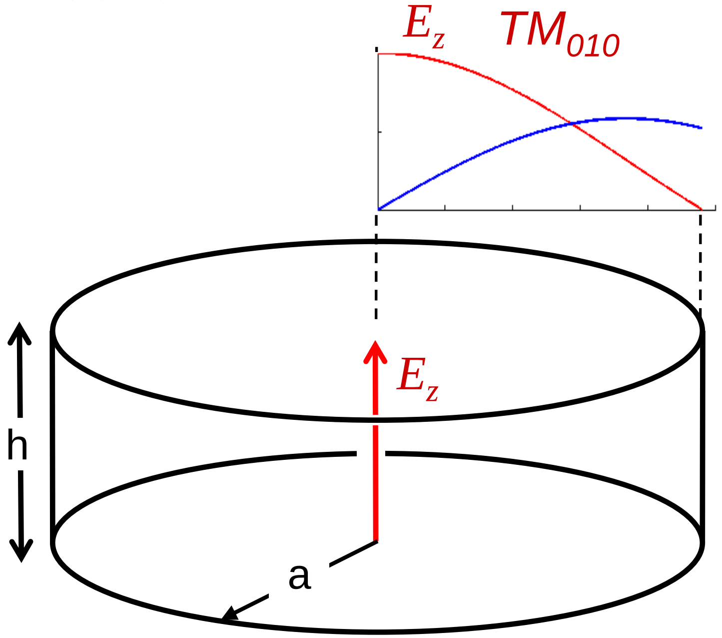
RF Engineering
Pillbox Resonator II
(& some small remarks)

Christine Völlinger (CERN) & Manfred Wendt (BNL)

Pillbox Cavity Resonators (5/6)

Picture source: Caspers,
JUAS lecture 2021



For the case of the TM_{010}-mode, we have no dependence on cavity height, so we get a much simpler formulae:

$$\boxed{0.383\,\lambda_{\mathrm{TM},010} = a}$$

$$Q = \left(0.383\frac{\lambda_{\mathrm{TM},010}}{\delta}\right)\left[1 + \left(0.383\frac{\lambda_{\mathrm{TM},010}}{h}\right)\right]^{-1}$$

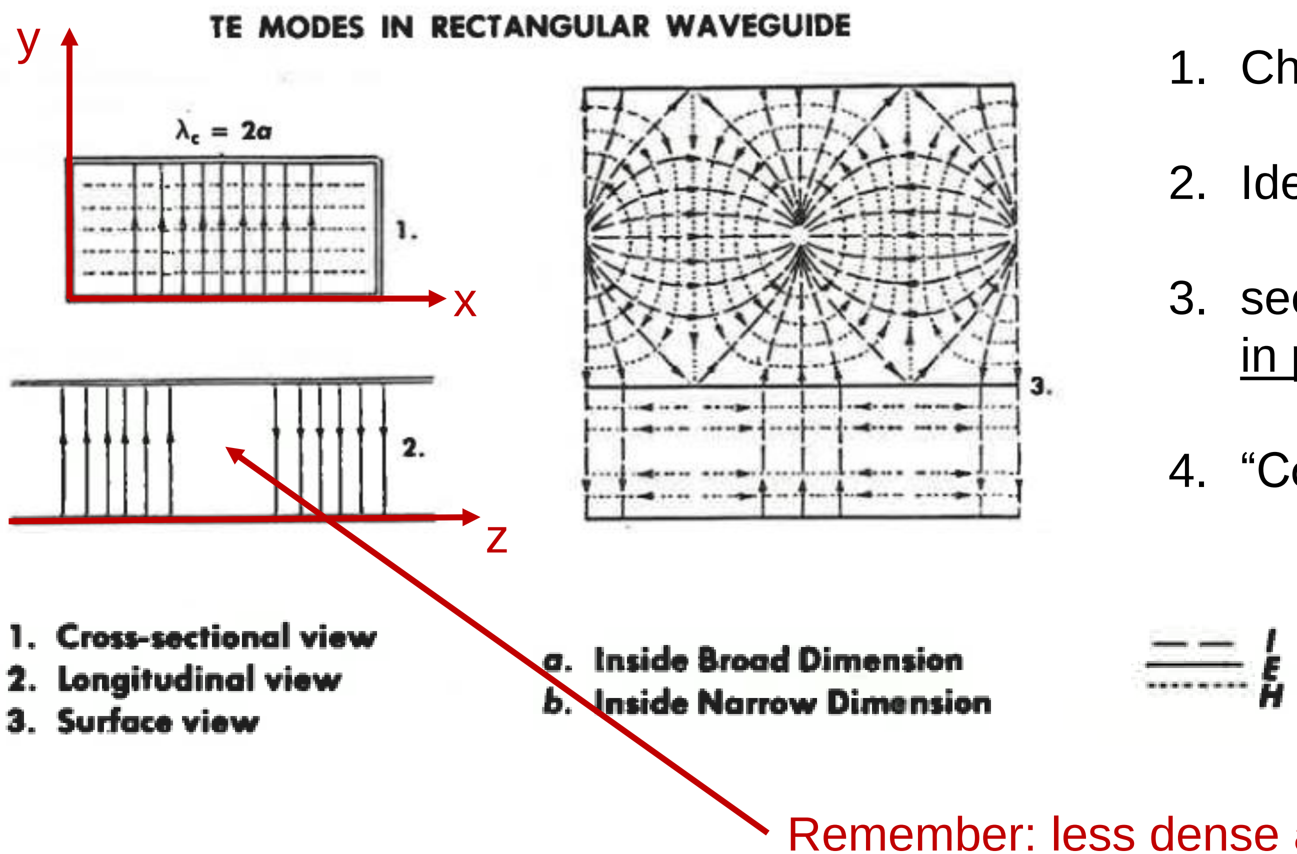
$$\boxed{Q = \frac{0.383\,\lambda_{\mathrm{TM},010}}{\delta}\left[1 + \frac{a}{h}\right]^{-1} = \frac{a}{\delta}\left[1 + \frac{a}{h}\right]^{-1}}$$

Typo in the slide of yesterday (now corrected)

