Joint Universities Accelerator School

JUAS 2025

18. – 24. February 2025

Normal-conducting accelerator magnets
Lecture 2: Basic principles

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CERN

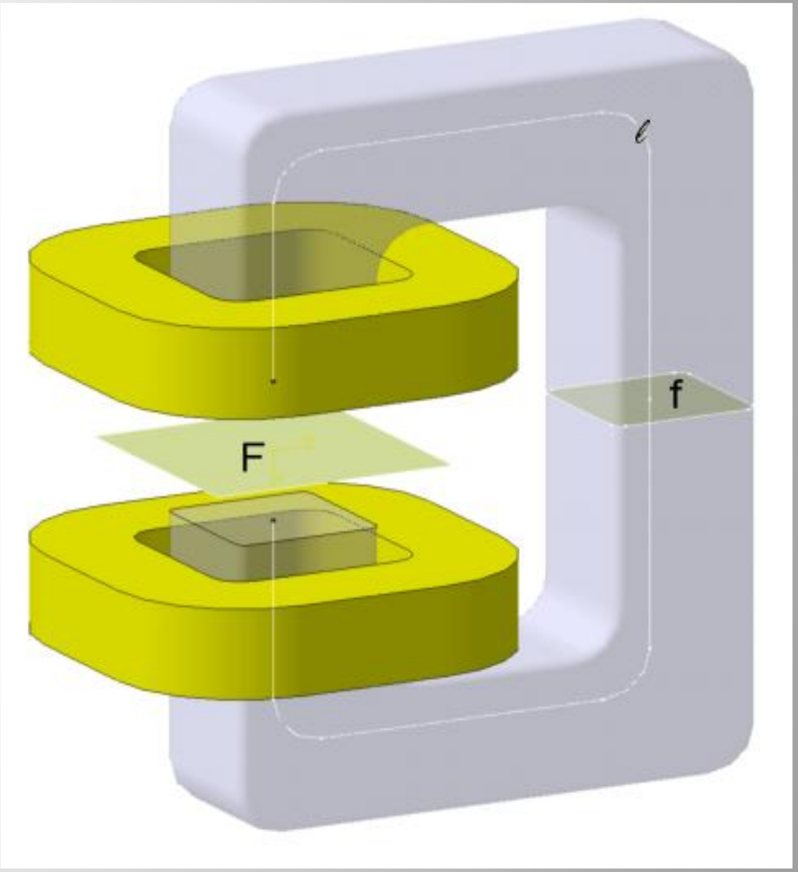
Basic principles

Coil dominated vs. Iron dominated

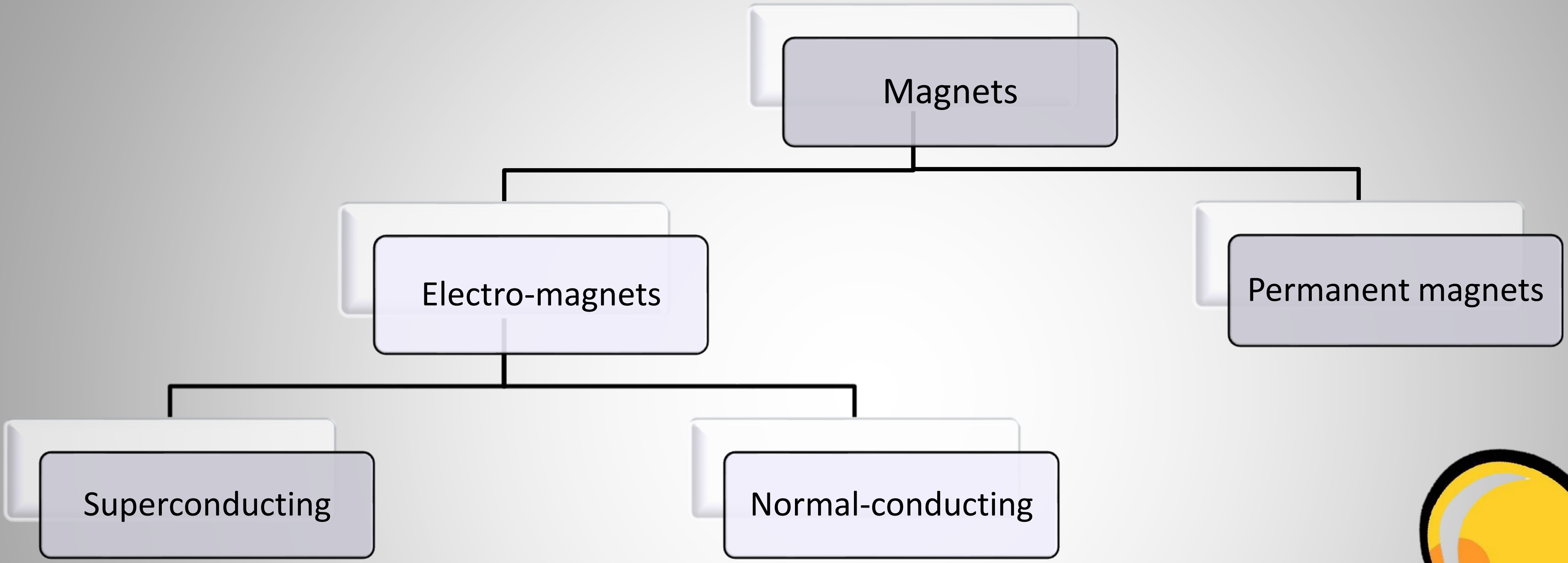
Purpose of (ideal) magnetic circuits

Real magnets and multipole errors

Iron properties



Magnet technologies


Magnets

Electro-magnets

Permanent magnets

Superconducting

Normal-conducting

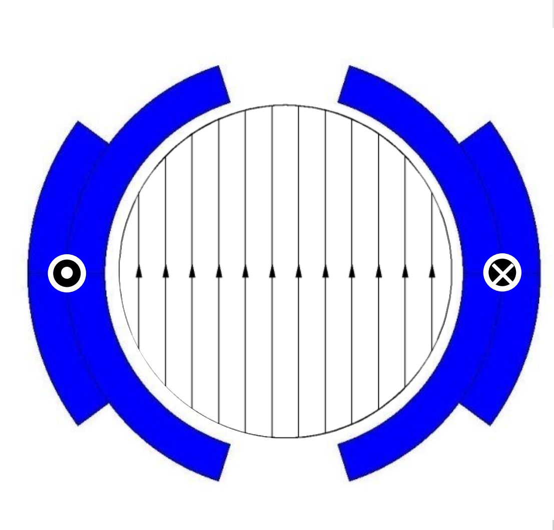
- zero electrical resistance
- no ohmic losses
- high current densities
- requires cryogenic cooling

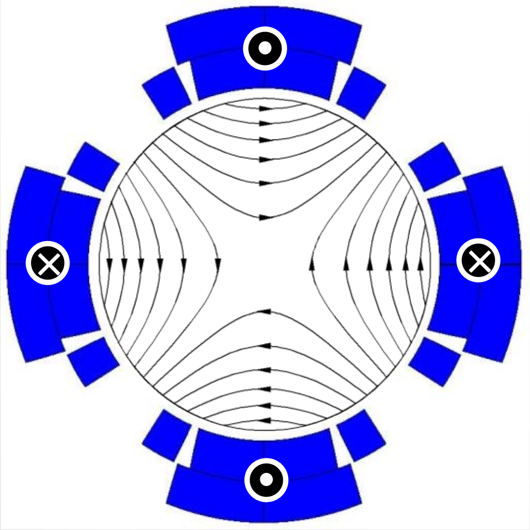
- limited by the ohmic losses
- only moderate current densities
- dissipated power to be removed
- requires air or water cooling

Coil dominated – Iron dominated

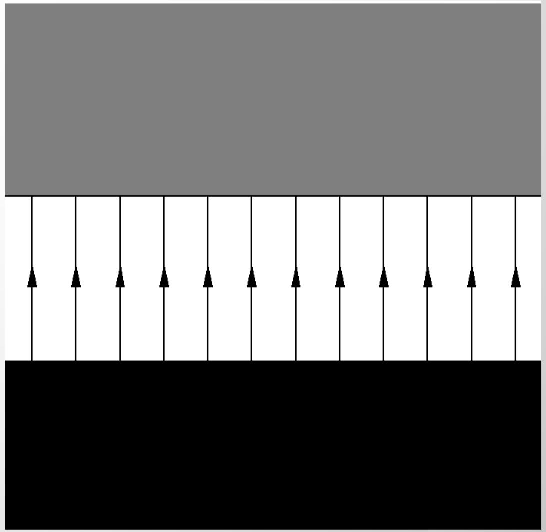
In coil-dominated magnets, the magnetic field in the aperture is shaped by the position of the conductors respectively the current distribution around the aperture

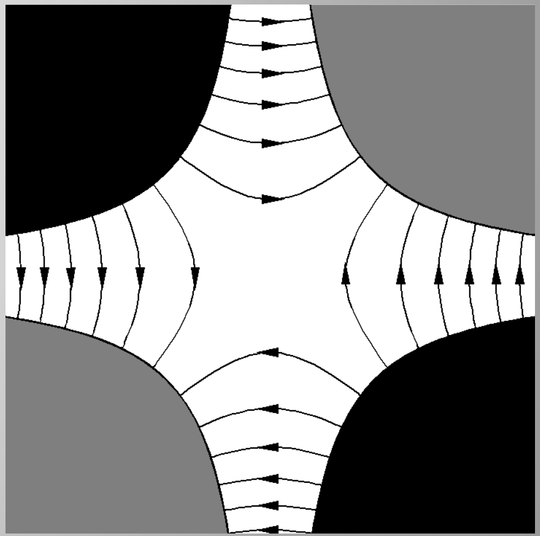
In iron-dominated magnets, the magnetic field is shaped by the geometry of the poles, which are surfaces of constant scalar potential


