Exercises for Beam Instrumentation and Diagnostics

JUAS 2025

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1 Calculation of currents

A facility is built of a LINAC and a synchrotron for protons or heavy ions.

1.1 LINAC

The average current is 0.1 mA of protons. The pulsed LINAC has a macro-pulse length of 1 ms and a macro-pulse period of 20 ms. The acceleration is done with a rf-frequency of 100 MHz. The particles are distributed 10 % in rf phase around the reference particle in longitudinal direction. The transverse beam size is $2x = 1$ cm assuming a rectangular shape .

Assumption for all parts of this exercise to simplify the calculations: Assume a homogeneous charge distribution with hard edges i.e. a rectangular shape. The beta-function β is constant everywhere and, hence, the transverse beam size $x = \sqrt{\beta \cdot \epsilon}$ changes only due to the variation of the emittance ϵ.

Calculate the pulse current, the number of particles within one macro-pulse and within one bunch. What is the particle density at $E_{kin} = 10$ MeV kinetic energy? Compare this density to that of the residual gas assuming a pressure of $p = 10^{-7}$ mbar $= 10^{-5}$ Pa ($\simeq 10^{-7}$ torr). Calculate the average distances between the protons.

Hint: Use the ideal gas theorem $p = nk_BT$, with n particle density, $T = 300$ K temperature and $k_B = 1.4 \cdot 10^{-23}$ J/K the Boltzmann-constant.

1.2 Synchrotron

With a kinetic energy of 10 MeV the particles are injected into a synchrotron with $L_{syn} = 220$ m circumference using multi-turn injection of $t_{inj} = 100$ μs length and 100 % efficiency. The final kinetic energy after acceleration is $E_{kin} = 1$ GeV. How many particles are injected, what is the electrical current at injection and extraction? Give also the revolution frequency.

Due to the multi-turn injection, the horizontal emittance ϵ is enlarged by a factor of 25. Calculate the density at injection and extraction for a de-bunched beam, assuming the conservation of the normalized emittance $\epsilon_{norm} = \frac{v_s}{c}\gamma\epsilon$ with v_s is the longitudinal velocity and γ the Lorentz factor. Compare the particle density to the residual gas density for a pressure of $p = 10^{-10}$ mbar. What are the average distances between the protons?

1.3 Extraction

a) Slow extraction: The stored particles are extracted within 10 s. Calculate the current and the mean distance between the particles in the transfer line. What is their density? Compare its density to the residual gas density for a pressure of $p = 10^{-9}$ mbar.

b) Fast extraction: For a transfer to an other synchrotron, all particles are collected within one bunch of $t_{bunch} = 100$ ns length. Give the current of this bunch within a transfer line; assume a rectangular bunch distribution for simplicity.

1.4 Changes for an ion accelerator

Assuming the same accelerator facility as above but for an Uranium beam. The kinetic energies are adopted by changing the unit from MeV to MeV/u (mass unit u= 938 MeV). At the ion source U^{4+} is generated; at $E_{kin} = 1$ MeV/u the ions are stripped to U^{28+}; additionally, at $E_{kin} = 10$ MeV/u the ions are stripped to U^{70+} and after extraction at $E_{kin} = 1$ GeV/u to the bare nuclei U^{92+}. For the LINAC the electrical current (in units of Ampere) stays the same as for protons.

What are the differences?

2 Methods of current measurements at proton accelerators

(from previous examination)

For the below-mentioned proton accelerators and beam parameters, an appropriate method for current measurement should be chosen; give the main argument for your choice:
a) Behind the ion source with an energy of 100 keV, a beam current of 100 mA and a pulse duration of 1 ms.
b) Behind the ion source with the same parameter as under **a)** but a current of 10 nA.
c) Behind a proton LINAC with an energy of 100 MeV, a beam current of 100 mA and a pulse duration of 1 ms.
d) The permanent monitoring during the 1 s long acceleration within a synchrotron from an energy of 100 MeV to 1 GeV and a current of the circulating beam of about 100 mA.
e) The circulating current within a synchrotron after acceleration and de-bunching (i.e. a continues current)
f) In a transport line behind a synchrotron where 10^{12} protons are extracted within 1 μs.
g) The same parameters as under **f)** but with a duration of 10 s using 'slow extraction' mode.

