Measurement of transverse Beam Emittance

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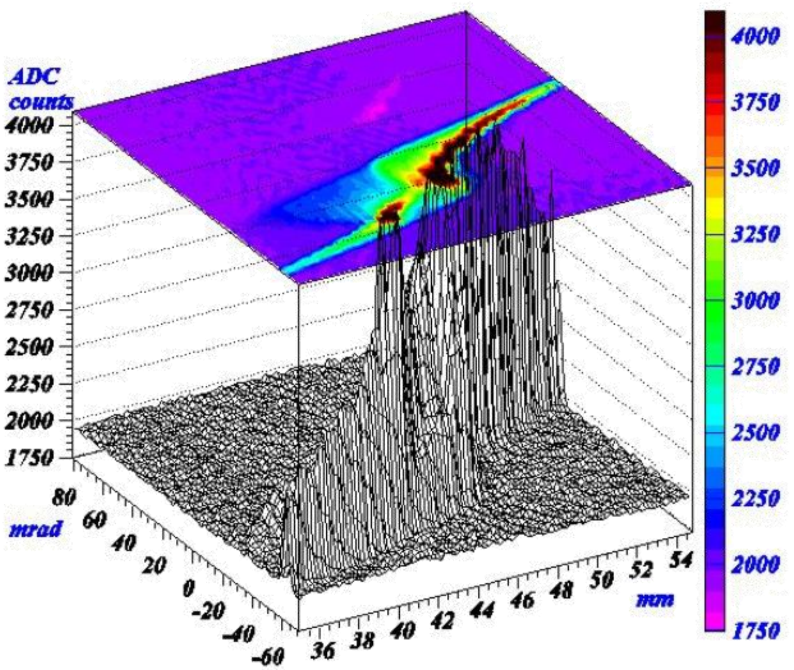


European Scientific Institute

Joint Universities Accelerator School

Measurement of transverse Emittance

The emittance characterizes the whole beam quality, assuming linear behavior as described by second order differential equation.

It is defined within the phase space as: $\varepsilon_x = \frac{1}{\pi}\int_A dx dx'$

The measurement is based on determination of:

either profile width $\boldsymbol{\sigma_x}$ and angular width $\boldsymbol{\sigma_x'}$ at one location
or $\boldsymbol{\sigma_x}$ at different locations and linear transformations.

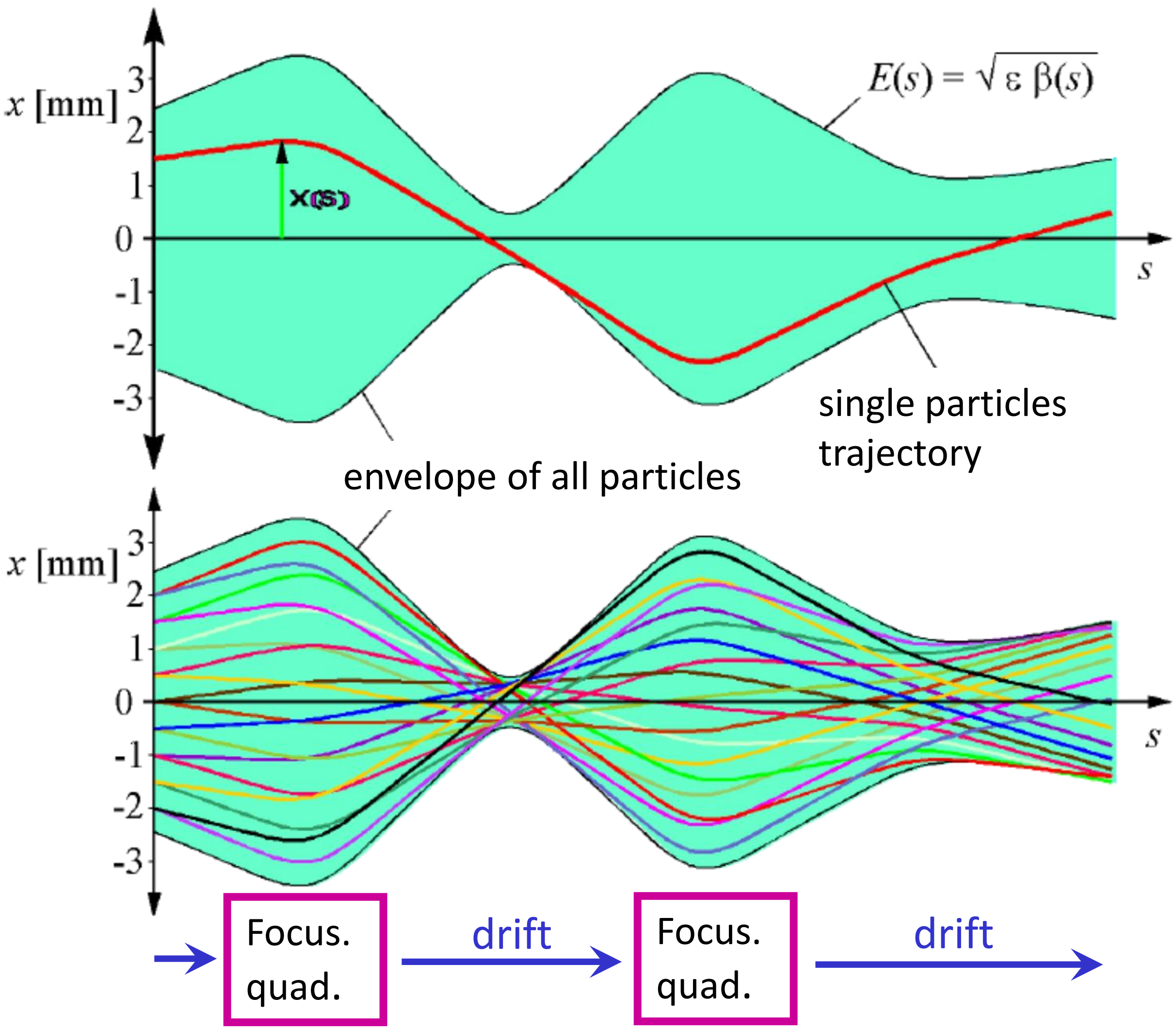
Different devices are used at transfer lines:

- All beams: Quadrupole variation & 'three grid' method using linear transformations (**not** well suited in the presence of non-linear forces)
- Lower energies E_{kin} < 100 MeV/u: slit-grid device, pepper-pot (suited in case of non-linear forces).
- ***Synchrotron:*** lattice functions results in stability criterion

$\Rightarrow$ beam width delivers emittance: $\varepsilon_x = \frac{1}{\beta_x(s)}\left[\sigma_x^2 - \left(D(s)\frac{\Delta p}{p}\right)\right]$ and $\varepsilon_y = \frac{\sigma_y^2}{\beta_y(s)}$

Outline:

- **Definition and some properties of transverse emittance**
- **Quadrupole strength variation and position measurement**
- **Slit-Grid device: scanning method**
- **Summary**



- ➢ Single particle trajectories are forming a beam
- ➢ They have a distribution of start positions and angles

⇒ Characteristic quantity is the **beam envelope**

⇒ **Goal:**

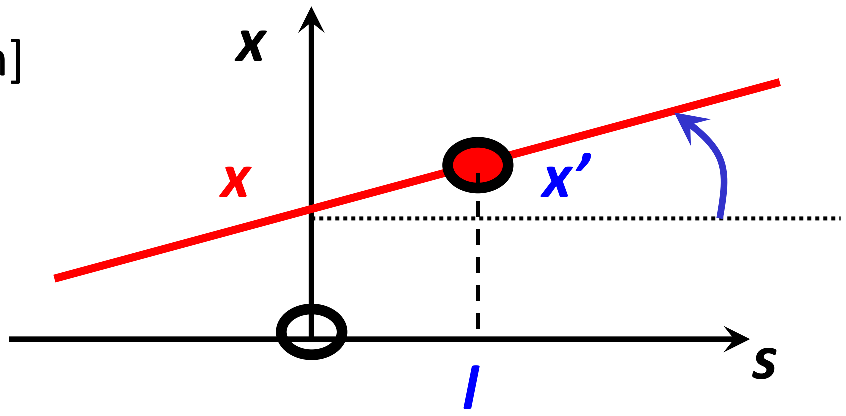
Behavior of whole ensemble

Plot: Wille

Horizontal and vertical coordinates at *s = 0* :

- ***x* :** Offset from reference orbit in [mm]
- ***x'*:** Angle of trajectory in unit [mrad]

  $x' = dx / ds$



Assumption: par-axial beams:

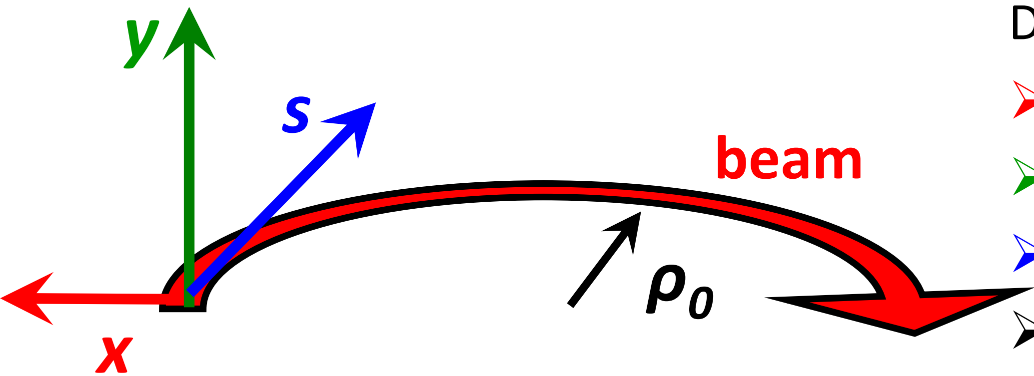
- x is small compared to dipole radius ρ_0
- Small angle with $p_x / p_s \ll 1$

Longitudinal coordinate:

- Longitudinal orbit difference: $l = -v_0 \cdot (t - t_0)$ in unit [mm]
- Momentum deviation: $\delta = (p - p_0) / p_0$ sometimes in unit [mrad] = [‰]

Reference particle: No offset $x \equiv y \equiv l \equiv 0$ & no angle $x' \equiv y' \equiv \delta \equiv 0$ for all s

For **continuous** beam: l has no meaning $\Rightarrow$ set $l \equiv 0$!



