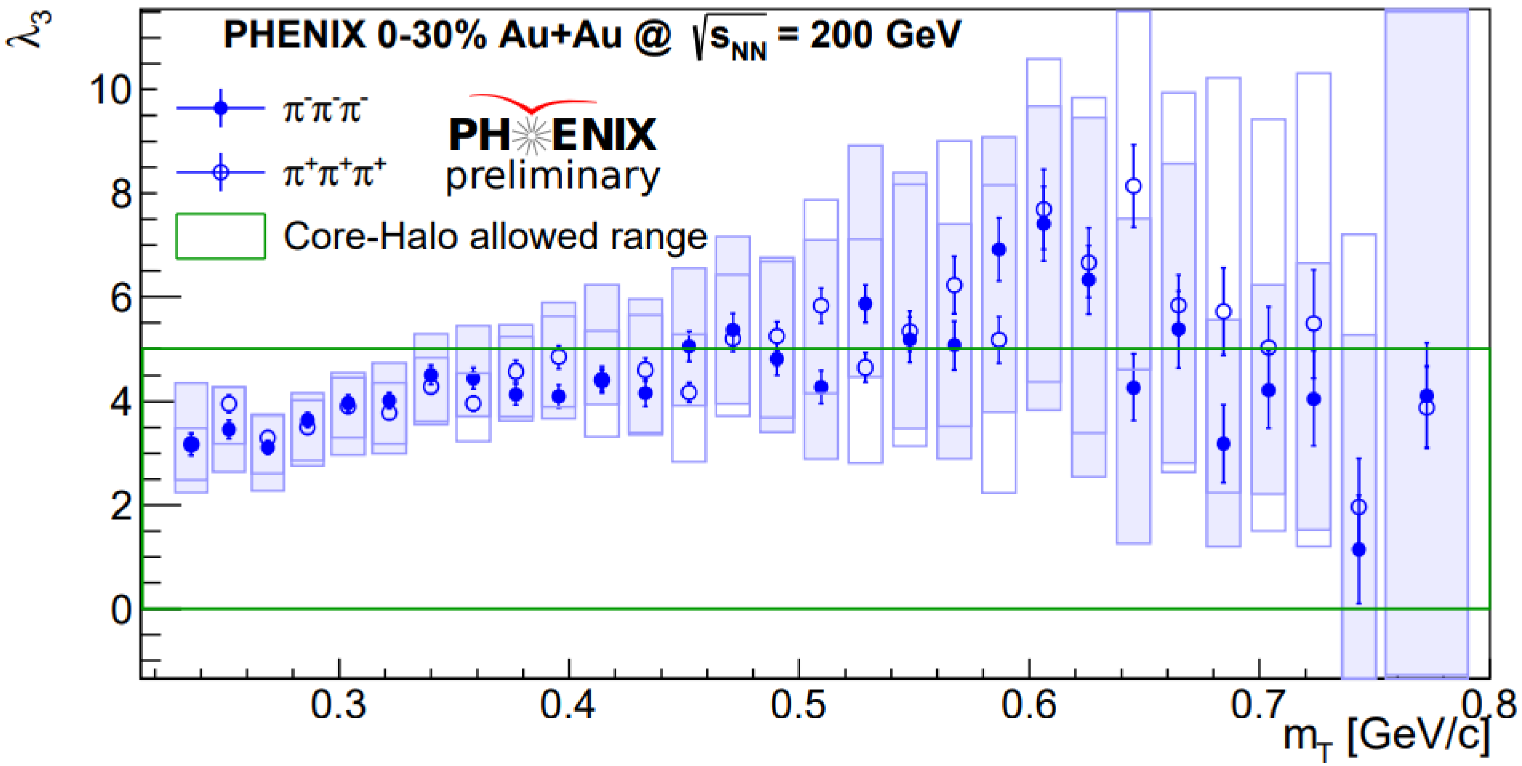
Partial coherence (p_c) vs fractional core (f_c)

- Simple theoretical model [5]: $\lambda_2(f_c, p_c)$, $\lambda_3(f_c, p_c)$
- Measured $\lambda_2^{\text{meas.}} \to$
  $\lambda_2^{\text{meas.}} = \lambda_2(f_c, p_c) \Longrightarrow f_c(p_c)$ (green lines)
- Measured $\lambda_3^{\text{meas.}} \to$
  $\lambda_3^{\text{meas.}} = \lambda_3(f_c, p_c) \Longrightarrow f_c(p_c)$ (blue lines)
- Example 2D plot at $m_T = 0.36$ GeV$/c^2$:



