Joint Universities Accelerator School

JUAS 2025

18. – 24. February 2025

Normal-conducting accelerator magnets
Lecture 1: Introduction

Thomas Zickler

CERN

Scope of the lectures

Overview of electro-magnet technology as used in particle accelerators considering *normal-conducting, iron-dominated* electro-magnets (generally restricted to direct current situations)

Main goal is to:

- create a fundamental understanding in accelerator magnet technology
- provide a guidebook with practical instructions how to start with the design of a conventional accelerator magnet
- focus on applied and practical design using 'real' examples
- guide students through an exhaustive case study designing a real normal-conducting accelerator magnet

Not covered:

- permanent magnet technology
- superconducting technology (see SC magnet lectures)

Literature

- CAS proceedings, Fifth General Accelerator Physics Course, University of Jyväskylä, Finland, September 1992, CERN Yellow Report 94-01
- International Conference on Magnet Technology, Conference proceedings
- Iron Dominated Electromagnets, J. T. Tanabe, World Scientific Publishing, 2005
- Magnetic Field for Transporting Charged Beams, G. Parzen, BNL publication, 1976
- Magnete, G. Schnell, Thiemig Verlag, 1973 (German)
- Field Computation for Accelerator Magnets: Analytical and Numerical Methods for Electromagnetic Design and Optimization, S. Russenschuck, Wiley-VCH, 2010
- Practical Definitions & Formulae for Normal Conducting Magnets, D. Tommasini, Sept. 2011
- CAS proceedings, Magnetic measurements and alignment, Montreux, Switzerland, March 1992, CERN Yellow Report 92-05
- CAS proceedings, Measurement and alignment of accelerator and detector magnets, Anacapri, Italy, April 1997, CERN Yellow Report 98-05
- The Physics of Particle Accelerators: An Introduction, K. Wille, Oxford University Press, 2000
- CAS proceedings, Magnets, Bruges, Belgium, June 2009, CERN Yellow Report 2010-004
- JUAS proceedings, Proceedings of the Joint Universities Accelerator School (JUAS)—Courses and exercises, November 2024, CERN Yellow Report 2024-003

Acknowledgements

Many thanks …

… to all my colleagues who contributed to this lecture, in particular J.Bauche, L.Bottura, M.Buzio, B.Langenbeck, N.Marks, A.Milanese, S.Russenschuck, D.Schoerling, C.Siedler, S.Sgobba, D.Tommasini, A.Vorozhtsov, A.Wolski

Timeline Lectures

Tuesday, 18.2.2025

Lecture 1	13:15 – 14:15
Introduction (50’)	
Lecture 2	14:15 – 15:15
Basic principles (50’)	
Lecture 3	15:15 – 16:15
Magnet types (50’)	

Wednesday, 19.2.2025

Lecture 4	13:15 – 14:15
Magnet construction (50’)	
Lecture 5	14:15 – 15:15
Analytical design (50’)	
Lecture 6	15:15 – 16:15
Numerical design (50’)	

Timeline Case Study I

Thursday, 20.2.2025

Case Study Intro	13:45 – 14:45
Introduction (50')	
Case Study #1	14:45 – 16:45*
Working Groups (120')	

Friday, 21.2.2025

Case Study #2	13:15 – 14:15
Working Groups (60')	
Case Study #3	14:15 – 15:15
Working Groups (60')	
Case Study #4	15:15 – 16:15
Working Groups (60')	

*) optional open end

Timeline Case Study II & Exam

Monday, 24.2.2025

Case study results I	11:00 – 12:00
Oral presentations by students	
15’ per group	
Case study results II	13:15 – 14:15
Tutorial	14:15 – 16:15
Magnet Optimization with FEMM (120’)	

Monday, 3.3.2025

Deadline for Written Reports*	12:00
Written Exam NC + SC Magnets	14:15 – 16:15

*) contributes to the final mark

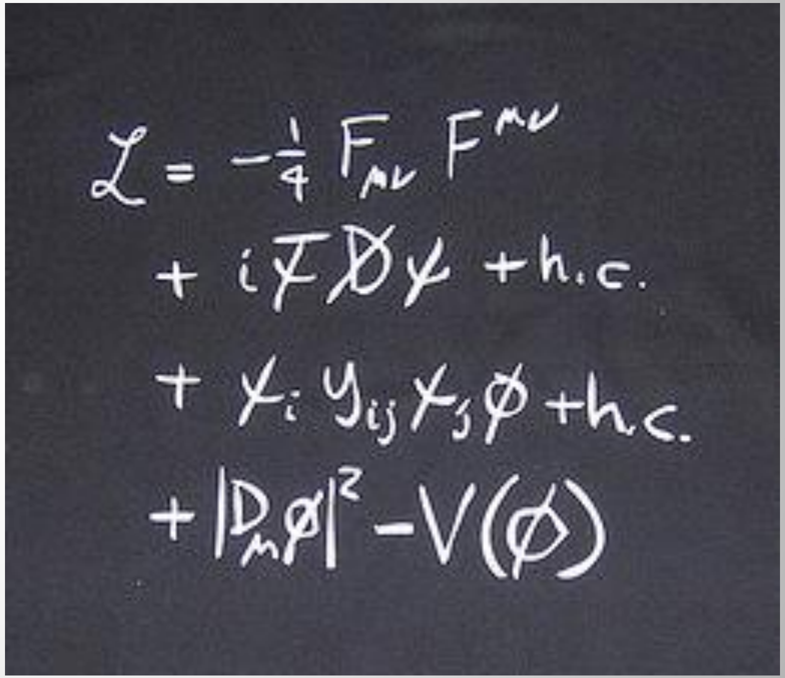
Introduction

Why do we need magnets?