Definition compared to **reference orbit**

- ***x*: horizontal**
- ***y*: vertical**
- ***s*: in beam direction**
- **ρ_0: radius of curvature**

$$F_L = F_z \Leftrightarrow \frac{1}{\rho_0} = \frac{e}{p} \cdot B(x=0, y=0, s)$$ on reference orbit

Taylor expansion of magnetic field at location ***s*** for field in ***y*** direction:

$$\frac{e}{p}B_y(x) = \frac{e}{p}B_y(0) + \frac{e}{p}\frac{dB_y}{dx}\cdot x + \frac{1}{2}\frac{e}{p}\frac{d^2B_y}{dx^2}\cdot x^2 + \frac{1}{6}\frac{e}{p}\frac{d^3B_y}{dx^3}\cdot x^3 + \ldots$$

$$\equiv \frac{1}{\rho_0} + kx + \frac{1}{2}mx^2 + \frac{1}{6}ox^3 + \ldots$$

dipole quadrupole sextupole octupole

Drift ($\rho_0 \to \infty$), dipoles and quadrupoles $\Rightarrow$ linear beam optics

The basic vector $\vec{x}$ is

6 dimensional:

$\vec{x} \in \mathbb{R}^6$

$$\vec{\boldsymbol{x}} = \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{x'} \\ \boldsymbol{y} \\ \boldsymbol{y'} \\ \boldsymbol{l} \\ \boldsymbol{\delta} \end{pmatrix} = \begin{pmatrix} \text{hori. spatial deviation} \\ \text{horizontal divergence} \\ \text{vert. spatial deviation} \\ \text{vertical divergence} \\ \text{long. deviation} \\ \text{momentum deviation} \end{pmatrix} = \begin{pmatrix} [\text{mm}] \\ [\text{mrad}] \\ [\text{mm}] \\ [\text{mrad}] \\ [\text{mm}] \\ [10^{-3}] \end{pmatrix}$$

The transformation of a single particle from a location s_0 to s_1 is given by the Transfer Matrix R: $\vec{\boldsymbol{x}}(\boldsymbol{s_1}) = \mathbf{R}(\boldsymbol{s_0} \to \boldsymbol{s_1}) \cdot \vec{\boldsymbol{x}}(\boldsymbol{s_0})$

Transfer Matrix $\mathbf{R}(s_0 \to s_1)$ is a 6x6 matrix ⇔ $\mathbf{R}(s) \in \mathbb{R}^{6x6}$

- The transformation can be done successive:

for $\mathbf{R_1 = R(} s_0 \rightarrow s_1 \mathbf{)}, \ldots, \mathbf{R_n = R(} s_{n-1} \rightarrow s_n \mathbf{)}$

It is $\mathbf{R = R_n \cdot R_{n-1} \cdot \ldots \cdot R_1}$ as it is a linear transformation

- The matrix elements describe the coupling between the components

$$R_{11} = (x|x), R_{12} = (x|x'), R_{13} = (x|y), R_{14} = (x|y'), R_{15} = (x|l), R_{16} = (x|\delta)$$

If all forces are symmetric along the reference orbit than the horizontal and vertical plane are decoupled:
$\Rightarrow$ sub-matrix is sufficient

$$\mathbf{R} = \begin{pmatrix} (x|x) & (x|x') & 0 & 0 & 0 & (x|\delta) \\ (x'|x) & (x'|x') & 0 & 0 & 0 & (x'|\delta) \\ 0 & 0 & (y|y) & (y|y') & 0 & 0 \\ 0 & 0 & (y'|y) & (y'|y') & 0 & 0 \\ (l|x) & (l|x') & 0 & 0 & (l|l) & (l|\delta) \\ 0 & 0 & 0 & 0 & (\delta|l) & (\delta|\delta) \end{pmatrix}$$

- It is **det(R) = 1** (Liouville's Theorem)
  i.e. **R** is invertible
- Sub-matrix
  here shown for drift ***L***:

$$R_x = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \quad R_y = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \quad R_s = \begin{pmatrix} 1 & L/\gamma^2 \\ 0 & 1 \end{pmatrix}$$

Some Examples for linear Transformations

The 2-dim sub-space (***x,x'***) can be used in case there is <u>**no**</u> coupling like dispersion $R_{16} = (x|\delta) = 0$

Important examples are:

- Drift with length ***L***: $\mathbf{R}_{\text{drift}} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$
- Horizontal **focusing** with quadrupole constant ***k*** end effective length ***l***:

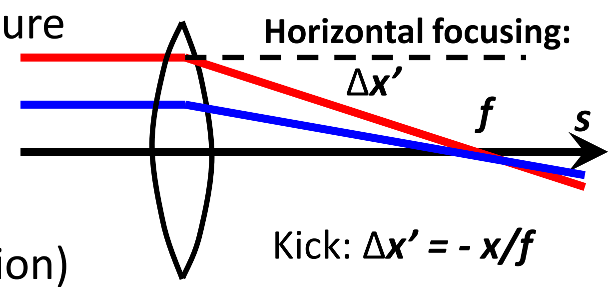
$$\mathbf{R}_{\text{focus}} = \begin{pmatrix} \cos\sqrt{k}\,l & \frac{1}{\sqrt{k}}\sin\sqrt{k}\,l \\ -\sqrt{k}\cdot\sin\sqrt{k}\,l & \cos\sqrt{k}\,l \end{pmatrix} \Rightarrow \mathrm{R}^{\text{thin lens}}_{\text{focus}} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

- Horizontal <u>**de**</u>**-focusing** with quadrupole constant ***k*** end effective length ***l***:

$$\mathbf{R}_{\text{de-focus}} = \begin{pmatrix} \cosh\sqrt{k}\,l & \frac{1}{\sqrt{k}}\sinh\sqrt{k}\,l \\ \sqrt{k}\cdot\sinh\sqrt{k}\,l & \cosh\sqrt{k}\,l \end{pmatrix} \Rightarrow \mathrm{R}^{\text{thin lens}}_{\text{de-focus}} = \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}$$

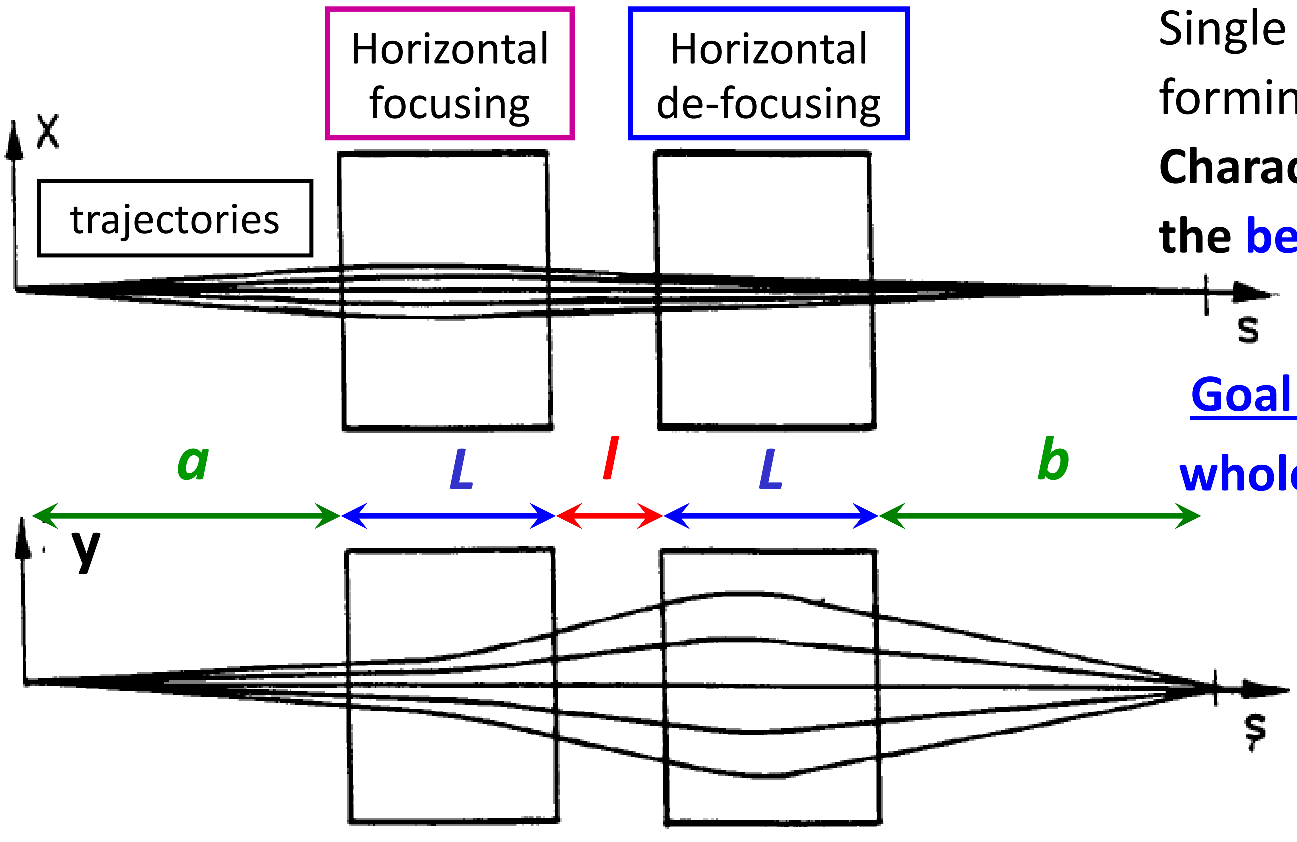
Ideal quad.: field gradient $g = B_{pole}/a$, B_{pole} field at poles, ***a*** aperture

$\rightarrow$ quadrupole constant $k = |g| / (B\rho)_0$



Thin lens approximation: $l \rightarrow 0 \;\Rightarrow\; kl \rightarrow \text{const} \;\Rightarrow\; kl \equiv 1/f$

$\Rightarrow$ simple transfer matrix (math. proof by 1st order Taylor expansion)



Single particle trajectories forming a beam

Characteristic quantity is the beam envelope

Goal: Description of whole ensemble behavior!

Transfer matrix:

$$R_{channel} = R_{drift}(b) \cdot R_{de\text{-}focus}(kL) \cdot R_{drift}(l) \cdot R_{focus}(kL) \cdot R_{drift}(a)$$

Plot: Rossbach, Schmüser

The basic 6 dimensional vextor is:

$$\vec{x} = \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{pmatrix} = \begin{pmatrix} \text{hori. spatial deviation} \\ \text{horizontal divergence} \\ \text{vert. spatial deviation} \\ \text{vertical divergence} \\ \text{long. deviation} \\ \text{momentum deviation} \end{pmatrix} = \begin{pmatrix} [\text{mm}] \\ [\text{mrad}] \\ [\text{mm}] \\ [\text{mrad}] \\ [\text{mm}] \\ [10^{-3}] \end{pmatrix}$$

Transformation of a single particle from s_0 to s_1 is given by the Transfer Matrix R:

$$\vec{x}(s_1) = \mathbf{R}(s_0 \rightarrow s_1) \cdot \vec{x}(s_0)$$

Transformation of a the envelope from s_0 to s_1 is given by the Beam Matrix σ:

$$\boldsymbol{\sigma}(s_1) = \mathbf{R}(s_0 \rightarrow s_1) \cdot \boldsymbol{\sigma}(s_0) \cdot \mathbf{R}^{\mathrm{T}}(s_0 \rightarrow s_1)$$

6-dim Beam Matrix with *decoupled* hor. & vert. plane:

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 & 0 & \sigma_{15} & \sigma_{16} \\ \sigma_{12} & \sigma_{22} & 0 & 0 & \sigma_{25} & \sigma_{26} \\ 0 & 0 & \sigma_{33} & \sigma_{34} & 0 & 0 \\ 0 & 0 & \sigma_{34} & \sigma_{44} & 0 & 0 \\ \sigma_{15} & \sigma_{25} & 0 & 0 & \sigma_{55} & \sigma_{56} \\ \sigma_{16} & \sigma_{26} & 0 & 0 & \sigma_{56} & \sigma_{66} \end{pmatrix}$$

horizontal
vertical
longitudinal
hor.-long. coupling
→ 10 values
all couplings
→ 21 values

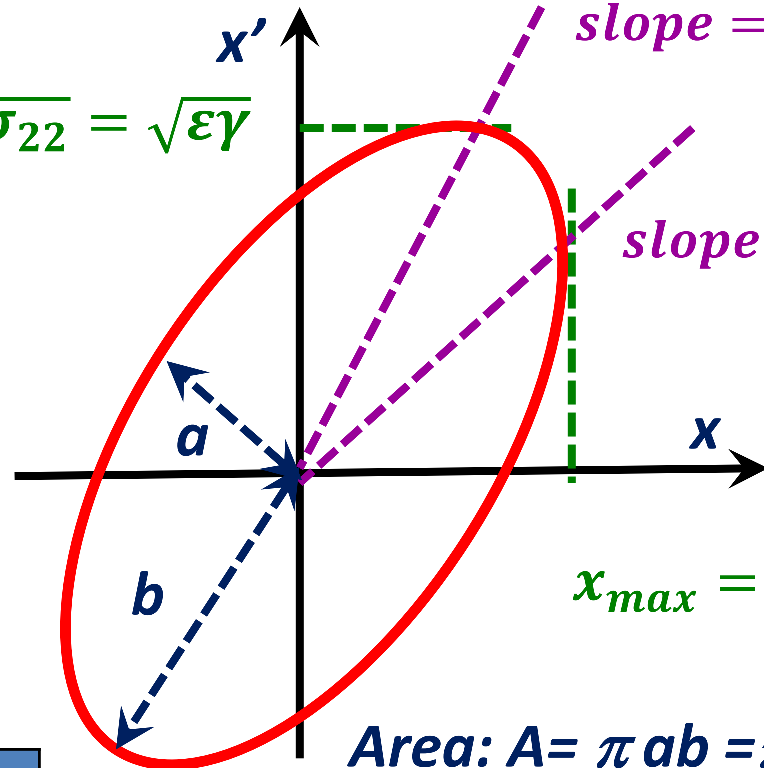
Horizontal beam matrix:

$$\sigma_{11} = \langle x^2 \rangle$$
$$\sigma_{12} = \langle x\, x' \rangle$$
$$\sigma_{22} = \langle x'^2 \rangle$$