$$\lambda_2 = f_c^2\left[(1-p_c)^2 + 2p_c(1-p_c)\right]$$

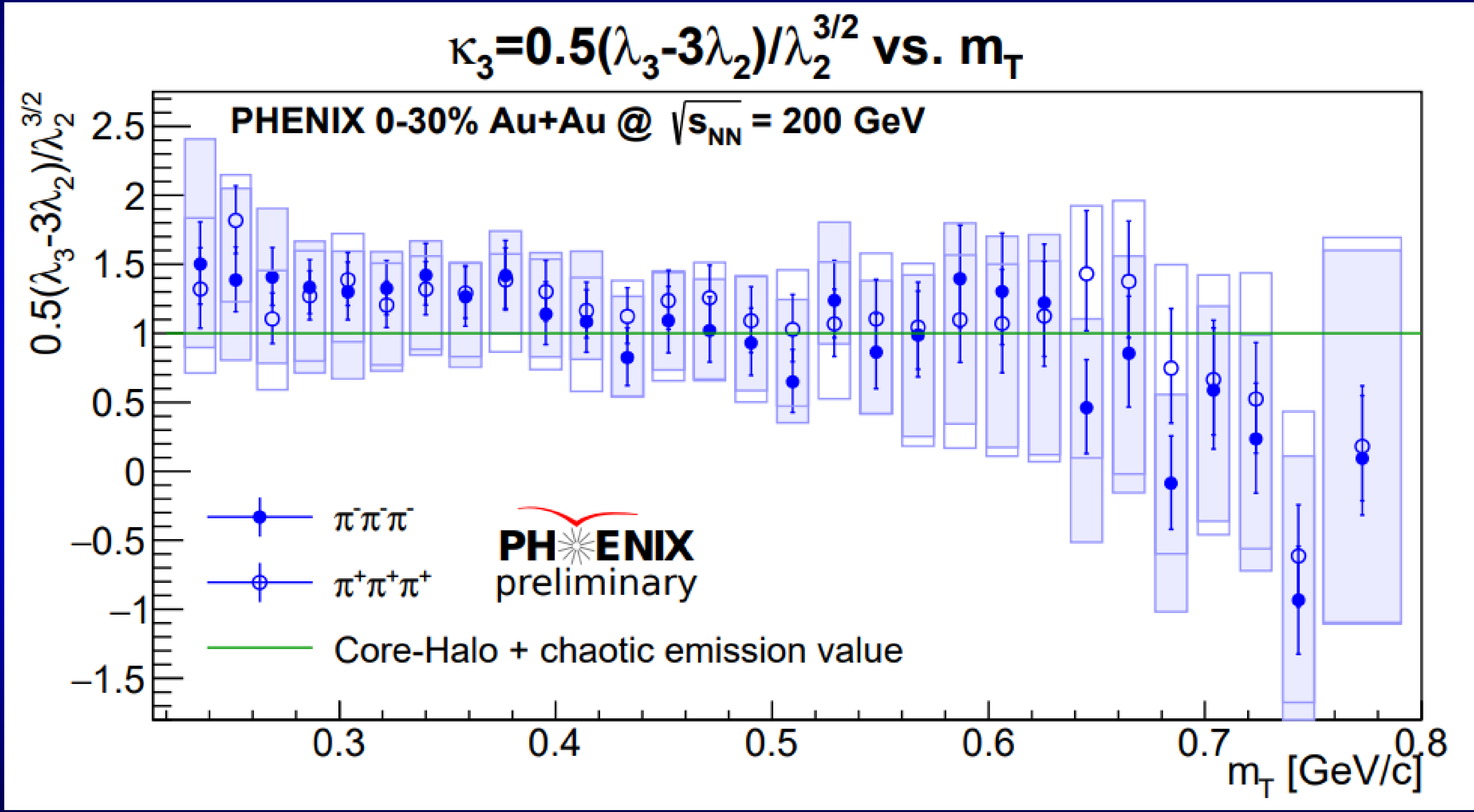
$$\lambda_3 = 2f_c^3\left[(1-p_c)^3 + 3p_c(1-p_c)^2\right] + 3f_c^2\left[(1-p_c)^2 + 2p_c(1-p_c)\right].$$



PHENIX preliminary data on three-pion Bose-Einstein: A.Bagoly, poster at QM17 Partial coherence measurement possible!

B. Kurgyis for the PHENIX Collaboration, *Phys.Part.Nucl.* 51 (2020) 3, 263-266, arXiv:1910.05019 [nucl-ex]

HBT: Two-particle symmetrization, chaotic source - or not ? Partial coherence: 3 vs 2 particle correlations