Remember? Skindepth is: $\delta = \sqrt{\dfrac{2}{\omega\sigma\mu}}$

Waveguide Mode Indices

Mode indices or “**how to see which mode it is?** (besides measuring the frequency…)”

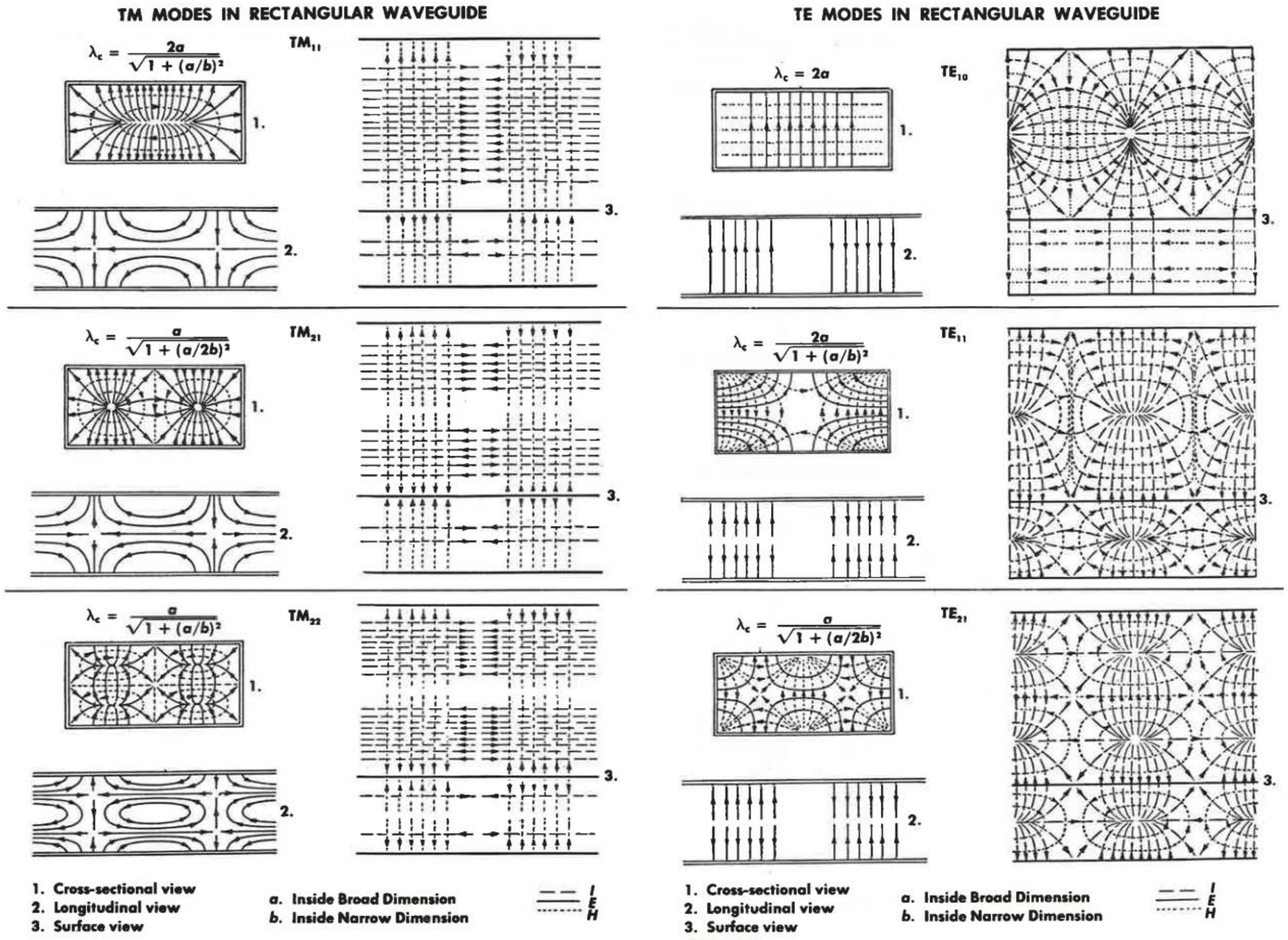


1. Chose your coordinate system.
2. Identify the electric field, and
3. see how the electric field is oriented <u>in propagation direction</u>.
4. “Count the zeros” in the respective directions.

Remember: less dense arrows means lower field strength!

Pictures taken from T.S. Saad, “Microwave Engineer’s Handbook”

Waveguide Mode Indices

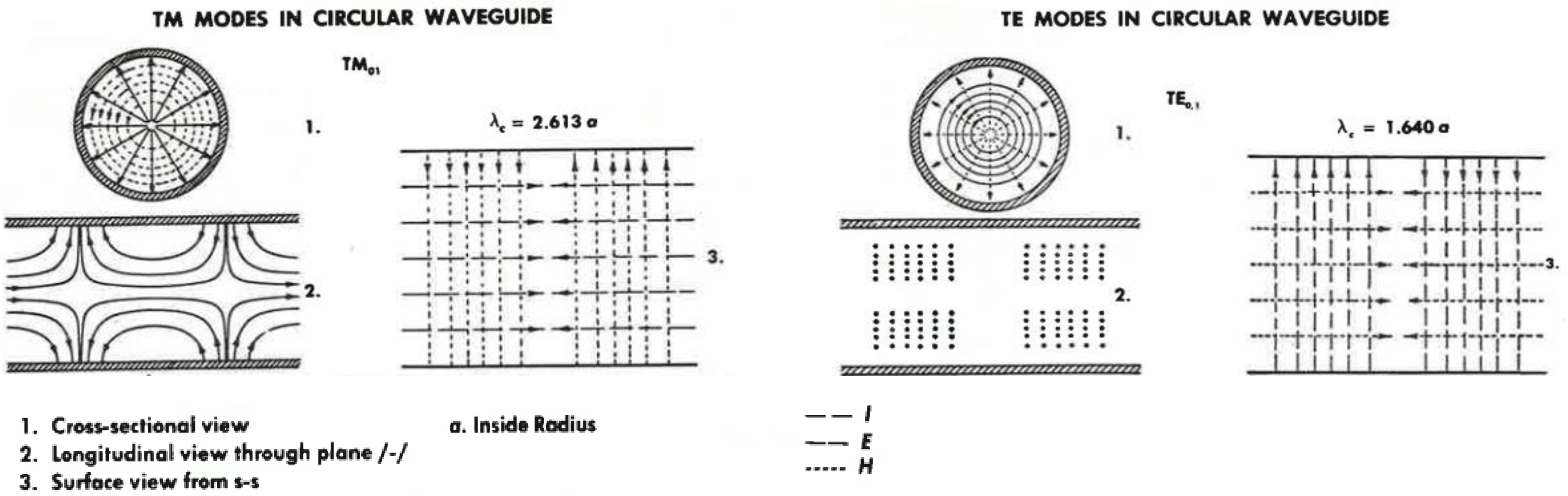


Pictures taken from T.S. Saad, “Microwave Engineer’s Handbook”

