RF Engineering Introduction

Christine Völlinger (CERN) & Manfred Wendt (BNL)

Introduction

- **Basic concepts of RF engineering, with some focus on particle accelerators**
- **This course material was presented several times at previous JUAS's, still evolving…**
  - **Topics, scope and presentation style is based on personal preferences**
    - and on some feedback from the JUAS 2023 and 2024
  - **Not meant to be complete or comprehensive, focus on the basics of 'classic' analog RF techniques**
    - Not covered: particle beam dynamics, LLRF, digital RF.
    - Additional material, examples and exercises can be found in *chapter II.2 – RF engineering* of the JUAS Book: https://e-publishing.cern.ch/index.php/CYRSP/article/view/1618
      - Highly recommended!!!
- **Let's try to learn interactive**
  - **For most sessions we tried to include some little quiz, please don't be shy and answer!**
  - **Try to work in teams with your colleagues, also for the exam preparation, but NOT during the exam!!!**
  - **Make use of additional material / demo-software with links for download provided on indico**
    - Also, please exercise on previous exams, and actively follow the tutorials
- **Active student participation**
  - **Quiz, Q&A, PC software examples (using CST Studio, Dellsperger Smith, Qucs, etc.)**
- **Please install and test the software**
  - **https://cernbox.cern.ch/s/mmvRrQhj0LdqAJG**
  - **The CST Studio Suite 2025 needs to be installed BEFORE the lecture!**
    - Please attend the RF engineering software installation session Wednesday, 19th Feb. 2025, 16:15, room 13/2-005

RF Engineering Lecture

- **Basic concepts**
  - **Introduction in RF engineering for particle accelerators**
  - **Transmission-lines (TEM and waveguides), cylindrical (“pill-box”) resonator**
  - **Standing waves, Smith-chart, scattering parameters**

- **Some RF engineering details and examples**
  - **Accelerating structures (standing / travelling wave)**
  - **RF measurement techniques**
    - diode detector, oscilloscope, spectrum analyzer, vector network analyzer
  - **RF components**
    - Transmission-line components, couplers, filters, amplifiers, power RF

- **Preparation for the RF exam**
  - **Tutorials, Q&A**

RF Engineering Schedule

Wednesday, 19th Feb., AM	Thursday, 20th Feb., AM	Friday, 21rst Feb., AM	Monday, 24th Feb., AM	Tuesday, 25th Feb., AM
	9:00 **RF Engineering** *Manfred*: Introduction to the *Smith* Chart	**9:00** **RF Engineering** *Christine*: Pillbox Resonator I	**9:00** **RF Engineering** *Christine*: Accelerating Structures	**9:00** **RF Engineering** *Manfred*: RF measurement techniques
		15 minutes break		
10:00 **RF Engineering** *Christine & Manfred*: Some practical notes *Christine*: RF Introduction	**10:00** **RF Engineering** *Christine*: Transmission-lines II	**10:00** **RF Engineering** *Michela*: 3D Numerical Analysis of Pillbox Eigenmodes	**10:00** **RF Engineering** *Manfred*: RF components – a selection	**10:00** **RF Engineering** *Michele, Manfred*: Tutorials (exam prep on S-parameters, Smith chart and pill-box resonator)
		15 minutes break		
11:00 **RF Engineering** *Christine*: Transmission-lines I	**11:00** **RF Engineering** *Manfred*: S-Parameters	**11:00** **RF Engineering** Manfred: PC exercise with Qucs *Christine*: Pillbox Resonator II		

RF Engineering Schedule

Monday, 3rd March, AM	Tuesday, 11th March, all day	Wednesday, 12th March, all day
9:00 Morning until 10:30 Written Examination RF Engineering	9:30 All day (CERN Prevessin) Practical Days RF Measurements *Michele*	9:30 All day (CERN Prevessin) Practical Days RF Measurements *Michele*

What is RF (Radio Frequency)?