Beam width for the three coordinates:

$$x_{rms} = \sqrt{\sigma_{11}}$$
$$y_{rms} = \sqrt{\sigma_{33}}$$
$$l_{rms} = \sqrt{\sigma_{55}}$$

The phase space distribution is described by an ellipse with the characteristics parameters



Corresponding parameters:

Beam Matrix	Statistic	Twiss Parameter
σ_{11}	$\langle x^2 \rangle$	$\varepsilon\beta$
σ_{22}	$\langle x'^2 \rangle$	$\varepsilon\gamma$
σ_{12}	$\langle xx' \rangle$	$-\varepsilon\alpha$

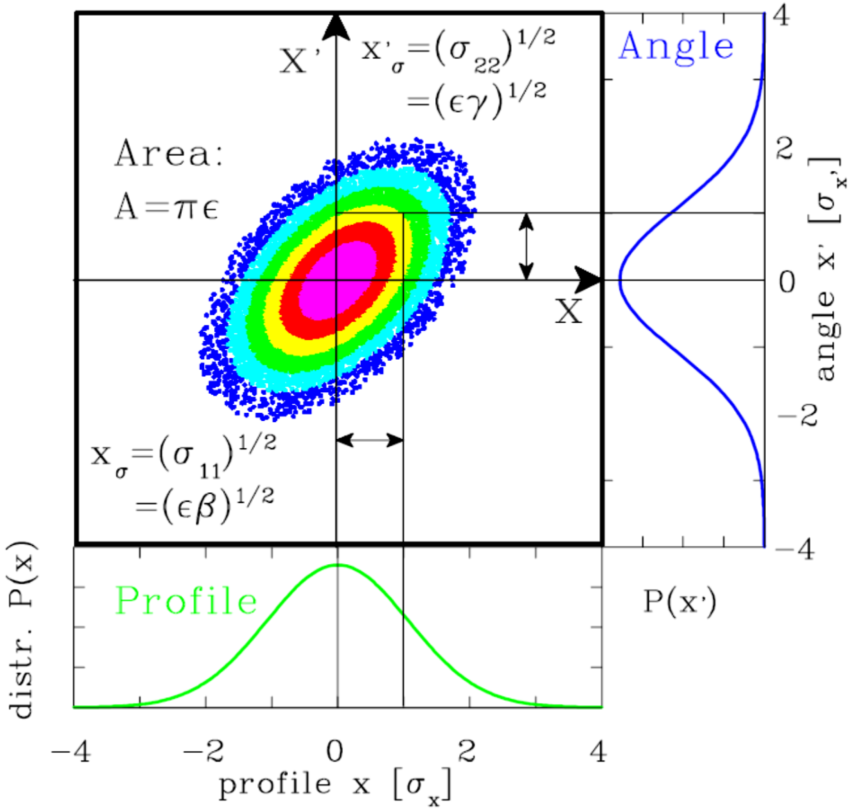
For quadratic matrices **A** and **B**:
$\det(\mathbf{A} \cdot \mathbf{B}) = \det(\mathbf{A}) \cdot \det(\mathbf{B})$

The determinate is preserved:

$$\varepsilon = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2} \Leftrightarrow 1 = \beta\gamma - \alpha^2$$

Proof: $\boldsymbol{\sigma}(s_1) = \mathbf{R} \cdot \sigma(s_0) \cdot \mathbf{R}^T$

$$\Rightarrow \det(\boldsymbol{\sigma}(s_1)) = \det(\mathbf{R}) \cdot \det(\boldsymbol{\sigma}(s_0)) \cdot \det(\mathbf{R}^T)$$
$$= \det(\boldsymbol{\sigma}(s_0))$$



Gaussian distribution for an ensemble of particles

The phase space distribution is

described by an ellipse

with the characteristics parameters

Corresponding parameters:

Beam Matrix	Statistic	Twiss Parameter
σ_{11}	$\langle x^2 \rangle$	$\varepsilon\beta$
σ_{22}	$\langle x'^2 \rangle$	$\varepsilon\gamma$
σ_{12}	$\langle xx' \rangle$	$-\varepsilon\alpha$

The determinate is preserved:

$$\varepsilon = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2} \Leftrightarrow 1 = \beta\gamma - \alpha^2$$

The beam distribution can be non-Gaussian, e.g. at:

- beams behind ion source
- space charged dominated beams at LINAC & synchrotron
- cooled beams in storage rings

General description of emittance

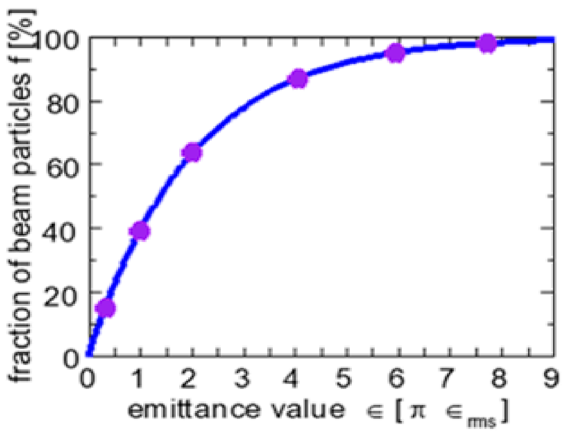
using terms of 2-dim distribution:

$$\varepsilon_{rms} = \sqrt{\underbrace{\langle x^2 \rangle \langle x'^2 \rangle}_{\text{Variances}} - \underbrace{\langle xx' \rangle^2}_{\text{Covariance i.e. correlation}}}$$

It describes the value for 1 standard derivation

For Gaussian beams only: ε_{rms} ↔ interpreted as area containing a fraction ***f*** of ions:

$$\varepsilon(f) = -2\pi\varepsilon_{rms} \cdot \ln(1-f)$$



Emittance ε(f)	Fraction f
$1 \cdot \varepsilon_{rms}$	15 %
$\pi \cdot \varepsilon_{rms}$	39 %
$2\pi \cdot \varepsilon_{rms}$	63 %
$4\pi \cdot \varepsilon_{rms}$	86 %
$8\pi \cdot \varepsilon_{rms}$	98 %

Care:
No common definition of emittance concerning the fraction ***f***

Poll concerning linear Transformations

Poll 4.1:

Which optical elements are **non-**linear?

1) Dipole
2) Quadrupole
3) Sextupole

Poll 4.2:

What is **wrong** concerning linear transformations?

1) The trajectories can be calculated by a matrix.
2) The coordinate vector is $\vec{x} = (x, x', y, y', l, \delta)^T$
3) All elements within a typical accelerator can be described by linear transformations.
4) The emittance is preserved.

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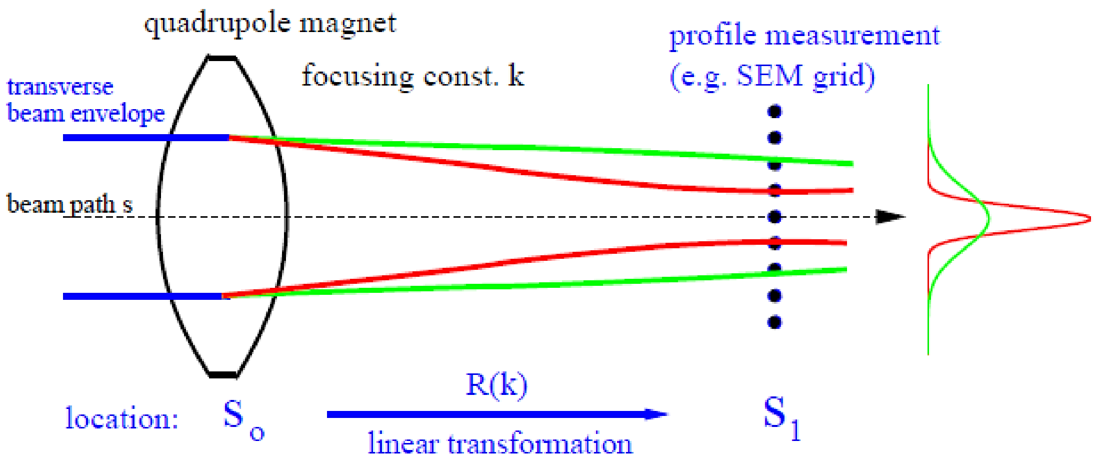
Outline:

- Definition and some properties of transverse emittance
- **Quadrupole strength variation and position measurement**

  emittance from several profile measurement and beam optical calculation
- **Slit-Grid device: scanning method**
- **Summary**

Emittance Measurement by Quadrupole Variation

From a profile determination, the emittance can be calculated via linear transformation, if a well known and constant distribution (e.g. Gaussian) is assumed.
The basis is beam matrix transformation $\boldsymbol{\sigma}\,(s_1) = \mathbf{R}(s_0 \to s_1) \cdot \boldsymbol{\sigma}(s_0) \cdot \mathbf{R}^{\mathrm{T}}(s_0 \to s_1)$



- Measurement of beam width

  $x^2_{max} = \sigma_{11}(1, k)$

  matrix **R(*k*)** describes the focusing.

- With the drift matrix the transfer is

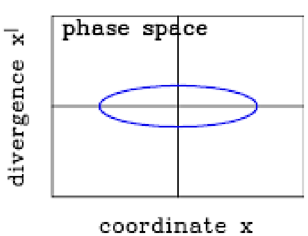
  $\mathbf{R}(k_i) = \mathbf{R}_{drift} \cdot \mathbf{R}_{focus}(k_i)$

- Transformation of the beam matrix

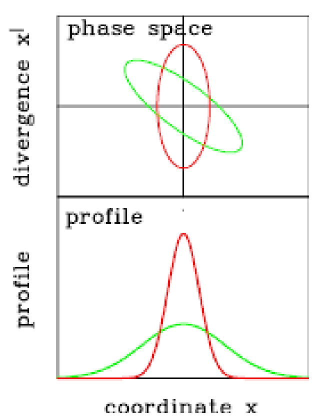
  $\sigma(1,k_i) = \mathbf{R}(k_i) \cdot \sigma(0) \cdot \mathbf{R}^{\mathrm{T}}(k_i)$

Task: Calculation of matrix $\sigma(0)$

at entrance s_0 i.e. all three elements



beam matrix:
(Twiss parameters)
$\sigma_{11}(0)$, $\sigma_{12}(0)$, $\sigma_{22}(0)$
to be determined



measurement:
$x^2(k) = \sigma_{11}(1,k)$

Measurement of transverse Emittance

- The beam width $x_{max}(s_1)$ at s_1 is measured ⇔ matrix element $\sigma_{11}(1, k_i) = x^2_{max}(k_i)$
- Different focusing of quadrupoles $k_1, k_2 \dots k_n$ are used ⇒ $R_{focus}(k_i)$
- After the drift the transfer matrix is $R(k_i) = R_{drift} \cdot R_{focus}(k_i)$
- **Task: Calculation of beam matrix $\sigma(0)$ at entrance s_0 (matrix elements give orientation)**
- **The transformation of the beam matrix is: $\sigma(1,k_i) = R(k_i) \cdot \sigma(0) \cdot R^T(k_i)$**

⇒ **Result: Redundant system of linear equations for matrix elements $\sigma_{ij}(0)$**

$$\sigma_{11}(1, k_1) = R^2_{11}(k_1) \cdot \sigma_{11}(0) + 2\, R_{11}(k_1)\, R_{12}(k_1) \cdot \sigma_{12}(0) + R^2_{12}(k_1) \cdot \sigma_{22}(0) \quad \text{focusing } k_1$$

