Overview of Magnetic Measurement Techniques

References

[1] A.K. Jain, “Overview of Magnetic Measurement Techniques”, US Particle Accelerator School, 2006 (most of the following slides)

[2] M. Buzio, “Fabrication and calibration of search coils”, Proceedings of CAS - CERN Accelerator School on Magnets, 2009

[3] A.K. Jain, “Measurements of Field Quality Using Harmonic Coils”, US Particle Accelerator School, 2001 (the annex)

[4] S. Russenschuck, “Field Computation for Accelerator Magnets: Analytical and Numerical Methods for Electromagnetic Design and Optimization”, Wiley, 2011

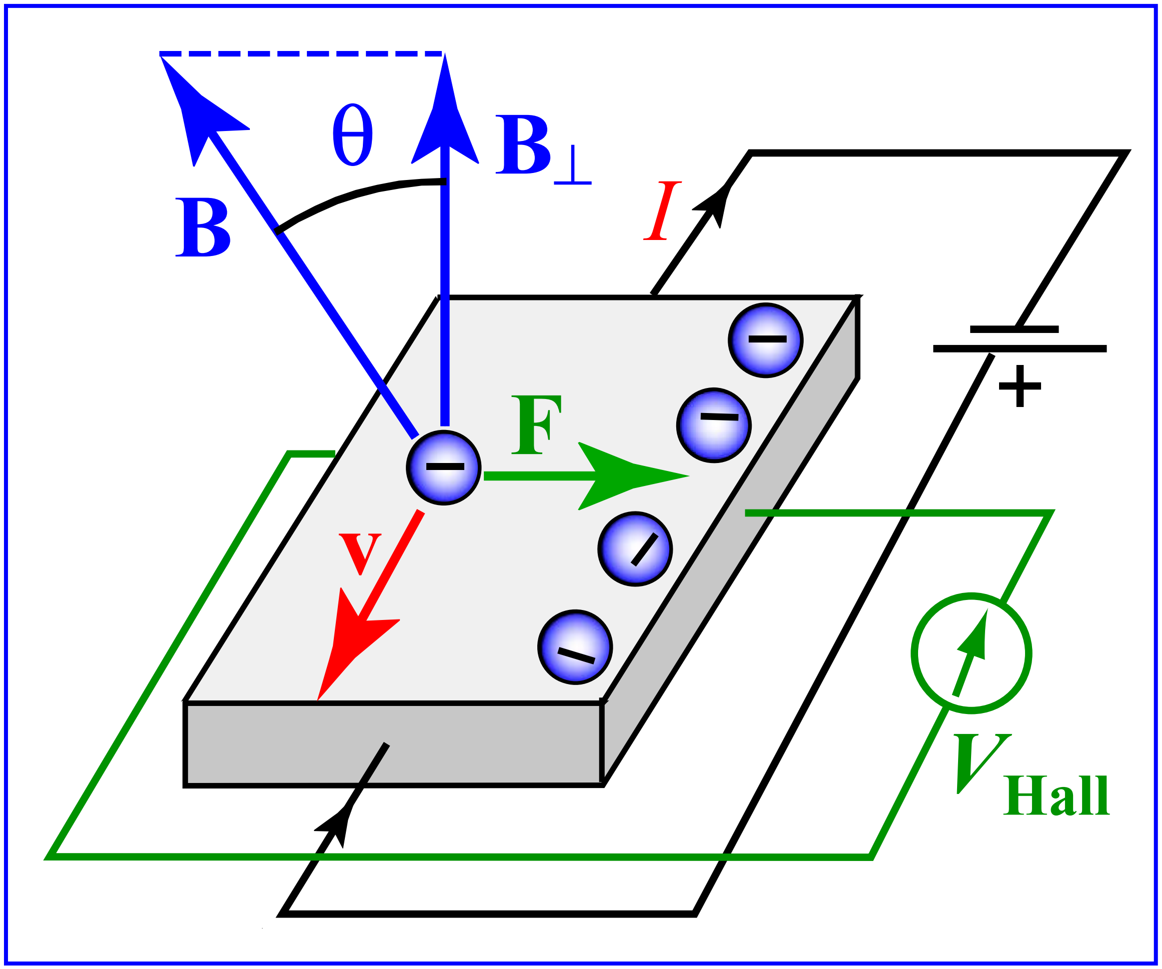
[5] CERN Accelerator School on Normal- and Superconducting Magnets, 2023

[6] Proceedings of CAS - CERN Accelerator School on Magnets, 2009

Outline

- Hall Probes
- Flux Measurements (with Coils and Wire)
- Nuclear Magnetic Resonance (NMR)

Hall Effect



Charge carriers experience a **Lorentz force** in the presence of a magnetic field.

This produces a steady state voltage in a direction perpendicular to the current and field.

$$V_{\text{Hall}} = G \cdot R_H \cdot I \cdot B \cos \theta$$

G = Geometric factor

R_H = Hall Coefficient

Hall Sensor Advantages

- ***Simple***, inexpensive devices, ***commercially available***.
- ***Small*** probe size makes it suitable for a large variety of applications.
- Can measure all ***field components***.
- Can be used for **fast** measurements.
- Can be used at low temperatures.
- Particularly suited for complex geometries, such as detector magnets.

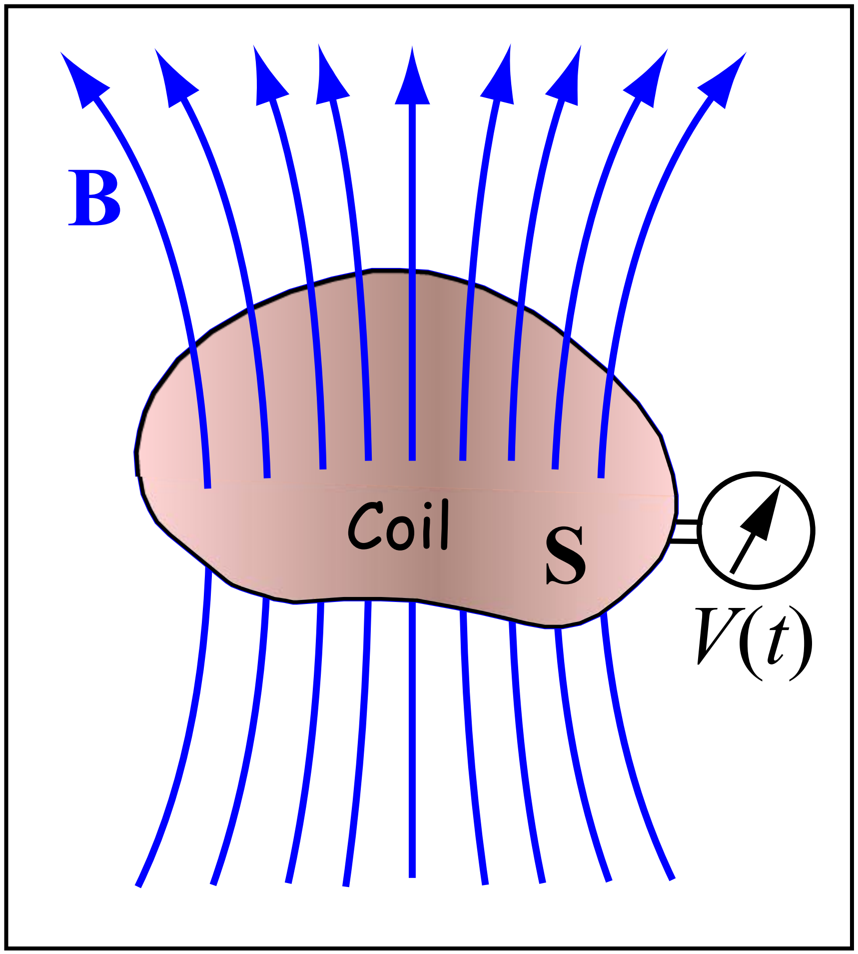
Hall Sensor Disadvantages

- ***Non-linear*** device, requires elaborate calibration of sensitivity for each probe.
- Sensitive to ***temperature***: Calibrate as a function of temperature; Keep temperature stable; Design compensated probes.
- Long term ***calibration drift***.
- ***Planar Hall effect*** can pose a problem for mapping 3-D fields. Special geometries are needed for measuring minor components.