Band name	Abbreviation	ITU band number	Frequency and Wavelength	Example Uses
Extremely low frequency	ELF	1	3–30 Hz 100,000–10,000 km	Communication with submarines
Super low frequency	SLF	2	30–300 Hz 10,000–1,000 km	C[illegible]ication with subma[illegible]
Ultra low frequency	ULF	3	300–3,000 Hz 1,000–100 km	Submarine communica[illegible]tion within mines
Very low frequency	VLF	4	3–30 kHz 100–10 km	Navigation, time signals, submarine co[illegible]atio[illegible]ss heart rate mo[illegible]
Low frequency	LF	5	30–300 kHz 10–1 km	[illegible]avigation, time signals, AM long[illegible]roadcasting (Europe an[illegible] of Asia)[illegible]o
Medium frequency	MF	6	300–3,000 kHz 1,000–100 m	AM (medium-wave) broadcasts, amateur radio, avalanche beacons
High frequency	HF	7	3–30 MHz 100–10 m	Shortwave broadcasts, citizens band radio, amateur radio and over-the-horizon aviation communications, RFID, over-the-horizon radar, automatic link establishment (ALE) / near-vertical incidence skywave (NVIS) radio communications, marine and mobile radio telephony
Very high frequency	VHF	8	30–300 MHz 10–1 m	FM, television broadcasts, line-of-sight ground-to-aircraft and aircraft-to-aircraft communications, land mobile and maritime mobile communications, amateur radio, weather radio
Ultra high frequency	UHF	9	300–3,000 MHz 1–0.1 m	Television broadcasts, microwave oven, microwave devices/communications, radio astronomy, mobile phones, wireless LAN, Bluetooth, ZigBee, GPS and two-way radios such as land mobile, FRS and GMRS radios, amateur radio, satellite radio, Remote control Systems, ADSB.
Super high frequency	SHF	10	3–30 GHz 100–10 mm	Radio astronomy, microwave devices/communications, wireless LAN, DSRC, most modern radars, communications satellites, cable and satellite television broadcasting, DBS, amateur radio, satellite radio.
Extremely high frequency	EHF	11	30–300 GHz 10–1 mm	Radio astronomy, high-frequency microwave radio relay, microwave remote sensing, amateur radio, directed-energy weapon, millimeter wave scanner, Wireless Lan 802.11ad.
Terahertz or Tremendously high frequency	THz or THF	12	300–3,000 GHz 1–0.1 mm	Experimental medical imaging to replace X-rays, ultrafast molecular dynamics, condensed-matter physics, terahertz time-domain spectroscopy, terahertz computing/communications, remote sensing

Source: en.wikipedia.org/wiki/Radio_spectrum

What is RF (Radio Frequency)?

Source: en.wikipedia.org/wiki/Radio_spectrum

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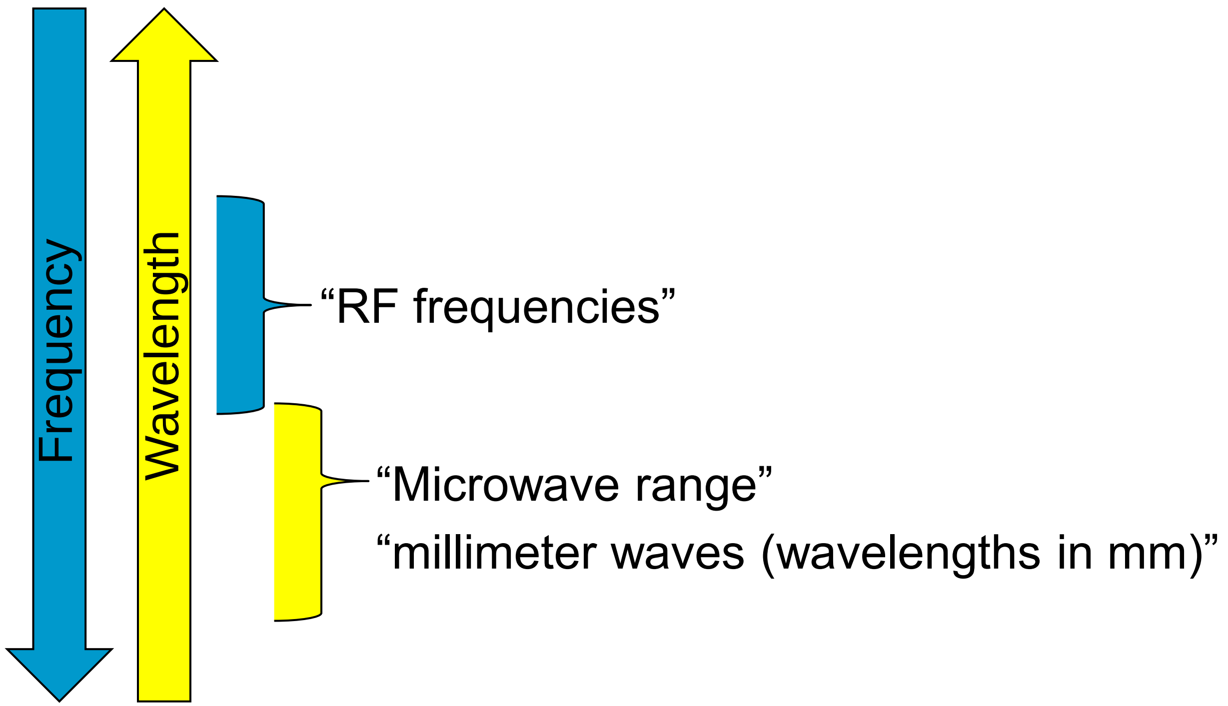
High frequency	HF	7	3–30 MHz 100–10 m
Very high frequency	VHF	8	30–300 MHz 10–1 m
Ultra high frequency	UHF	9	300–3,000 MHz 1–0.1 m
Super high frequency	SHF	10	3–30 GHz 100–10 mm
Extremely high frequency	EHF	11	30–300 GHz 10–1 mm
Terahertz or Tremendously high frequency	THz or THF	12	300–3,000 GHz 1–0.1 mm



What you should know by heart:

Hz "Hertz" is the unit of frequency: sec^{-1}

1 MHz = 10^6 Hz (Megahertz)
1 GHz = 10^9 Hz (Gigahertz)

Frequency f and wavelength λ are inversely proportional:
Large frequency means small wavelength.

