Longitudinal Parameter Measurement

JUAS 2025, ESI-Archamps at CERN

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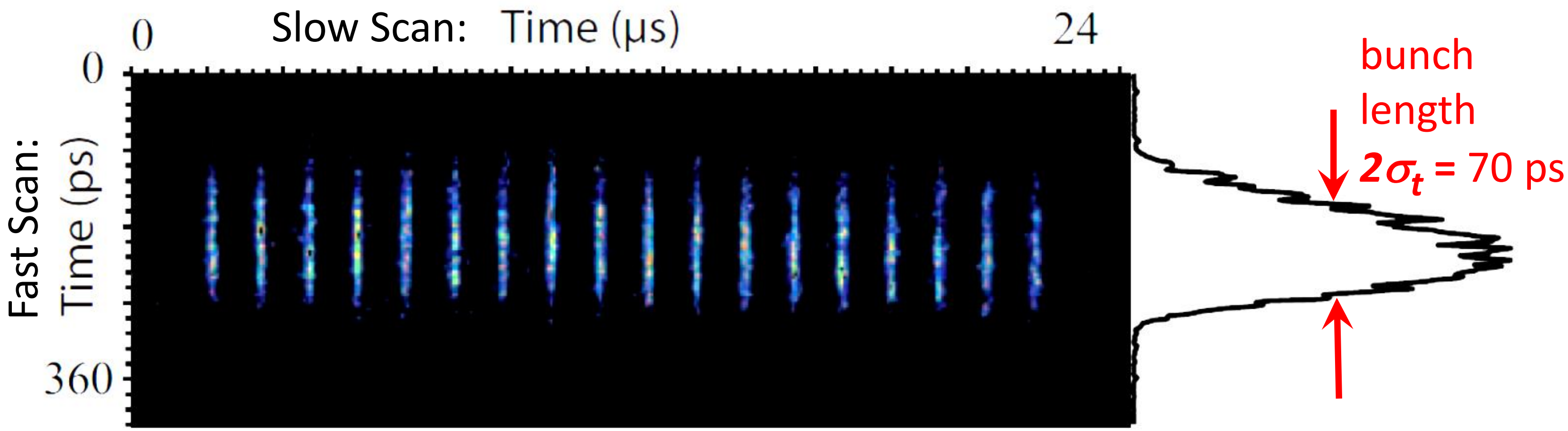


Measurement of longitudinal parameter:

- Definition of longitudinal phase space
- Proton LINAC: Determination of mean energy
- Determination of longitudinal emittance
- Bunch length measurement for non-relativistic beams
- Bunch length measurement for relativistic beams
- Bunch length from beam deflection by rf cavity
- Summary

Longitudinal ↔ transverse correspondences:

- position relative to rf ↔ transverse center-of-mass
- bunch structure in time ↔ transverse profile in horizontal and vertical direction
- momentum or energy spread ↔ transverse divergence
- longitudinal emittance ↔ transverse emittance.

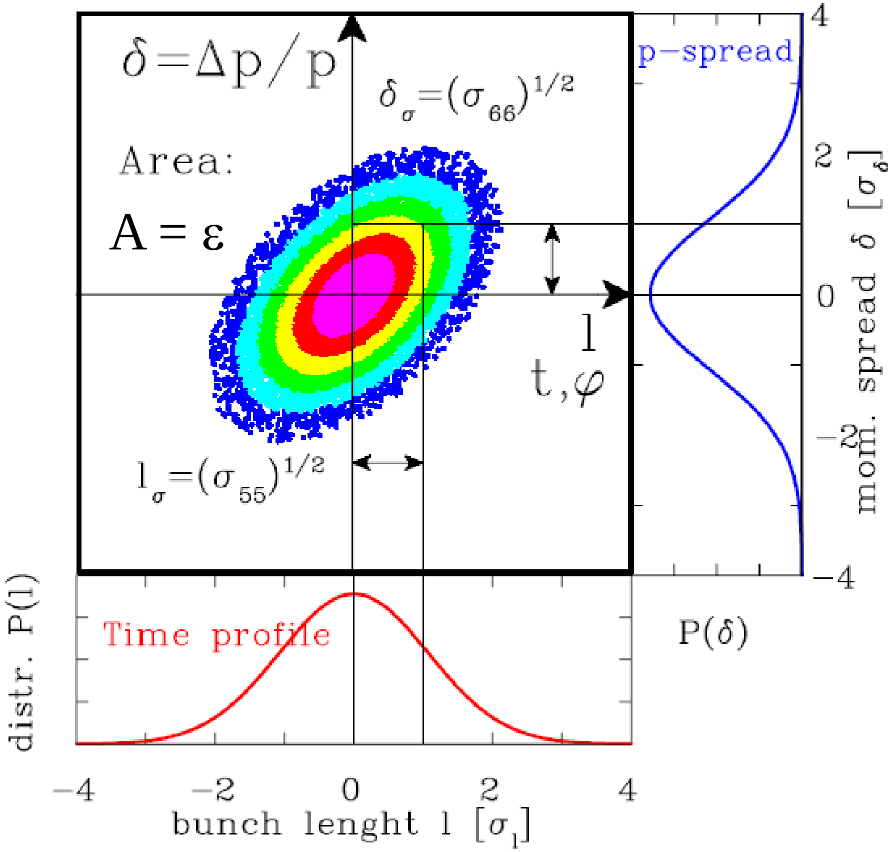
The longitudinal dynamics is described by the longitudinal emittance as given by:

- Spread of the bunches ***l*** in time, length ***or*** rf-phase.
- Momentum spread ***δ = Δp/p***, or energy spread ***ΔW/W***

$$\Rightarrow \varepsilon_{long} = \frac{1}{\pi}\int_A dl \cdot d\delta$$

The normalized value is preserved:

$$\varepsilon_{long}^{norm} = \beta\gamma \cdot \varepsilon_{long}$$



The Bunch Position measured by a Pick-Up

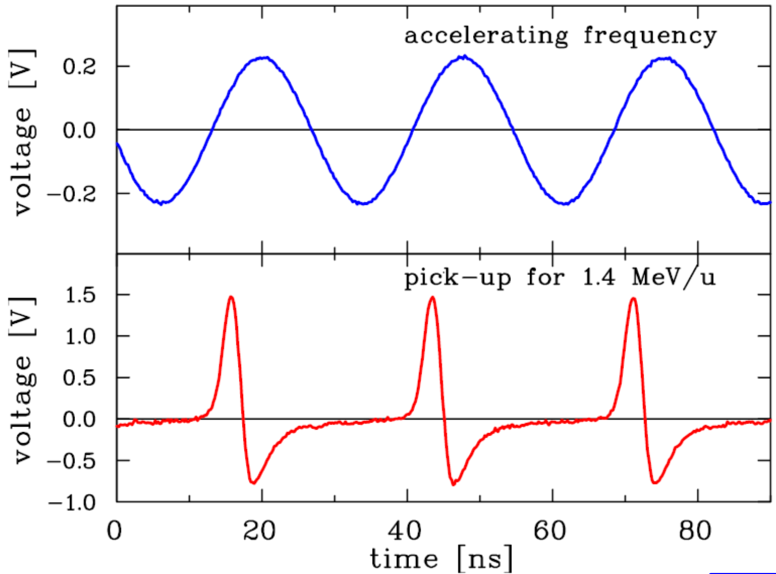
The ***bunch position*** is given relative to the accelerating rf.

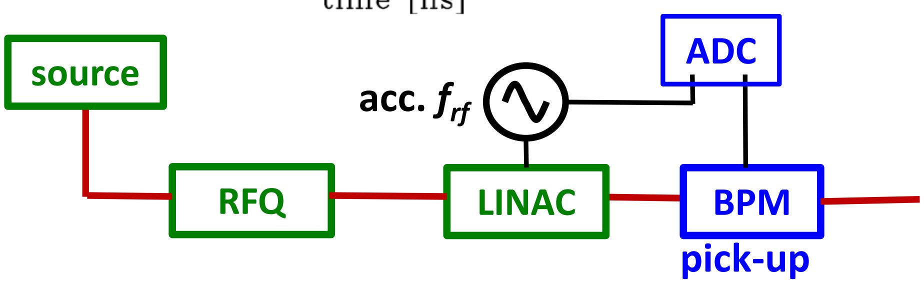
e.g. φ_{ref}=-30^{o} inside a rf cavity

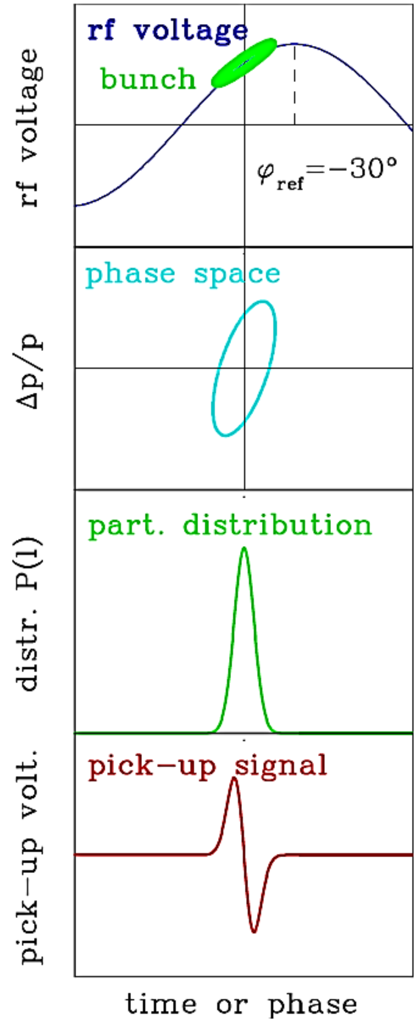
must be well aligned for optimal acceleration

Transverse correspondence: Beam position

Example: Pick-up signal for f_{rf}=36 MHz rf at GSI-LINAC:







Outline:

- Definition of longitudinal phase space
- Proton LINAC: Determination of mean energy

  used for alignment of cavities phase and amplitude
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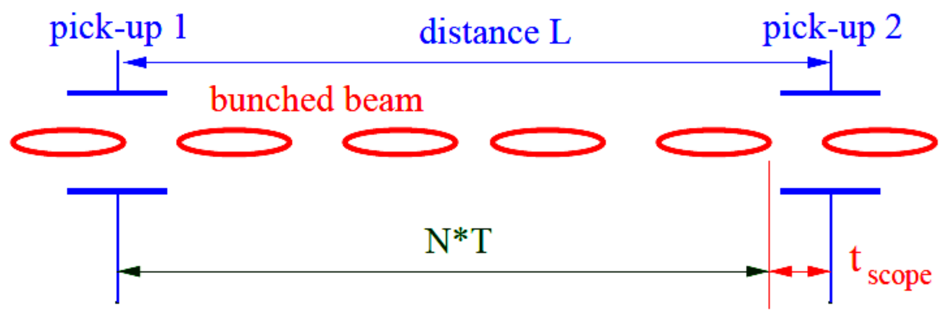
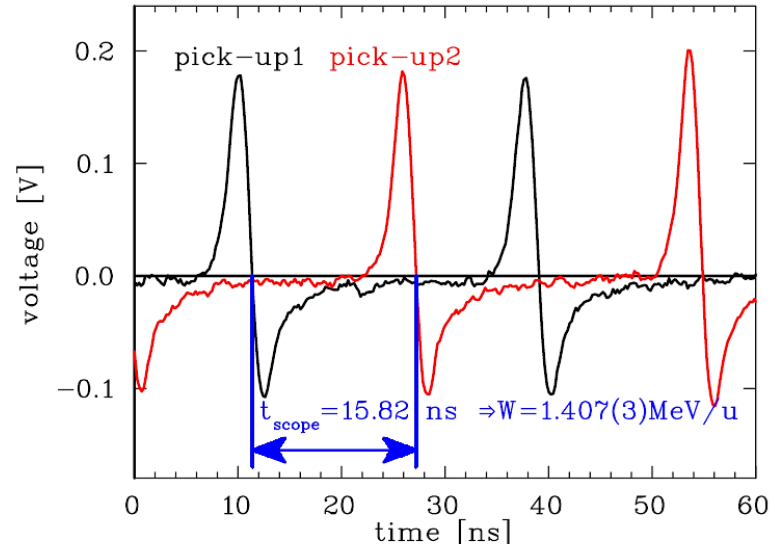
The energy delivered by a LINAC is sensitive to the mechanics, rf-phase and amplitude.

For non-relativistic energies at proton LINACs **time-of-flight** (ToF) with two pick-ups is used:

$$\beta c = \frac{L}{NT + t_{\text{scope}}}$$

$\rightarrow$ the velocity β is measured.





Example: Time-of-flight signal from two pick-ups at 1.4 MeV/u:

The reading is $\boldsymbol{t_{scope}}$= 15.82(5) ns with

$\boldsymbol{f_{rf}}$ = 36.136 MHz ⇔ $\boldsymbol{T}$ = 27.673 ns

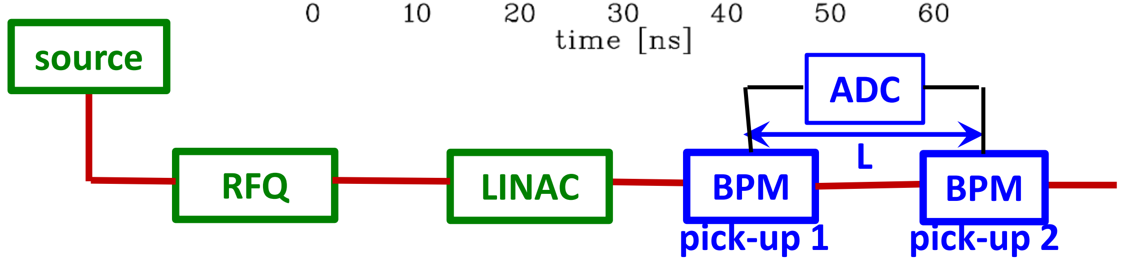
$\boldsymbol{L}$ = 1.629(1) m & $\boldsymbol{N}$ = 3

⇒ $\boldsymbol{\beta}$ = 0.05497(7)

⇔ $\boldsymbol{E_{kin}}$ =1.407(3) MeV/u

The accuracy is typically 0.1 %

i.e. comparable to ***Δp/p***



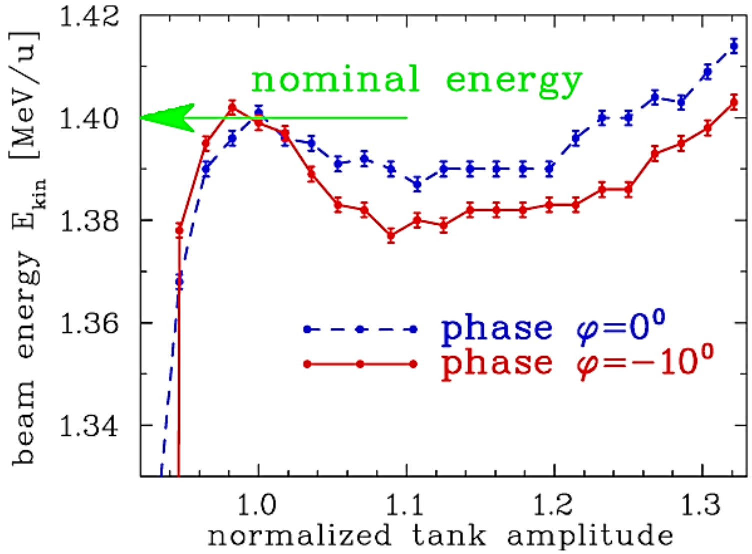
Cavity Alignment using a TOF Measurement

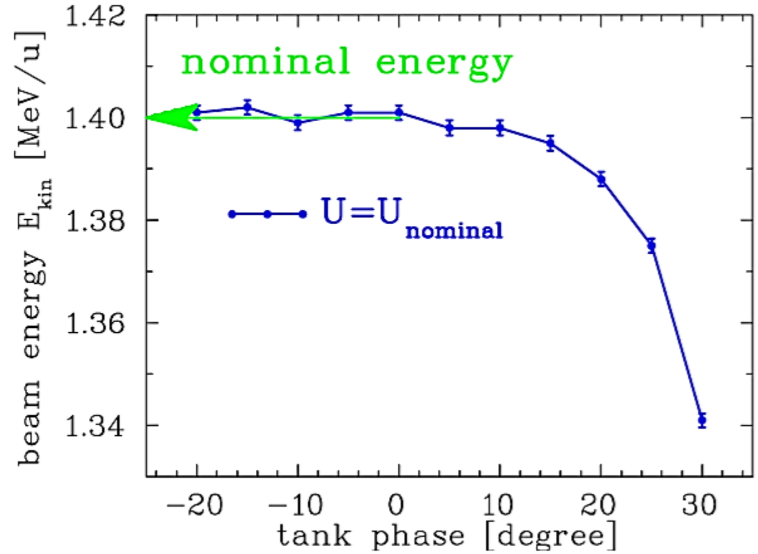
The mean energy is important for the matching between LINAC module.