B_1: normal dipole


B_2: normal quadrupole

High currents -> often superconducting

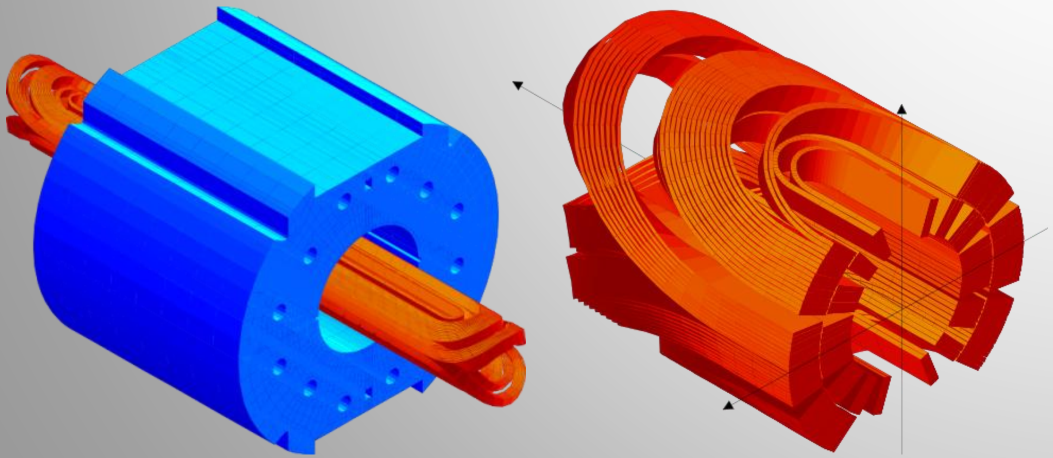

B_1: normal dipole

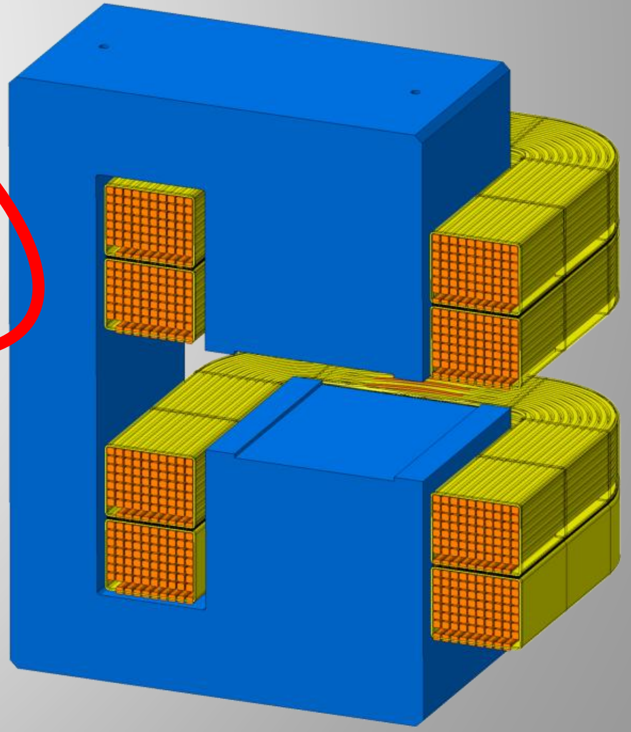

B_2: normal quadrupole

Moderate currents -> often normal-conducting

Magnet technologies

- Magnets
  - Electro-magnets
    - Superconducting
      - **Coil dominated** $B < 11$ T
      - Iron dominated $B < 2$ T
    - Normal-conducting
      - Coil dominated $B < 1$ T
      - **Iron dominated** $B < 2$ T
  - Permanent magnets

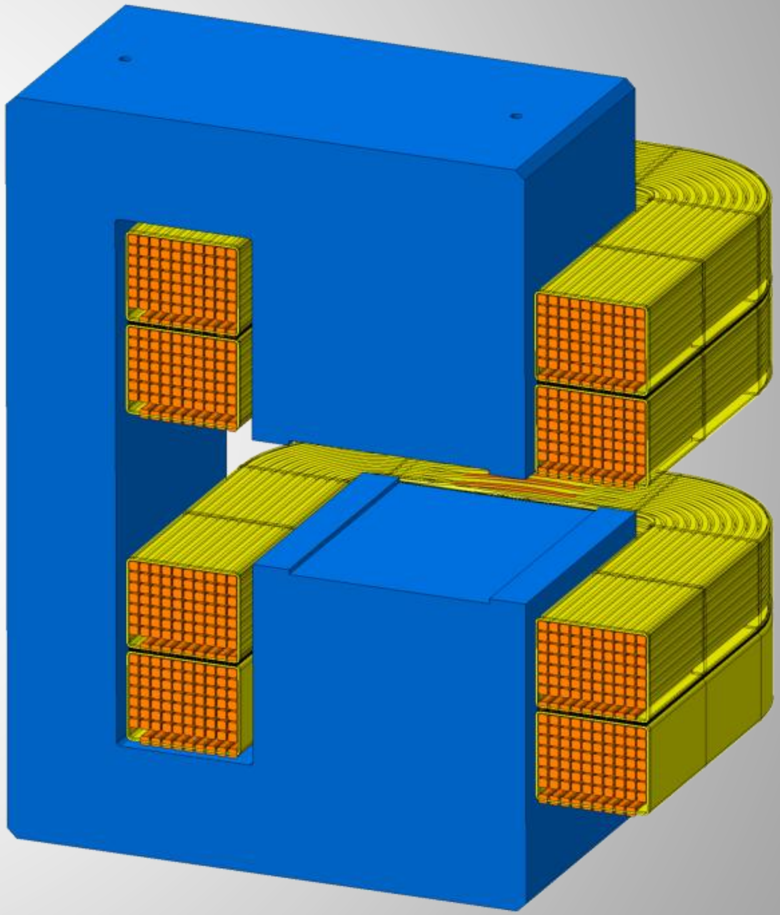




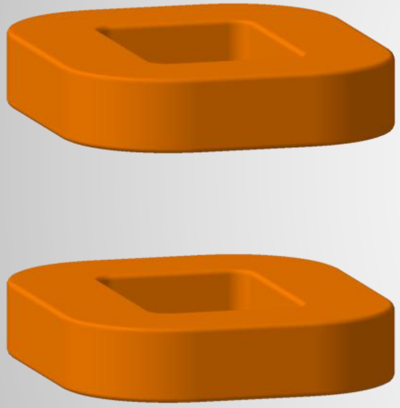
The magnetic (iron) circuit

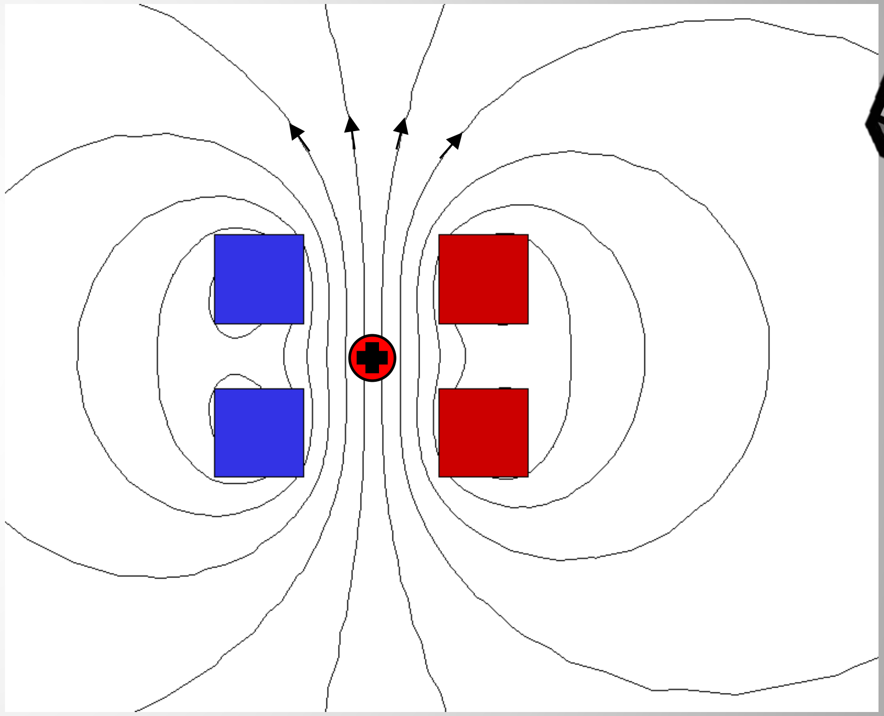
The magnetic (iron) circuit serves several purposes:

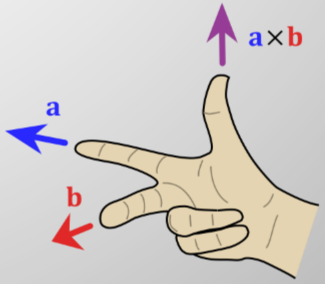
- confine the magnetic flux in the circuit to avoid stray flux
- enhance the magnetic effect induced by currents in the coils
- shape the magnetic field distribution in the region of interest



Creating the magnetic field

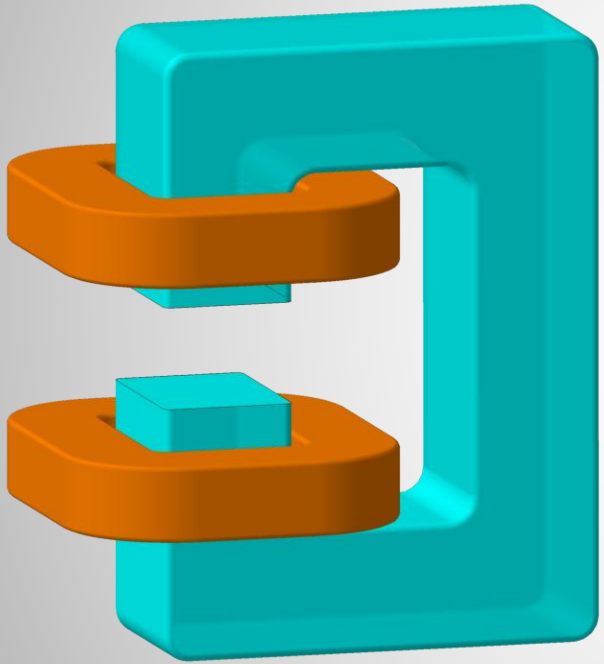


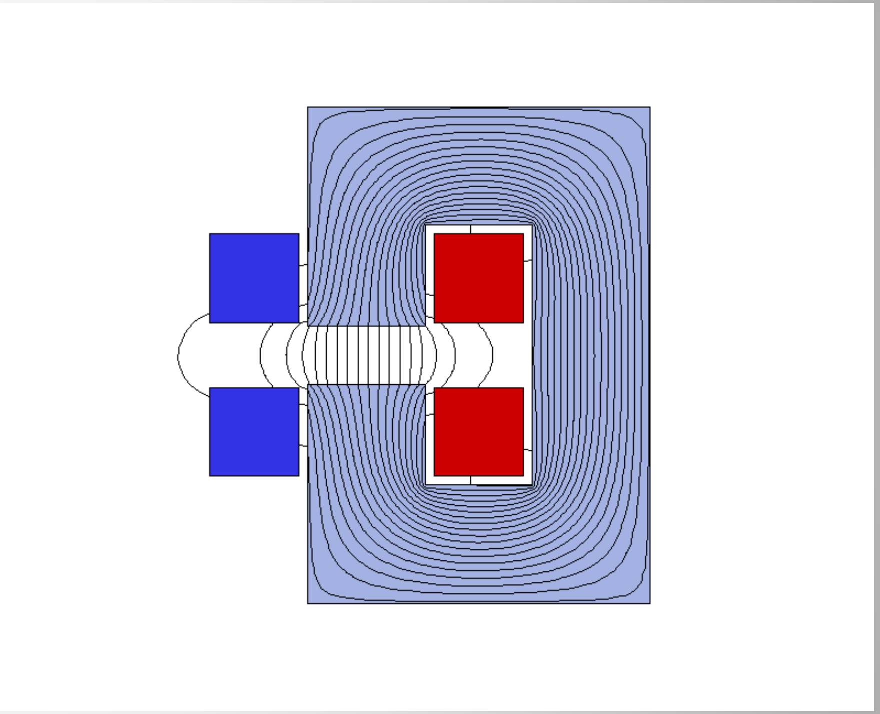




Flux lines represent the magnetic field
Coil colors indicate the current direction

