$\mu = \mu_0 \mu_r$
$\mu_0 = 4\pi \times 10^{-7}$ H/m
$\varepsilon = \varepsilon_0 \varepsilon_r$
$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m
$c_0 = 2.998 \times 10^{8}$ m/s
$e = 1.602 \times 10^{-19}$ C

JUAS 2023 – RF Exam

Name: ______________________________ Points: ______ of 20

Please calculate and write your formulas and results clear and legibly, on a separate sheet of paper where appropriate.

1. Single choice questionnaire (½ point each= 4 points)

Please tick the correct answer like this: ☒.

a) A cylindrical cavity has the diameter equal to its height, and therefore:

- ○ $f_{TM010} > f_{TE111}$
- ○ $f_{TM010} < f_{TE111}$
- ☒ $f_{TM010} \cong f_{TE111}$

b) The transit-time of a cavity resonator is defined as the

- ○ time it takes for the electromagnetic field in the cavity to buildup.
- ☒ time it takes for a charged particle of constant velocity to pass the gap of the cavity.
- ○ time difference between minimum and maximum energy gain of a charged particle in the gap of the cavity.

c) The characteristic impedance Z_0 of a coaxial cable:

- ☒ is defined by the ratio of outer/inner conductor.
- ○ is always 50 Ω.
- ○ needs to be as small as possible to minimize losses and allow the maximum power transmission

d) The TEM mode:

- ○ is the fundamental mode of rectangular waveguides.
- ○ can be propagated in all the waveguides (circular, rectangular, arbitrary shaped).
- ☒ can be propagated just in a two-conductor geometry.

e) During characterization of a RF circuit, the measured value shows a variation of 3 dBm. Is this measurement reliable?

- O yes.
- X no.
- O OK, if we measure S-parameters.

f) If the measured power of the system is $P = 6\ mW$, what is the power expressed in dBm?

- O +6 dBm.
- X $+8\ dBm$.
- O -10 dBm.

g) The measurement trace in the *Smith*-chart shows a circle. From this, we can deduce that we measure:

- O a high amplifying gain.
- O a good signal transmission.
- X the locus of a resonance.

h) If a resonator is critically coupled to the feeding port, its trace on the Smith-chart will show me…?

- X the trace on the *Smith*-chart will pass through the center point of the chart.
- O the trace on the *Smith*-chart will encircle the center point of the chart.
- O the trace on the *Smith*-chart will stay on the outer contour.

2. S-Parameters (4 points)

a) Give the S-parameters in matrix form for the following components:
(assume "ideal" components, operating at a single, representative frequency)

a. Ideal amplifier of voltage gain $g = 20\ dB$ (½ point)

$$(S) = \begin{pmatrix} 0 & 0 \\ 10 & 0 \end{pmatrix}$$

b. Ideal circulator (½ point)

$$(S) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

c. Ideal $6\ dB$ attenuator (½ point)

$$(S) = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix}$$

d. Ideal $-20\ dB$ directional coupler (½ point)

$$(S) = \begin{pmatrix} 0 & -j0.995 & 0.1 & 0 \\ -j0.995 & 0 & 0 & 0.1 \\ 0.1 & 0 & 0 & -j0.995 \\ 0 & 0.1 & -j0.995 & 0 \end{pmatrix}$$

b) Please list:

a. Which of those ideal components are matched? (½ point)

$S_{ii} = 0$
all (a, b, c, d)

b. Which of the components are reciprocal and symmetric? (½ point)

$S_{ij} = S_{ji} \ \wedge \ S_{ii} = S_{jj}$
attenuator and directional coupler (c, d)

c. Which of the components are passive? (½ point)

circulator, attenuator and directional coupler (b, c, d)

d. Which of the components are passive and lossless? (½ point)

$$(S)^\dagger (S) = (S)^T (S)^* = (I) \ \Rightarrow \sum_{k=1}^{N} S_{ki} S_{ki}^* = 1\ \forall\ i = j \ \wedge \ \sum_{k=1}^{N} S_{ki} S_{kj}^* = 0\ \forall\ i \neq j$$

circulator, directional coupler (b, d)