Hall Sensor Specifications

- Typical Range: < 1 mT to 30 T
- Typical Accuracy: ~ 0.01% to 0.1%
- Typical dimensions: ~ mm
- Frequency response: DC to ~ 20 kHz (~ a few Hz for fully compensated signal)
- Time Stability: ±0.1% per year

Flux Measurements: Induction Law



Flux through a coil defined by the surface **S** is:

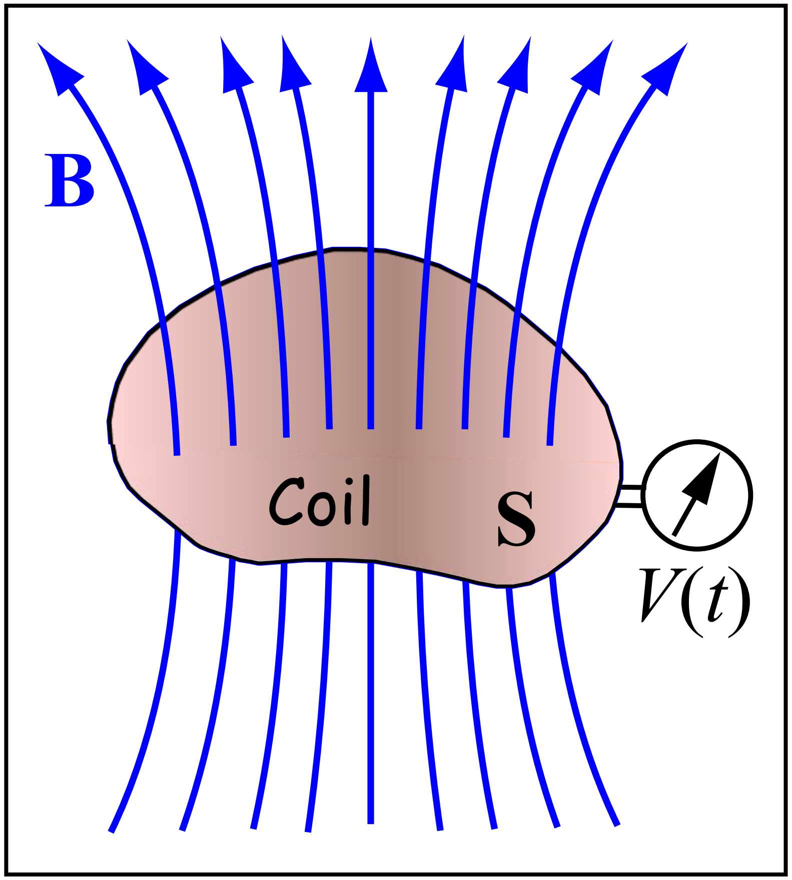
$$\Phi = \int_S \mathbf{B} \cdot d\,\mathbf{S}$$

If the flux linked varies with time, a loop voltage is induced, given by:

$$V(t) = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left[\int_S \mathbf{B} \cdot d\,\mathbf{S}\right]$$

The time dependence may be caused by either a varying field or a varying surface area vector, or both.

Flux Measurements



Time dependence of flux gives:

$$V(t) = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left[\int_S \mathbf{B} \cdot d\,\mathbf{S}\right]$$

The change in flux is given by:

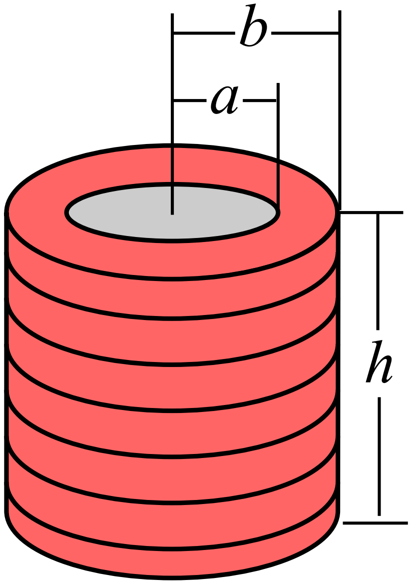
$$\Phi_{end} - \Phi_{start} = -\int_{t_{start}}^{t_{end}} V(t) \cdot dt$$

and can be measured by integrating the voltage signal.

To know the flux at a given instant, one needs to know Φ_{start}

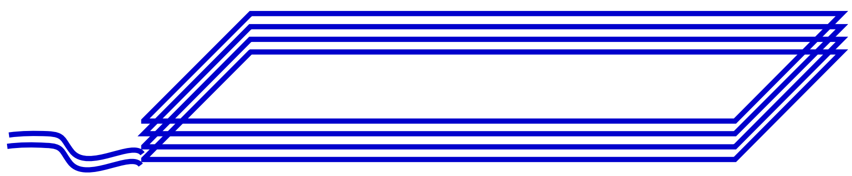
$\Rightarrow$ (1) Use $\Phi_{start} = 0$; (2) Flip Coil/Rotating coil: $\Phi_{end} = \mp\Phi_{start}$

Coil Geometries



Point Coil

Insensitive up to 4th order spatial harmonic with proper choice of height and radii.

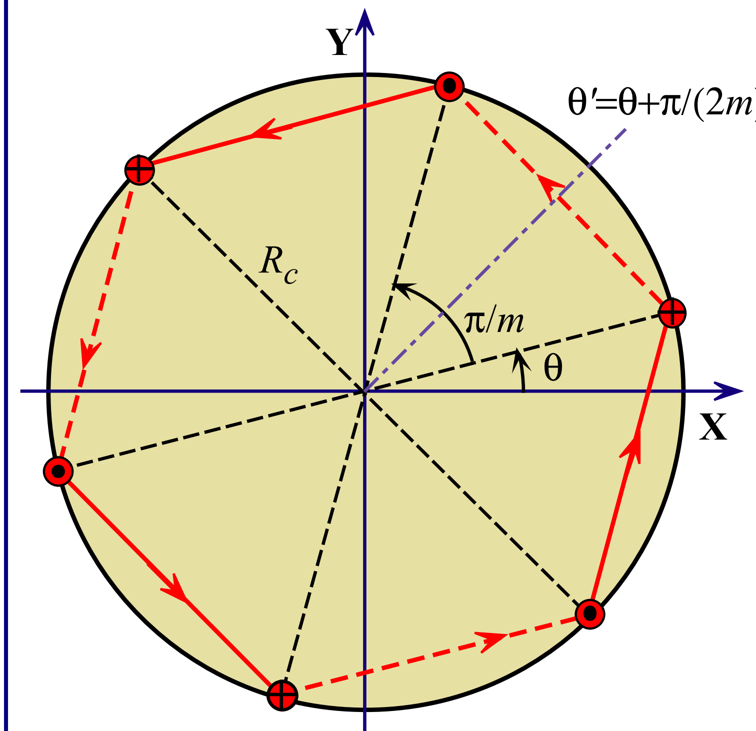


Flat Coil (Line or Area Coil)