It depends on phase and amplitude of the rf wave inside the cavities.

Example: Energy at GSI LINAC (nominal energy 1.400 MeV/u):

(distance between pick-ups: L = 1.97 m $\Rightarrow$ N = 4 bunches)





- ***Proton LINACs:*** Amplitude and phase should be carefully aligned by precise TOF
- ***Electron LINACs***: Due to relativistic velocity, TOF is not applicable.

Poll concerning longitudinal Phase Space and ToF

Poll 6.1:

Compare the longitudinal to the transverse phase space:
The relative momentum spread $\Delta p/p_0$ corresponds to the transverse …

1) beam centre
2) beam width
3) beam divergence
4) beam emittance

Poll 6.2:

The mean energy can be measured via…

1) two BPMs for relativistic beams only
2) two BPMs for non-relativistic beams only
3) two SEM-Grids and the know dispersion

Poll 6.3:

What is **correct** for a proton facility comprising of several LINAC cavities?
ToF measurement are used to align …

1) the beam position
2) the quadrupole settings
3) **all** cavity amplitudes
4) **only** the last cavity amplitude to reach the correct final energy

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Outline:

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- Determination of longitudinal emittance
  LINAC: variation of bunch length
  Synchrotron: Topographic reconstruction
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- Bunch length measurement for relativistic beams
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The particle trajectory is described with the 6-dim vector $x^t = (x, x', y, y', l, \delta)$

For linear beam behavior the 6x6 transport matrix R is used:

Transformation from location s_0 to s_1 for a single particle is:

$$\vec{x}(s_1) = \mathrm{R} \cdot \vec{x}(s_0)$$

$$\vec{x}(s_1) = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & \dots & \dots & \dots & \dots \\ R_{31} & \dots & R_{33} & R_{34} & \dots & \dots \\ R_{41} & \dots & R_{43} & R_{44} & \dots & \dots \\ R_{51} & \dots & \dots & \dots & R_{55} & R_{56} \\ R_{61} & \dots & \dots & \dots & R_{65} & R_{66} \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{pmatrix}$$

= 0 (R_{15}, R_{16} block); =0 (rows 3–4, columns 5–6); = 0 (R_{51}, R_{61} block); =0 (rows 5–6, columns 3–4)

Envelope i.e. emittance defined by beam matrix:

$$\sigma(s_1) = \mathrm{R} \cdot \sigma(s_0) \cdot \mathrm{R}^T$$

R separates in 3 matrices only **<u>if</u>** the transverse and longitudinal planes do **<u>not</u>** couple,

e.g. no dispersion ***D = -R*** $_{16}$ ***= 0***

The longitudinal beam matrix **σ** is **<u>then</u>** a 2 x 2 matrix

with bunch length $l_{rms} = \sqrt{\sigma_{55}}$ & momentum spread $\frac{\Delta p}{p} = \delta_{rms} = \sqrt{\sigma_{66}}$

Longitudinal focusing:

Variation of the bunch shape by a rf-buncher
$\rightarrow$ components 5 and 6 from 6-dim phase-space

Transversal correspondence:

Quadrupole variation

- Transfer matrix of buncher & drift:

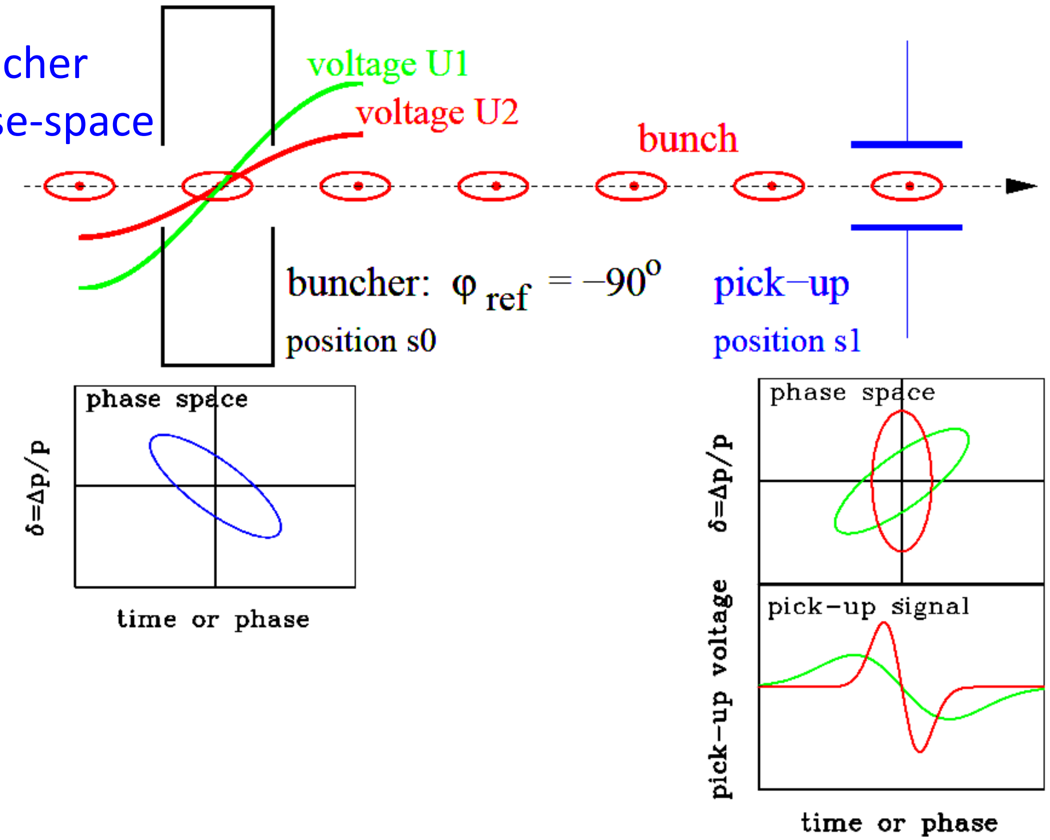
$$R_{buncher} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

$$R_{drift} = \begin{pmatrix} 1 & L/\gamma^2 \\ 0 & 1 \end{pmatrix}$$

with focal length $\frac{1}{f} = \frac{2\pi f_{rf}}{Apv^2} \cdot U$:

- Variation of buncher amplitude ***U***

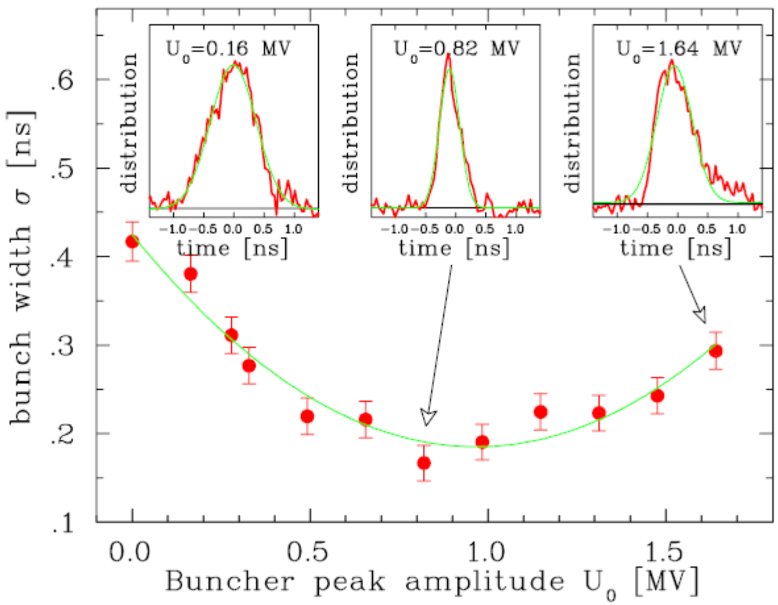
$\Rightarrow$ different bunch width at pick-up location



Result of a longitudinal Emittance Measurement

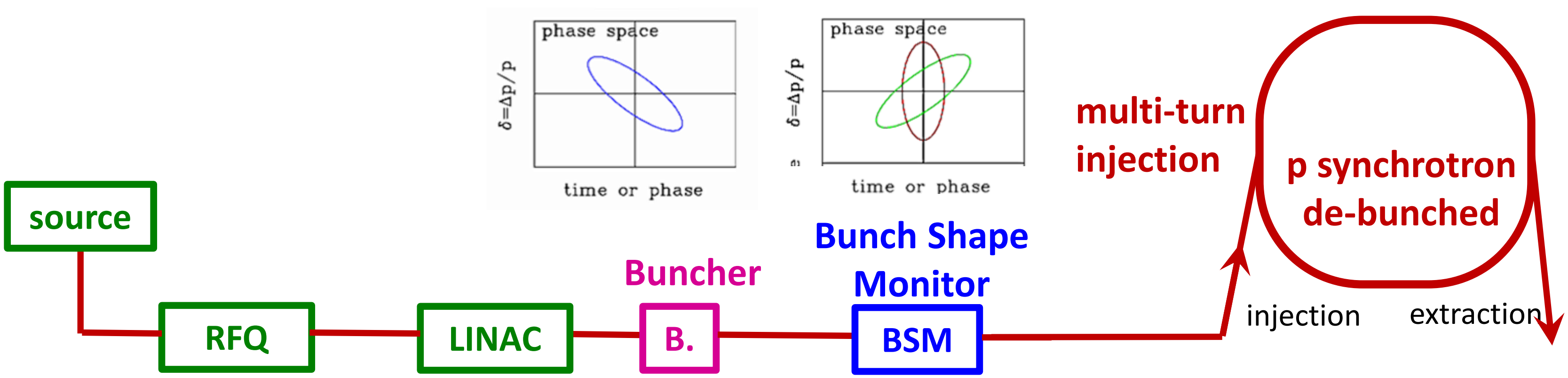
Example GSI LINAC: Voltage variation at buncher for 11.4 MeV/u Ni^{14+} beam, 31 m drift:

- The structure of short bunches can be determined with special monitor
- This example: The resolution is better than 50 ps or 2° for 108 MHz
- Typical bunch length at proton LINACs:
  $\sigma_{bunch} \approx$ 10 to 300 ps
- Determination of longitudinal emittance possible



Application for synchrotron injection:

Shaping of longitudinal phase space by buncher
i.e. long bunches ⇔ low momentum spread
to match to the synchrotron long acceptance

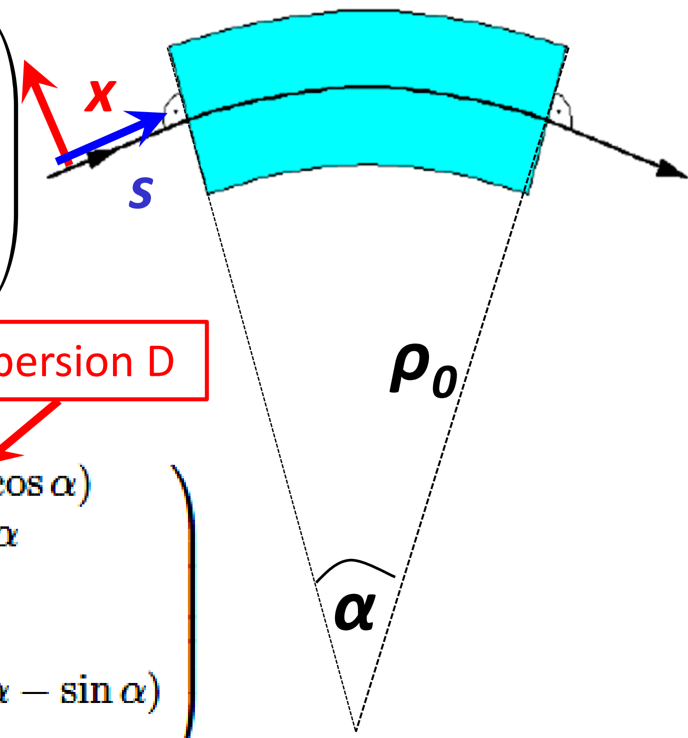


Recap: Transfer Matrix of a Sector Dipole Magnet

Horizontal sector dipole magnet (i.e. edge rectangular to reference orbit)

Basis vector for trajectory:

$$\vec{x} = \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{pmatrix} = \begin{pmatrix} \text{hori. spatial deviation} \\ \text{horizontal divergence} \\ \text{vert. spatial deviation} \\ \text{vertical divergence} \\ \text{long. deviation} \\ \text{momentum deviation } \Delta p/p \end{pmatrix} = \begin{pmatrix} [\text{mm}] \\ [\text{mrad}] \\ [\text{mm}] \\ [\text{mrad}] \\ [\text{mm}] \\ [10^{-3}] \end{pmatrix}$$



Transformation of trajectory: $\vec{x}(s_1) = \mathbf{R} \cdot \vec{x}(s_0)$

Transfer matrix of a sector dipole:

$$R = \begin{pmatrix} \cos\alpha & \rho_0 \sin\alpha & 0 & 0 & 0 & \rho_0(1-\cos\alpha) \\ -\frac{\sin\alpha}{\rho_0} & \cos\alpha & 0 & 0 & 0 & \sin\alpha \\ 0 & 0 & 1 & \rho_0\alpha & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\sin\alpha & -\rho_0(1-\cos\alpha) & 0 & 0 & 1 & \rho_0\frac{\alpha}{\gamma^2} - \rho_0(\alpha - \sin\alpha) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Contribution to beam width via $\boldsymbol{\sigma}(s_1) = \mathbf{R} \cdot \boldsymbol{\sigma}(s_0) \cdot \mathbf{R}^T$

$$x_{rms}(s_1) = \sqrt{\sigma_{11}(s_1)} \propto \sqrt{\varepsilon\beta + \left(D\frac{\Delta p}{p}\right)^2}$$ i.e. adding in quadrature

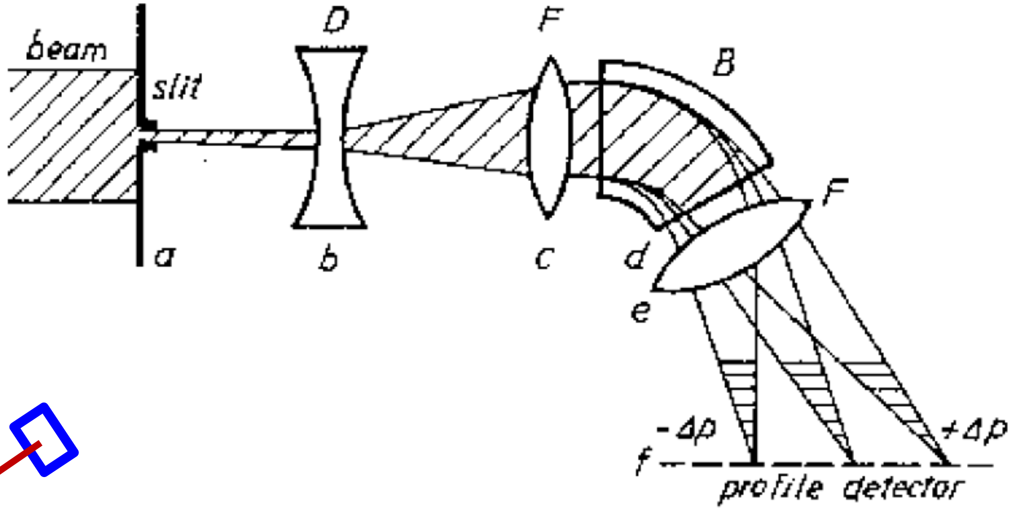
Result: Fast particle → less bending & slower particle → more bending

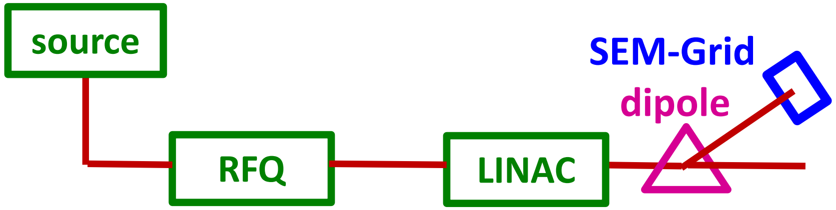
⇔ Coupling $\boldsymbol{x}$ and $\boldsymbol{\delta}$ due to $\boldsymbol{R_{16} \neq 0}$ ⇒ can be used as spectrometer

Measurement of Energy Spread by magnetic Spectrometer

Transfer line: The mom. spread $\boldsymbol{\delta = \Delta p/p}$ can be determined by a magnetic spectrometer: via dispersion, the momentum is shifted to a spatial distance.