Confining the magnetic field



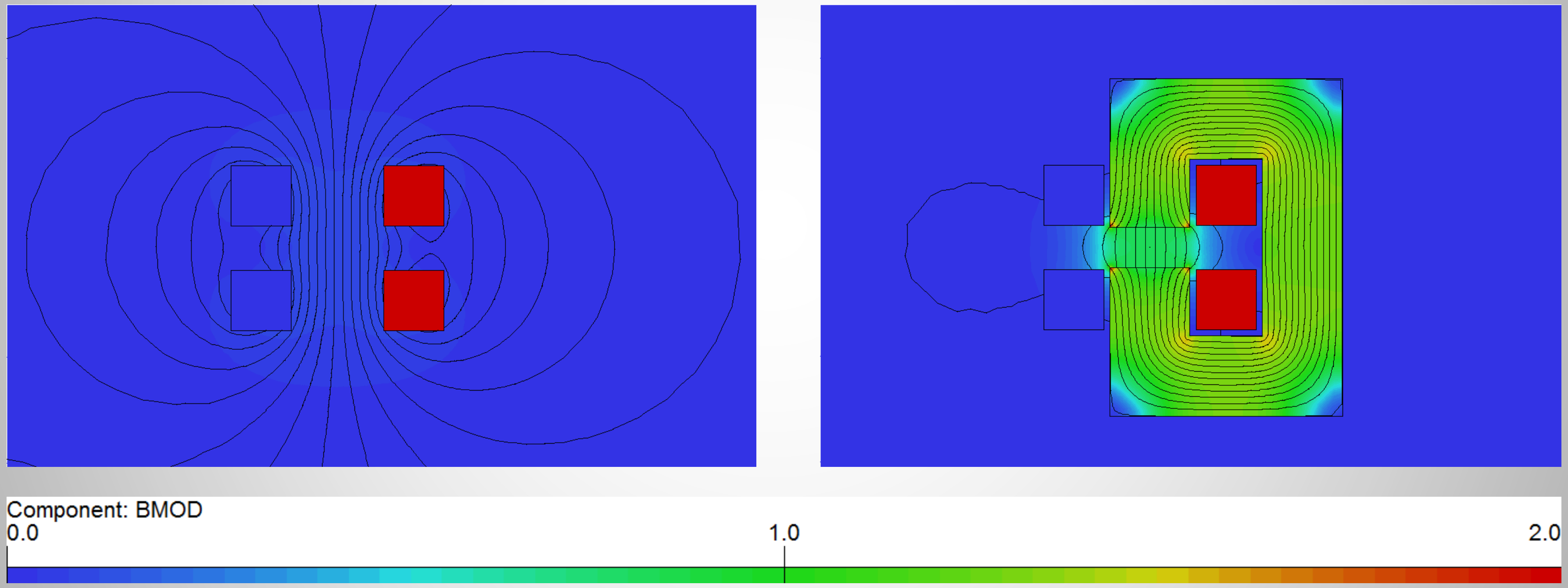


Coils hold the electrical current
Iron holds the magnetic flux

Enhancing the magnetic field

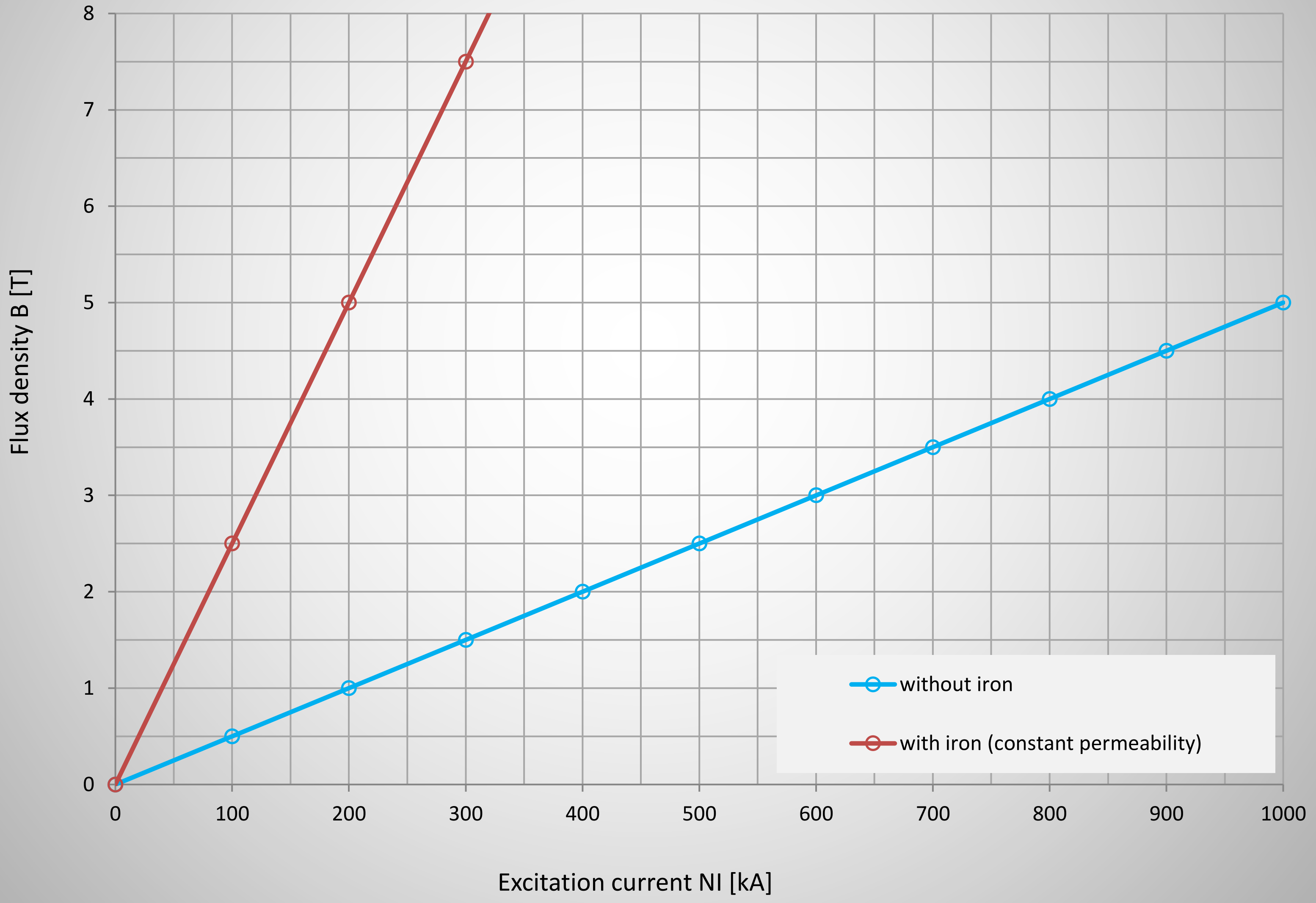
$I = 32$ kA
$B_0 = 0.09$ T

$I = 32$ kA
$B_0 = 0.80$ T



The presence of a magnetic circuit can increase the flux density in the magnet aperture by factors!

Transfer function (ideal magnet)



Excitation current in a dipole

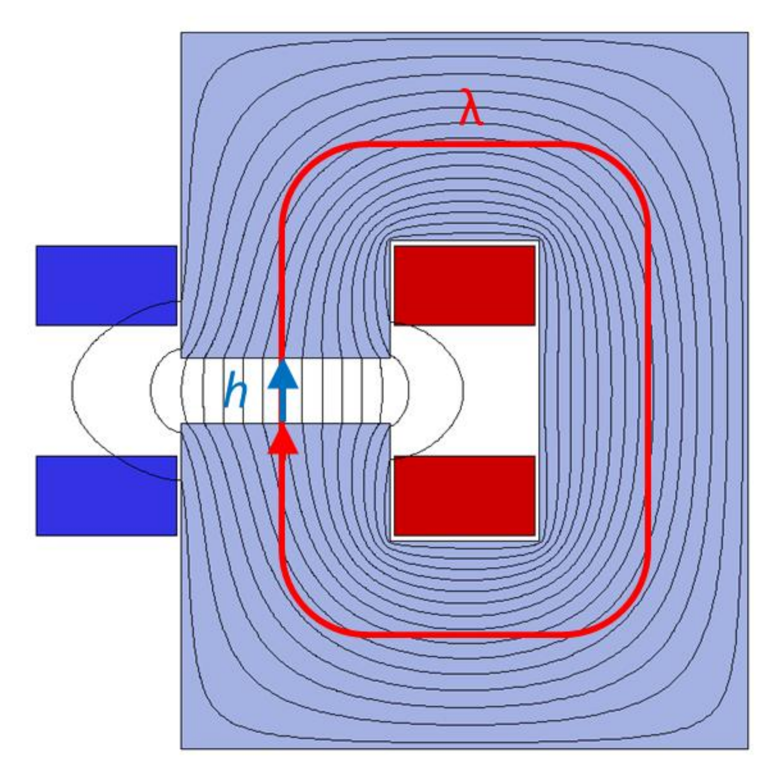
Ampere's law $\oint \vec{H} \cdot d\vec{s} = NI$ and $\vec{B} = \mu \vec{H}$ with $\mu = \mu_0 \mu_r$

leads to $$NI = \oint \frac{\vec{B}}{\mu} \cdot d\vec{s} = \int_{air} \frac{\vec{B}}{\mu_{air}} \cdot d\vec{s} + \int_{iron} \frac{\vec{B}}{\mu_{iron}} \cdot d\vec{s} = \frac{Bh}{\mu_{air}} + \frac{B\lambda}{\mu_{iron}}$$

