

# The Technology & Applications of Particle Accelerators

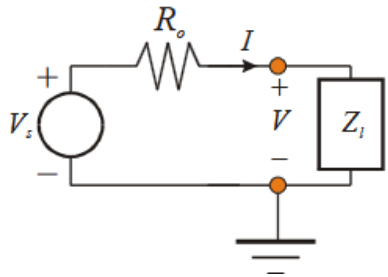
## RF engineering course

Tutorial on S parameters and Smith chart

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# Reflection coefficient and Smith chart



## Reflection coefficient

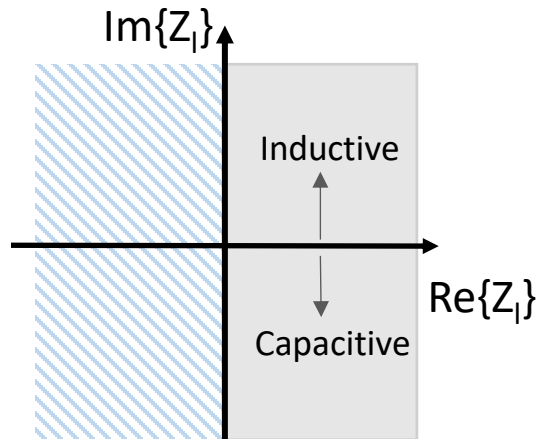
$$\Gamma_l(R_0) \stackrel{\text{def}}{=} \frac{I^-}{I^+} \stackrel{\text{def}}{=} \frac{V^-}{V^+} = \frac{Z_l - R_0}{Z_l + R_0} = \frac{z - 1}{z + 1}$$