An appropriate optic must b e chosen to separate the transverse and longitudinal parameters



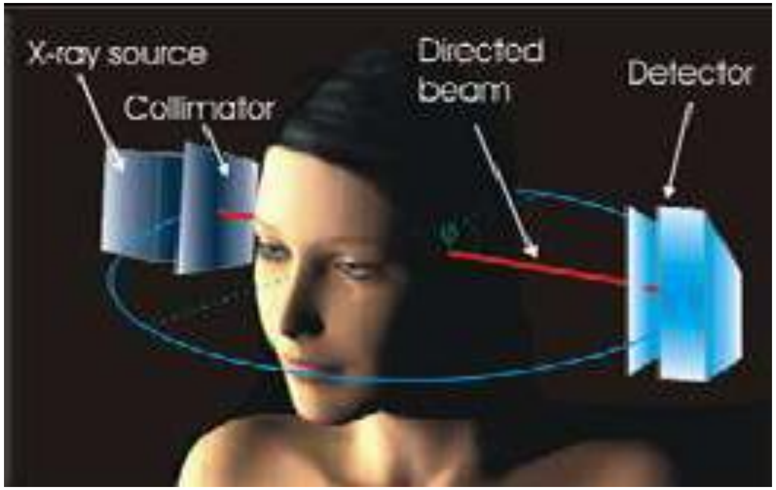


Longitudinal Emittance using tomographic Reconstruction

Tomography is medical image method

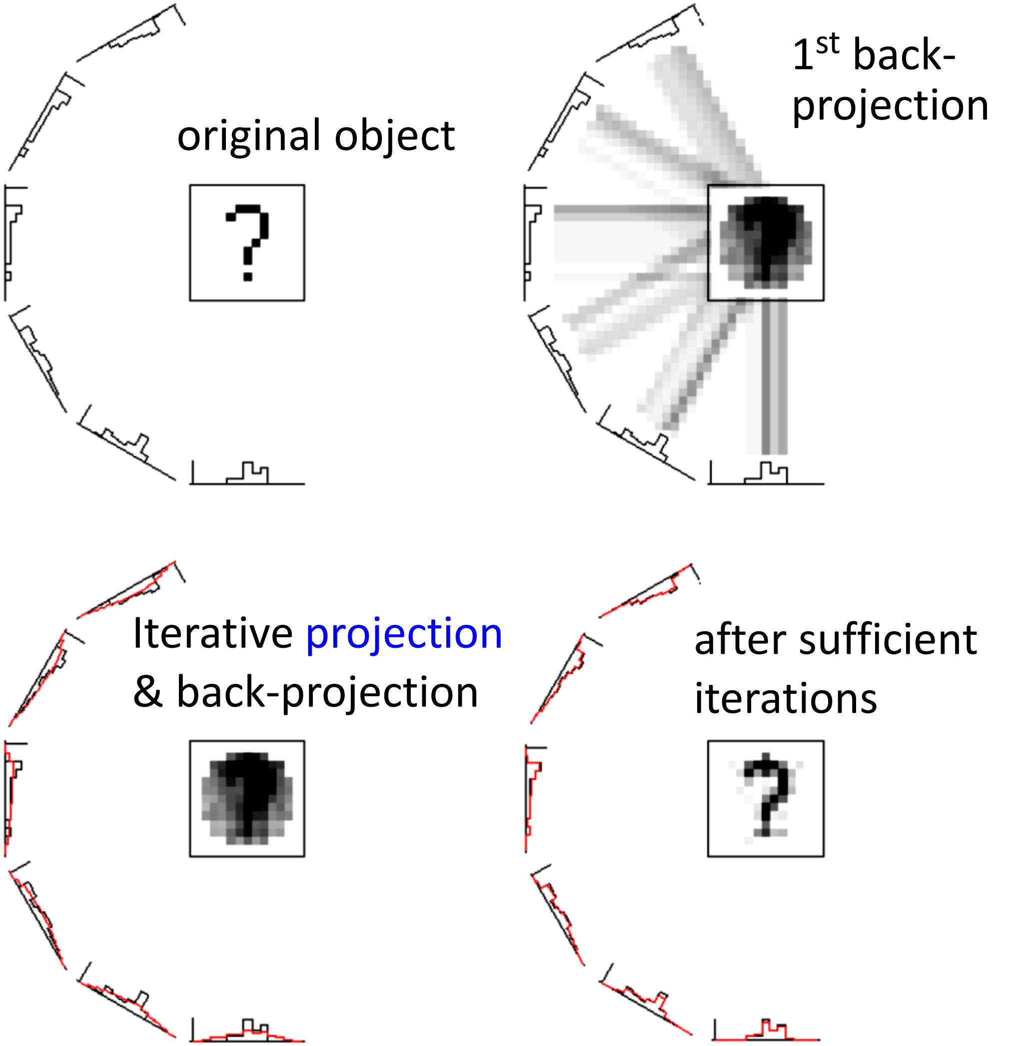
Tomography:

2-dim reconstruction of
sufficient 1-dim projections



Algebraic back projection:

Iterative process by redistributing
the 2-dim image and considering the
differences to the previous iteration step.



Longitudinal Emittance using tomographic Reconstruction

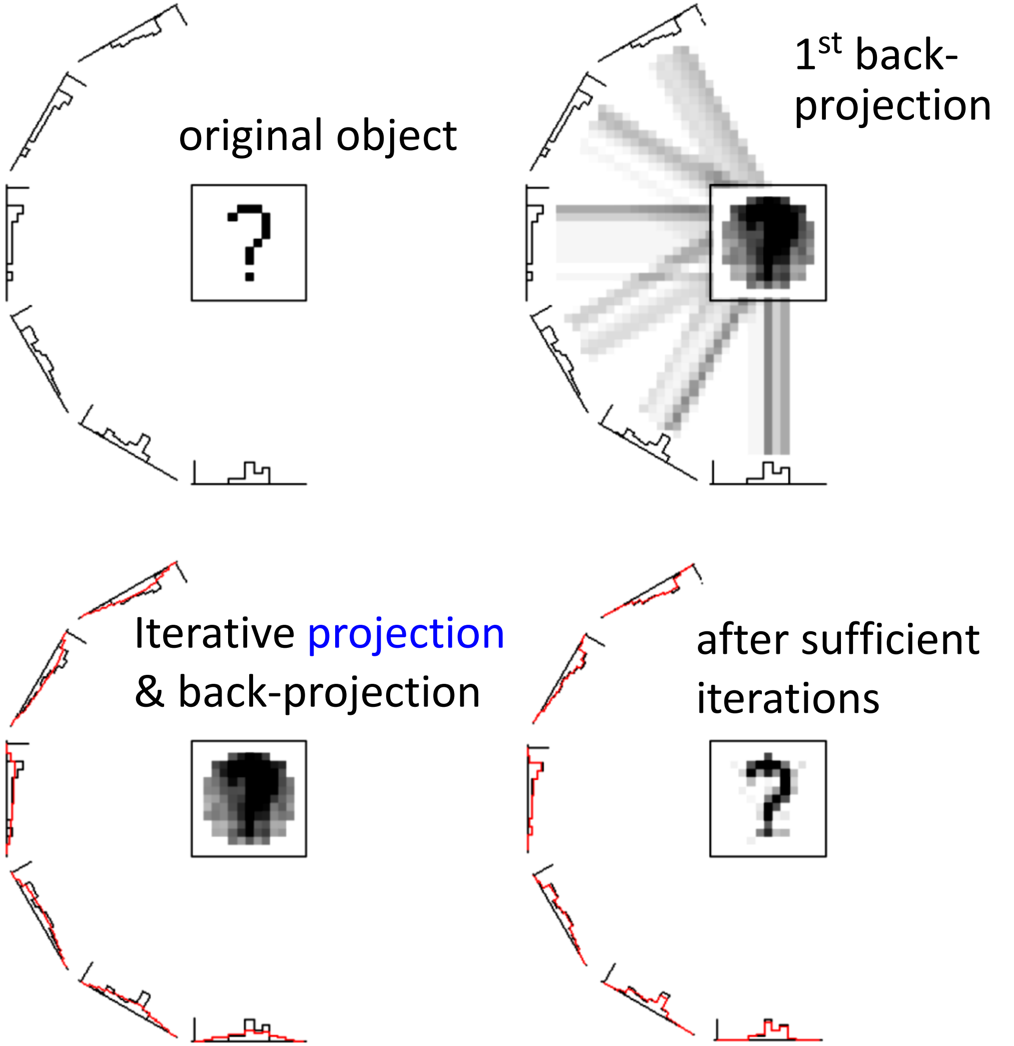
Tomography is medical image method

Tomography:
2-dim reconstruction of sufficient 1-dim projections

Application at accelerators:
Longitudinal emittance evolution in synchrotrons.

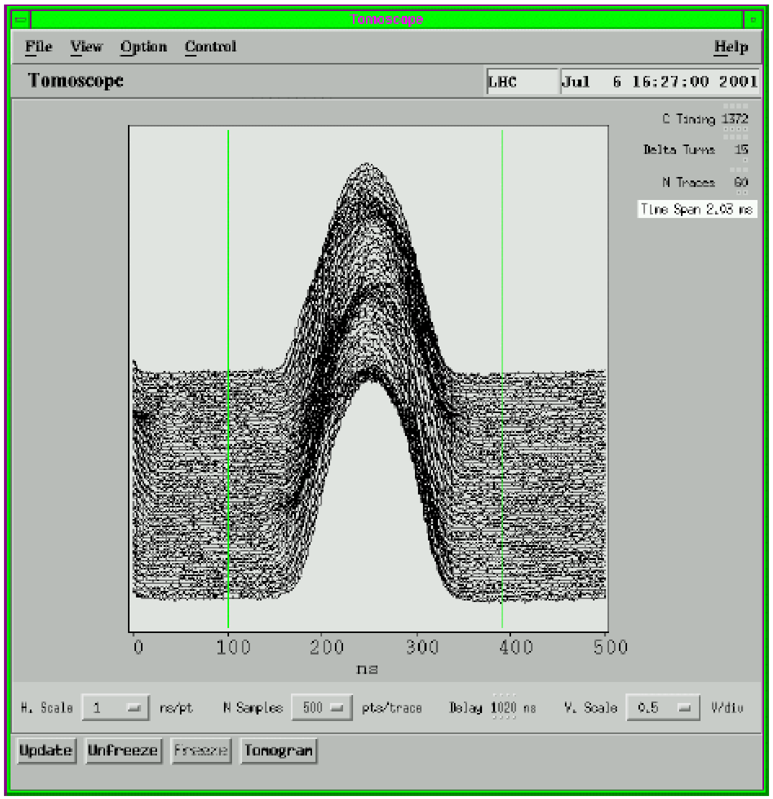
Bunch observation:
Each revolution,
the bunch shape changes a bit
due to synchrotron oscillations.
Fulfilled condition: $f_{synch} << f_{rev}$.

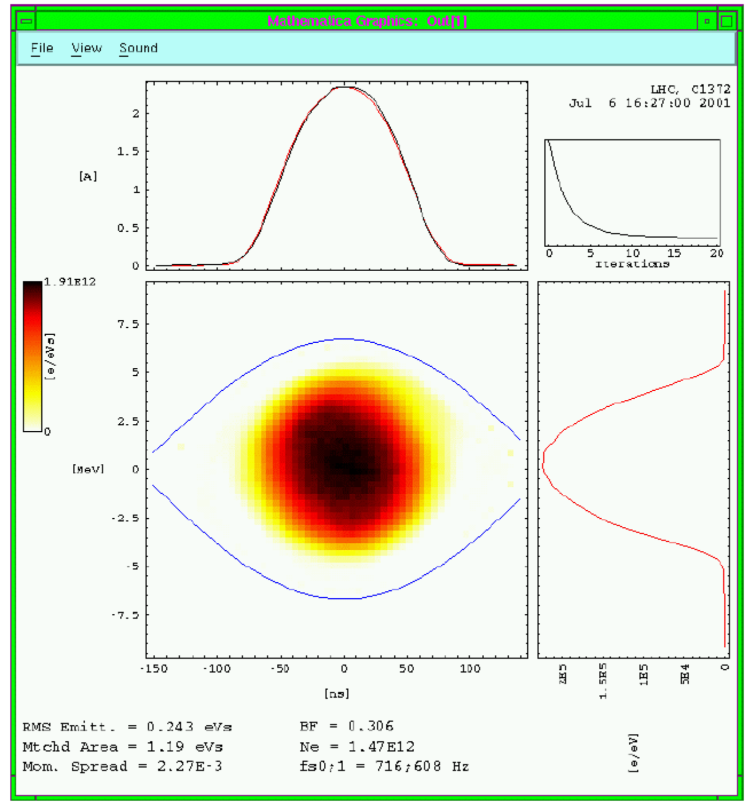
Algebraic back projection:
Iterative process by redistributing the 2-dim image and considering the differences to the previous iteration step.



Results of tomographic Reconstruction at a Synchrotron I

Bunches from 500 turns at the CERN PS and the phase space for the first time slice, measured with a wall current monitor:

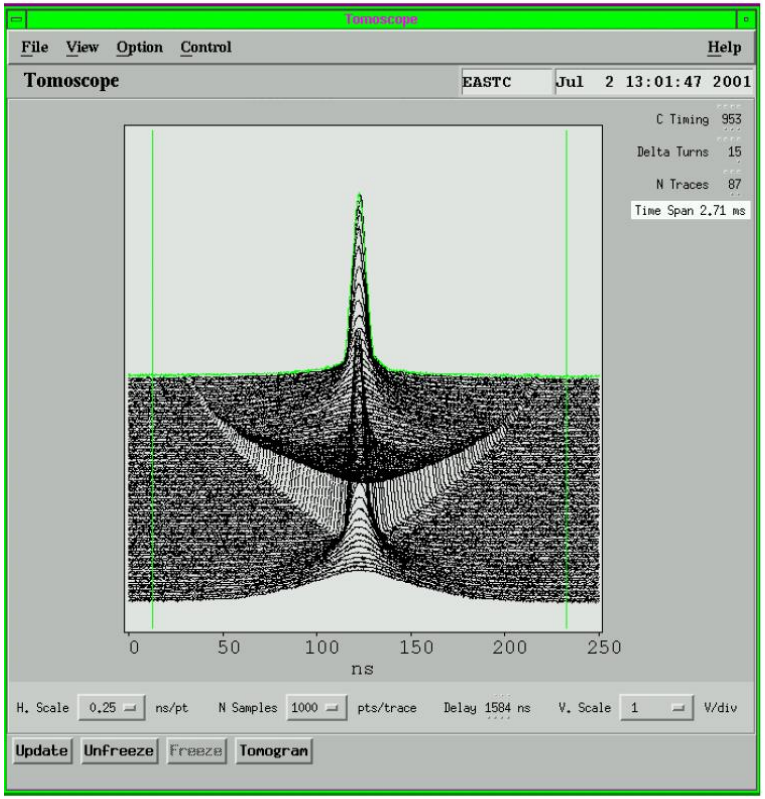


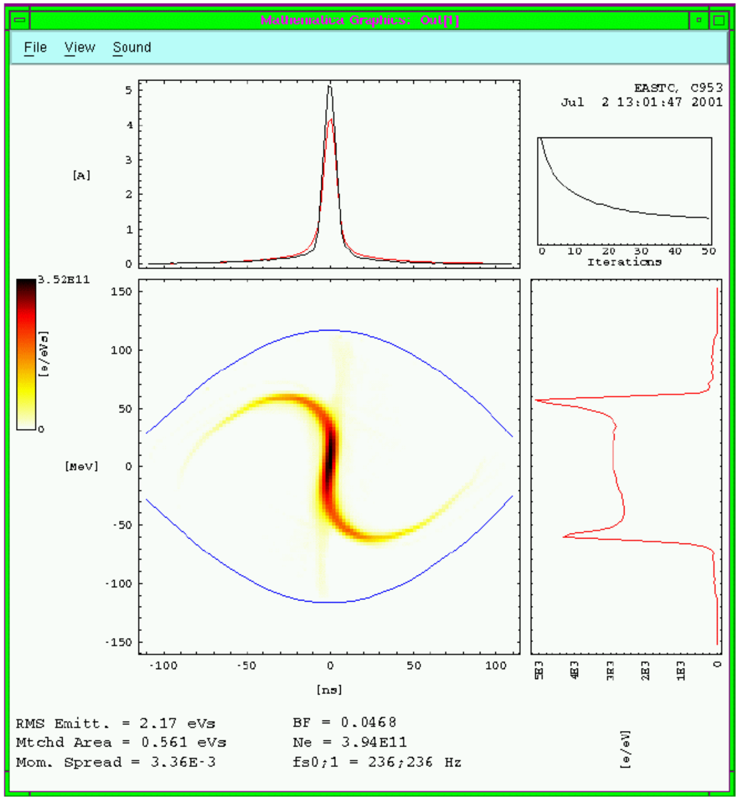


Typical bucket filling. Important knowledge for bunch 'gymnastics'.