A bit of history…

Maxwell and friends

Field description by multipoles



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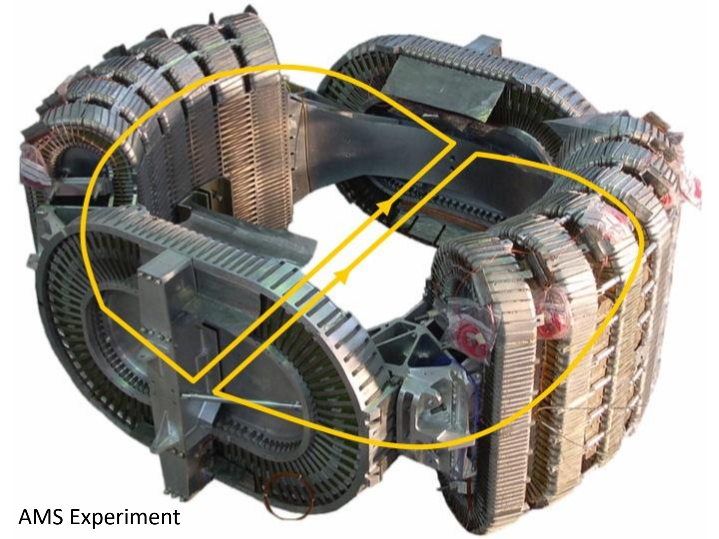
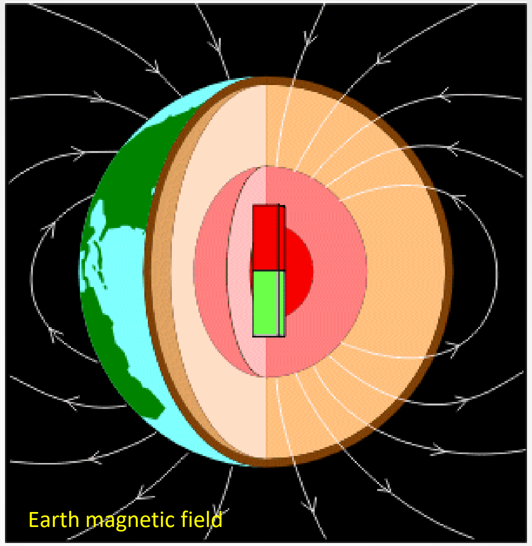
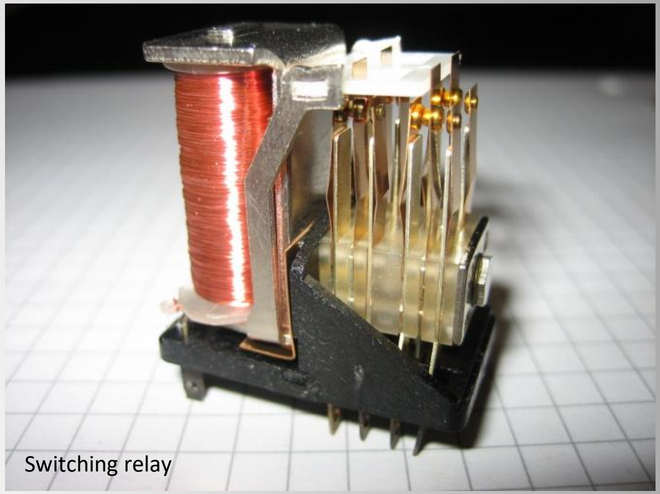
Magnets everywhere…

