LÉVY-STABLE SOURCES FROM SPS THROUGH RHIC TO LHC

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WPCF 2024, TOULOUSE

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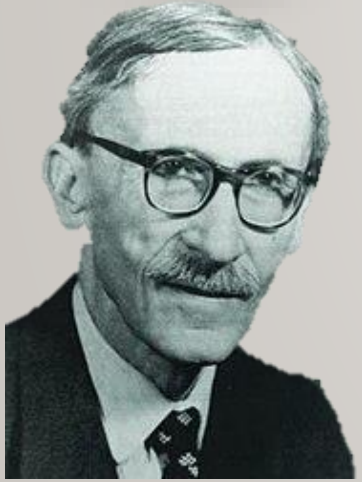


OVERVIEW OF LÉVY FEMTOSCOPY RESULTS AT WPCF

1. Monday 11:35: Máté Csanád, this talk ☺
2. Tuesday 17:00: Sneha Bhosale, 3D measurements of pion HBT correlations at STAR
3. Tuesday 17:25: Barnabás Pórfy, Energy scan results with Lévy type femtoscopy at NA61/SHINE
4. Tuesday 17:40: Márton Nagy, Coulomb and strong final state interactions with Lévy sources
5. Wednesday 16:05: Dániel Kincses, Lévy walk of pions in heavy-ion collisions
6. Wednesday 16:20: Emese Árpási, 3D source sizes and shapes of hadron emission in EPOS
7. Wednesday 16:35: László Kovács, Event-by-event two-kaon source function with EPOS
8. Friday 16:15: Sándör Lökös, Centrality dependence of Lévy-stable two-pion correlations at PHENIX

\+ Friday 15:30: Balázs Kórodi, Pseudorapidity distributions with the STAR EPD at BES-II energies

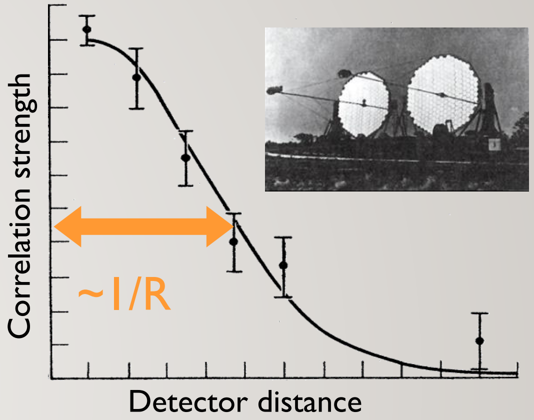
HBT OR FEMTOSCOPY IN HIGH ENERGY PHYSICS

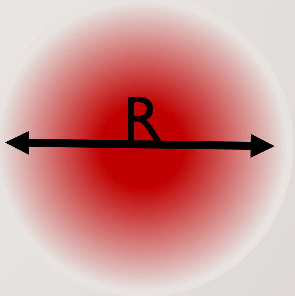
- R. Hanbury Brown, R. Q. Twiss - observing Sirius with radio telescopes
  - Intensity correlations vs detector distance ⇒ source size
  - Measure the sizes of apparently point-like sources!
- Goldhaber et al: applicable in high energy physics
- Understanding: Glauber, Fano, Baym, …
  Phys. Rev. Lett. 10, 84; Rev. Mod. Phys. 78 1267, …
  - Momentum correlation $C(q)$ related to source $S(r)$

    $C(q) \cong 1 + \left| \int S(r) e^{iqr} dr \right|^2$

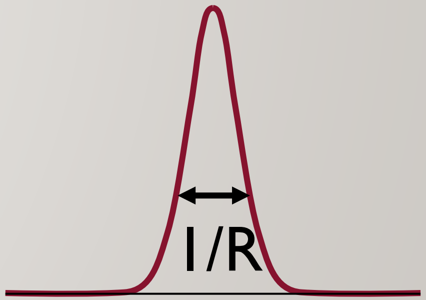
    (under some assumptions)
  - Also with distance distribution $D(r)$:

    $C(q) \cong 1 + \int D(r) e^{iqr} dr$
  - Neglected: pair reconstruction, final state interactions, multi-particle correlations, coherence, …
- What is the source shape? Can be explored via femtoscopy





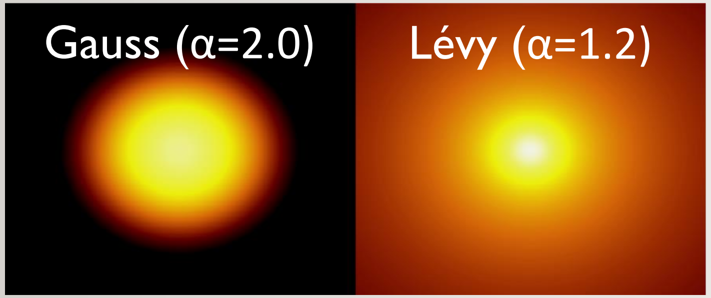
source function S(r)

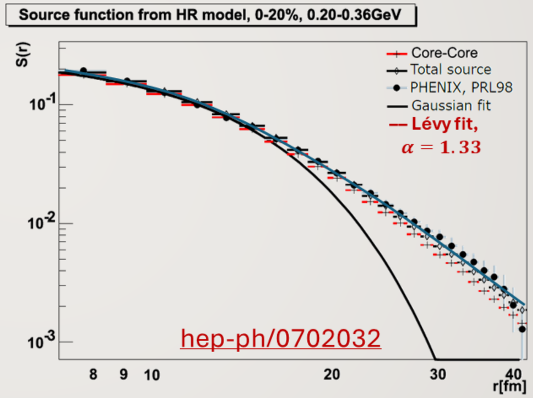


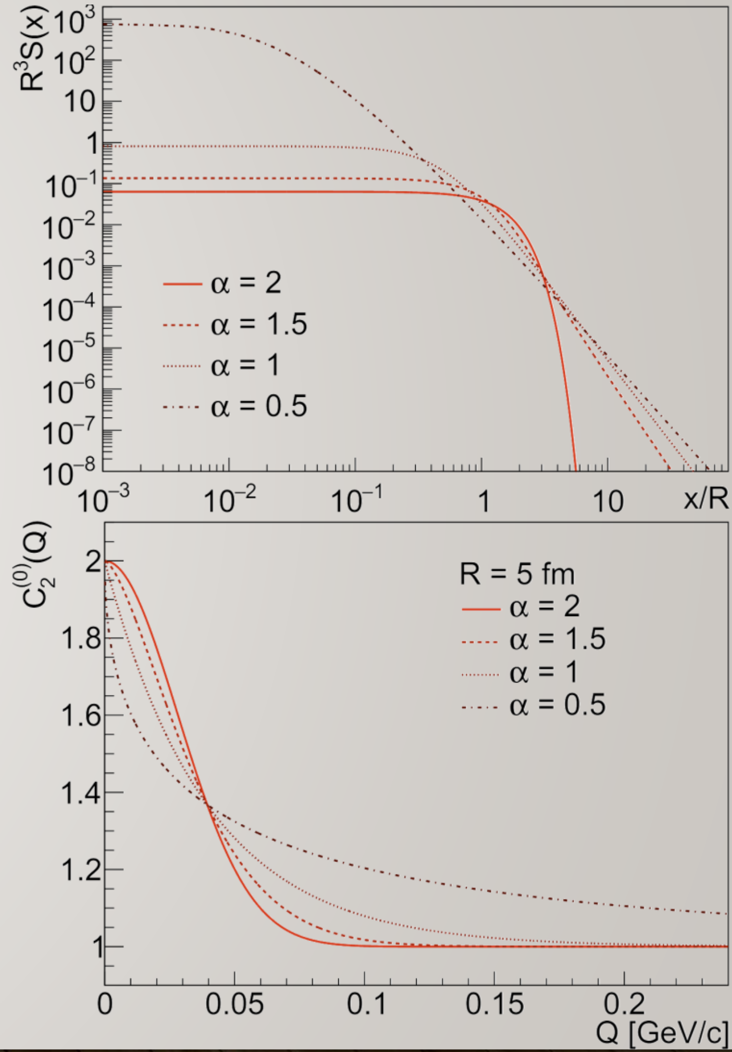
correlation funct. C(q)