→  $|\Gamma_l| \leq 1$  for  $\text{Re}\{Z_L\} \geq 0$

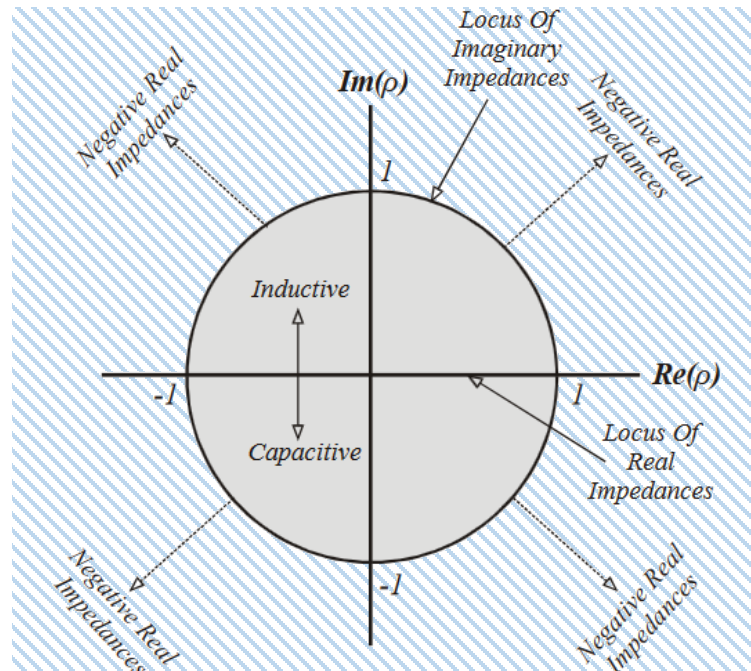
$R_0$  Reference impedance

$z = \frac{Z_l}{R_0}$  Normalized impedance

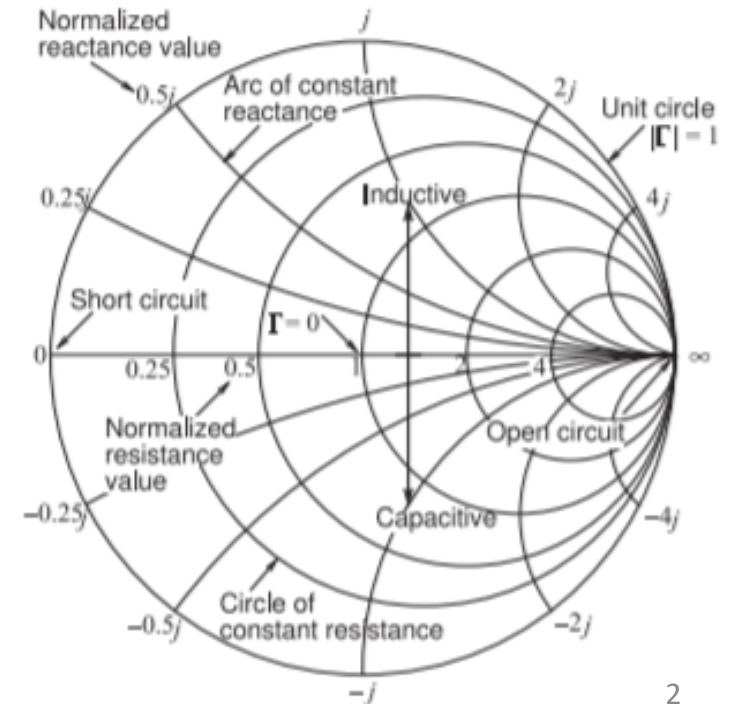