What is Radio Frequency (for accelerators)?

Source: en.wikipedia.org/wiki/Radio_spectrum

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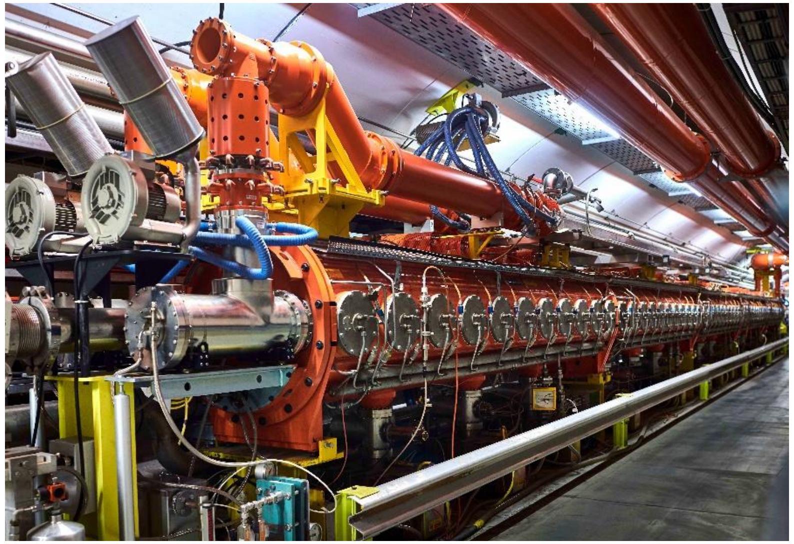
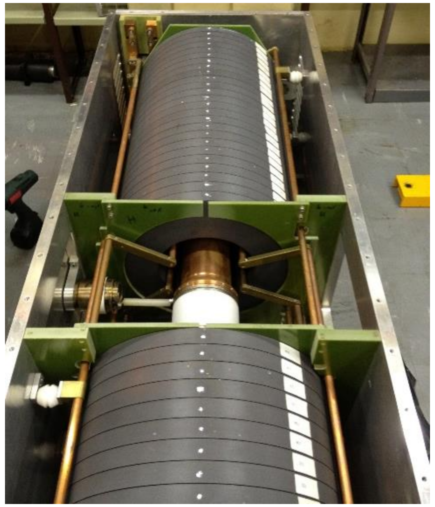


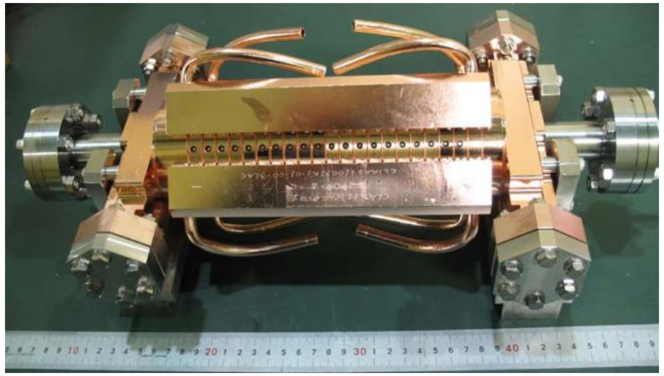
Travelling wave cavity, freq = 200 MHz
Total length: 16 m. (CERN SPS)

approx. 2 m

Accelerating Cavity, freq = 80 MHz
(CERN PS)



Ferrite Loaded Cavity,
freq = 3 – 8 MHz
(CERN PS Booster)