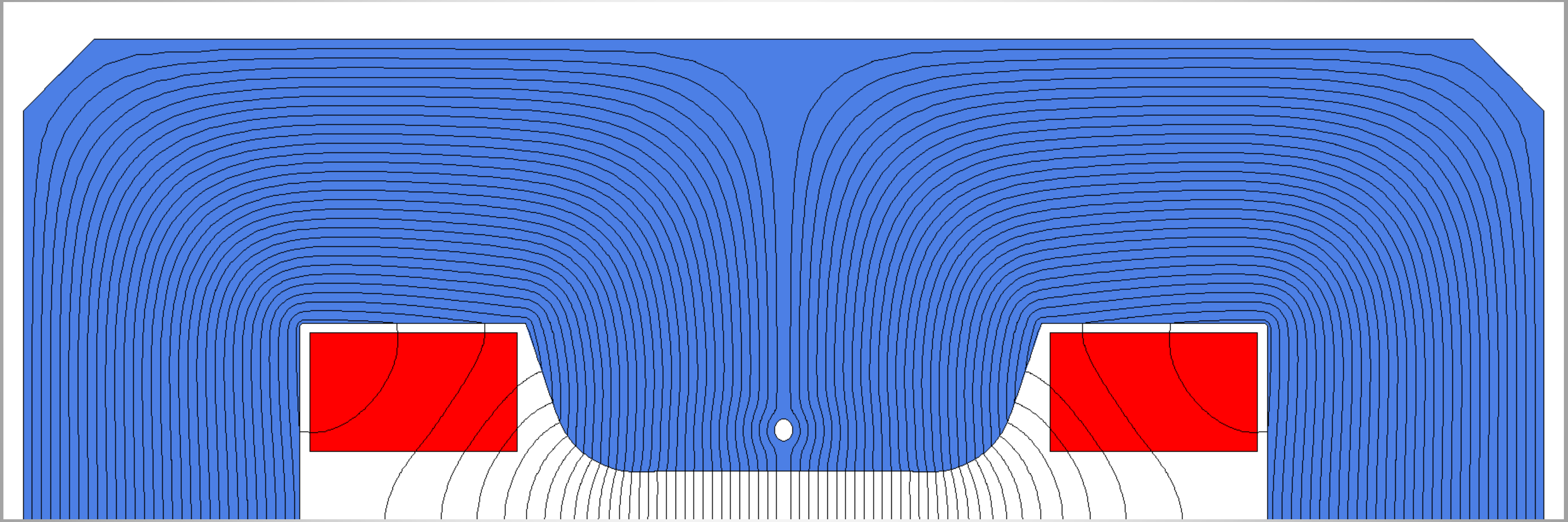
assuming, that *B* is constant along the path

If $\mu_{iron} \to \infty$ then $\frac{\lambda}{\mu_{iron}} \to 0$

then: $$\boxed{NI = \frac{Bh}{\mu_0}}$$



Shaping the magnetic field



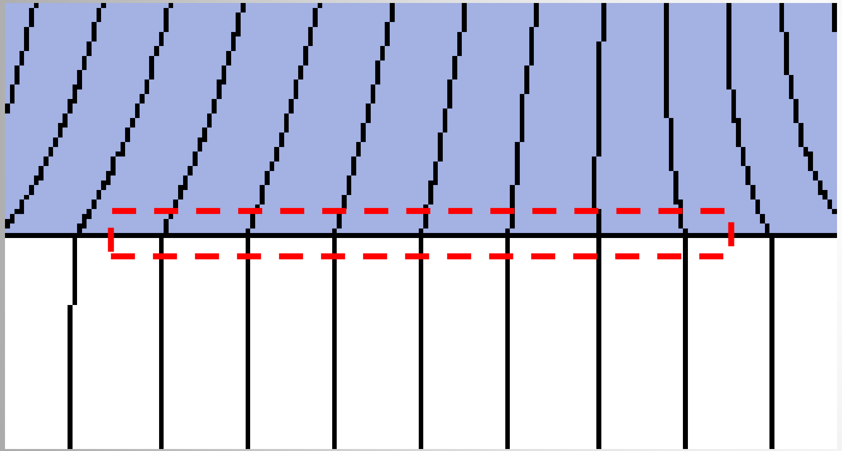
Ideal H-shape dipole with infinite permeability: $\mu_{iron} \rightarrow \infty$

Why flux lines do always enter a material with infinite permeability perpendicular to the surface?

Shaping the magnetic field

Because they obey to Maxwell's equations:

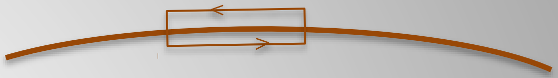
‘ideal’ iron $\mu_r = \infty$



air $\mu_r = 1$

$\boxed{H_{\parallel} = constant}$ $\boxed{B_{\perp} = constant}$ $\boxed{B_{\parallel} = 0}$

$\nabla x \vec{H} = 0$



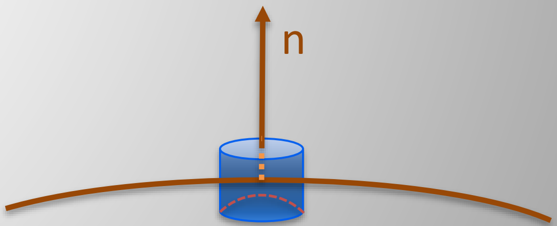
The integral of the field strength equals zero

$\boxed{H_{\parallel,\text{ air}} = H_{\parallel,\text{ iron}}}$

$\vec{B} = \mu_0 \mu_r \vec{H}$

$\boxed{B_{\parallel,\text{ air}} = \frac{B_{\parallel,\text{ iron}}}{\mu_{r,\text{iron}}} = 0}$

$\nabla \cdot \vec{B} = 0$



The flux which enters must be equal to the flux which exits

$\boxed{B_{\perp,\text{ air}} = B_{\perp,\text{ iron}}}$

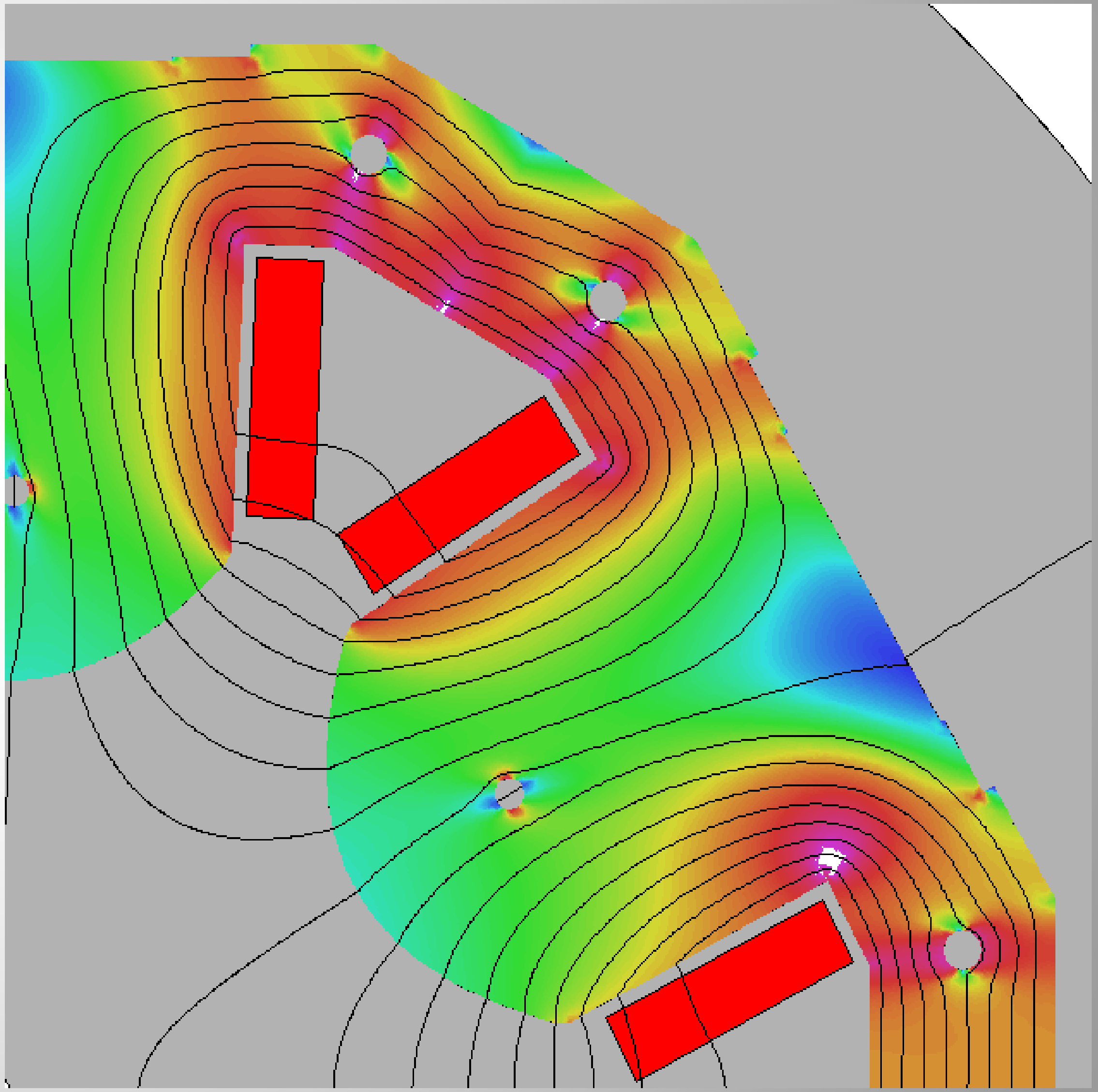
Shaping the magnetic field

Conclusion: flux lines in free space always meet a material with infinite permeability perpendicular to the surface.

Conversely: if we can shape a material with infinite permeability such that its surface is everywhere perpendicular to our desired field configuration, then the only field that can exist around the material will be this desired field!

These shapes perpendicular to our desired field configuration are called surfaces of constant scalar potential.

The field distribution in the aperture of a magnet can be controlled by such surfaces of constant scalar potential, also called “poles faces”.