## Impedance plane



## Reflection coefficient plane



## Impedance Smith chart



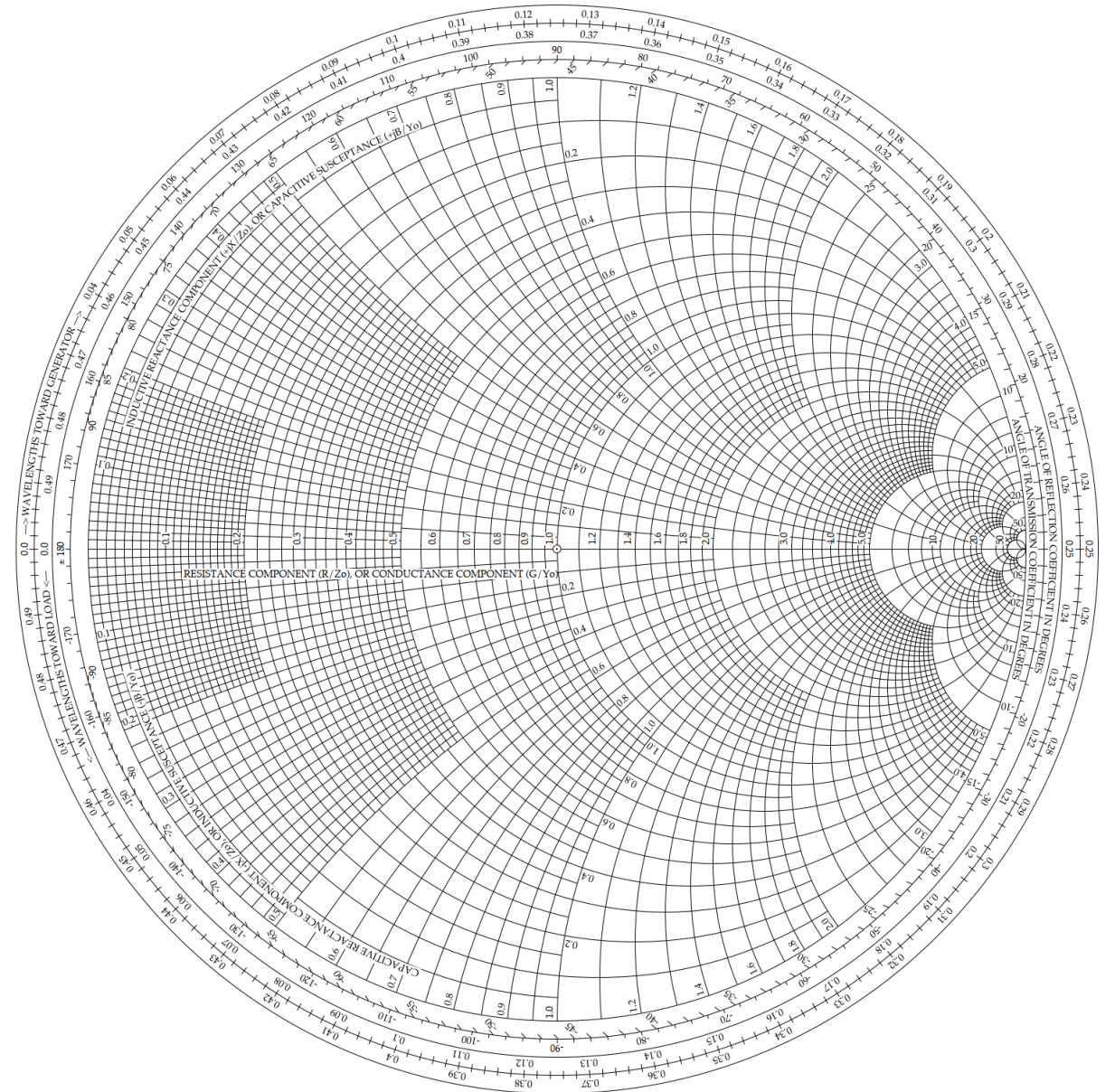
$$\left\{ \begin{array}{l} Z_l = \frac{V}{I} \\ I = I^+ - I^- \\ V = V^+ + V^- \\ \frac{V^+}{I^+} = \frac{V^-}{I^-} = R_0 \end{array} \right.$$

