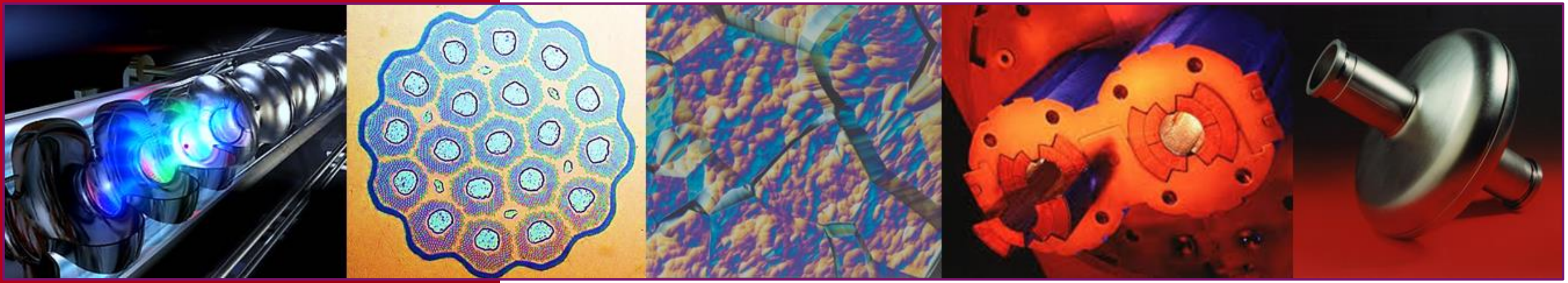
DE LA RECHERCHE À L'INDUSTRIE

SUPERCONDUCTIVITY IN ACCELERATORS
PART I : INTRODUCTION



JUAS 2025| Claire ANTOINE

MAGNETS VS RF CAVITIES

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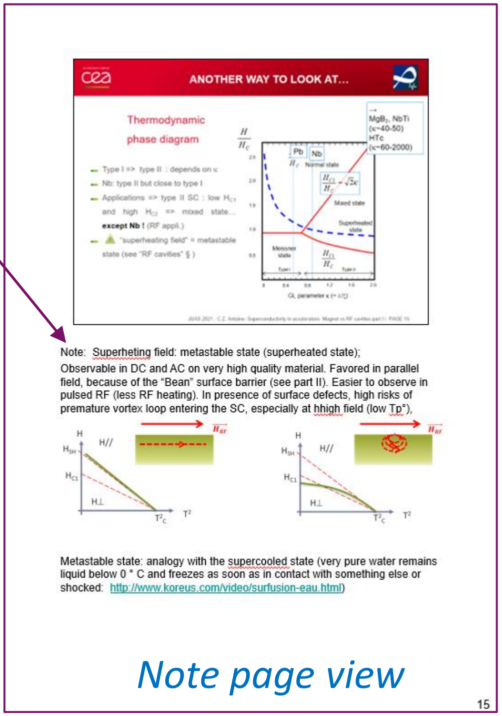
PREAMBLE :

- **The aim of this lecture is to provide to physicists non expert in material science a glimpse on how materials are tailored (and must be chosen) for a particular application**
- **The pptx files are meant to be a “handbook” to recall this kind of details, they contain complementary notes**
  + slides not shown during oral presentation

ArXiv : [2310.09097] Superconductivity in particle accelerators (arxiv.org)

- **Outlook:**
  - Introduction : where SC is found in accelerators and why
  - I Back to basics : main facts about superconductivity
  - Theories (no complete theory exists!)
  - Vortex penetration
  - II Vortex in presence of electric field or current
  - Pinning of vortex
  - Optimization of SC cables
  - III Optimization of SRF material
  - Some (impressive) examples



ANOTHER WAY TO LOOK AT...

Thermodynamic phase diagram

- Type I => type II : depends on κ
- Nb: type II but close to type I
- Applications => type II SC : low H_{C1} and high H_{C2} => mixed state... **except Nb !** (RF appl.)
- "superheating field" = metastable state (see "RF cavities" §)

Note: Superheting field: metastable state (superheated state);
Observable in DC and AC on very high quality material. Favored in parallel field, because of the “Bean” surface barrier (see part II). Easier to observe in pulsed RF (less RF heating). In presence of surface defects, high risks of premature vortex loop entering the SC, especially at hhigh field (low Tp°),

Metastable state: analogy with the supercooled state (very pure water remains liquid below 0 ° C and freezes as soon as in contact with something else or shocked: http://www.koreus.com/video/surfusion-eau.html)

Note page view

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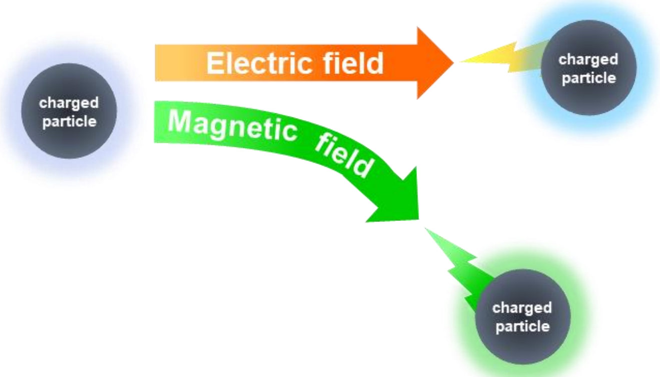
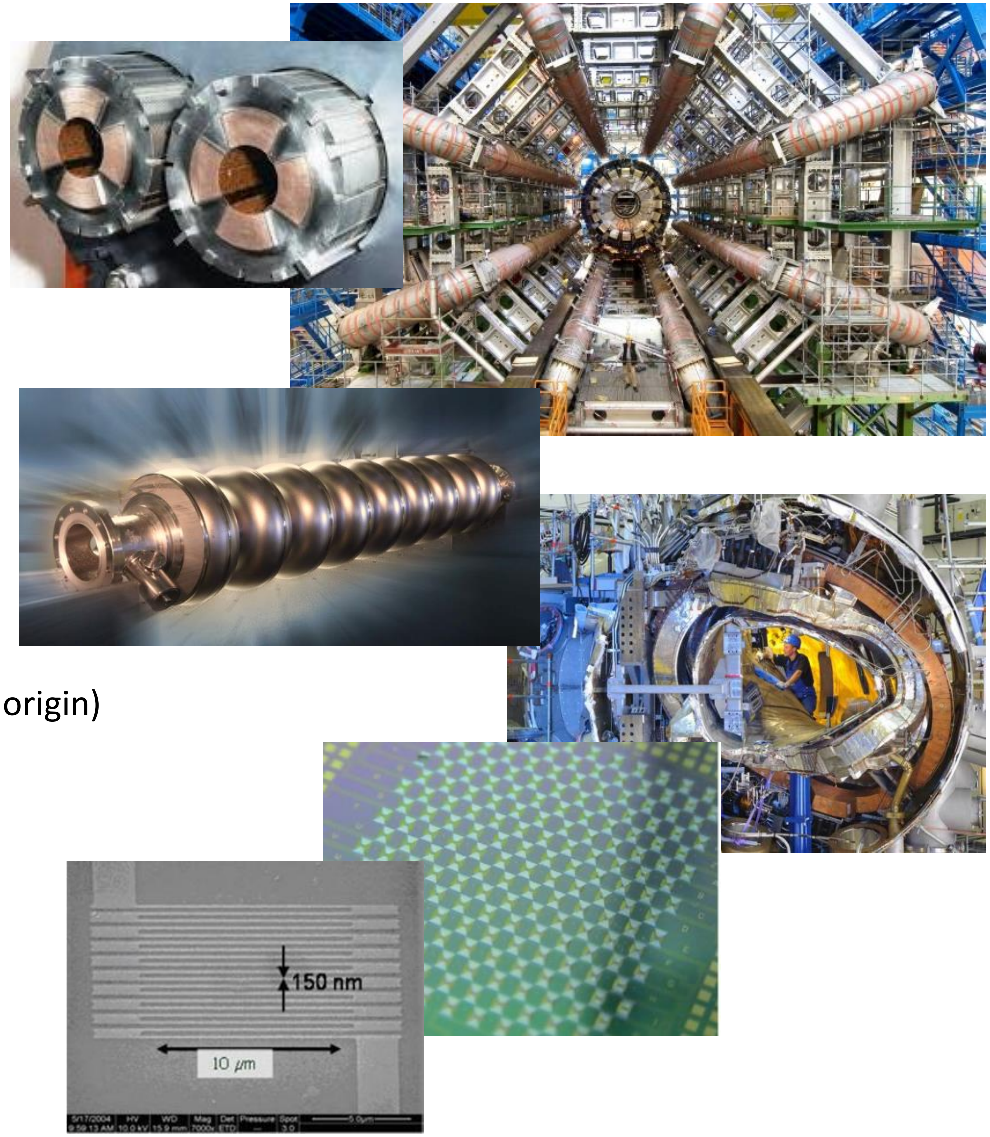
INTRODUCTION

SC IN ACCELERATORS: WHERE ?