$$\kappa_3 = \frac{\lambda_3 - 3\lambda_2}{2\sqrt{\lambda_2^3}}.$$

κ_3 ~ 1 within errors

→

No signal for partial coherence (p_c > 0), so

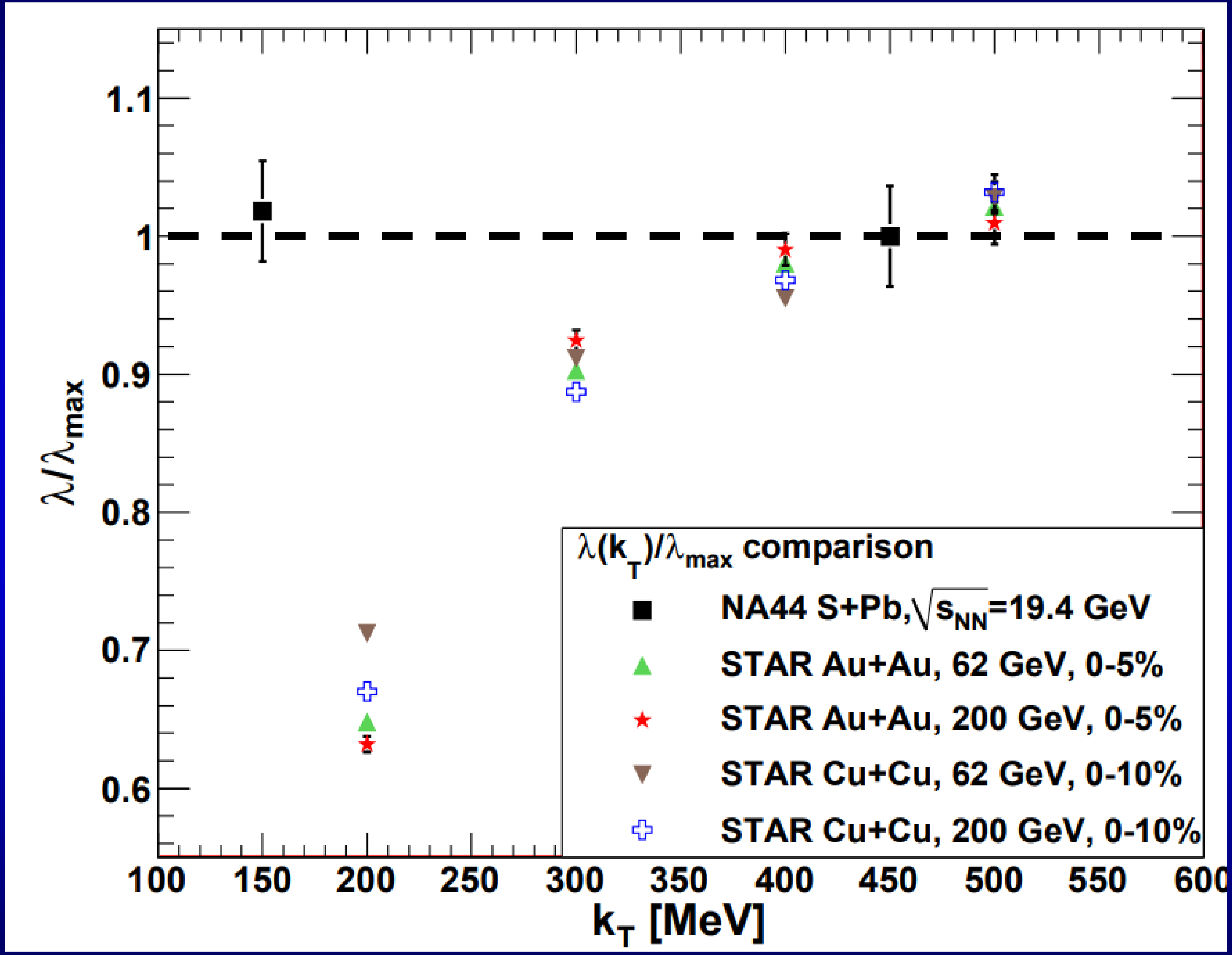
→

p_c cannot mask $U_A(1)$

B. Kurgyis for the PHENIX Collaboration, *Phys.Part.Nucl.* 51 (2020) 3, 263-266, arXiv:1910.05019 [nucl-ex]

Can λ/λ_{max} $U_A(1)$ restoration signal be switched off?