...

$$\sigma_{11}(1, k_n) = R^2_{11}(k_n) \cdot \sigma_{11}(0) + 2\, R_{11}(k_n)\, R_{12}(k_n) \cdot \sigma_{12}(0) + R^2_{12}(k_n) \cdot \sigma_{22}(0) \quad \text{focusing } k_n$$

- To have an error estimation at least three measurements must be done

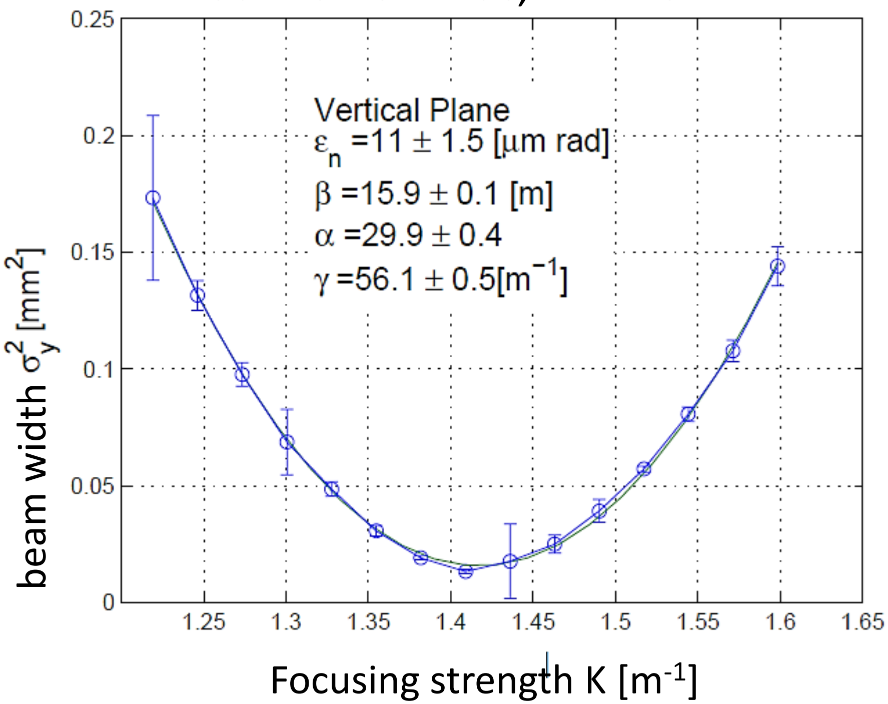
Assumptions:
- Constant emittance, in particular no space-charge broadening
- Only elliptical shaped beam distribution is considered
- No misalignment, i.e. beam center equals center of the quadrupoles
- If **not** valid: A self-consistent algorithm can be used.

Using the 'thin lens approximation' i.e. the quadrupole has a focal length of ***f***:

$$\mathrm{R}_{focus}(K) = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ K & 1 \end{pmatrix} \Rightarrow \mathrm{R}(L,K) = \mathrm{R}_{drift}(L)\cdot \mathrm{R}_{focus}(K) = \begin{pmatrix} 1+LK & L \\ K & 1 \end{pmatrix}$$

Measurement of the matrix-element $\sigma_{11}(1,K)$ from $\sigma(1,K) = R(K) \cdot \sigma(0) \cdot R^T(K)$

Example: Square of the beam width at ELETTRA 100 MeV e⁻ Linac, YAG:Ce:



G. Penco (ELETTRA) et al., EPAC'08

For completeness: The relevant formulas

$$\begin{aligned}\sigma_{11}(1,K) = {}& L^2\sigma_{11}(0)\cdot K^2 \\ & + 2\cdot(\ L\sigma_{11}(0) + L^2\sigma_{12}(0)\ )\cdot K \\ & + L^2\sigma_{22}(0) + \sigma_{11}(0) \\ \equiv {}& a\cdot K^2 - 2ab\cdot K + ab^2 + c \\ = {}& a\cdot(K-b)^2 + c\end{aligned}$$

The three matrix elements at the quadrupole:

$$\sigma_{11}(0) = \frac{a}{L^2}$$

$$\sigma_{12}(0) = -\frac{a}{L^2}\left(\frac{1}{L} + b\right)$$

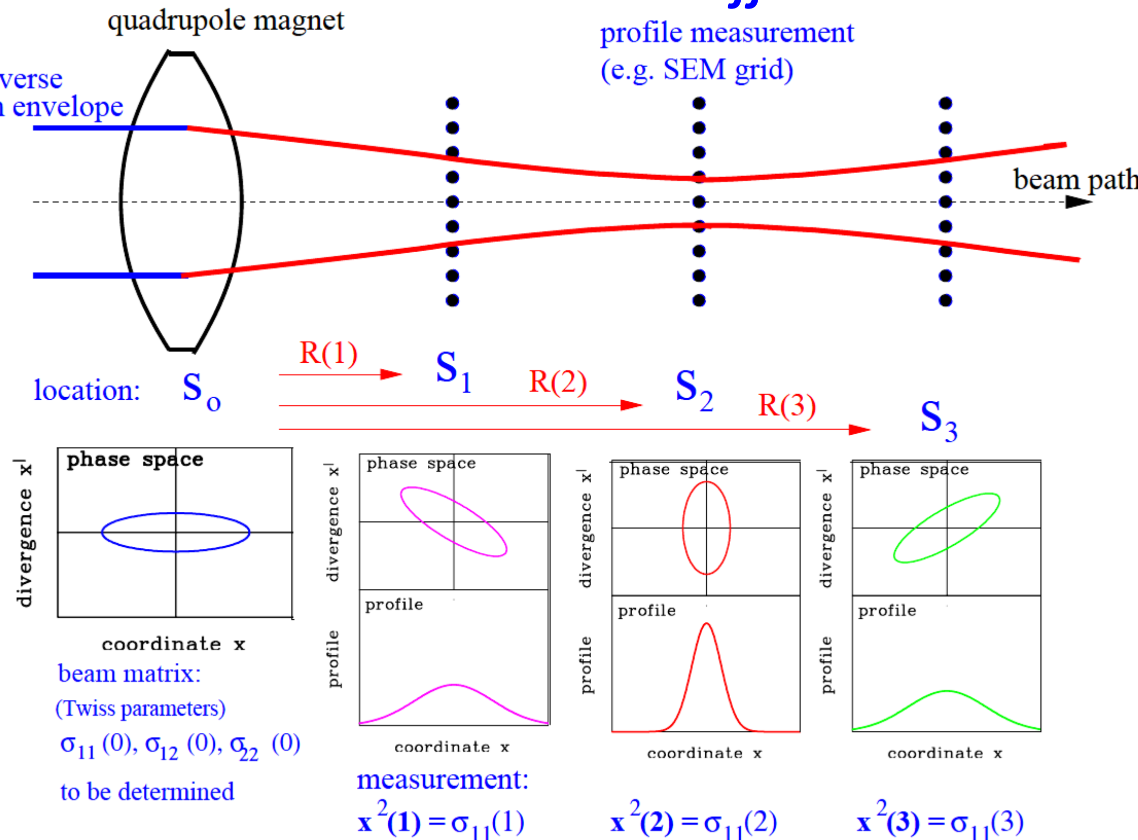
$$\sigma_{22}(0) = \frac{1}{L^2}\left(ab^2 + c + \frac{2ab}{L} + \frac{a}{L^2}\right)$$

$$\varepsilon_{rms} \equiv \sqrt{\det \sigma(0)} = \sqrt{\sigma_{11}(0)\cdot\sigma_{22}(0) - \sigma_{12}^2(0)} = \sqrt{ac}\,/\,L^2$$

Instead of quadrupole variation, the beam width is measured at ***different*** locations:

The procedure is:

- Beam width ***x(i)*** measured at the locations s_i
  $\Rightarrow$ beam matrix element $x^2(i) = \sigma_{11}(i)$.
- The transfer matrix **R(*i*)** is known. (without dipole a 3 × 3 matrix.)
- The transformations are:
  $\sigma(i) = R(i) \cdot \sigma(0) \cdot R^T(i)$
  $\Rightarrow$ redundant equations:



$\Rightarrow$ **Result: at least equations for elements $\sigma_{ij}(0)$**

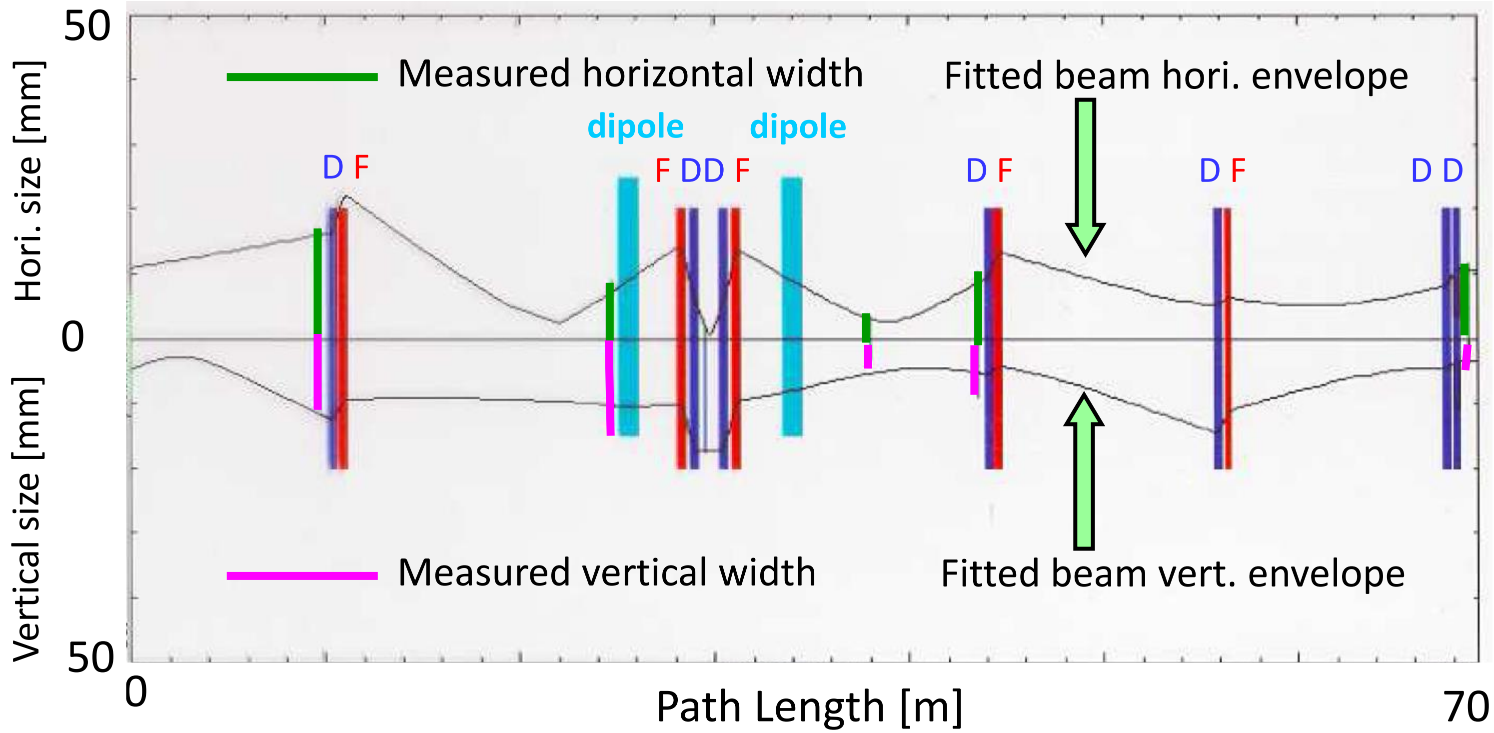
$$\sigma_{11}(1) = R_{11}^2(1) \cdot \sigma_{11}(0) + 2\, R_{11}(1)\, R_{12}(1) \cdot \sigma_{12}(0) + R_{12}^2(1) \cdot \sigma_{22}(0) \quad \text{for } R(1)\text{: } s_0 \rightarrow s_1$$

...

$$\sigma_{11}(n) = R_{11}^2(n) \cdot \sigma_{11}(0) + 2\, R_{11}(n)\, R_{12}(n) \cdot \sigma_{12}(0) + R_{12}^2(n) \cdot \sigma_{22}(0) \quad \text{for } R(n)\text{: } s_0 \rightarrow s_n$$

Solution: Solving the linear equations like for quadrupole variation or fitting the profiles with linear optics code (e.g. MADX).