…some more field patterns…

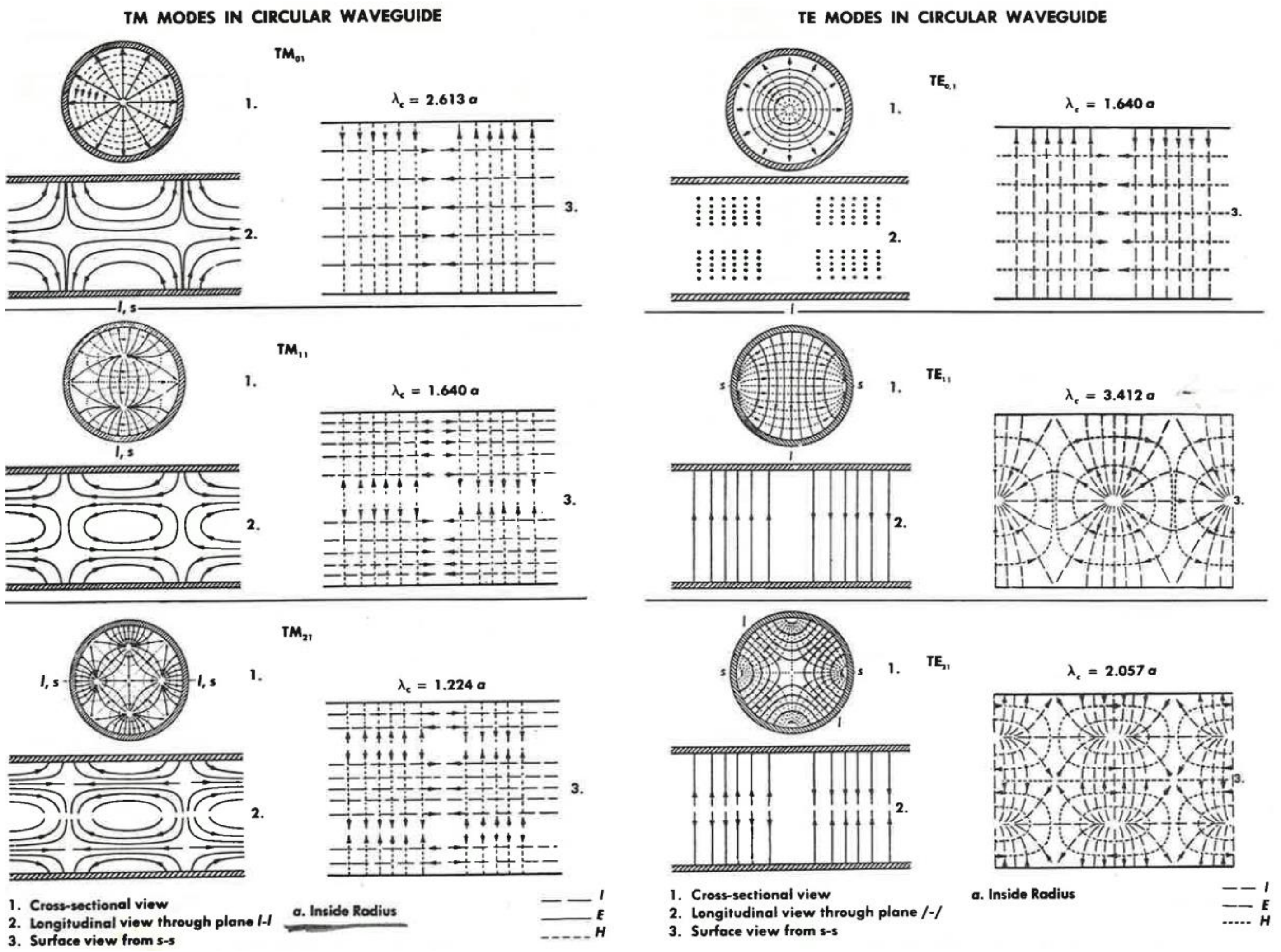
Waveguide Mode Indices

Pictures taken from T.S. Saad, “Microwave Engineer’s Handbook”



For circular waveguide → more difficult to see, but we can at least easily identify the TE/TM modes, and then “count” for f, r as indices.

Waveguide Mode Indices

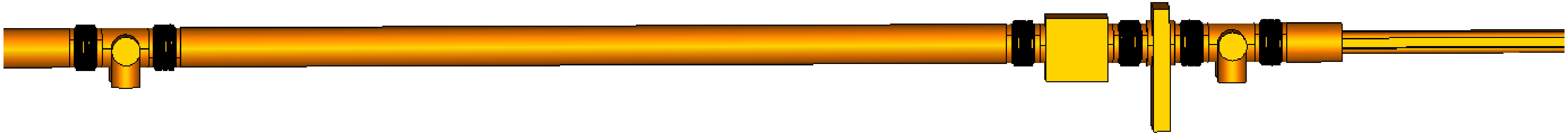


Pictures taken from T.S. Saad, “Microwave Engineer’s Handbook”

…some more field patterns…

Remark to cut-off frequency… (1/2)

Why did we so stress the cut-off frequency calculations?



Example:

Bellows are everywhere in your vacuum beam pipe to allow alignment and to compensate longitudinal movements.