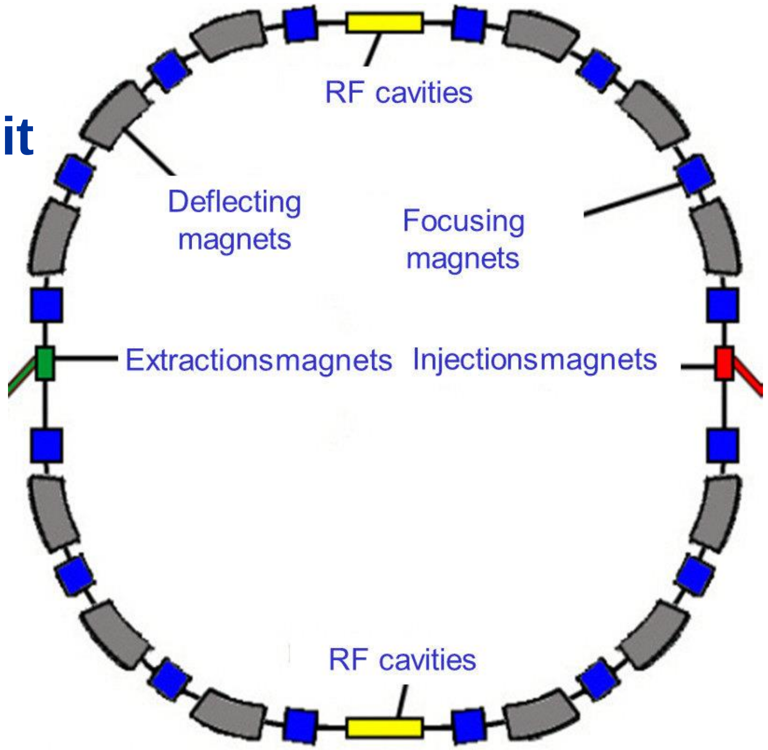


CLIC structure, freq = 12 GHz

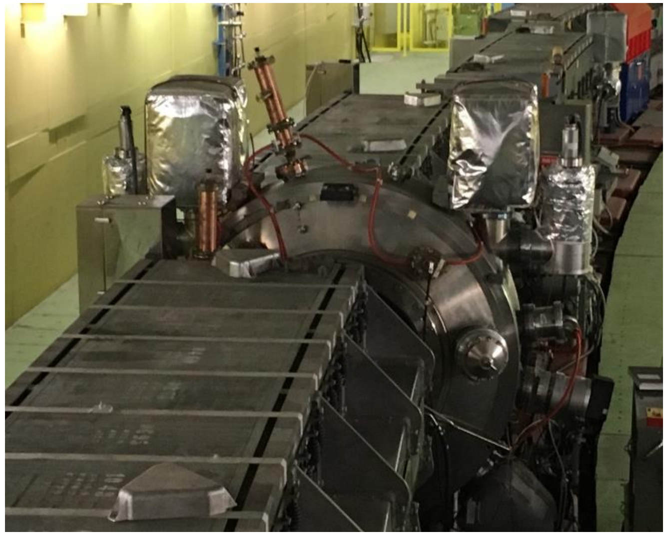
What is Radio Frequency (for accelerators)?

- **RF domain is huge!**
- **The radiofrequency (RF) system does the actual acceleration - it transforms a string of magnets into an accelerator.**

- **Cavity is the most visible part of an RF system**
  - → **On top of the RF system food chain**
  - → **Interacts directly with beam**
  - → **Provides acceleration and RF manipulations, but also some unwanted effects (e.g. beam coupling impedance contribution)**
  - → **Cavity is no “stand-alone” element…**
  - → **… requires RF signals generated (LLRF system) to be operated successfully and power input to transfer energy to the beam.**



A simplified RF System



PS single cell cavity
(“pillbox”)