3 Transformer for a pulsed LINAC

A macro-pulse of 1 ms length has to be measured by a current transformer with an allowed droop of 3 % and a upper cut-off frequency of $f_{high} = 100$ kHz (why is this bandwidth needed?). The core has the size: inner radius $r_i = 30$ mm and outer radius $r_o = 60$ mm and length in beam direction of 4 cm. The permeability of the torus is $\mu_r = 10^5$ and $\mu_0 = 4\pi \cdot 10^{-7}$ Vs/Am. Calculate the droop time τ_{droop} and the corresponding lower cut-off frequency f_{low}.

Passive transformer: The load resistor of a passive transformer is $R = 1$ kΩ, having a temperature of 300 K. The loss resistivity is $R_L = 10\ \Omega$.
Calculate the required number of windings for the given droop, the sensitivity (meaning the current-to-voltage conversion [V/A], also called trans-impedance) and the detection threshold for a signal-to-noise of $S/N = 1$ for pure thermal contributions. Use the thermal noise voltage $U_{eff} = \sqrt{4k_B T R \Delta f}$ with Boltzmann-constant $k_B = 1.4 \cdot 10^{-23}$ J/K, temperature $T = 300$ K and bandwidth $\Delta f = f_{high} - f_{low}$.
What is the corresponding minimal detectable beam current?
Active transformer: Calculate for the same properties of an active transformer of the same size with an open-loop gain of $A = 10^6$, a feedback resistor $R_f = 1$ MΩ and the same loss resistivity $R_L = 10\ \Omega$.
Fast passive transformer: Passive transformers are normally used for short bunch measurements as transferred between synchrotrons. They have a load resistor of $R = 50\ \Omega$ for smooth signal transmission. For an allowed droop of 3 % per μs and a bandwidth of 100 MHz calculate the same properties as above. Because the permeability is frequency dependent $\mu_r \propto 1/f$ for $f > 100$ kHz, take an average value of $\mu_r \simeq 10^3$.

4 Slow extraction current measurement

Assuming the stored number of particles from exercise 1 at a kinetic energy of 1 GeV/u and 1 s extraction time for proton ($1.25 \cdot 10^{12}$ stored) and Uranium ($1.8 \cdot 10^{10}$ stored) case. A current measurement is done with a 0.5 cm long ionization chamber, filled with Argon at atmospheric pressure. The energy loss of protons is

$$\frac{dE_p}{\rho dx} = 0.00144 \frac{\text{MeV}}{\text{mg/cm}^2}.$$

The scaling for heavy ions in this energy range is $dE/dx \propto Z^2$. For Argon, the W-value is 26.3 eV, and the mass is 40 amu. (Avogadro-number $N_A = 6.0 \cdot 10^{23}$ mol^{-1} and volume of 1 mol gas is $V_{mol} = 22.4$ l.)
Calculate the secondary currents for both ions. (The nuclear charge of Uranium is $Z = 92$ and mass $A = 238$.)
Calculate the secondary current using a secondary electron monitor (SEM) with the energy loss in aluminium for protons of

$$\frac{dE_p}{\rho dx} = 0.00177 \frac{\text{MeV}}{\text{mg/cm}^2}$$

and the secondary electron yield $Y = 27.4\ e^-/(\text{MeV/mg/cm}^2)$. What is the secondary current for the proton beam? What is the secondary current for the Uranium beam using the scaling $dE/dx \propto Z^2$?

5 Beam power at a LINAC

Calculate the absorbed power for a beam delivered by a LINAC with proton or Uranium $^{238}\text{U}^{28+}$ at $E_{kin} = 10$ MeV/u stopped in a water cooled Faraday cup. The electrical macro-pulse current for both ions is $I_{macro} = 2$ mA, and the pulse duration is $t_{macro} = 1$ ms with a repetition rate of 50 Hz. What is the average power? What is the required flow of cooling water for a maximal increase of the water temperature of 60 ^{o}C. The specific heat of water is $c = 1\text{cal/gK} = 4.2\text{J/gK}$.
What is the peak power?

