The Technology & Applications of Particle Accelerators

RF engineering course

Tutorial on S parameters and Smith chart

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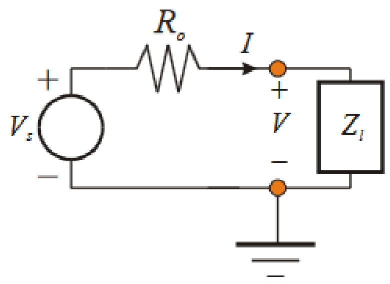
Reflection coefficient and Smith chart



Reflection coefficient

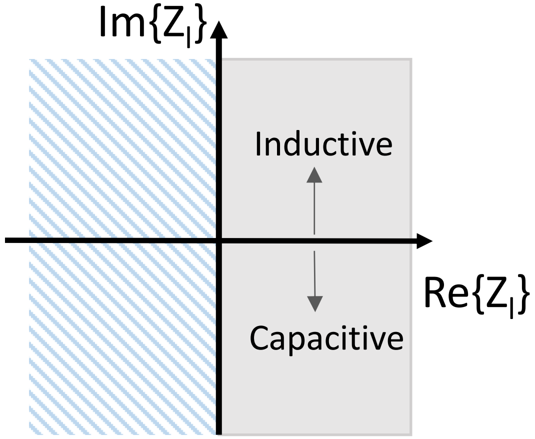
$$\Gamma_l(R_0) \overset{\text{def}}{=} \frac{I^-}{I^+} \overset{\text{def}}{=} \frac{V^-}{V^+} = \frac{Z_l - R_0}{Z_l + R_0} = \frac{z-1}{z+1}$$

$\rightarrow$ $|\Gamma_l| \leq 1$ for $\mathrm{Re}\{Z_L\} \geq 0$

R_0 **Reference impedance**

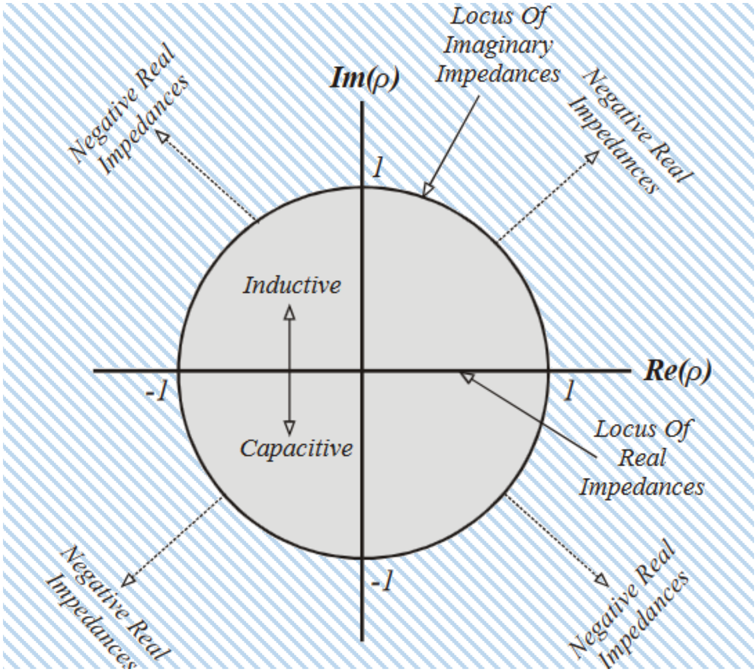
$$z = \frac{Z_l}{R_0}$$ **Normalized impedance**

Impedance plane

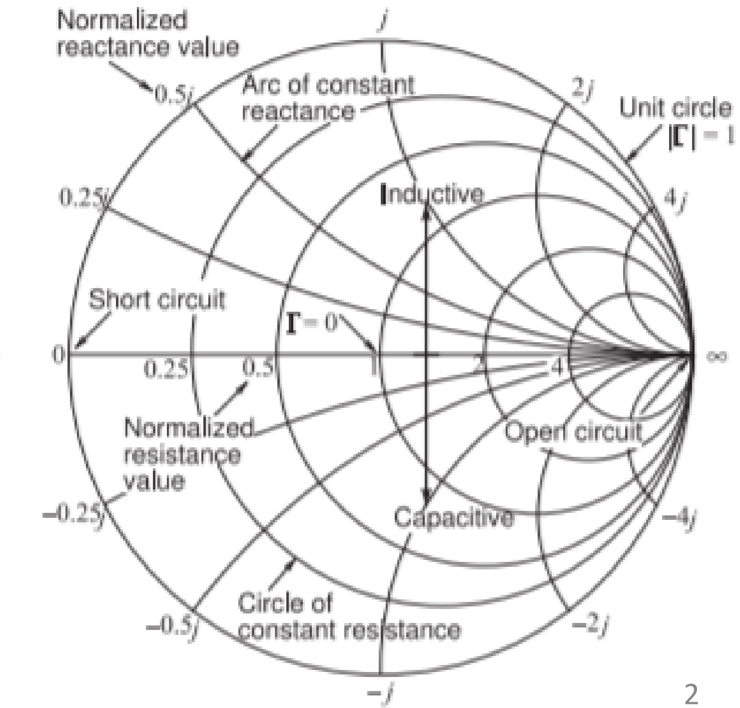


$$\begin{cases} Z_l = \dfrac{V}{I} \\ I = I^+ - I^- \\ V = V^+ + V^- \\ \dfrac{V^+}{I^+} = \dfrac{V^-}{I^-} = R_0 \end{cases}$$