Results of tomographic Reconstruction at a Synchrotron II

Bunches from 500 turns at the CERN PS and the phase space for the first time slice, measured with a wall current monitor:





Mismatched bunch shown oscillations and filamentation due to 'bunch-rotation'.

Poll concerning longitudinal Mismatch, long. Emittance

Poll 6.4:

Assume a LINAC facility.

The bunch length variation by a buncher followed by a bunch shape measurement corresponds to an transverse emittance measurement of type

1) Quadrupole variation
2) Three grid method
3) Slit-grid method

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Poll 6.5:

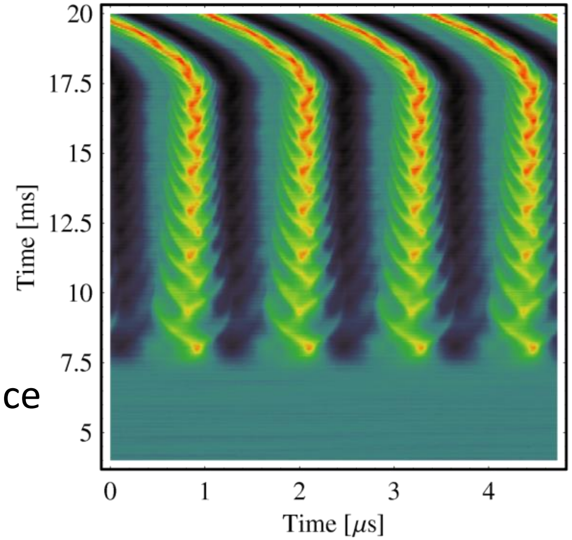
Assume an injection from a proton LINAC to a synchrotron.

What happens if injection energy is **not** correct?

(i.e. the particles' revolution frequency differs from the acc. frequency)

1) The particles perform coherent synchrotron oscillations
2) The particles perform <u>**in**</u>coherent synchrotron oscillations
3) The tune is changed significantly
4) Nothing, as long as the particles stay within the longitudinal acceptance

f_{acc} shift by 0.2% of nominal value
⇒ Coherent longi. oscillation



→ Streak camera

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- **Bunch length measurement for non-relativistic beams**
  Determination of particle arrival
- **Bunch length measurement for relativistic beams**
- Bunch length from beam deflection by rf cavity
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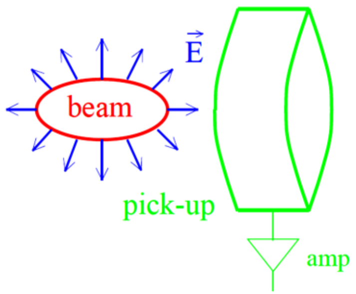
Bunch Structure at low E_{kin}: Not possible with Pick-Ups

Pick-ups are used for:

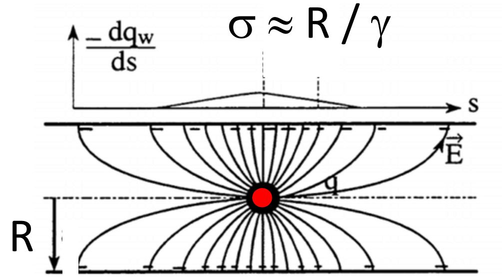
- precise for bunch-center relative to rf
- course image of bunch shape

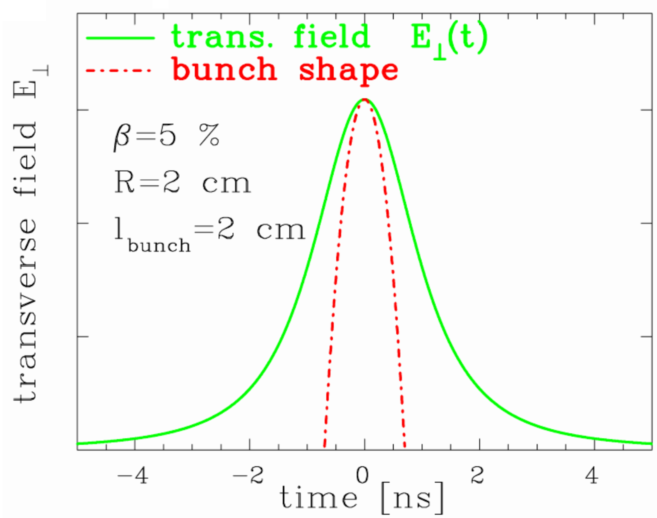
But:

For $\beta << 1 \rightarrow$ long. ***E***-field significantly modified:



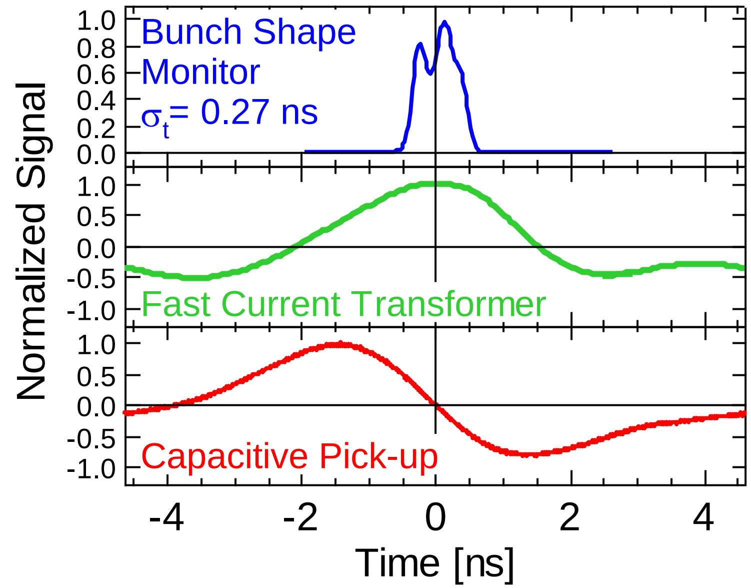
ampl.





Example: Comparison pick-up – particle counter:

Ar beam of 1.4 MeV/u (β = 5.5%) , f_{rf} = 108 MHz



$1/f_{rf}$ = 9.2 ns

⇒ the pick-up signal is insensitive to bunch 'fine-structure'

Lorentz transformation of single point-like charge:

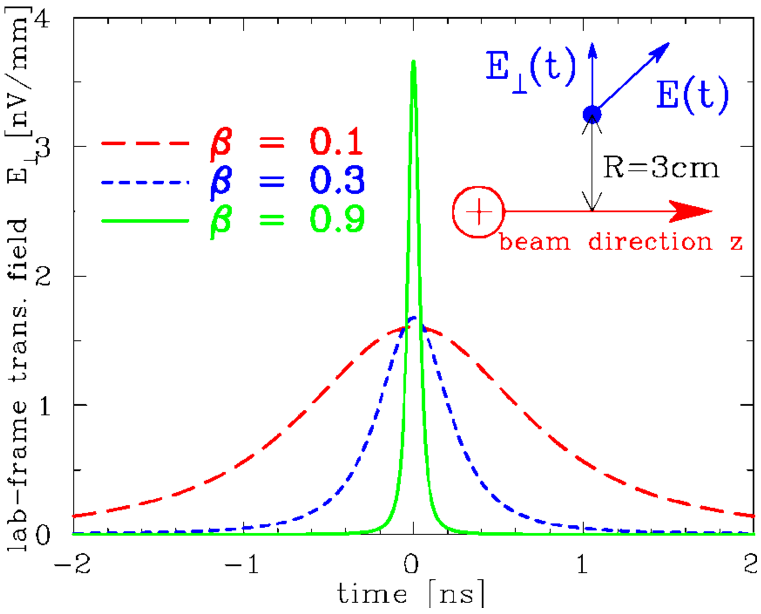
Lorentz boost ***and*** transformation of time: $E_\perp(t) = \gamma \cdot E'_\perp(t')$ and $t \to t'$

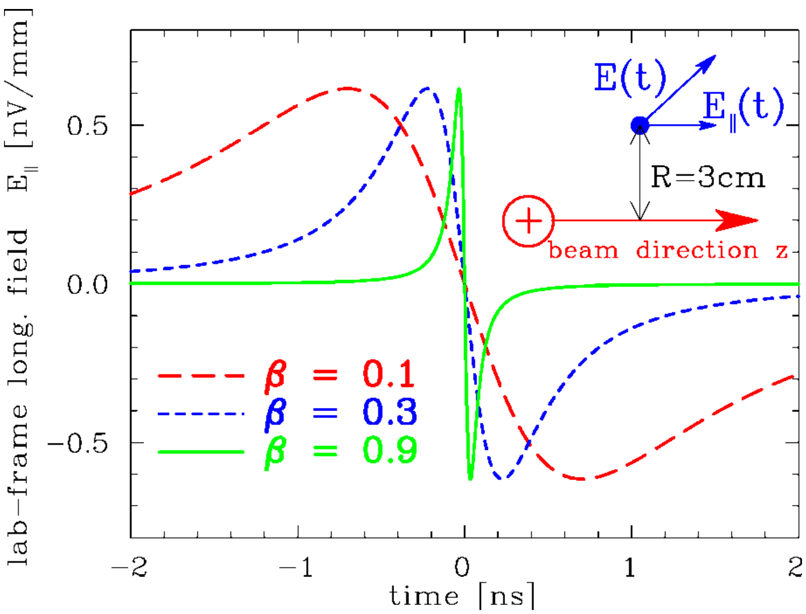
Trans. $\boldsymbol{E_\perp}$ lab.-frame of a point charge:

$$E_\perp(t) = \frac{e}{4\pi\varepsilon_0} \cdot \frac{\gamma R}{\left[R^2 + (\gamma\beta ct)^2\right]^{3/2}}$$

Long. $\boldsymbol{E_{||}}$ lab.-frame of a point charge:

$$E_\parallel(t) = -\frac{e}{4\pi\varepsilon_0} \cdot \frac{\gamma\beta ct}{\left[R^2 + (\gamma\beta ct)^2\right]^{3/2}}$$





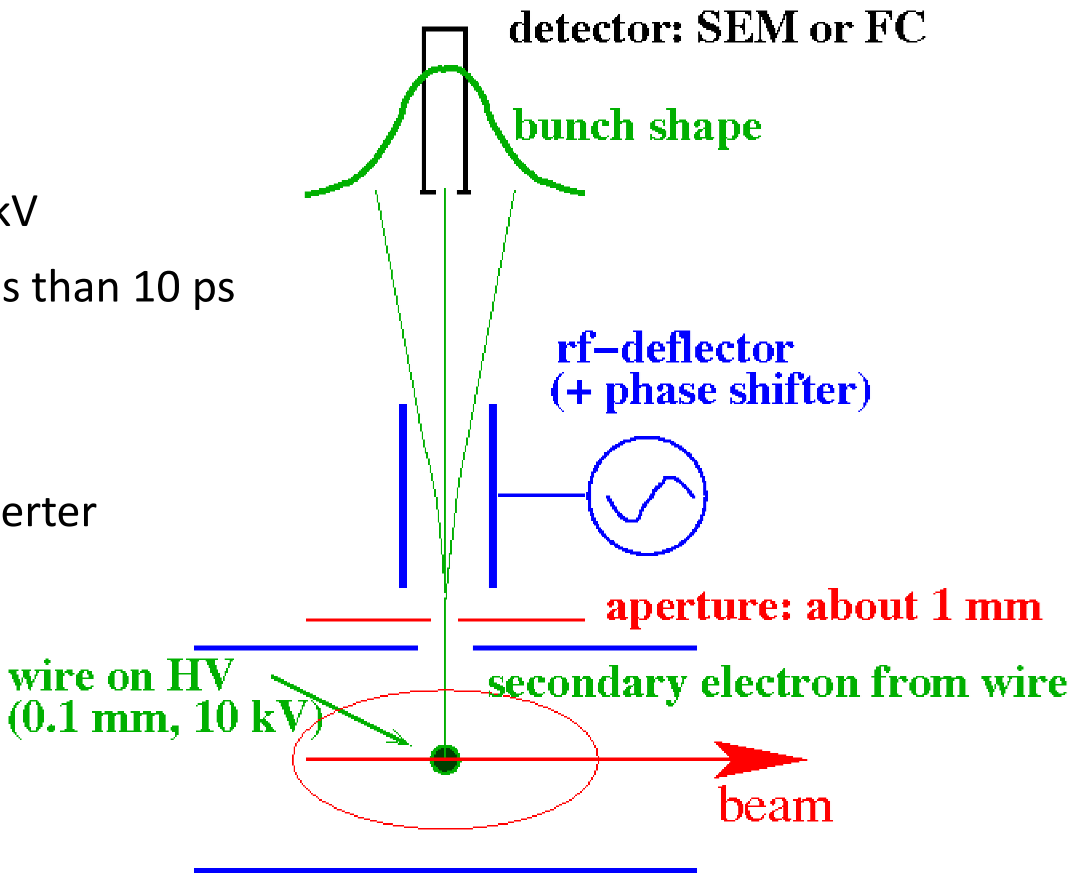
Secondary e⁻ liberated from a wire carrying the time information.

→ Bunch Shape Monitor (BSM)

Working principle:

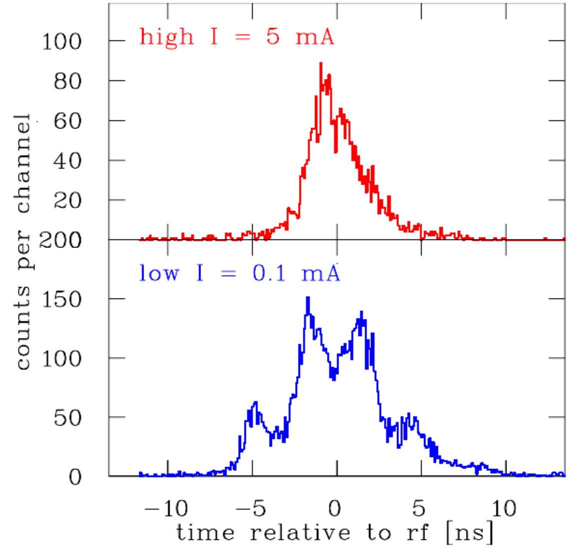
- insertion of a 0.1 mm wire at ≈ 10 kV
- emission of secondary e⁻ within less than 10 ps
- secondary e⁻ are accelerated
- toward an rf-deflector
- rf-deflector as 'time-to-space' converter
- detector with a thin slit
- slow shift of the phase
- resolution ≈ 10 ps ≈ 1° @ 280 MHz
- Measurements are comparable

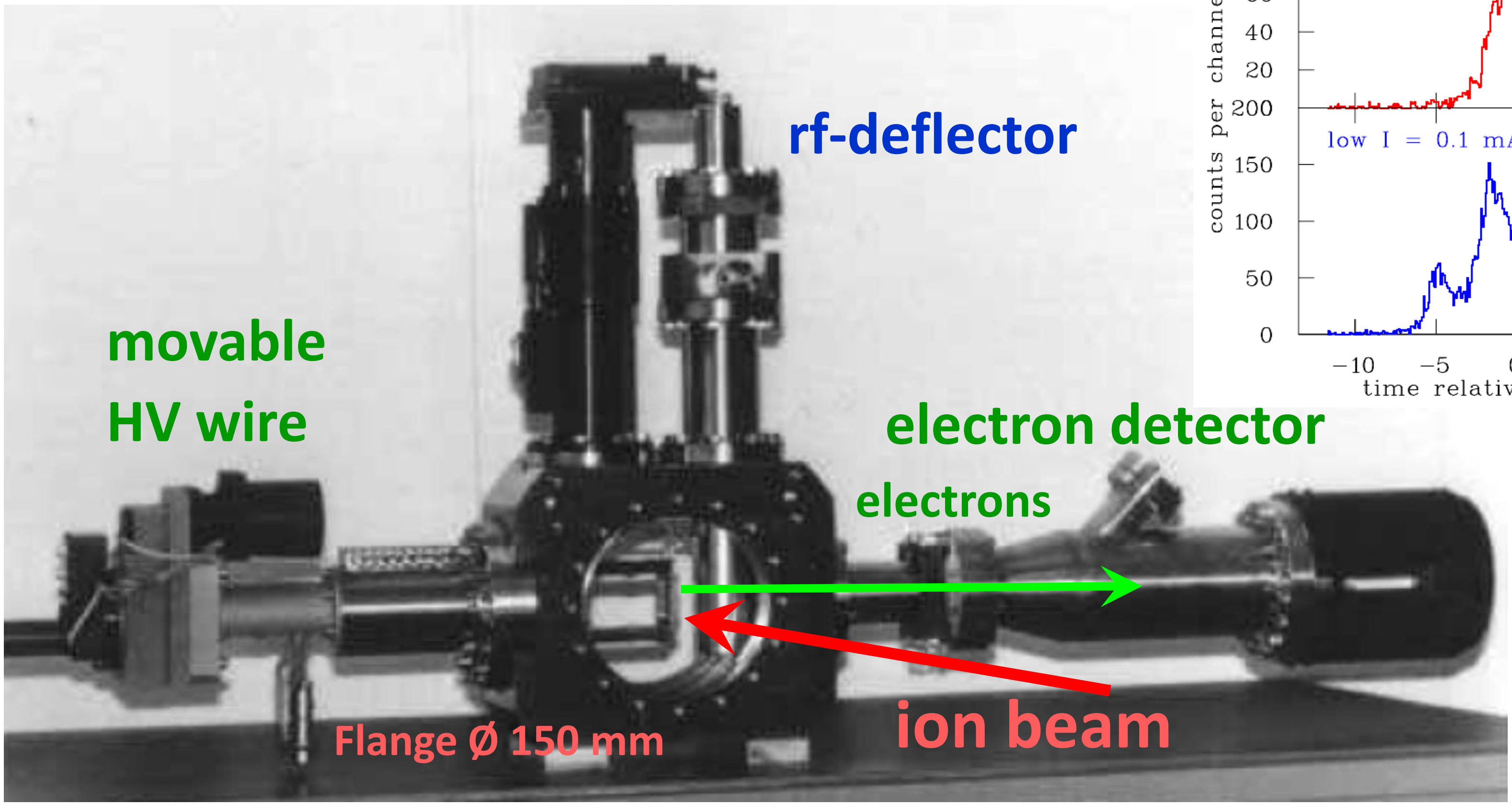
  to that obtained with particle detectors.