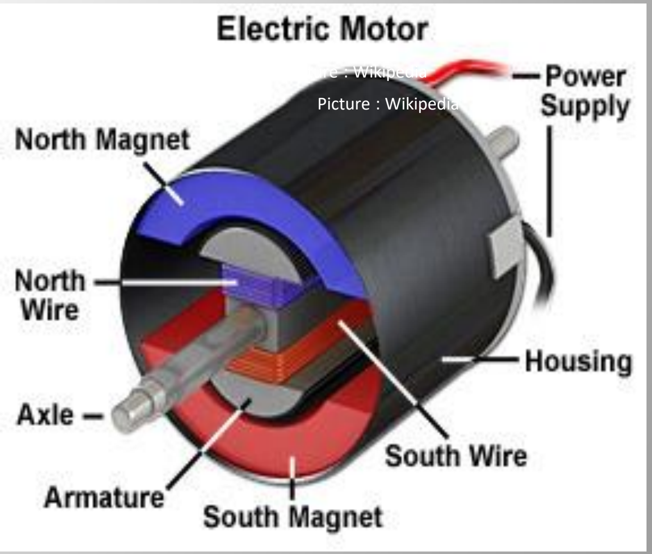

AMS Experiment


Earth magnetic field


Switching relay


MAGLEV train

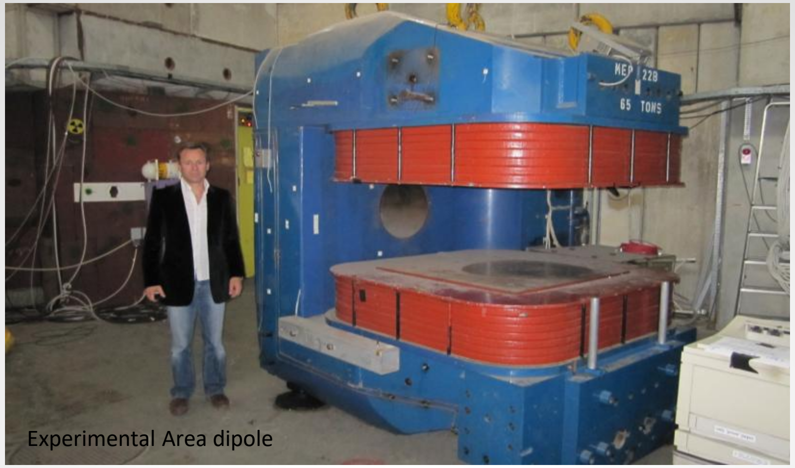

Waste sorting

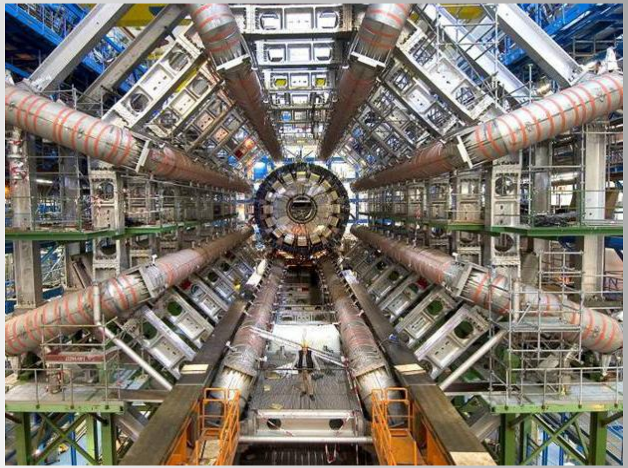


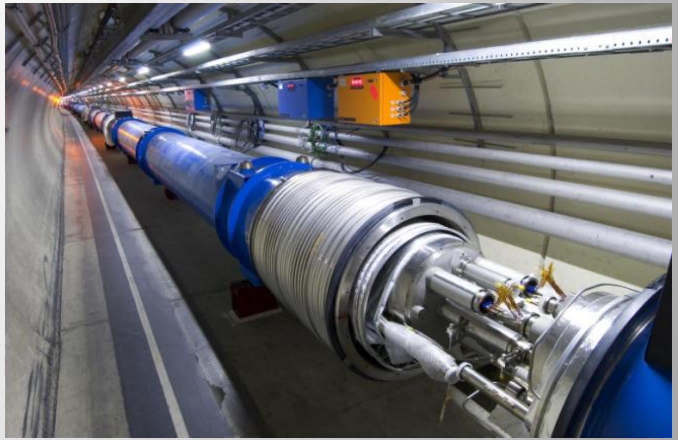
Picture : Wikipedia

Magnets at CERN


Linac4 quadrupole


Experimental Area dipole

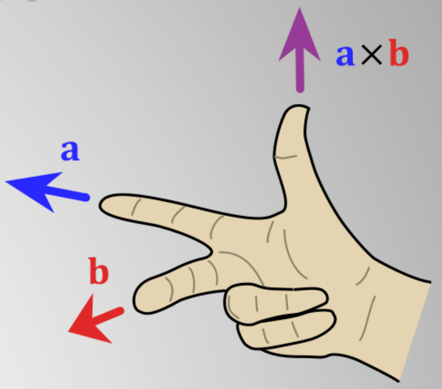




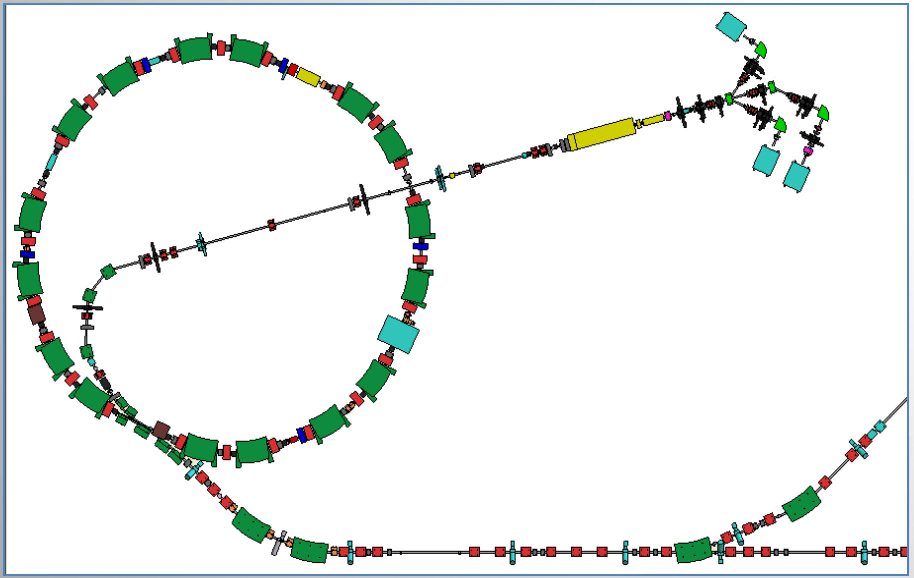
Normal-conducting magnets: 4800 magnets (50 000 t) are installed in the CERN accelerator complex
Superconducting magnets: 10 000 magnets (50 000 t) mainly in LHC
Permanent magnets: 150 magnets (4 t) in Linacs & Experimental Areas

Why do we need magnets?

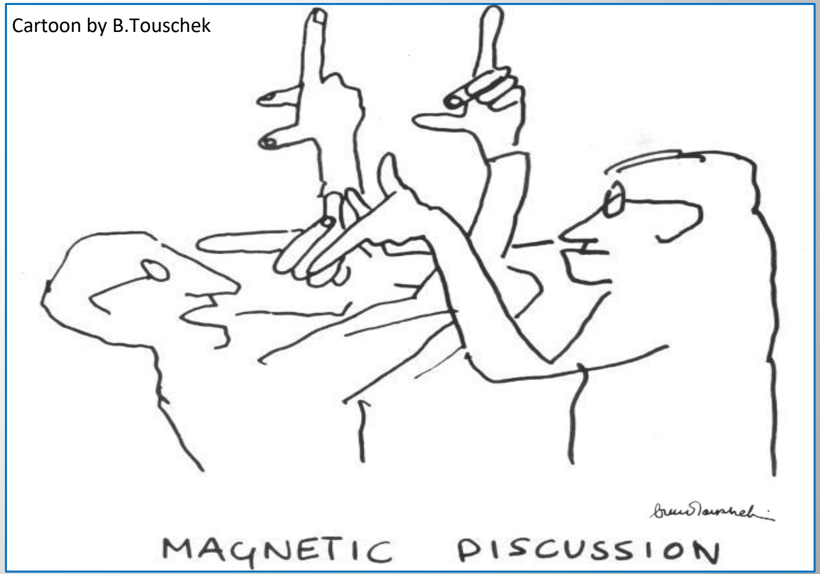
- Interaction with the beam
  - guide the beam to keep it on the orbit
  - focus and shape the beam
- Lorentz's force: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
  - for relativistic particles this effect is equivalent if $\vec{E} = c\vec{B}$
  - if $B = 1$ T then $E = 3\cdot10^8$ V/m(!)