LÉVY DISTRIBUTIONS IN HEAVY ION PHYSICS

- Central limit theorem (diffusion) and thermodynamics lead to Gaussians
- Measurements suggest phenomena beyond Gaussian distribution
- Lévy-stable distribution: $\mathcal{L}(\alpha,R;r)=\frac{1}{2\pi}\int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$
  - From generalized central limit theorem, power-law tail ~ $r^{-(1+\alpha)}$
  - Special cases: α = 2 Gaussian, α = 1 Cauchy







- Shape of the correlation functions with Lévy source:
  - $C_2(q)=1+\lambda\cdot e^{-|qR|^\alpha}$; $\alpha=2$: Gaussian; $\alpha=1$: exponential
    Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67-78
- Sidenote: Lévy ≠ not Gaussian (but ⊂ {not Gaussian})
- A possible reason for Lévy source: Lévy flight or Lévy walk

LÉVY PROCESSES IN NATURE AND IN SCATTERING

- Lévy walk and Lévy flight: known in ecology, climatology, etc
  - If stepsize distribution has no finite width:
    generalized central limit theorem, Lévy-stable limiting distributions
- In HIC: increasing mean free path, stepsize increase
  - Seen in expansion under Coulomb potential in solid-state physics
- Indeed, observed in UrQMD [arXiv:2409.10373]
  - See talk by D. Kincses on Wednesday

E. I. Kiselev, Phys. Rev. B 103, 235116 (2021)

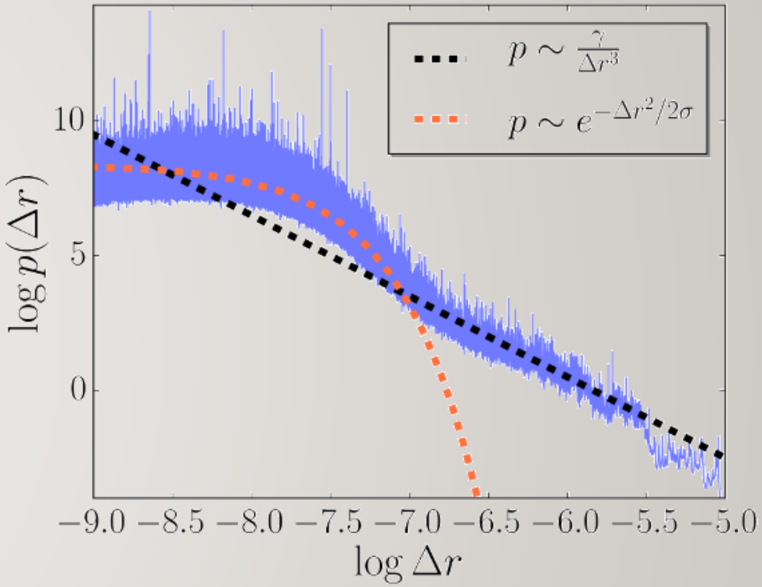
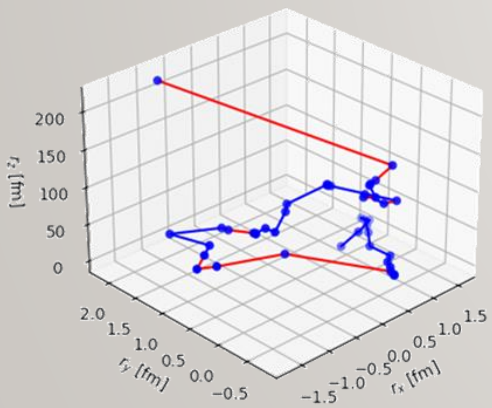
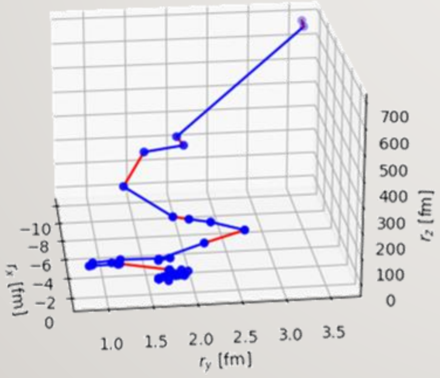
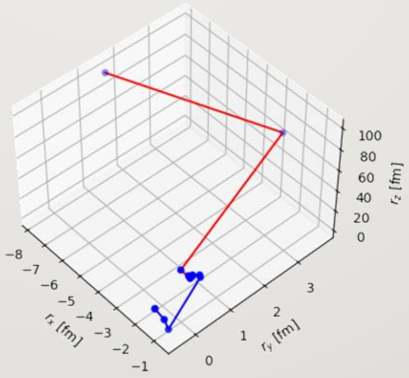


Figure 1. The Figure shows the step size distribution $p(\Delta r)$ of a random walk as performed by Coulomb interacting, diffusing particles in two dimensions. At large step sizes, the distribution clearly follows the $p \sim \Delta r^{-3}$ power-law which leads to the superdiffusive dynamics described by Eq. (1). The data was obtained by integrating the system of coupled Langevin equations of Eq. (56).

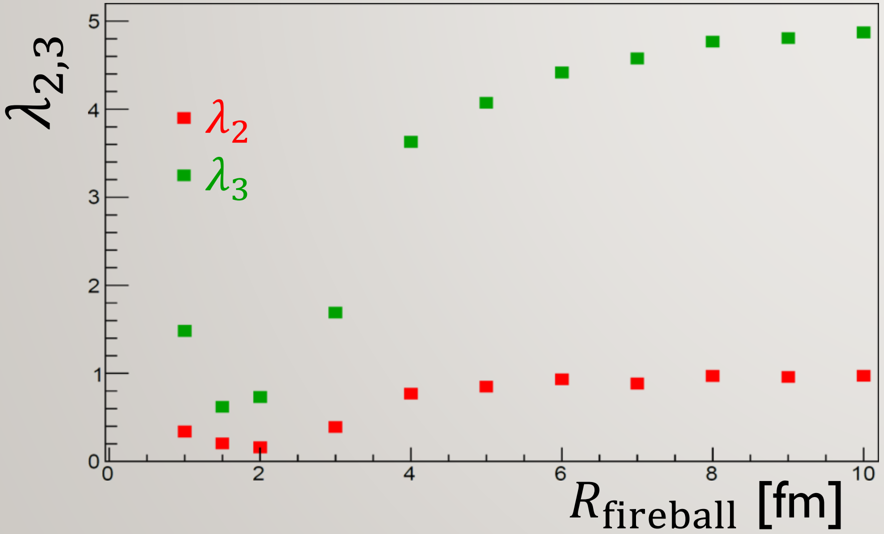






CHARGED CLOUD: OTHER INTERESTING EFFECTS

- Coulomb potential: infinite range, affecting evolution for a long time
- Solid-state physics (as mentioned on previous slide): may cause Lévy flight and power-law tails
  - Maybe also in heavy-ion physics?
- Another interesting effect: distortion of flight paths after kinetic freeze-out
  - Similarly to an Aharonov-Bohm effect (arXiv:2007.07167 and arXiv:2410.15525)
- Decreases correlation strengths for very dense sources