1. Place the impedance  $Z$  on the  $50\Omega$  normalized smith chart

	$Z$
A	$50 + j0$
B	$20 - j15$
C	$10 + j25$
D	$0 - j50$

2. Using the smith chart, find the normalized impedance  $z$  for the given reflection coefficients

	Reflection coefficient $\Gamma$
a	1
b	$1 \angle 90^\circ$
c	$1 \angle 180^\circ$
d	0.5



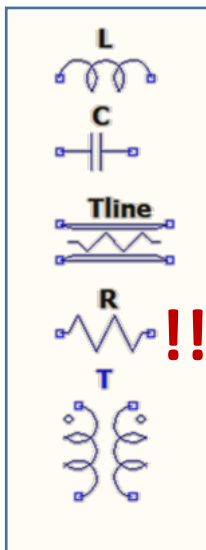
# Impedance matching using the impedance Smith chart

**Definition:** Designing source and load impedances to maximize power transfer (=minimize signal reflection).

↓

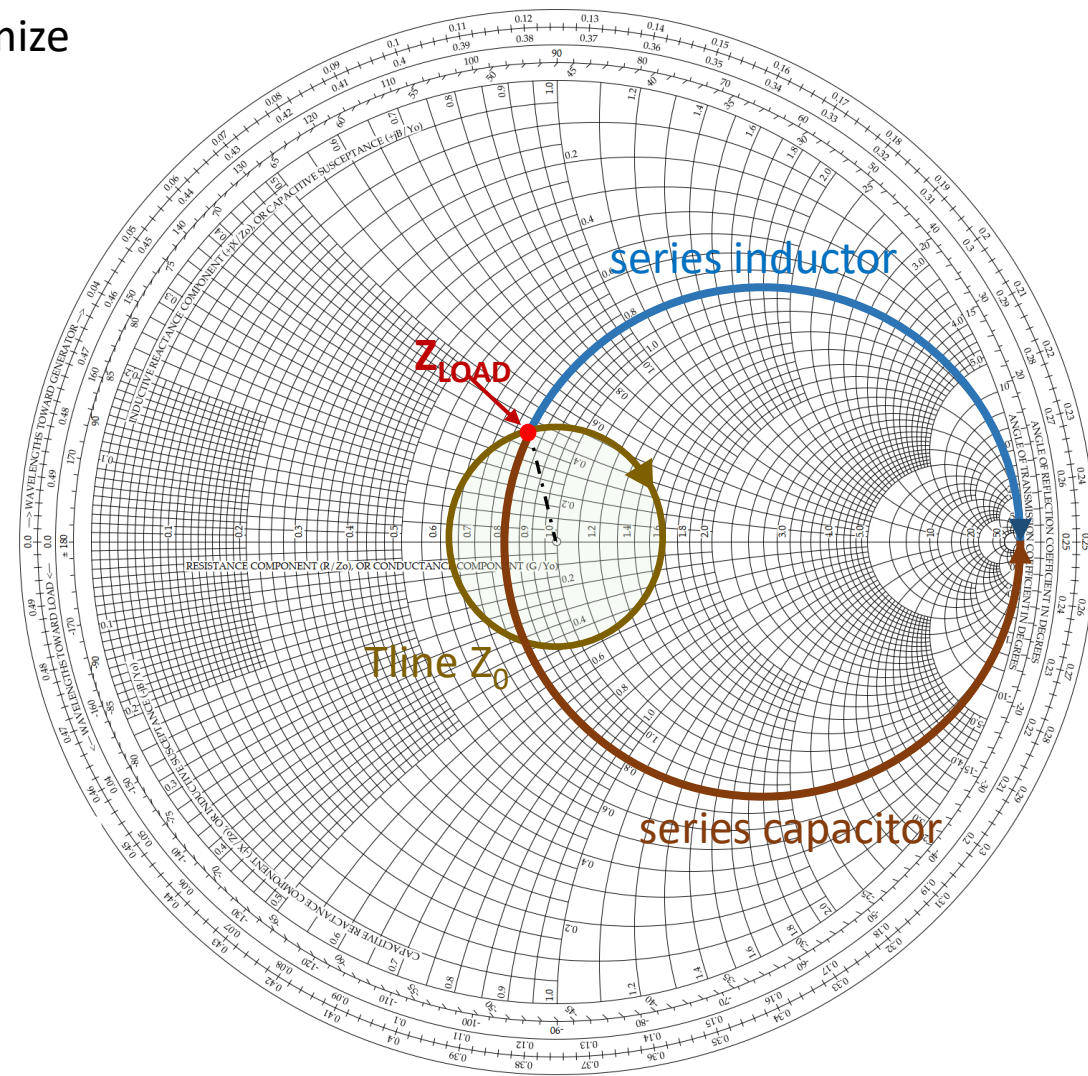
$$Z_{load} = Z_{source}^* (= R_{source} \text{ in most cases})$$