Reflection coefficient plane



Impedance Smith chart



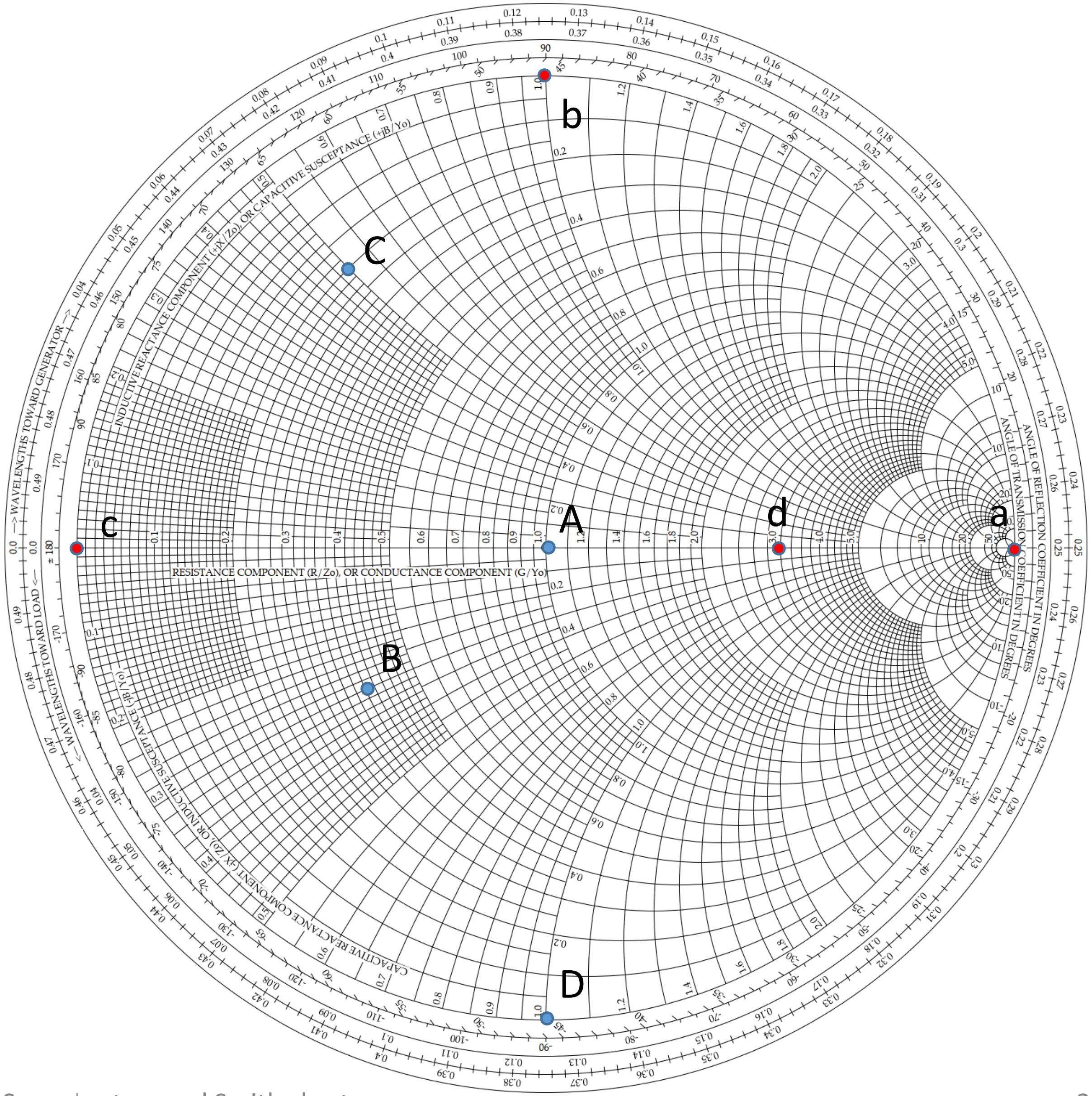
Smith chart excercises

1. Place the impedance ***Z*** on the 50Ω normalized smith chart

	Z
A	50 + j0
B	20 – j15
C	10 + j25
D	0 – j50

2. Using the smith chart, find the normalized impedance ***z*** for the given reflection coefficients

	Reflection coefficient Γ
a	1
b	1 ∠ 90°
c	1 ∠ 180°
d	0.5

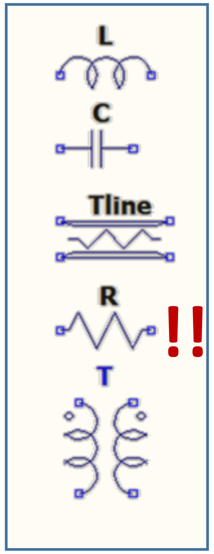


Impedance matching using the impedance Smith chart

Definition: Designing source and load impedances to maximize power transfer (=minimize signal reflection).