Ideal pole faces

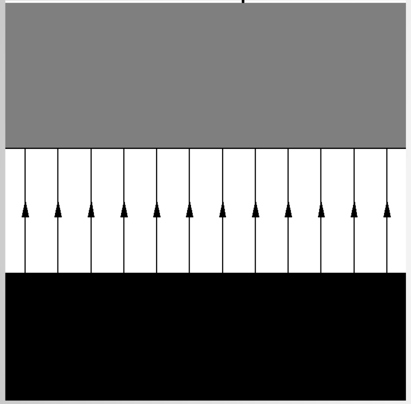
Each fundamental n-pole has its well-defined pole face (=surface of constant scalar potential)

Ideal pole faces are extended to infinity in all directions

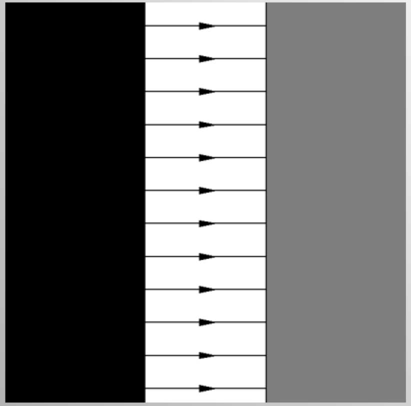
Surfaces of constant scalar potential are two parallel planes

$$y = \pm r$$

normal dipole



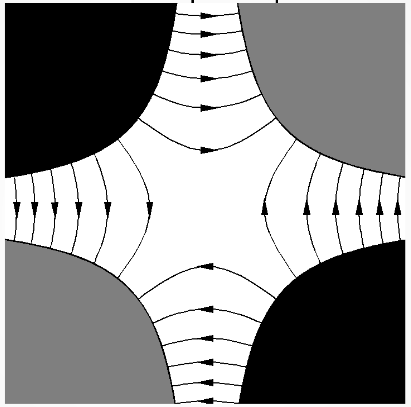
skew dipole



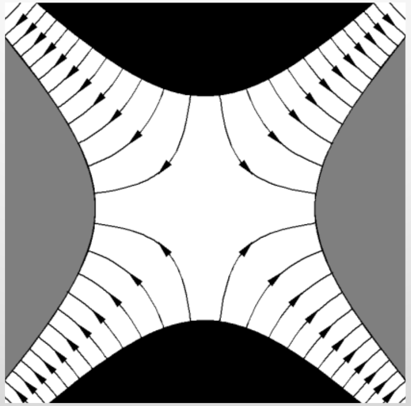
Surfaces of constant scalar potential are four hyperbolas

$$2xy = \pm r^2$$

normal quadrupole

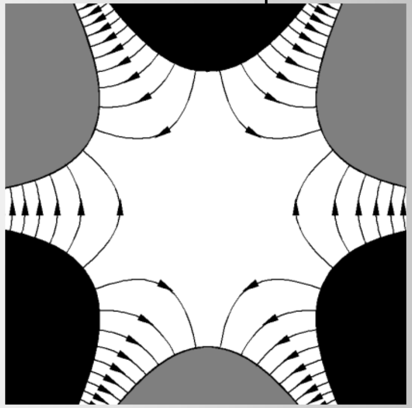


skew quadrupole

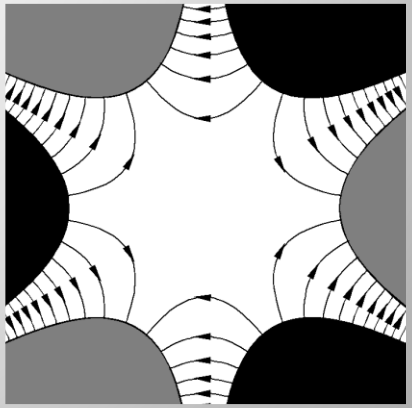


$$3x^2y - y^3 = \pm r^3$$

normal sextupole

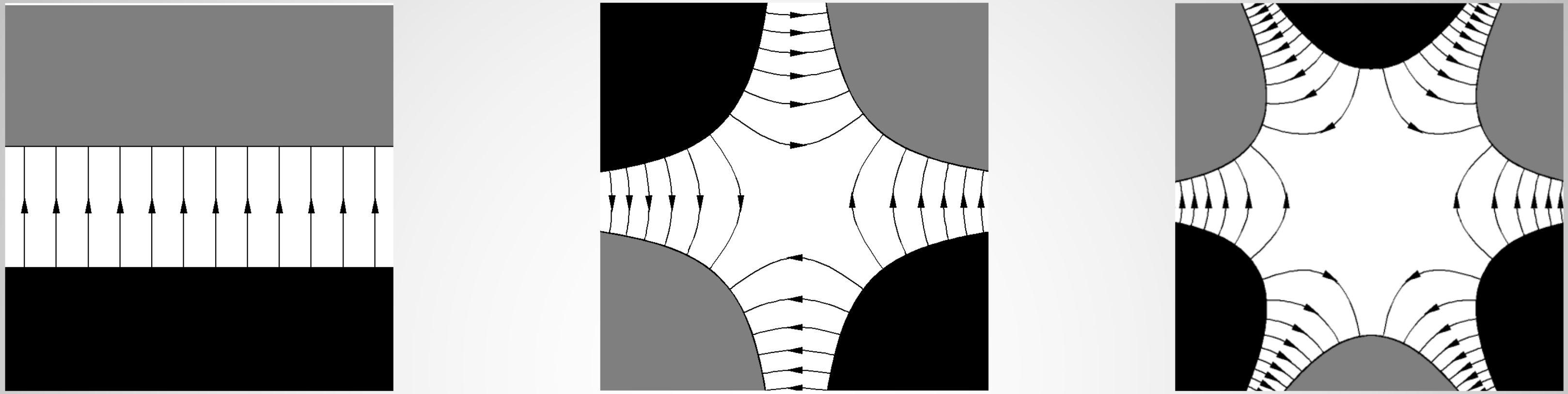


skew sextupole

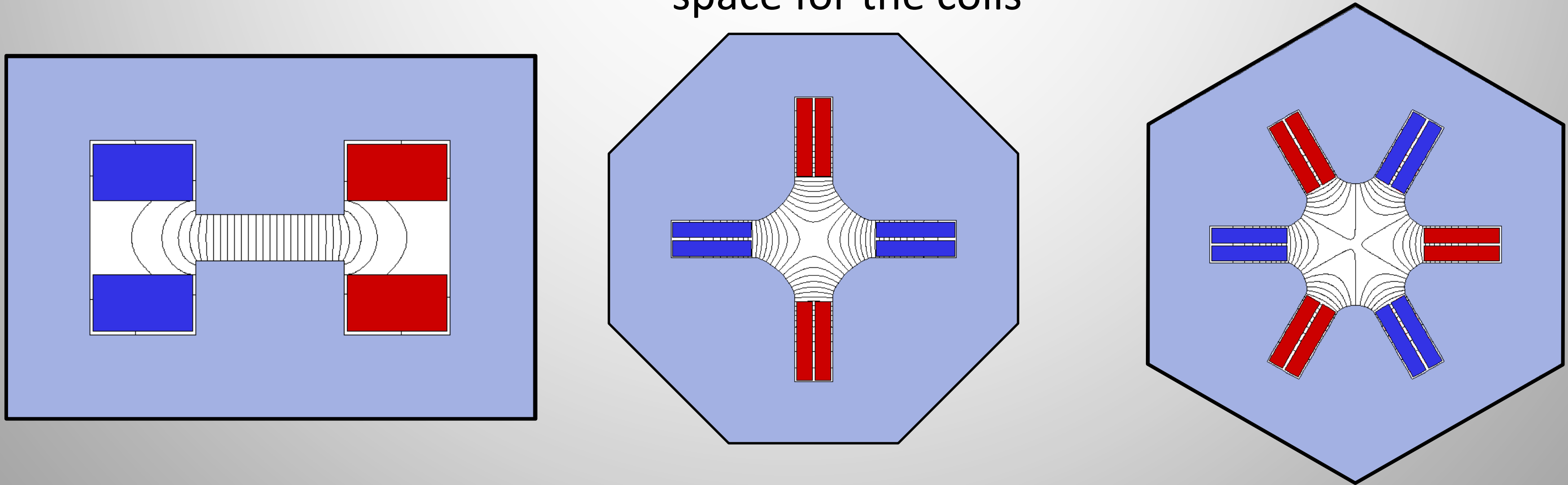


Ideal vs. real magnets

Ideal magnets have infinite permeability, and the poles are extended to infinity in all directions



Real magnets have high, but finite and non-linear permeability, and the poles are truncated to provide space for the coils



Multipole errors

If we take the multipole expansion from the previous lecture

$$B_y + iB_x = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{x+iy}{R_{ref}}\right)^{n-1}$$

and introduce the dimensionless normalized multipole coefficients (normal and skew):

$$b_n = \frac{B_{\mathrm{n}}}{B_{\mathrm{ref}}} 10^4 \quad \text{and} \quad a_n = \frac{A_n}{B_{\mathrm{ref}}} 10^4$$

with B_{ref} being the fundamental field of a magnet: $B_{\mathrm{ref\,(dipole)}} = B_1$; $B_{\mathrm{ref\,(quad)}} = B_2$; …

we can describe each magnet by its ideal fundamental field and higher order harmonic distortions:

$$B_y + iB_x = \underset{\uparrow \atop \text{Fundamental field}}{B_{ref}} \underbrace{\frac{1}{10^4} \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x+iy}{R_{ref}}\right)^{n-1}}_{\text{Harmonic distortions or Multipole errors}}$$

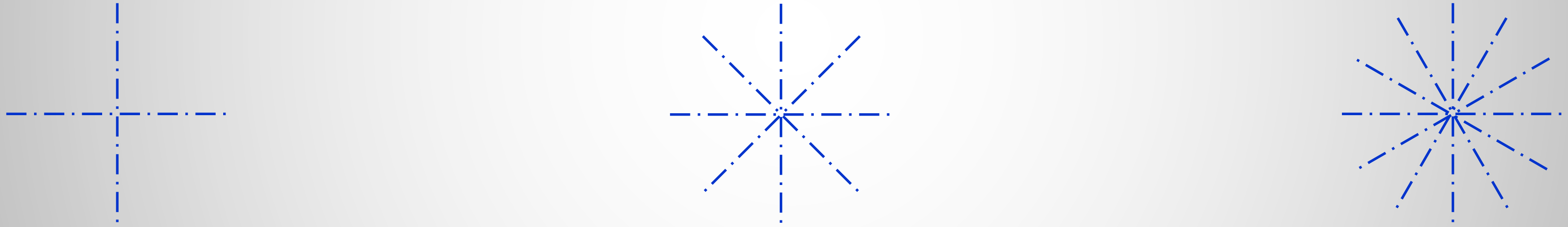
Multipole errors

Multipole errors can be divided into two families:

‚Allowed‘ multipoles are design intrinsic and result from the finite size of the poles

$$n = N(2m + 1)$$

n: order of multipole component
N: order of the fundamental field
m: integer number (m≥1)



<u>fully symmetric dipole</u>
allowed: b_3, b_5, b_7, b_9, etc.
non-allowed: all others

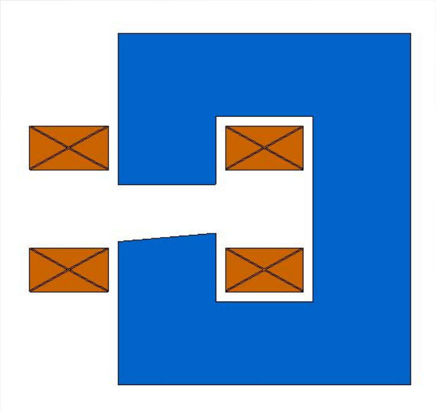
<u>fully symmetric quadrupole</u>
allowed: b_6, b_{10}, b_{14}, b_{18}, etc.
non-allowed: all others

<u>fully symmetric sextupole</u>
allowed: b_9, b_{15}, b_{21}, etc.
non-allowed: all others

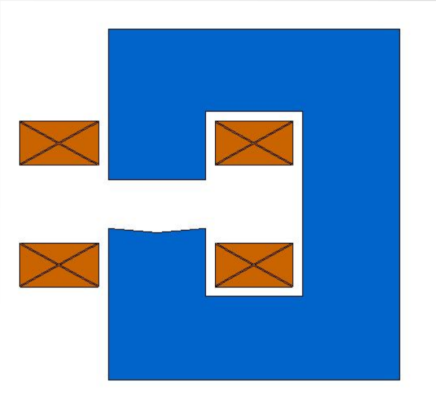
‚Non-allowed‘ multipoles result from a violation of symmetry and indicate a fabrication or assembly error

Asymmetries

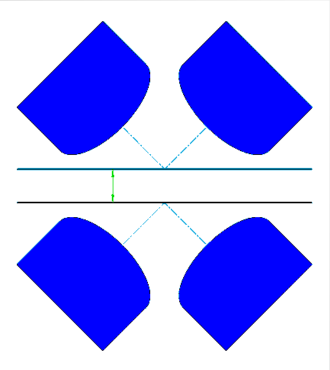
Asymmetries generating ‘non-allowed’ harmonics



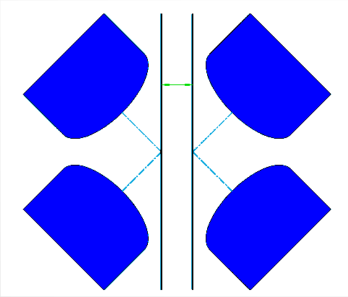
n = 2, 4, 6, ...