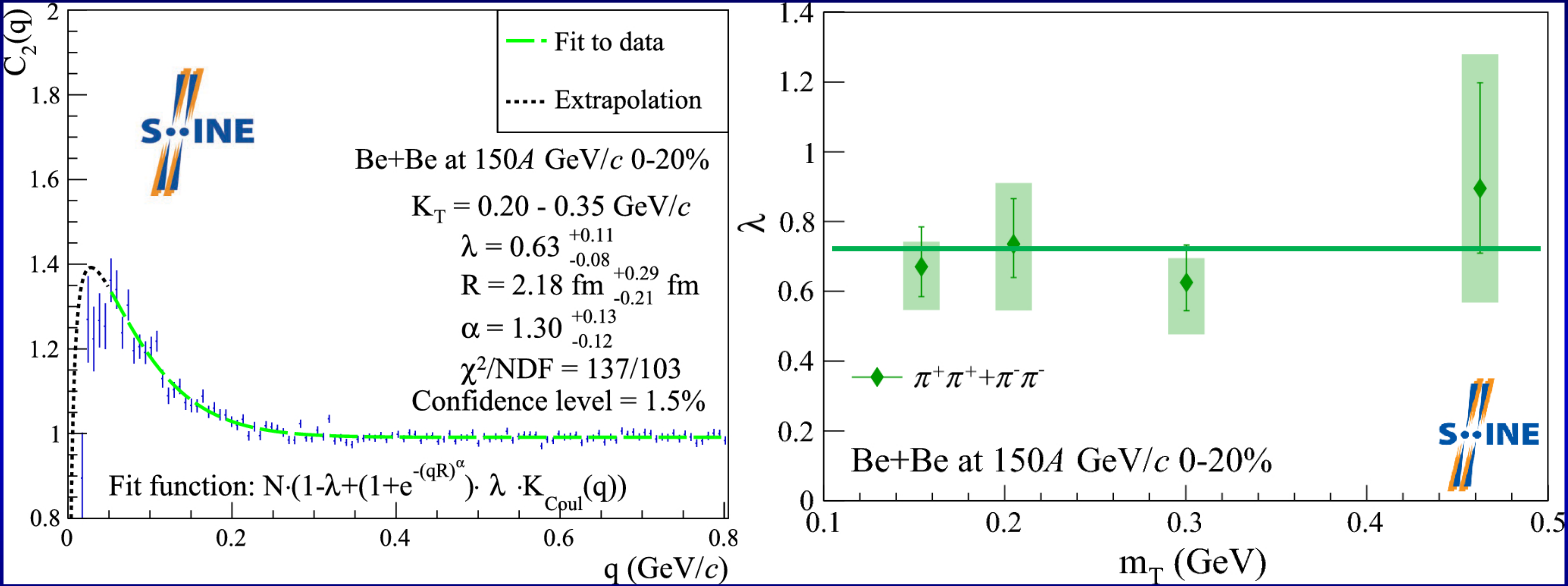
Yes, as known from the first papers!



NA44 data on charged pion Bose-Einstein correlation indicate a null effect in S+Pb at $\sqrt{s_{NN}}$ = 19.4 GeV ! Contrasted to STAR data on charged pion B-E correlation in $\sqrt{s_{NN}}$ = 62 and 200 GeV Au+Au collisions: suppression signal of $U_A(1)$ restoration. R. Vértesi, T.Cs., J. Sziklai , arXiv:2307.09573 [nucl-ex]

Can λ/λ_{max} $U_A(1)$ restoration signal be switched off?

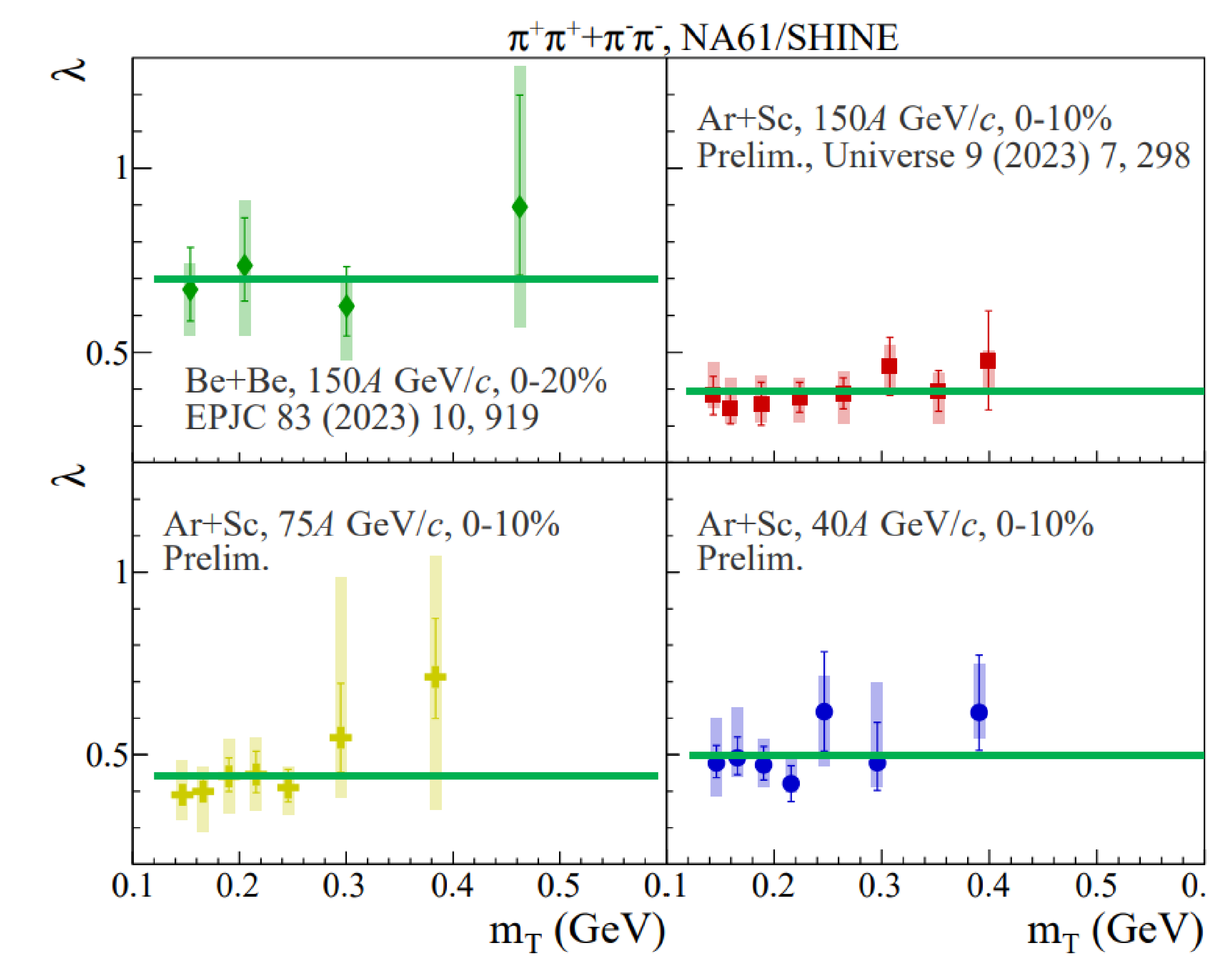
Yes, as known from the first papers, but confirmed by NA61!