6 Material destruction for intense beams

A beam of 1 MeV/u Uranium $^{238}\text{U}^{4+}$ with a macro-pulse current of $I_{macro} = 10$ mA and duration of $t_{macro} = 200\ \mu$s is stopped on a copper plate e.g., inside a Faraday Cup. The penetration depth is $R_{Cu} = 9.5\ \mu$m. Assume a round beam of radius r with a constant beam density. Estimate the minimal possible beam spot to avoid melting of the stopper material. The required constants for copper are given in Table 1.
What does change if such a beam hits a tungsten surface of a high-power Faraday cup? The range in tungsten is $R_W = 7.4\ \mu$m; for further material constants see Table 1.
What does change if the beam hit the vacuum pipe made of iron? The range in iron is $R_{Fe} = 9.7\ \mu$m; other material constants are given in Table 1.
Having a beam of radius $r_{beam} = 5$ mm. What is the maximum allowed pulse length to avoid melting?
What is a rough scaling for the minimal radius as a function of beam energy?
Calculate the same properties for a 1 MeV proton beam. The ranges are $R_{Cu} = 6.7\ \mu$m in copper, $R_W = 5.38\ \mu$m in tungsten, and $R_{Fe} = 6.4\ \mu$m i iron.

	Range R_{mat}	density ρ	melting T_{melt}	sp. heat c	melt. heat a_{melt}
Copper Cu	9.5 μm	8.96 g/cm^3	1083 ^{o}C	0.44 J/gK	210 J/g
Tungsten W	7.4 μm	19.25 g/cm^3	3380 ^{o}C	0.13 J/gK	192 J/g
Iron Fe	9.7 μm	7.87 g/cm^3	1535 ^{o}C	0.45 J/gK	275 J/g

Table 1: Ranges of the ions and material constants required for the calculations: Range R for 1 MeV/u Uranium, density ρ, melting temperature T_{melt}, specific heat c and melting heat a_{melt}.

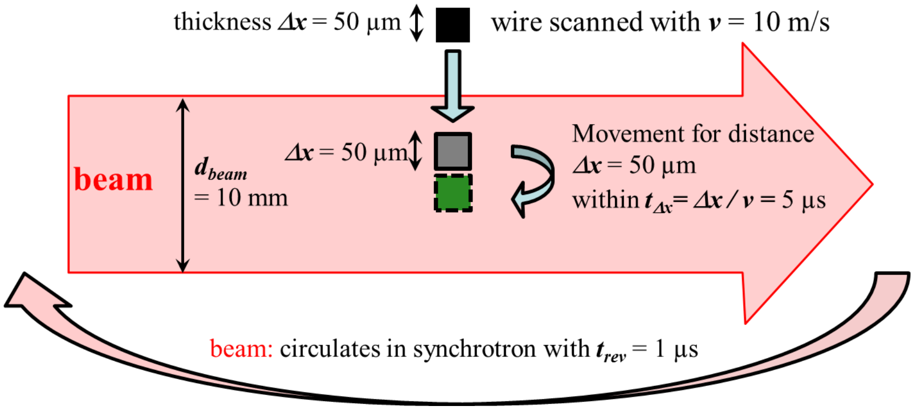

Figure 1: Simplified scheme of a flying wire measurement in a synchrotron.

7 Transverse profile by flying wire scanner

In a synchrotron, a beam of protons or Uranium at an energy of $E_{kin} = 1$ GeV/u is stored. A thin wire is scanned through the beam to monitor the transverse profile. The beam diameter is Ø 10 mm and the revolution time is $t_{rev} = 1.0$ μs. The wire is made of Graphite with 50×50 μm cross section and scanned with a velocity of $v = 10$ m/s. The setup is schematically depicted in Fig. 1.
The energy loss in Graphite of protons is $dE/dx = 4.29$ MeV/cm and for Uranium of mass 238 amu is $dE/dx = 35640$ MeV/cm.
The number of stored particles are for protons $N_{stored} = 10^{12}$ and for Uranium $N_{stored} = 10^9$.
Assumption to simplify the calculation: The beam and wire have a rectangular cross section, and the particles doesn't execute betatron oscillations.
Calculate the energy loss per passage of the ion through the wire and the calculate the ratio with respect to the ion's kinetic energy.
Calculate the average number of the ion's passages through the wire.
Hint: Calculate first the time it takes for the wire passage to pass a length of 50 μm (i.e. the wire thickness)! What is the total energy loss for an individual ion after these passages? Do you think, the particles can be used for further acceleration inside the synchrotron?
What is the total thermal energy absorbed by the wire? Does the wire withstand the energy deposition without melting assuming a maximal power rating of 1 W/mm?
What happens, if not fully stripped Uranium is stored?