-Fixed coil; Varying field

-Flip Coil/Moving Coil; Static field

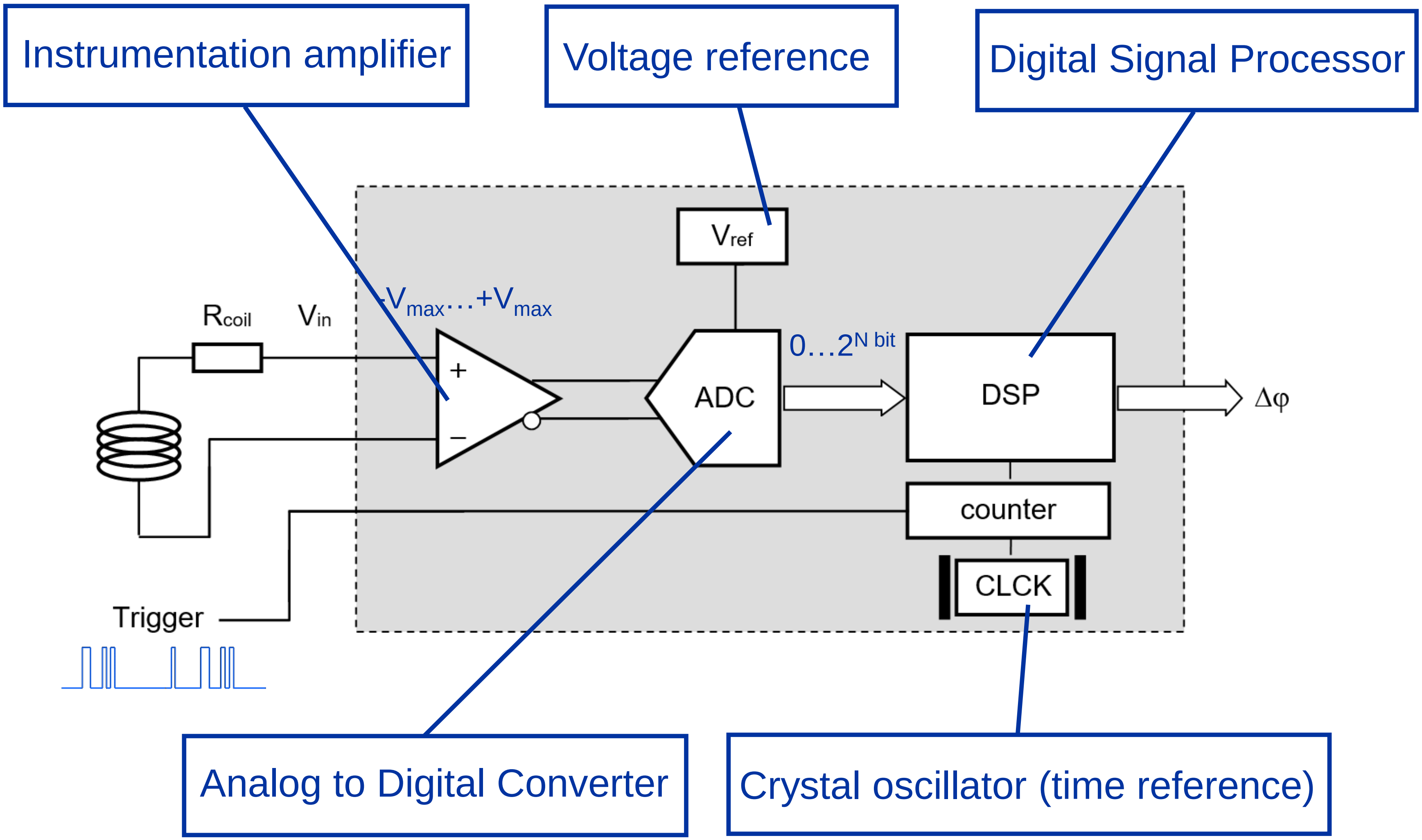
-Rotating Tangential/Radial



Multipole Coil

Sensitive to only odd multiples of a specified harmonic (Morgan Coils)

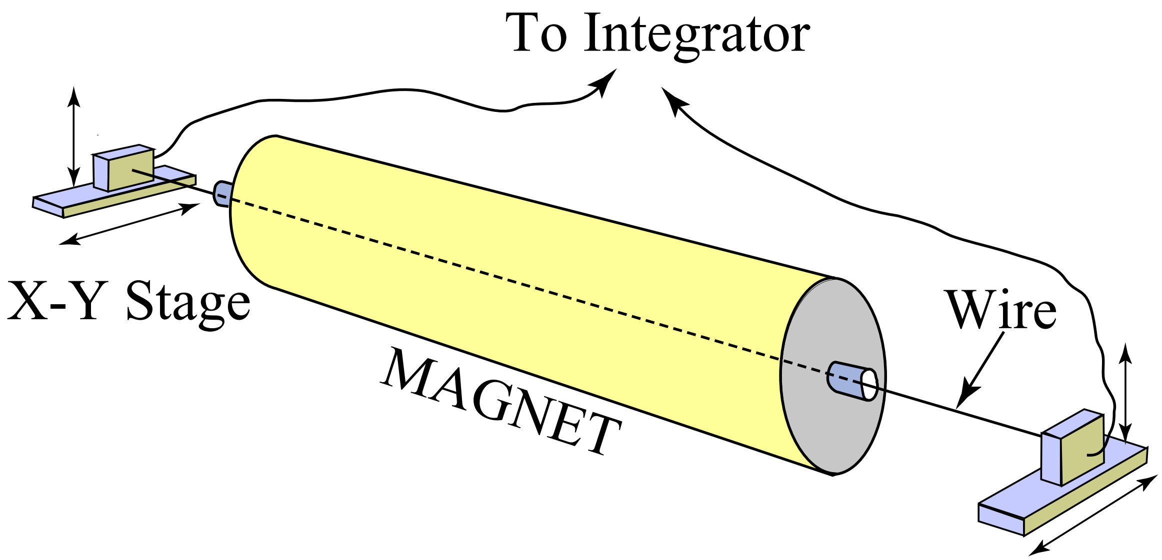
Voltage Integrator



Measurements with Coils

- ***Simple***, ***passive***, ***linear***, ***drift-free*** devices.
- Require ***change in flux*** ⇒ ramp field with static coil, or move coil in a static field. Pay attention to ramping/moving details.
- Measure ***flux***, not ***field***. ⇒ ***Calibration of geometry*** very important; limits *accuracy*.
- Field variations across the coil area must be accounted for ⇒ ***harmonic analysis***.

A special case: Stretched Wire

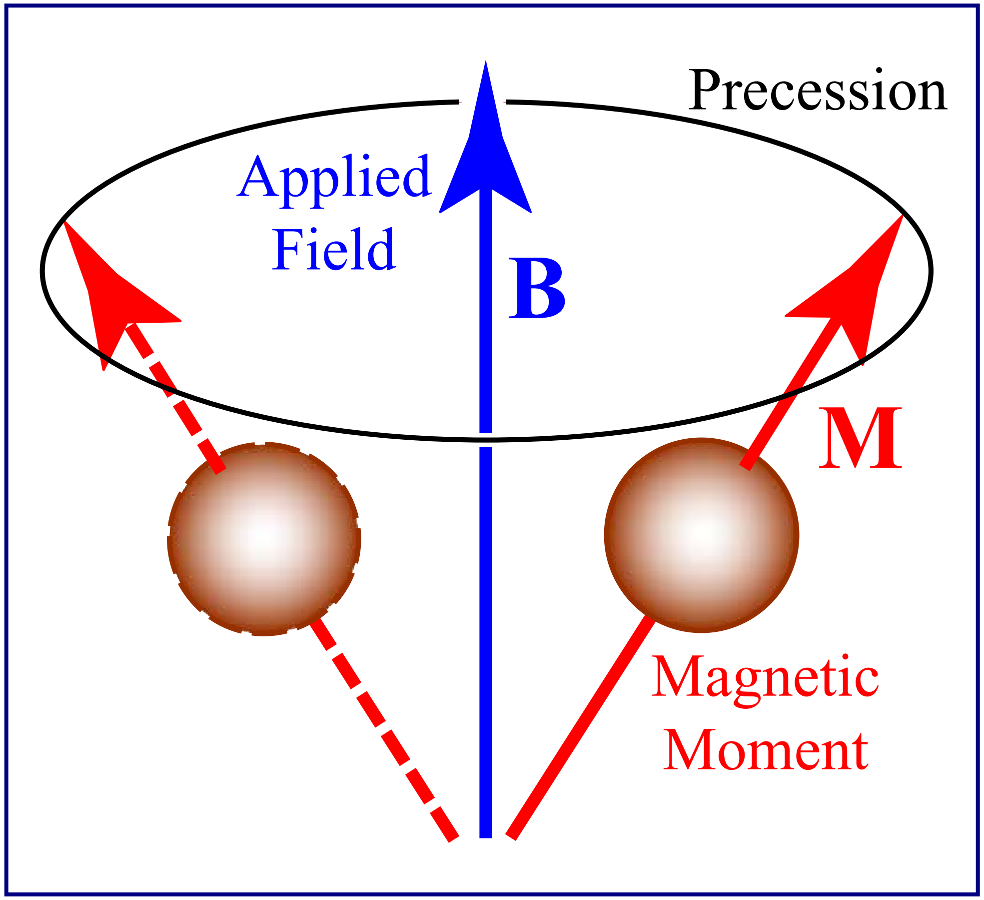


Stretched Wire Measurement

- Move a stretched wire in a magnet
- Measure change in flux for various types of motion
- Use expected field symmetry to locate the magnetic center