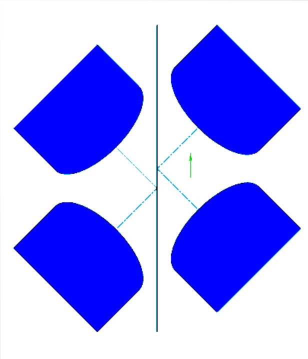
n = 3, 6, 9, ...



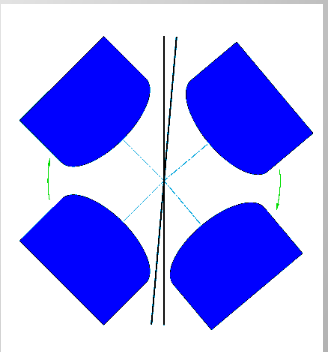
n = 4 (neg.)



n = 4 (pos.)



n = 3



n = 2, 3

These errors can seriously affect machine behaviour and must be controlled!

Finite permeability

Ferro-magnetic materials have a high (μ_r>>1), but not infinite permeability!

This finite permeability in the iron has two effects:

$$NI = \oint \frac{\vec{B}}{\mu} \cdot d\vec{s} = \int_{air} \frac{\vec{B}}{\mu_{air}} \cdot d\vec{s} + \int_{iron} \frac{\vec{B}}{\mu_{iron}} \cdot d\vec{s} = \frac{Bh}{\mu_{air}} + \frac{B\lambda}{\mu_{iron}}$$

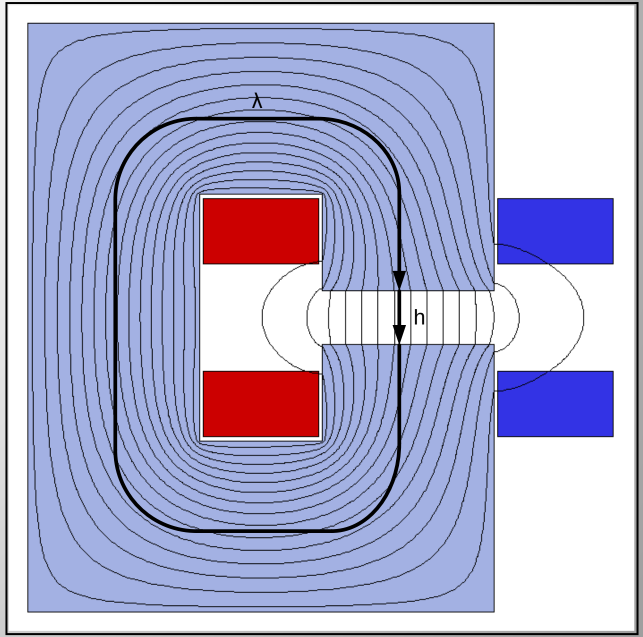
1) The path in the iron can no longer be fully neglected

However, if the iron is not saturated, then: $\dfrac{h}{\mu_{air}} \gg \dfrac{\lambda}{\mu_{iron}}$

and:

$$\boxed{NI \approx \frac{Bh}{\mu_0}}$$

2) The flux lines will no longer be perpendicular to a surface of constant scalar potential (poles) resulting in field errors



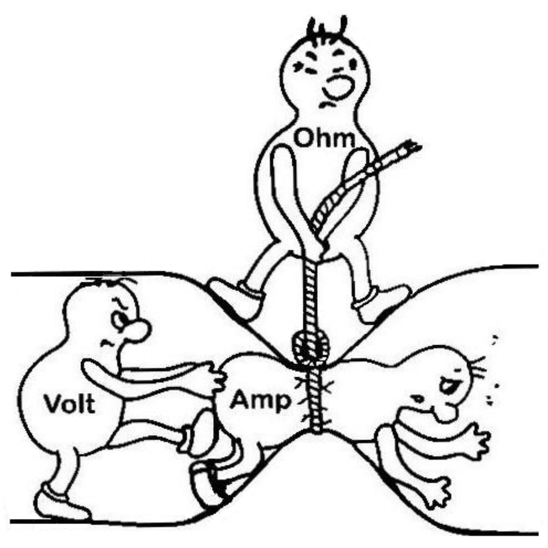
Reluctance

Similar to electrical circuits, one can define the ‘resistance’ of a magnetic circuit, which is called ‘reluctance’:

Ohm’s law:

$$R = \frac{U}{I} = \frac{l_E}{a_E \sigma}$$

- Resistance R [Ω]
- Electro-motive force U [V]
- Electric current I [A]
- Conductor length l_E [m]
- Conductor cross section a_E [m^2]
- El. conductivity σ [S m^{-1}]



Hopkinson’s law:

$$R_M = \frac{NI}{\Phi} = \frac{l_M}{a_M \mu_0 \mu_r}$$

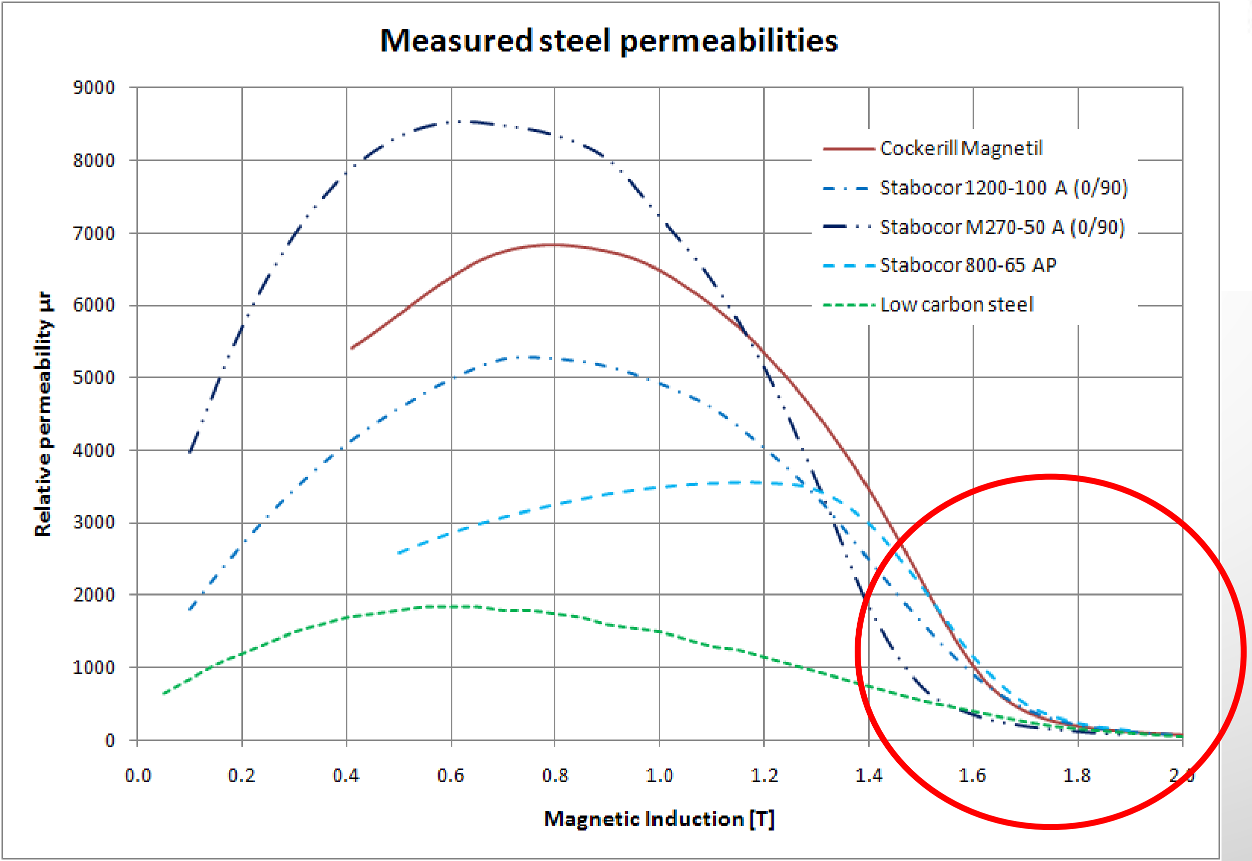
- Reluctance R_M [A V^{-1} s^{-1}]
- Magneto-motive force NI [A]
- Magnetic flux Φ [Wb]
- Flux path length in iron l_M [m]
- Iron cross section a_M [m^2]
  (perpendicular to flux)
- Permeability μ [V s A^{-1} m^{-1}]

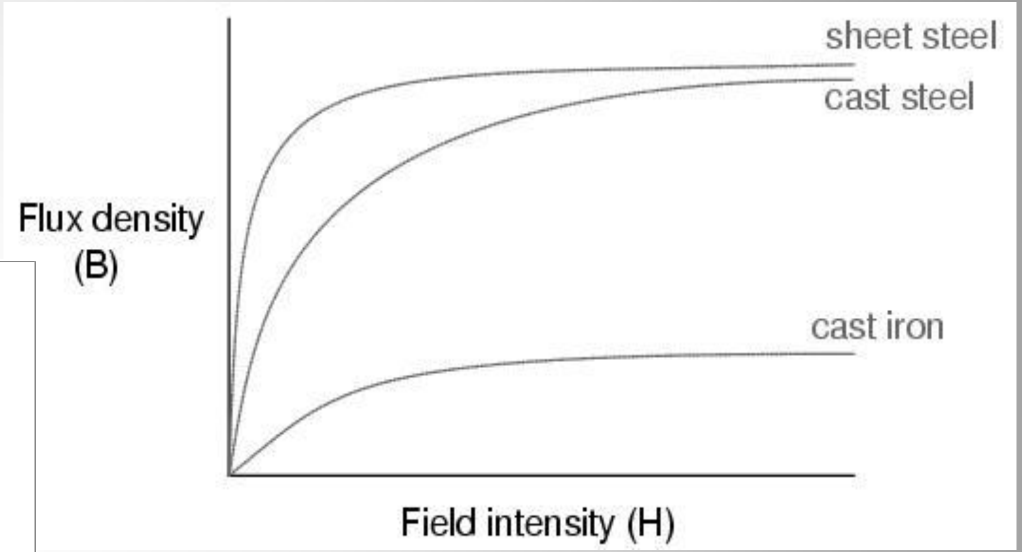
…but: μ_{iron} is in general not constant!