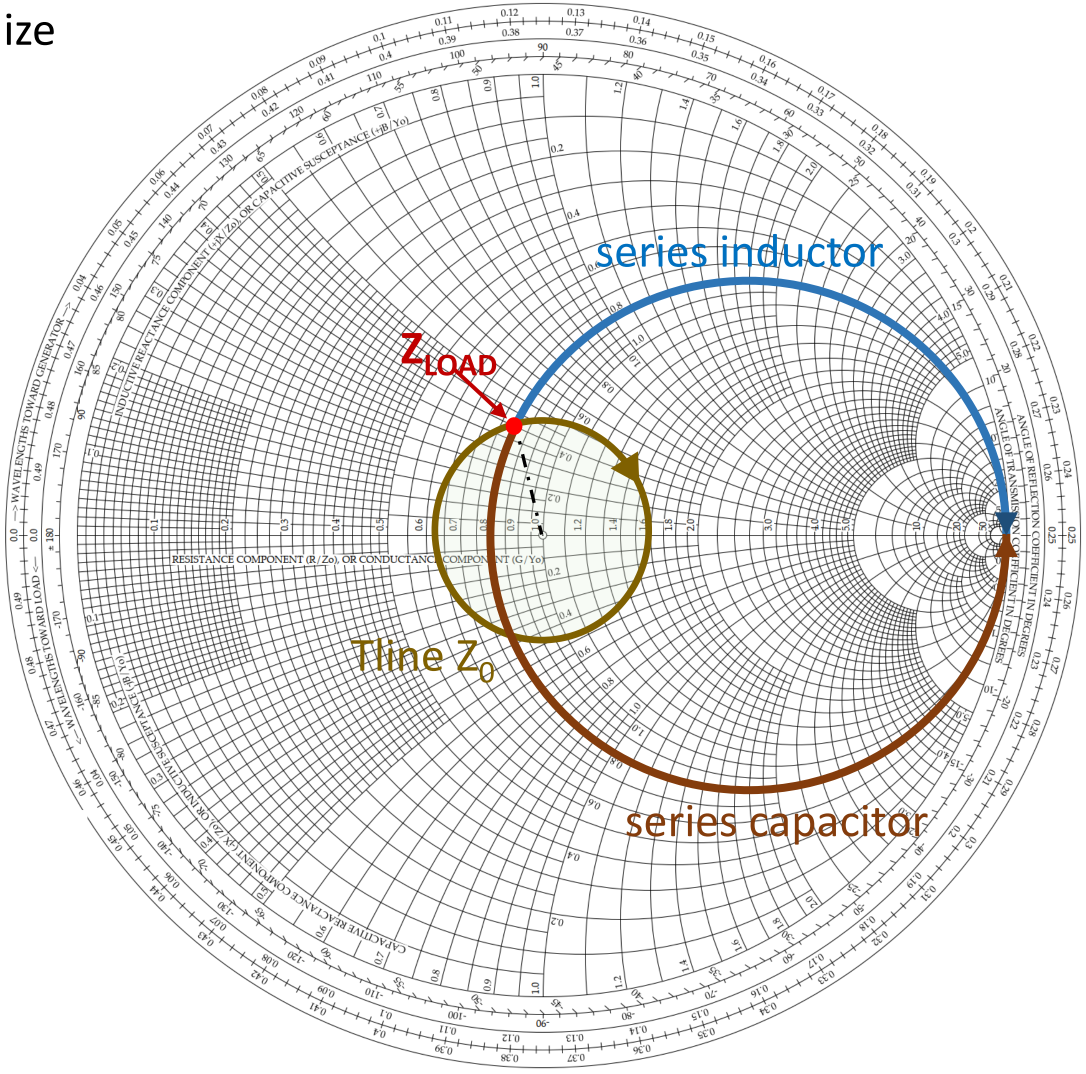
$$Z_{load} = Z^*_{source} (= R_{source} \text{ in most cases})$$