8 Signal estimation for an ionization profile monitor

8.1 LINAC

The transverse profile of a LINAC beam of $I = 1$ mA Ar^{10+} and pulse length $t_{macro} = 1$ ms at energy $E_{kin} = 1.4$ MeV/u should be measured with an ionization profile monitor. The length of the collecting strips are $l = 100$ mm. The residual gas contain 90 % N$_2$ and 10 % H$_2$ at a pressure of $p = 10^{-7}$ mbar. Give the resulting voltage after a current-to-voltage converter using a trans-impedance amplifier with a conversion of $Z_{trans} = 10^8$ V/A $\equiv 0.1$ V/nA (referred to as trans-impedance).
Assumption: Take a rectangular beam cross section and assume that 10 % of the beam is covered by one strip.
The needed quantities are: Density at *normal* pressure for H$_2$ is $\rho_H = 0.09 \cdot 10^{-3}$ g/cm^3 and for N$_2$ it is $\rho_N = 1.25 \cdot 10^{-3}$ g/cm^3. The energy loss for H$_2$ is

$dE_H/\rho dx = 66$ MeV/(mg/cm^2) and for N$_2$ it is $dE_N/\rho dx = 22$ MeV/(mg/cm^2). As a first approximation, assume that the average energy for the production of one e^--ion pair is 36 eV.

8.2 Synchrotron

Assume that the beam is injected to a synchrotron with 10 fold multi-turn efficiency with respect to the current. The residual gas contains now only H$_2$ and has a pressure of $p = 10^{-10}$ mbar. Calculate the current per strip for this case measured within $t_{meas} = 1$ ms.
How can one achieve a higher secondary current without changing the vacuum pressure?

9 Signal estimation for broad-band BPM

The beam position in a proton synchrotron has to be measured by a linear-cut type BPM. The pick-up electrode has a length of $l = 20$ cm, a distance from the beam centre to the plates is $a = 10$ cm and a total capacity of $C = 100$ pF, as used for the calculation of Fig. 5.3 and 5.4 in the lecture notes. The beam has a velocity of $\beta = 50$ % and a bunch length of $\sigma = 100$ ns. For the position read-out, assume a linear response $U_\Delta/U_\Sigma = x/a$ as a function of displacement x. Calculate the transfer impedance Z_t of a half-cylindrical plate using a 1 MΩ termination. Give the sum voltage for a peak value of the beam current of $I_{beam} = 1$ A. What is the difference voltage for a $x = 1$ mm displacement in this case? What are the corresponding values for U_Σ and U_Δ for a 50 Ω termination assuming $Z_t(50\Omega) = Z_t(1\text{M}\Omega)/20$, corresponding to a frequency of 1 MHz as shown in Fig. 5.3?
Remark: In principle, for the transfer impedance, the Fourier-transformation is required (Why?). For a Gaussian function in time with standard deviation σ_t the Fourier-transformation is a half-Gaussian function (for positive frequencies centred at $f = 0$) having a standard deviation $\sigma_f = \frac{1}{2\pi\sigma_t}$.
What is the thermal noise at the $R = 50$ Ω and $R = 1$ MΩ amplifier input-impedance for a bandwidth of $\Delta f = 100$ MHz? Use the thermal noise voltage $U_{eff} = \sqrt{4k_B T R \Delta f}$ with the temperature $T = 300$ K and Boltzmann-constant $k_B = 1.4 \cdot 10^{-23}$ J/K. What is the minimal beam current for $x = 1$ mm displacement for a signal-to-noise ratio $S/N = 2$ using only this thermal noise? (For a real amplifier, the noise contribution is at least a factor 2 bigger.)
For longer bunches, the transfer impedance has a scaling $Z_t \propto \omega$ for the 50 Ω termination. At which bunch length the position sensitivity of the 1 MΩ and the 50 Ω terminations are equal?
How can the position sensitivity be improved, and what is the main reason?