- **Magnets (deviation)**
  - Curvature, focalization magnets
  - Detectors
- **RF Cavities (acceleration)**
- **Other close applications**
  - Magnets for medical imaging (at development origin)
  - Fusion magnets
  - Electrical engineering (high challenges!!!)
  - Josephson Junctions
    - SC electronics (RSFQ logic)
    - SQUIDS
      - Magnetic field detection
      - Bolometers
  - Nanodetectors (constrictions, wires…)



WHY ?

DC

Driving some10 000 A :

Copper Wire

NbTi SC wire (He cooled)

Electromagnets

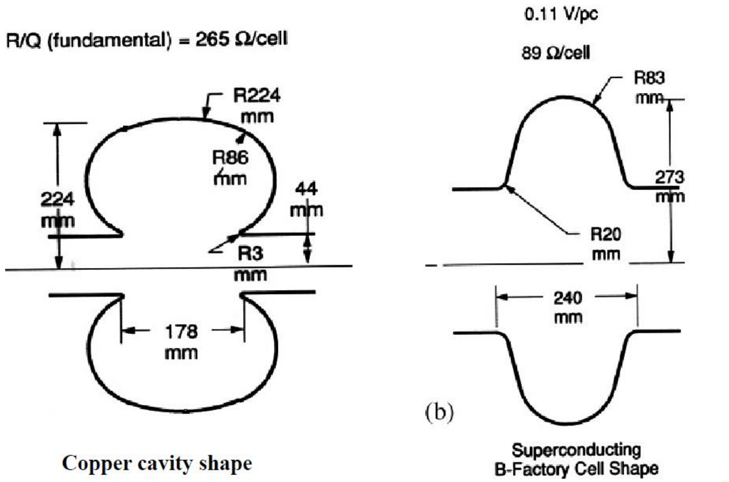
- **Examples**
  - LHC @ CERN, NbTi magnets:
    - 27 km circumference, power ~1 energy plant (only cooling, cryogenic power).
    - If magnets = Cu (+Fe) : 100 km circumference, power supply ~4 power plants,
  - In detectors: allows increasing « transparency »
    - Magnets needed for trajectory curvature. Occupied volume => no detection elements

WHY?

RF Cavities

- **More complex, depends a lot upon duty cycle**
  - Duty cycle = 1 (power 100% of the time) <= only superconductivity
- **Example**
  - Proton linacs (neutrons sources): (700MHz, CW)
- **Other advantages**
  - Larger aperture => lower impedance => easier alignment and lower wake field / better emittancy

	Copper	Niobium (SC)
Surface resistance: R_S	7mΩ	10nΩ
Foreseeable accelerating field: E_{acc}	1,6 MV/m	10 MV/m
RF efficiency $h_{RF}=P_{beam}/(P_{beam}+P_{cav})$	15%	100%
Cryogenic efficiency ($h_{cryo}=h_{Carnot}$ x h_{thermo} avec $h_{Carnot}=T_{froid}/(T_{chaud}-T_{froid})$)	100%	0,2%
>Global efficiency $P_{fournie\ au\ faisceau}/P_{à\ la\ prise}$	**7.5%**	**49%**
Actual length to gain 1 GeV	**833 m**	**286 m**



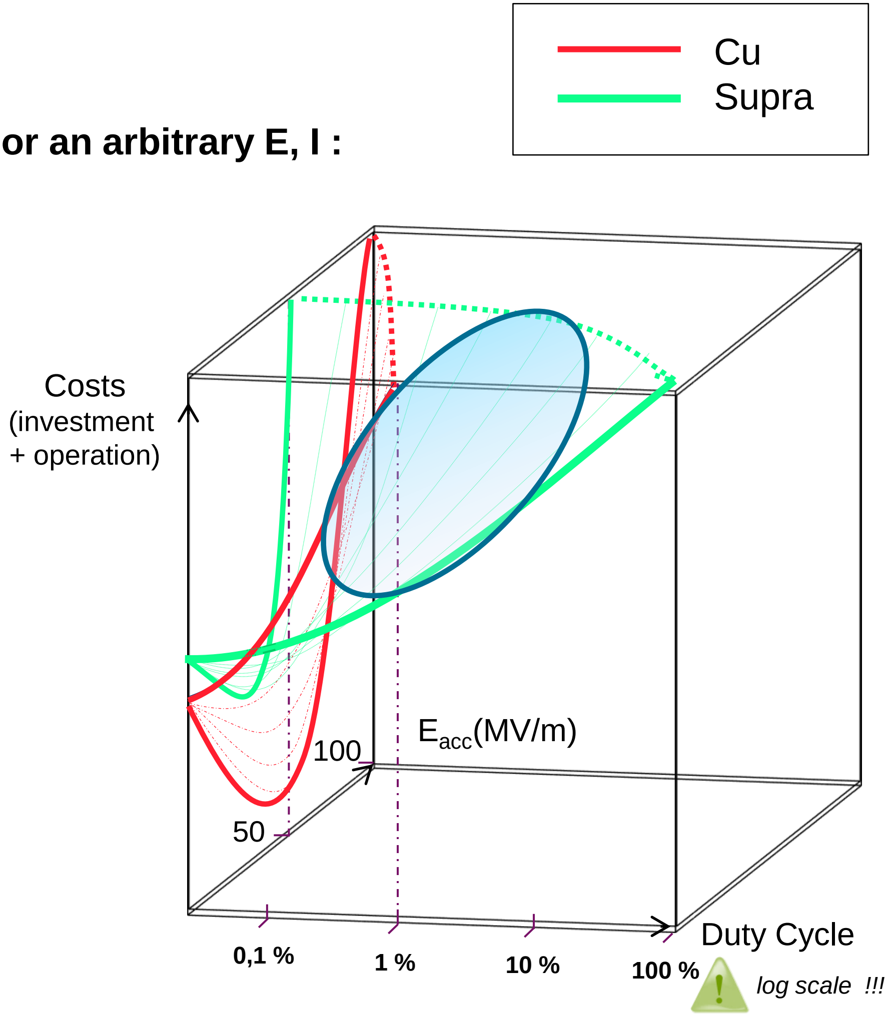
Copper cavity shape

Superconducting B-Factory Cell Shape

SOME COSTS ELEMENTS

- **Linac costs**
  - ~ 1/3 tunnel, construction
  - ~ 1/3 niobium, cryo
  - ~ 1/3 RF, beam control
- **∃ Optimum accelerating field**
  - Low field => longer machine => cost ↑
  - High field => RF costs ↑ for Cu and cryogenic costs ↑ for SC
- **Cost↑ with Duty Cycle.**
- **Other example: CLIC vs ILC** *(e+/e- collider, Higgs factory)*
  - ILC : D.C. = 0,5 % @ 1,3 GHz
  - CLIC : D.C. = 0,001 % @ 12 GHz

For an arbitrary E, I :