3. Coaxial transmission-line **(3 points)**

a) A coaxial cable has a characteristic impedance of 75 Ω. The diameter of the outer conductor (at the boundary with the dielectric) was measured and is 15 mm. The material between inner and outer conductor is Teflon, which has dielectric constant of $\varepsilon_r = 2.1$.
What is the diameter of the inner conductor? (1 point)

$$Z_0 = \frac{\eta_0}{2\pi\varepsilon_r}\ln\left(\frac{D}{d}\right) \quad \Longrightarrow d = D\exp\left(-\frac{2\pi Z_0\sqrt{\varepsilon_r}}{\eta_0}\right)$$

$$d = 15x10^{-3}m\exp\left(-\frac{2\pi\times 75\times\sqrt{2.1}}{377}\right) = 2.45\ mm$$

b) The cable is used for transmission of a 3 GHz signal between the accelerator tunnel and the klystron gallery, the length of the cable is 25 m. What attenuation (in dB or in $Neper$) due to the losses, as sum of conductor and dielectric losses, can we expect? Both conductors are made of copper, which has a conductivity of $\sigma_{Cu} = 5.8\times 10^7$ S/m. For Teflon, the dielectric filling material, we find $\tan\delta = 0.00028$ @ 3 GHz. (1 point)

There unfortunately is a discrepancy in the lectures of Christine (Transmission Lines I, Page 20) and Manfred (RF Components, Page 7) wrt. the equations of the conductor losses α_c (or α_R) in a coaxial line. While this needs to be resolve in future, of course, both results are valid!

$$\rho_{Cu} = \frac{1}{\sigma_{Cu}} = \frac{\Omega m}{5.8\times 10^7} = 1.72414\times 10^{-8}\ \Omega m = \rho_D = \rho_d, \qquad \mu_r = 1 = \mu_{rD} = \mu_{rd}$$

Attenuation in $Neper$

$$\alpha_c = \frac{\sqrt{f\mu_0/\pi}}{2Z_0}\left(\frac{\sqrt{\mu_{rD}\rho_D}}{D} + \frac{\sqrt{\mu_{rd}\rho_d}}{d}\right)$$
$$= \frac{\sqrt{3\times 10^9\ s^{-1}\ 4\pi\times 10^{-7}\ VsA^{-1}m^{-1}/\pi}}{2\times 75\ VA^{-1}}\left(\frac{\sqrt{1\times 1.72414\times 10^{-8}\ VA^{-1}m}}{15\times 10^{-3}\ m}\right.$$
$$\left. + \frac{\sqrt{1\times 1.72414\times 10^{-8}\ VA^{-1}m}}{2.45\times 10^{-3}\ m}\right) = 0.0144075\ Neper/m$$

$$\alpha_d = \pi f\sqrt{\mu_0\varepsilon_0\varepsilon_r}\tan\delta$$
$$= \pi\times 3\times 10^9\ s^{-1}\sqrt{4\pi\times 10^{-7}\ VsA^{-1}m^{-1}\ 8.854\times 10^{-12}\ AsV^{-1}s^{-1}\ 2.1}$$
$$\times 0.00028 = 0.0127561\ Neper/m$$

$$\alpha = \alpha_c + \alpha_d = 0.0271637\frac{Neper}{m} \quad \Longrightarrow \quad \alpha l = 0.0271637\frac{Neper}{m}\times 25m$$
$$= 0.679091\ Neper$$