Example: The hor. and vert. beam envelope and the beam width at a transfer line:



Assumptions:
- constant emittance, in particular no space-charge broadening
- 100 % transmission i.e. no loss due to vacuum pipe scraping
- no misalignment, i.e. beam center equals center of the quadrupoles.

Poll 4.3:

What is **correct** concerning the emittance measurement by linear transformations?

1) Only the 2nd statistical moments are determined to characterize the beam distribution.
2) Any beam distribution can be reliably determined by this methods.
3) The transfer line can contain any optical element.
4) In all cases, three measurements are sufficient for an emittance determination.

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  emittance from several profile measurement and beam optical calculation
- **Slit-Grid device: scanning method**

  scanning slit → beam position & grid → angular distribution
- **Summary**

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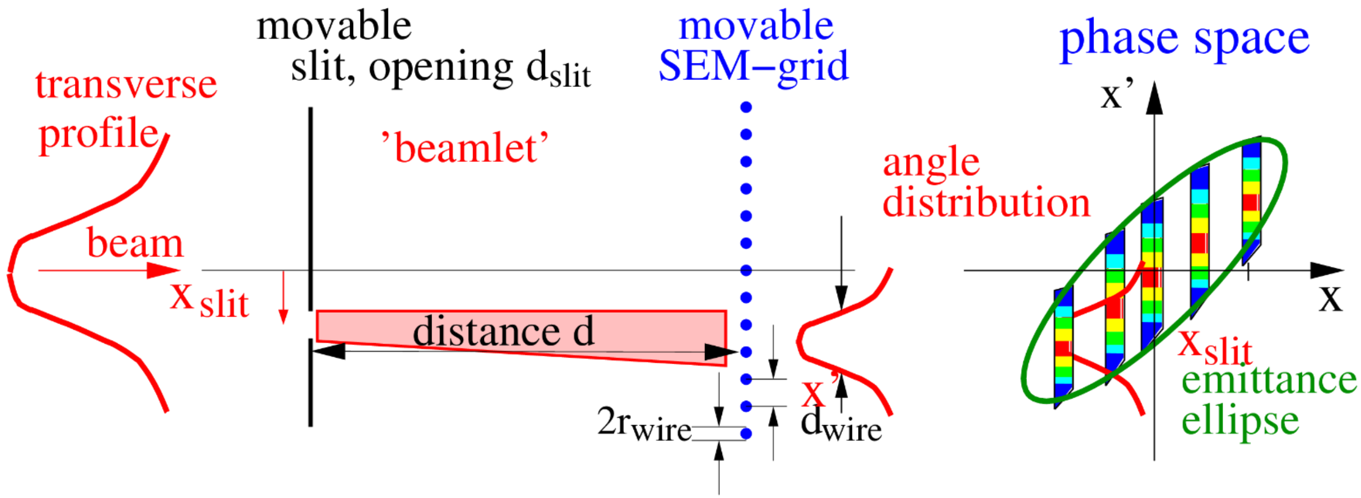
The Slit-Grid Measurement Device

Slit-Grid: Direct determination of position and angle distribution.

Used for protons with E_{kin} < 100 MeV/u $\Rightarrow$ range R < 1 cm.

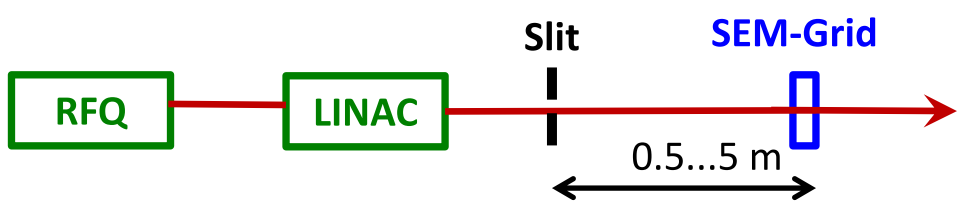
Hardware **Analysis**



Slit: position ***P(x)*** with typical width: 0.1 to 0.5 mm

Distance: typ. 0.5 to 5 m (depending on beam energy 0.1 ... 100 MeV)

SEM-Grid: angle distribution ***P(x')***



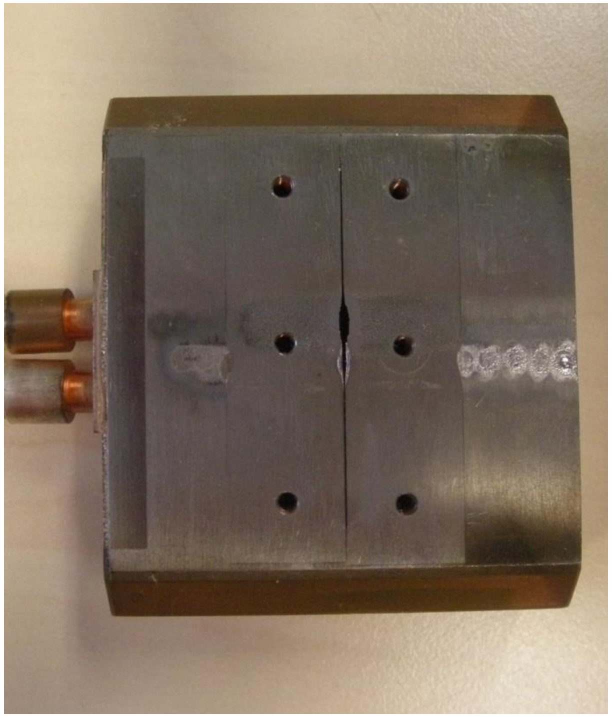
Slit & SEM-Grid

Slit with e.g. 0.1 mm thickness

$\rightarrow$ Transmission only from ***Δx***.

Example: Slit of width 0.1 mm (defect)

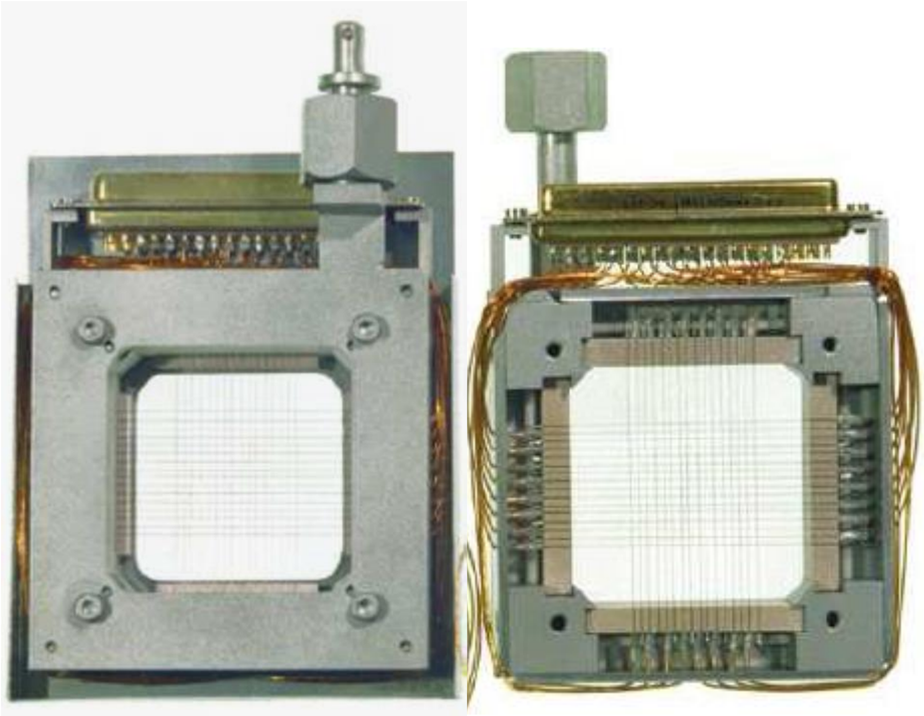
Moved by stepping motor, 0.1 mm resolution



Beam surface interaction: e^- emission

$\rightarrow$ measurement of current.

Example: 15 wire spaced by 1.5 mm:



Each wire is equipped with one I/U converter

different ranges settings by $\boldsymbol{R_i}$

$\rightarrow$ very large dynamic range up to 10^6.

The distribution of the ions is depicted as a function of

- Position [mm]
- Angle [mrad]

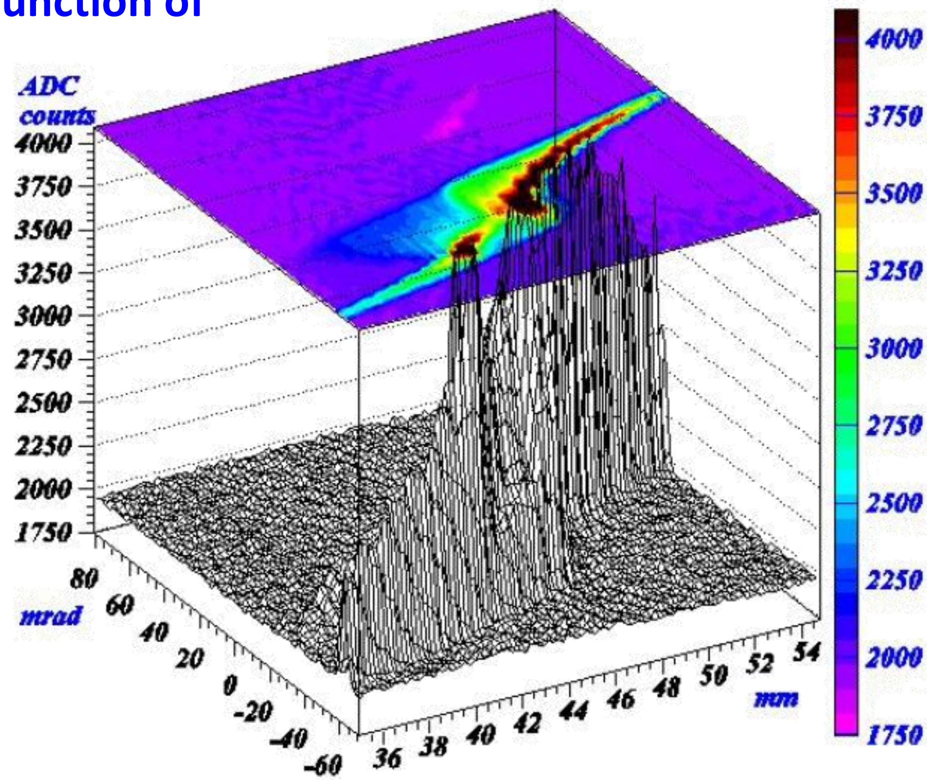
The distribution can be visualized by

- Mountain plot
- Contour plot

Calc. of 2nd moments $\langle x^2 \rangle$, $\langle x'^2 \rangle$ & $\langle xx' \rangle$

Emittance value ε_{rms} from

$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$



Problems:

- Finite **binning** results in limited resolution
- **Background** $\rightarrow$ large influence on $\langle x^2 \rangle$, $\langle x'^2 \rangle$ and $\langle xx' \rangle$

Or fit of distribution i.e. ellipse to data

$\Rightarrow$ **Effective emittance only**

Modern alternative: Tomographic reconstruction

Beam: Ar^{4+}, 60 KeV, 15 µA

at Spiral2 Phoenix ECR source.