„*Right hand rule*" applies

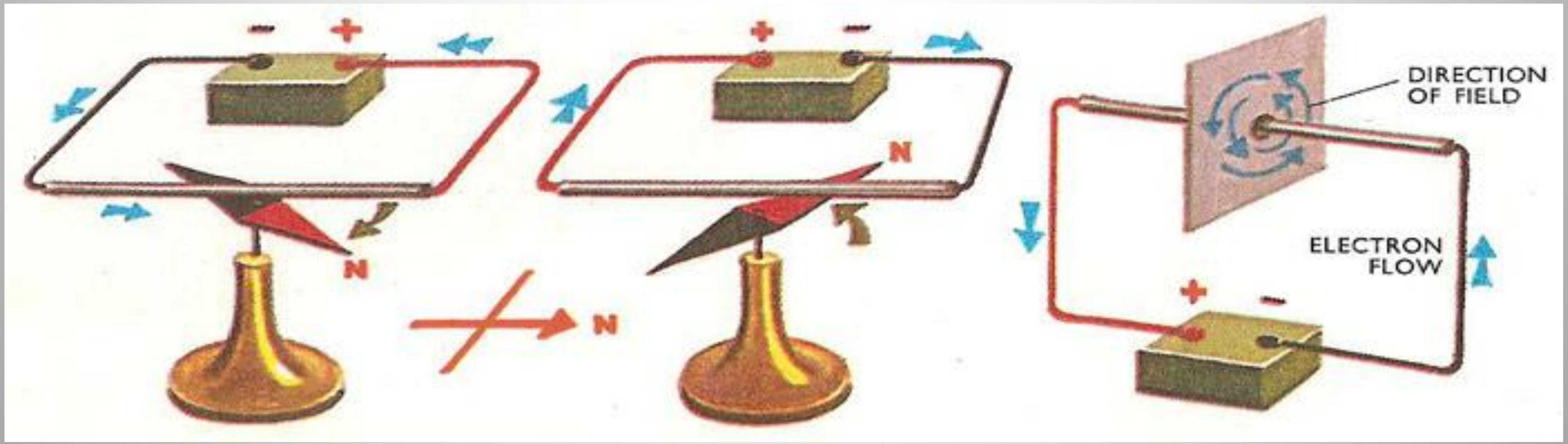


Cartoon by B.Touschek



A bit of history…

1820: Hans Christian Ørsted (1777-1851) finds that electric current affects a compass needle

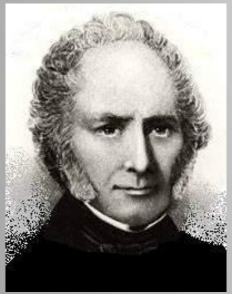


"Electricity and magnetism are somehow related…"

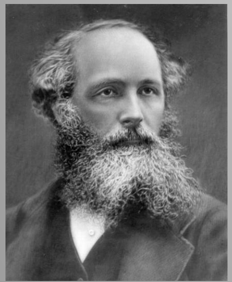
A bit of history...

1820: Andre Marie Ampere (1775-1836) in Paris finds that wires carrying current produce forces on each other

1820: Michael Faraday (1791-1867) at Royal Society in London develops the idea of electric fields and studies the effect of currents on magnets and magnets inducing electric currents

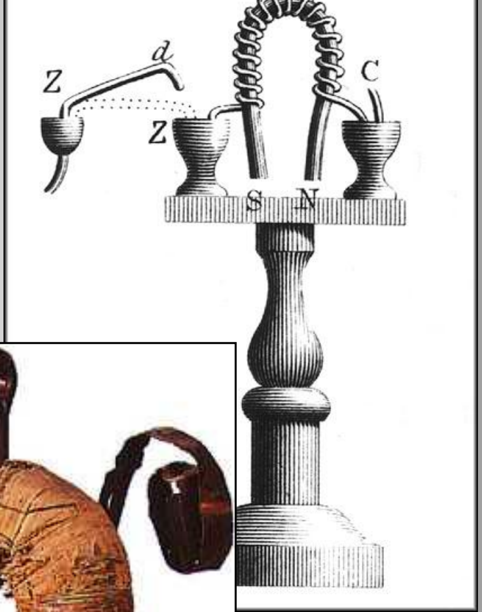


1825: British electrician, William Sturgeon (1783-1850) invented the first electromagnet



1860: James Clerk Maxwell (1831-1879), a Scottish physicist and mathematician, puts the theory of electromagnetism on mathematical basis

Pictures: Wikipedia

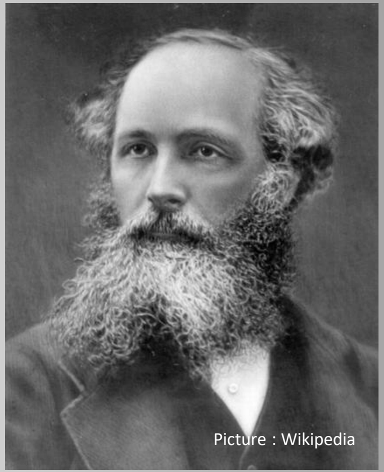


William Sturgeon

Joseph Henry

Maxwell's equations

In 1873, Maxwell published "Treatise on Electricity and Magnetism" in which he summarized the discoveries of Coulomb, Ørsted, Ampere, Faraday, et. al. in four mathematical equations (valid in free space):


Picture : Wikipedia

Gauss' law for electricity: $\nabla \cdot \vec{D} = \rho$ $\quad \oiint_{\partial V} \vec{D} \cdot d\vec{a} = \rho$

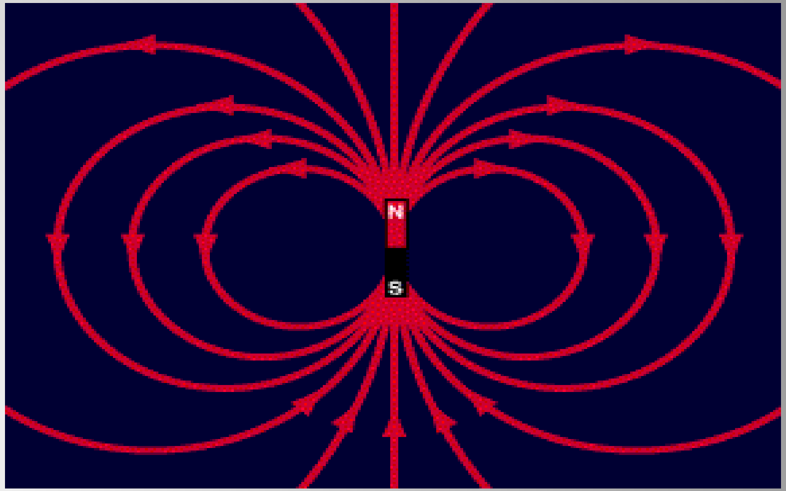
Gauss' law of flux conservation: $\nabla \cdot \vec{B} = 0$ $\quad \oiint_{\partial V} \vec{B} \cdot d\vec{a} = 0$