Our tools

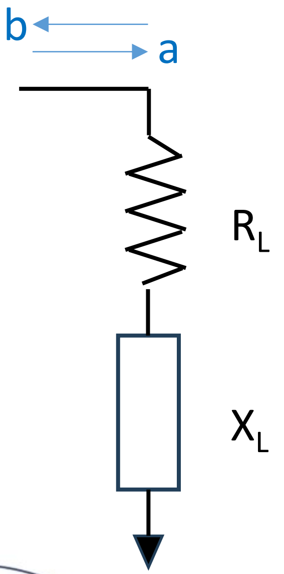


Remarks

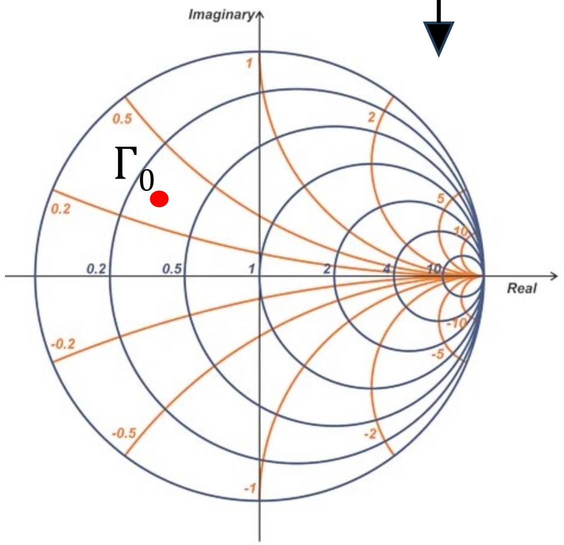
1. Lossless matching → no R
2. Transformers have power and BW limitations in the real world
3. Elements can be connected in series and in parallel (using the admittance smith chart)
4. Single frequency matching

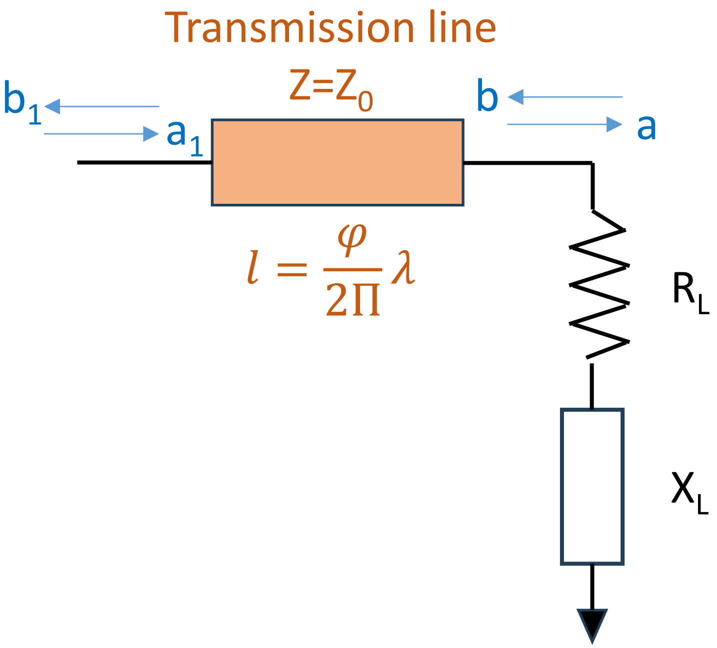


Series transmission line effects

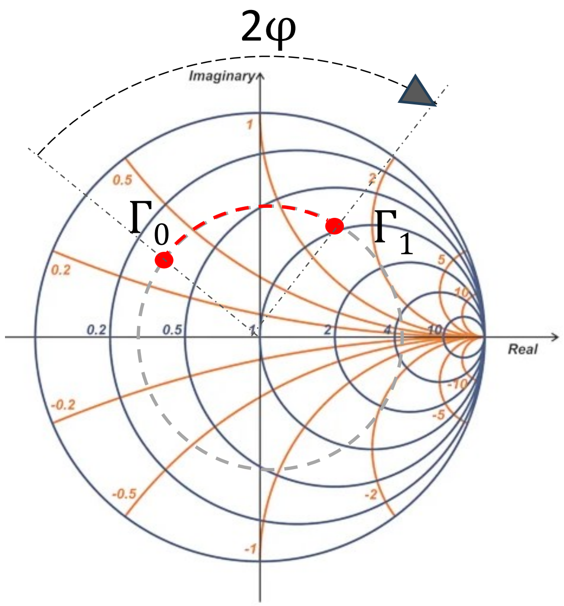


$$\Gamma_0 = \frac{b}{a}$$





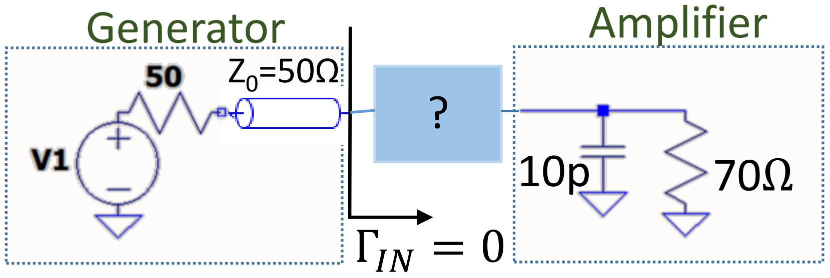
$$\Gamma_1 = \frac{b_1}{a_1} = \frac{b \cdot e^{+j\varphi}}{a \cdot e^{-j\varphi}} = \frac{b}{a} e^{j2\varphi}$$



- A transmission line in series **with characteristic impedance equal to the reference impedance** is a delay element that changes only the phase of the reflection coefficient

Impedance matching exercise

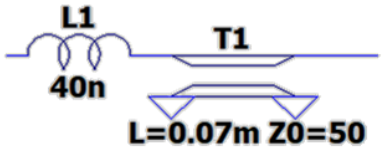
An RF amplifier has an equivalent input impedance of 70Ω in parallel with 10pF at the operating frequency of 100MHz. Calculate a lossless matching network for the maximum power transfer from the 50Ω generator to the amplifier?