Nuclear Magnetic Resonance

Isotope with a nonzero nuclear spin



I = Spin

γ = Gyromagnetic ratio

$\mathbf{M}$ = Magnetic Moment

= $\gamma.h.I$

Energy = $\mathbf{B.M}$

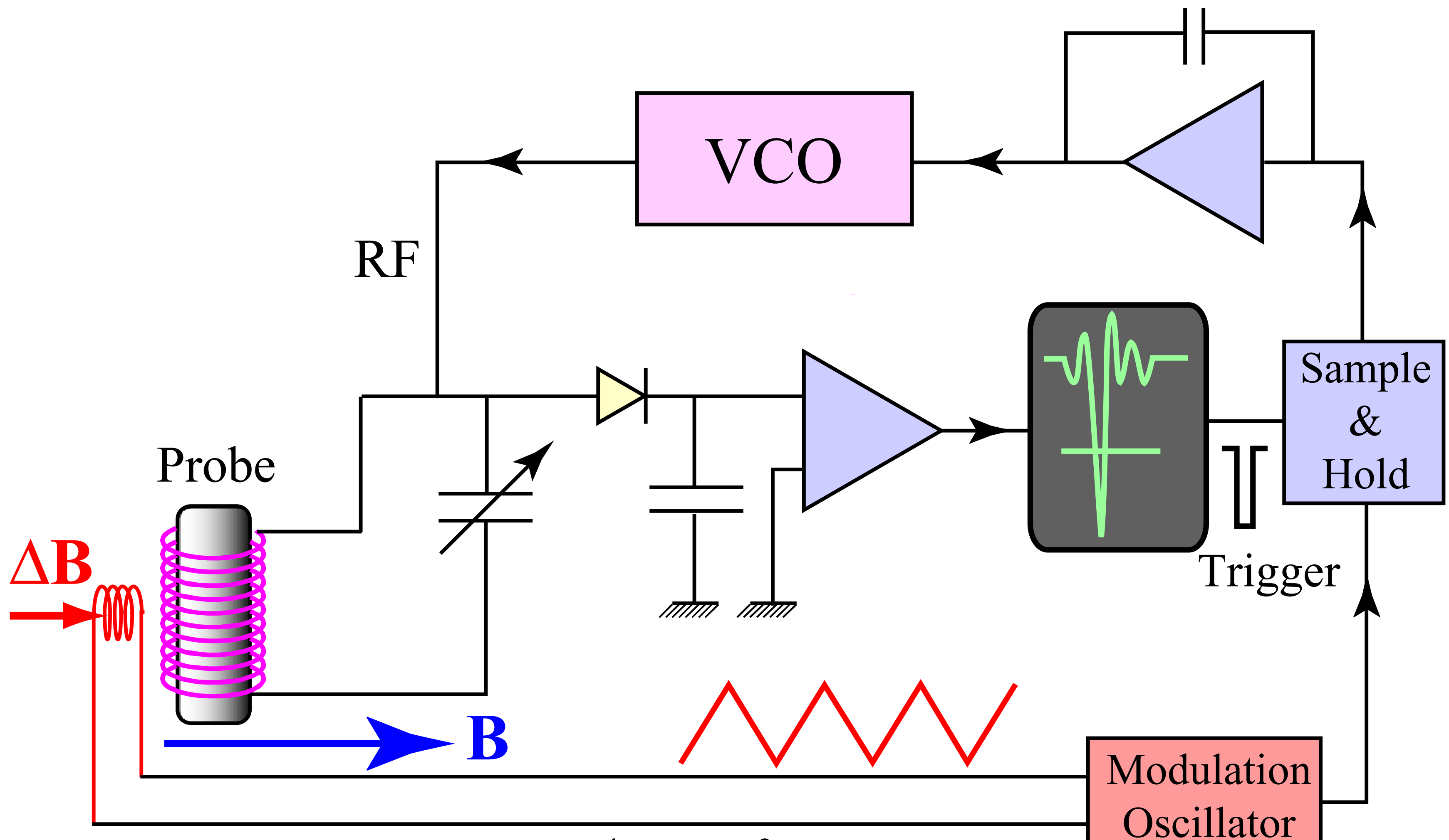
Spin component along the field direction can take integral values from $-I$ to $+I$. $\Rightarrow$ Energy gap = $\gamma.h.B$

Frequency = $\gamma.B$

Gyromagnetic Ratio

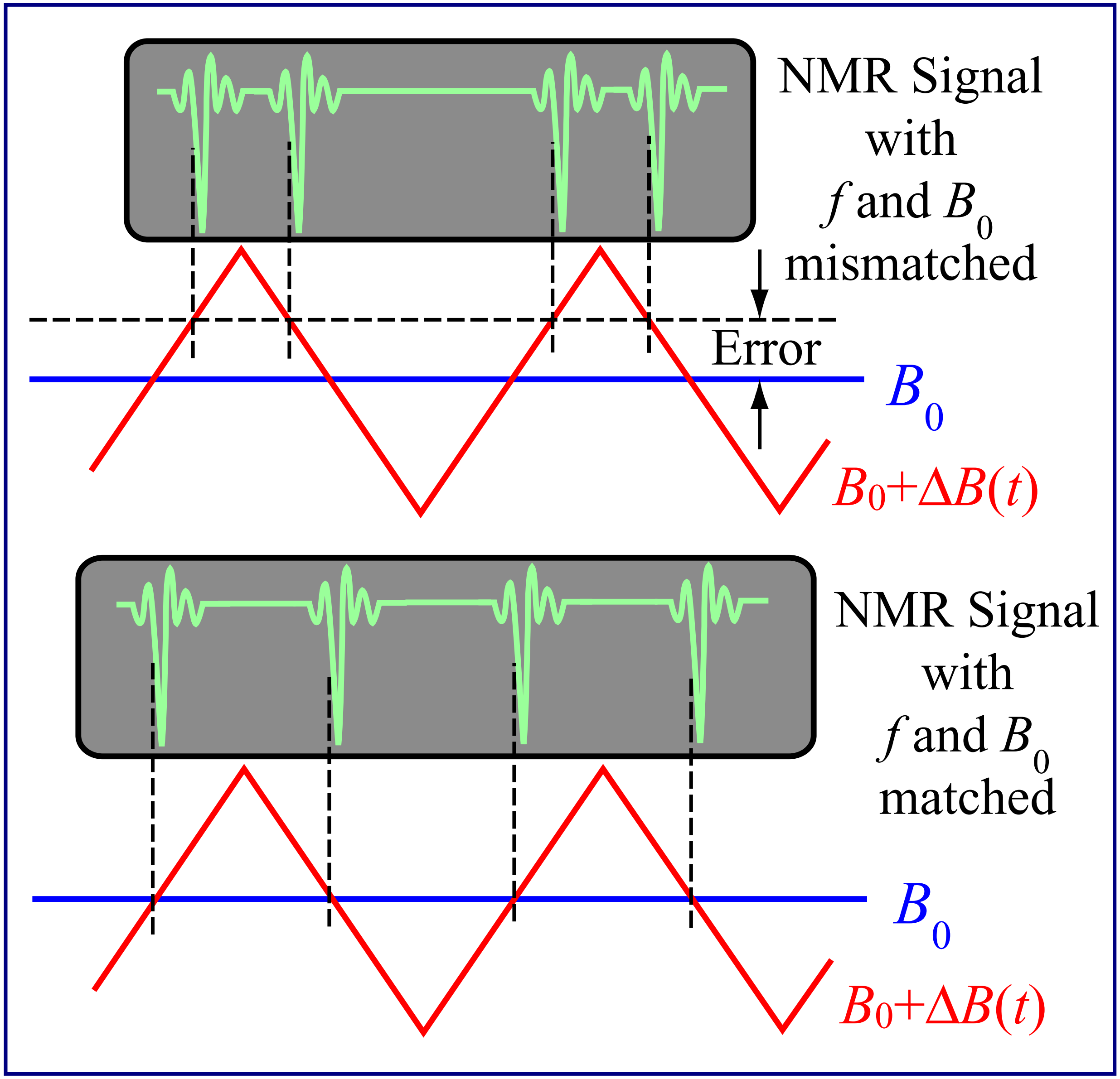
Particle	γ (MHz/T)	Application
e^-	28026.5	0.5 to 3.2 mT
^1H	42.576396	0.04 to 2 T
^2H	6.53569	2 T to 14 T
^3He	32.4336	Cryogenic
^{27}Al	11.0942	Cryogenic

NMR Magnetometer



$\Delta B/B \sim 10^{-4}$ to 10^{-3}; 30-70 Hz

Locking RF to NMR Resonance



Resonance occurs at non-zero value of modulating signal.

NMR signals arrive at uneven intervals.

Resonance occurs at Zero value of modulating signal.

NMR signals arrive at even intervals.

Requirements for NMR

NMR can provide measurement of magnetic field with ***absolute accuracy*** of 0.1 ppm. However, certain requirements must be met:

- Field must be ***stable*** (< 1% per second).
- Field must be ***homogeneous*** (< 0.1% per cm):
  - The signal deteriorates; difficult to lock
  - Probe positioning accuracy becomes critical.

Summary

- Numerous methods exist for measurement of magnetic fields. Only some of them are in common use for measuring accelerator magnets.
- NMR technique is the standard for absolute accuracy, but can not be used in all situations.
- Hall probes are very popular for point measurements, such as for field mapping of detector magnets.
- A variety of pick up coils are the most often used tools for characterizing field quality in accelerator magnets.
- Innovative techniques have been developed for alignment measurements to suit various applications.