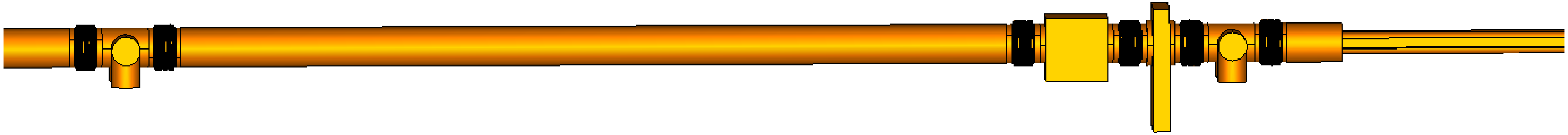
Bellows are slightly larger than the beam pipe itself → in the area of the bellow, the cut-off frequency is a bit lower → new mode will build up, but cannot propagate.

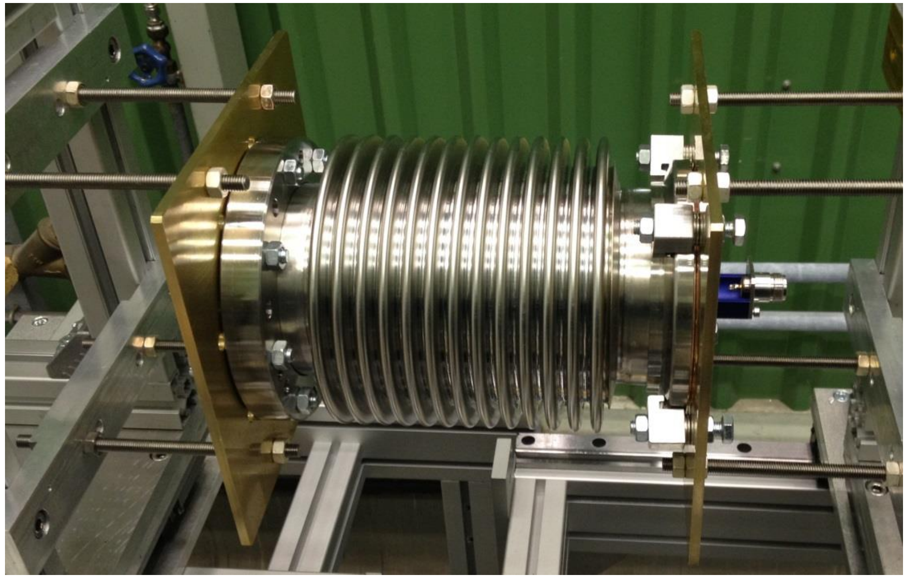
It will build a resonance…. Hanging in the bellow… seen by the next particle bunch passing the vacuum beam pipe…

This is called a “trapped mode”. The only mitigation is to shield the bellow!

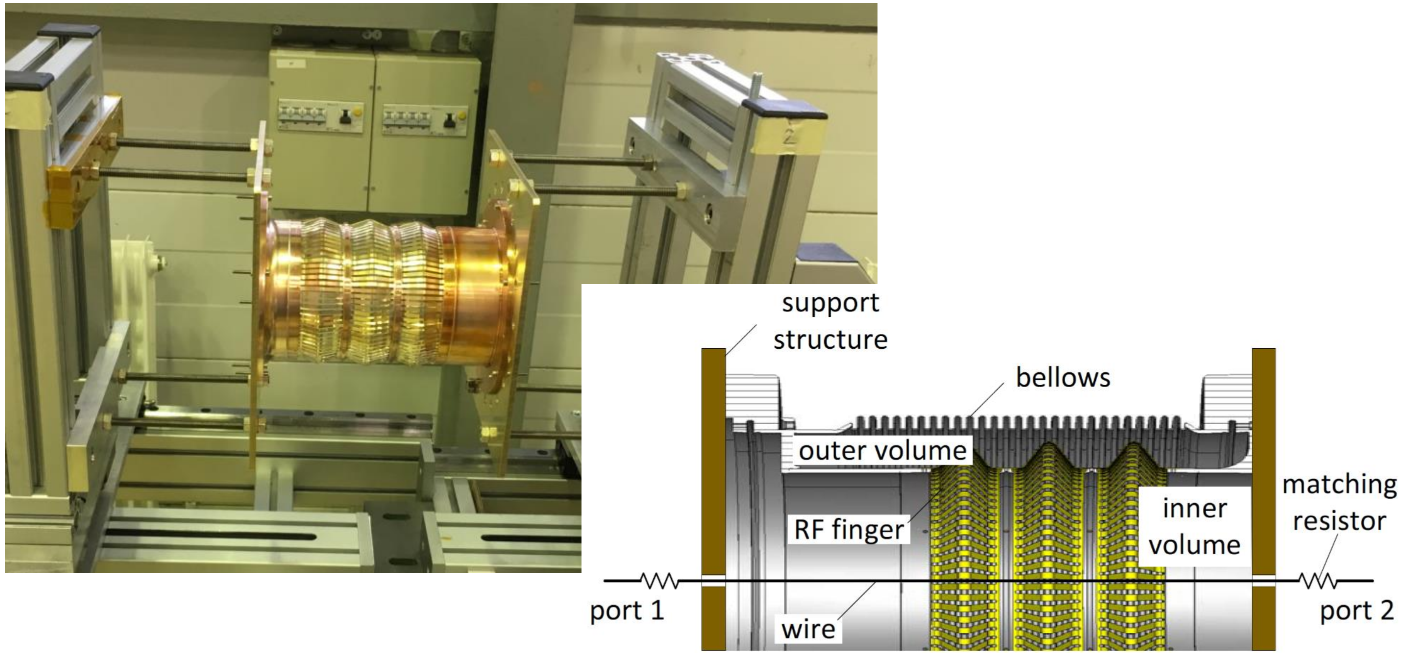
Remark to cut-off frequency… (2/2)

Calculating the cut-off frequency of the beam pipe and its bellows gives you a good idea of which modes you can expect.