Faraday's law of induction: $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\quad \oint_{\partial a} \vec{E} \cdot d\vec{s} = -\frac{d\Phi}{dt} = -\frac{d}{dt} \iint_{a} \vec{B} \cdot d\vec{a}$

Ampere's law: $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ $\quad \oint_{\partial a} \vec{H} \cdot d\vec{s} = \iint_{a} \vec{J} \cdot d\vec{a} + \frac{d}{dt} \iint_{a} \vec{D} \cdot d\vec{a}$

Constitutional laws: $\vec{D} = \varepsilon \vec{E}$ $\quad \vec{B} = \mu \vec{H}$

Magnet vocabulary



IEEE defines the following terms and units:

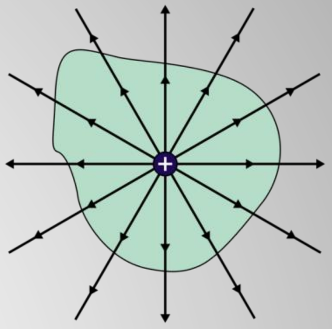
- Magnetic field strength:
  - H (vector) [A m^{-1}]
  - magnetizing force produced by electric currents
- Magnetic flux density:
  - B (vector) [T or kg A^{-1} s^{-2})]
  - density of magnetic flux driven through a medium by the magnetic field
  - Note: both, magnetic field strength and flux density are frequently referred to as "Magnetic Field“
  - H, B and μ relates by the constitutive law for materials: $B = \mu H$
- Permeability:
  - $\mu = \mu_0 \mu_r$
  - permeability of free space $\mu_0 = 4 \cdot \pi \cdot 10^{-7}$ [V s A^{-1} m^{-1} or kg A^{-1} s^{-2}]
  - relative permeability μ_r (dimensionless): $\mu_{\text{air}} = 1$; $\mu_{\text{iron}} > 1000$ (not saturated)
- Magnetic flux:
  - ϕ [Wb or kg m^2 A^{-1} s^{-2}]
  - surface integral of the flux density component perpendicular trough a surface

Special case: Magnetostatics

Gauss' law for electricity:

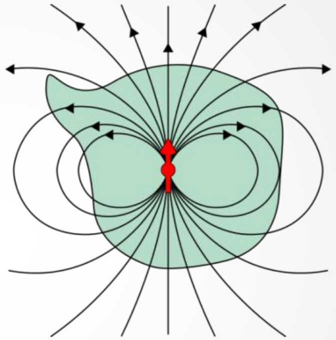
$$\nabla \cdot \vec{D} = \rho$$

$$\vec{D} = \varepsilon \vec{E}$$



Gauss' law of flux conservation:

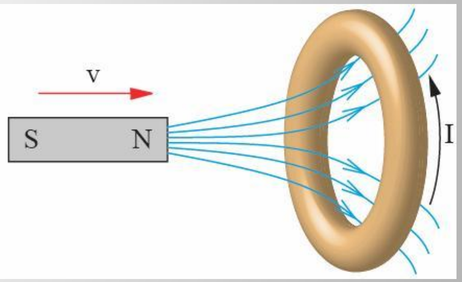
$$\nabla \cdot \vec{B} = 0$$



Faraday's law of induction:

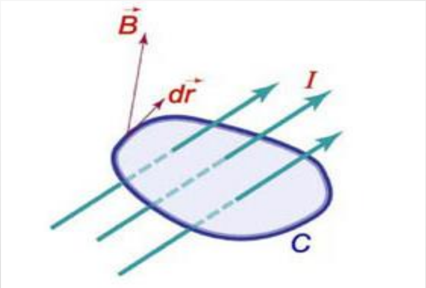
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{B} = \mu \vec{H}$$



Ampere's law:

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$



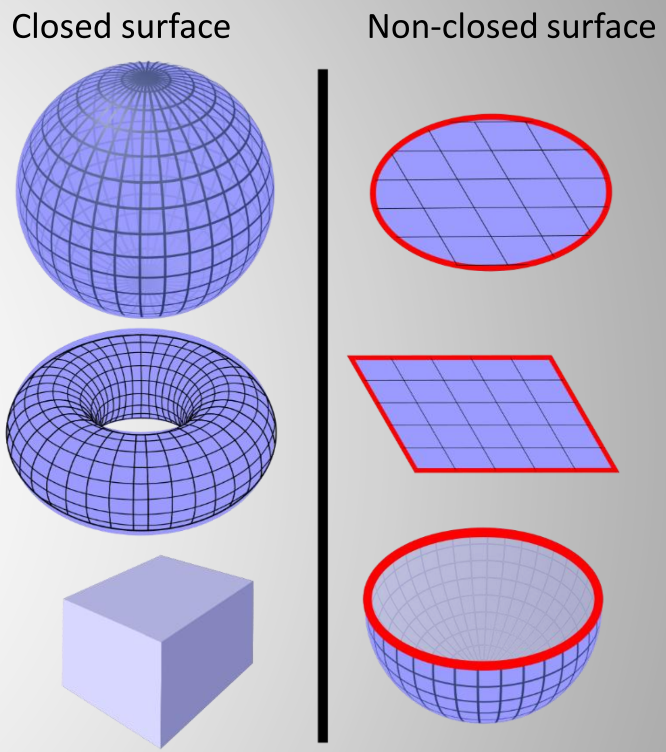
Gauss' law for magnetism

The B field is divergence free:

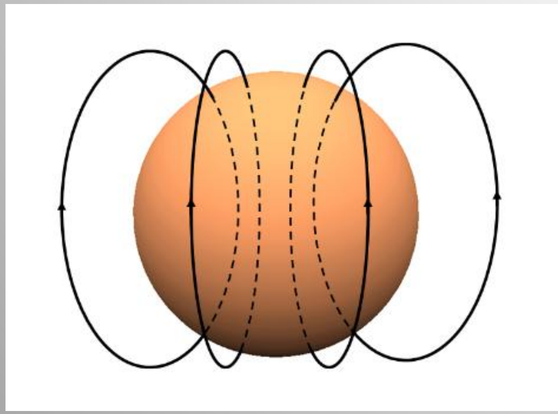
$$\nabla \cdot \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