SEM: secondary electron multiplier

Example: The bunch shape behind RFQ with120 keV/u:





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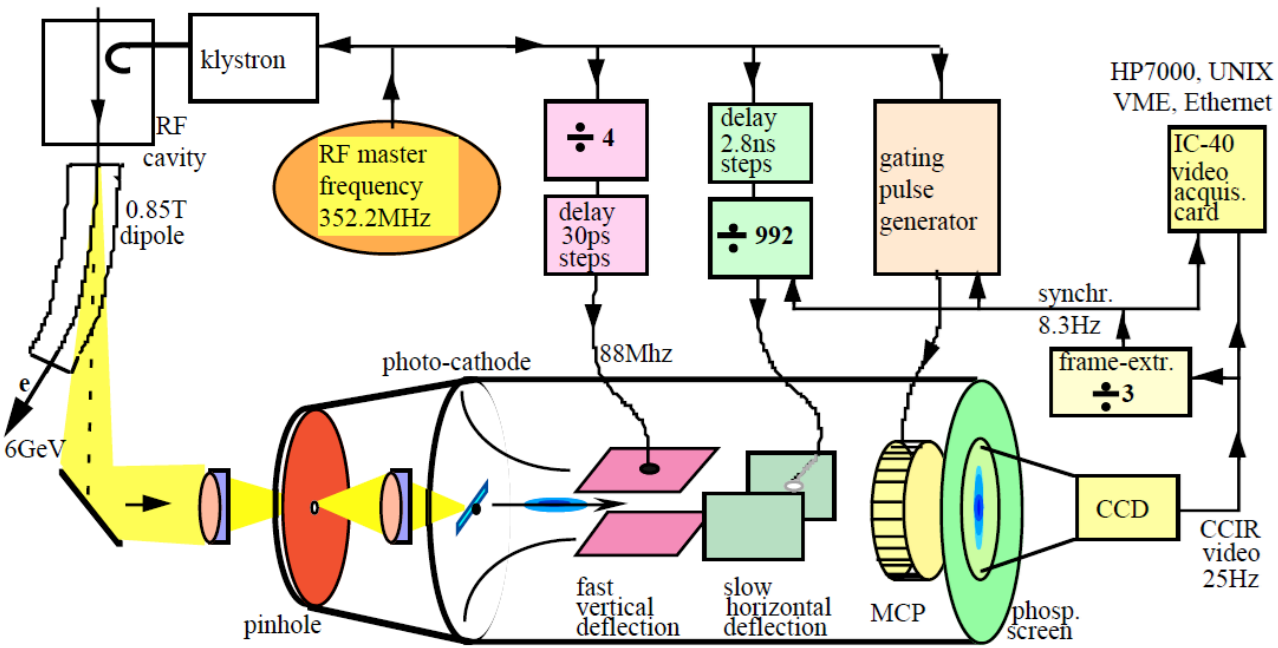
  Synchrotron light monitor and electro-optical modulation of a laser beam
- Bunch length from beam deflection by rf cavity
- **Summary**

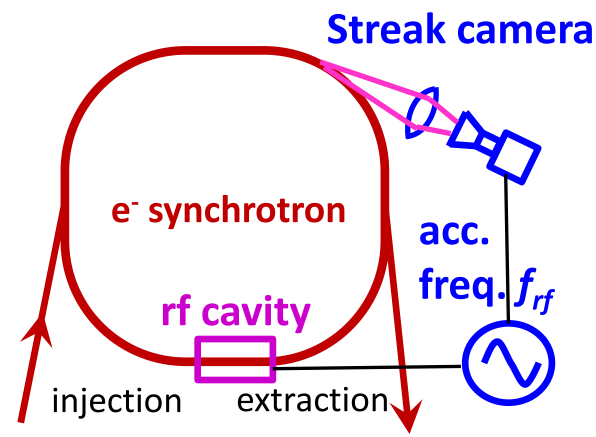
Bunch Length Measurement for relativistic Electrons

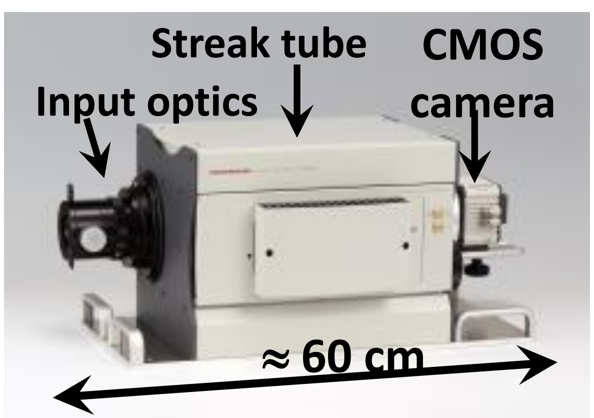
Electron bunches are too short (σ_t < 100 ps) to be covered by the bandwidth of pick-ups (f < 3 GHz ⇔ t_{rise} > 100 ps) for structure determination.

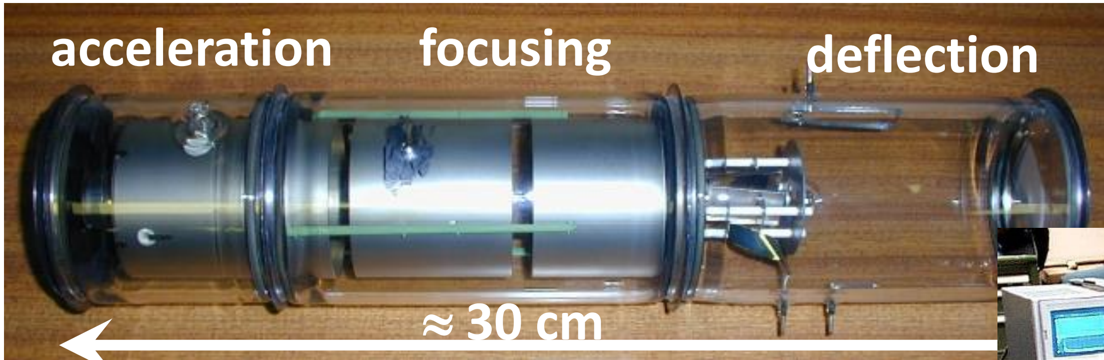
→ Time resolved observation of synchr. light with a streak camera: Resolution ≈ 1 ps.

Scheme of a streak camera



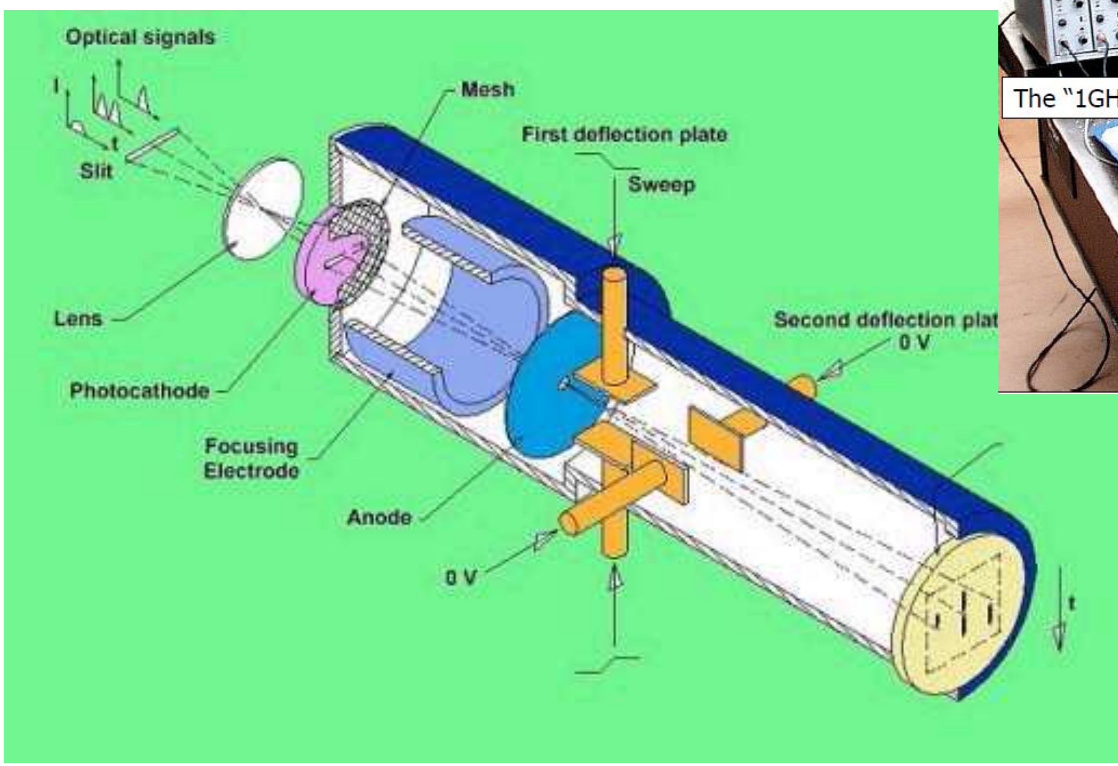


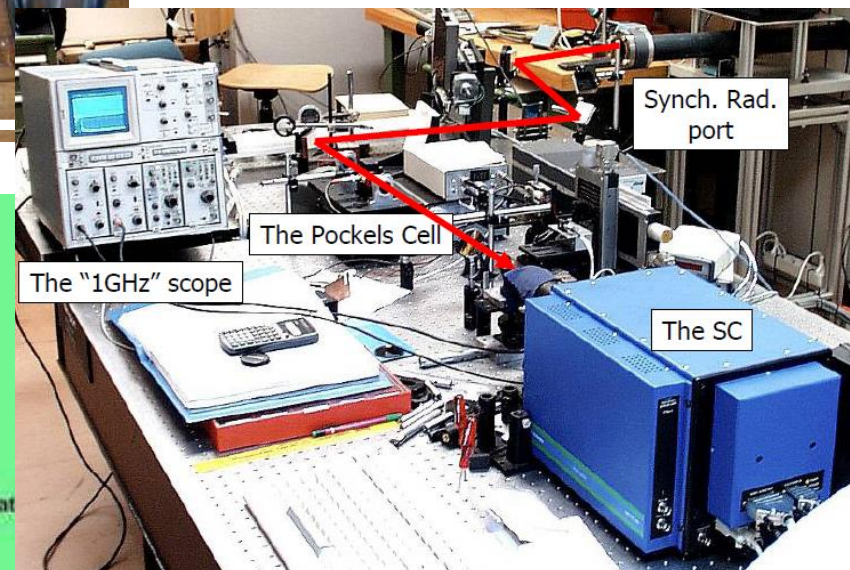


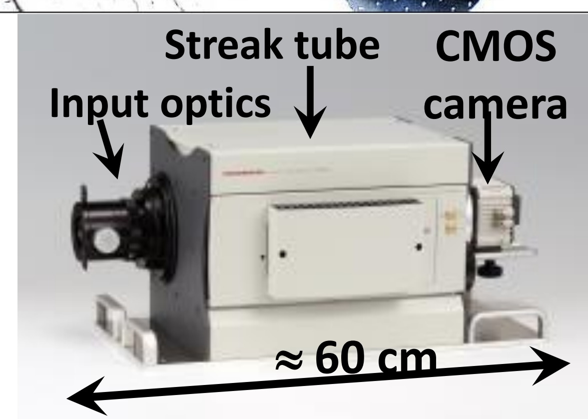


Hardware of a streak camera

Time resolution down to 0.5 ps







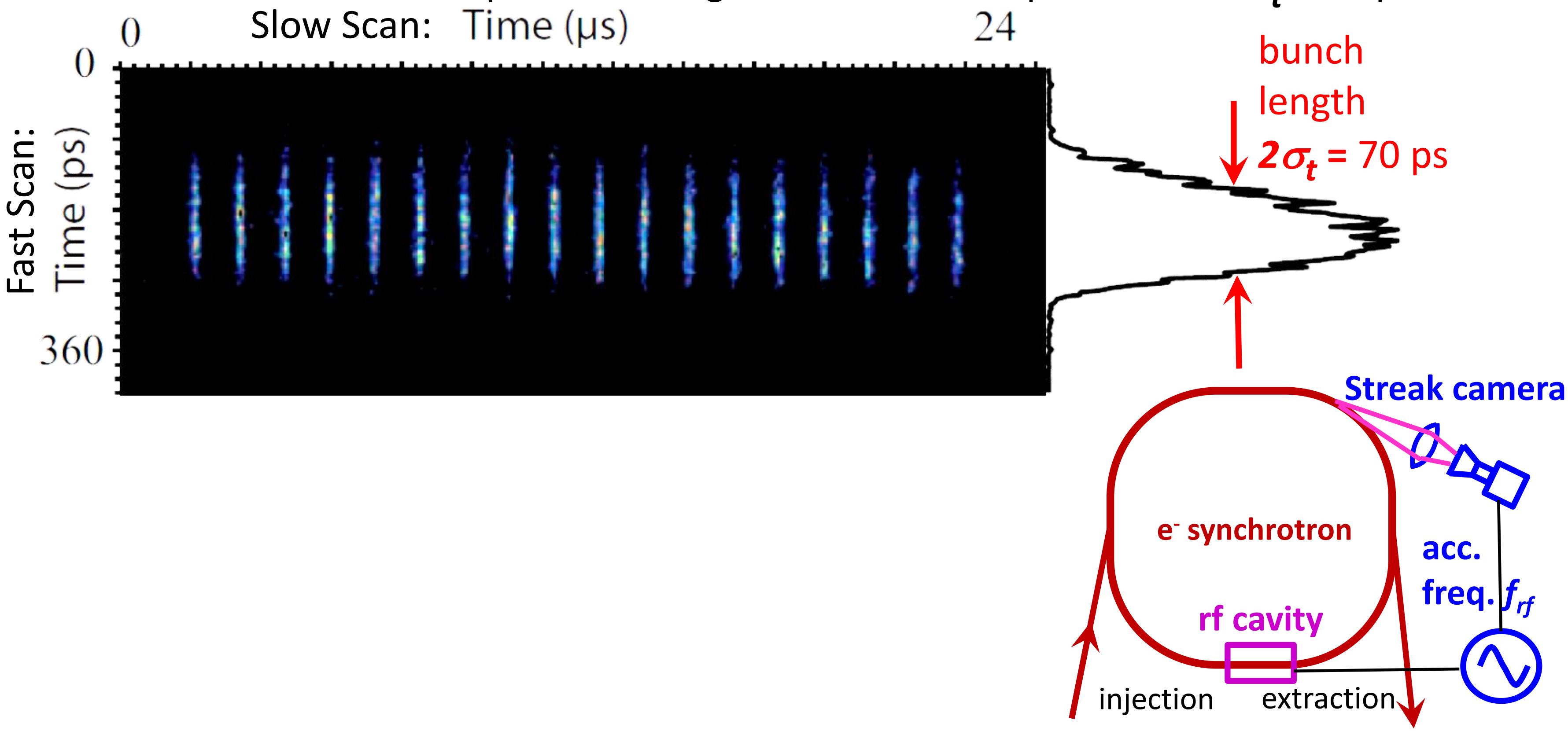
Results of Bunch Length Measurement by a Streak Camera

The streak camera delivers a fast scan in vertical direction (here 360 ps full scale)

and a slower scan in horizontal direction (24 µs).