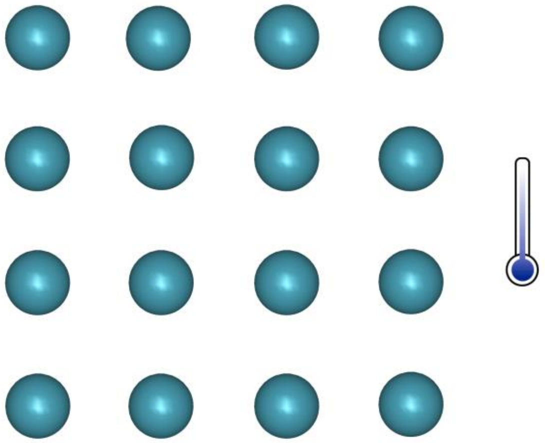


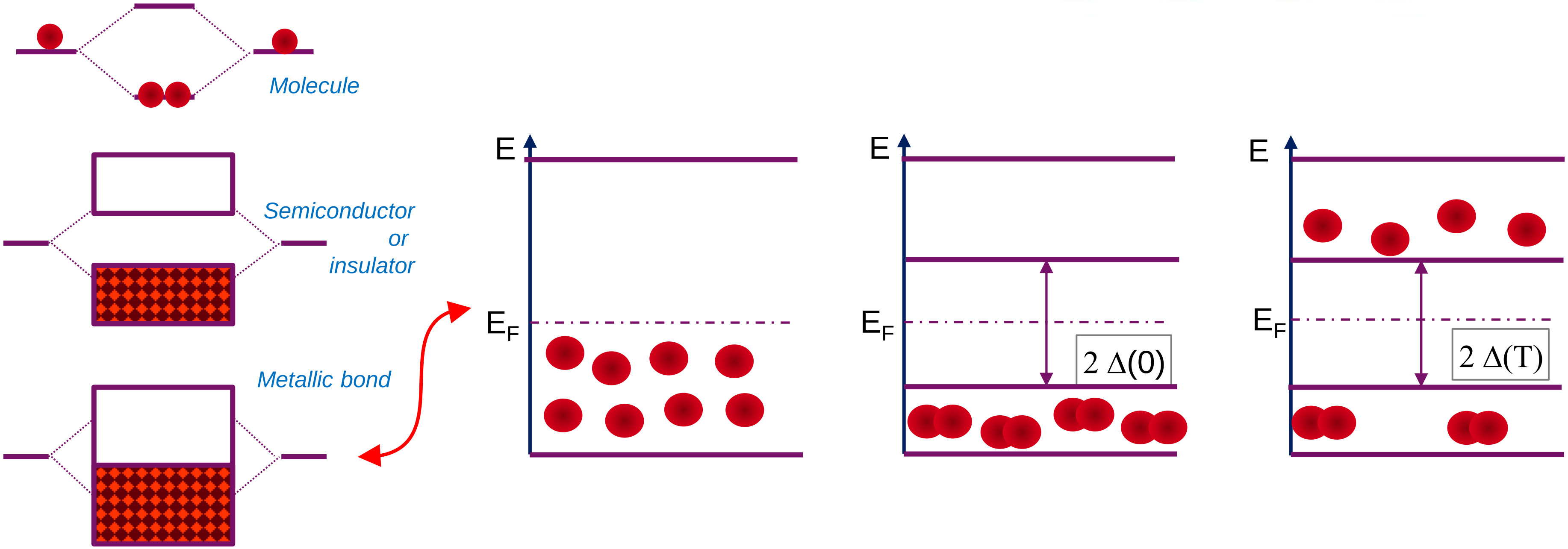
BACK TO BASICS

SUPERCONDUCTIVITY

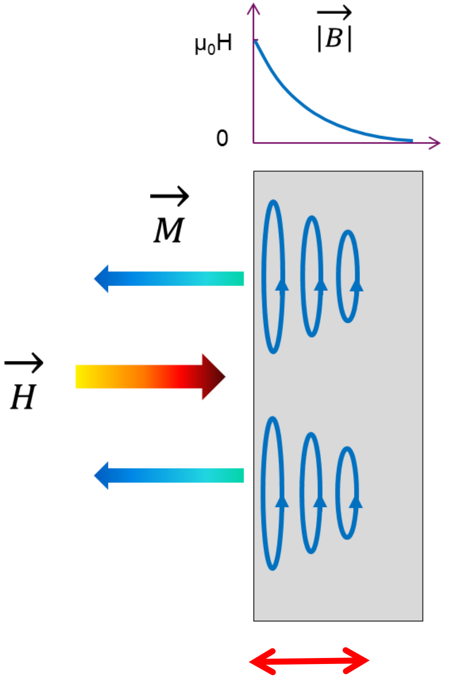
Simplified principle:

- Coupling of e- via phonons
- Single e- = fermions, Cooper pairs = bosons
- Condensation (Bose-Einstein) + gap opening
- 1! Wave function for all paired e- : macroscopic coherent state w. R=0 (⚠ : not always true !!!)





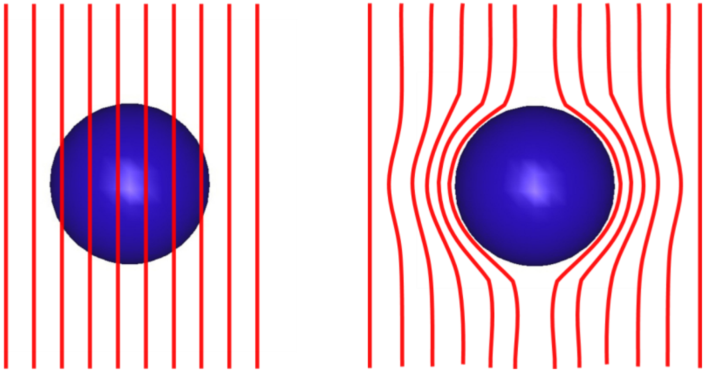
MEISSNER STATE, TYPE I & II SC

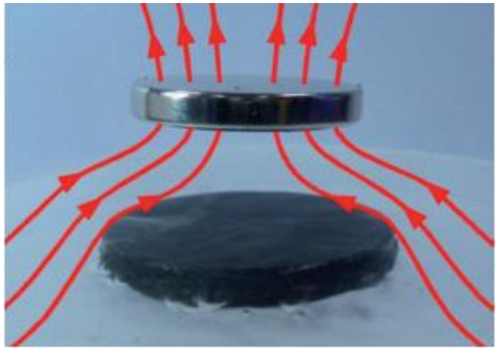


λ ~ 50 nm (Nb)

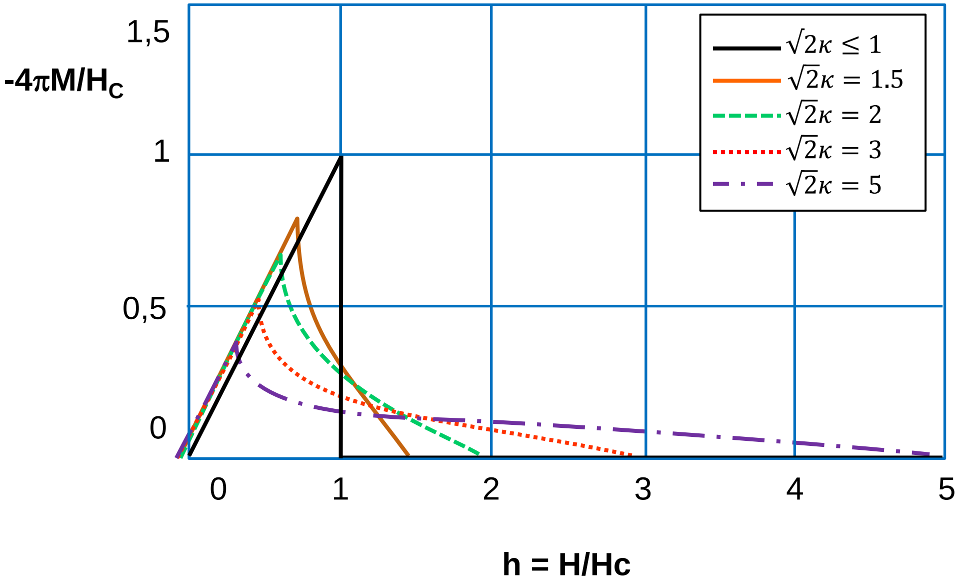
Supercurrent >> GA/m^2

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = 0$$





Magnetic levitation



Type I

Type II

K :a sc figure of merit (see below)