NA61 data: no signal of decrease of λ/λ_{max} for $m_T < 0.5$ GeV, no signal of $U_A(1)$ symmetry restoration
Small systems (Be+Be) and relatively low energy, $\sqrt{s} < 20$ GeV.

NA61 data on charged $\pi\pi$ correlation in 150 AGeV Be+Be collisions
Eur.Phys.J.C 83 (2023) 10, 919, e-Print: 2302.04593 [nucl-ex]

Can λ/λ_{max} $U_A(1)$ restoration signal be switched off?

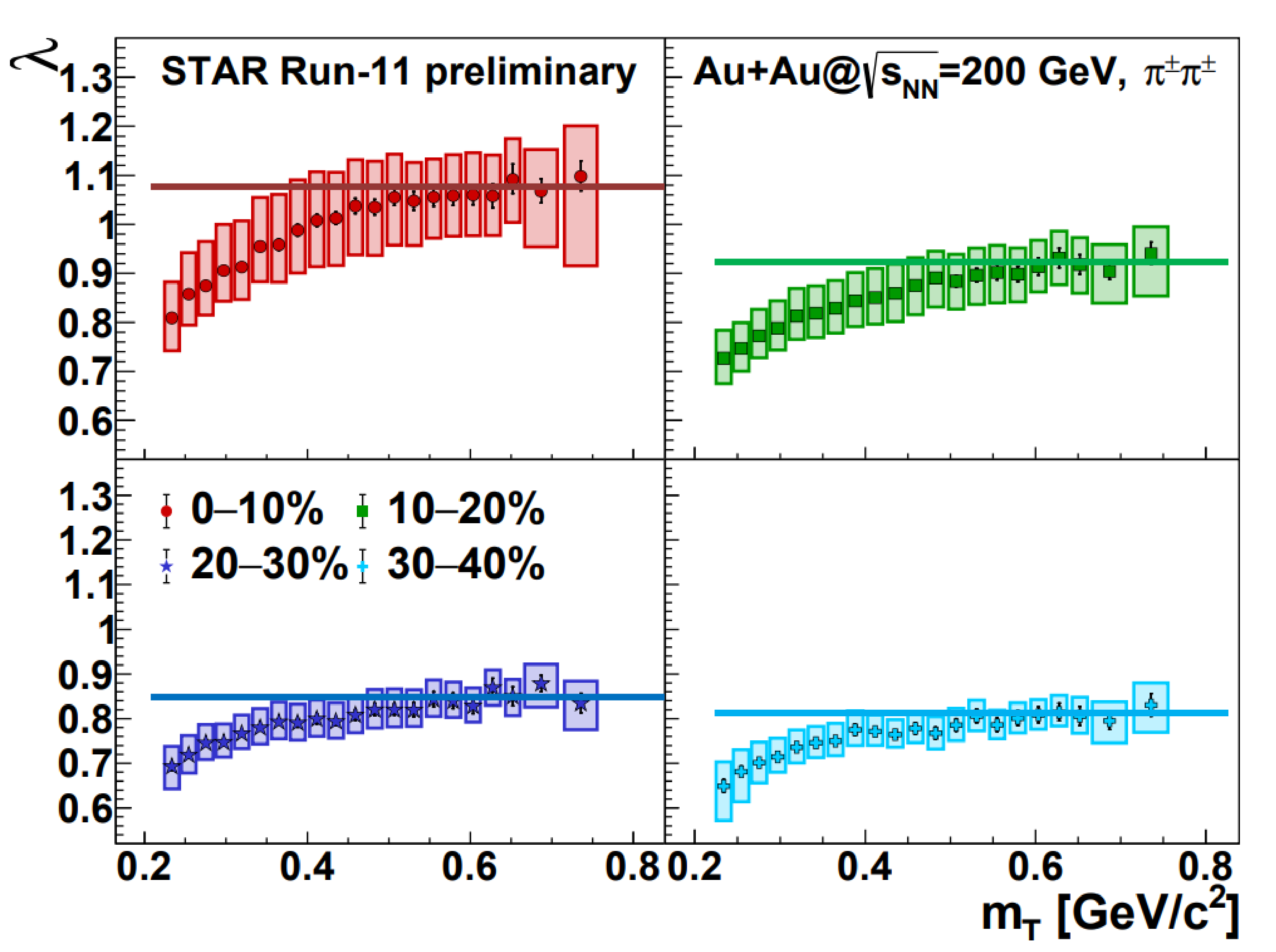


NA61: YES!