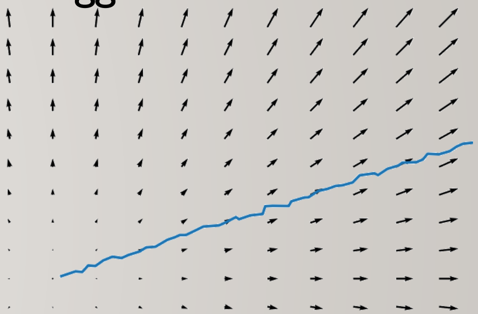


$\lambda_3 = 5$

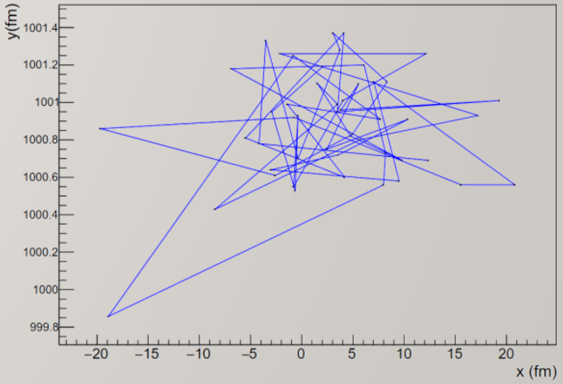
No A-B effect, pure core, fully chaotic source

$\lambda_2 = 1$

exaggerated illustration

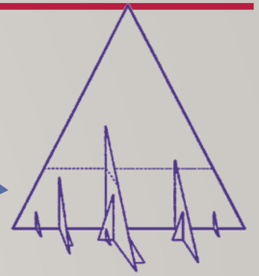


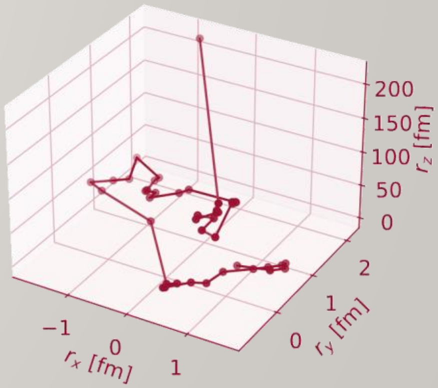
simulated transverse path



WHY DO LÉVY SHAPES APPEAR, WHY IS IT IMPORTANT?

- A more comprehensive list of possible reasons:
  - Jet fragmentation (Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon. B36 (2005) 329-337)
    - See also talk by Yacine Mehtar-Tani at ExploreQGP workshop in Belgrade
  - Critical phenomena (Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828 (2006) no.1, 525-532)
  - Directional or event averaging, non-sphericality (Cimerman et al., Phys.Part.Nucl. 51 (2020) 282)
  - Lévy walk (BJP37(2007); PRB103(2021), Entropy24(2022); PLB847(2023); arXiv:2409.10373)
- Importance of utilizing Lévy sources:
  - Measuring α and R
    - Order of quark-hadron transition, critical point search, understanding source dynamics
  - Measuring λ also requires correct shape assumption
    - In-medium mass modification, coherent pion production

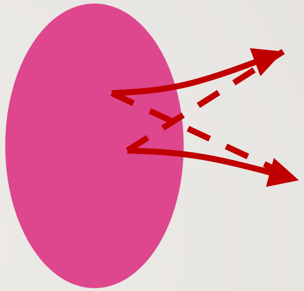




INTERACTIONS: THE COULOMB-EFFECT

- Plane-wave result, based on $\left|\Psi_2^{(0)}(r)\right|^2 = 1 + e^{iqr}$, for pair source $D(r)$

$$C_2(q,K) \cong \int D(r,K)\left|\Psi_2^{(0)}(r)\right|^2 dr = 1 + \int D(r,K)e^{iqr}dr$$



- If there are interactions, solve Schrödinger eq:

$$\Psi_2^{(0)}(r) \rightarrow \Psi_2^{(\mathrm{int})}(r_1, r_2)$$

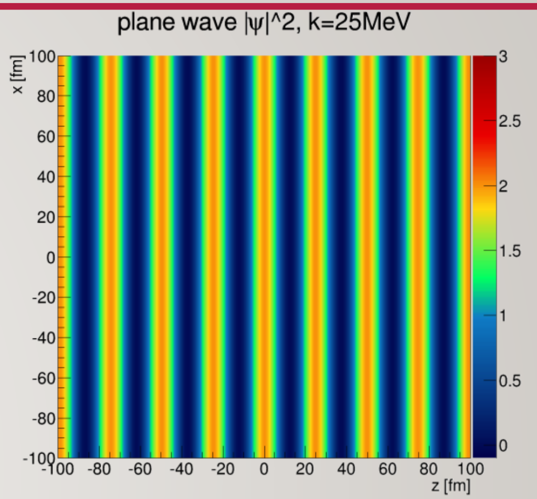
- For Coulomb, solution is known:

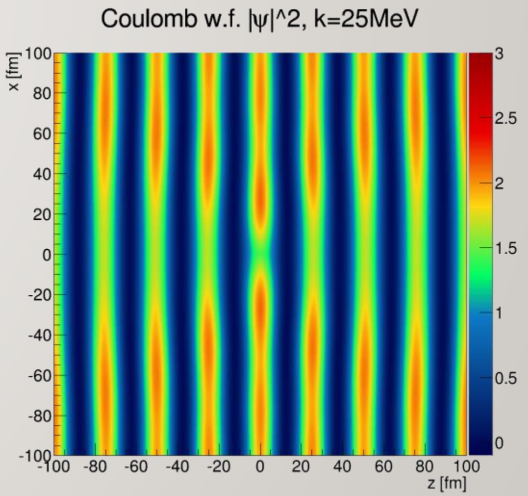
$$\left|\Psi_2^{(C)}(r)\right|^2 = \frac{\pi\eta}{e^{2\pi\eta}-1} \cdot \left(\text{complicated hypergeometric expression}\right)$$

- Direct fit with this, or the usual iterative Coulomb-correction:

$$C_{\mathrm{Bose-Einstein}}(q)K(q), \text{ where } K(q) = \frac{\int D(r,K)\left|\Psi_2^{(C)}(r)\right|^2 dr}{\int D(r,K)\left|\Psi_2^{(0)}(r)\right|^2 dr}$$