Matching means transforming the load impedance to the line impedance

$$Z_{IN} = 70\Omega \,//\, 10pF = \frac{70 \cdot (j \cdot 2 \cdot \pi \cdot 1e8 * 10e-12)^{-1}}{70 + (j \cdot 2 \cdot \pi \cdot 1e8 * 10e-12)^{-1}}$$

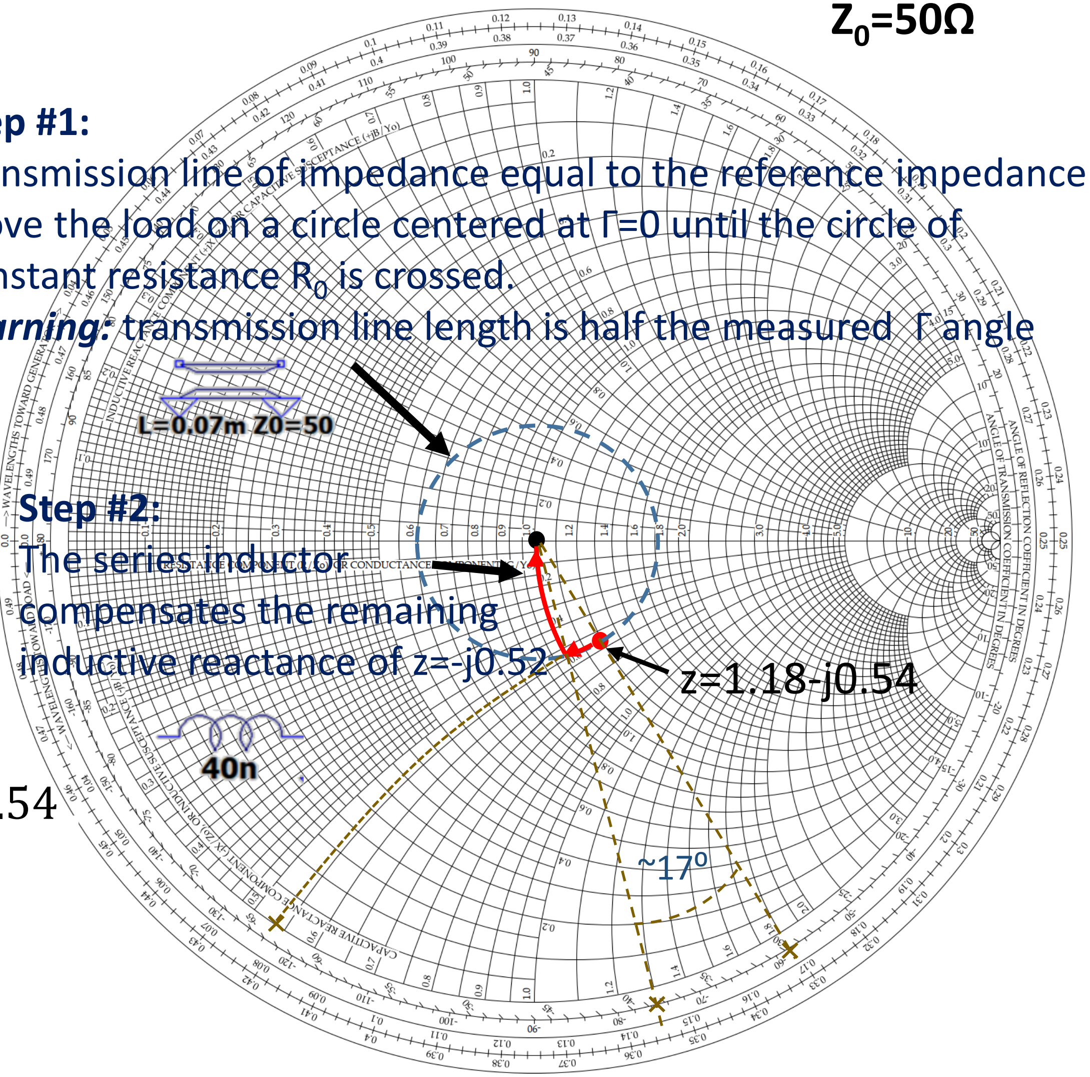
$$\cong \frac{-70 \cdot j166}{70 - j166} \cong 59 - j27\Omega \rightarrow z_{IN} = \frac{Z_{IN}}{Z_0} = 1.18 - j0.54$$