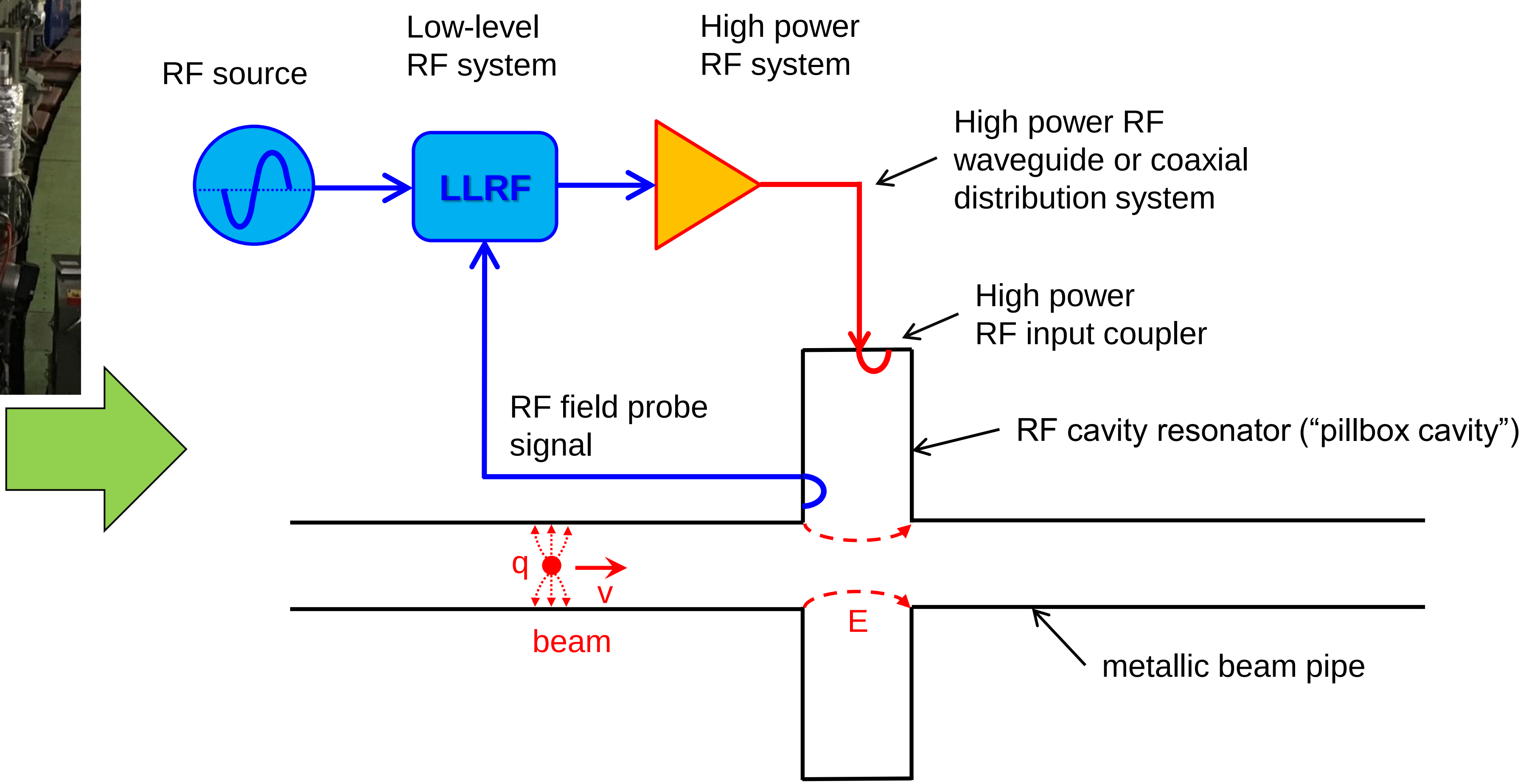


Illustration stolen from M. Wendt

Purpose of this lecture

From the introduction of Maxwell's equation (lecture of Andrea Mostacci), you know already:

- What are the sources of electro-magnetic fields.
- Differential form of Maxwell's equations.

Now, we will look into:

- Lorentz' force (direct application of M.E. to particle accelerators).
- Time-varying fields and the wave equation.
- Plane waves as simplest solution of the wave equation, and corresponding terms like:
  - Frequency f, radian frequency ω and wavelength λ
  - Propagation constant k (real and complex), attenuation constant α and phase constant β
  - Phase velocity v_p and wave impedance
  - Skin depth (penetration depth) δ

Purpose of this lecture

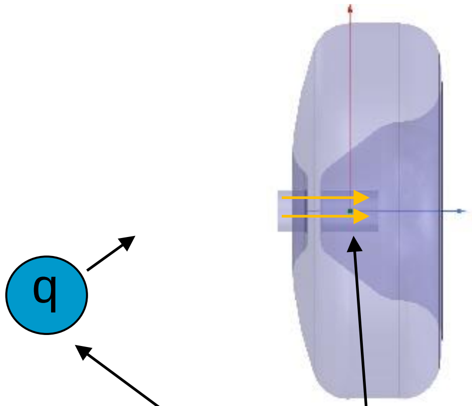
From M.E. in differential form to Helmholtz' equation… these give the basis of our field description.

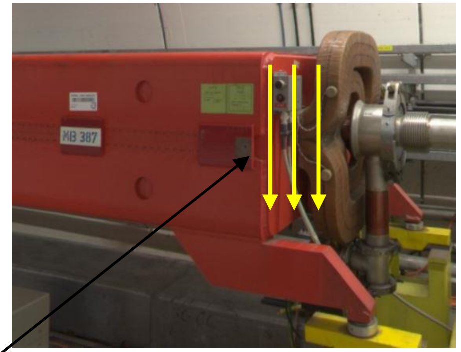
$$\nabla \cdot \vec{D} = \rho \quad \text{or} \quad \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$





$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Accelerating force and *bending force* in a circular accelerator.

Basic Wave Equation & Figures of Merit

… can't spare you some maths, but
at least it is not new…

From Maxwell's curl equations ("differential form") →

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{H} = \mu\epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

Wave equation or *Helmholtz' equation*

(see talk of Andrea Mostacci for details)

What you should know:

Equally, we can write *Helmholtz' equation* in phasor notation (source-free, linear media, harmonic time dependence):

$$\nabla^2 \vec{E} + \omega^2 \mu\epsilon \vec{E} = 0$$

$$\nabla^2 \vec{H} + \omega^2 \mu\epsilon \vec{H} = 0$$

define →

Propagation constant k
(or: phase constant, or wave number, or separation constant), with unit 1/m

$$k = \omega\sqrt{\mu\epsilon}$$

$$\omega = 2\pi f$$

is the *radian frequency*

Basic Wave Equation & Figures of Merit

$$\nabla^2\vec{E} + \omega^2\mu\epsilon\vec{E} = 0$$

This is the equation to solve…

Simplest solution is: Plane wave (no variation in x and y).
We assume prop

Quiz!

Test your knowledge.

y domain

$$\frac{d^2E_x}{dz^2} + k^2E_x$$

$\cos(\omega t + kz)$

n in time domain

E^+ and E^- denote the wave amplitudes for travelling in positive and negative z-direction.