Additional slides

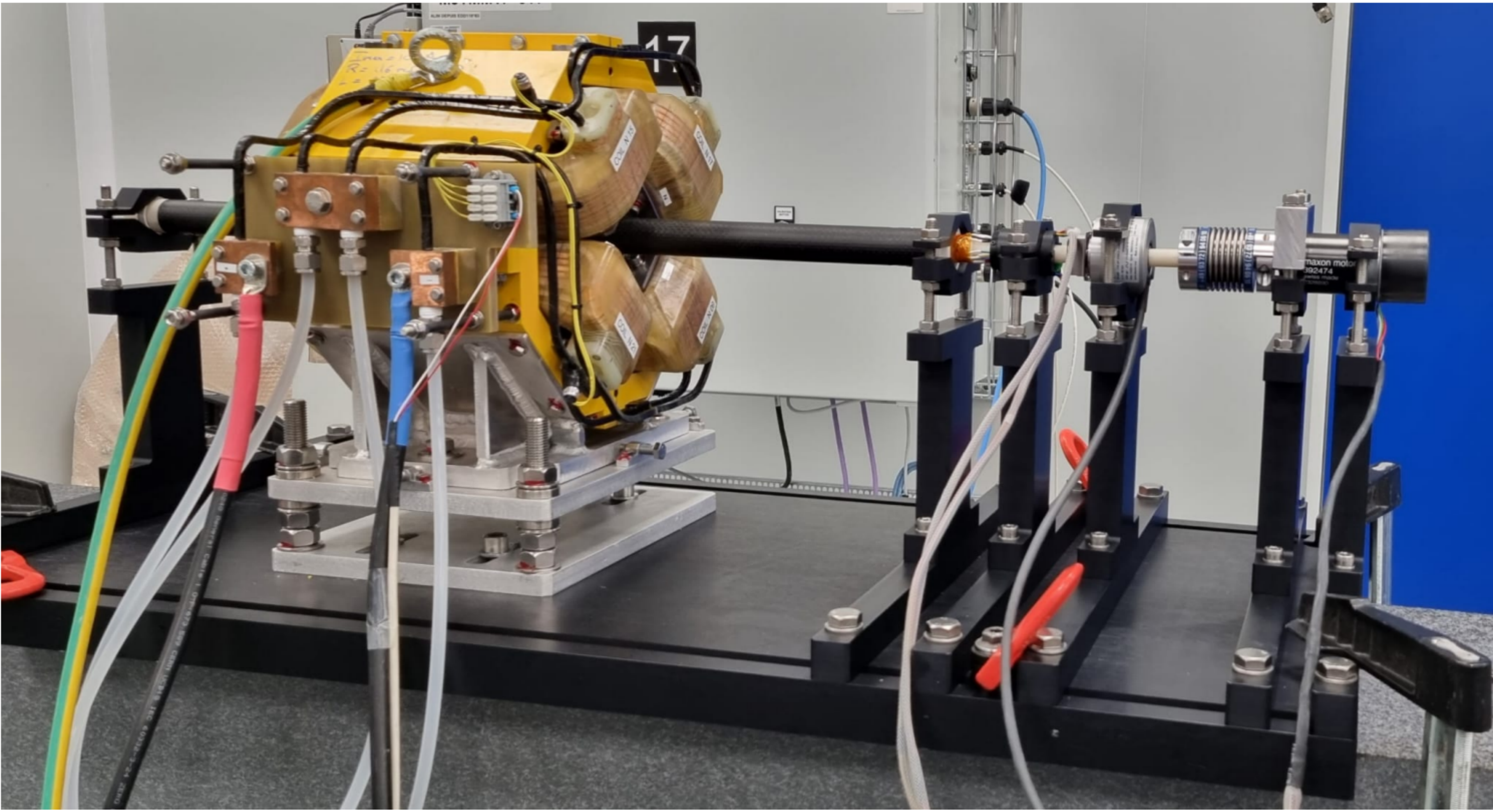
Rotating coils

- For most accelerator magnets, a harmonic description of the field is often used, both for characterizing the field quality, as well as for particle tracking studies.
- The "Harmonic Coil" technique, employing rotating coils, is the most convenient, accurate, and widely used technique for the measurement of harmonic coefficients in accelerator magnets.

Basic Principle

- The harmonic coefficients are related to the azimuthal variation of the field components.
- A rotating coil (loop of wire) measures the azimuthal variation of the intercepted flux. The field harmonics are then deduced using a knowledge of the geometry of the coil.
- A coil often uses several loops of different geometries to improve the accuracy of measurements by a process of ***"bucking"***.

A Typical Rotating Coil Setup



Multipoles

In a region of space
- free of magnetic sources (currents or magnetic materials)
- where the longitudinal component of B is constant

B(x,y) can be simply described by a **series of coefficients** B_1, A_1, B_2, A_2, B_3, A_3,... The so-called **harmonics**, or **multipoles**.

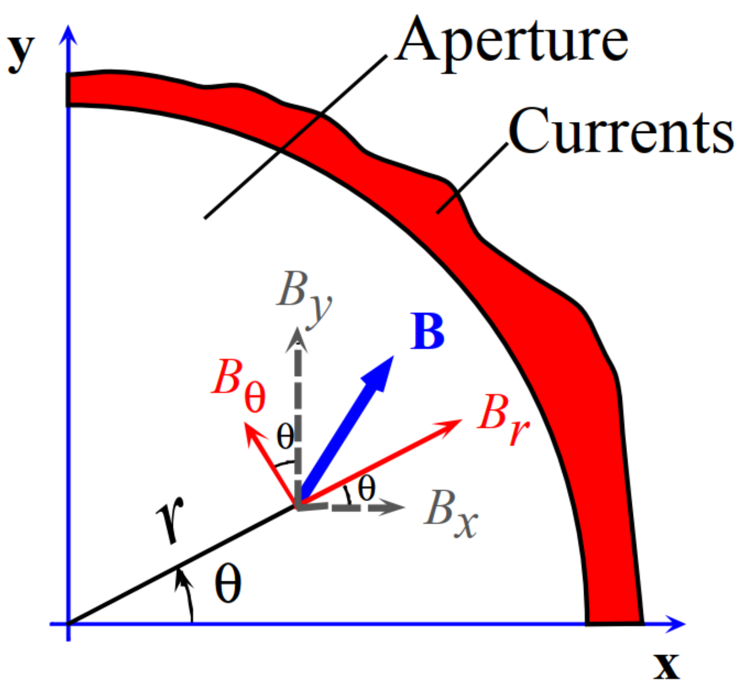
$$B_r = \sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^{n-1} [B_n \sin(n\theta) + A_n \cos(n\theta)]$$

$$B_\theta = \sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^{n-1} [B_n \cos(n\theta) - A_n \sin(n\theta)]$$

R is a reference radius and the units are ***tesla***

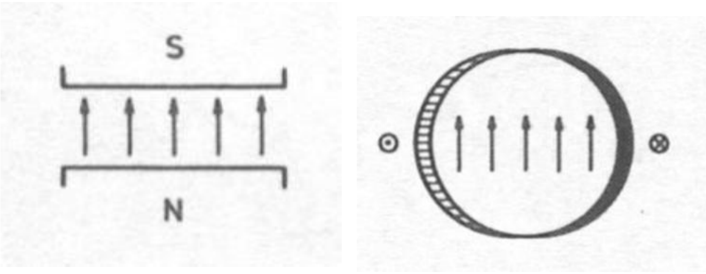
Or by using the complex notation

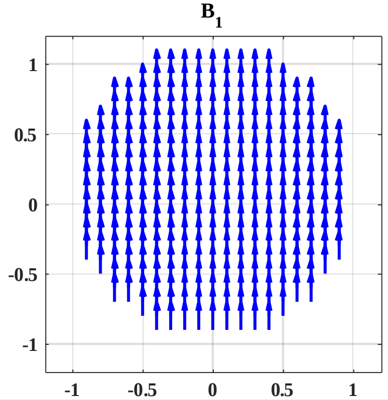
$$B_y + iB_x = \sum_{n=n_0}^{\infty} B_n + iA_n \left(\frac{x+iy}{R_{ref}}\right)^{n-1}$$