Attenuation in dB

$$\alpha_c = 6\times 10^{-9}\sqrt{f\varepsilon_r}\frac{D+d}{dD\ln(D/d)}$$

$$= 6\times 10^{-9}\sqrt{3\times 10^9\ \times\ 2.1}\frac{15\times 10^{-3}m + 2.45\times 10^{-3}m}{2.45\times 10^{-3}m\ 15\times 10^{-3}m\ln(15\times 10^{-3}m/2.45\times 10^{-3}m)}$$

$$= 0.124826\ dB/m$$

$$\alpha_d = 91\times 10^{-9} f\sqrt{\varepsilon_r}\tan\delta = 91\times 10^{-9}\times 3\times 10^9\ \sqrt{2.1}\times 0.00028 = 0.110772\ dB/m$$

$$\alpha = \alpha_c + \alpha_d = 0.235598\frac{dB}{m} \quad\Longrightarrow\quad \alpha l = 0.235598\frac{dB}{m}\times 25\ m = 5.88996\ dB$$

Sanity check

$$\alpha = 0.0271637\frac{Neper}{m} = 0.0271637\frac{20}{\ln 10}\frac{dB}{m} = 0.23594\frac{dB}{m}$$

Christine's equation for α_c:

$$\alpha_c = \frac{\sqrt{\varepsilon_r}}{Z_0\ln(D/d)}\left(\frac{1}{d}+\frac{1}{D}\right)\sqrt{\frac{\mu\omega}{2\sigma}}$$

$$= \frac{A\sqrt{2.1}}{75\ V\ \ln(15/2.45)}\left(\frac{1\ m^{-1}}{2.45\times 10^{-3}} + \frac{1\ m^{-1}}{15\times 10^{-3}}\right)\sqrt{\frac{4\pi\times 10^{-7}\ Vs\ 2\pi\cdot 3\times 10^9\ Vm}{Am\ s\ 2\cdot 5.8\times 10^{-7}\ A}} = 0.07237\ Neper/m$$

$$\alpha = \alpha_c + \alpha_d = 0.085126\ \frac{Neper}{m} \Longrightarrow\ \alpha l = 0.085126\frac{Neper}{m}\times 25m = 2.1282\ Neper$$

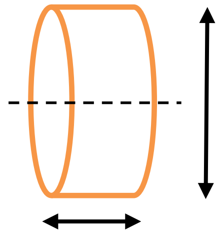
$$\alpha_{dB} = \frac{20\ dB}{\ln 10}\alpha = 8.686\ dB\ \times\ 2.1282 = 18.485\ dB$$

c) What dielectric constant ε_r would the filling material need to have to get to a characteristic impedance of 50 Ω for this coaxial cable with the same dimensions of outer and inner conductor? (1 point)

$$Z_0 = \frac{\eta_0}{2\pi\sqrt{\varepsilon_r}}\ln\left(\frac{D}{d}\right) \quad\Longrightarrow\quad \varepsilon_r = \left[\frac{\eta_0}{2\pi Z_0}\ln\left(\frac{D}{d}\right)\right]^2 = \left[\frac{377\ \Omega}{2\pi\times 50\ \Omega}\ln\left(\frac{15\ mm}{2.45\ mm}\right)\right]^2 = 4.725$$

4. Resonant Cavity (4 points)

An "empty" (air), cylindrical "pillbox" cavity is made of copper, the conductivity of copper is $\sigma_{Cu} = 5.8 \times 10^7$ S/m. The eigenmode used for the acceleration of protons, which have a constant velocity of $v = 2 \times 10^8\ m/s$, should have a resonant frequency of $f_{acc} = 352\ MHz$.
We assume a simplified, "ideal" cylindrical resonator (no beam ports) for this analysis!



a) The ratio of cavity diameter / height is selected as $2a/h = 1.75$. (½ point)
What is the ratio of the corresponding frequencies for the accelerating mode and the nearest, unwanted higher-order mode, f_{HOM}/f_{acc}?
(Hint: Make use of the mode chart!
We are not looking for a precise value of the frequency ratio.)