## Our tools



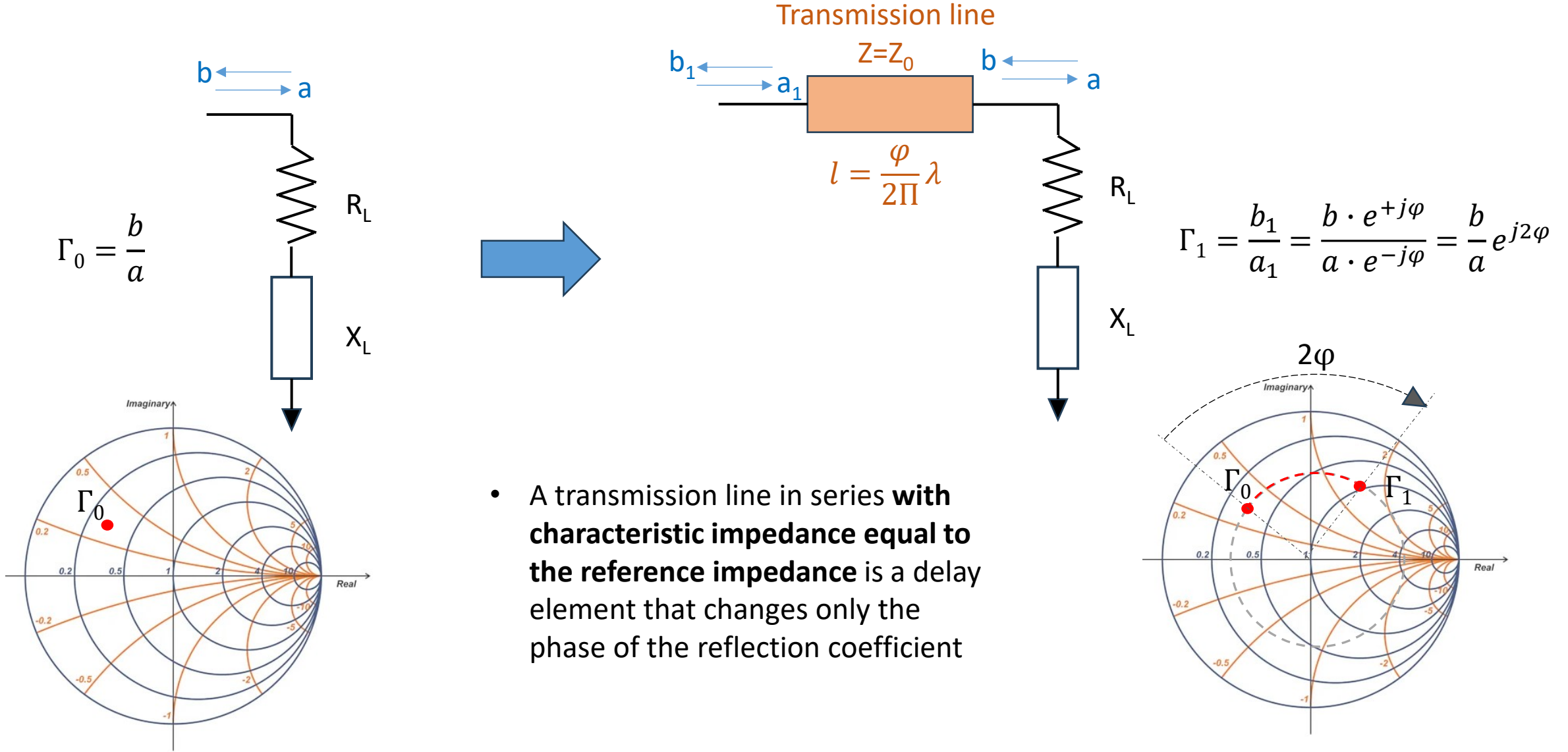
## Remarks

1. Lossless matching → no R
2. Transformers have power and BW limitations in the real world
3. Elements can be connected in series and in parallel (using the admittance smith chart)
4. Single frequency matching