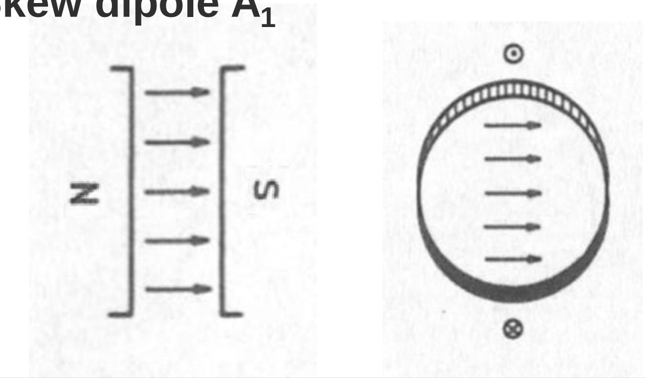
View from the Lead End of the Magnet

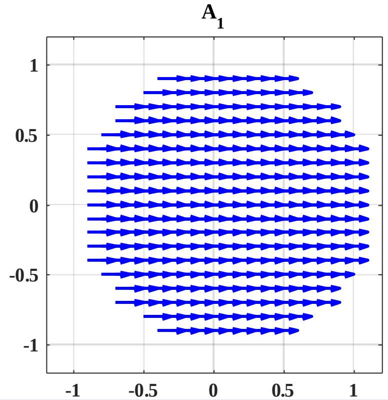
Normal dipole B_1



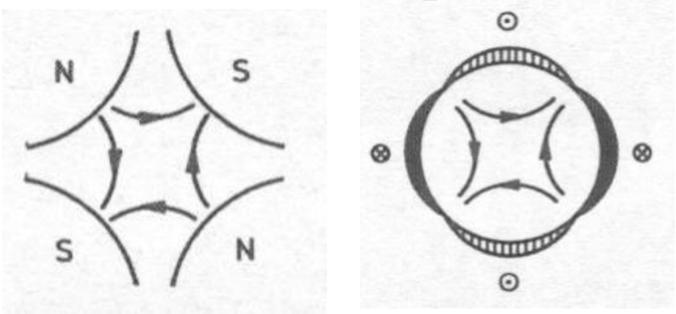


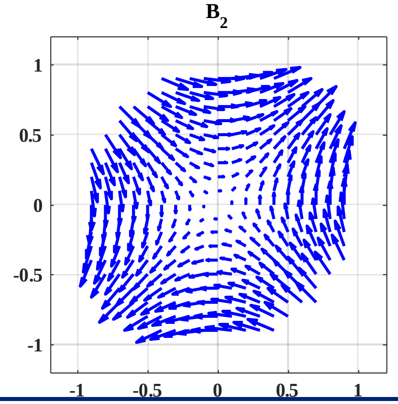
Skew dipole A_1



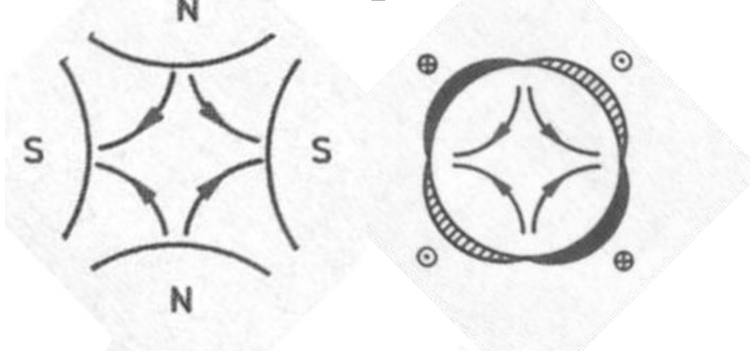


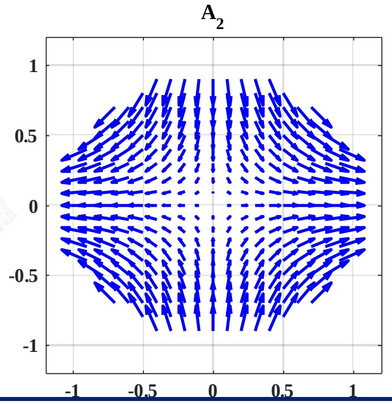
Normal quadrupole B_2



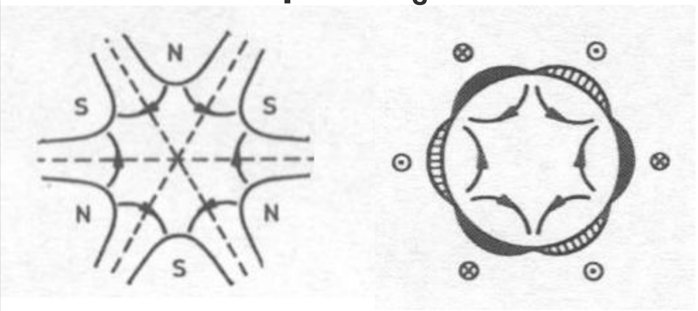


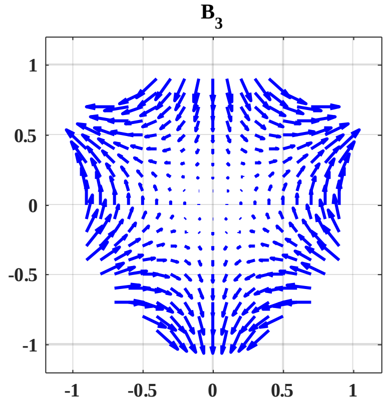
Skew quadrupole A_2



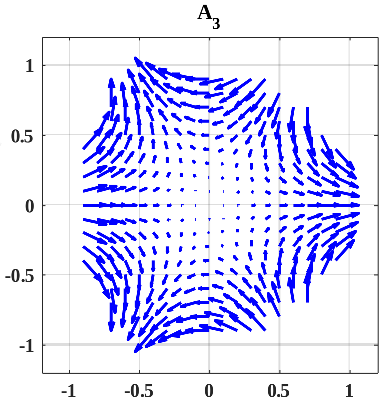


Normal sextupole B_3





Skew sextupole A_3



Multipoles

By assuming that the magnet is mainly generating one field component (main field), we can factorize

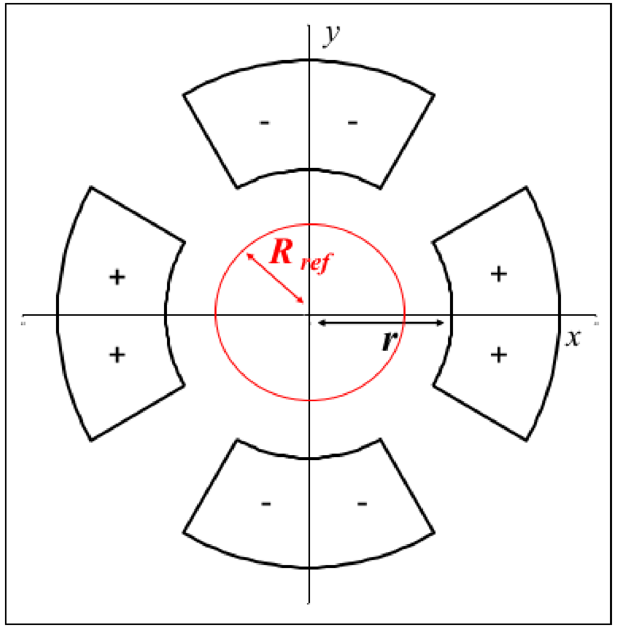
- the main field component (B_1 for dipoles, B_2 for quadrupoles, …)
- 10^{-4} to get numerical values ~1 since the expected deviations from the ideal field are small (~0.01%)

$$B_y + iB_x = 10^{-4} B_1 \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x+iy}{R_{ref}} \right)^{n-1}$$

The coefficients b_n, a_n are called **normalized multipoles** given in *units* at the reference radius R_{ref}

- $\mathbf{b_n}$ are the **normal** multipoles
- $\mathbf{a_n}$ are the **skew** multipoles

In general, only a small set of coefficients (**n<10**) is sufficient to have an accurate description of the field in the region of interest.