We will now consider 3 different cases to introduce the yet unknown figures of merit.

1. *Plane wave in a lossless medium,*
2. *Plane wave in lossy medium,*
3. *Plane wave in a good conductor.*

Quiz no. 1

1. **What is the unit of frequency?**
   - m
   - 1/sec
   - mm/sec
   - Ohm

2. **With increasing frequency, does the wavelength of a microwave reduce or increase?**

   - wavelength reduces with increasing frequency
   - wavelength increases with increasing frequency
   - wavelength is a constant and does not change

3. **What is the phase velocity of a wave travelling in vacuum?**

   - $v_p = \lambda \cdot f$
   - c_0 (speed-of-light)
   - ω

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   - ω

Basic Wave Equation & Figures of Merit

1. Solution for plane wave in a lossless medium

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0 \qquad k = \omega\sqrt{\mu\epsilon} \qquad \omega = 2\pi f$$

$$E_x(z,t) = Re\{E_x(x,\omega)e^{j\omega t}\} = E^+ \underbrace{\cos(\omega t - kz)} + E^- \underbrace{\cos(\omega t + kz)}$$

wave travelling in positive z-direction — wave travelling in negative z-direction

How can we "see" that?

$$(\omega t - kz)$$

Keep the cosine argument constant to maintain a fixed point of the wave: for increasing time, z has to increase as well → this describes a propagation in positive z-direction. ☺

Basic Wave Equation & Figures of Merit

1. Solution for plane wave in a lossless medium

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wave travelling in positive z-direction — wave travelling in negative z-direction

What you should know

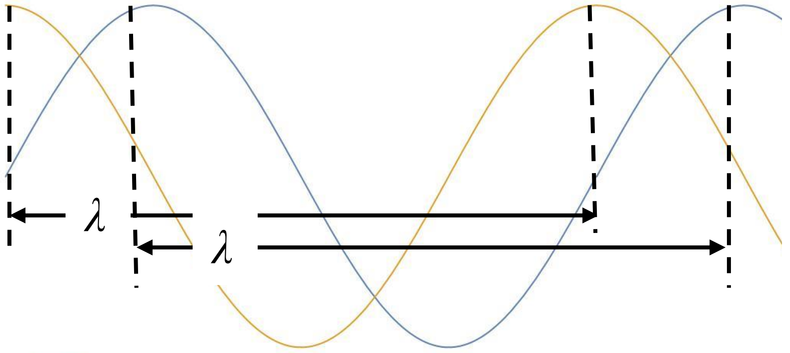
We define Phase velocity v_p *as the velocity at which a fixed phase point travels:*

$$(\omega t - kz) = constant!$$

$$v_p = \frac{dz}{dt} = \frac{d}{dt}\left(\frac{\omega t - constant}{k}\right) = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}}$$

In free-space, this would be the speed of light: $v_p = c_0 = \frac{1}{\sqrt{\mu_0\epsilon_0}}$

Basic Wave Equation & Figures of Merit



1. Solution for plane wave in a lossless medium

We define → Wavelength λ as the distance between two successive reference points on the wave at a fixed instant in time:

$$(\omega t - kz) - [\omega t - k(z+\lambda)] = 2\pi$$

Position 'z'   Position 'z+ λ'

Hence: $$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\omega\sqrt{\mu\epsilon}} = \frac{2\pi v_p}{\omega} = \frac{v_p}{f}$$

What you should know by heart:

$$\lambda f = v_p$$

We define → Wave impedance for plane waves = intrinsic impedance of a medium

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\ \Omega$$

in vacuum,
also called *vacuum impedance*

2. Solution for plane wave in a conductive (lossy) medium

Take the plane wave description for harmonic time dependence (slide 9), and the propagation constant k.
In the case of conductive medium, we observe losses and k becomes complex.

Remember? From: $\nabla^2 \vec{E} + \omega^2 \mu\epsilon \vec{E} = 0$ ➡ $\frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0$ $\quad k = \omega\sqrt{\mu\epsilon}$

If we now take conductive (lossy) medium, *conductivity σ remains in the equations*.

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$
$$\nabla \times \vec{H} = j\omega\epsilon\vec{E} + \sigma\vec{E}$$

➡ $\nabla^2 \vec{E} + \omega^2 \mu\epsilon \left(1 - j\frac{\sigma}{\omega\epsilon}\right) \vec{E} = 0$ ➡ $\frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0$

Complex propagation constant

define ➡ $\gamma = j\omega\sqrt{\mu\epsilon}\sqrt{1 - j\frac{\sigma}{\omega\epsilon}} = \alpha + j\beta$

phase constant (β)

attenuation constant (α)

2. Solution for plane wave in a conductive medium (lossy)

What you should know

define → $$\gamma = j\omega\sqrt{\mu\epsilon}\sqrt{1 - j\frac{\sigma}{\omega\epsilon}} = \alpha + j\beta$$

- γ: *Complex propagation constant*
- β: *phase constant*
- α: *attenuation constant*

Important remarks:

1. The same equation for the complex propagation constant was derived in the lecture of Andrea Mostacci by using a complex permittivity.
2. If the conductivity is zero, we obtain: $\gamma = j\omega\sqrt{\mu\epsilon} = jk$
3. In textbooks, formulae for attenuation constant differs, depending on the used definition.
   Be careful and check the naming convention used in the different books before you start calculating!