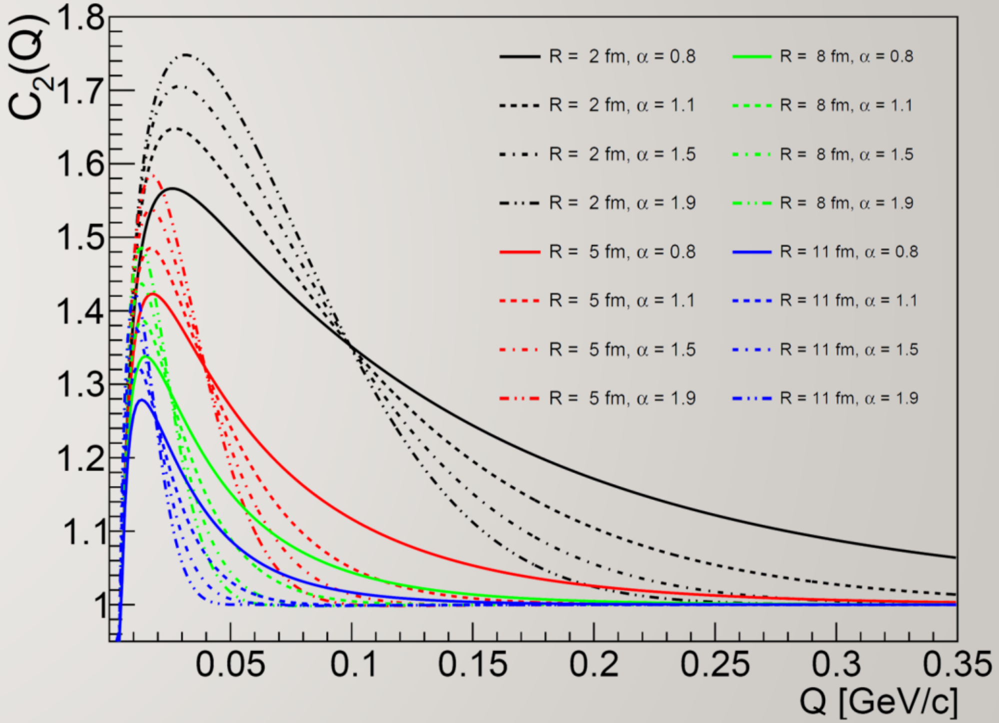
- Complication: need for integrating power-law tails
- Many new results: see talk by M. Nagy on Tuesday





HOW TO CALCULATE THE COULOMB EFFECT

- Calculating correlation functions with the Coulomb effect included: time consuming in the past
- Method used in early analyses: Coulomb correction calculated for fixed radius and shape
  - For example, fixing $R = 5$ fm and $\alpha = 2$
- More consistent method: correlation function with Coulomb FSI precalculated in a tabular form
  - Iterative fitting, see e.g., PHENIX, PRC97 (2018) 6, 064911
- Convenient, but somewhat restricted method: interpolating functional form, in a limited R, α range
  - See Csanád, Lökös, Nagy, Phys.Part.Nucl. 51 (2020) 238, used in arXiv:2306.11574 [CMS], arXiv:2302.04593 [NA61]

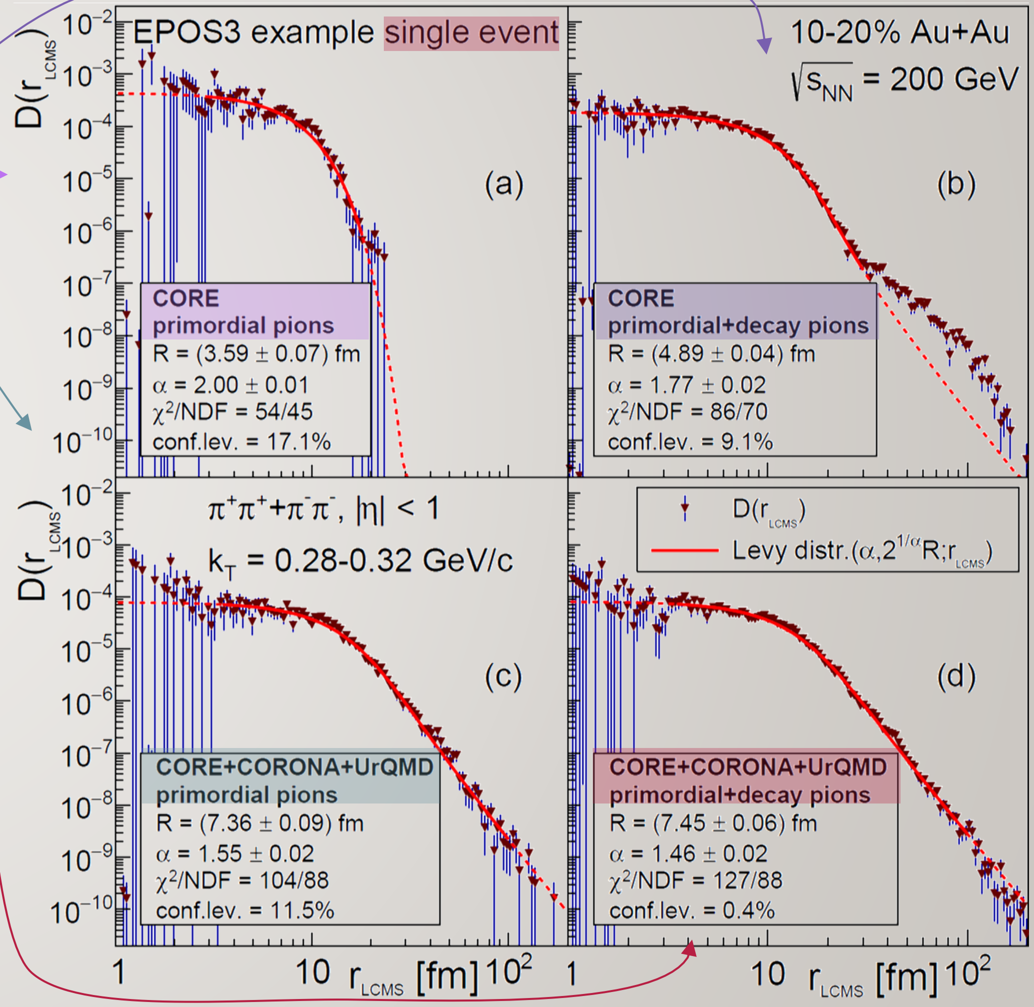


- Recent method: see talk by Márton Nagy
  - Nagy, Purzsa, Csanád, Kincses Eur. Phys. J. C 83, 1015 (2023), code at github.com/csanadm/CoulCorrLevyIntegral
  - Recent developments: 3D calculation, protons, see talk by M. Nagy on Wednesday

LÉVY SHAPES IN SINGLE EPOS EVENTS, 1D

- EPOS model: parton-based Gribov-Regge theory (PBGRT)
  - Werner et al., PRC82 (2010) 044904, PRC89 (2014) 064903, …
- Source observed in four stages:
  - a) CORE, primordial pions: close to Gaussian
  - b) CORE, with decay products: power-law structures
  - c) CORE+CORONA+UrQMD, primordial pions: Lévy shape
  - d) CORE+CORONA+UrQMD, with decay products: Lévy shape
  - Radii in the four stages (one example event)
    3.59 fm → 4.89 fm → 7.36 fm → 7.45 fm
  - Shape (α) change: 2.00 → 1.77 → 1.55 → 1.46
- Can one relate the observed HBT radii to the hydro phase homogeneity lengths?
- More investigations needed…
- Related talks by D. Kincses, E. Árpási, L. Kovács on Wed