NA61 data: no signal of decrease of λ/λ_{max} for $m_T < 0.5$ GeV, no signal of $U_A(1)$ symmetry restoration
Small AND intermediate systems (Be+Be and Ar+Sc) and relatively low energy, $\sqrt{s} < 20$ GeV.

NA61 data on charged ππ correlation in 150 AGeV Be+Be and $E_{lab} \leq 150$ GeV Ar+Sc collisions
B. Pórffy for the NA61 Collaboration, e-Print: 2406.022423 [nucl-ex]

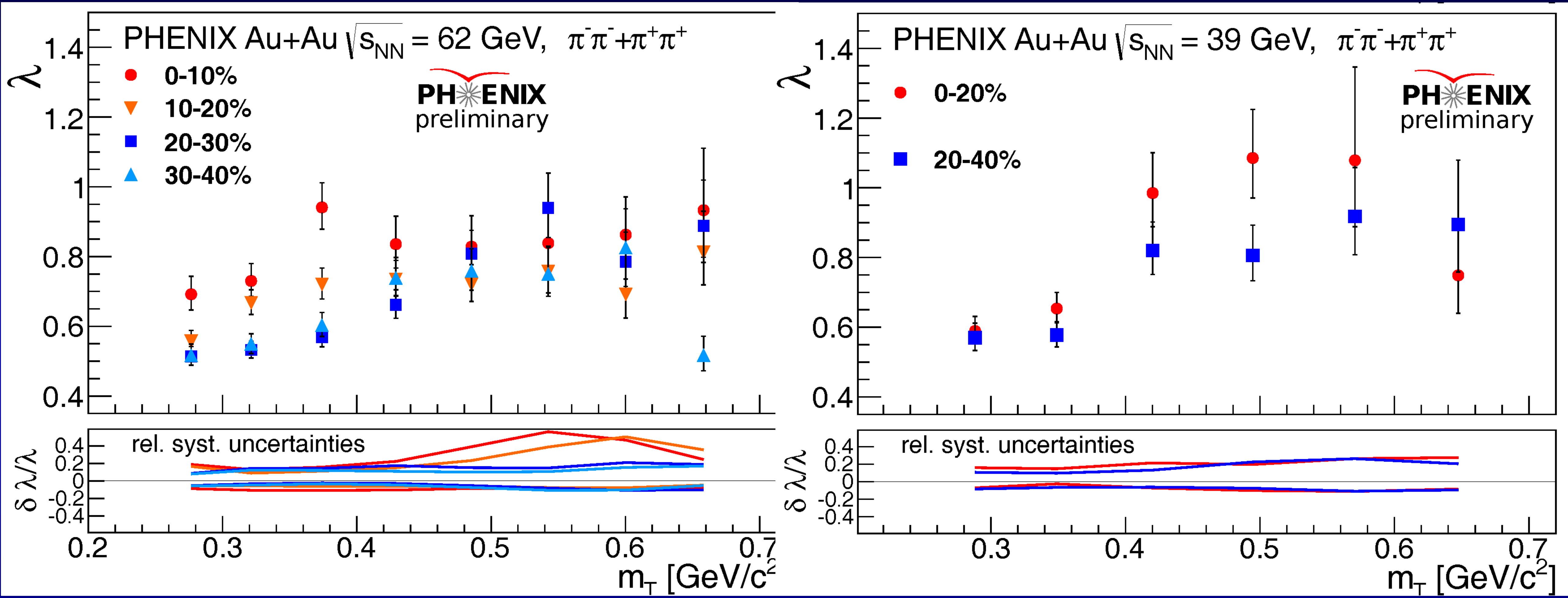
Is $\lambda(m_T)/\lambda_{max}$ confirmed in $\sqrt{s_{NN}}$ = 200 GeV Au+Au?



STAR preliminary: YES!