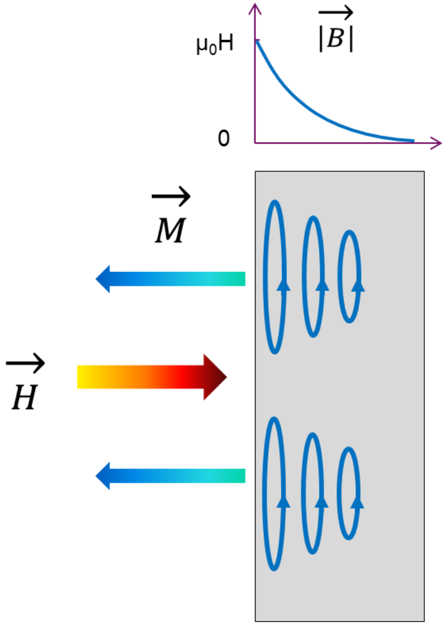
Magnetization of defect free SC

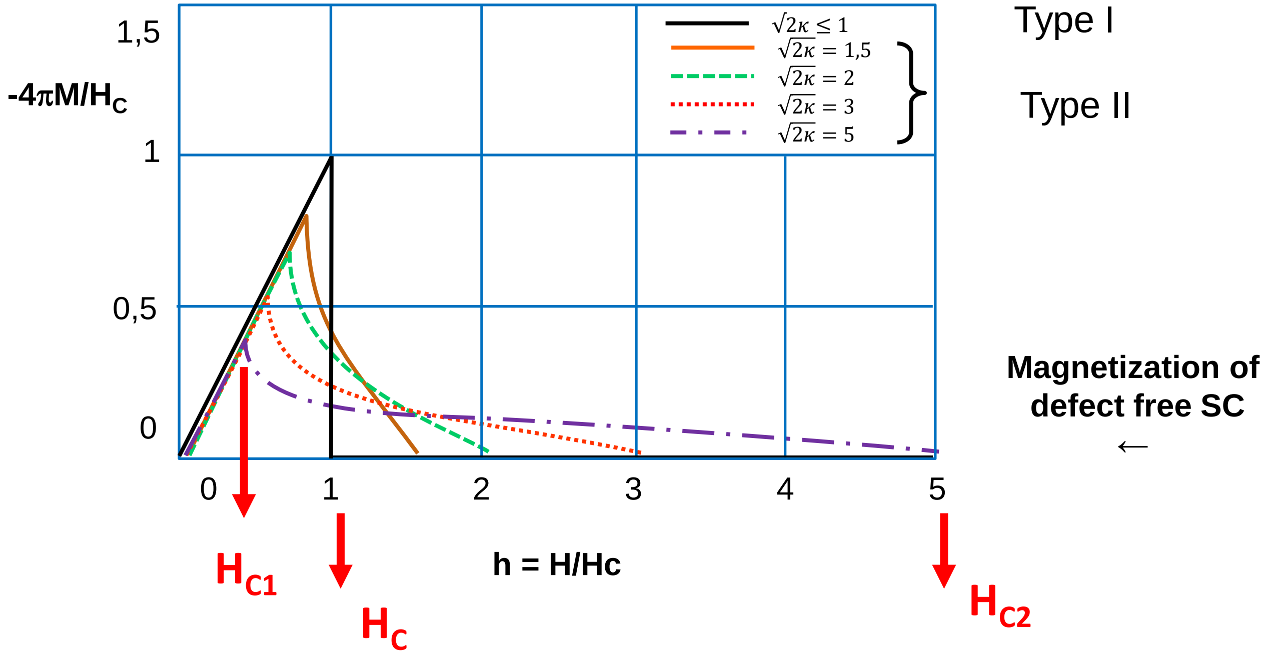
←

TYPE I & II

Important parameters:

- H_C : thermodynamic critical field (intrinsic)
- H_{C1} : 1st critical field (transition: Meissner state => Mixed state, depends on material's state)
- H_{C2} : 2nd critical field (transition Mixed state => Normal Conducting state, depends on material's state)
- κ : λ/ξ: Ginzburg-Landau parameter (depends on material's state, T independent)
- λ : field penetration depth, ξ : Cooper pairs coherence length (see below)

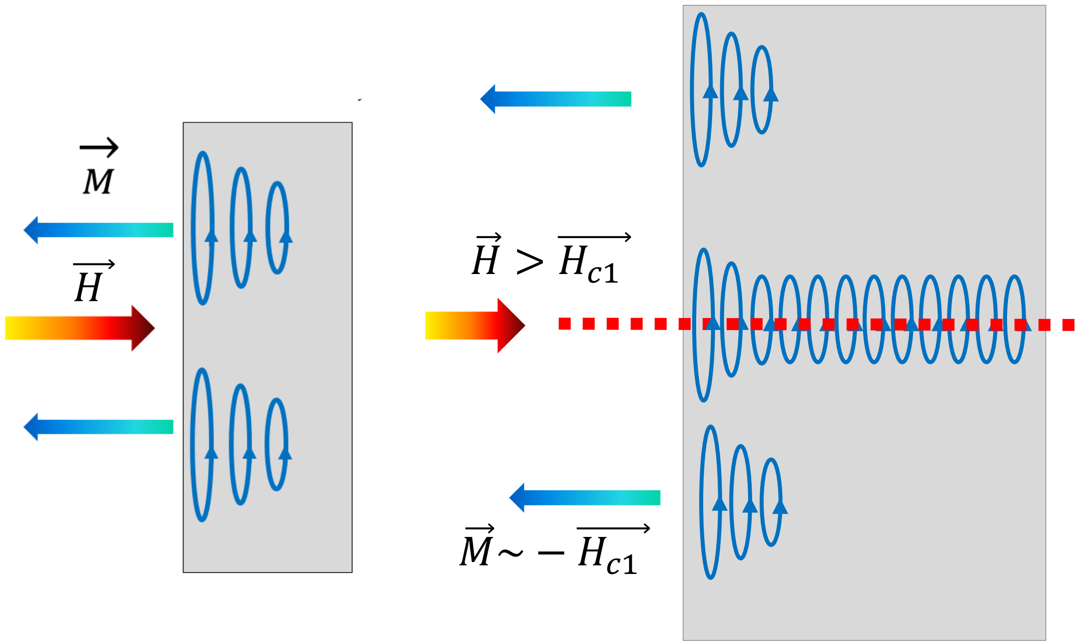


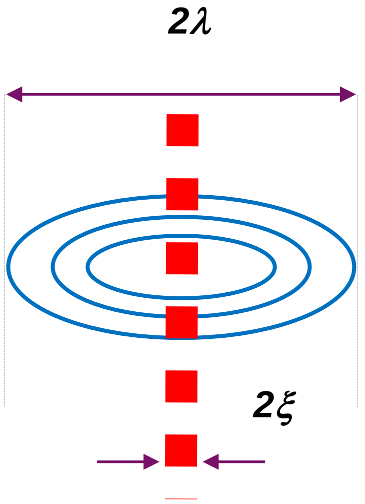


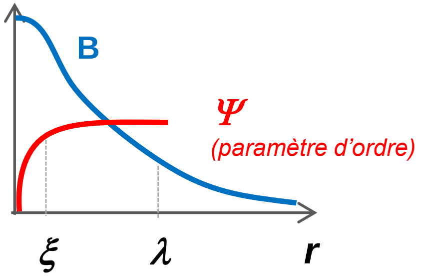
MIXED STATE AND VORTEX

Mixed state : H > H_{C1}

- Field is not fully screened anymore
- Flux lines enter the material => normal zones surrounded by screening current (≡ vortex)
- Remaining material still in the superconducting state







- Each votex contains 1! Quantum of flux Φ_0
- Screning currents extend over r ~ λ

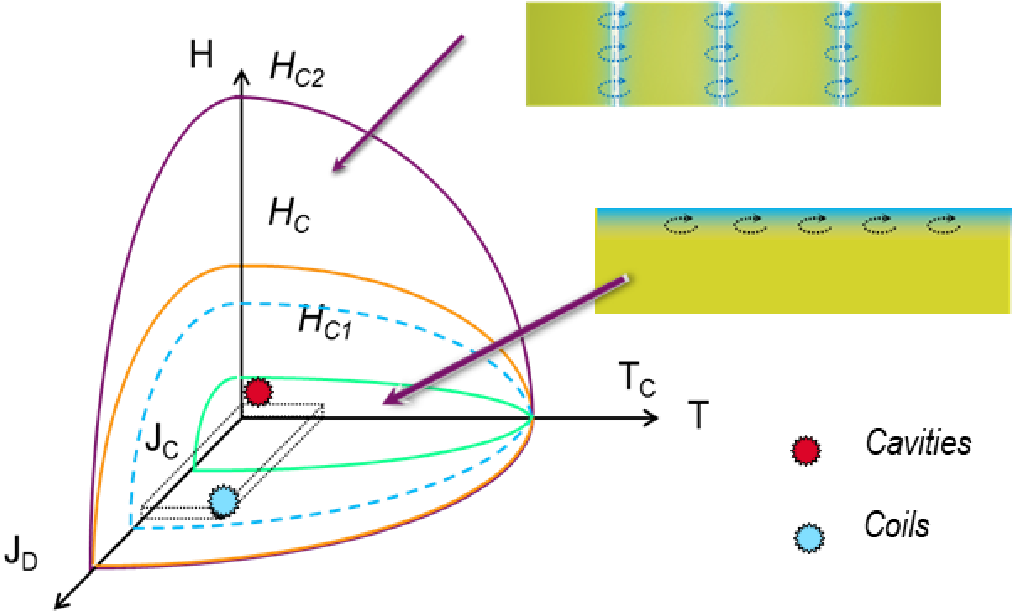
NB : do not confuse *intermediary state* (type I SC, coexistence of normal zone and SC zones, WITHOUT vortex, if H is non uniform) with mixed state (vortex state, for type II SC)

Φ_0 = h/(2e) = 2,06783376 × 10^{-15} Weber (T/m²)

A COMPLEX PHASE DIAGRAM

Applications…

- **Type II SC : 3 main Phases**
  - Meissner state (SC, B=0)
  - Mixed state (SC + vortices (B≠0))
  - Normal conducting State



- **∃ thousands of SC**
- **In practice:**
  - <10 are actually used
  - They are all type II : low H_{C1} and high H_{C2} => mixed state… **EXCEPT Nb !** (RF appli.)