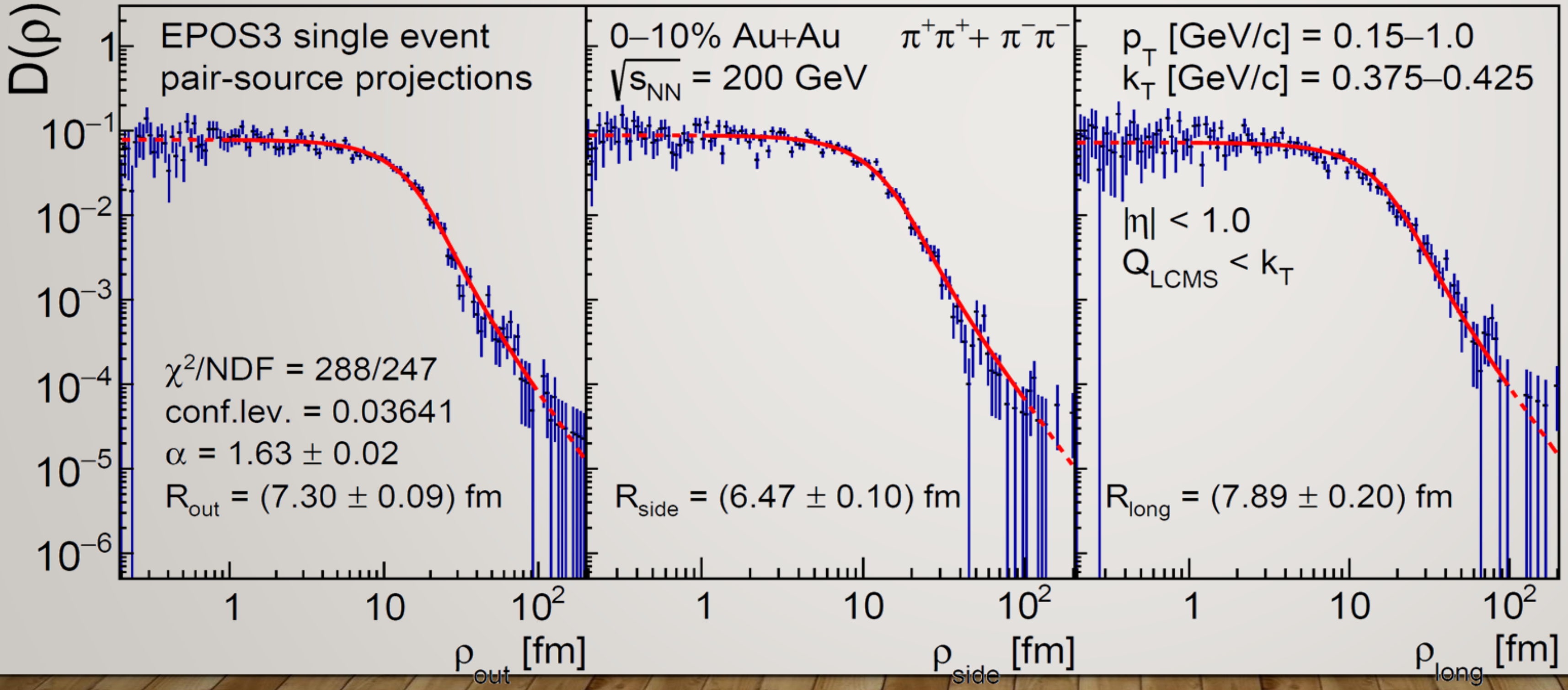
arXiv:2201.07962

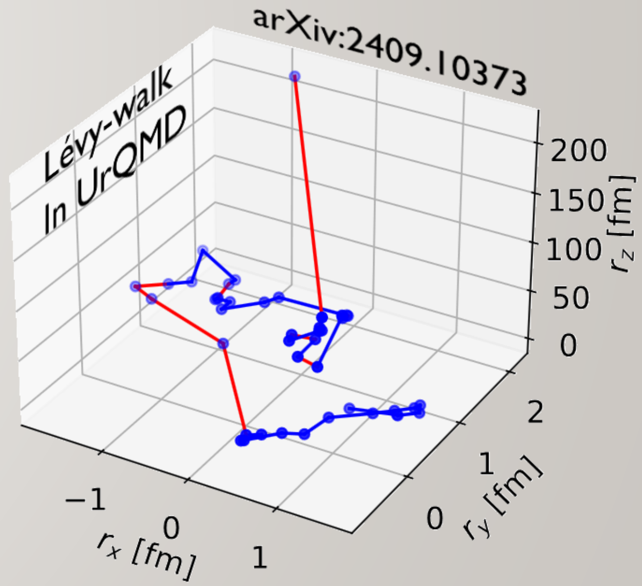


LÉVY SHAPES IN SINGLE 3D EPOS EVENTS, 3D

- What if the Lévy shapes appeared only because of directional averaging?
- Let's check 3D event shapes in EPOS! → Also Lévy, with similar α and radii (as those in 1D)
- Clear physical reason: Lévy walk (see talk by D. Kincses on Wednesday)