Reference impedance
Z_0=50Ω

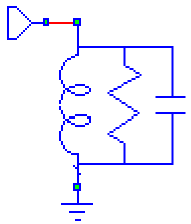
Step #1:
Transmission line of impedance equal to the reference impedance move the load on a circle centered at Γ=0 until the circle of constant resistance R_0 is crossed.
Warning: transmission line length is half the measured Γ angle

Step #2:
The series inductor compensates the remaining inductive reactance of z=-j0.52



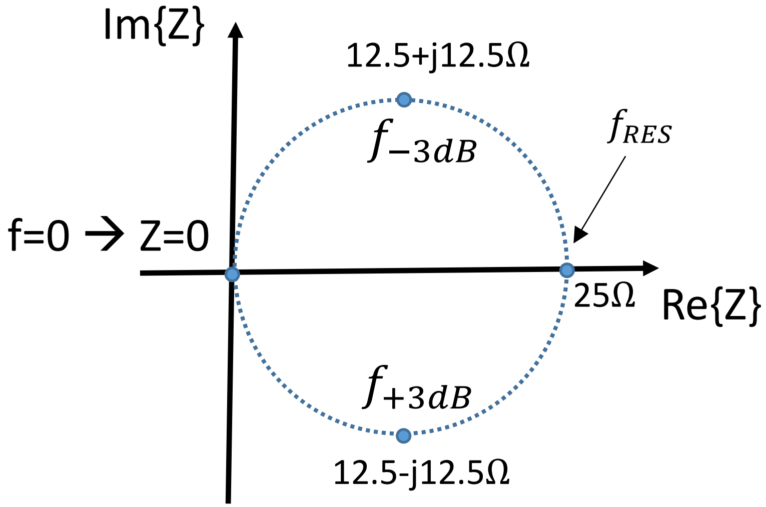
Resonator (1)

Given the parallel resonator



R=25Ω
L=5nH
C=2000pF

a) Calculate Q factor, bandwidth and -3dB frequencies
b) Draw the relevant points into the impedance plane
c) Do the same in the smitch chart



$$f_{RES} = \frac{1}{2\pi\sqrt{LC}} = 50.3MHz$$

$$Q = \frac{R}{\omega L} = 15.8$$

$$BW = \frac{f_{RES}}{Q}$$

$$f_{\pm 3dB} = f_{RES}\left(1 - \frac{1}{2Q}\right) = \begin{cases} 51.9MHz \\ 48.7MHz \end{cases}$$

Let's derive the impedance locus for the resonator:

$$Z_{IN} = \frac{1}{\frac{1}{R} + j\omega C + \frac{1}{j\omega L}} = \frac{1}{\frac{1}{R} + \frac{j}{\omega L}(\omega^2 LC - 1)} = \frac{\omega RL}{\omega L + jR(\omega^2 LC - 1)}$$

$$\left|Z_{IN} - \frac{R}{2}\right| = \left|\frac{R}{2} \cdot \frac{\omega L - j(\omega^2 LC - 1)}{\omega L + j(\omega^2 LC - 1)}\right| = \frac{R}{2} \longrightarrow$$

Circle of radius R/2 centered in R/2

Resonator (2)

What's the reflection coefficient locus?

$$\Gamma = \frac{z-1}{z+1}$$

Resonator in impedance plane (circle): $|z - z_0| = r$

Equivalent to: $zz^* - zz_0^* - z^*z_0 + \underbrace{(z_0^*z_0 - r^2)}_{\text{constant}} = 0$

After shifting reference planes and changing variable :