STAR preliminary, charged ππ correlation in 0-10%, 10-20%, 20-30% and 30-40% Au+Au @ 200 GeV
D. KIncses for the STAR Collaboration, Universe 10 (2024) 3, 102, e-Print: 2401.11169 [nucl-ex]

Can λ/λ_{max} $U_A(1)$ restoration signal be switched off?

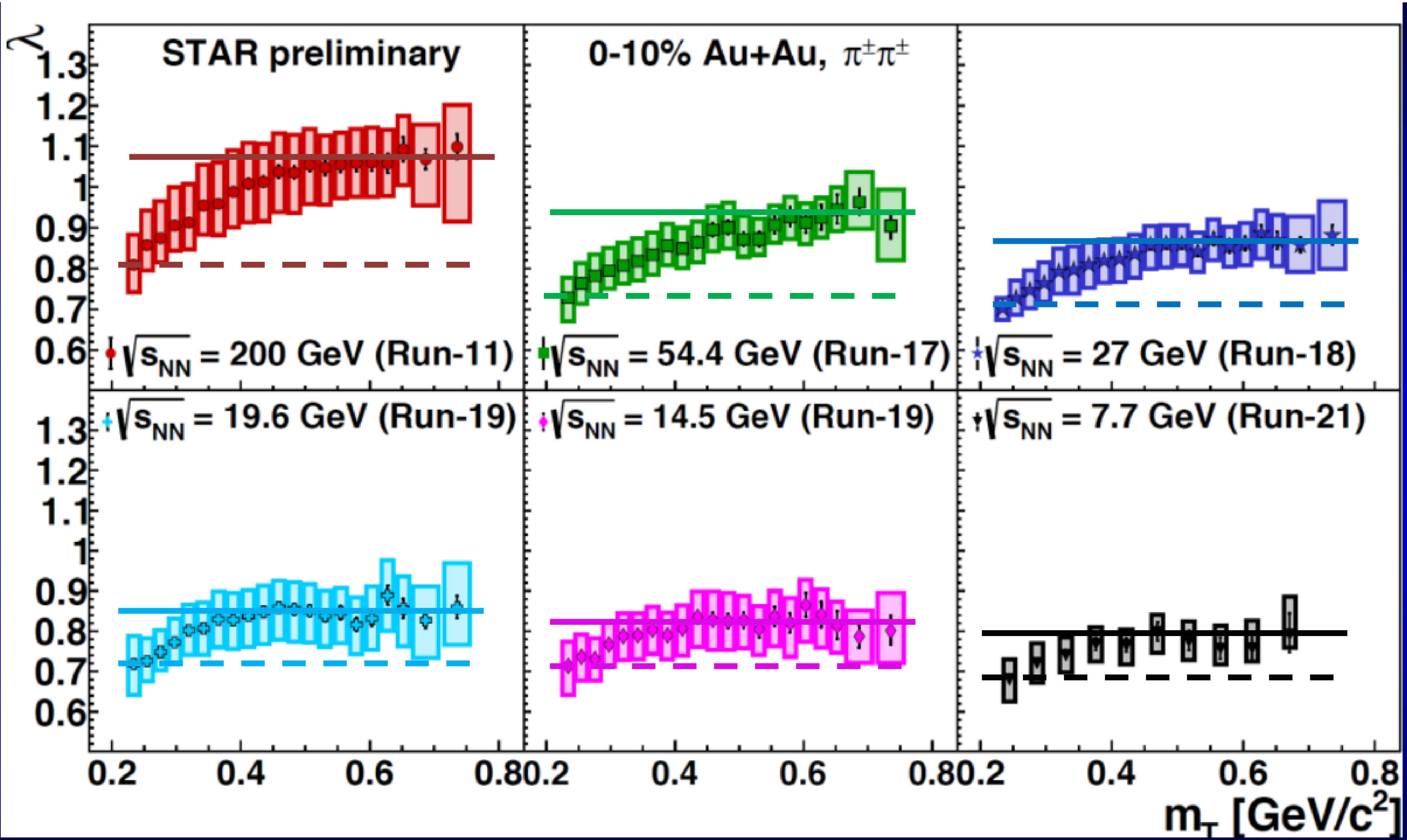


PHENIX preliminary data: qualitatively a of decrease of λ/λ_{max} for $m_T < 0.5$ GeV, but limited statistics!

both at $\sqrt{s_{NN}}$ = 39 and 62 GeV: greater magnetic field, less momentum resolution at low m_T as compared to Run-10 Au+Au data.

D. Kincses for the PHENIX Collaboration, Universe 4 (2018) 1, 11, e-Print: 1711.06891 [nucl-ex]

Excitation function of $\lambda(m_T)/\lambda_{max}$ in Au+Au@RHIC BES?