Non-linear permeability

Permeability: correlation between magnetic field strength *H* and magnetic flux density *B*

$$\vec{B} = \mu \vec{H} \qquad \mu = \mu_0 \mu_r$$





Ferro-magnetic materials: high permeability (μ_r>>1), but not constant

The permeability decreases at low <u>and</u> at high magnetic induction

→ Iron saturation!

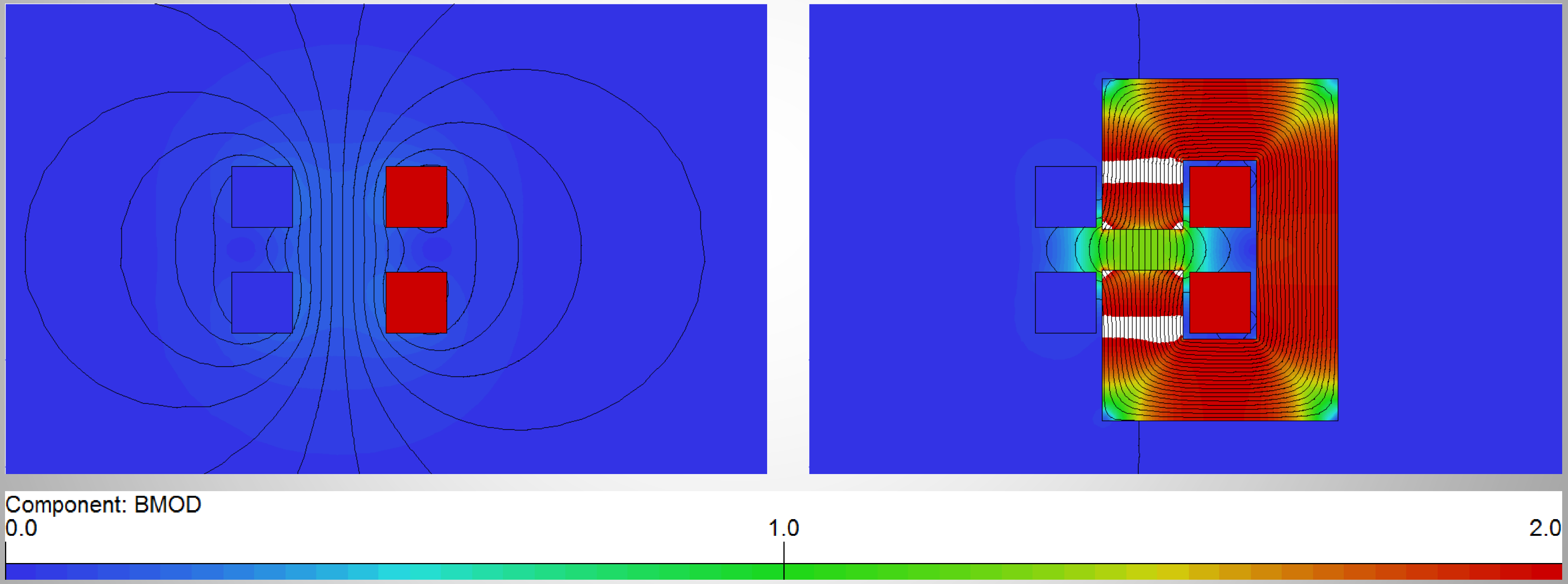
Iron Saturation

$I = 32$ kA
$B_0 = 0.09$ T

$I = 64$ kA
$B_0 = 0.18$ T

$I = 32$ kA
$B_0 = 0.80$ T

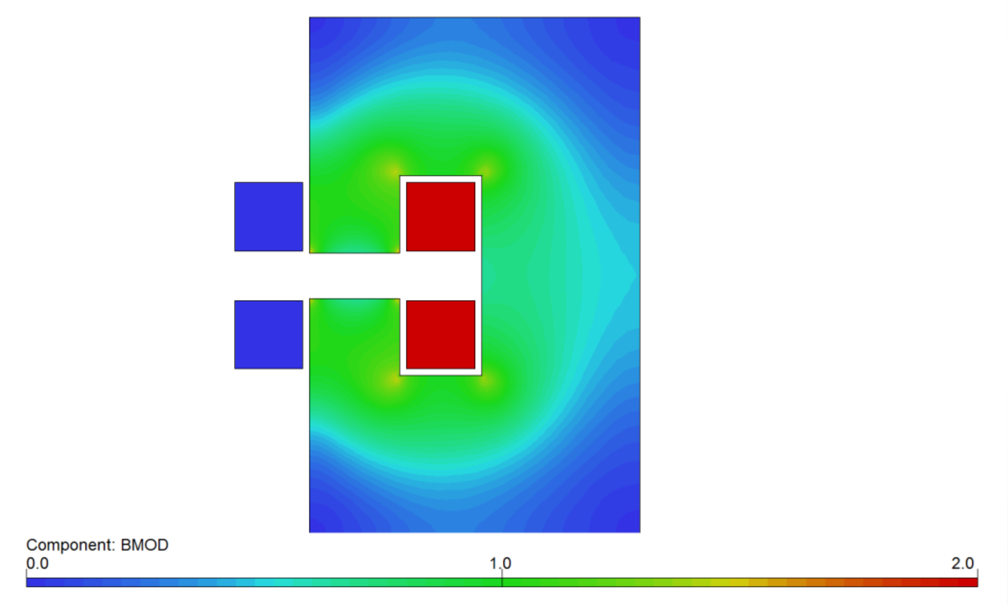
$I = 64$ kA
$B_0 = 1.30$ T



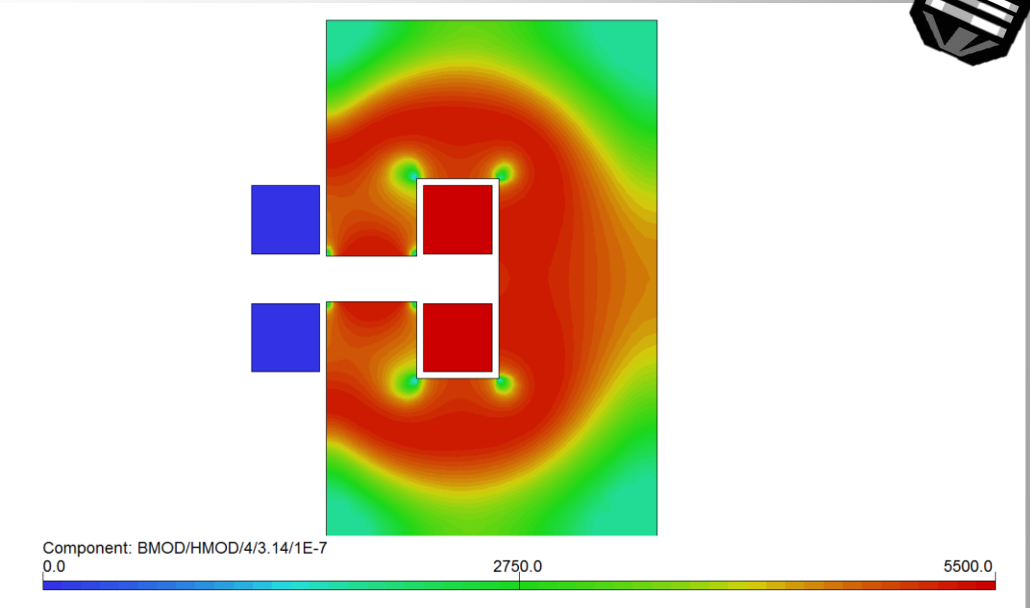
Iron saturation (small μ_{iron}) leads to non-linearities