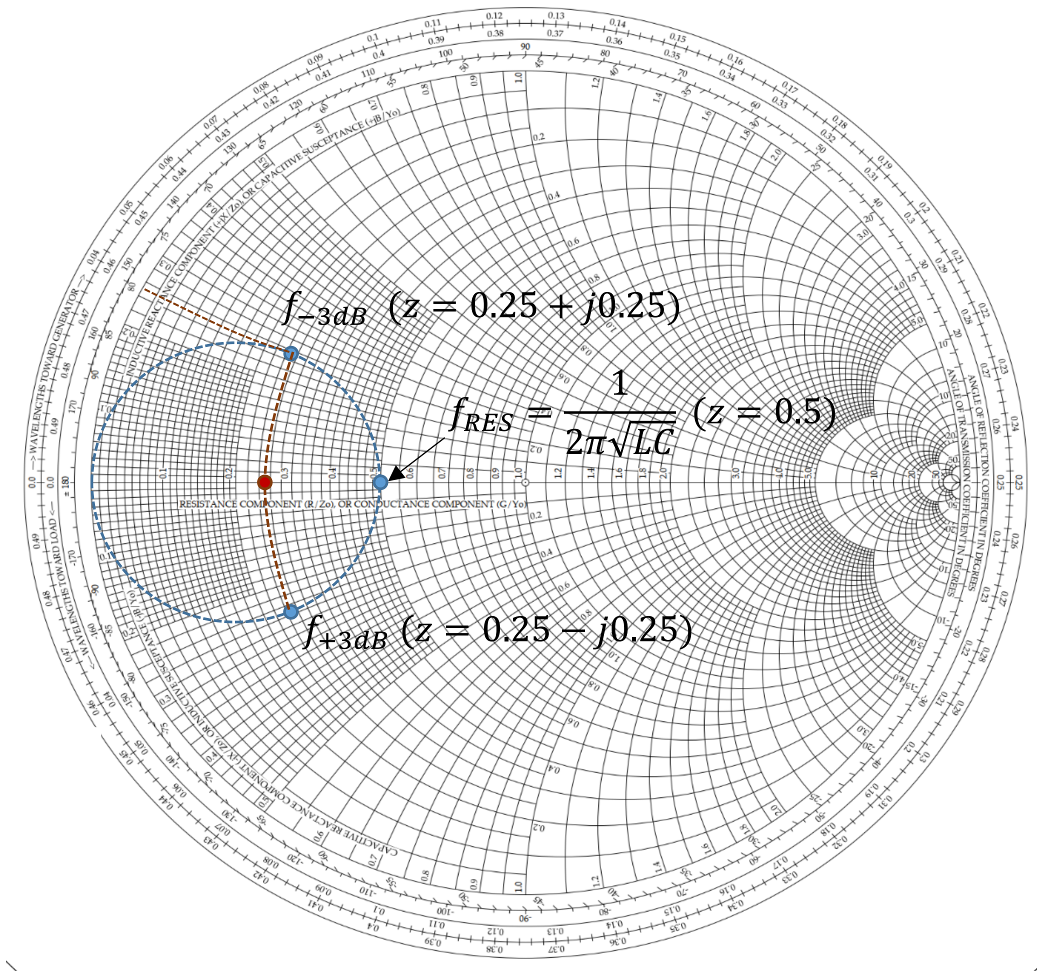
$$\frac{1}{\Gamma}\frac{1}{\Gamma^*} - \frac{1}{\Gamma}\frac{1}{\Gamma_0^*} - \frac{1}{\Gamma^*}\frac{1}{\Gamma_0} + K = 0$$

equivalent to (after algebric manipulations):

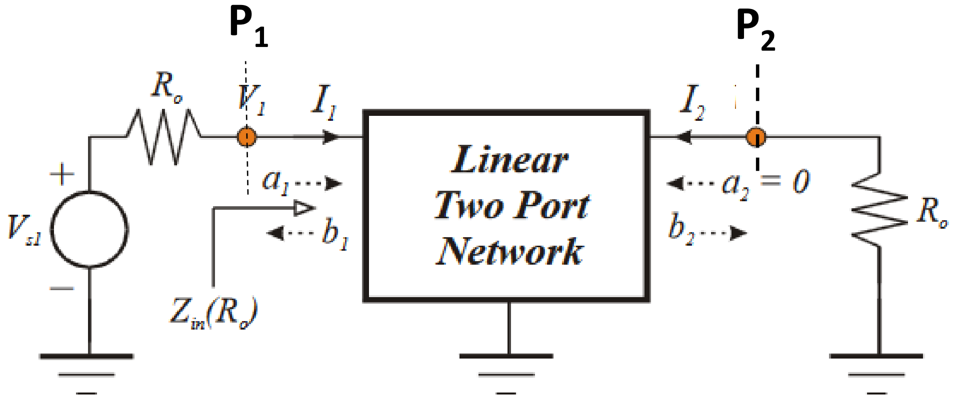
$$\Gamma\Gamma^* - \Gamma^*\Gamma_0 - \Gamma\Gamma_0^* + \Gamma_0^*\Gamma_0 = J^2 \longrightarrow |\Gamma - \Gamma_0| = J$$

Circles in impedance plane are mapped to circles in reflection coefficient plane

Using Z_0=50Ω (arbitrary choice)



S parameters



Reference (line) impedance : R_0

Voltage @ P_N : $V_N = V_{Ni} + V_{Nr}$

Incident wave @ P_N : $a_N = \frac{V_{Ni}}{\sqrt{R_0}}$

Reflected wave @ P_N : $b_N = \frac{V_{Nr}}{\sqrt{R_0}}$

2 ports S parameters $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ $\longrightarrow$ $S_{11} = \frac{b_1}{a_1}\Big|_{a_2 = 0} = \Gamma_1\Big|_{a_2 = 0}$

S_{xx} is the reflection coefficient of port X only when all the other port are terminated on the reference impedance

Other linear network parameters exist, e.g.: impedance (**Z**), admittance (**Y**), hybrid (**H**)

While measuring systems based on the other parameters exist, at high frequency only the S parameters are used.

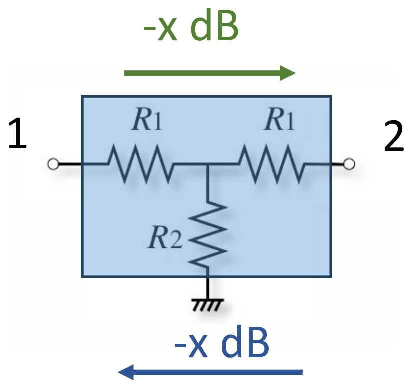
Z,Y,H parameters are based on open and/or short circuits at the ports, impractical at high frequency (an open circuit for example radiates energy). Also extreme reflections (open and short) can make unstable active circuits.