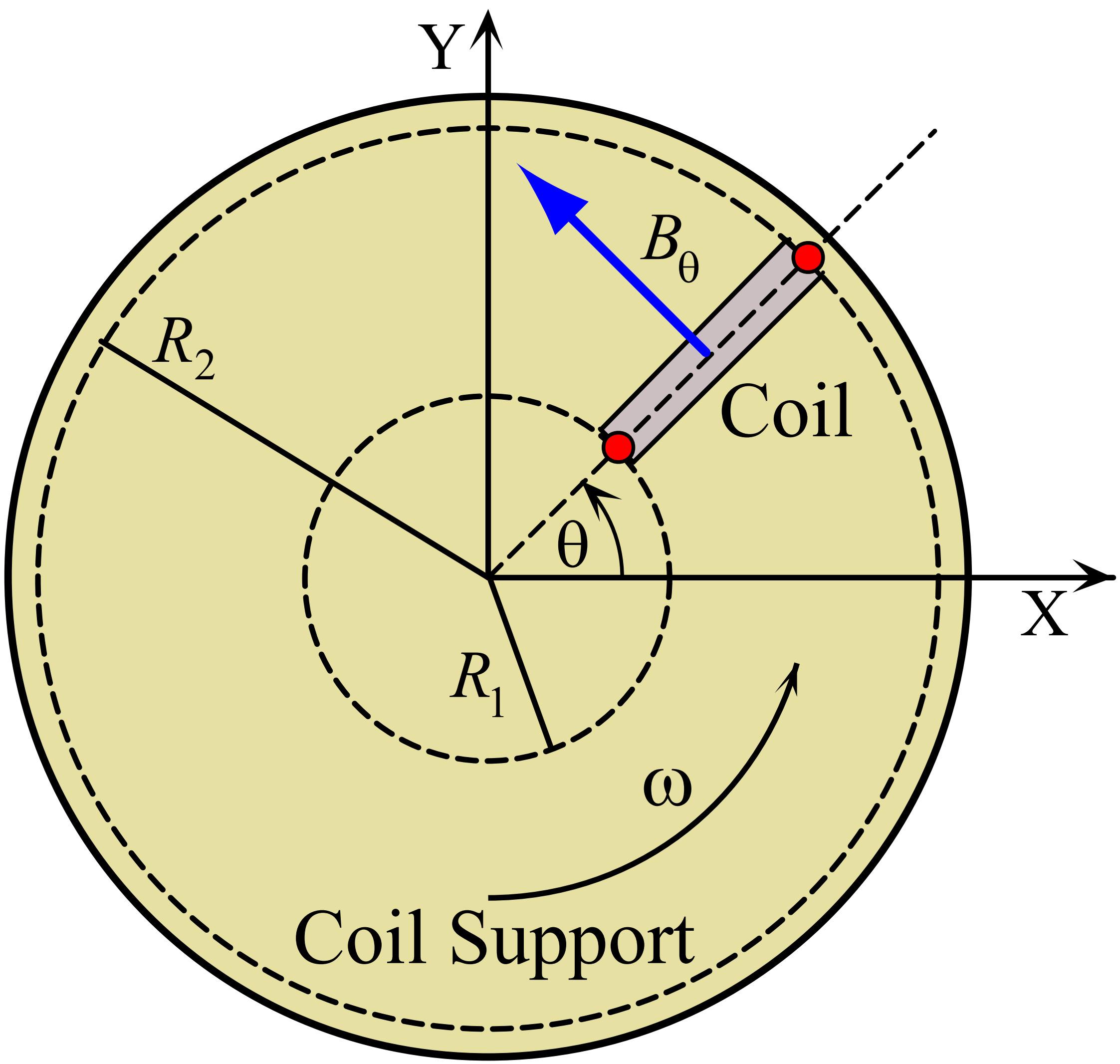


The reference radius is usually chosen as 2/3 of the aperture radius.

Coil Geometries

- All geometries employ a loop of wire, with one pair of sides parallel to the magnet axis.
- The plane of the loop can be oriented in an arbitrary direction, but two specific geometries, known as ***"radial"*** or ***"tangential"*** coils, are most common due to ease of fabrication, characterization and data analysis.
- Special geometries to measure specific harmonics are also used.

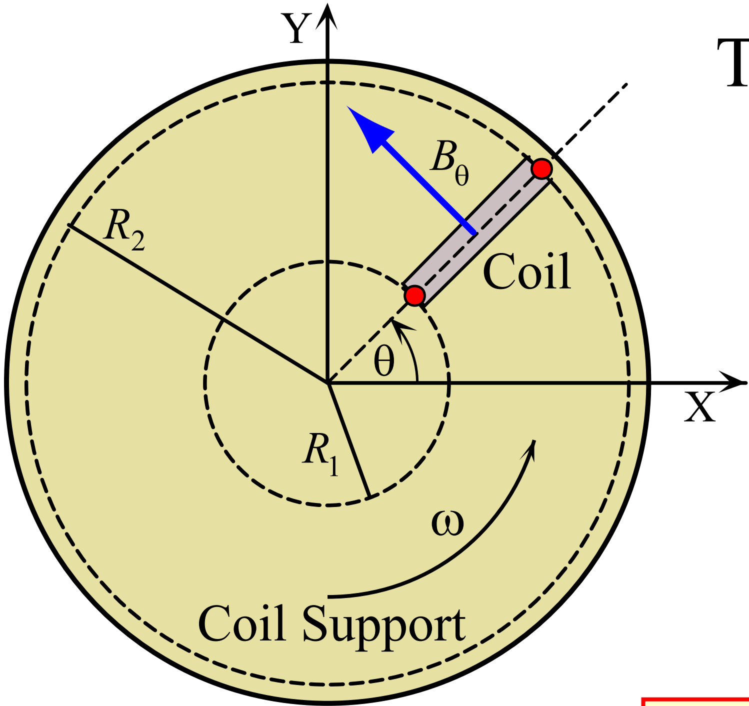
Cross Section of a Radial Coil



Loop of wire is placed in a radial plane to produce a RADIAL COIL.

A radial coil is sensitive to the AZIMUTHAL component of the field

Signal from a Radial Coil



The voltage signal at time t is:

$$V(t) = -\left(\frac{d\Phi}{dt}\right)$$

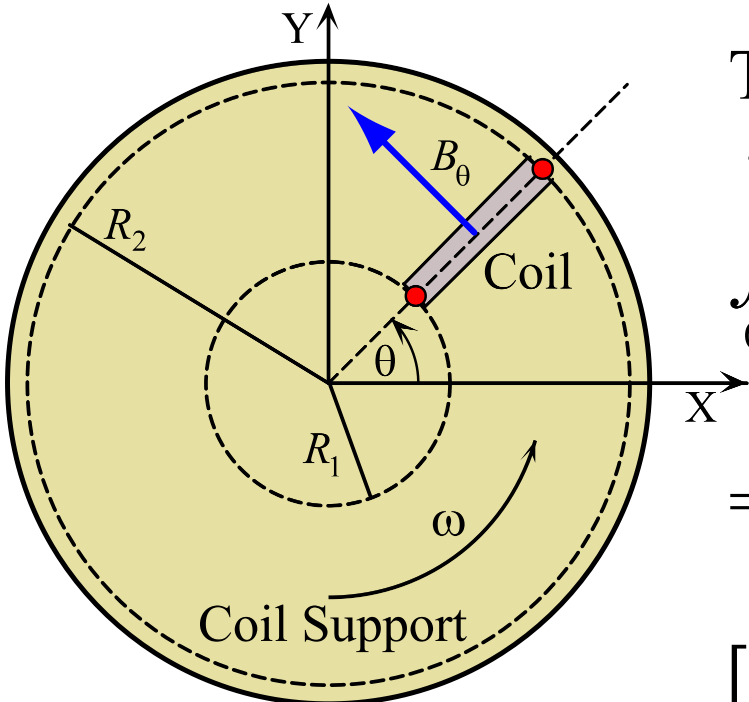
$$= \sum_{n=1}^{\infty} NLR_{ref}\,\omega \left[\left(\frac{R_2}{R_{ref}}\right)^n - \left(\frac{R_1}{R_{ref}}\right)^n\right] \times$$

$$\left[B_n \sin(n\omega t + n\delta) + A_n \cos(n\omega t + n\delta)\right]$$

N = No. of turns
L = Length
δ = angle at ($t = 0$)
ω = angular velocity
$\theta = \omega t + \delta$

The periodic variation of coil voltage is also described by a Fourier series, whose coefficients are related to the Normal and Skew harmonics, the geometric parameters of the coil, and angular velocity.