Plot from P. Ausset, DIPAC 2009

The Resolution of a Slit-Grid Device

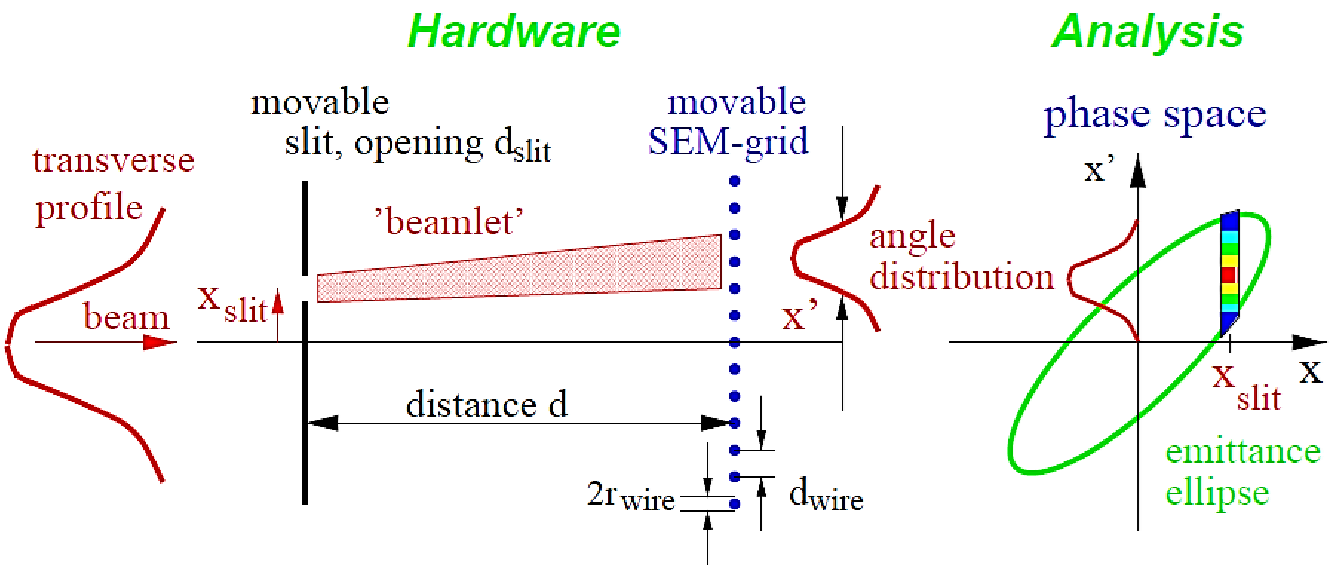
The width of the slit d_{slit} gives the resolution in space $\Delta x = d_{slit}$.

The angle resolution is $\Delta x' = (d_{wire} + 2r_{wire})/d$

$\Rightarrow$ discretization element $\Delta x \cdot \Delta x'$.

By scanning the SEM-grid the angle resolution can be improved.

Problems for small beam sizes or parallel beams.



For pulsed LINACs: Only one measurement each pulse $\rightarrow$ long measuring time required.

Poll concerning normalized Emittance

Poll 4.4:

A typical slit-grid emittance installation can be applied for …

1) **all beams** and all energies
2) **electron** beams for all energies
3) **proton** beam energies **below** 100 MeV only
4) **proton** beam energies **above** 100 MeV only

Poll 4.5:

Assuming an electron and proton beam of 10 MeV (starting from the same source). What do expect concerning the emittance of both beams?

1) The electron beam has a **smaller** emittance than the proton beam.
2) The electron beam has a **larger** emittance than the proton beam.
3) Both beams have the **same** emittance as it relates to the beam energy.

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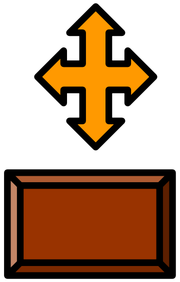
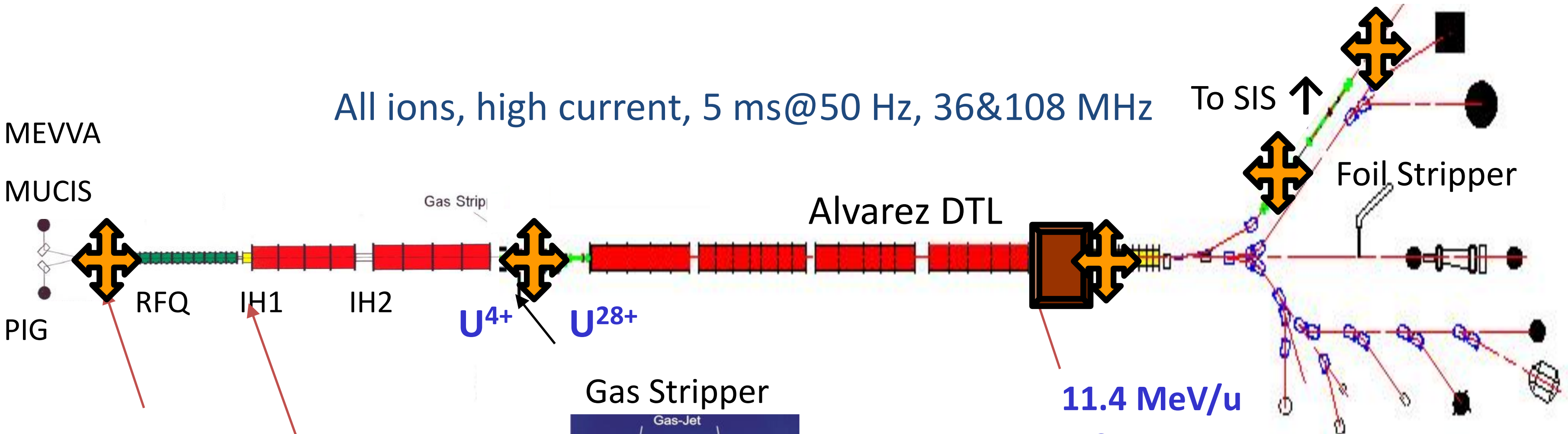
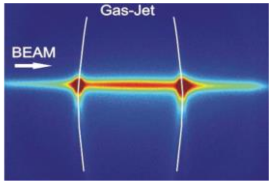


Slit Grid Emittance: Standard device, total 9 device

Pepper-pot Emittance: Special device, total 1 device



Gas Stripper



1.4 MeV/u ⇔ β = 0.054

Constructed in the 70th, Upgrade 1999, further upgrades in preparation

Remark: For higher energies where ions cannot be stopped, **quadrupole variation** is used.

→ Conclusion

Poll concerning normalized Emittance

Poll 4.6:

In a synchrotron no special emittance measurement for a **stable circulating** beam is discussed. What is the reason?

1) The emittance has **no** practical meaning.
2) The emittance of a circulating beam is **always** constant.
3) For emittance measurement, the beam **must be** extracted from the synchrotron and methods in transfer lines must be applied
4) For a stable circulating beam, the emittance is directly **linked** to the transverse profile $x = \sqrt{\varepsilon\beta}$ as related to a relation between Twiss parameter α, β and γ at one location.

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Summary for transverse Emittance Measurement

Emittance measurements are very important for comparison to theory.

It includes size (value of $\boldsymbol{\varepsilon}$) and orientation in phase space ($\boldsymbol{\sigma_{ij}}$ or $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$)

i.e three independent values $\varepsilon_{rms} = \sqrt{\sigma_{11} \cdot \sigma_{22} - \sigma_{12}} = \sqrt{< x^2 > \cdot < x'^2 > - < xx' >^2}$

Low energy beams → direct measurement of x- and x'-distribution

- ***Slit-grid:*** movable slit → ***x***-profile, grid → ***x'***-profile
- Variances exists: slit-slit, slit-kick, pepperpot .... method

All beams → profile measurement + linear transformation:

- ***Quadrupole variation:*** one location, different setting of a quadrupole
- ***'Three grid method':*** different locations
- ***Assumptions:***
  - well aligned beam, no steering
  - no emittance blow-up due to space charge.

Important remark: For a synchrotron with a *stable beam storage*,

width measurement is sufficient using $x_{rms} = \sqrt{\varepsilon_{rms} \cdot \beta}$

Backup slides

Quiz related to Beam Emittance and Slit-Grid Method

- What is meant by beam quality = emittance, i.e.

  what type of quantitative description is used?
- What is the geometrical meaning of a phase space distribution for $-\varepsilon \cdot \alpha \equiv \sigma_{12} \equiv \langle xx' \rangle \neq 0$?
- Expectation: Is the emittance of a 10 MeV electron beam smaller or larger than a proton beam?
- Describe shortly the measurement principle of a slit-grid method!
- Do you know variances of the slit-grid method?
- Why is a slit-grid measurement of 100 MeV electron and proton beam not meaningful?

→ Back to quadrupole variation

Quiz related to Beam Emittance Measurement

- The emittance is defined via statistical parameters $<x^2>$, $<xx'>$, $<x'^2>$. What does this mean?
- What might causes the beam to be different from a Gaussian (i.e. simple) distribution?
- What is the principle of an emittance measurement for quadrupole variation?
- What is the principle for the 3 grid method?
- Can you imaging how a quadrupole variation or 3-grid method must be modified if the beam is 'non-Gaussian'?

The emittance characterizes the whole beam quality: $\varepsilon_x = \frac{1}{\pi}\int_A dx\,dx'$

Ansatz:

Beam matrix at one location: $\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} = \varepsilon \cdot \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$ with $\vec{x} = \begin{pmatrix} x \\ x' \end{pmatrix}$

It describes a 2-dim probability distr.

The value of emittance is:

$$\varepsilon_x = \sqrt{\det \boldsymbol{\sigma}} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}$$

For the profile and angular measurement:

$$x_\sigma = \sqrt{\sigma_{11}} = \sqrt{\varepsilon\beta} \quad \text{and}$$

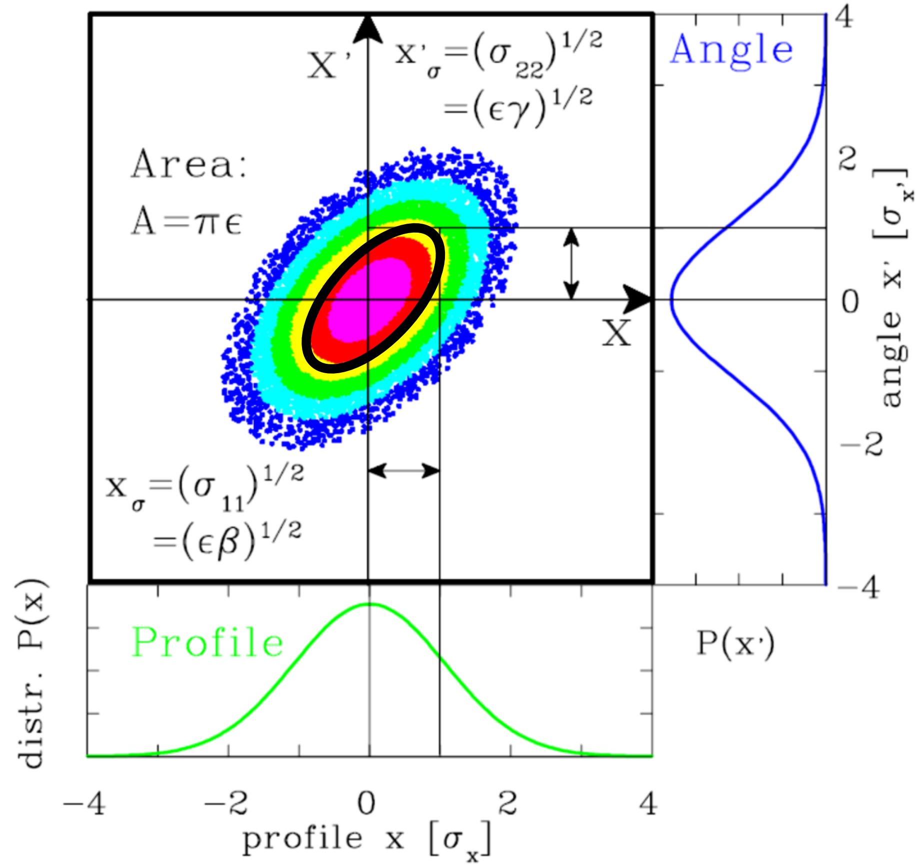
$$x'_\sigma = \sqrt{\sigma_{22}} = \sqrt{\varepsilon\gamma}$$