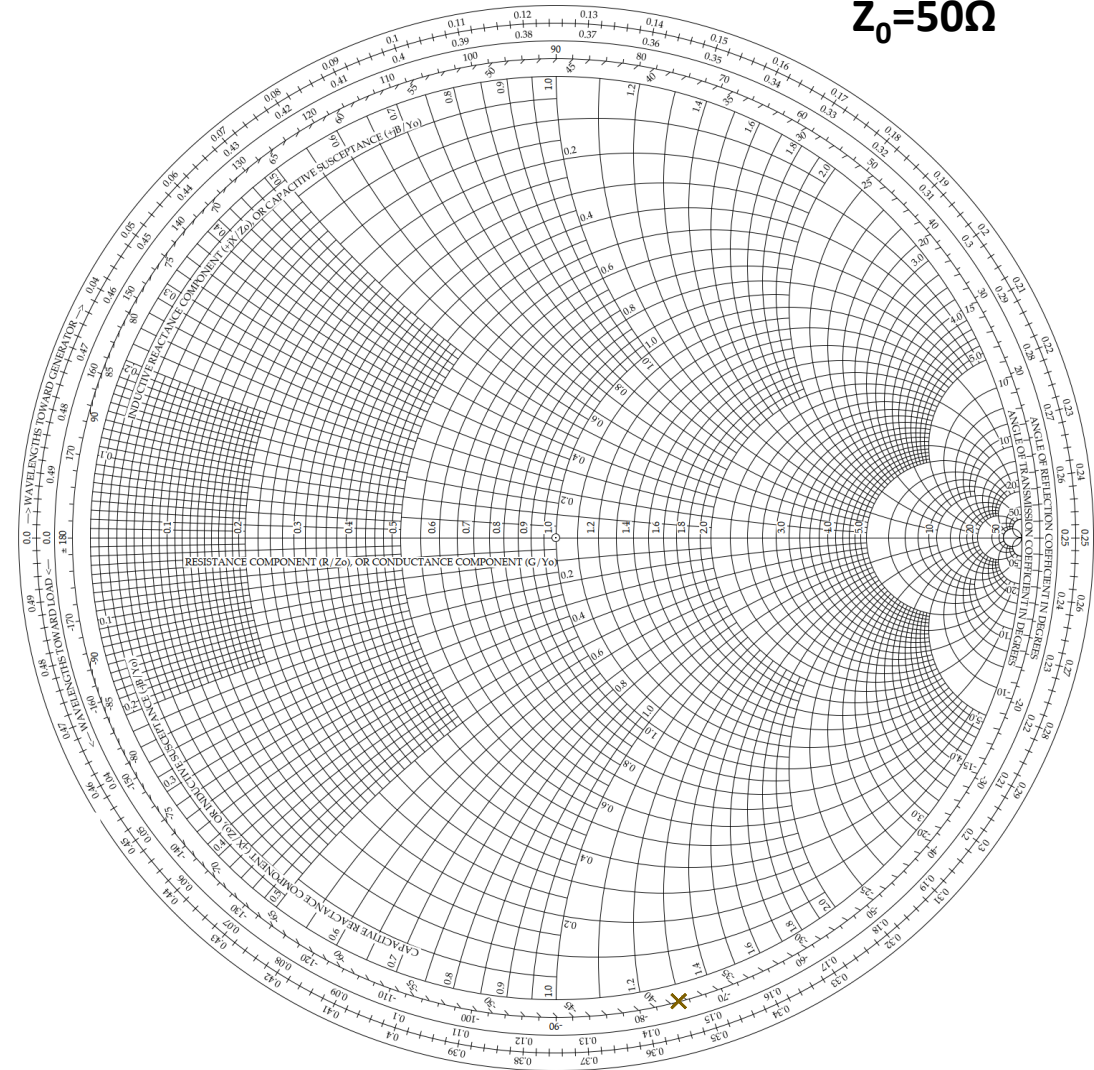
# Series transmission line effects



# Impedance matching exercise

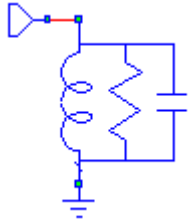
An RF amplifier has an equivalent input impedance of  $70\Omega$  in parallel with  $10\text{pF}$  at the operating frequency of  $100\text{MHz}$ . Calculate a lossless matching network for the maximum power transfer from the  $50\Omega$  generator to the amplifier?

Reference impedance  
 $Z_0=50\Omega$



# Resonator (1)

Given the parallel resonator



$$R=25\Omega$$

$$L=5\text{nH}$$

$$C=2000\text{pF}$$

- Calculate Q factor, bandwidth and -3dB frequencies
- Draw the relevant points into the impedance plane
- Do the same in the smith chart

# Resonator (2)

What's the reflection coefficient locus?

$$\Gamma = \frac{z - 1}{z + 1}$$

Resonator in impedance plane (circle):  $|z - z_0| = r$

Equivalent to:  $zz^* - zz_0^* - z^*z_0 + \underbrace{(z_0^*z_0 - r^2)}_{\text{constant}} = 0$

After shifting reference planes and changing variable :