From the mode chart we find for the TM010 accelerating mode:
$(2af_{acc})^2 \cong 5.3 \times 10^8\ (MHz\ cm)^2 \implies a \cong 32.7\ cm$

For the higher-order mode frequency we find from the mode chart:
$\left(\frac{2a}{h}\right)^2 = 3.0625 \implies (2af_{HOM})^2 \cong 10 \times 10^8\ (MHz\ cm)^2$
$\implies f_{HOM} \cong 483.5\ MHz$ (exact: $484.277\ MHz$)

$f_{HOM}/f_{acc} \cong 1.37$ (exact: 1.37579)

b) What type of mode is used for the acceleration of the protons? (½ point)

TM010 or E010

c) What type of mode is the nearest higher-order mode? (½ point)

TE111 or H111

d) Compute the values of the dimensions of the cylindrical cavity, diameter $2a$ and height h, for a frequency of 352 MHz for the acceleration mode (½ point)

$$a = \frac{p_{01}c}{2\,\pi\,f_{TM010}} = 0.383\lambda_{TM010} = 0.383\frac{c}{f_{TM010}} = 0.383\frac{2.998 \times 10^8\ ms^{-1}}{352 \times 10^6\ s^{-1}} = 0.326\ m$$

$$\text{diameter} = 2a = 0.6519\ m$$
$$\text{height} = h = \frac{2a}{1.75} = 0.37254\ m$$

e) What is the Q-factor of the cavity? (½ point)
(Hint: Start by calculating the skin depth for copper at the resonant frequency)

$$\omega_{TM010} = 2\pi f_{TM010} = 2\pi \cdot 352 \times 10^6\ s^{-1} = 2.21168 \times 10^9\ s^{-1}$$

$$\delta = \sqrt{\frac{2}{\omega_{TM010}\sigma_{Cu}\mu_0}} = \sqrt{\frac{2\, m\, s\, V\, A\, m}{2.21168\times10^9 \cdot 5.8\times10^7 A \cdot 4\pi\times10^{-7} V\, s}} = 3.52237\ \mu m$$

$$Q = \frac{a}{\delta}\frac{1}{1+{}^{a}/_{h}} = \frac{0.326\ m}{3.52\times10^{-6}\ m}\frac{1}{1+0.875} = 49357$$

f) Calculate the value of the R/Q (R-over-Q) for the pillbox resonator. (½ point)
(Hint: Please use the analytical expression for the R/Q, you also may verify if you can use the approximation.)

$$\frac{p_{01}}{2}\frac{h}{a} = \frac{2.405}{2}\frac{2}{1.75} = 1.3742 \quad \sin\left(\frac{p_{01}}{2}\frac{h}{a}\right) = \sin(1.3742) = 0.9807$$

The approximation cannot be used!

$$\frac{R}{Q} = \frac{4\eta_0}{\pi p_{01}^3 J_1^2(p_{01})}\frac{\sin^2\left(\frac{p_{01}}{2}\frac{h}{a}\right)}{h/a}$$

Numerical values for the *Bessel* functions and zeros are:
$p_{01} \cong 2.405\,,\ \ J_1(p_{01}) = J_1(2.405) \cong 0.52$

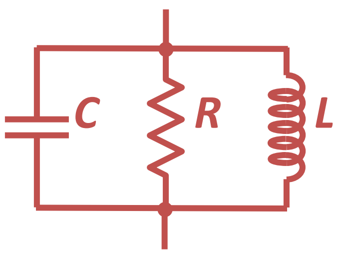
$$\Longrightarrow\ \frac{R}{Q} \cong \frac{4\times377\ \Omega}{\pi\times\ 2.405^3\times0.52^2}\frac{\sin^2\left(\frac{2.405}{2}\frac{2}{1.75}\right)}{\frac{2}{1.75}} = 107.7\ \Omega$$

g) Calculate the values of the parallel equivalent circuit of the accelerating mode of the cavity and sketch the circuit. (½ point)

$$R = \frac{R}{Q}\,Q = 107.7\ \Omega\ \times\ 49357 \cong 5.31575\ M\Omega$$

$$C = \frac{1}{\omega_{TM010}R/Q} = \frac{1\ s\ A}{2.21168\times10^9\ 107.7\ V} \cong 4.19815\ pF$$

$$L = \frac{R/Q}{\omega_{TM010}} = \frac{107.7\ s\ V}{2.21168\times10^9\ A} \cong 48.6964\ nH$$



h) What is the value of the "effective shunt impedance" of this cylindrical cavity? (½ point)
(Hint: To answer this question you first need to calculate the transit-time factor.)