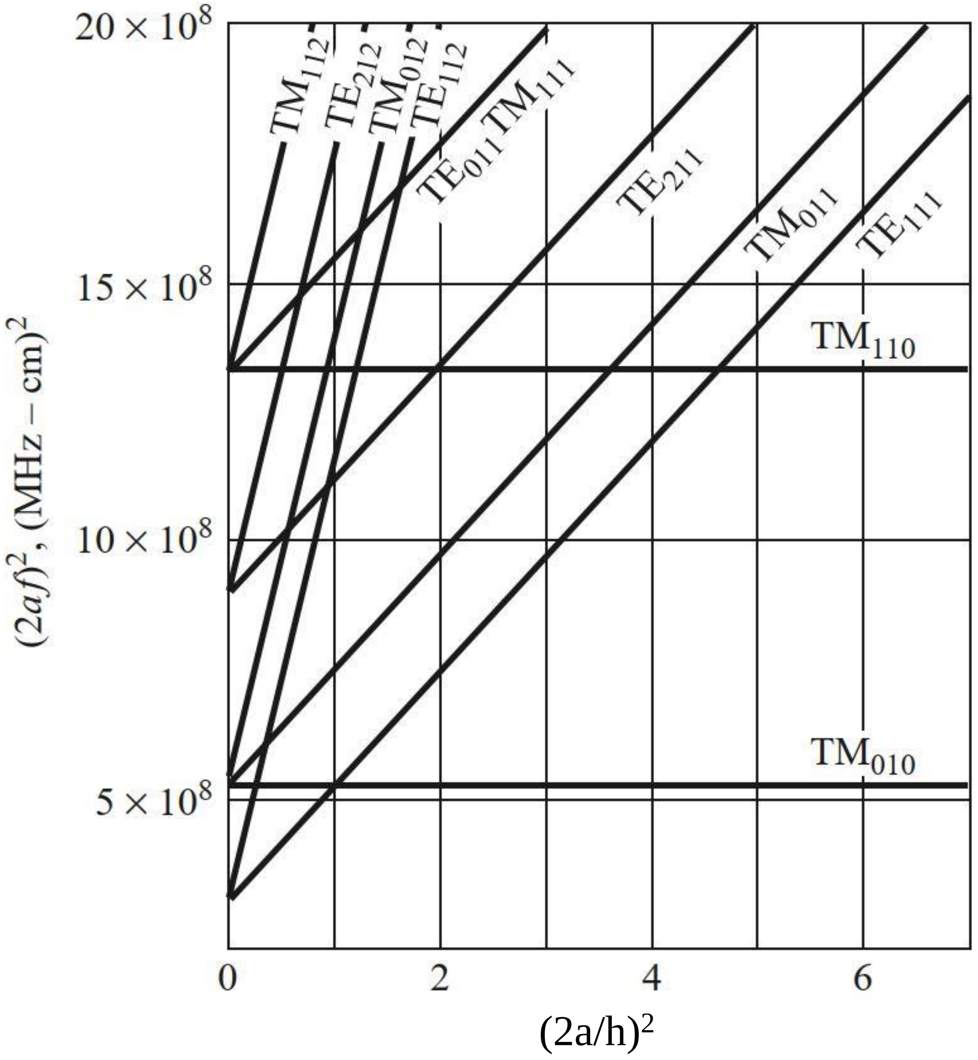


RF-shield design from CERN,
TE-VSC, J. Perez Espinos, C. Garion



Typical problem for beam coupling impedance contribution → see more on Monday, next week!

Pillbox Cavity resonators (6/6, of yesterday)



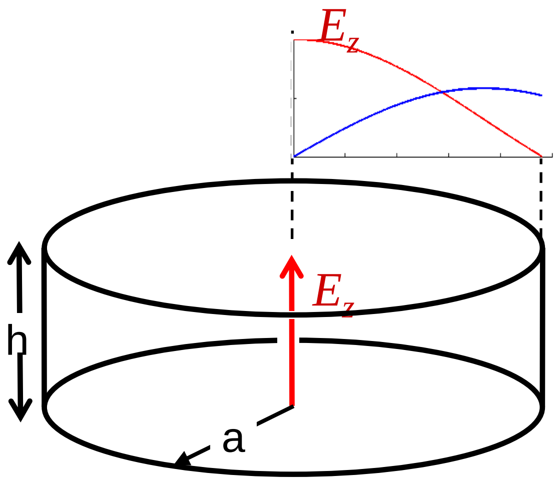
Resonant chart for a general cylindrical cavity showing the excited modes as a function of cavity dimensions.

→ 1st TE-mode is TE_{111}

→ 1st TM-mode is TM_{010},
and shows up for ratios $(2a/h)^2 > 1$

Source: Pozar, *Microwave engineering*, 4th ed., Wiley

Pillbox Cavity Resonators (1/2)



Remember?

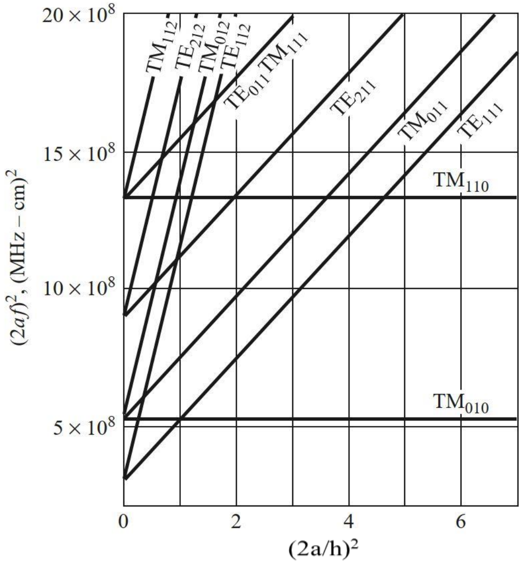
- Mode used for acceleration is TM010.
- Resonant frequencies for TE$_{nml}$-mode:

$$f_{\mathrm{TE},nml} = \frac{c_0}{2\pi}\sqrt{\left(\boxed{\frac{p'_{nm}}{a}}\right)^2 + \left(\frac{l\pi}{h}\right)^2}$$

- Resonant frequency for TM$_{nml}$-modes:

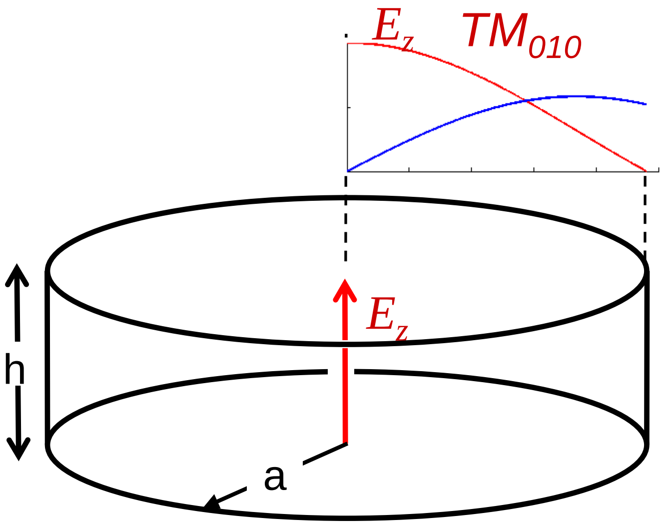
$$f_{\mathrm{TM},nml} = \frac{c_0}{2\pi}\sqrt{\left(\boxed{\frac{p_{nm}}{a}}\right)^2 + \left(\frac{l\pi}{h}\right)^2}$$

Roots of B.F.



Note that the TM$_{010}$- mode is *independent of the cavity height h* (until the TE$_{111}$ mode shows up, roughly at *2a/h=1*).

Pillbox Cavity Resonators (2/2)



For the case of the TM_{010}-mode, when we have no dependence on cavity height, we can use simpler formulae for the wavelength:

$$0.383\,\lambda_{\mathrm{TM},010} = a$$

And we can use the Q-calculation (equally simplified!):

$$Q = \frac{0.383\,\lambda_{\mathrm{TM},010}}{\delta}\left[1+\frac{a}{h}\right]^{-1} = \frac{a}{\delta}\left[1+\frac{a}{h}\right]^{-1}$$

Skindepth: $\delta = \sqrt{\frac{2}{\omega\sigma\mu}}$

$\eta_0 = 120\,\pi\,\Omega$

Accelerator-figure-of-Merit:

$$R/Q = \frac{4\eta_0}{\pi\,p_{01}^3\,J_1^2(p_{01})}\,\frac{\sin^2\left(\frac{p_{01}}{2}\frac{h}{a}\right)}{h/a}$$

$p_{01} = 2.405$

$J_1(p_{01}) = 0.51911$

$$R/Q = 128\,\frac{\sin^2\left(\frac{p_{01}}{2}\frac{h}{a}\right)}{h/a} \approx 185\,h/a$$

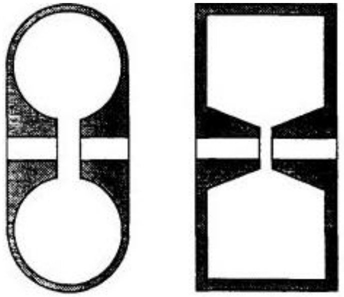
For small arguments of Sinus-function

Source: Pozar, *Microwave Engineering*, 4th ed., Wiley

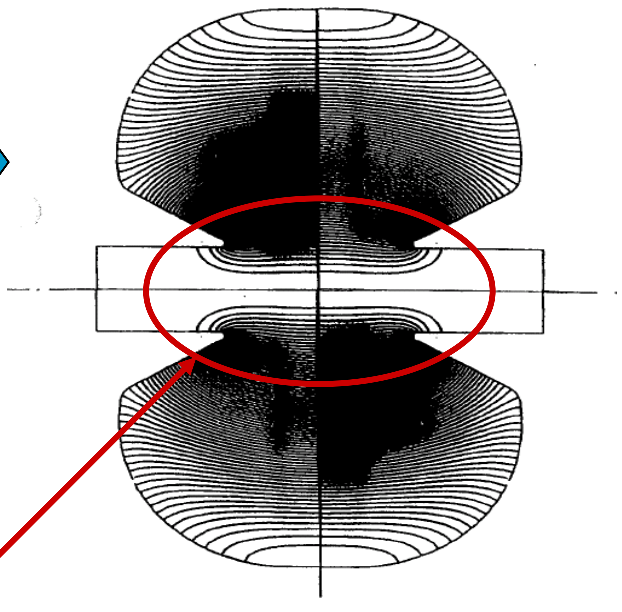
Pillbox Cavity Design Feature

In practice, a “pure” pillbox cavity is not very efficient for acceleration.
A simple shape modification can be done by using so-called “nose cones”.

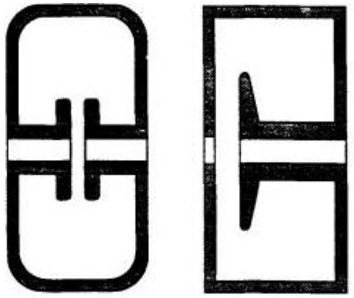
Nose cone is a protrusion on the cavity wall that causes a concentration of the electrical field in the gap.