10 Profile measurement with a scraper

(from previous examination)

Inside a synchrotron, the transverse beam profile is measured sometimes destructively using a scraper. It consist of a plate moved in the beam path, see Fig. 2. For the profile determination, the scraped position is drawn on the horizontal axis, i.e. its position is the independent variable. The time needed to pass the beam is orders of magnitude longer (typically 1 s) than the revolution time (typically several μs). During the measurement, no refilling of the synchrotron occurs.
Discuss this method:
a) What is the primary *beam* quantity to be measured?

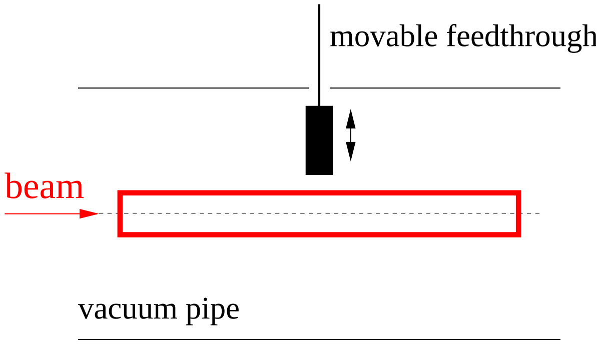

Figure 2: Scheme of a profile measurement at a synchrotron using a scraper.

b) Propose a method to determine this quantity directly or indirectly and give the reason for your choice.
c) How is the beam *profile* generated from the measured primary signal and the scraper position? Make a principle drawing for the position dependence of the primary signal versus the position and the deduced profile.
d) Why does it makes only sense to go up to the middle of the beam?
e) This method can also be used at a LINAC. What is the change of the primary signal as a function of the scraper position? Which scraper movement should be applied to measure the entire transverse profile?

11 Beam energy measurement by time-of-flight
(from previous examination)

At a proton LINAC, the coarse final energy is known between 5 MeV and 10 MeV. The accelerator is operated with an rf-frequency of $f_{acc} = 100$ MHz. The distance of the two BPMs used for the time-of-flight measurement is 4 m.
a) To what limits does one have to know the energy to get a unique solution?
b) A third pick-up should be installed for the coarse energy determination. What is the maximal distance to achieve a unique solution?