Basic Wave Equation & Figures of Merit

3. Solution for plane wave in a good conductor (good, but not perfect!)

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$
$$\nabla \times \vec{H} = j\omega\epsilon\vec{E} + \sigma\vec{E}$$

➡ $$\nabla^2\vec{E} + \omega^2\mu\epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right)\vec{E} = 0$$ ➡ $$\frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0$$

Good conductor means that *displacement current is considerably smaller than conduction current:* $\sigma \gg \omega\epsilon$

➡ $$\gamma = \alpha + j\beta \simeq j\omega\sqrt{\mu\epsilon}\sqrt{\frac{\sigma}{j\omega\epsilon}} = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}$$

What you should know

define ➡ skin depth (= penetration depth): $\delta_s = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$

Depth over which amplitude of the EM-field will decrease by a factor of 1/e.

Side remark:

$$\sqrt{j} = \frac{1+j}{\sqrt{2}}$$

Plane wave propagation at a glance…

TABLE 1.1 Summary of Results for Plane Wave Propagation in Various Media

	Type of Medium		
Quantity	Lossless ($\epsilon'' = \sigma = 0$)	General Lossy	Good Conductor ($\epsilon'' \gg \epsilon'$ or $\sigma \gg \omega\epsilon'$)
Complex propagation	$\gamma = j\omega\sqrt{\mu\epsilon}$	$\gamma = j\omega\sqrt{\mu\epsilon}$	$\gamma = (1+j)\sqrt{\omega\mu\sigma/2}$
[illegible]	[illegible]	[illegible]	[illegible]
Wavelength	$\lambda = 2\pi/\beta$	$\lambda = 2\pi/\beta$	$\lambda = 2\pi/\beta$
Phase velocity	$v_p = \omega/\beta$	$v_p = \omega/\beta$	$v_p = \omega/\beta$

2nd quiz!

Test your knowledge.

Source: Pozar, Microwave engineering, 4th ed., Wiley

Quiz no. 2

The table shows the different electrical conductivities of copper, gold and aluminum:

Material	Resistivity, ρ, at 20 °C (Ω·m)	Conductivity, σ, at 20 °C (S/m)
Silver[d]	1.59×10^{-8}	6.30×10^{7}
Copper[e]	1.68×10^{-8}	5.96×10^{7}
Annealed copper[f]	1.72×10^{-8}	5.80×10^{7}
Gold[g]	2.44×10^{-8}	4.11×10^{7}
Aluminium[h]	2.65×10^{-8}	3.77×10^{7}
Calcium	3.36×10^{-8}	2.98×10^{7}

Source: https://en.wikipedia.org/wiki/Electrical_resistivity_and_conductivity

$$\delta_s = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

1. **Which metal has the lowest field penetration depth?**
   - Silver
   - Calcium
   - Gold

2. **Does the skin depth increase or reduce with increasing frequency?**
   - skin depth increases with increasing frequency
   - skin depth reduces with increasing frequency

Quiz no. 2

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1. **Which metal has the lowest field penetration depth?**
   - Silver (correct answer)
   - Calcium
   - Gold

2. **Does the skin depth increase or reduce with increasing frequency?**
   - skin depth increases with increasing frequency
   - skin depth reduces with increasing frequency (correct answer)

RF Engineering terms…decibels: *dB*

- *Decibels* are used to express large number ranges by *using the base 10 logarithm of numbers.*
- *Decibels* are very handy to cover several orders of magnitude, for example power from *mW* to *MW*…

$$P_1\,[dB] = 10\log_{10}\left(\frac{P_1}{P_{\text{ref}}}\right)$$

- Although the ratio gives a dimensionless expression, we call the result *"Bel"* - after Alexander Bell. The factor of "10" then just compensates for *deci-Bel (dB)*. *(yes, with only one 'l', must have been a physicist typing…)*

Let's develop this:

$$P_1 = 90W \quad P_{\text{ref}} = 1W \;\Rightarrow\; P_1\,[B] = \log_{10}\left(\frac{P_1}{P_{\text{ref}}}\right) = \log_{10}\left(\frac{90\ W}{1\ W}\right) = 1.95 \;\Rightarrow\; P_1\,[dB] = 19.5$$

Trivial. Just like *1 deci-meter = 0.1 meter*.