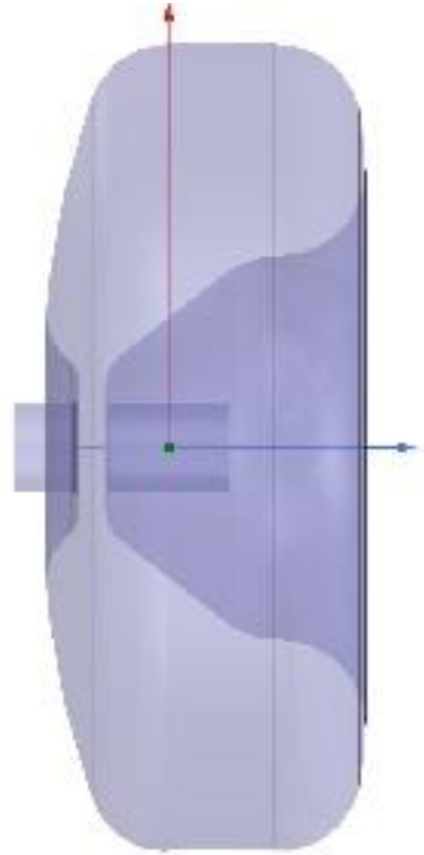
Source: Puglisi,
RF-CAS 92, CERN



Enhancement of E-field



Source: Puglisi,
RF-CAS 92, CERN

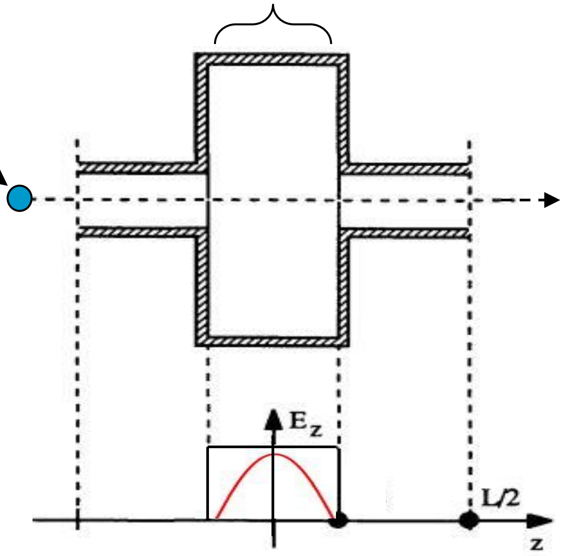


PS 80 MHz cavity

Transit Time Factor (1/2)

accelerating gap

particle with
velocity v



$E(0, z)$

Field along
the z-axis
(r=0)

$$\text{transit-time factor } T = \frac{\textit{energy gained in time-varying RF-field}}{\textit{energy gained in a DC field of voltage } V_0}$$

$$T = \frac{\int_{-L/2}^{L/2} E(0,z)\cos(2\pi z/(\beta\lambda))\,dz}{\int_{-L/2}^{L/2} E(0,z)\,dz}$$

Distance the particle travelled
in an RF-period

- Particle in the harmonic time-varying field will see less energy gain compared to a constant DC field.
- This is called transit-time effect and is described by a factor T (so-called transit-time factor).

remark
to cosine argument: $\omega t = \omega \frac{z}{v} = 2\pi f \frac{z}{\beta c_0} = \frac{2\pi z}{\beta\lambda}$

Transit Time Factor (2/2)

$$\boxed{T = \frac{\int_{-L/2}^{L/2} E(0,z)\cos(2\pi z/\beta\lambda)\,dz}{\int_{-L/2}^{L/2} E(0,z)\,dz}}$$

Transit time factor

$E(0,z)$ = constant

$$T = \frac{\int_{-L/2}^{L/2} \cos(2\pi z/\beta\lambda)dz}{\int_{-L/2}^{L/2} dz} = \frac{2\sin\left(\frac{2\pi}{\beta\lambda}\frac{L}{2}\right)\left(\frac{\beta\lambda}{2\pi}\right)}{L}$$

$$\boxed{T = \frac{\sin\left(\frac{\pi L}{\beta\lambda}\right)}{\frac{\pi L}{\beta\lambda}}}$$

gap length

- To achieve max. energy gain from this formulae, we want that $\boldsymbol{T = 1 \rightarrow L = 0}$
- Leads to design request: *gap as small as possible*.
- But other considerations as: risk of RF electric breakdown also impact on optimum gap geometry.
- Note that it is assumed that the particle does not change velocity along the gap length.

Accelerator efficiency figures-of-merit (1/2)

Several figures-of-merit are commonly used to characterize accelerating cavities:

- *Q-value*
  unloaded Q_0 - measure of the resonance quality $$Q_0 = \frac{\omega U}{P}$$

energy stored in the resonator

energy dissipated in the resonator

- *Shunt impedance [MΩ]*
  measure of effectiveness to produce an axial voltage V_0 $$r_s = \frac{V_0^2}{P}$$

Usual design goal is a high shunt impedance

- *Effective Shunt impedance [MΩ/m]*
  Measure of effectiveness per unit power loss to deliver energy to a particle $$r_{s,\mathrm{eff}} = \frac{(V_0 \dot{T})^2}{P} = r_s T^2$$

- *R-over-Q [Ω]*
  Measure of cavity acceleration efficiency at a given frequency $$r/Q = \frac{(V_0 \dot{T})^2}{\omega U}$$

Accelerator efficiency figures-of-merit (2/2)

Typical values for different cavities:

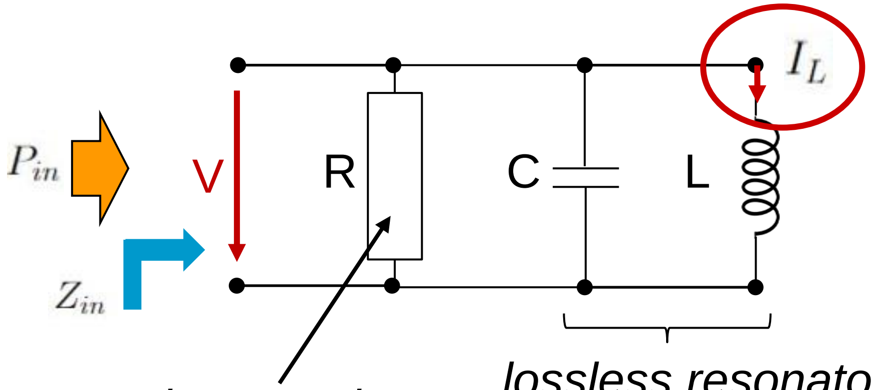
Cavity type	***R/Q***	**Q_0**	***R***
Ferrite loaded cavity (low frequency, rapid cycling)	4 kΩ	50	200 kΩ
Room temperature copper cavity (type 1 with nose cone)	192 Ω	$30 * 10^3$	5.75 MΩ
Superconducting cavity (type 2 with large iris)	50 Ω	$1 * 10^{10}$	500 GΩ

Following Caspers et al., JUAS 2021

Cavity equivalent circuit (1/4)

At frequency near resonance, the cavity resonator can be modeled by a lumped-element circuit.