12 Beam diagnostics design for a proton facility

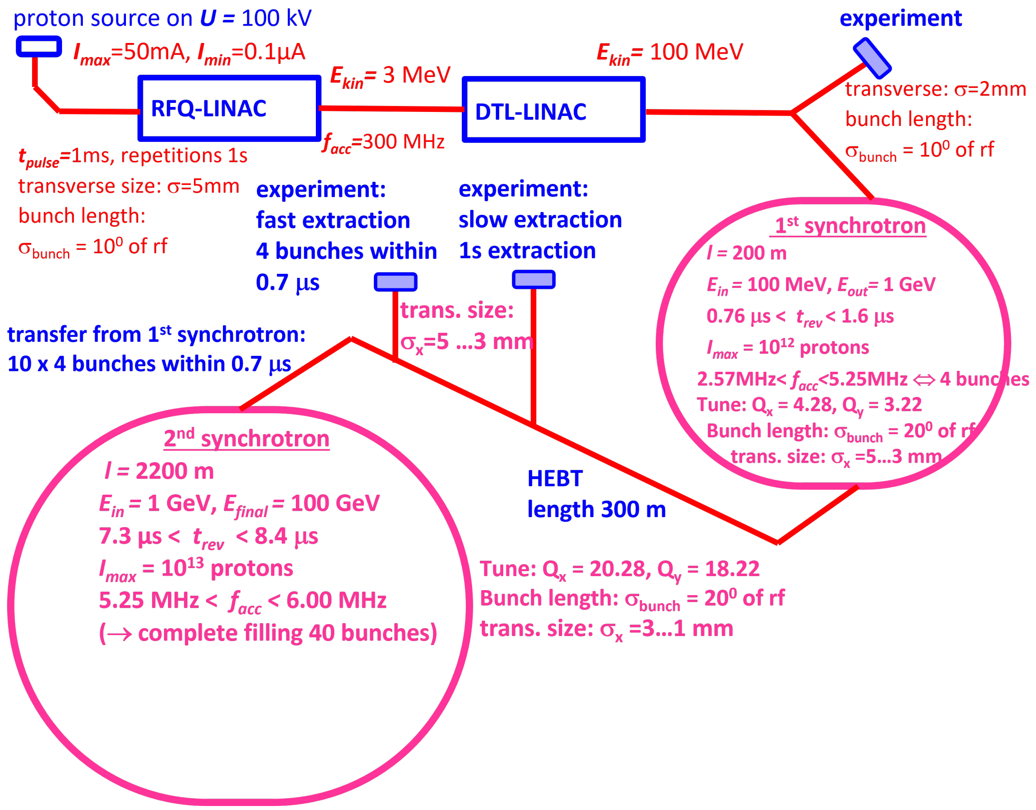

Figure 3: The simplified proton facility for beam diagnostics design.

The task for the group work is the design of beam diagnostics for a proton facility built of a LINAC, a booster-synchrotron and a high-energy synchrotron as depicted in Fig. 3. The facility is outlined below, which only provides the fundamental parameters. It is a realistic task to ask for actual facts and numbers required for an adequate layout of the beam instrumentation and diagnostics methods.

The basic parameters for the facility are:

1. The proton source is located on a high voltage platform with $U = 100$ kV, i.e. the protons' energy is $E_{kin} = 100$ keV. The maximal current is $I = 50$ mA, but also lower currents down to $I = 100$ nA are sometimes needed for the commissioning of the facility. Within a Lowe Energy Beam transport (LEBT) line the beam is transported to the LINAC; the beam size in the LEBT is $\sigma = 8$ mm (σ refers always to one standard deviation).
2. A low energy LINAC (e.g. RFQ) is used for the acceleration to $E_{kin} = 3$ MeV. It is driven in a pulsed mode with a maximal pulse length of $t_{pulse} = 1$ ms and $f_{rep} = 1$ Hz repetition rate. The accelerating frequency is $f_{acc} = 300$ MHz. After the LINAC, the transverse beam size is in the order of $\sigma = 5$ mm. The bunch length is $\sigma_{bunch} = 10^o$ of the acceleration frequency.
3. In the second part of the LINAC, protons are accelerated to $E_{kin} = 100$ MeV. The transverse beam size is in the order of $\sigma = 2$ mm; the bunch length is $\sigma_{bunch} = 10^o$ of the acceleration frequency.
4. The $E_{kin} = 100$ MeV proton beam is injected in a synchrotron of $l = 200$ m circumference which delivers a maximum final kinetic energy of $E_{kin} = 1$ GeV. A time of

$t_{cycle} = 1$ s is needed for acceleration from 100 MeV to 1 GeV. The acceleration frequency is in between 2.57 MHz $< f_{acc} <$ 5.25 MHz resulting in 4 circulating bunches. The beam velocity at injection is $v/c = 43$ % and at extraction it is $v/c = 87$ % which corresponds to a time for one revolution from $t_{rev} = 1.6$ μs to $t_{rev} = 0.76$ μs. The bunch length is $\sigma_{bunch} = 20^o$ of the acceleration frequency. The beam size varies from injection to extraction from $\sigma = 5$ to 3 mm (σ refers always to one standard deviation). The horizontal and vertical tunes are $Q_x = 4.28$ and $Q_y = 3.22$, respectively. At maximum 10^{12} protons can be stored in this synchrotron.

5. The beam is extracted towards the High Energy Beam Transport HEBT line of 300 m length. Three different target locations are foreseen to be delivered with beam, either by slow extraction within 1 s or by single-turn extraction within 0.76 μs to a fixed target as well as a transfer to a 2^{nd} synchrotron. The transverse beam size is typically $\sigma = 3$ to 5 mm (one standard deviation).
6. The beam can be injected into a second synchrotron of $l = 2200$ m length for further acceleration from 1 GeV to 100 GeV. The protons from the 1^{st} synchrotron are injected by a so-called 'bunch-to-bucket' transfer, i.e. 10 successive fillings are used to accumulate 10^{13} protons in the 2^{nd} synchrotron before acceleration is started. The acceleration takes $t_{cycle} = 1$ s, and then the beam is extracted to a target. The resulting bunch length is $\sigma_{bunch} = 20^o$ of the acceleration frequency (one standard deviation). The tunes are $Q_x = 20.28$ and $Q_y = 18.22$, respectively. The beam size varies from injection to extraction from $\sigma = 3$ to 1 mm (σ refers always to one standard deviation).