RF Engineering terms…decibels: *dB* and *dBm*

- Backwards conversion is also possible, of course:

$$a\,[dB] = 10\log_{10}\left(\frac{P_1}{P_2}\right) \quad \Rightarrow \quad \frac{P_1}{P_2} = 10^{\frac{a[dB]}{10}}$$

- Very often, the *power of 1 mW is taken as reference quantity*. In this case, we speak of *"dBm"*:

$$P_1\,[dBm] = 10\log_{10}\left(\frac{P_1}{1\ mW}\right)$$

RF Engineering terms…decibels: *dB*

- Finally, let's speak about *voltage decibels* and *power decibels* (that's just nonsense!).
- **There is only one type of decibels as it is a ratio. A value given in dB is always the same.**
- However, if we express our power values in voltages…. we could create *voltage decibels*… ☠

Remember?

$$P_1\,[dB] = 10\log_{10}\left(\frac{P_1}{P_{\text{ref}}}\right)$$

$$P_1 = \frac{1}{2}\frac{V_1^2}{R_1}$$

$$P_2 = \frac{1}{2}\frac{V_2^2}{R_2}$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{V_1^2}{R_1}\cdot\frac{R_2}{V_2^2} = \frac{V_1^2}{V_2^2}$$

If (!): $R_1 = R_2$

Let's use the identity:

$$\log_{10}(x^y) = y\ \log_{10}(x) \Rightarrow 10\ \log_{10}\left(\frac{V_1^2}{V_2^2}\right) = 2\cdot 10\ \log_{10}\left(\frac{V_1}{V_2}\right) = 20\ \log_{10}\left(\frac{V_1}{V_2}\right)$$

☠ "voltage decibels"

RF Engineering terms…decibels: *dB*

This can be confusing, if you are not used to it.

- Remember what we said before:
  There is only one type of decibels as it is a ratio. A value given in dB is always the same.

$$P_1\,[dB] = 10\log_{10}\left(\frac{P_1}{P_{\mathrm{ref}}}\right)$$

(Power value as input)

$$V_1\,[dB] = 20\log_{10}\left(\frac{V_1}{V_{\mathrm{ref}}}\right)$$

(Voltage value as input)

Example:

$$V_1 = 1\,V \quad V_2 = 3\,V \quad R = 10\,\Omega$$

$$P = \frac{1}{2}\frac{V^2}{R}$$

$$20\log_{10}\left(\frac{V_1}{V_2}\right) = 20\log_{10}\left(\frac{1}{3}\right) = -9.54\,dB$$

$$10\log_{10}\left(\frac{P_1}{P_2}\right) = 10\log_{10}\left(\frac{1}{9}\right) = -9.54\,dB$$

RF Engineering terms…decibels: *dB*

- S-parameters, the standard measurement value used in RF (more details later) are often expressed in dB. Per definition, S-parameters are already ratios thus no explicit reference is needed.
- Power ratio = voltage ratio squared.
- Some common values expressed in dB:

	power ratio	V, I, E or H ratio, S-parameter
-20 dB	0.01	0.1
-10 dB	**0.1**	**0.32**
-3 dB	**0.50 (-50%)**	**0.71**
-1 dB	0.794 (about -20%)	0.891 (about-11%)
0 dB	**1**	**1**
1 dB	1.258 (about+26%)	1.122 (about+12%)
3 dB	**2.00 (+100%)**	**1.41**
10 dB	**10**	**3.16**
20 dB	100	10
n * 10 dB	10^{n}	$10^{n/2}$

RF Engineering terms…decibels: *dB*

- Let's have a closer look and **let's talk RF-engineering**:

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3 dB	**2.00 (+100%)**	
10 dB	**10**	
20 dB	100	
n * 10 dB	10^n	

"I measure 3dB less" → The measured power is only half of the reference power.

$$-3dB\ @\ 1W = P_{ref} \Rightarrow 0.5W = P_{meas}$$

$$W = P_{meas}$$

Last quiz!

Test your knowledge.

" We see a 3dB increase..

But: the measured voltage is higher by a factor ~1.5.

" I hardly see any signal anymore" → (Yup, I put in a 20 dB attenuator)
… so our signal went down by a factor of….?

"Upps, 33 dB more power, is that enough?" → 33dB = 20dB + 10dB + 3dB (*so your power increases by: 100*10*2 = **2000***)

Quiz no. 3

1. **If I have -5dB on a signal, is this amplifying or reducing the signal?**
   - amplifying the signal
   - reducing the signal
   - both

2. **If my amplifier gives an amplification of 0dB, can I be happy with this?**
   - yes, this is a good amplification value
   - no, this is a poor amplification value
   - this is not amplifying

3. **If we see a 3dB increase, the measured power is:**
   - half the reference power
   - twice the reference power
   - power does not change, only the voltage changes

Quiz no. 3

1. **If I have -5dB on a signal, is this amplifying or reducing the signal?**
   - amplifying the signal
   - reducing the signal (correct answer)
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   - half the reference power
   - twice the reference power (correct answer)
   - power does not change, only the voltage changes

Thank you for your attention!

Let's have a break!

Further reading…

1. POZAR, David M., *“Microwave Engineering”*, 4th edition, Wiley and sons.
2. ZHANG, Keqian, “*Electromagnetic Theory for Microwaves and Optoelectronics*”, 2nd edition, Springer