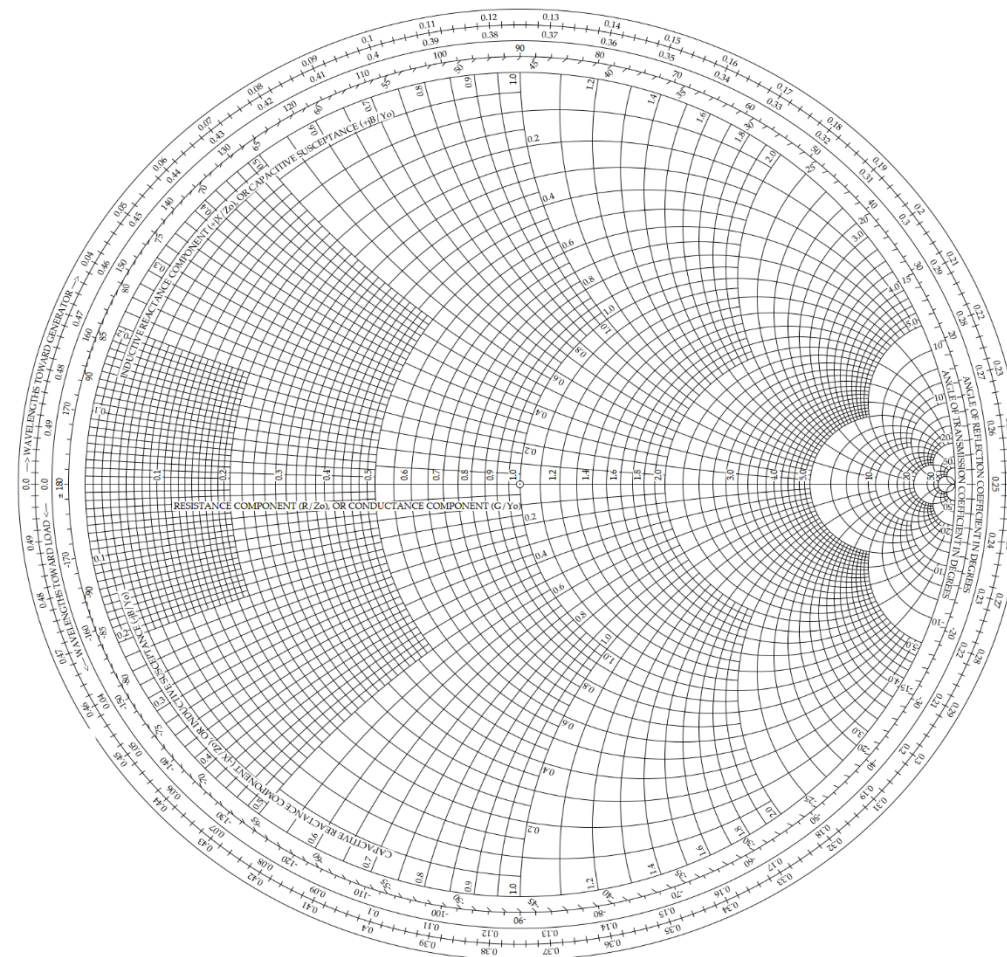
$$\frac{1}{\Gamma} \frac{1}{\Gamma^*} - \frac{1}{\Gamma} \frac{1}{\Gamma_0^*} - \frac{1}{\Gamma^*} \frac{1}{\Gamma_0} + K = 0$$

equivalent to (after algebraic manipulations):

$$\Gamma\Gamma^* - \Gamma^*\Gamma_0 - \Gamma\Gamma_0^* + \Gamma_0^*\Gamma_0 = J^2 \rightarrow |\Gamma - \Gamma_0| = J$$

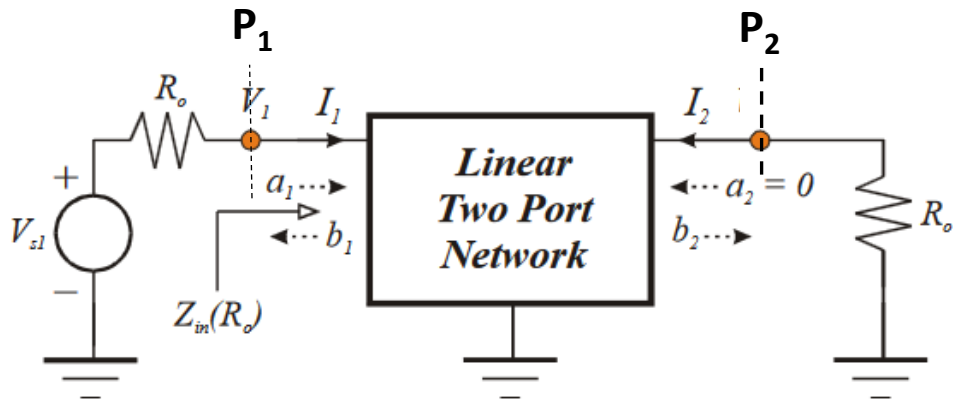
**Circles in impedance plane are mapped to circles in reflection coefficient plane**