Gauss' theorem relates the flux of a vector field through a closed surface to the divergence of the field in the enclosed volume

$$\oiint_{\partial V} \vec{B} \cdot d\vec{a} = \iiint_V \nabla \cdot \vec{B} dV = 0$$



Picture : Wikipedia



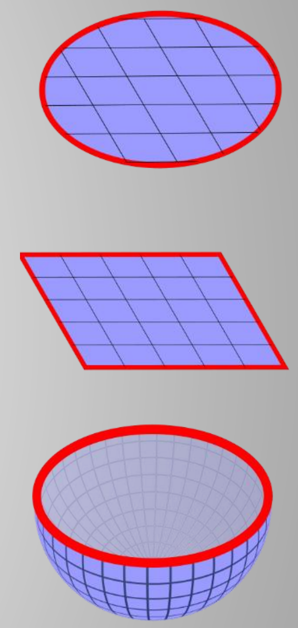
The sum of the total magnetic flux through any Gaussian (closed) surface is zero, or, the magnetic field is a solenoidal vector field with neither sources nor sinks

Ampere's circuital law

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\vec{i_x} + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\vec{i_y} + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\vec{i_z} = \vec{J}$$

(Open) Surface with closed boundary

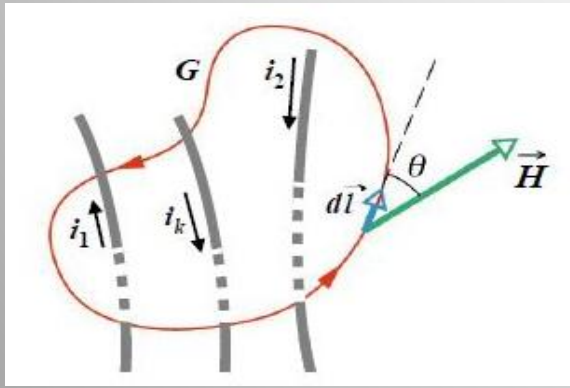


Picture : Wikipedia

Stokes' theorem relates the integral of the normal component of the curl of a vector field over an open surface to the line integral of the vector field around the boundary of the surface

$$\iint_a \nabla \times \vec{H}\, da = \oint_{\partial a} \vec{H} \cdot d\vec{s} = \iint_a \vec{J}\, da = NI$$

$$\oint_{\partial a} \vec{H} \cdot d\vec{s} = NI$$

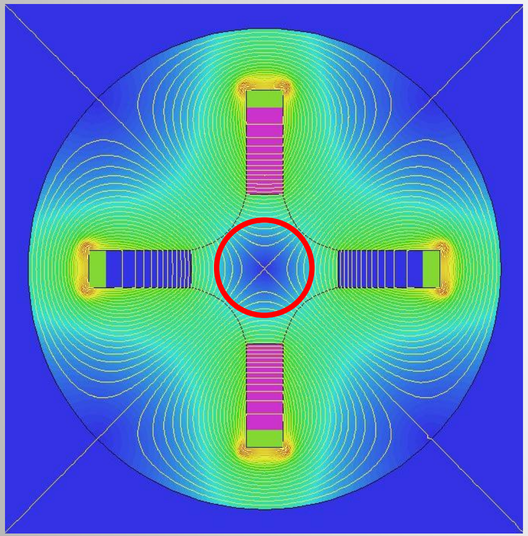


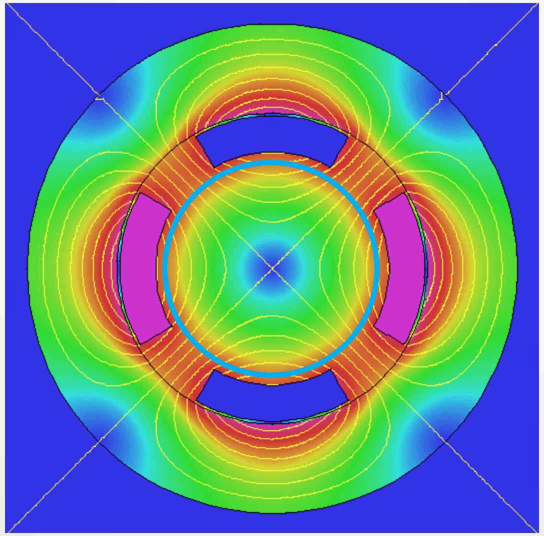
The line integral of the magnetic field around a closed boundary equals the total current passing through a surface that is surrounded by that boundary

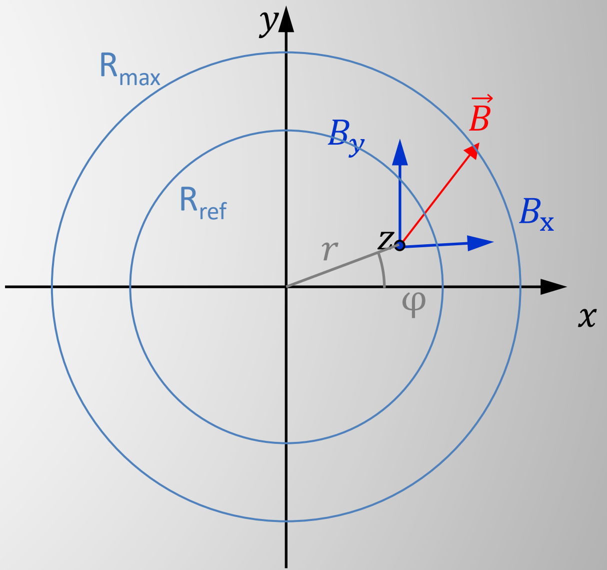
Field description

How can we conveniently describe $\vec{B} = B_y + iB_x$ in the aperture?