Geometrical interpretation:

All points $\boldsymbol{x}$ fulfilling $\boldsymbol{x}^t \cdot \boldsymbol{\sigma}^{-1} \cdot \boldsymbol{x} = 1$

are located on a **ellipse**

$$\sigma_{22}x^2 - 2\sigma_{12}xx' + \sigma_{11}x'^2 = \det \boldsymbol{\sigma} = \varepsilon_x^2$$



The Emittance for Gaussian Beams

The density function for a 2-dim Gaussian distribution is:

$$\rho(x,x') = \frac{1}{2\pi\epsilon}\exp\left[-\frac{1}{2}\ \vec{x}^T\ \sigma^{-1}\ \vec{x}\ \right]$$

$$= \frac{1}{2\pi\epsilon}\exp\left[\frac{-1}{2\det\sigma}\left(\sigma_{22}x^2 - 2\sigma_{12}xx' + \sigma_{11}x'^2\right)\right]$$

It describes an ellipse with the characteristics

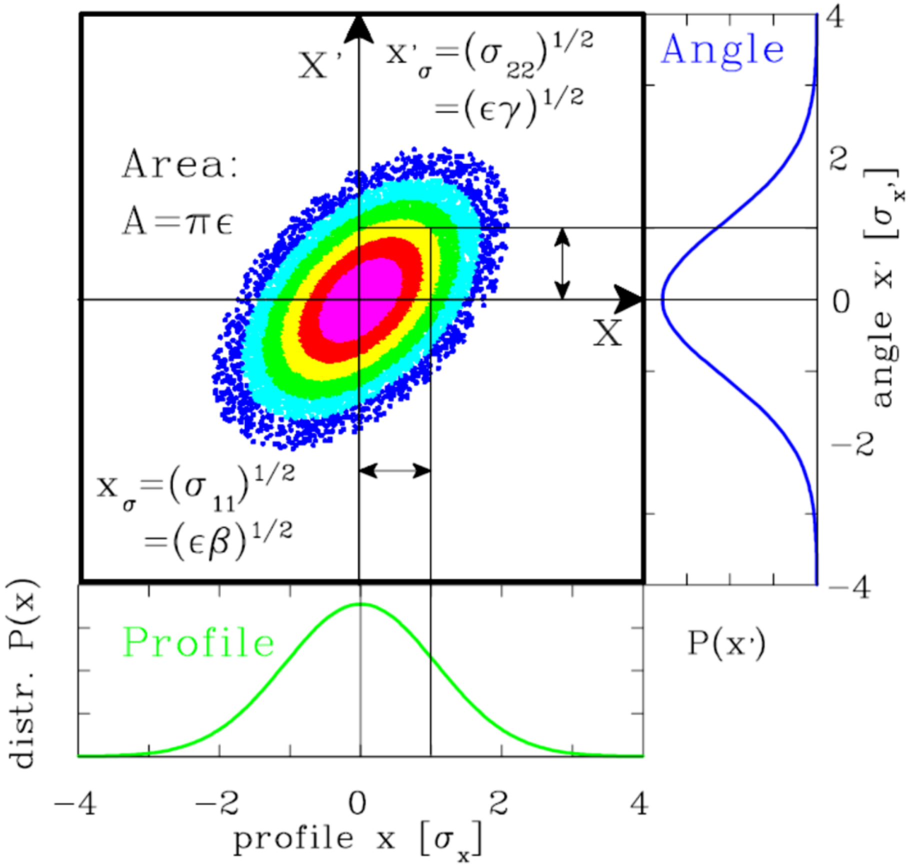
profile and angle Gaussian distribution of width

$$x_\sigma \equiv \sqrt{\langle x^2 \rangle} = \sqrt{\sigma_{11}} \quad \text{and}$$

$$x'_\sigma \equiv \sqrt{\langle x'^2 \rangle} = \sqrt{\sigma_{22}}$$

and the correlation or covariance

$$\text{cov} \equiv \sqrt{\langle xx' \rangle} = \sqrt{\sigma_{12}}$$



For $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ it is $\mathbf{A}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

assuming $\det(\mathbf{A}) = ad\text{-}bc \neq 0 \Leftrightarrow$ matrix invertible

The Emittance for Gaussian and non-Gaussian Beams

The beam distribution can be non-Gaussian, e.g. at:

- beams behind ion source
- space charged dominated beams at LINAC & synchrotron
- cooled beams in storage rings

General description of emittance

using terms of 2-dim distribution:

$$\varepsilon_{rms} = \sqrt{\underbrace{\langle x^2 \rangle \langle x'^2 \rangle}_{\text{Variances}} - \underbrace{\langle xx' \rangle^2}_{\text{Covariance i.e. correlation}}}$$

Variances **Covariance i.e. correlation**

It describes the value for 1 standard derivation

$$\langle x \rangle \equiv \mu = \frac{\iint x \cdot \rho(x,x')\, dxdx'}{\iint \rho(x,x')\, dxdx'} \qquad \langle x' \rangle \equiv \mu' = \frac{\iint x' \cdot \rho(x,x')\, dxdx'}{\iint \rho(x,x')\, dxdx'}$$

$$\langle x^n \rangle = \frac{\iint (x-\mu)^n \cdot \rho(x,x')\, dxdx'}{\iint \rho(x,x')\, dxdx'} \qquad \langle x'^n \rangle = \frac{\iint (x'-\mu')^n \cdot \rho(x,x')\, dxdx'}{\iint \rho(x,x')\, dxdx'}$$

$$\text{covariance}: \ \langle xx' \rangle = \frac{\iint (x-\mu)(x'-\mu') \cdot \rho(x,x')\, dxdx'}{\iint \rho(x,x')\, dxdx'}$$

For discrete distribution:

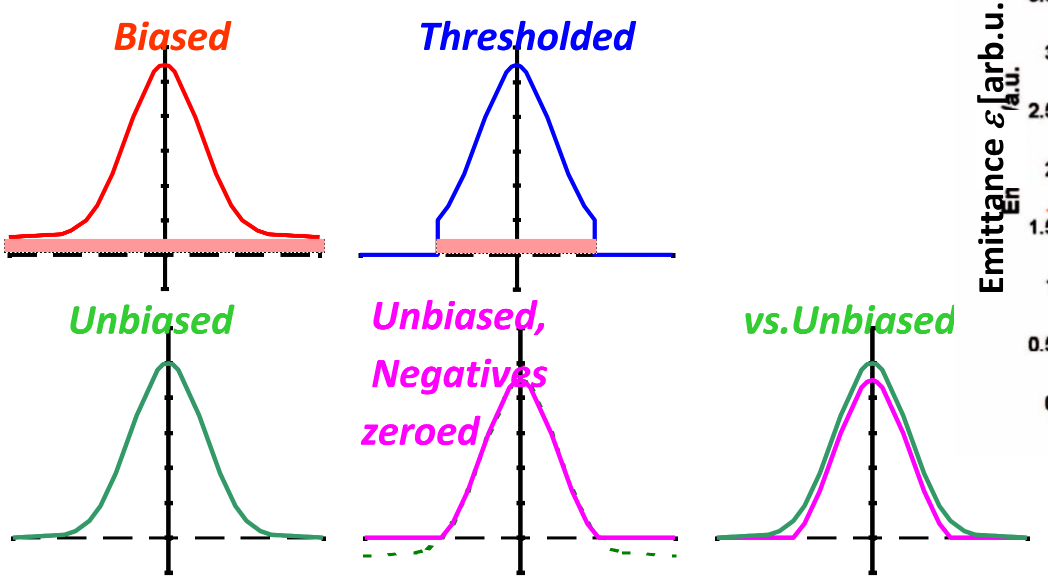
$$\langle x \rangle = \frac{\sum_{i,j} \rho(i,j) \cdot x_i x'_j}{\sum_{i,j} \rho(i,j)}$$

and correspondingly for all other moments

The Noise Influence for Emittance Determination

A real measurement of beamlets contains:

- **Noise i.e. fluctuation of the output**
- **Bias i.e. electrical offset from amplifier**

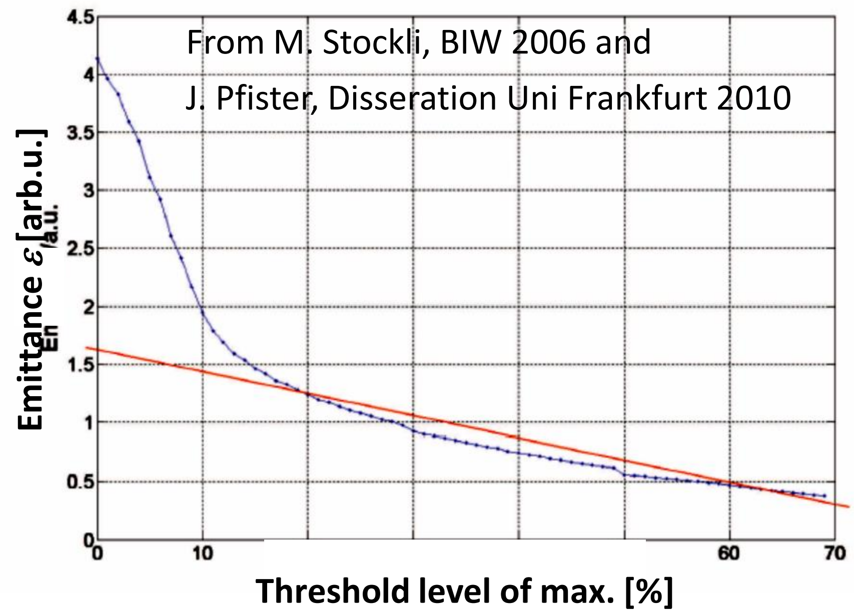


→ Strong influence of noise reduction to numerical values of ***<x>, <x'^{2}>*** and ***<xx'>*** and on $\boldsymbol{\varepsilon_{rms}}$

⇒Algorithm & cut-level must be given for evaluation

General: Typical error ***Δε/ε > 10 %***

Example: Dependence of $\boldsymbol{\varepsilon_{rms}}$ on threshold



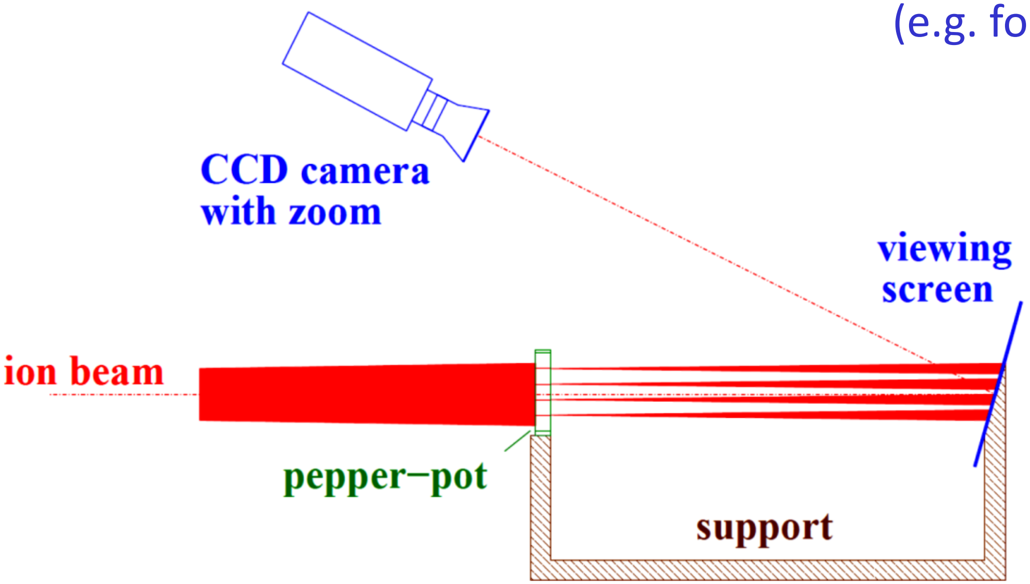
$$\langle x'^2 \rangle = \frac{\int x'^2 \cdot \rho(x,x')\,dx\,dx'}{\int \rho(x,x')\,dx\,dx'} \quad \text{for continous values}$$

