Particle Source

Tutorial

1.Thermal emittance of an ion source

A H^+ beam is created from a plasma with a round aperture of R=5 mm radius. Particles are assumed to be extracted uniformly from the disk surface. The ions follow Maxwell-Boltzman momentum distribution function.? (see lecture slide #15).

1.1 Show that $\sigma_x = \frac{R}{2}$.

hint : The probability distribution function of particles on the disk is:

$f(r,\theta) = \frac{1}{\pi R^2}, \forall r \leq R, \forall\, \theta \in [0,2\pi]$

1.2 Show that : $\sigma_{x'} = \frac{1}{\gamma\beta}\sqrt{\frac{kT}{mc^2}}$

hints: $\int_{-\infty}^{+\infty} e^{-ax^2} dx = \sqrt{\frac{A}{\pi}}$ *and* $\int_{-\infty}^{+\infty} x^2 e^{-ax^2} dx$ *can be integrated by parts*

1.3 The ion temperature is 300K. The H+ kinetic energy is 100 keV. Calculate the ion beam normalized emittance in units of $\pi.mm.mrad$ (see slide)

2. Cyclotron frequency

We propose to demonstrate the formula : $B\rho = mv/q$. *Consider a particle of mass m and charge q located at* $\overrightarrow{OM}(0, y(0) = \frac{vm}{qB} = \frac{v}{\omega}, 0)$ *and with velocity*$\vec{v}(v, 0,0)$ *at time t=0 in a cartesian frame (0,x,y,z). The magnetic field is uniform with* $\vec{B}(0,0,B)$.

2.1 Solve Newton's Law and find that :

$v_x(t) = v.cos(\omega t)\,, v_y(t) = -v.sin(\omega t)\,, v_z(t) = 0$ *and* $\omega = \frac{qB}{m}$

2.2 Integrate to find the trajectory:

$x(t) = \rho.\sin(\omega t)\,, y(t) = \rho.\cos(\omega t), z(t) = 0\,, \rho = v/\omega$

2.3 deduce that : $B\rho = mv/q$

2.4 what is the electron cyclotron frequency (in Hz) in a field of 1 Tesla?

2.5 What magnetic field is needed to curve an electron of 100 keV kinetic energy in a dipole with a 1m bending radius?

3. Charge breeding

A 1+ ion source delivers a ^{39}K beam. The ion source potential is set to 10 kV.

3.1 What is the ion velocity? Is it relativistic?

3.2 The ions are injected into an ECR charge breeder whose plasma is set to the high voltage of 9,90 kV. At what energy does the ^{39}K ion pass into the plasma? What is their velocity?

3.3 The mean electron energy in the plasma is 90 eV. The ionization potential to pass from $^{39}K^{+}$ to $^{39}K^{+2+}$ is $P_{1\to 2} = 31.63$ eV, and the number of electron on the subshell is $g_i = 6$. Calculate the electron impact cross section σ of $^{39}K^{+}$ to form $^{39}K^{2+}$. *(see formula in the **table 1** below).*

3.4 The plasma electron density is n=$10^{12}\ cm^{-3}$. Calculate The mean free path $\lambda = \frac{1}{\sigma n}$ associated with the collision. The plasma length is *L=50 cm*. Demonstrate that the number of remaining $^{39}K^{+}$ beam after the crossing of the plasma is :

$$N(L) = N(0)e^{-L/\lambda}$$

And deduce the actual ratio of K^{+} converted to K^{2+}

Table 1: *Electron impact formula*

Lotz formula: Ion-Electron impact cross section from charge state q to q+1	$\sigma_{i\to i+1} \sim 4.5 \times 10^{-14}\ g_i . \frac{ln\left(\frac{T}{P_{i+1}}\right)}{TP_{i+1}}$	cm^2
Incoming Electron Kinetic Energy	T	eV
Atom Ionization potential to switch from charge state i to i+1	P_{i+1}	eV

4. Electron Beam Ion Source

4.1 An EBIS is equipped with a planar LaB6 thermionic cathode heated at 2000K with a disk diameter of 4 mm. What is the electron beam intensity generated?

4.2 The electron beam is compressed by the EBIS magnetic field by a factor of 10. What is the electron current density and beam diameter in the source?

4.3 The electron beam is accelerated by a -6 kV high voltage. A population of N 1+ Argon atom is trapped in the EBIS. Ar2+ ions are created by electron impact. Show that the evolution of $N^{2+}(t)$ ions follows:

$$N^{2+}(t) = N(1 - e^{-\nu t})$$

Calculate the time to have half of the the Argon 1+ population to be ionized to 2+.

Hint: $j_e = en_e v_e$, use the collision rate $\nu = n_e \sigma_{1+\to 2+} v_e$

4.4 Same question for the 17+ to 18+ ionization.

4.5 The electron beam voltage is set to 900V. What is the maximum argon charge achievable in the source? See **Table 2**.

***Table 2**: ionization potential of argon electron cloud*

I.P.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*
eV	*16*	*28*	*41*	*60*	*75*	*91*	*124*	*143*	*422*	*479*	*539*	*618*	*686*	*756*	*855*	*918*	*4121*	*4426*

4.6 The EBIS electrostatic trapping of ions is opened at t=0 s. A bunch of ions is then extracted under a voltage of +10 kV. A faraday cup is located 2 m away from the source. At what time does the ion bunches of Ar8+, Ar10+ and Ar 12+ arrive in the cup?

5. Magnetic emittance of an ion source

The ion source described in the exercise 1 used the electron cyclotron resonance to ionize gas and produce the beam. The magnetic field intensity at the beam extractin hole is B=0.1 T. What is the normalized transverse magnetic emittance for protons? (see slides 6,7,13,17).

$\epsilon_{mag,x} = \sigma_x \, \sigma_{x\prime} = \sigma_x^2 \, \frac{1}{2} \frac{q|B|}{\gamma m v}$ *(RMS, not normalized)*