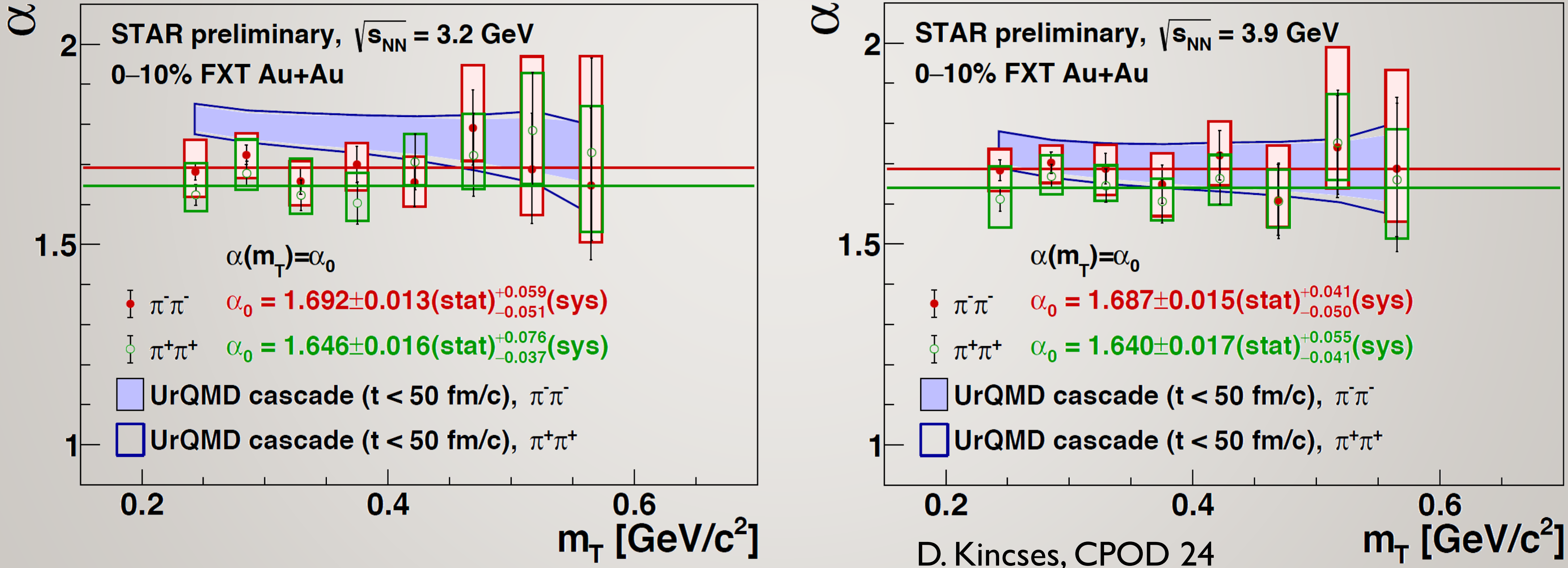
arXiv:2409.10373





URQMD COMPARED TO STAR DATA AT 3.2 AND 3.9 GEV

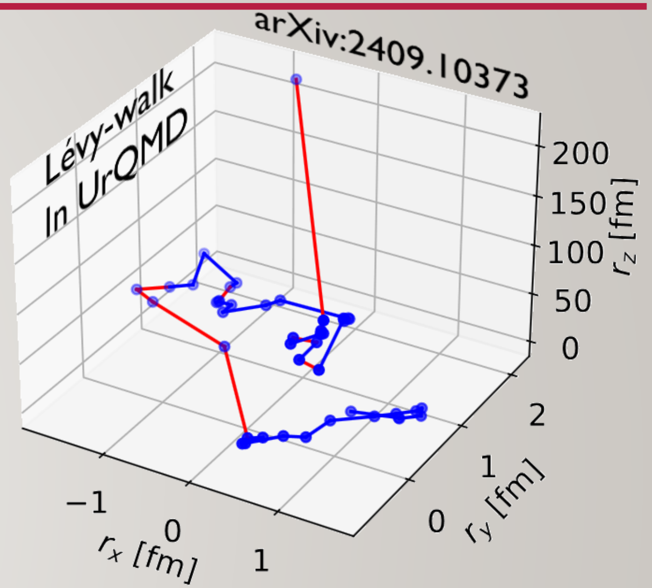
- Non-Gaussian values ($\alpha < 2$); small systematic difference between $\pi^-\pi^-$ and $\pi^+\pi^+$ pairs
- 3.9 and 3.2 GeV compatible, no m_T dependence observed
- UrQMD within uncertainties – no other effect but rescattering and decays, good agreement

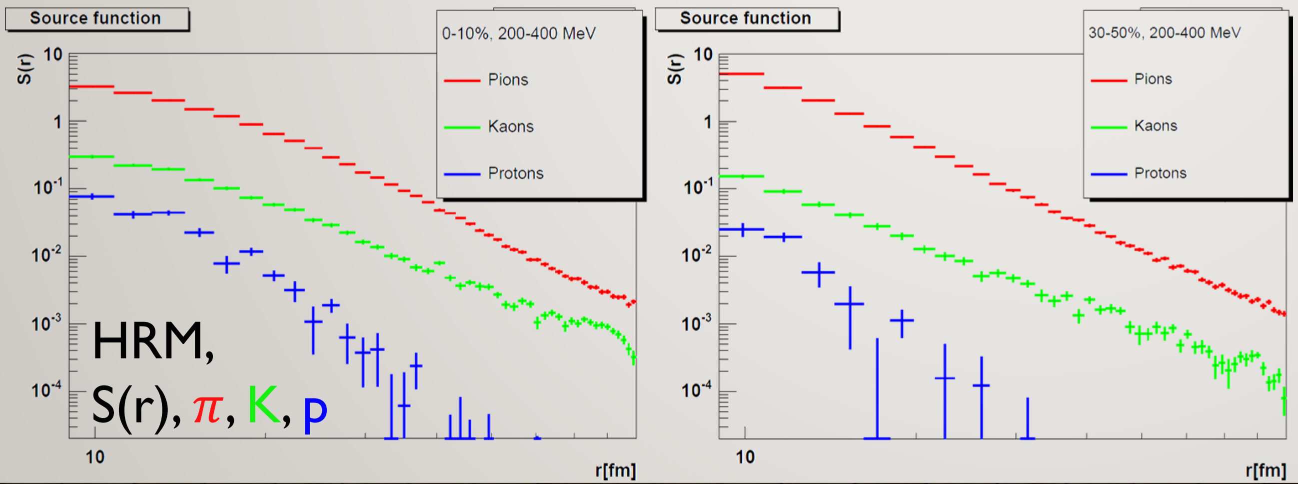


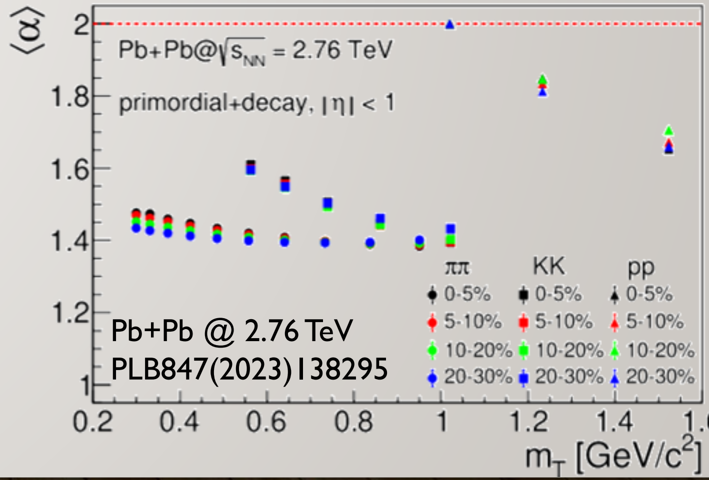
D. Kincses, CPOD 24

SO WHEN DO THE POWER-LAW TAILS FORM?

- Based on EPOS: apparently Gaussian in hydro phase
- Power-law tails due to Lévy-walk: scattering processes
  - 2-by-2, decay, coalescence, etc
- How to test? Particle type dependence!
  - Based on cross-sections: $\alpha(p) > \alpha(\pi) > \alpha(K)$
    Humanic, IJMPE15(2006)197, Csanád, Csörgő, Nagy, BJP37(2007)1002
  - Not confirmed by EPOS! Role of decays?