Calculate the relative velocity of the proton beam:

$$\beta = \frac{v}{c} = \frac{2 \times 10^8 m/s}{2.998 \times 10^8 m/s} \cong 0.667128$$

We need the wavelength for the TM010 mode frequency:

$$\lambda_{TM010} = \frac{c}{f_{TM010}} = \frac{2.998 \times 10^8\ m\ s}{352 \times 10^6 s} \cong 0.851683\ m$$

Now we can calculate the transit-time factor:

$$T = \frac{\sin\left(\frac{\pi h}{\beta \lambda_{TM010}}\right)}{\frac{\pi h}{\beta \lambda_{TM010}}} = \frac{\sin\left(\frac{\pi \times 0.37254\ m}{0.667128 \times 0.851683\ m}\right)}{\frac{\pi \times 0.37254\ m}{0.667128 \times 0.851683\ m}} \cong 0.428563$$

And finally, the effective shunt impedance:

$$r_{s,eff} = r_s T^2 = 5.31575 \times 10^6\ \Omega \times 0.428563^2 \cong 976.324\ k\Omega$$

5. *Smith* chart **(5 points)**

Given are the following complex impedances Z or reflection coefficients Γ, each associated with an operating frequency f:

point	A	B	C	D	E	F
Z	$j50\,\Omega$	$(35+j75)\,\Omega$	$(50-j50)\,\Omega$	$50\,\Omega$	$(25+j25)\,\Omega$	$(25-j25)\,\Omega$
Γ	$1\angle 90°$	$0.67\angle 60°$	$0.45\angle -63.4°$	0	$0.45\angle 116.5°$	$0.45\angle -116.6°$
f	$500\,MHz$	$1\,GHz$	$100\,MHz$	$200\,MHz$	$200.005\,MHz$	$199.995\,MHz$

a) Indicate points A, B, C, D, E and F into the "Impedance *Smith* Chart", using $Z_0 = 50\,\Omega$ as reference impedance for normalization.
With help of the Smith chart, or by other calculation tools, fill the missing information for Z and Γ for each point in the table. (1½ points)

b) For points A, B and C, calculate the values of the equivalent lumped element or series circuit of resistive and reactive elements using the given frequency. (1½ points)

$$L_A = \frac{\Im[Z_A]}{2\pi f_A} = \frac{50\,Vs}{2\pi \times 500 \times 10^6\,A} = 15.9\,nH$$

$$R_B = \Re[Z_B] = 35\,\Omega$$

$$L_B = \frac{\Im[Z_B]}{2\pi f_B} = \frac{75\,Vs}{2\pi \times 1 \times 10^9\,A} = 11.9\,nH$$

$$R_C = \Re[Z_C] = 50\,\Omega$$

$$C_C = \frac{1}{2\pi f_C \Im|Z_C|} = \frac{1\,As}{2\pi \times 100 \times 10^6 \times 50\,V} = 31.8\,pF$$

c) What type of circuit element must be added in series to circuit C, to match the impedance to $Z_0 = 50\,\Omega$?
Want is the value of this circuit element? (1 point)

An inductor in series must be added.

$$L_C = \frac{\Im|Z_C|}{2\pi f_C} = \frac{50\,Vs}{2\pi \times 100 \times 10^6\,A} = 79.6\,nH$$

d) Points D, E and F belong together with the detuned short location to the locus of the resonant mode of a cavity, measured as S_{11} in the *Smith* chart.
Please sketch the locus and evaluate the unloaded Q-value, Q_0, of the resonant mode based on the frequencies associated with the points given in the table. (1 point)

$$Q_0 = \frac{f_E + f_F}{f_E - f_F} = \frac{(200.005 + 199.95)\,MHz}{(200.005 - 199.995)MHz} = 40000$$

IMPEDANCE SMITH CHART