Using  $Z_0=50\Omega$   
(arbitrary choice)





# S parameters



Reference (line) impedance :  $R_0$

Voltage @  $P_N$  :  $V_N = V_{Ni} + V_{Nr}$

Incident wave @  $P_N$  :  $a_N = \frac{V_{Ni}}{\sqrt{R_0}}$

Reflected wave @  $P_N$  :  $b_N = \frac{V_{Nr}}{\sqrt{R_0}}$

**2 ports S parameters**  $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \rightarrow S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2 = 0} = \Gamma_1 \Big|_{a_2 = 0}$

$S_{xx}$  is the reflection coefficient of port X only when all the other port are terminated on the reference impedance

Other linear network parameters exist, e.g.: impedance (**Z**), admittance (**Y**), hybrid (**H**)

While measuring systems based on the other parameters exist, at high frequency only the S parameters are used.

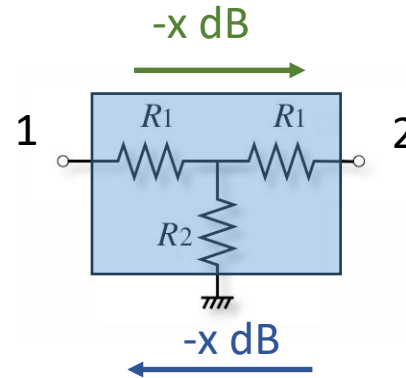
Z,Y,H parameters are based on open and/or short circuits at the ports, impractical at high frequency (an open circuit for example radiates energy). Also extreme reflections (open and short) can make unstable active circuits.

# Basic S parameters exercises

- Parts of the S-matrix of an ideal attenuator are given.  
Fill the missing matrix elements.

What is the nominal attenuation value in dB written on the component?

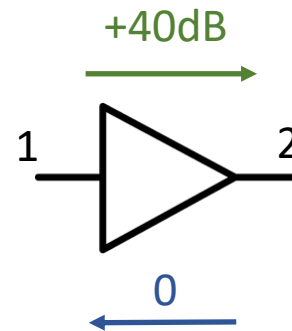
$$S = \begin{bmatrix} 0 & 0.1 \\ ? & ? \end{bmatrix}$$



- An amplifier is perfectly matched at input and output, i.e. no reflection at the input and output ports. It has a gain of 40 dB and no reverse transmission.

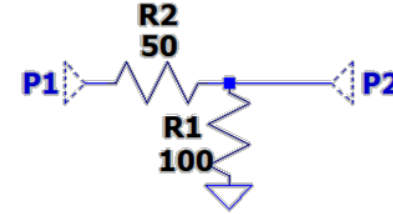
Write the S matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$



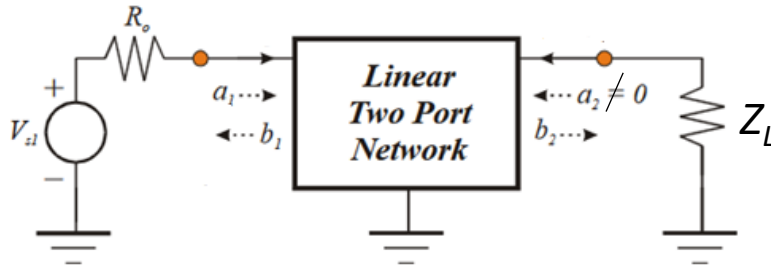
# S parameters exercise

Calculate the S matrix of the 2-port network in a  $50\ \Omega$  reference system.



# General 2-port reflection coefficient

Given the S-parameters, find the input reflection coefficient of a 2-port network for a generic load impedance.



$$\Gamma_1 = \frac{b_1}{a_1} \quad \Gamma_L = \frac{a_2}{b_2}$$