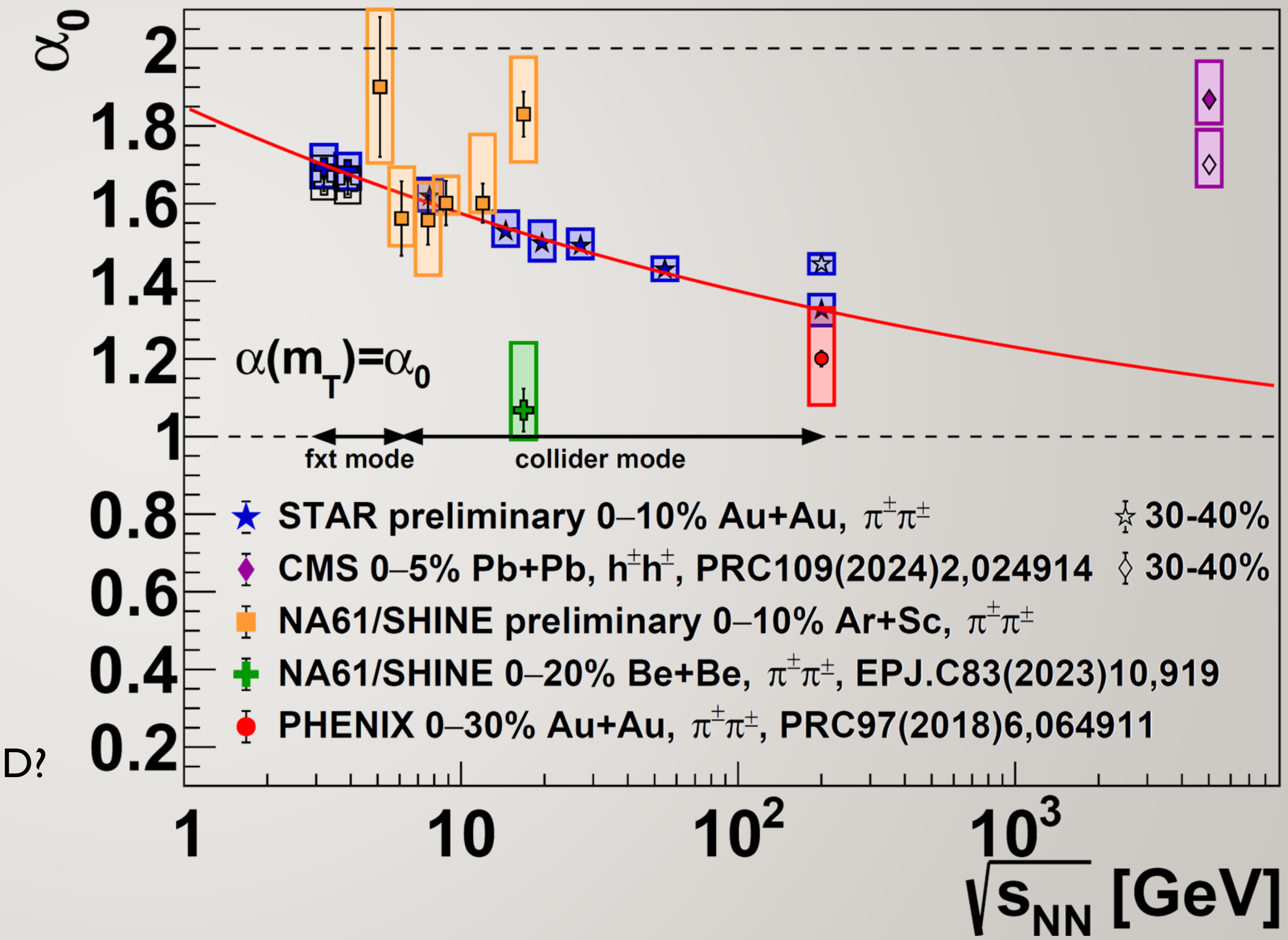






PION SOURCE SHAPE EVOLUTION, SPS→RHIC→LHC

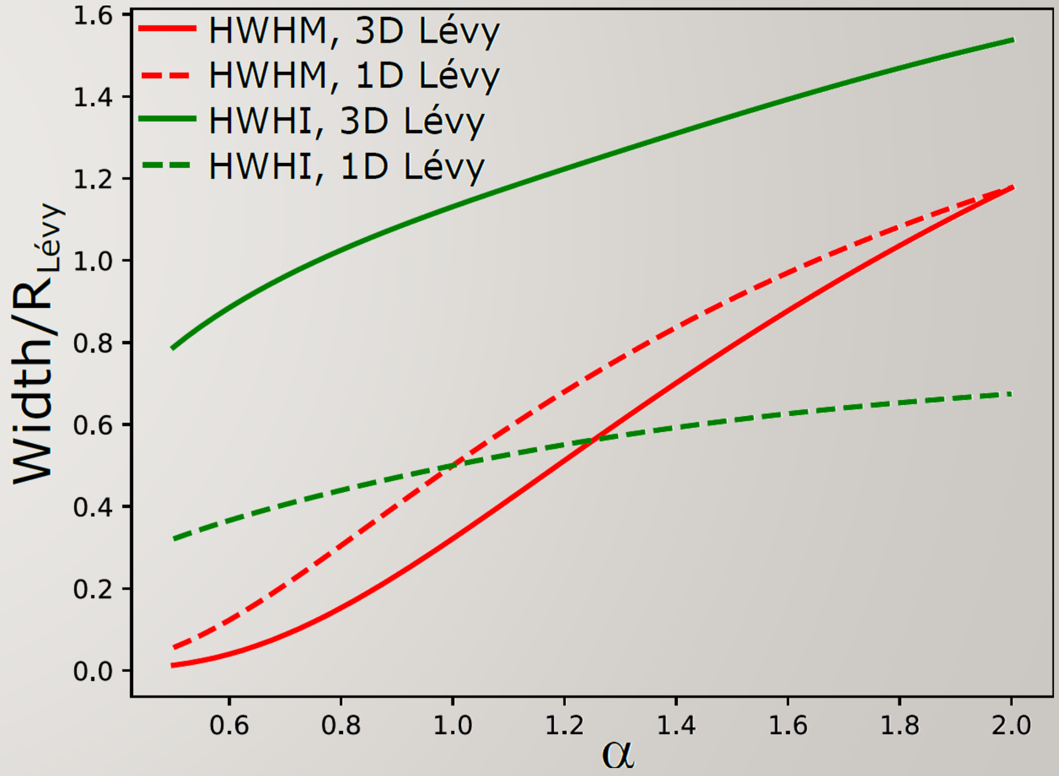
- NA61: Be+Be at 17 GeV and Ar+Sc at 5-17 GeV
- CMS: Pb+Pb at 5 TeV
- Opposite centrality trends?
  - Larger alpha in more central in CMS
  - Larger alpha in more peripheral in STAR
- Opposite energy dependence trends?
  - Minimum at NA61 in Ar+Sc
  - Decrease with energy at STAR in Au+Au
  - Increase from STAR to CMS
- Origin of nonmonotonicity?
  - Difference in momentum acceptance & PID?
- Possible analysis in O+O vs Au+Au



SOURCE SIZE MEASURE CHANGE WITH α

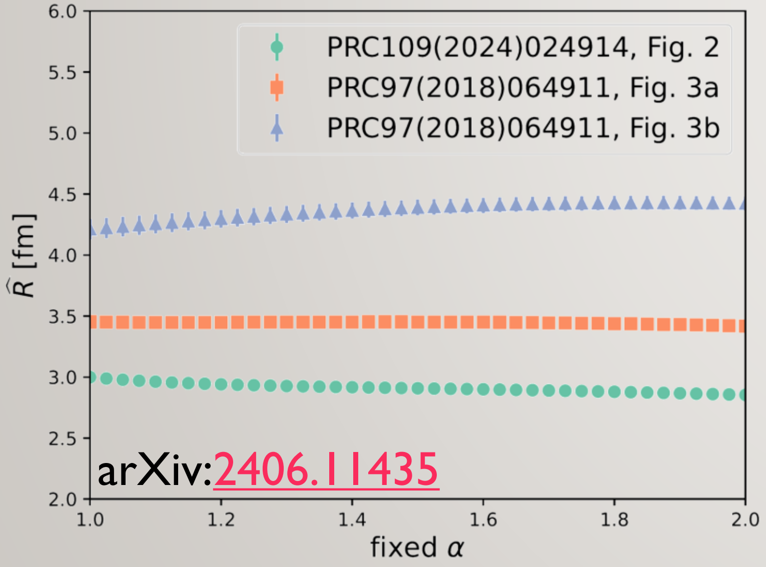
- No tail if $\alpha = 2$, power law if $\alpha < 2$; tail depends on α
  - If $S(r)$ Lévy, pair source $D(r)$ also Lévy with same α and $R \to 2^{1/\alpha}R$
- RMS = ∞ if $\alpha < 2$, in practice: depends on cutoff
- What do Gaussian HBT radii mean?
  Important also w.r.t. CEP search
- Alternative measures:
  - HWHI: (half) width at half integral
  - HWHM: (half) width at half max
  - Large difference between 1D and 3D relative width
  - Width (normalized by R) nontrivially depends on α
  - If $\alpha = 2$ or $\alpha = 1$ assumed: deviation from true scale