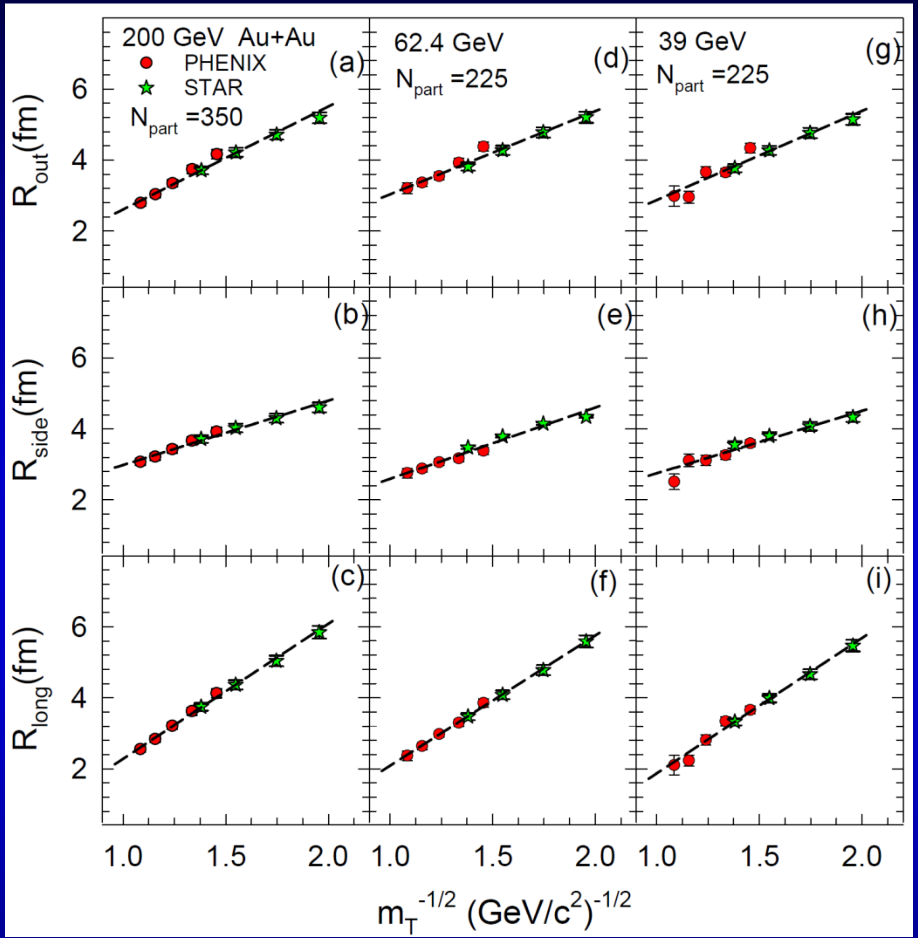


STAR preliminary: in 0-10% Au+Au, $\lambda_{min}/\lambda_{max}$ decreases with decreasing $\sqrt{s_{NN}}$

STAR preliminary, charged $\pi\pi$ correlation in 0-10% Au+Au @ 200, 54.4, 27, 19.6, 14.5 and 7.7 GeV

D. KIncses for the STAR Collaboration, Universe 10 (2024) 3, 102, e-Print: 2401.11169 [nucl-ex]

HBT: Signals of 3d hydro flow, ONLY for $\alpha = 2$



$$\frac{1}{\Delta\overline{\eta}^2} = \frac{1}{\Delta\eta^2} + \frac{M_t}{T_0},$$
$$\overline{R}_\perp^2 = \frac{R_G^2}{1 + \frac{M_t}{T_0}\left(\langle u_t\rangle^2 + \langle \frac{\Delta T}{T}\rangle_r\right)},$$

$$R_l^2 = \overline{\tau}^2\Delta\overline{\eta}^2,$$
$$R_o^2 = \overline{R}_\perp^2 + \beta_t^2\Delta\overline{\tau}^2,$$
$$R_s^2 = \overline{R}_\perp^2$$

$$R_s^2 = R_{..}^2,$$
$$R_o^2 = R_{:}^2 + \beta_t^2[\cosh^2(\overline{\eta})R_=^2 + \sinh^2(\overline{\eta})R_\parallel^2],$$
$$R_{ol}^2 = -\beta_t\sinh(\overline{\eta})\cosh(\overline{\eta})(R_=^2 + R_\parallel^2),$$
$$R_l^2 = \cosh^2(\overline{\eta})R_\parallel^2 + \sinh^2(\overline{\eta})R_=^2,$$

Theory challenge for Levy $\alpha < 2$

Indication of hydro scaling behaviour of Gaussian R(side,out,long) at low m_T

$$\frac{1}{R^2} = \frac{1}{R_0^2} + \frac{m_T}{T_0}\cdot H_0^2$$

R_{long} m_t-scaling: Yu. Sinyukov and A. Makhlin: Z.Phys. C39 (1988) 69

R_{side} , R_{out} , R_{long} m_t-scaling: T. Cs, B. Lörstad, hep-ph/9509213 (shells of fire vs fireballs)

S. Chapman, P. Scotto, U. W. Heinz, hep-ph/9408207