Example: Bunch length at the synchrotron light source SOLEIL for U_{rf} = 2 MV

for slow direction 24 µs and scaling for fast scan 360 ps: measure σ_t = 35 ps.



Courtesy of M. Labat et al., DIPAC'07

→ conclusion

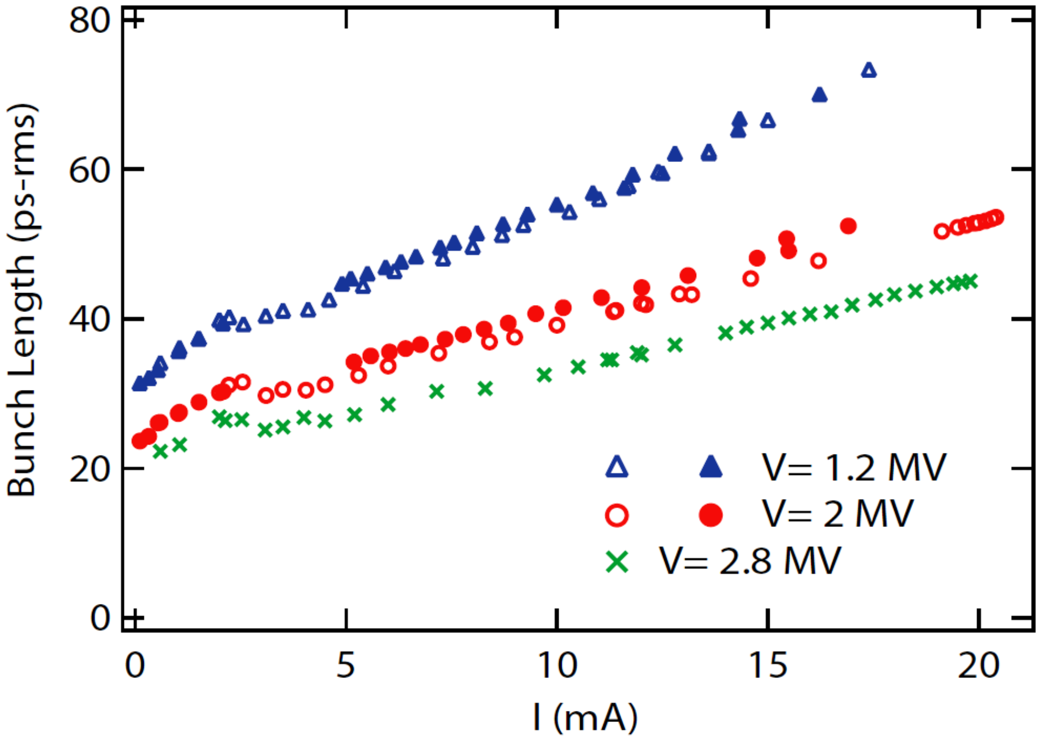
The Importance of Bunch Length by Streak Camera

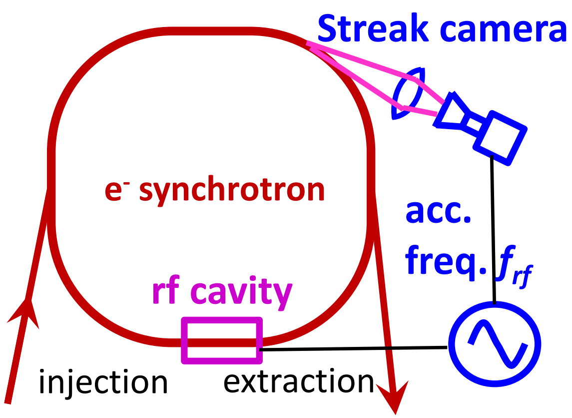
Short bunches are desired by the synchrotron light users for time resolved spectroscopy.

The bunch focusing is changed by the rf-amplitude.

Example: Bunch length σ_t as a function of stored current

(space-charge de-focusing, impedance broadening) for different rf-amplitudes at SOLEIL:

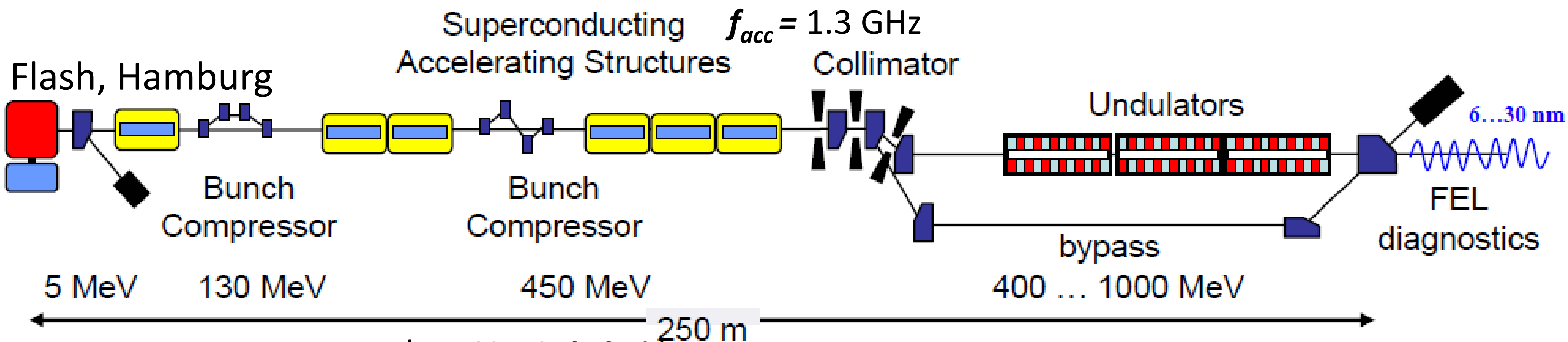


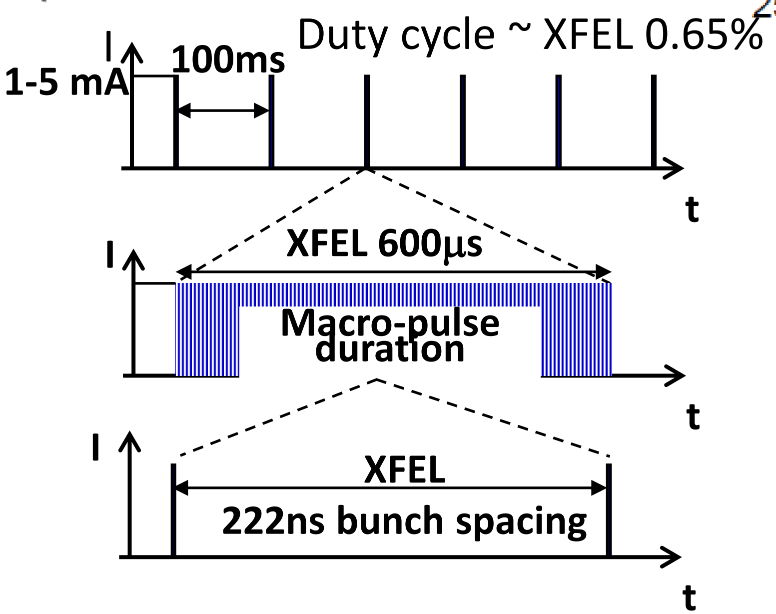


The Artist View of a Streak Camera

4th Generation Light Sources: LINAC based, single pass with large energy loss

$E_{electron} \approx 1 \ldots 18$ GeV, **coherent** light from undulator, $E_\gamma < 1000$ keV, temporally short pulse



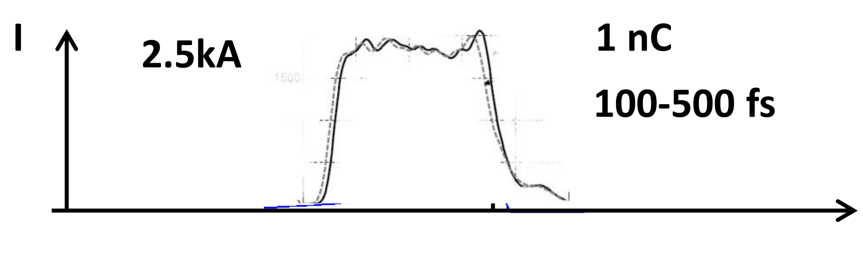


Goal: Short bunches with **high** number of particles

→ short, intense laser pulses for electron generation

Requirement: Position stability ⇒ resolution < 1 µm

Single bunch duration < 1 ps



For comparison: time t_{bunch} = 100 fs ⇔ length l_{bunch} = 30 µm

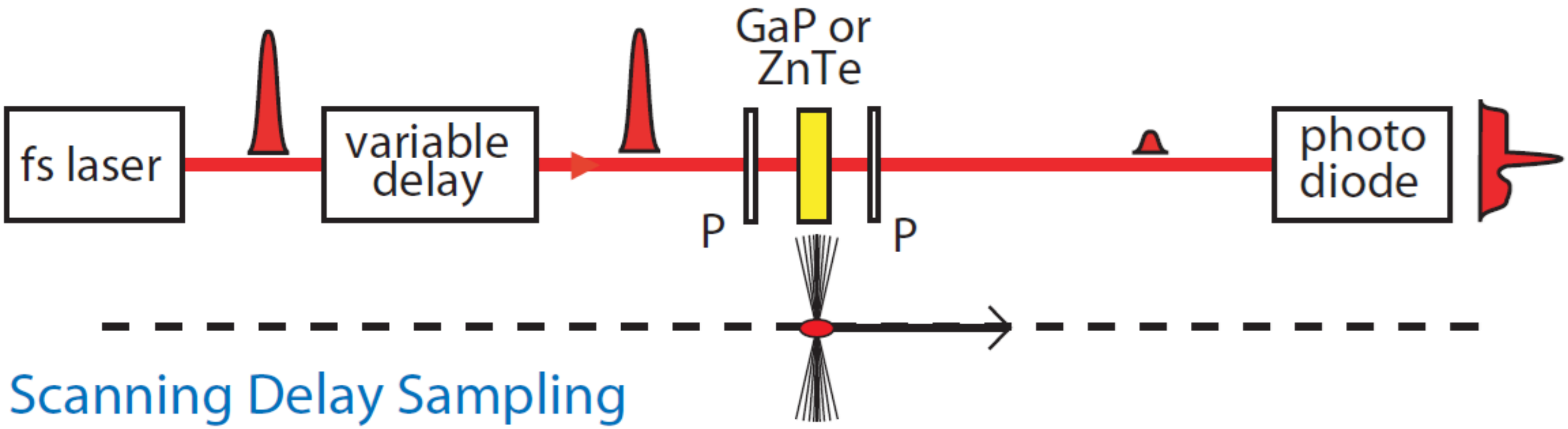
For Free Electron Lasers → bunch length below 1 ps is achieved

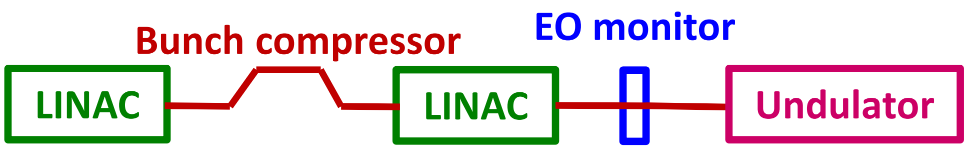
- Below the resolution of streak camera
- Short laser pulses with ***t*** **≈ 10 fs** and electro-optical modulator

Electro optical modulator: Birefringent, rotation angle depends on external electric field

Relativistic electron bunch: transverse ele. field $E_{\perp,lab} = \gamma E_{\perp,rest}$ carries the time information

Scanning of delay between bunch and laser → time profile after several pulses.





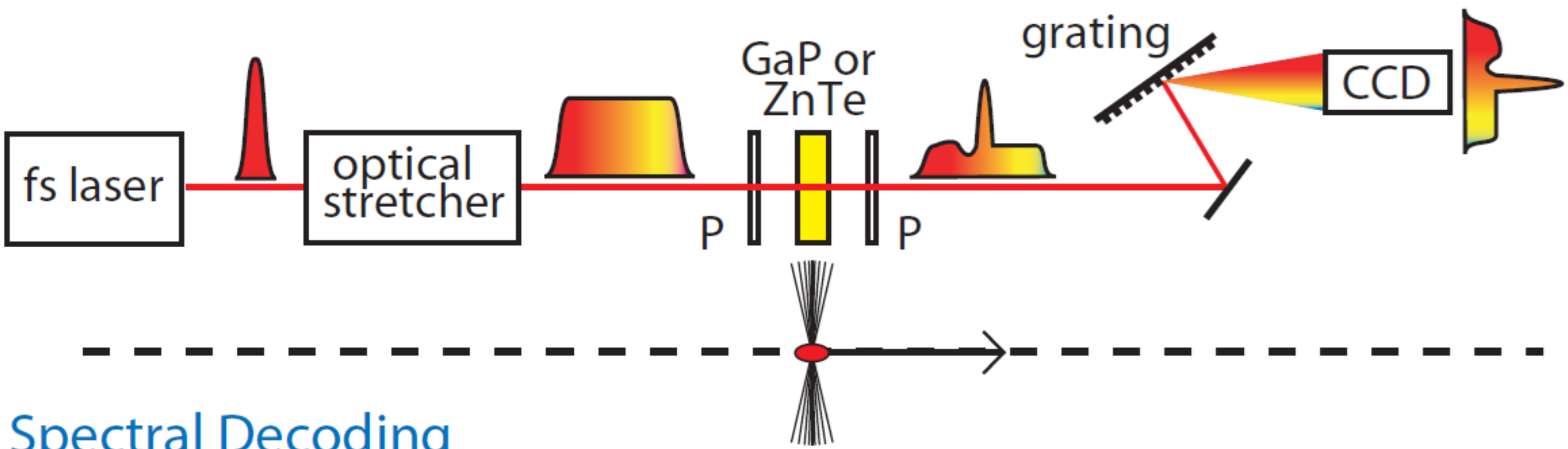
Courtesy S.P.Jamison et al., EPAC 2006

For Free Electron Lasers → bunch length below 1 ps is achieved

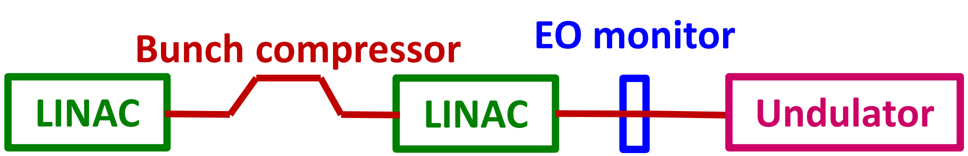
Short laser pulse ⇔ broad frequency spectrum (property of Fourier Transformation)