- at any point $\boldsymbol{z} = x + iy = re^{i\varphi}$ in a 2D plane, where B_z = constant
- for any field configuration
- regardless of the magnet technology and how the field is produced







$\boldsymbol{B} = B_y + iB_x = \boldsymbol{C}_n\, \boldsymbol{z}^{n-1}$ with the complex coefficient $\boldsymbol{C}_n = B_n + iA_n$ and $\boldsymbol{z} = x + iy$ is a possible solution to Maxwell's equations for a magnetostatic situation in free space. Magnetic fields conform to this description are called multipole fields

"we can describe a complex field vector by a complex scalar coefficient and the complex position vector"

Multipole fields

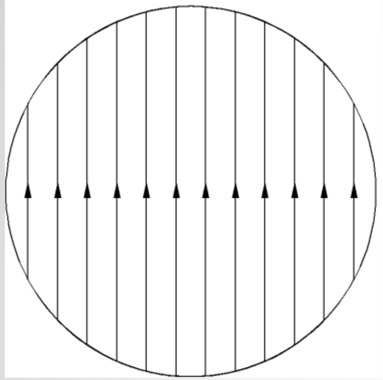
What is the meaning of $C_n = B_n + iA_n$?

''Normal'' field

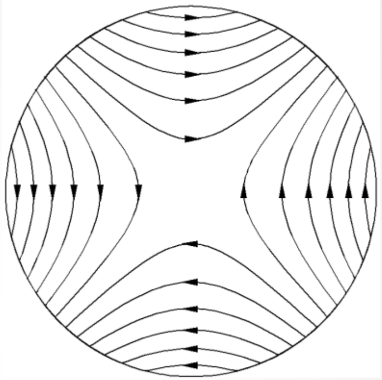
Rotation by $\pi/2n$

'Skew'' field

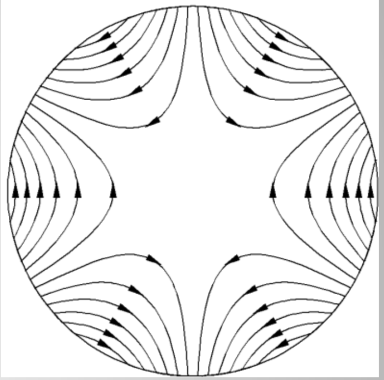
C_1= real: normal dipole



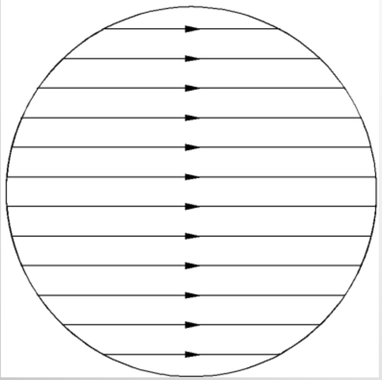
C_2 = real: normal quadrupole



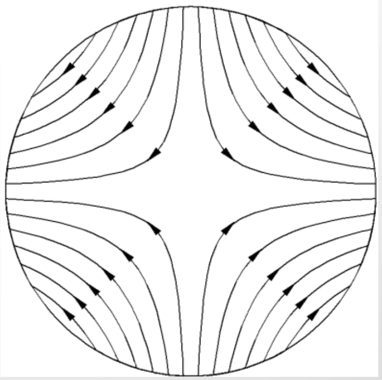
C_3 = real: normal sextupole



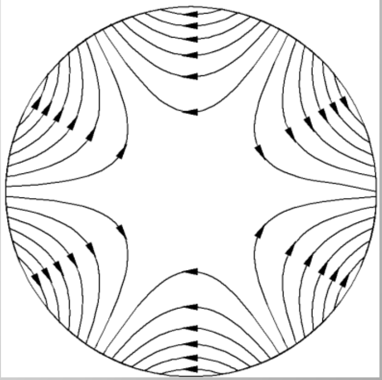
C_1 = imaginary: skew dipole



C_2 = imaginary: skew quadrupole



C_3 = imaginary: skew sextupole



Flux lines are vector equipotential lines and $\vec{B}$ is always tangential to the flux lines.