NB : J_D depairing current dens. (intrinsic)
≠ J_C critical current density (technical limit)
Jc has no meaning in RF (see part II)

ANOTHER WAY TO LOOK AT…

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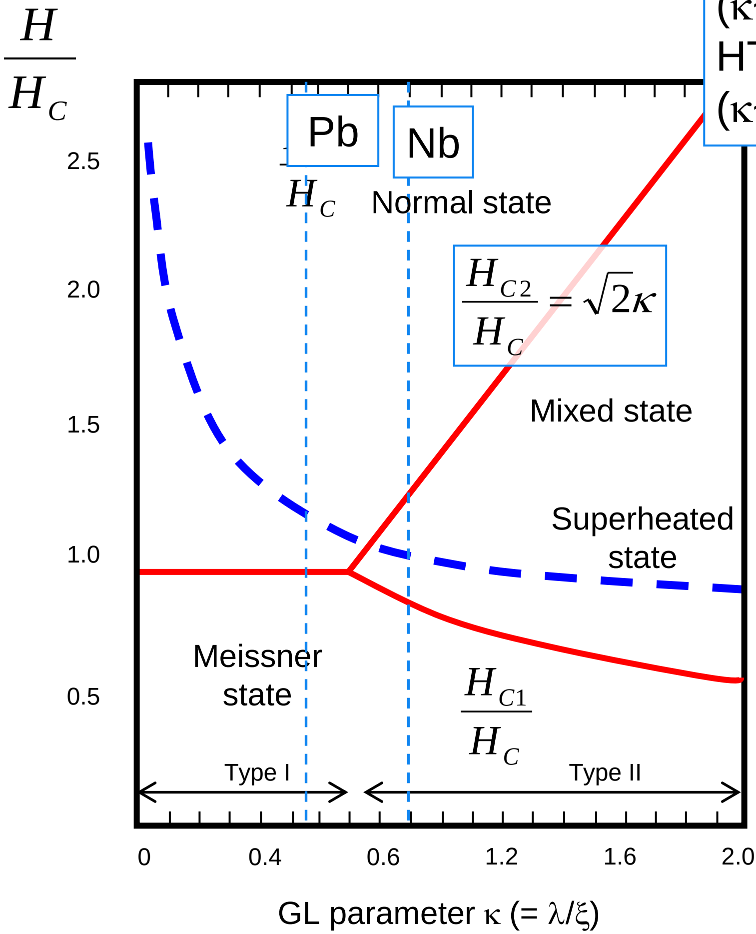
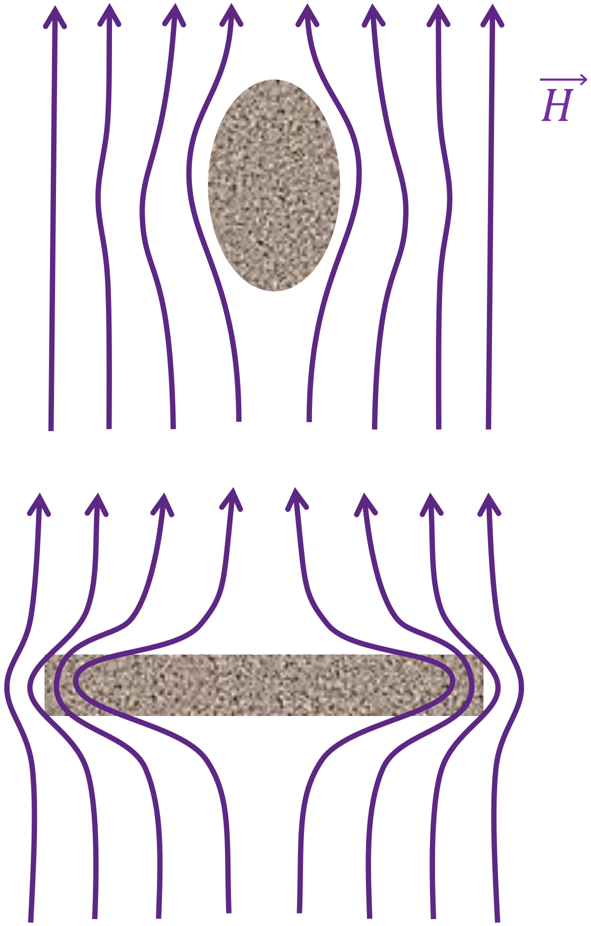
- ⚠ "superheating field" = metastable state (see "RF cavities" §)



→
MgB_2, NbTi
(κ~40-50)
HTc
(κ~60-2000)

DEMAGNETIZATION COEFFICIENT



Geometrical efects

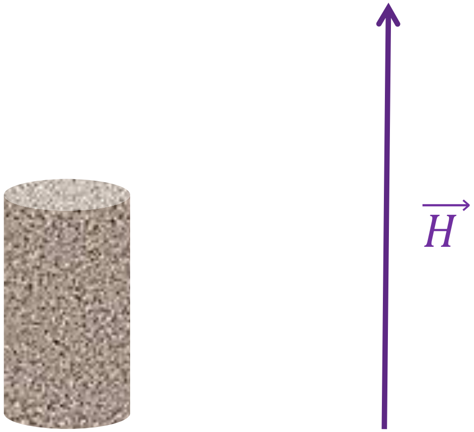
- **Elliptical samples : H = H_C everywhere**
- **Arbitrary shape : consider local deformation of flux lines**

$$\vec{B} = \mu_0 \, (\vec{H} + \vec{M}) \quad => \quad \vec{B} = \mu_0 \, (\vec{H} + (1\text{-}D) \, \vec{M})$$

- Infinitively thin strip **D → 1**

- Sphere : **D = 1/3**

- Infinite cylinder: **D = 0**

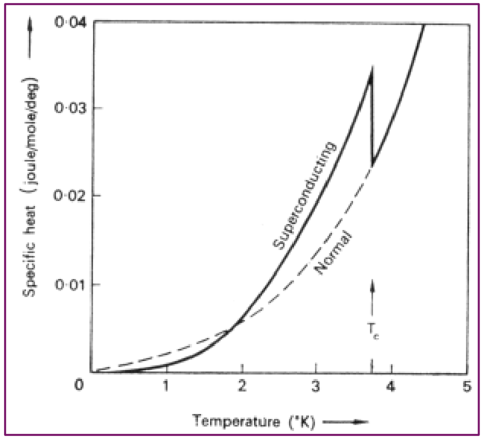


⇒ If field/shape is not uniform, some areas may transit before others !!!

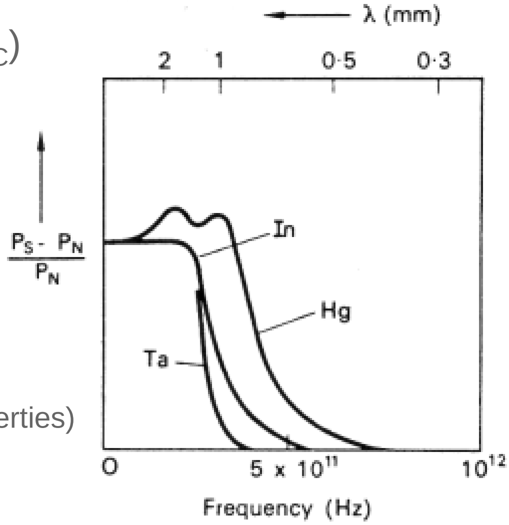
THEORIES - GENERALITIES

OTHER AFFECTED PROPERTIES

Transition normal => SC



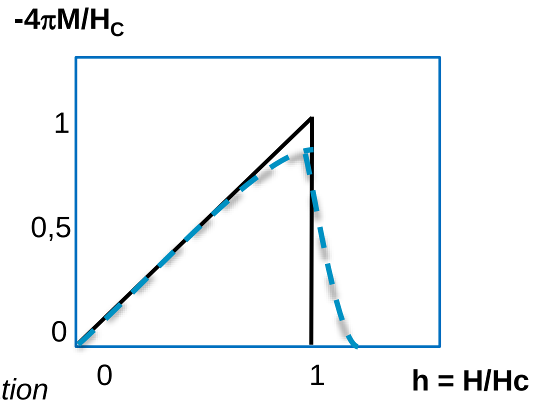
- Jump on specific heat but not on latent heat (@ B=0)
  - => 2nd order transition (thermodynamics)
  - Electronic contribution changes, but not the crystalline lattice one
- Isotopic Effect :
  - Tc ~ $1/\sqrt{m}$ (there is a link with crystalline structure, but X-rays show crystalline lattice is not affected)
- e- : long range order :
  - Abrupt transition @ Tc : many e- concerned
  - (Proximity effect (within ξ @ NC-SC interface))
- No change in visible light absorp^n (related to ρ_n)
  - But absorption band @ ~10^{11} -10^{12} Hz (~ 10^{-3} -10^{-4} eV~ some K~ T$_C$) => SC gap, bounding energy related to Tc
- Thermal conductivity ↓
  - Some of the e- that were involved in thermal conductivity are now in the form of Cooper pairs
- Mechanical state influence
  - Isostatic pressure can play on Tc (e,g, organic SC, RT SC)
  - Deformation, residual stress: plays on m.f.p.. ℓ (ℓ huge role on SC properties)
- No thermoelectric effects in type I SC
  - Because Cooper pairs do not transport entropy (but Vx do !)