13 Beam diagnostics design for a LINAC FEL facility

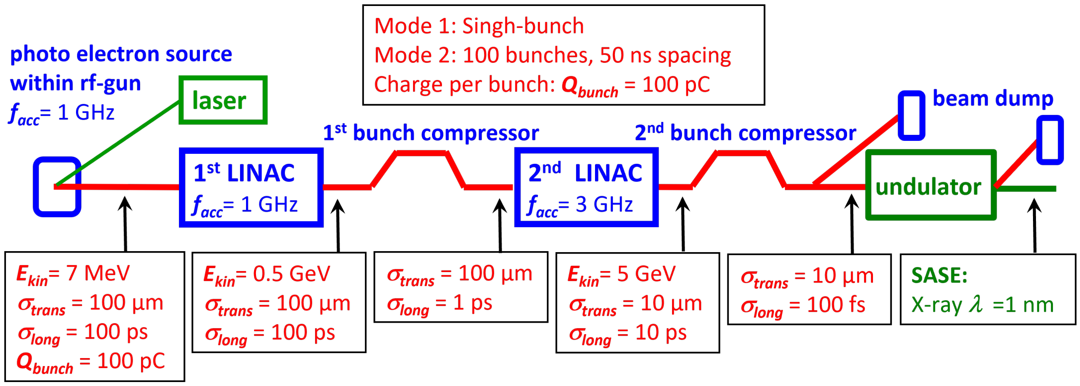

Figure 4: Simplified electron LINAC FEL facility for beam diagnostics design.

The task for this group work is to discuss the beam instrumentation for an electron LINAC, which is used as a light source of coherent X-rays referred to as Free Electron Laser depicted in Fig. 4. The light is generated in a long undulator, and the coherence is based on the SASE process; SASE stands for Self Amplified Spontaneous Emission. The simplified facility is oriented on the SwissFEL at Paul Scherrer Institute (PSI) in Switzerland.

Tow modes of operation should be assumed:

1. Single-bunch operation where only one bunch is generated
2. Bunch train of 100 bunches separated 50 ns.

The related pulse structure is generated by the pulsing of the laser beam, which illuminates the photo-cathode. The charge per bunch is $Q_{bunch} = 100$ pC. The repetition rate is 100 Hz.

The simplified facility comprises:

1. The electron source, the so-called rf-gun, is equipped with a photo-cathode illuminated by a short, intensive laser beam. The photo-cathode is installed an rf-cavity operated with $f_{acc} = 1$ GHz. The kinetic energy of the electrons out of the rf-gun is $E_{kin} = 7$ MeV.
2. After a drift for transverse focusing, longitudinal matching, and beam diagnostics, the beam is injected into a first LINAC. The LINAC is operated with a frequency of $f_{acc} = 1$ GHz. The final energy is $E_{kin} = 500$ MeV. The typical transverse beam size is $\sigma_{trans} = 100$ μm, and the bunch length is $\sigma_{long} = 100$ ps. (σ refers always to one standard deviation)
3. A first bunch compressor shorten the bunch to $\sigma_{long} = 1$ ps. (The bunch compressor uses the variation of path lengths for electrons with different energies such that the electrons reach the input of the subsequent LINAC with a compressed time spread, i.e. a short bunch length.)
4. The second LINAC operates at $f_{acc} = 3$ GHz and accelerates to $E_{kin} = 5$ GeV. The transverse beam size $\sigma_{trans} = 10$ μm.
5. A second bunch compressor shorten the bunch to $\sigma_{long} = 100$ fs; the transverse beam size stays at $\sigma_{trans} = 10$ μm.
6. The beam is then sent to a beam dump or a 30 m long undulator. The SASE light in the soft X-ray range ($\lambda = 1$ nm) is emitted from the undulator. Behind this undulator, the electron beam is bent toward a second beam dump.