Optical stretcher: Separation of colors by different path length

⇒ different colors at different time ⇒ **single-shot observation**

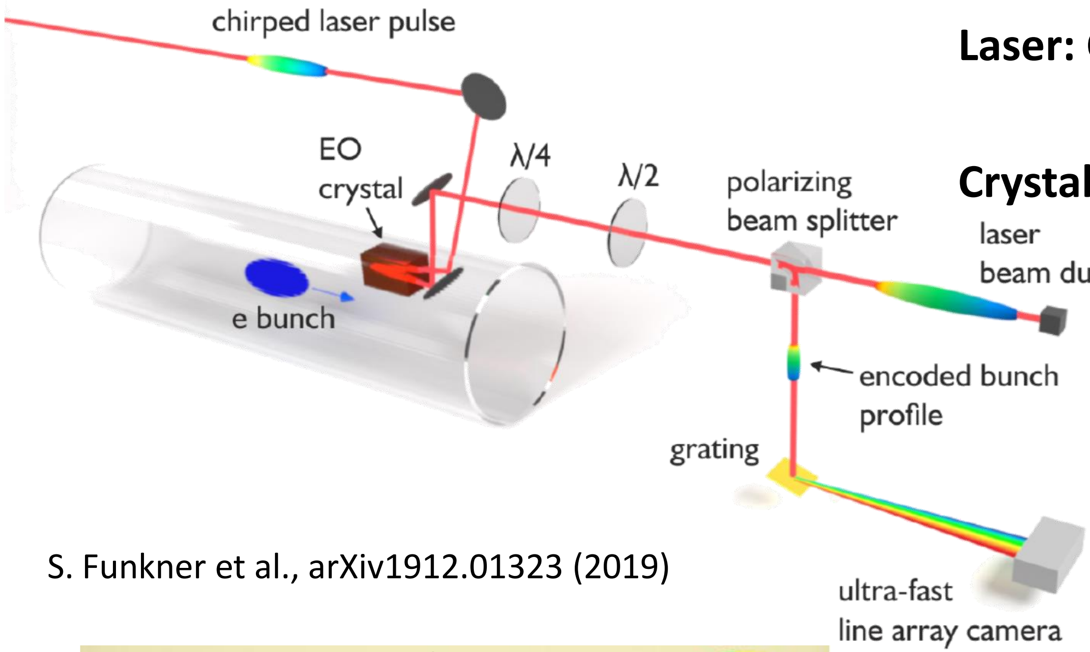


Spectral Decoding



Courtesy S.P.Jamison et al., EPAC 2006

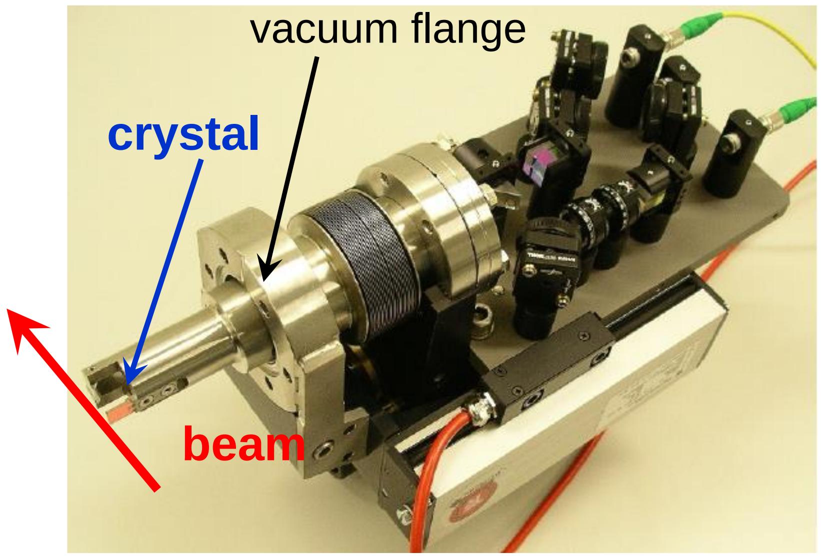
Hardware of a spectral-decoded EOSD Scanning Setup

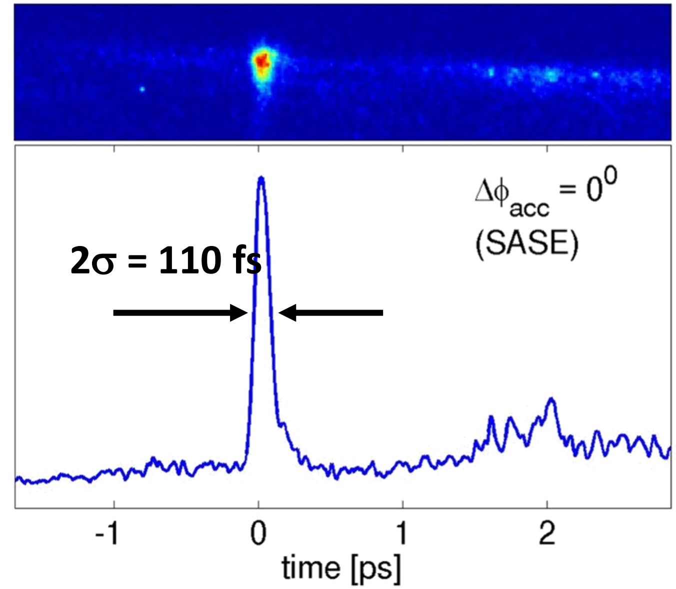


Laser: Commercial Ti:Sa or Yb-fibre,
10 fs duration, near IR,
Crystal: GaP or ZnTe, 100 μm thickness

S. Funkner et al., arXiv1912.01323 (2019)

Example: Bunch length at FLASH
100 fs bunch duration = 30 μm length!





B. Steffen et al, DIPAC 2009
B. Steffen et al., Phys. Rev. AB 12, 032802 (2009)

Poll 6.6:

Optical synchrotron light is emitted from …

1) electrons only
2) electron everywhere along their path
3) highly relativistic neutral particles if the Lorentz factor is larger than 1000
4) any charged particles on a curved trajectory

Poll 6.7:

A question related to a typical bunch length at circular synchrotron light sources with $\boldsymbol{f_{rf}}$=500 MHz: What is the time resolution of a typical streak camera?

1) 1 fs
2) 1 ps
3) 1 ns

www.menti.com

code: 1208 6026

Poll 6.8:

The short bunches with 100 fs length at FEL-facilities are measured with electro-optical methods. Could the same method be applied for 10 MeV proton beams with a bunch length of 100 ps?

1) **Yes**, assuming the number of particle per bunch is sufficient

2) **Yes**, but the method is very cumbersome and more simple method are available

3) **No**, protons at that energy don't emit synchrotron light in the optical range

4) **No**, the transverse electric field does not represent the particle distribution

Summary of longitudinal Measurements

Longitudinal ↔ transverse correspondences:

- position relative to rf ↔ transverse center-of-mass
- bunch structure in time ↔ transverse profile in space
- momentum or energy spread ↔ transverse divergence.

Determination uses:

Broadband pick-ups:
- position relative to rf, mean energy
- emittance at transfer lines or synchrotron via tomography
  assumption: bunches longer than pick-up.

Particle detectors:
- TOF or secondary e^- from wire
  → for non-relativistic proton beams
  reason: ***E***-field does not reflect bunch shape.

Streak cameras:
- time resolved monitoring of synchrotron radiation
  → for relativistic e^--beams, $t_{bunch} < 1$ ns
  reason: too short bunches for rf electronics.

Laser scanning:
- Electro-optical modulation of short laser pulse
  → very high time resolution

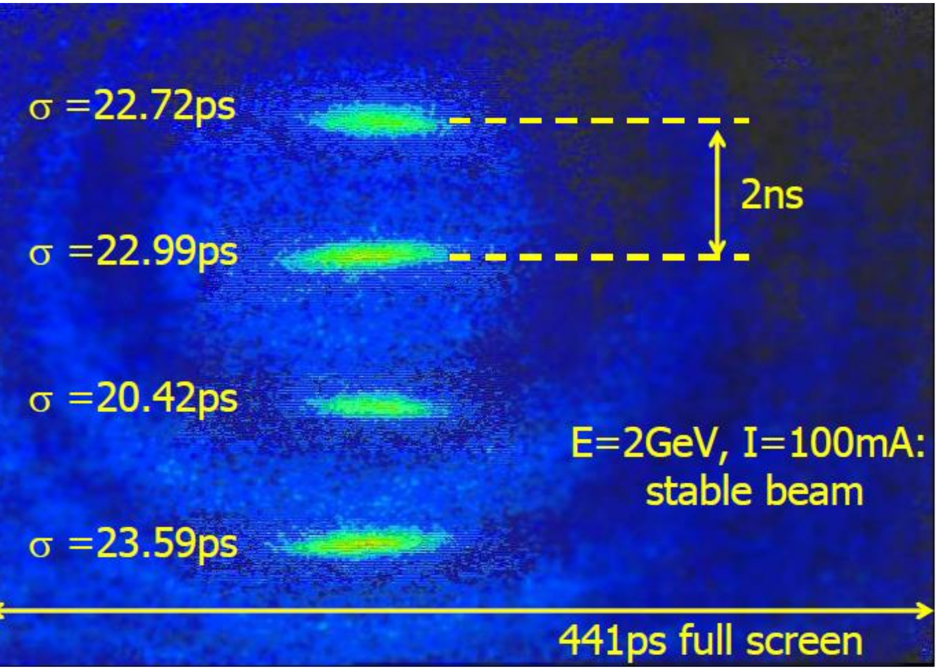
Beam deflection:
- Transverse deflection of primary beam
  → very high time resolution, but most expensive 'device'.

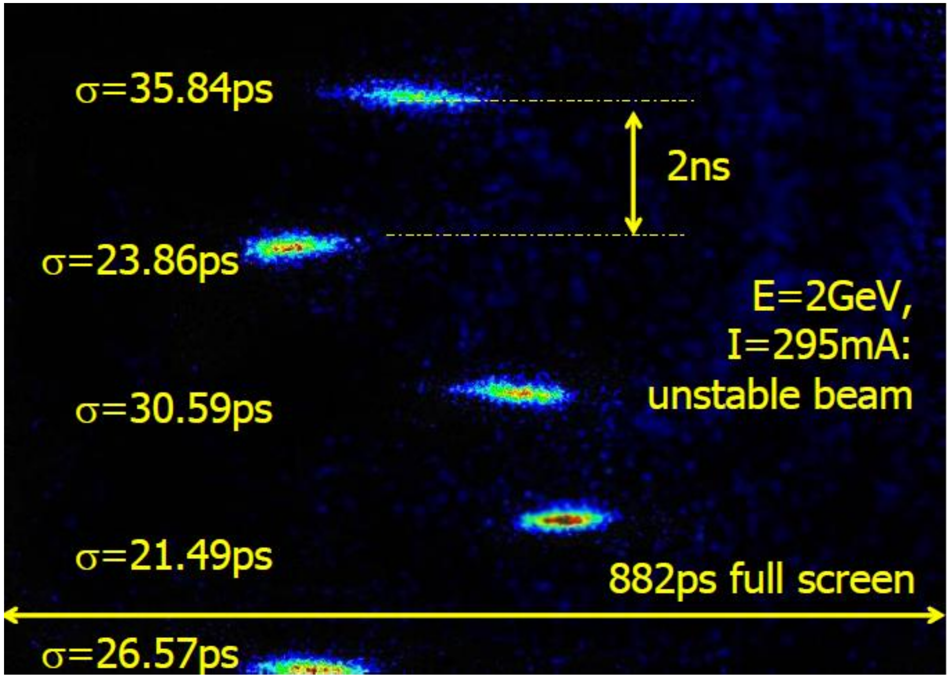
Backup slides

Bunch observation by Streak Camera

Longitudinal mismatched injection lead to coherent synchrotron oscillations, coupled bunch motion and bunch shape oscillations

Example: Bunch shape oscillations at Elettra, Trieste:





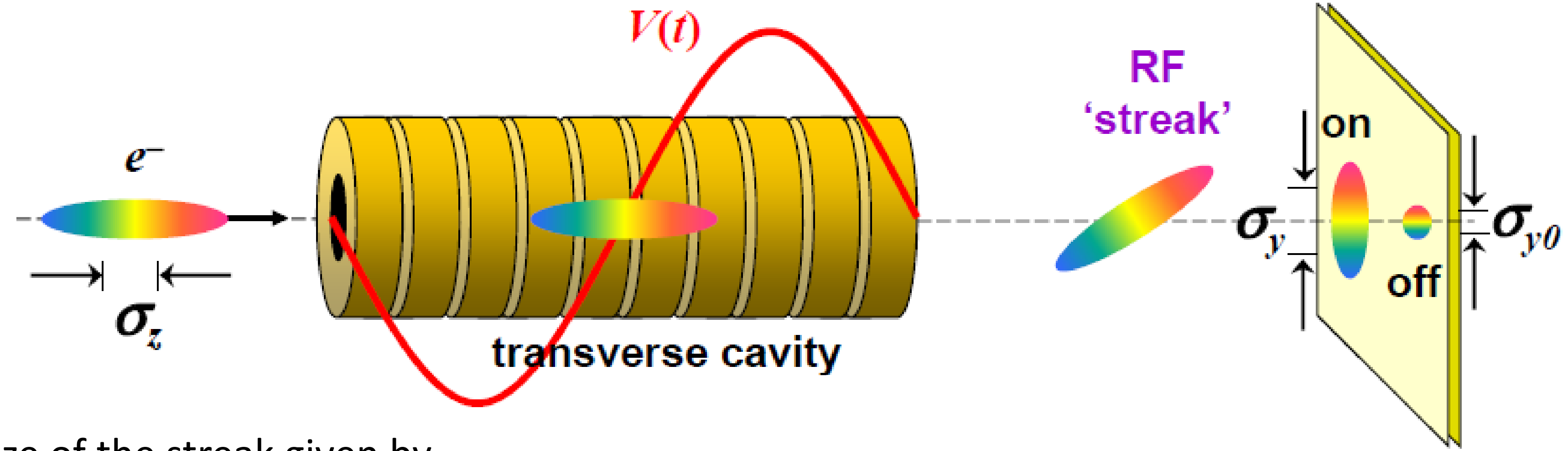
From M. Ferianis, CAS'07

Outline:

- Definition of longitudinal phase space
- Proton LINAC: Determination of mean energy
  used for alignment of cavities phase and amplitude
- Determination of longitudinal emittance
  LINAC: variation of bunch length
  Synchrotron: Topographic reconstruction
- Bunch length measurement for non-relativistic beams
  Determination of particle arrival
- Bunch length measurement for relativistic beams
  Synchrotron light monitor and electro-optical modulation of a laser beam
- **Bunch length from beam deflection by rf cavity**
- **Summary**