THEORIES… (1)

Macroscopic models (most used…)

- London (1935)
  - 2-fluids model, inspired from superfluid Helium behavior
  - Classical theory, supposes uniform n_S *(not always true)*
  - Does not work well for thin films
  - Valid $\forall$ T, but low B (at high B: non linear behavior)
  - Explains Meissner state and predicts λ (within a factor 2)



Thin film magnetization

(also exists for bulk mater. in a lesser extent)

- Ginzburg Landau (1950)
  - Introduces quantum aspects : 1! Wave function, order parameter ψ, coherence length ξ
  - Introduces non-linear behavior
  - Expansion valid only at T~ T_C *(or if T~ 0 K , H~H_{C2})*
  - Numerical predictions similar to London, but better prediction of thin films behavior
  - GLAC (Ginzburg-Landau-Abrikosov- Gor'kov) : extension of G.L. for κ >> 1,2 (ℓ very small ≡ "dirty" SC)

Notes:

Both model apply to low J ($J<<J_D$)

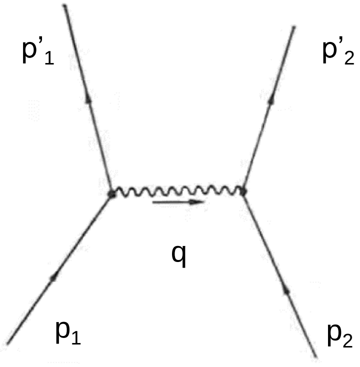
Rather apply to types II (for types I : Pippard)

GLAC also works at low T

THEORIES… (2)

Most complete…

- BCS (Bardeen-Schrieffer-Cooper) : (1957)
  - Local, quantum theory
  - In general not used for engineering predictions (too complex),
  - Cooper pairs = electrons with opposed moments p and spins (at t)
  - 1 ! Wave function for all SC e-, correlated system, many exchanges between pairs
  - Works for low coupling factor (type II)
  - Allows to explain the main aspects of SC (gap, T_C, ….)



$$q = h\nu_q / s$$

Fréq, phonons

Vit, du son

Other refinements

- Eliashberg
  - Valid ∀ T , complex calculation
  - High coupling factor (applies to Nb, although type II ?)
- Eilenberg
  - Semi-classical theory, still not valid at lower T
  - "clean " type II : takes into account some inhomogeneity
- RF :
  - Response to EM wave: Mattis-Bardeen @ weak field
  - High current density ($J \sim J_D$) : "non linear R_{BCS}"

  (clean type II)

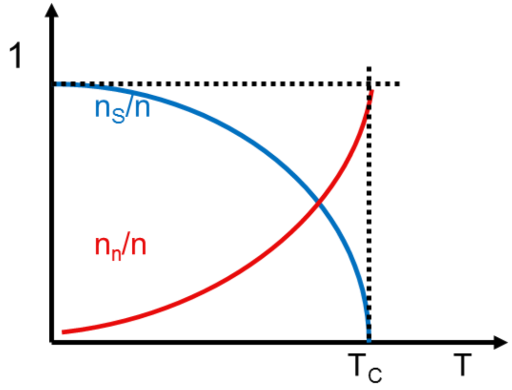
SC:

- No complete theory ☹
- Many particular, complex developments
- Be careful : limited range of validity
- Many contradictions in literature
- Sometimes far from real world

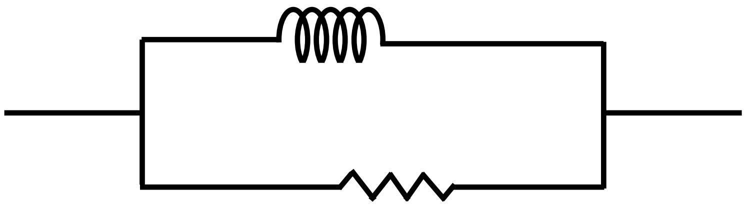
THEORIES – MORE DETAILS

LONDON (I)

2 fluids model



- Conductivity has 2 components
  - Normal electron density $n_n/n = n\left(\frac{T^4}{T_C^4}\right)$
  - Superfluid electrons density $n_s/n = 1 - n_n/n = n\left(1-\frac{T^4}{T_C^4}\right)$
  - $\vec{j} = \vec{j_n} + \vec{j_s}$
  - $\sigma = \sigma_1 + i\,\sigma_2 = \frac{ne^2\tau}{m}\left(\frac{T^4}{T_C^4}\right) - i\frac{ne^2}{m\omega}\left(1-\frac{T^4}{T_C^4}\right)$ *with m effective electron mass, e elementary charge, τ relaxation time constant, ω RF frequency*
- Equivalent circuit:
  - NC component is short circuited in DC
  - Only SC component is accelerated by E,
  - $J_s = n_s\, e\, v_s$
  - $m\dot{\vec{v_s}} = -e\vec{E}$ *e- electrodynamics (acceleration by $\vec{E}$)*



Λ *Specific inductance,* $\Lambda = \frac{m}{ne^2}$

ρ_n *Normal conducting resistivity*

- 1^{rst} London Equation (≡ Maxwell + 2 fluids) *replaces Ohm's law , for SC (no e- scattering)*

$$\boxed{\frac{d\vec{J_S}}{dt} = \frac{e^2 n_s}{m}\vec{E} = \lambda_L^2\vec{E}} \quad \text{or} \quad \vec{J_S} = \frac{\vec{A}}{\mu_0\lambda_L^2} \quad \Longrightarrow \quad J(x) = \frac{H(0)}{\lambda_L}e^{\left(-x/\lambda_L\right)}$$

Electric field differs from 0 only if current density varies

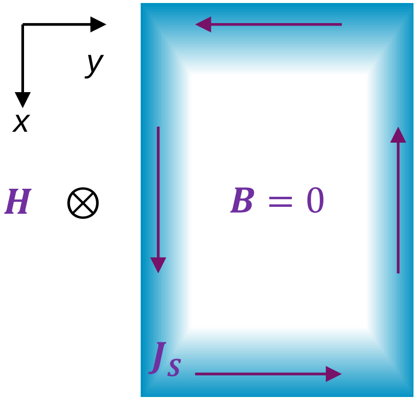
LONDON (II)

- 2^{nd} London Equation (rot 1^{rst} eq. + Maxwell)

$$\nabla \times \frac{d\vec{J_S}}{dt} = \nabla \times \lambda_L^2 \vec{E} \xrightarrow[\nabla \times \vec{E} = -\mu_0 \frac{d\vec{H}}{dt}]{\nabla \times \vec{H} = \vec{J_S}} \boxed{\nabla^2 \vec{H} = \frac{\vec{H}}{\lambda_L^2}} \longrightarrow H(x) = H(0)e^{\left(-x/\lambda_L\right)}$$

Perfect diamagnetism when x>>λ_L

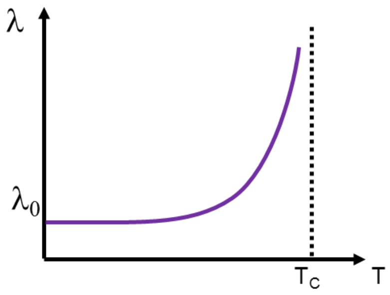
- London explains Meissner state
  - Field penetrates over λ_L
  - Within this depth => complete screening of H_0
  - B = 0 in the core of the SC
  - Maximum current density cannot exceed J_D (depairing current density):



$$\vec{B} = \vec{B}(0)e^{\left(-x/\lambda_L\right)}$$

$$\boxed{J_D(T) = \frac{H_C(T)}{\lambda_L(T)} \sim J_0\left(1 - \frac{T^2}{{T_C}^2}\right)^{3/2}}$$

$$\lambda_L = \left(\frac{m}{e^2 n_s(T)\mu_0}\right) \longrightarrow$$



Note: predicted $\lambda_L << \lambda$ experimental (London does not take into account non –local effects)

GINZBURG LANDAU MODEL (II)

Gauge theory/ Landau approach/ mean field

(universal approach for all critical phenomena, e.g. phase transitions)

- Find a macroscopic order parameter ψ
  - Describes all freedom degrees from the critical point
  - Can be guided by geometrical consideration (Gauge theory)

    Gauge theory : field theory based on a local symmetry group (Gauge group) which defines a “gauge invariance” .