Radial Coil Analysis: Integrators



The integrated voltage at time t is:

$$\int_0^t V(t)dt = \Phi(0) - \Phi(\theta)$$

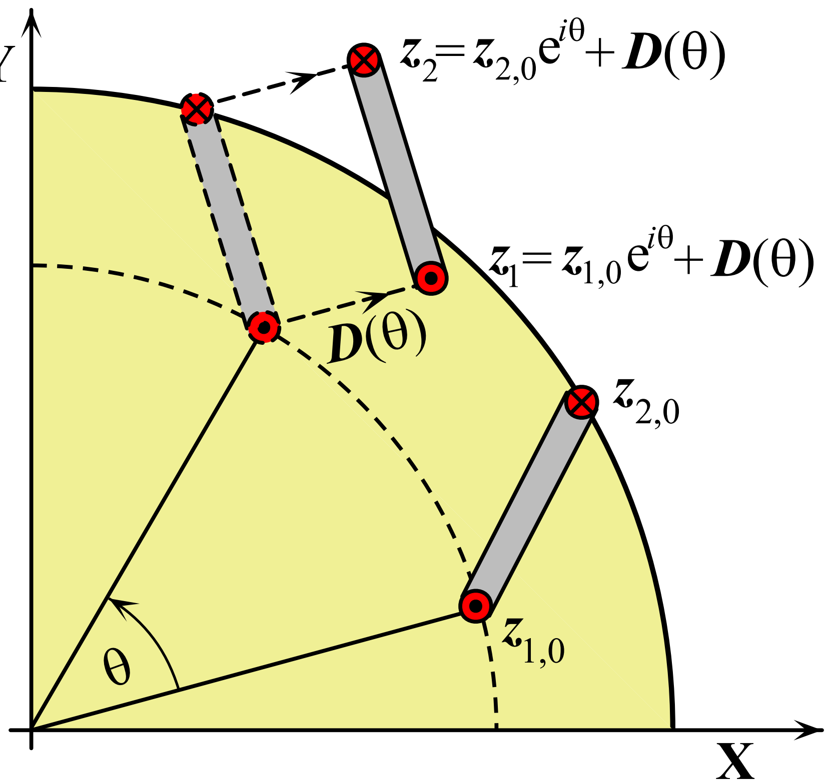
$$= \Phi(0) - \sum_{n=1}^{\infty} \frac{NLR_{ref}}{n}\left[\left(\frac{R_2}{R_{ref}}\right)^n - \left(\frac{R_1}{R_{ref}}\right)^n\right] \times$$

$$\left[B_n \cos(n\theta + n\delta) - A_n \sin(n\theta + n\delta)\right]$$

N = No. of turns
L = Length
δ = angle at ($t = 0$)
ω = angular velocity
$\theta = \omega t + \delta$

The integrator mode has the advantage that the signal is independent of the rotational speed. The integrator drift, however, can be a problem, and needs correction.

Transverse Vibrations

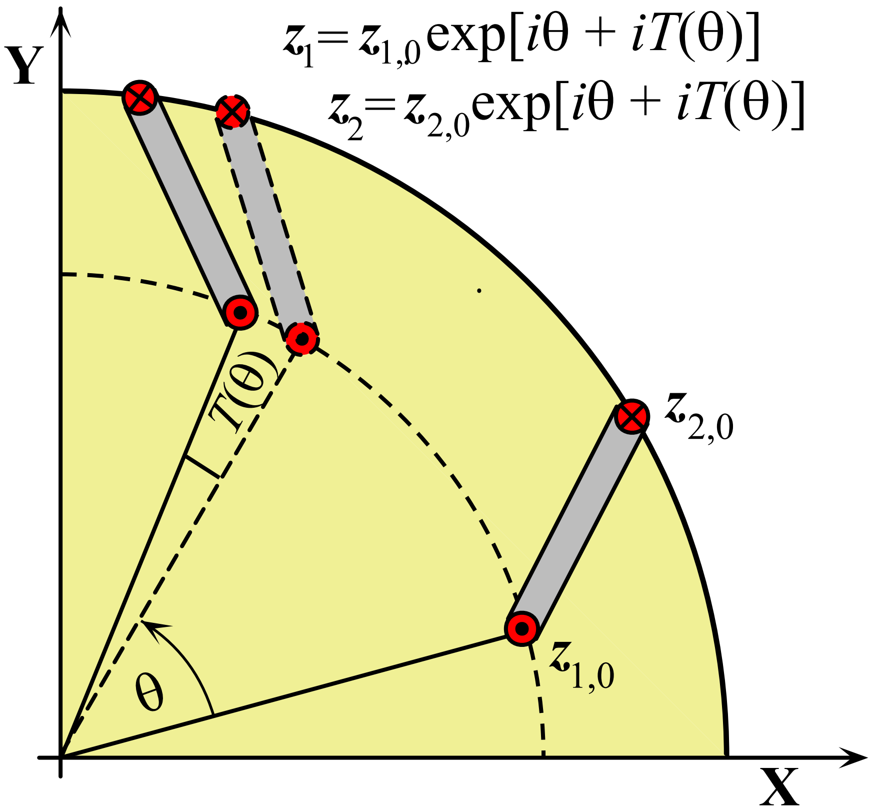


The coil is displaced from the ideal position by a vector $\boldsymbol{D}(\theta)$ when the coil rotates through θ.

In a pure $2n$-pole field, the amount of spurious harmonics in the coil signal is roughly proportional to the sensitivity of the coil to the $(n-1)$th harmonic.

The effect of transverse vibrations in a $2n$-pole magnet can be minimized by using a coil system whose sensitivity to the $(n$-1)th harmonic is zero.

Torsional Errors

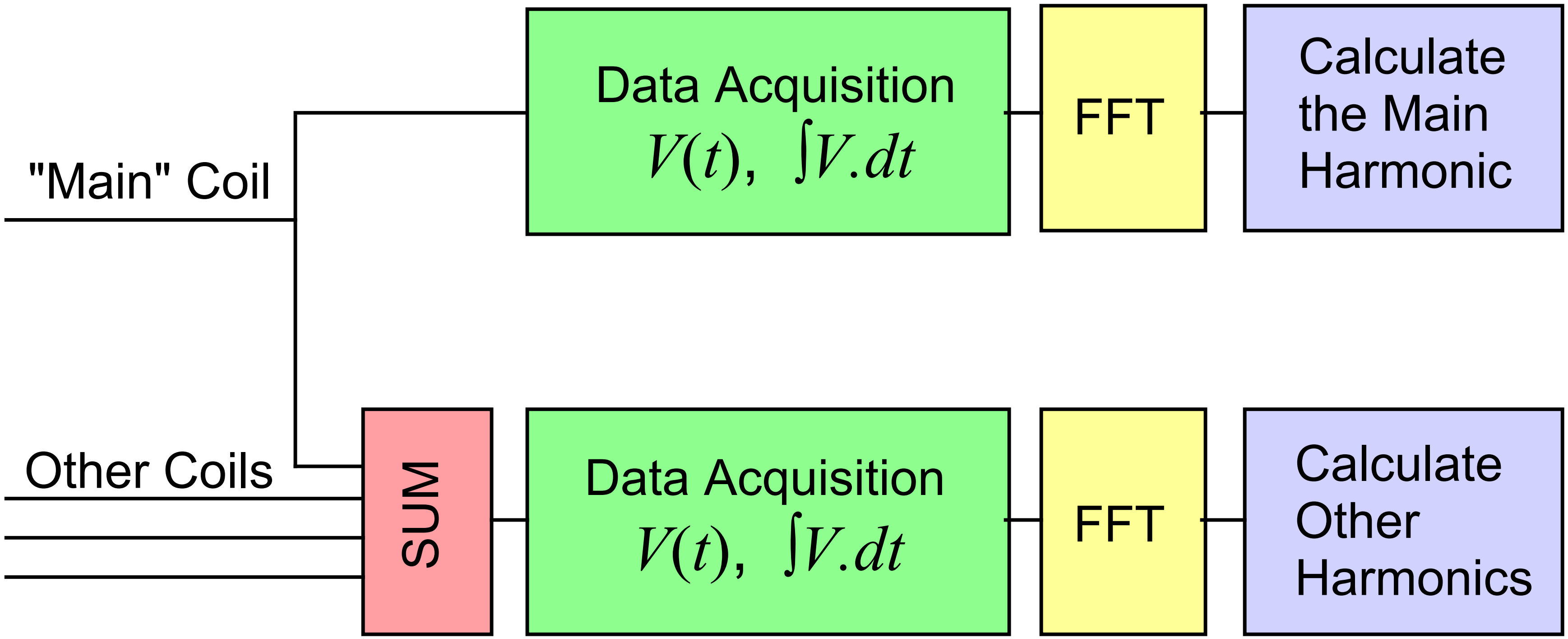


The coil angular position is $\theta + T(\theta)$ when it should have been θ.

In a pure 2*n*-pole field, the amount of spurious harmonics in the coil signal is roughly proportional to the sensitivity of the coil to the *n*-th harmonic.

The effect of torsional errors in measuring a 2*n*-pole magnet can be minimized by using a coil system whose sensitivity to the *n*-th harmonic is zero.

Analog Bucking



In *Analog Bucking*, one (or more) of the coils is used to measure the most dominant harmonic term (the "Main" term). The outputs of various coils are summed *before* recording the data.

Calibration of Rotating Coils

- For accurate work, suitable ***calibration*** of the coil is as important as precise fabrication.
- Generally, most parameters of interest can be obtained by carrying out measurements in ***known*** (strength and angle) dipole and quadrupole fields.
- For a coil of radius ~20 mm, it is possible to attain an absolute accuracy of ***~0.02% for the main field*** in dipoles.
- With good calibration, and data analysis, errors in ***higher harmonics*** (at the coil radius) are ***below 10 ppm*** of the main field.