Basic S parameters exercises

1. Parts of the S-matrix of an ideal attenuator are given.
   Fill the missing matrix elements.
   What is the nominal attenuation value in dB written on the component?

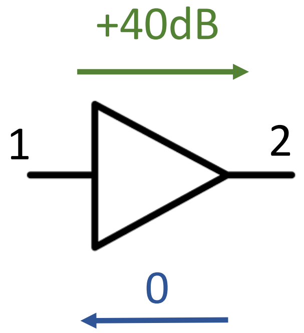
$$S = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}$$

20dB attenuator



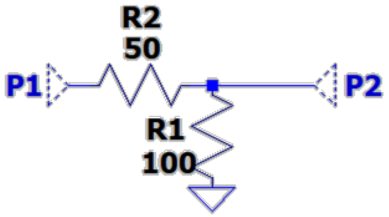
2. An amplifier is perfectly matched at input and output, i.e. no reflection at the input and output ports.
   It has a gain of 40 dB and no reverse transmission.
   Write the S matrix

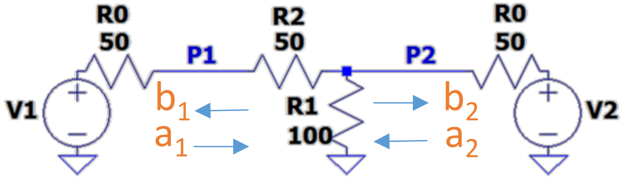
$$S = \begin{bmatrix} 0 & 0 \\ 100 & 0 \end{bmatrix}$$



S parameters exercise

Calculate the S matrix of the 2-port network in a 50 Ω reference system.





$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

What do we expect for S_{21} and S_{12} ?

1. With V_2=0:

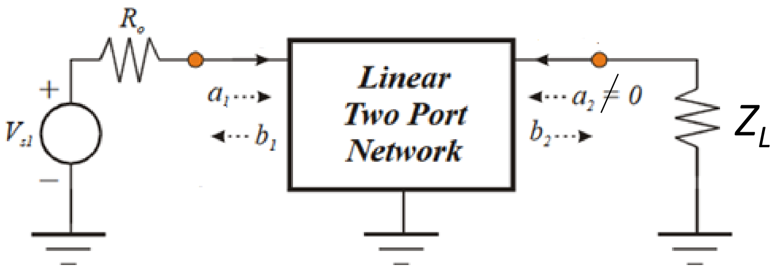
- $S_{11} = \frac{b_1}{a_1} = \Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$ Z_1=(50//100)+50=83.3Ω $\longrightarrow$ $\boldsymbol{S_{11} = 0.25 = ?\ dB}$
- $S_{21} = \frac{b_2}{a_1} = \frac{V_{P2}}{a_1} = \frac{V_{P2}}{V_{P1}} \cdot \frac{V_{P1}}{a_1}$
- $\frac{V_{P2}}{V_{P1}} = \frac{50//100}{50+50//100} = 0.4$
- $V_{P1} = a_1 + b_1 \longrightarrow \frac{V_{P1}}{a_1} = 1 + \frac{b_1}{a_1} = 1 + S_{11} = 1.25 \longrightarrow \boldsymbol{S_{21} = S_{12} = 0.5 = ?\ dB}$

2. With V_2=0:

- $S_{22} = \frac{b_2}{a_2} = \Gamma_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0} = 0$ = **? dB**

General 2-port reflection coefficient

Given the S-parameters, find the input reflection coefficient of a 2-port network for a generic load impedance.



$$\Gamma_1 = \frac{b_1}{a_1} \qquad \Gamma_L = \frac{a_2}{b_2}$$

Expanding the 2 ports S matrix:

$$\begin{cases} b_1 = S_{11}a_1 + S_{12}a_2 \quad \rightarrow \quad \Gamma_1 = \dfrac{b_1}{a_1} = S_{11} + S_{12}\dfrac{a_2}{a_1} \\ b_2 = S_{21}a_1 + S_{22}a_2 \quad \rightarrow \quad \dfrac{a_1}{a_2} = \left(\dfrac{b_2}{a_2} - S_{22}\right)\dfrac{1}{S_{21}} = \left(\dfrac{1}{\Gamma_L} - S_{22}\right)\dfrac{1}{S_{21}} \end{cases}$$

$$\Gamma_1 = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - \Gamma_L S_{22}}$$

For which conditions the input reflection coefficient is independent of the load impedance? Why?

From the reflection coefficient, find the input impedance for a generic load.

$$Z_1 = R_0\left(\frac{1+\Gamma_1}{1-\Gamma_1}\right) = R_0\left[\frac{(1-\Gamma_L S_{22})(1+S_{11}) + S_{12}S_{21}\Gamma_L}{(1-\Gamma_L S_{22})(1-S_{11}) - S_{12}S_{21}\Gamma_L}\right]$$