    Applies well for phase transition: usually the high temperature side is symmetric, and there is a symmetry breaking at the transition (decreasing T).

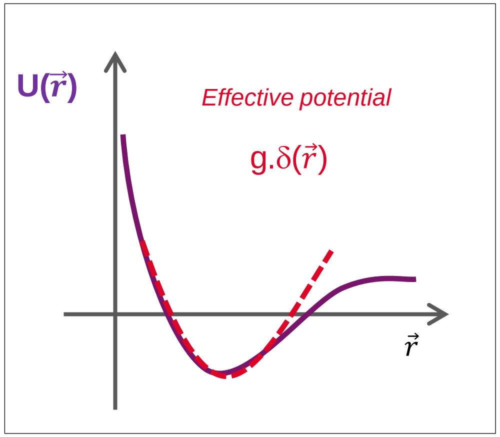
- Build up an effective free energy formula F[ψ] (mean field approxim^n)
  - Spatial and thermodynamic fluctuations of ψ are supposed small / system size
  - U($\vec{r}$) can be replaced by gxδ($\vec{r}$) + weak perturbation (small d° in ψ Taylor expansion)
  - One chooses g = effective potential with same symmetry properties in the considered energy range

    Either with intuition

    Either by minimizing the Free energy

    => Gives fundamental configuration ψ_0
  - Allows reconstructing the phase diagram
  - F[ψ] must respect microscopic symmetries of the system
  - Equivalent to an effective Hamiltonian (so-called “Ginzburg-Landau” $\mathcal{H}$)

“Gauge Invariant ”



GINZBURG LANDAU EQUATIONS

ψ = 0 if T > T_C, ψ ↑↑ if T → 0, and ψ ~ 0 @ T_C=>Taylor expansion

$$\mathcal{F}_s(T,\Psi,\vec{A}) = \mathcal{F}_n(T) + a(T-T_c)|\Psi|^2 + \frac{b}{2}|\Psi|^4 + \underbrace{\frac{1}{2m^*}|(\frac{\hbar}{i}\vec{\nabla}}_{} - \underbrace{e^*\vec{A})\Psi|^2 + \frac{B^2}{2\mu_0} - \vec{H}_0\cdot\vec{B}}_{}$$

- NOTES
  - Spatial fluctuations of n_s in the term $i\hbar\vec{\nabla}$
  - Odd terms ≠ 0 only if 1^{rst} order (field H)
  - F_0 must be Gauge invariant : invariant for symmetry operation inside the disordered phase
  - At H=0, J=0 and ψ is real **=> ξ can be derived**
  - Limit condition at surface : **J=0**, **B-H =0**

$$\psi(\mathbf{r}) \longrightarrow \psi(\mathbf{r}) \exp -i\frac{e^*}{\hbar c} f(\mathbf{r})$$

$$\boxed{\Psi = |\Psi| e^{i\theta} \text{ avec } |\Psi| = \sqrt{n_s}}$$

Minimization.

Long and complex calculation

$$\boxed{\begin{array}{l} \alpha\Psi + \beta\Psi|\Psi|^2 + \frac{1}{2m}[\frac{\hbar}{i}\vec{\nabla} - q\vec{A}]^2\Psi = 0 \\ \vec{J} = \frac{i\hbar q}{2m}[\Psi\vec{\nabla}\Psi^* - \Psi^*\vec{\nabla}\Psi] - \frac{q^2\vec{A}}{m}|\Psi|^2 \end{array}}$$

Ginzburg Landau Equations

Oops ! coupled non linear differential equations…
=> Numerical treatments only

- Accounts for Meissner state in type I et II SC (supercurrents), flux quantification, macroscopic quantic effects…
- Allows λ and ξ estimation

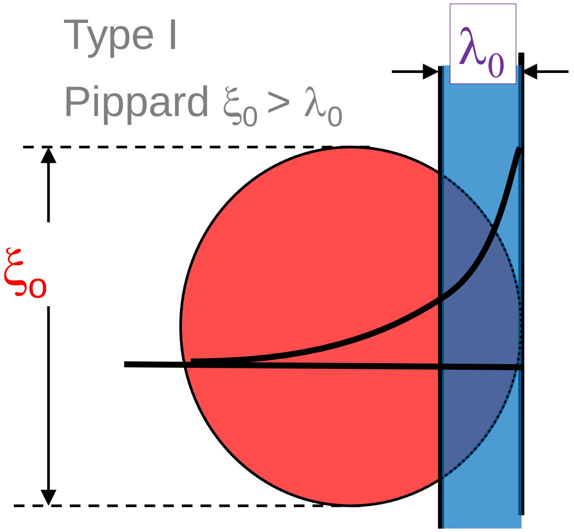
equations valid only close to transition !!!

e.g. H~ H_{C2} and T ~0 or H~0 and T~T_C

CHARACTERISTIC LENGTHS ξ_0, λ_0, ℓ

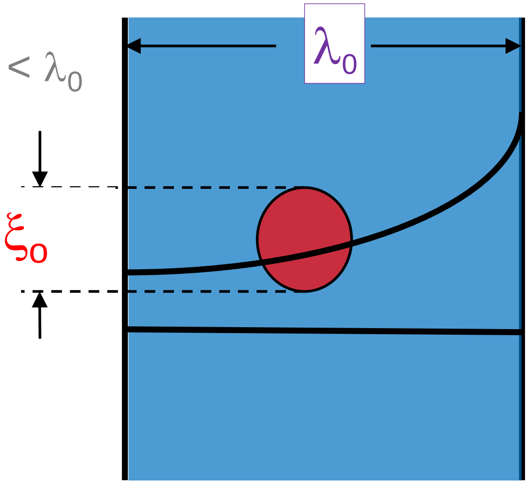
Type I

Pippard $\xi_0 > \lambda_0$



Type II

London $\xi_0 < \lambda_0$



- $\xi = \xi_0$ if $\ell \rightarrow \infty$ otherwise

$$\boxed{\frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{\ell}}$$

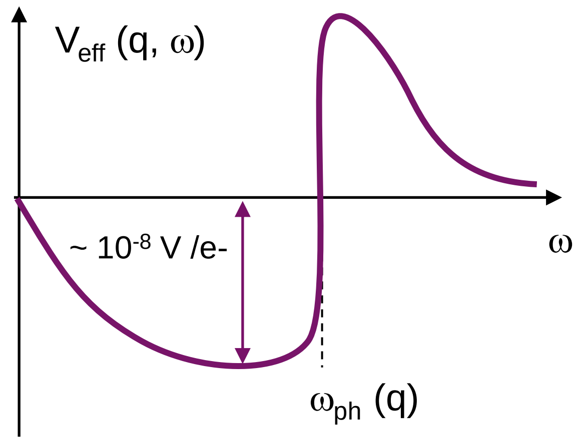
ℓ *mean free path*

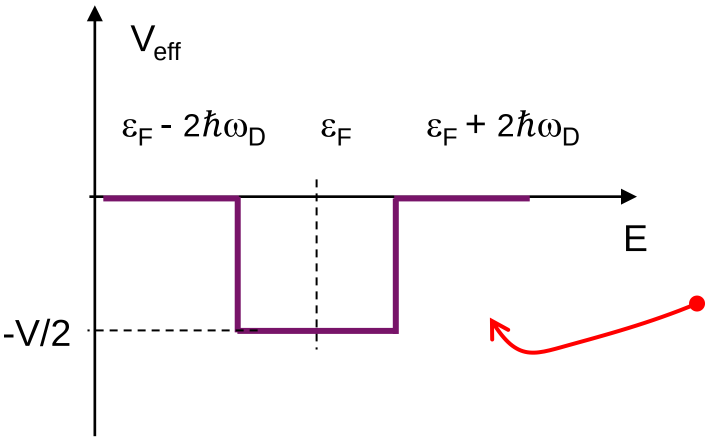
- if ℓ ↘ then:
  - ξ ↘
  - λ ↗
  - κ ↗↗
- and:

$$\boxed{\lambda = \lambda_L \cdot \left(\frac{\xi_0}{\xi}\right)^{\frac{1}{2}} = \lambda_L \cdot \left(1 + \frac{\xi_0}{\ell}\right)^{\frac{1}{2}}}$$