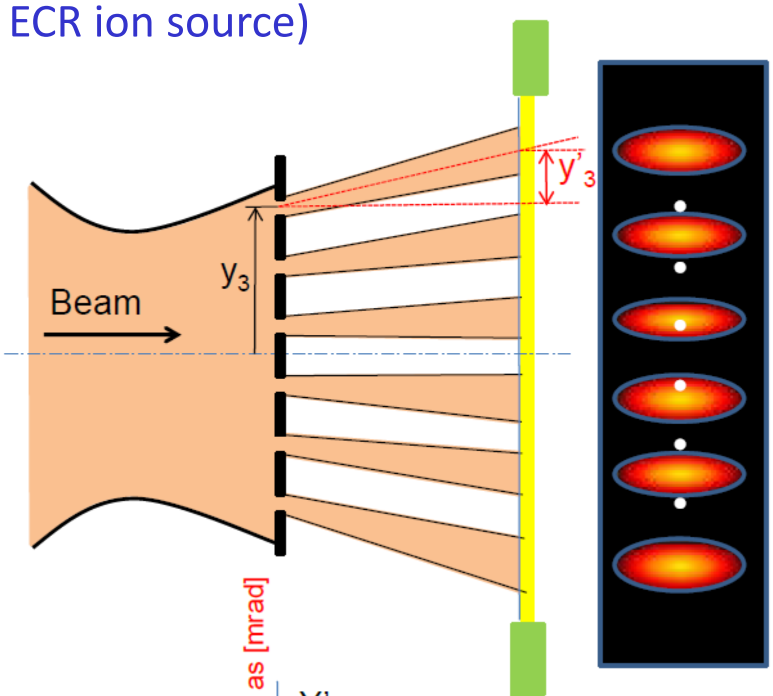
$$= \frac{\sum_{i,j} x'^{\,2}_{ij} \cdot P(x_{ij}, x'_{ij})}{\sum_{i,j} P(x_{ij}, x'_{ij})} \quad \text{for discrete values}$$

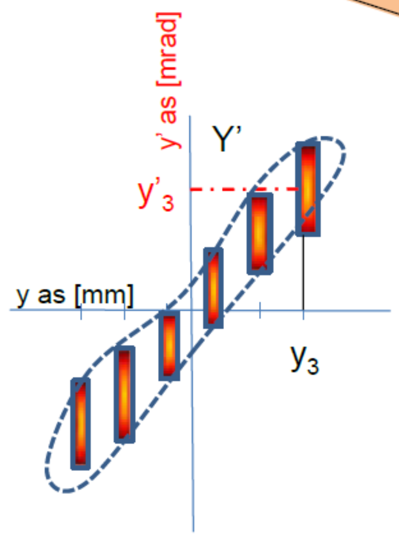
$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

The Pepperpot Emittance Device

- For pulsed LINAC: Measurement within one pulse is an advantage
- If horizontal and vertical direction coupled → 2-dim evaluation **required** (e.g. for ECR ion source)





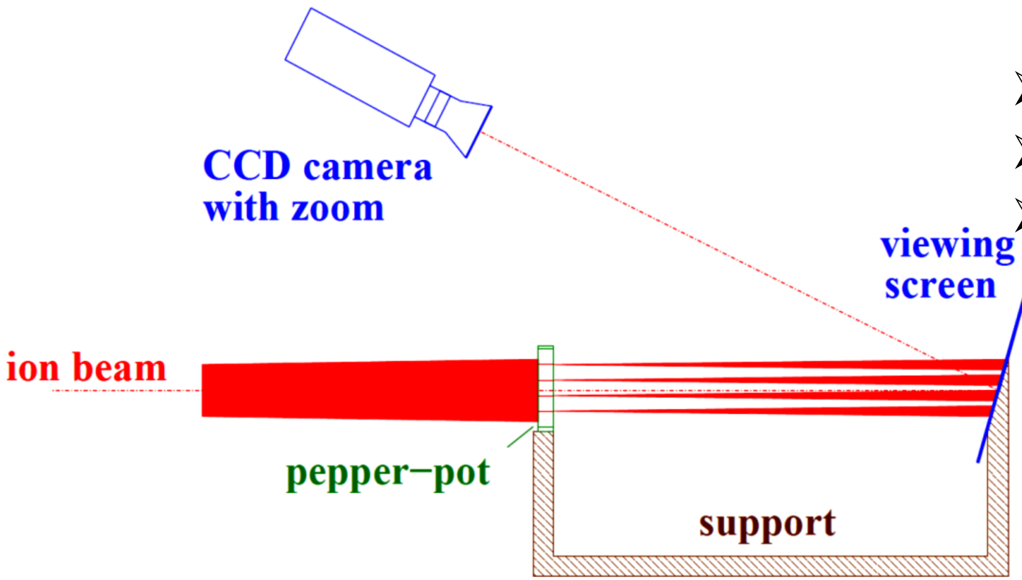


Good **spatial** resolution if many holes are illuminated.

Good **angle** resolution ***only*** if spots do not overlap.

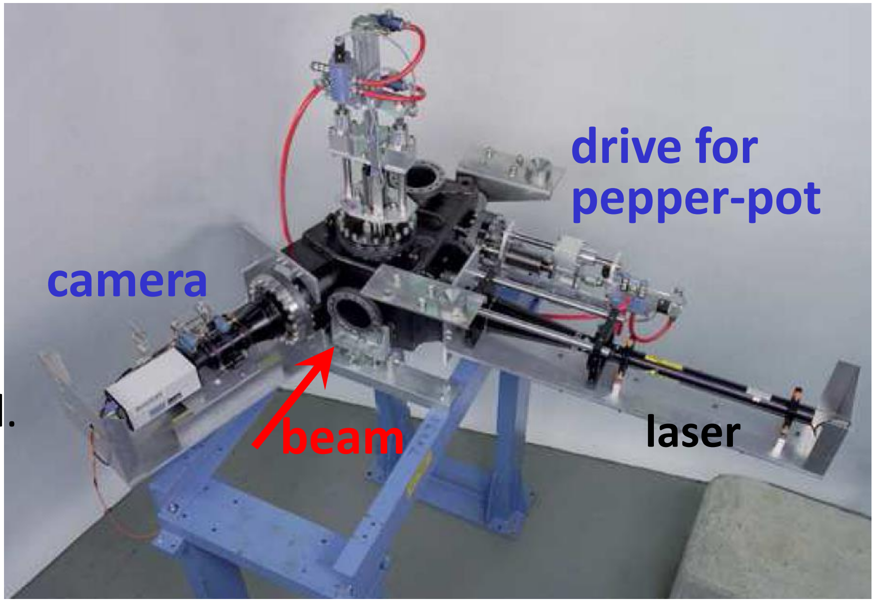
Partly from H.R. Kremers et al., ECRIS 2010

The Pepperpot Emittance Device at GSI UNILAC



Example GSI-LINAC 0.12 to 11 MeV/u:

- ***Pepper-pot***: 15 × 15 holes with Ø 0.1mm on a 50 × 50 mm^2 copper plate
- ***Distance***: pepper-pot-screen: 25 cm
- ***Screen:*** Al_2O_3, Ø 50 mm
- ***Data acquisition***: high resolution camera

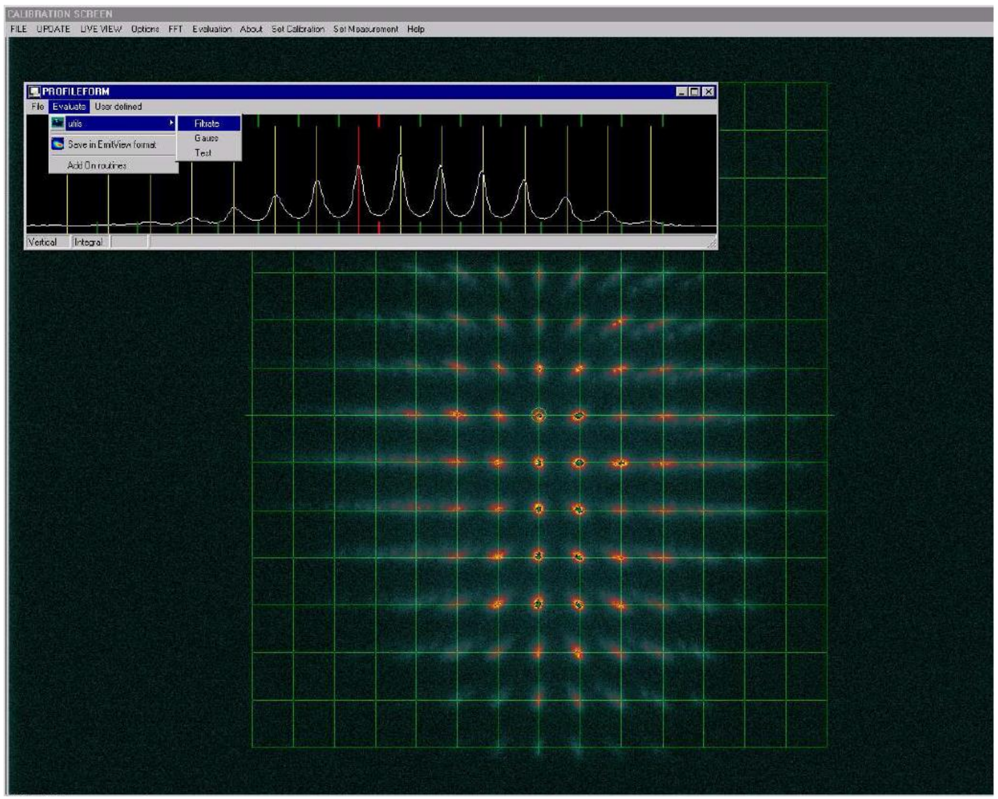


Good **spatial** resolution if many holes are illuminated.

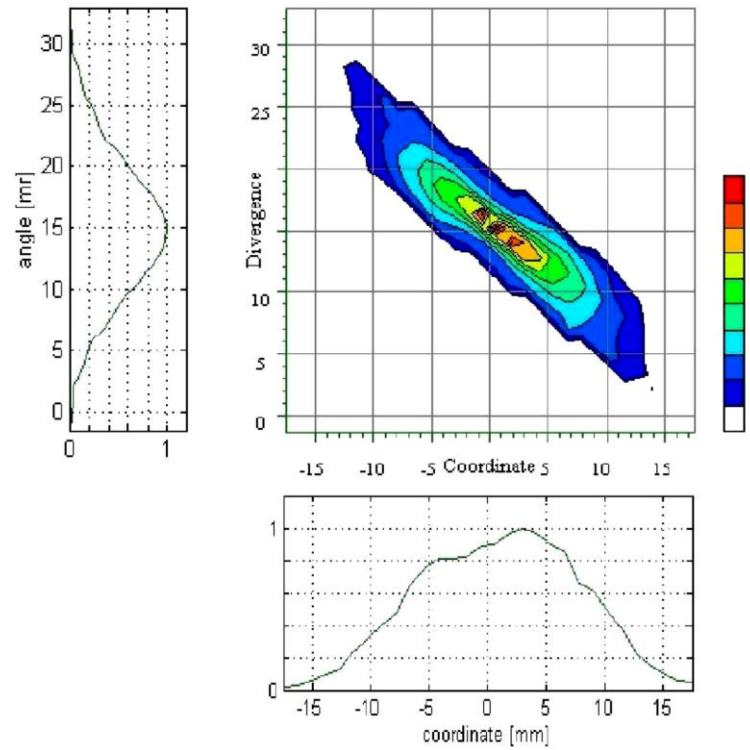
Good **angle** resolution ***only*** if spots do not overlap.

Result of a Pepperpot Emittance Measurement

Example: Ar $^{1+}$ ion beam at 1.4 MeV/u,
screen image from single shot at GSI:



Data analysis:
Projection on
horizontal and vertical plane
→ analog to slit-grid.



The Artist View of a Pepperpot Emittance Device