Bunch Length by rf-Deflection: Principle

Transversal deflection of the bunch i.e. time-to-space conversion

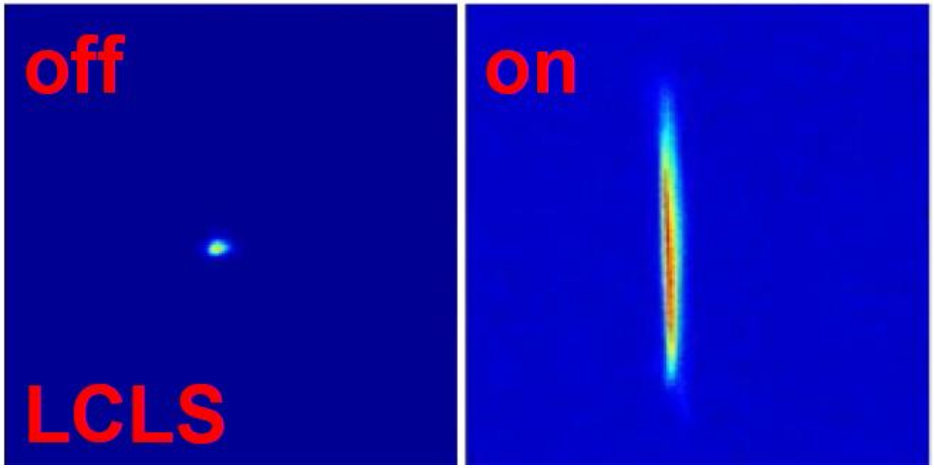


Size of the streak given by

$$\sigma_y = \sqrt{\sigma_{y0}^2 + R_{35} \cdot k \cdot \sigma_z^2}$$

k is determined by the rf-power

$$k = \frac{2\pi e \cdot U_{rf}}{\lambda_{rf} E}$$



From D. Xiang, IPAC'12

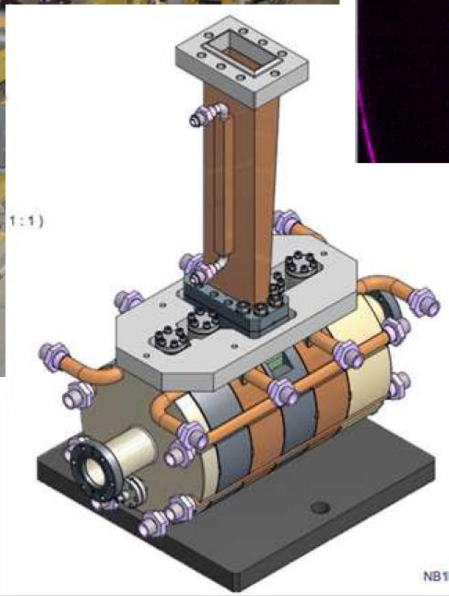
Bunch Length by rf-Deflection: Hardware

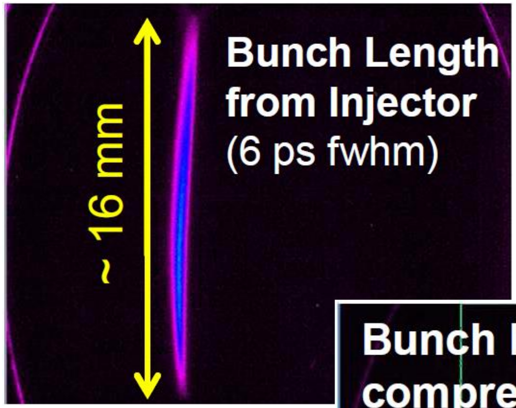
Transversal deflection of the bunch

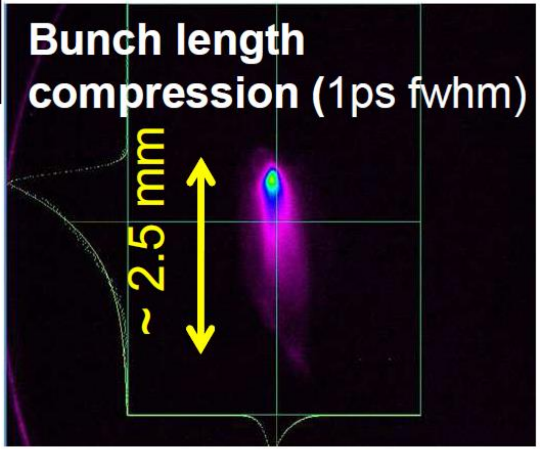
i.e. time-to-space conversion

Example: Cavity at FERMI, Trieste, Italy

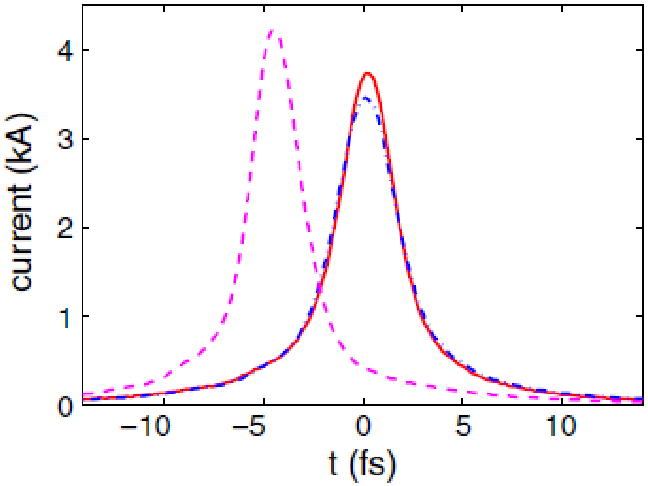
Beam energy	320 MeV
Typical beam size	0.2 mm
Length	0.5 m
Frequency	2.998 GHz
Max. rf power	5 MW
Total trans. volt.	4.9 MV
Time resolution	70 fs

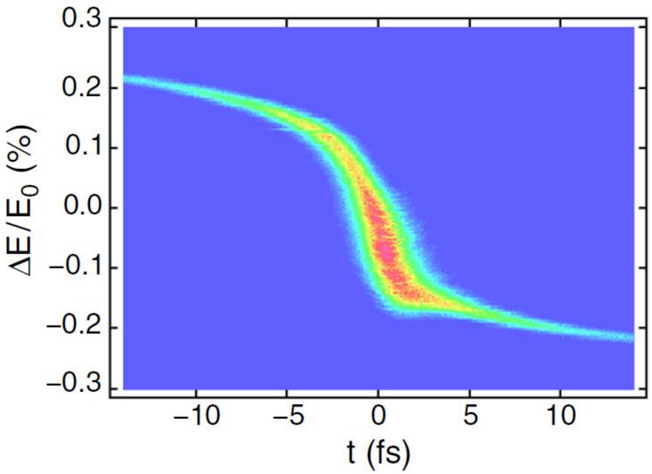






From M. Veronese, BIW'12





From D. Xiang, PRST-AB, 13, 094001 (2010)

Example: Short bunch generation at Free Electron Laser Facility LCLS, Stanford, USA

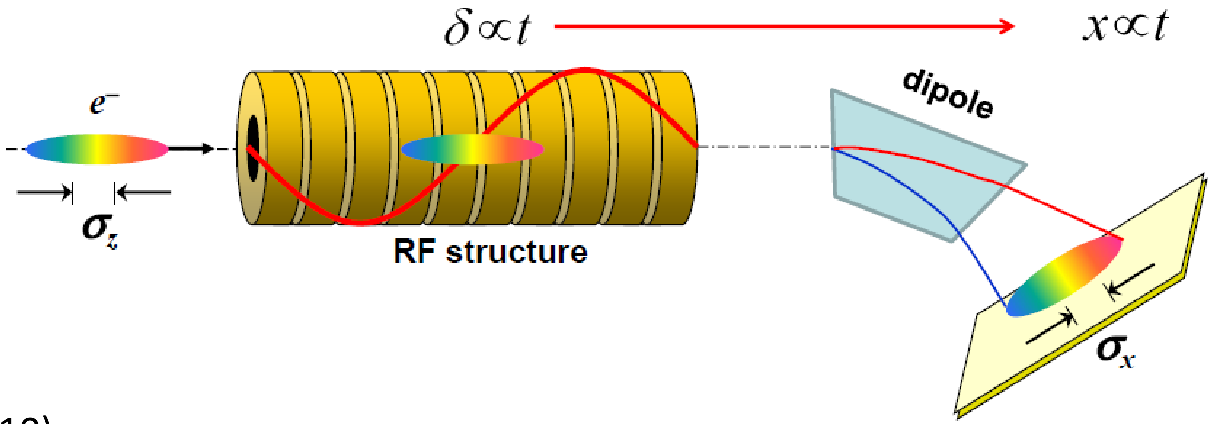
Time resolution down to (true) fs range!

Vertical deflection with cavity → time spectum

Horizontal bending

→energy (momentum) distribution via dispersion

⇒ Longitudinal phase space determined!



Broadband coaxial Faraday Cups for Bunch Structure

The bunch structure can be observed with cups, having a bandwidth up to several GHz.

Bandwidth and rise time: BW [GHz] = **0.3/t_{rise}**[ns]

Impedance of a

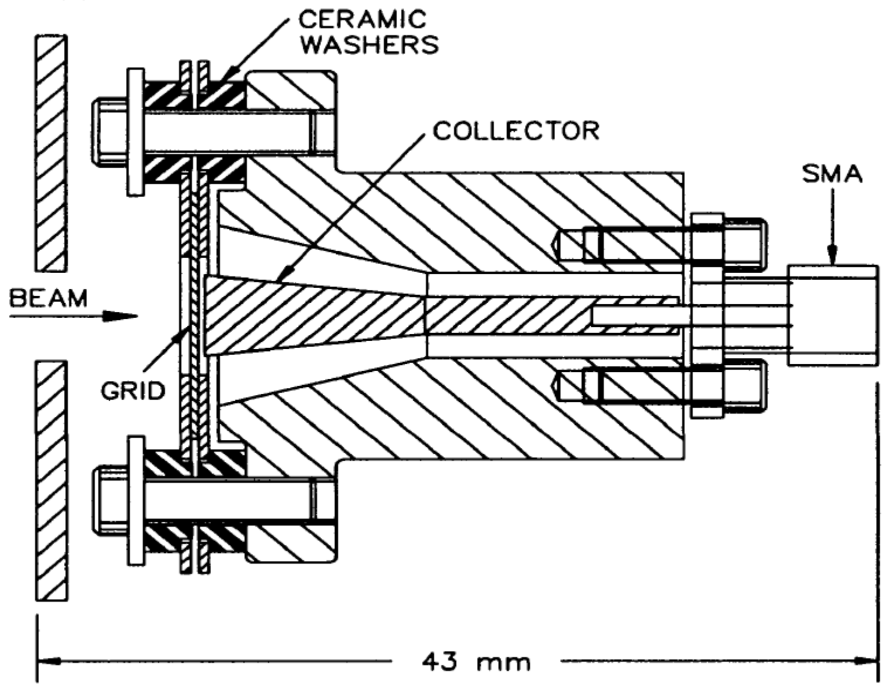
coaxial transmission line:

$$Z_0 = \frac{Z_c}{2\pi} \cdot \ln \frac{r_{\text{shield}}}{r_{\text{coll}}}$$

with $Z_c = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}$

for vacuum $Z_C = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega$

$\rightarrow$ impedance matching to prevent for reflections



Voltage reflection: $\rho_V = \frac{Z - Z_0}{Z + Z_0}$

Voltage Standing Wave Ratio: $\text{VSWR} = \frac{Z}{Z_0} = \frac{1+\rho_V}{1-\rho_V}$

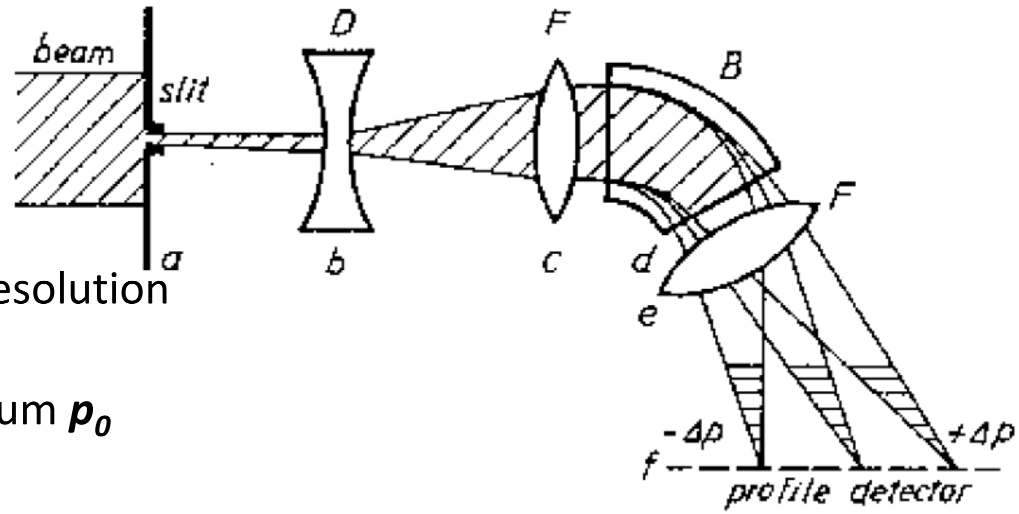
***Z* = *Z*$_0$**: no reflection. ***Z* = 0** $\Rightarrow$ **ρ_V** = −1: short circuit. ***Z* = ∞** $\Rightarrow$ **ρ_V** = 1: open circuit.

Realization of a Broadband coaxial Faraday Cup

Measurement of Energy Spread by magnetic Spectrometer

Transfer line: The mom. spread $\boldsymbol{\delta = \Delta p/p}$ can be determined by a magnetic spectrometer: via dispersion, the momentum is shifted to a spatial distance.

An appropriate optic must b e chosen to separate the transverse and longitudinal parameters

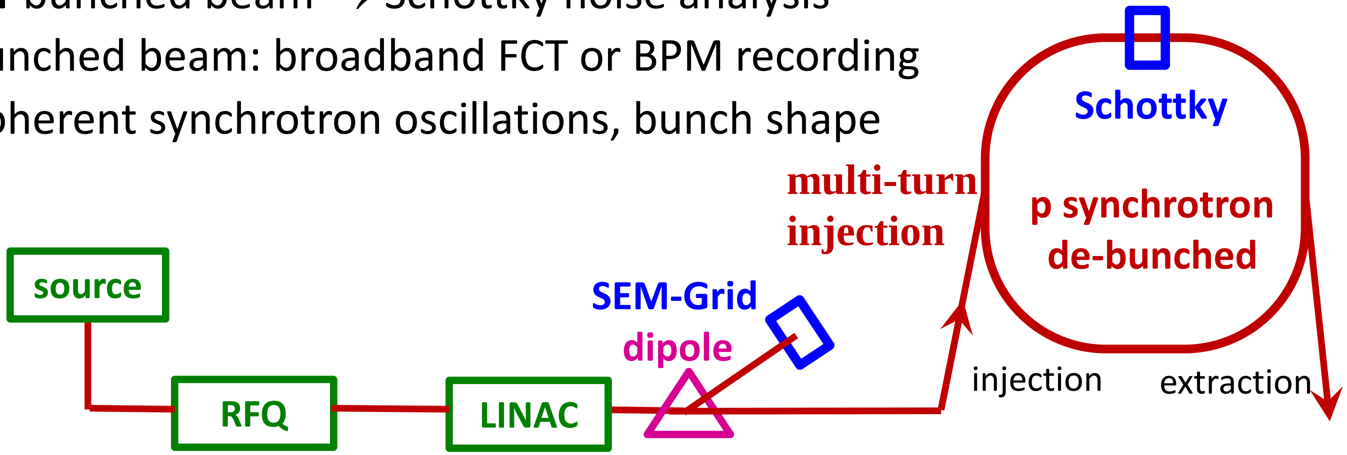


However, a synchrotron is a very high resolution spectrometer

Goal: Measurement of central momentum $\boldsymbol{p_0}$

and momentum spread $\Delta \boldsymbol{p} / \boldsymbol{p_0}$

- **un**-bunched beam → Schottky noise analysis
- bunched beam: broadband FCT or BPM recording
  coherent synchrotron oscillations, bunch shape



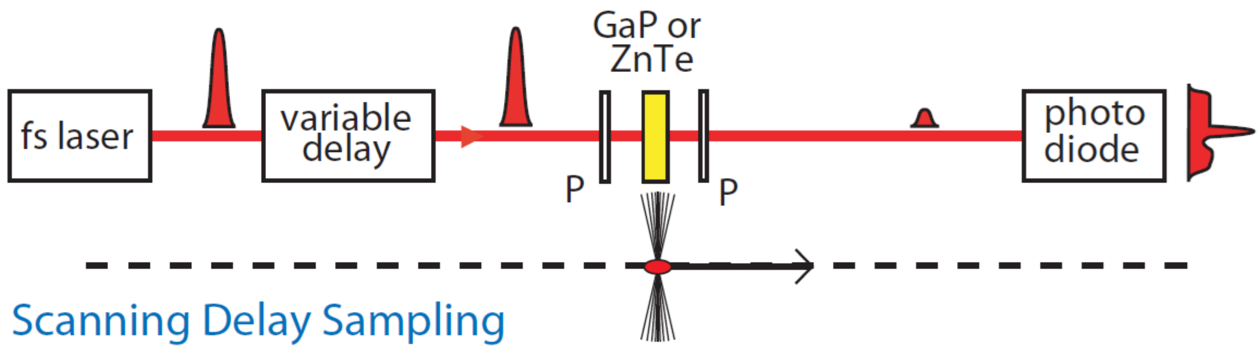
FELs → bunch length below 1 ps is achieved, i.e. below the resolution of streak camera

- Short laser pulses with ***t*** **≈ 10 fs** and electro-optical modulator

Electro optical modulator: birefringent, rotation angle depends on external electric field

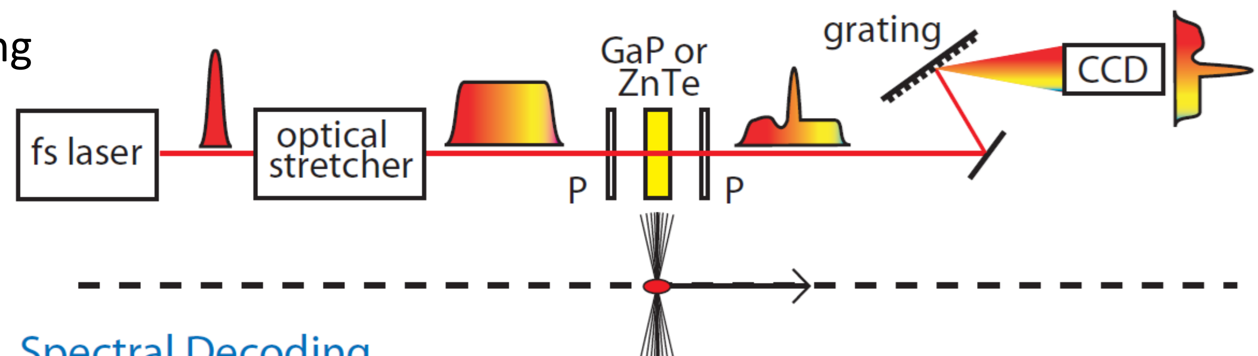
Relativistic electron bunches: transverse field $\boldsymbol{E_{\perp\, lab} = \gamma E_{\perp\, rest}}$ carries the time information.

Measurement by **scanning**
Short laser pulses ***t*** **< 10 fs**
and delay line
⇒ scanning method



Scanning Delay Sampling

Measurement spectral decoding
Short laser pulses ***t*** **< 10 fs**
has broad frequency spectrum
and delay line
⇒ **single shot** determination



Spectral Decoding

Further methods used!

From S.P.Jamison et al., EPAC 2006