For a cavity with the desired high shunt impedance, only a parallel resonant circuit is suited
→ require to model a large voltage.



lossy resistor

lossless resonator

$$P_{in} = \frac{1}{2} V I^* = \frac{1}{2}|V|^2 \frac{1}{Z_{in}^*}$$

power to the resonator circuit

$$Z_{in} = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right)^{-1}$$

input impedance

$$P_{in} = |V|^2 \left(\frac{1}{R} + j\frac{1}{\omega L} - j\omega C\right)$$

Power dissipated in the resistor: $P_{\text{loss}} = \frac{1}{2}\frac{|V|^2}{R}$

Energy stored in capacitor: $W_e = \frac{1}{4}|V|^2 C$

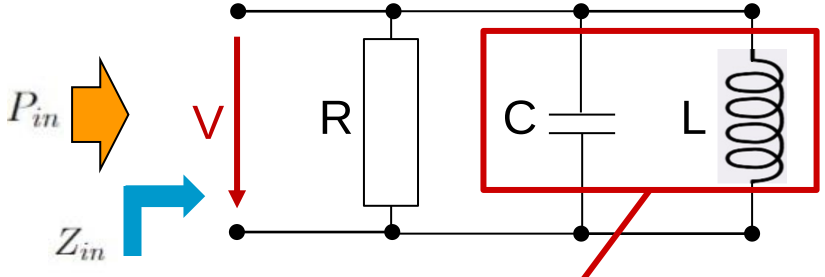
Energy stored in inductor: $W_m = \frac{1}{4}|I_L|^2 L = \frac{1}{4}|V|^2 \frac{1}{\omega^2 L}$

Cavity equivalent circuit (2/4)

At frequency near resonance, the cavity resonator can be modeled by a lumped-element circuit.

For a cavity with the desired high shunt impedance, only a parallel resonant circuit is suited
→ require to model a large voltage.

resonance case is: $W_e = W_m$



$$P_{in} = |V|^2\left(\frac{1}{R} + j\frac{1}{\omega L} - j\omega C\right) = P_{\text{loss}} + 2j\omega\cancel{(W_e - W_m)}$$

power to the resonator circuit

Power dissipated in the resistor: $P_{\text{loss}} = \frac{1}{2}\frac{|V|^2}{R}$

$$Z_{in} = \frac{P_{\text{loss}}}{\frac{1}{2}|I|^2} = R$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

$$\boxed{Q_0 = \omega_0\frac{2W_m}{P_{\text{loss}}} = \frac{R}{\omega_0 L} = \omega_0 RC}$$

☺ trivial!

Cavity equivalent circuit (3/4)

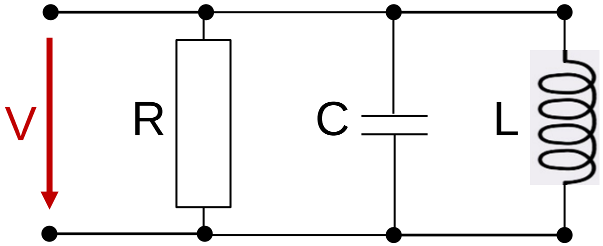
At frequency near resonance, the cavity resonator can be modeled by a lumped-element circuit.

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

resonance frequency

$$\boxed{Q_0 = \omega_0 \frac{2W_m}{P_{\text{loss}}} = \frac{R}{\omega_0 L} = \omega_0 RC}$$

unloaded Q



R in equivalent circuit == shunt impedance in cavity

C and L in equivalent circuit == resonance mechanism

$$\boxed{C_{\text{par}} = \frac{Q_0}{\omega_0 r_{\text{shunt}}}}$$

$$\boxed{L_{\text{par}} = \frac{r_{\text{shunt}}}{\omega_0 Q_0}}$$

Cavity equivalent circuit (4/4)

Connecting the cavity to the outer world…

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

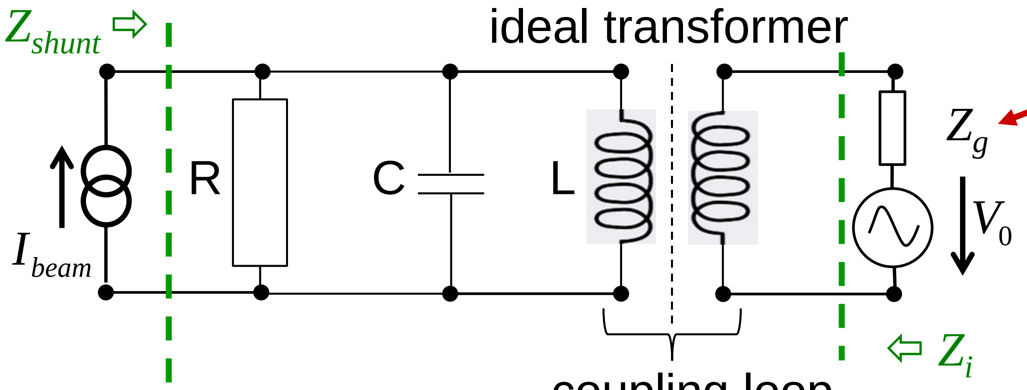
resonance frequency

$$\boxed{Q_0 = \omega_0 \frac{2W_m}{P_{\mathrm{loss}}} = \frac{R}{\omega_0 L} = \omega_0 RC}$$

unloaded Q

$$\boxed{C_{\mathrm{par}} = \frac{Q_0}{\omega_0 r_{\mathrm{shunt}}}} \quad \boxed{L_{\mathrm{par}} = \frac{r_{\mathrm{shunt}}}{\omega_0 Q_0}}$$

parallel capacitance and inductance



ideal transformer

Generator impedance
(generally complex due to matching requirement)

coupling loop
to generator (RF amplifier)

- Beam is usually modelled as a current source and sees a (generally complex) Z_{shunt}.
- Via the transformer, the coupling to the cavity can be adjusted to *“matching”*.

Thank you for your attention.

Let’s have a break!

Further reading…

1. POZAR, David M., *“Microwave Engineering”*, 4th edition, Wiley and sons.
2. ZHANG, Keqian, “*Electromagnetic Theory for Microwaves and Optoelectronics*”, 2nd edition, Springer
3. BHAT, Shiban, *“Stripline-like transmission Lines for Microwave Integrated Circuits”*, New Age International Publishers
4. HOFFMANN, *“Integrated Microwave Circuits”*, Springer
5. SAAD, *“Microwave Engineers’ Handbook”*, vol. 1,