arXiv:2401.01249

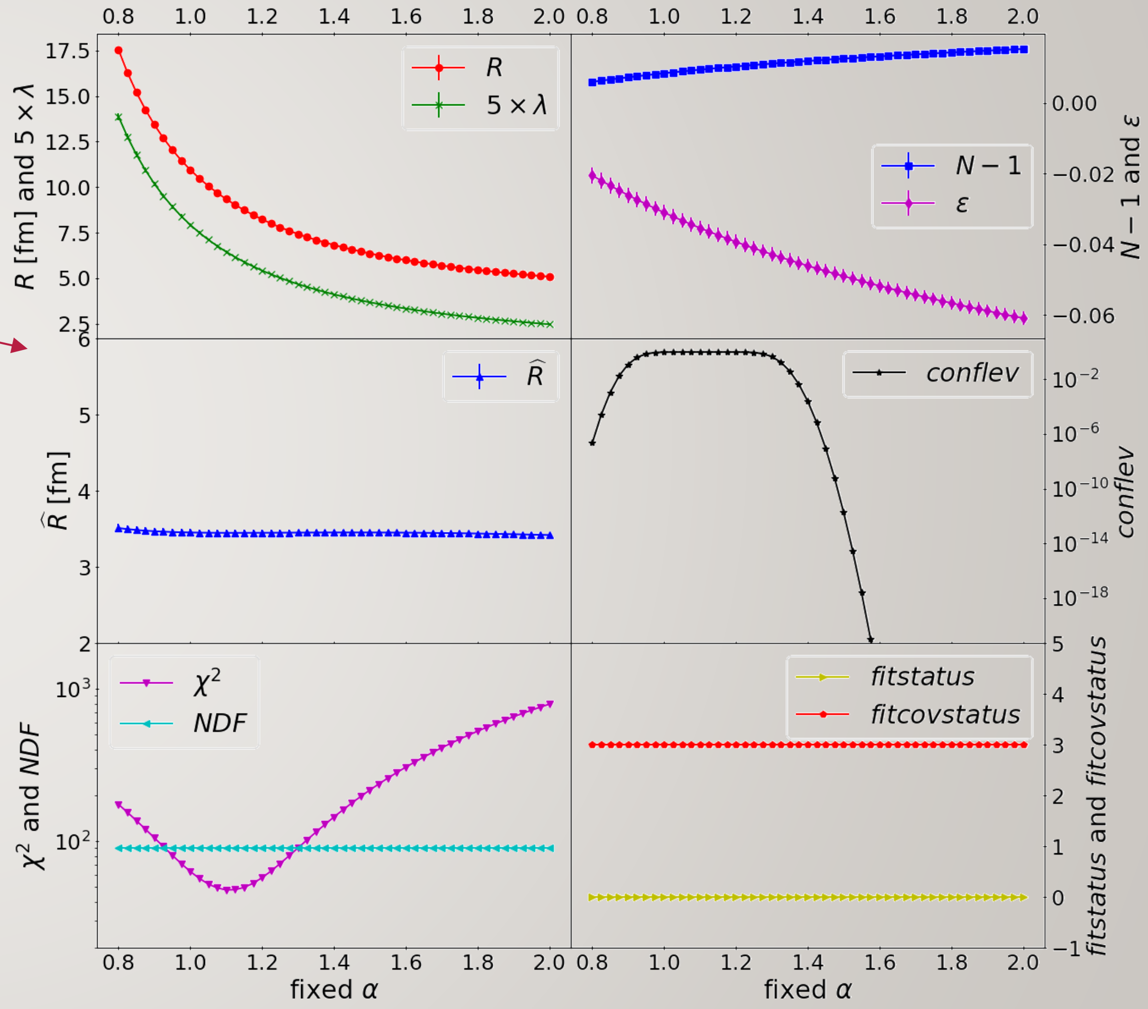


16 RESCALING HBT RADII FROM GAUSS TO LÉVY

- Source shape and size entangled in Gaussian radii
- Fits possible with many α values
  - Some statistically acceptable, some not
  - Fits to PHENIX HBT paper PRC 2018, Fig 3a
- $\hat{R} = R/[\lambda(1+\alpha)]$ scaling observed generally



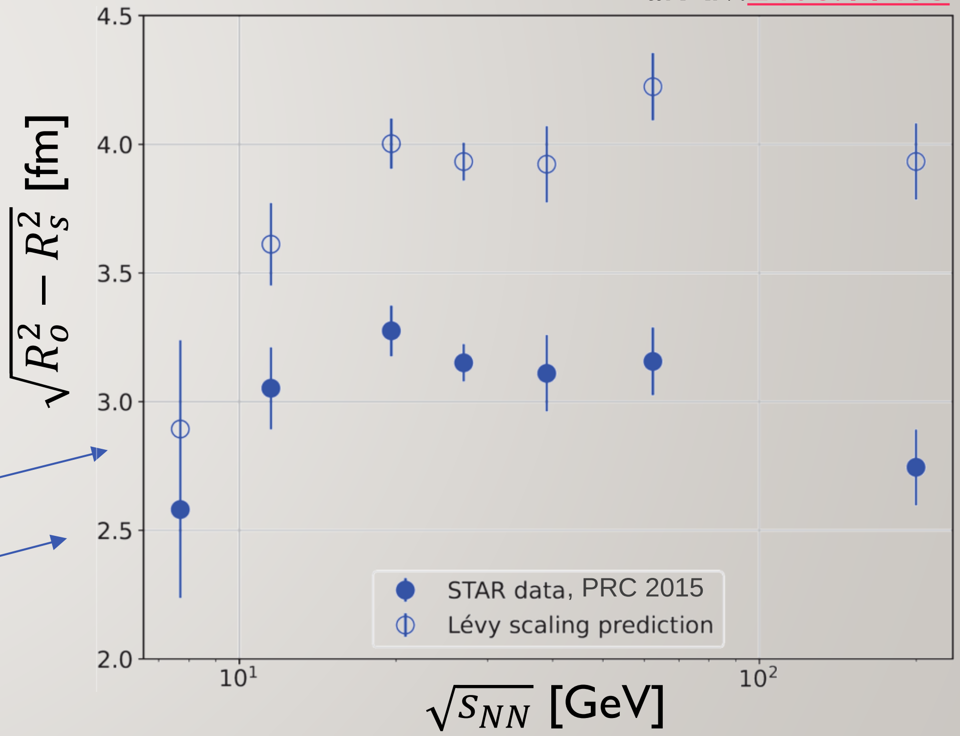
arXiv:2406.11435



ENERGY DEPENDENCE OF LÉVY SOURCE SIZE?

- $\hat{R} = \frac{R}{\lambda(1+\alpha)}$ doesn't depend on α, can estimate $R_{\text{free }\alpha} = R_{\text{Gauss}} \frac{\lambda_{\text{free }\alpha}(1+\alpha)}{\lambda_{\text{Gauss}}(1+2)}$
  - Assuming trends of α and λ as $A \cdot \sqrt{s_{NN}}^{\,B}$, with $A_\alpha = 1.85, B_\alpha = -0.06, A_\lambda = 0.6, B_\lambda = 0.06$
- Different trends of guesstimated $R_{\text{Lévy}}$ and R_{Gauss}
- Caused by shape change with $\sqrt{s_{NN}}$
- Connection of $\sqrt{R_o^2 - R_s^2}$ to emission duration: based on Gaussian sources
- Importance of measuring $R_{o,s,l}$ with free α

arXiv:2406.11435



$\hat{R}$ scaling guesstimate for Lévy radii

Original Gaussian Radii