Multipole fields

"Normal" field

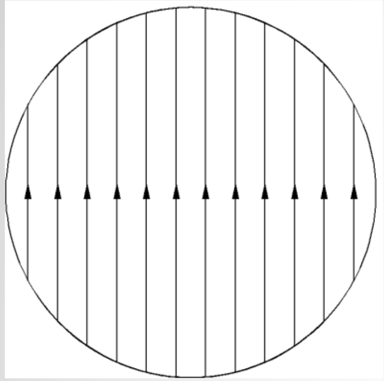
↓

Rotation by $\pi/2n$

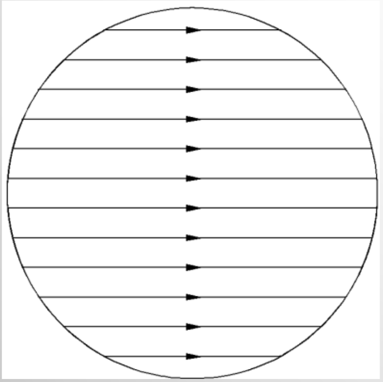
↓

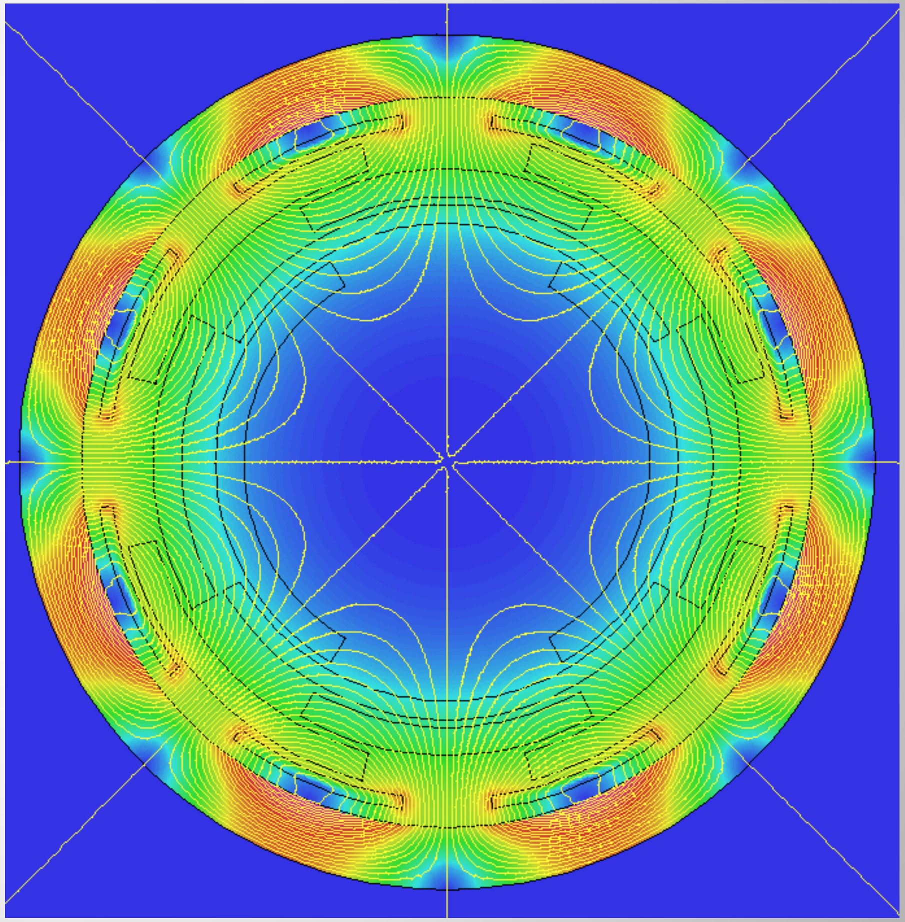
'Skew" field

C_1= real: normal dipole



C_1 = imaginary: skew dipole





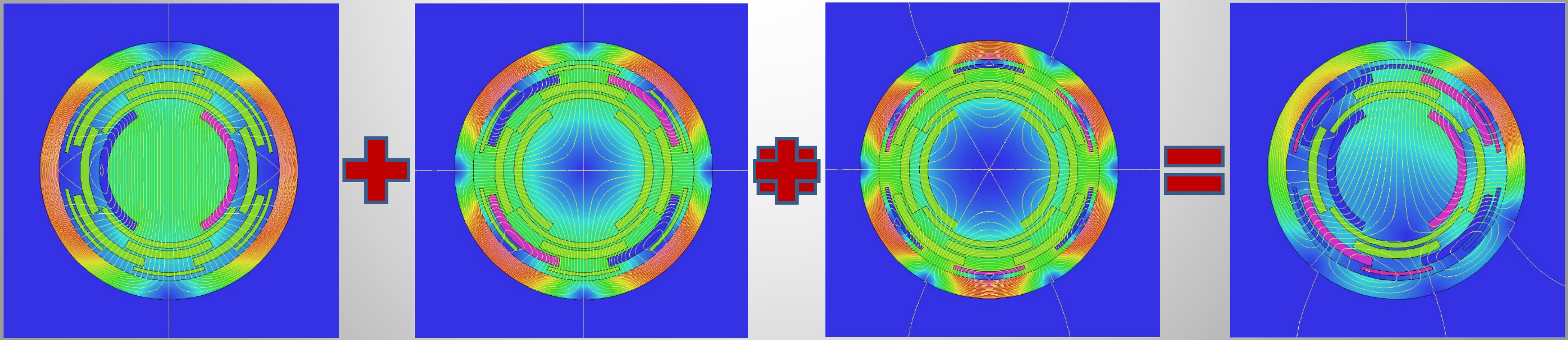
Linearity and superposition

Maxwell's equation are linear:

$$\nabla \cdot \left(\overrightarrow{B_1} + \overrightarrow{B_2}\right) = \nabla \cdot \overrightarrow{B_1} + \nabla \cdot \overrightarrow{B_2}$$

$$\nabla \times \left(\overrightarrow{H_1} + \overrightarrow{H_2}\right) = \nabla \times \overrightarrow{H_1} + \nabla \times \overrightarrow{H_2}$$

Hence, we can apply the principle of superposition to construct complicated magnetic fields just by adding together a set of simpler fields



Field description

Since Maxwell's equation are linear, we can superpose any number of multipole fields and obtain a valid solution to Maxwell's equation

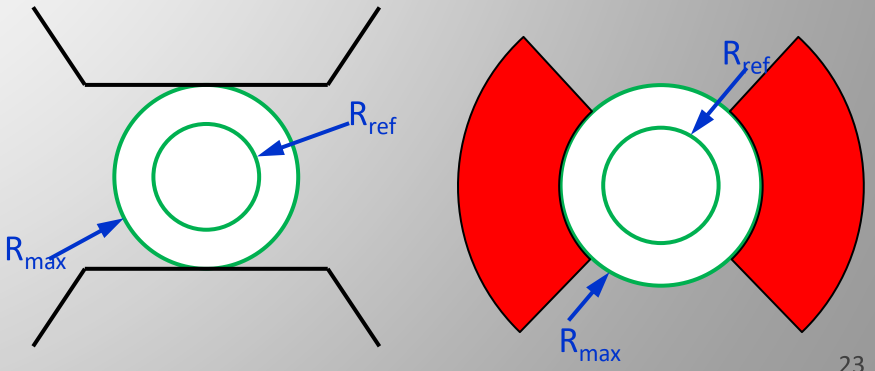
Hence, we can describe any arbitrary 2D vector field within a circle of validity R_{max} by a series of scalar coefficients:

$$B_y + iB_x = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{x + iy}{R_{ref}} \right)^{n-1}$$

The B_n are the "normal multipole coefficients" and the A_n are the "skew multipole coefficients". R_{ref} is a reference radius, which might be chosen arbitrarily; however, the value of B_n and A_n depends on R_{ref}

This 2D decomposition holds only in a region of space:

- without magnetic materials ($\mu_r = 1$)
- without currents
- when B_z is constant



Summary

- Magnets in an accelerator or beam line are needed to steer and focus the charged particle beam
- The magnetic flux density or magnetic induction $\boldsymbol{B}$ is the density of magnetic flux driven through a medium by the magnetic field $\boldsymbol{H}$
- Gauss' law and Ampere's law are the two fundamental Maxwell's equation in magneto-statics
- Magnetic fields and magnetic induction are linear and can be superposed
- A 2D vector field can be expressed by a series of multipole coefficients

Thanks for your attention…