Iron Saturation

Flux density plot



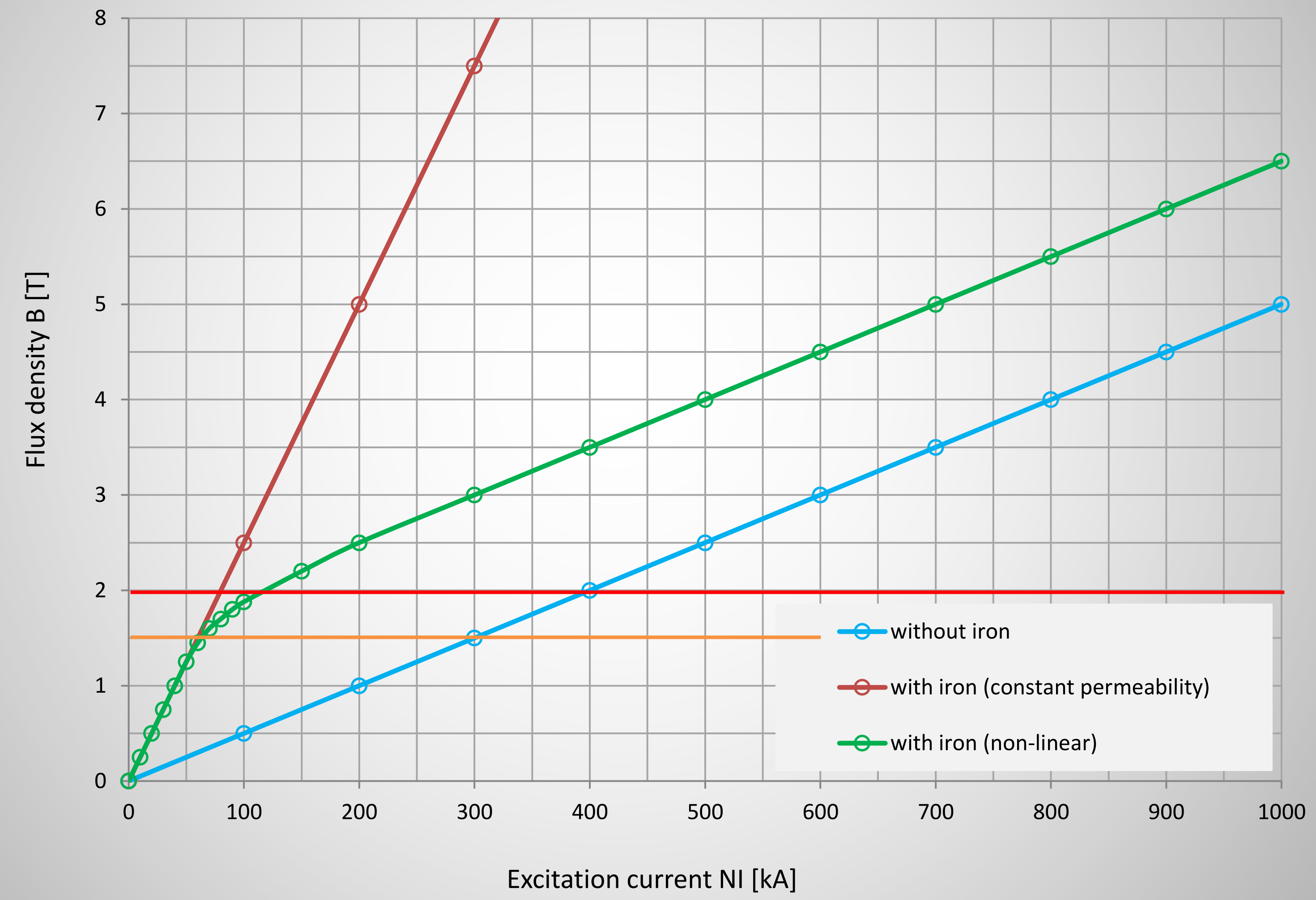
Relative permeability plot



Iron saturation (small μ_{iron}) leads to inefficiencies

Iron Saturation

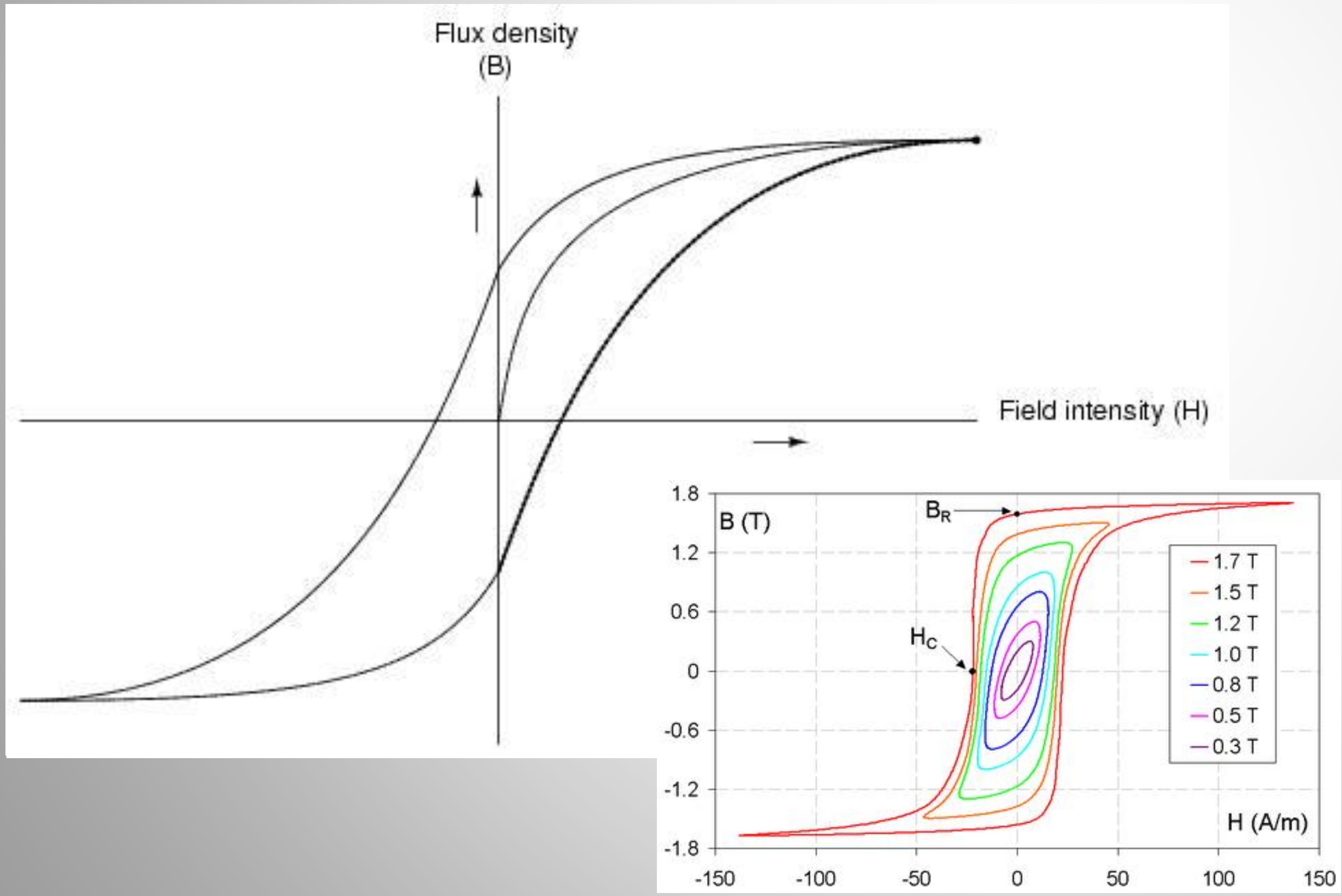
$$\vec{B} = \mu_0 \vec{H} + \vec{J} = \mu_0 \mu_r \vec{H}$$



Increase of *B* above 1.5 T in iron requires non-proportional increase of *H*

Steel hysteresis

The flux density $B(H)$ as a function of the field strength is different, when increasing and decreasing excitation

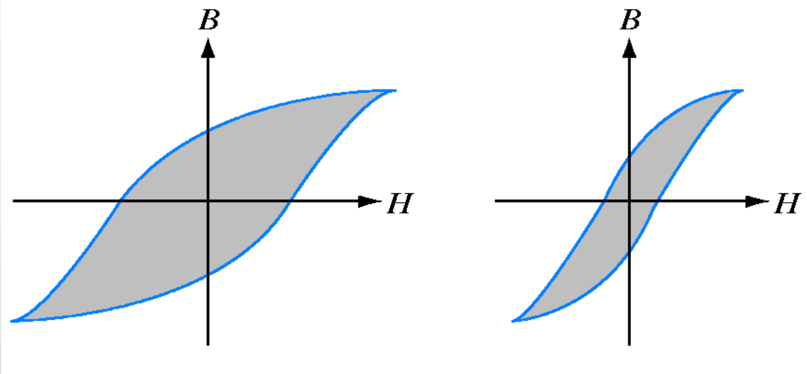


Remanent field (Retentivity):

$$H = 0 \rightarrow B = B_r > 0$$

Coercivity or coercive force:

$$B = 0 \rightarrow H = H_c < 0$$



(a) Hard material

(b) Soft material

$H_c > 1000$ A/m

$H_c < 1000$ A/m

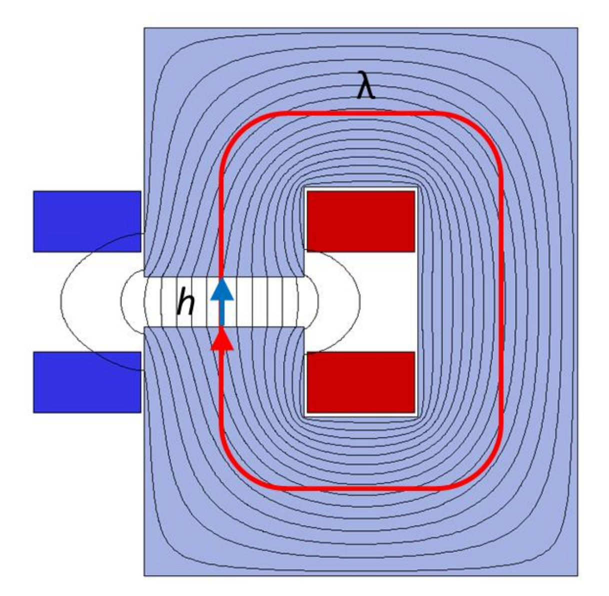
Residual field in a magnet

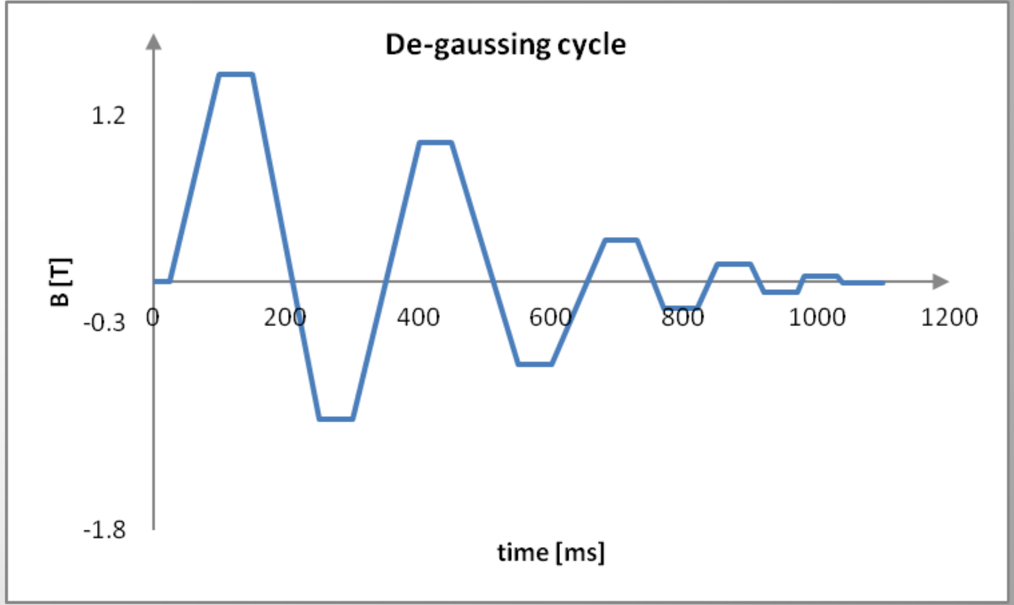
In a continuous ferro-magnetic core the residual field is determined by the remanent field B_r

In a magnet core (gap), the residual field B_{res} is determined by the coercivity H_c

Assuming the coil current $I = 0$: $\oint \vec{H} \cdot d\vec{s} = \int_{air} \vec{H}_{air} \cdot d\vec{s} + \int_{iron} \vec{H}_c \cdot d\vec{s} = 0$

$$\boxed{B_{res} = -\mu_0 H_c \frac{\lambda}{h}}$$





The previous cycle history can have an influence on residual field in a magnet

Summary

- Coils carry the electrical current, the iron yoke carries and enhances the magnetic flux
- In iron-dominated magnets, the magnetic field is shaped by the geometry of the poles
- Magnetic steel is characterized by its relative (non-linear) permeability μ_r and its coercivity H_c
- Iron saturation influences the efficiency and linearity of the magnetic circuit and has to be taken into account in the design

Thanks for your attention…