By playing with ℓ (crystalline state, impurities) one can modify the SC properties

BCS THEORY ELEMENTS (I)





Effective interaction

(nucleus screening by e-, lattice vibrations, etc…)

- Jellium model :
  - 1) Effective potential due to all charged particles (ions + electrons ~ continuous media) ≡ *Screened Coulomb interaction*
  - 2) one consider delays due to phonons *(ions move slower than electrons)*
- Effective Hamiltonian H = H_0 + V
  - H_0 = Hamiltonian for system without interaction *(jellium)*
  - V = Perturbative potential *(Interaction with phonons)*
- Only electrons close to the Fermi level do “feel” a positive interaction => bound states *(Cooper pairs)*
- Next simplification :
  - only e- with energy $E_v = \varepsilon_F \pm 2\hbar\omega_D$ are submitted to a potential ~ –V/2. ω_D = Debye frequency

BCS THEORY ELEMENTS (II)

Matrix diagonalization :

- => gives new wave functions with lower energy (difference = Δ) *compared to W.F. without interaction*
- Variational Wave function *describes the creation of Cooper pairs with $c^\dagger_{k,\uparrow}c^\dagger_{-k,\downarrow}$operator*

$$|\psi\rangle=\prod_k(u_k + v_k c^\dagger_{k,\uparrow}.c^\dagger_{-k,\downarrow})|0\rangle$$

Cooper pairs creation operator

H_0 matrix elements *V matrix elements*

- It explains:
  - Superconducting state : Cooper pair condensation (~bosons)
  - Phase lock-in
  - Particles number is not fixed (uncertainty principle)
  - Gap existence $\Delta \sim \langle c^\dagger_{k,\uparrow}.c^\dagger_{-k,\downarrow}\rangle$

Same symmetry as the GL order parameter ψ …,
but not exactly the same value for the gap Δ_{BCS}

Calculations too complex in practice, => simpler formalisms are more often used (London, G.L.…)

AND IN AC OR RF ?

RESISTIVE BEHAVIOR IN AC/RF

Cooper pairs inertia : field is not anymore perfectly screened within λ

- NC case : normal e⁻ are accelerated and dissipate
- Reminder : normal metal in AC

In the SC normal e- ~ quasiparticles, e.g Cooper pairs thermally broken at T>0 K

$$\boxed{Z_n = \frac{1\_i}{\sigma_n \delta} = (1-i)\frac{\rho_n}{\delta}}$$

$\sigma_n = 1 / \rho_n$ = DC conductivity @ T

δ = skin depth

- Extension to SC:
  - 2 fluids model (London) : $\sigma_1 - i\sigma_2$ in place of σ_n ⟶
  - BCS => σ_1/σ_n and σ_2/σ_n can be numerically estimated (Mattis et Bardeen integrals)
  - London approach: σ_1/σ_n and σ_2/σ_n can be approximated:

$$\boxed{\frac{R_s}{R_n} \sim \frac{1}{\sqrt{2}} \frac{\frac{\sigma_1}{\sigma_n}}{\left(\frac{\sigma_2}{\sigma_n}\right)^{\frac{3}{2}}}}$$

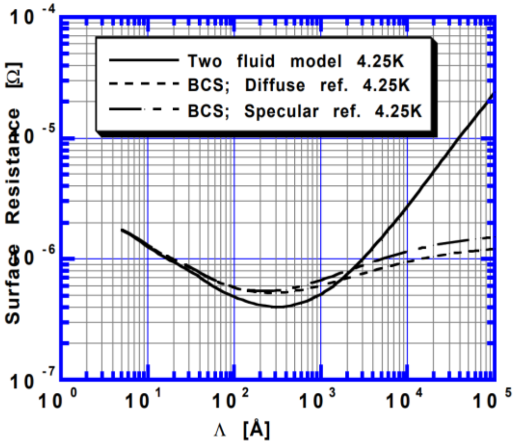
$$\boxed{\frac{\sigma_2}{\sigma_n} \sim \frac{\pi\Delta}{\omega}\tanh\frac{\Delta}{2K_BT}}$$

and

$$\boxed{\frac{\sigma_1}{\sigma_n} \sim \left[\frac{\frac{2\Delta}{K_BT}}{\left(1+e^{-\Delta/K_BT}\right)^2}\right] e^{-\Delta/K_BT} \ln\frac{\Delta}{\hbar\omega}}$$

- RF surface resistance R_S

$$\boxed{R_{BCS} = A(\lambda_L^4, \xi_F, \ell, \sqrt{\rho_n})\frac{\omega^2}{T}e^{-\Delta/kT}}$$



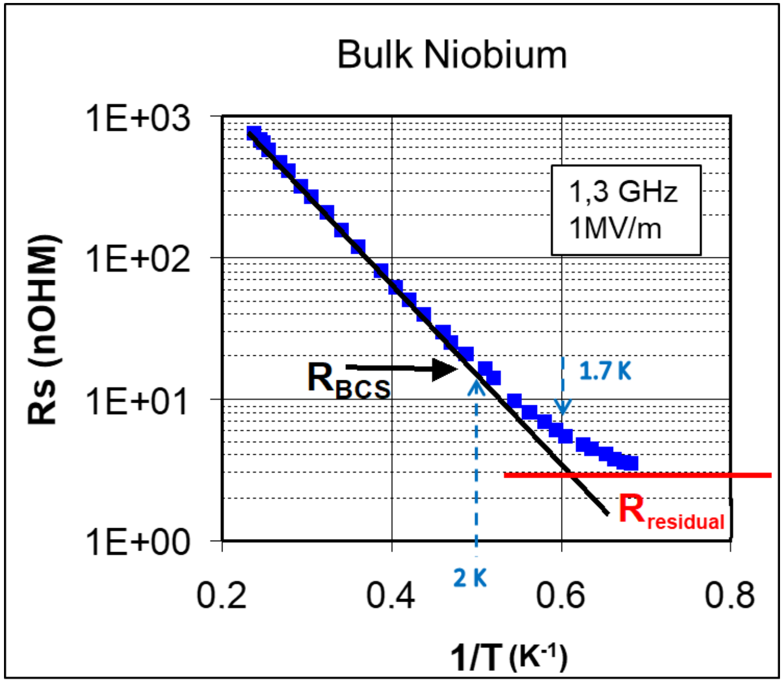
Only valid at small field !!!

RF RESIDUAL RESISTANCE OR…

… the limits of predictive theories

Vacancies, dislocations…
Explained in part II

- Rs (sometimes quoted R_0): Not predicted by BCS!
- Contains the (initially) “unknown”:
- Flux trapped during cooldown:
  - Existence of pinning center *(e.g. remnant of damage layer, thermal strain, poor recrystallization…)*
  - Mitigation: Magnetic hygiene *(non magnetic steel for nuts and bolt. Active and passive magnetic shielding of the cryostat .Cooldown procedures.* => Q_0 x10 !
- Other pair breaking mechanism:
  - Existence of magnetic impurity *(e.g. vacancies in the oxide layer)*
  - Proximity effect *(∃ a metallic layer at the metal oxide interface: NbO => SC by proximity effect but affects the gap)*
  - Other suspects (not yet identified)



The metal – (natural) oxide interface not easily modeled : lattice mismatch => stress => dislocations, impurities segregation… What are the exact SC parameters there ? Difficult to include in simple model

FIRST PART CONCLUSION

Superconductivity :

- Quantic phenomenon observable at macroscopic scale
- Coupling of 2 e- (fermions) into a boson (Cooper pair)
  - Coupling mechanism only elucidated for conventional SC *(metals and alloys)*
- No existing complete theory
  - Many specific developments *(phenomenological)*
  - Be careful with validity limits
  - Many contradiction found in the literature *(most of them due to misuse of the proper model)*
- Theory often very far from real world *(not very conclusive)*
- In practice all applications use type II SC
- All of them in mixed state… except SRF acceleration application

NB. they are some SC RF applications in SC electronics …@ mixed state

NEXT:
PART II : MIXED STATE & VORTEX