$\mu = \mu_0 \mu_r$
$\mu_0 = 4\pi \times 10^{-7}$ H/m
$\varepsilon = \varepsilon_0 \varepsilon_r$
$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m
$c = 2.998 \times 10^{8}$ m/s
$e = 1.602 \times 10^{-19}$ C

JUAS 2024 – RF Exam

Name: ________________________________ Points: ______ of 20

Please calculate and write your formulas and results clear and legibly, on a separate sheet of paper where appropriate.

1. Single choice questionnaire (½ point each= 4 points)

Please tick the correct answer like this: ☒.

a) The phase velocity on a dielectric-loaded TEM transmission-line ($\varepsilon_r > 1$) is always

- ○ faster than the velocity of speed-of-light
- ○ equal to the velocity of speed-of light
- ☒ slower than the velocity of speed-of-light

b) Length and frequency of a sinusoidal RF wave

- ○ are proportional to each other.
- ☒ are inverse proportional to each other.
- ○ are independent properties.

c) A cylindrical resonator is called *pillbox* cavity if:

- ☒ $f_{TM010} < f_{TE111}$
- ○ $f_{TM010} > f_{TE111}$
- ○ $f_{TM010} = f_{TE111}$

d) A resonant cavity is in critical coupling to the feed-port if:

- ○ higher-order modes are perfectly damped.
- ○ the S_{11} *Smith* chart measurement shows a circle.
- ☒ no reflections appear at the feed-port.

e) A 10 W RF generator is connected via a 20 dB attenuator to a 50 Ω load impedance.
At the load we measure:

- O $1\ W$
- O $-20\ dBW$
- X $+20\ dBm$

f) What is true for 2-conductor TEM transmission-lines:

- X Can be used for a broad range of frequencies, including DC (0 Hz).
- O The signal transmission is based on eigenmodes with a specific cutoff frequency.
- O Low losses at high frequencies, therefore ideal for high-power RF transmission applications.

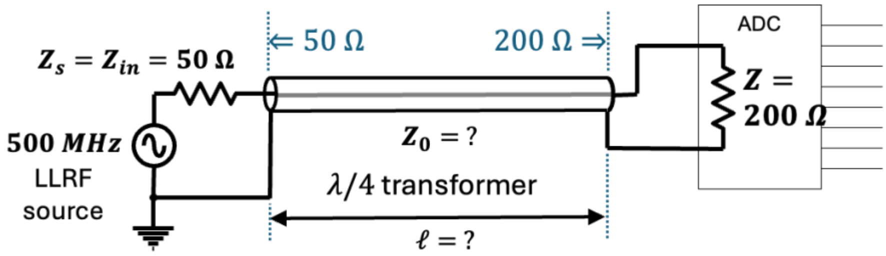
g) A 1-port RF network is measured with a vector network analyzer (VNA). The instrument display shows $|S_{11}| = 0\ dB$ at the frequency of interest. What does this result mean?

- X The RF power supplied by the VNA is fully reflected at the input of the RF network, back to the VNA port.
- O No power is reflected, the RF network is perfectly matched.
- O Both above statements can be true because no information of the S_{11} phase was provided.

h) The 1 dB compression point of a RF amplifier:

- O is always higher than the third-order intercept (TOI) point.
- X expresses the power at the output (or input) at which the gain of the amplifier deviates by 1 dB from the gain of an ideal amplifier.
- O expresses the power lost due the non-linear saturation of the amplifier.

2. Transmission-lines: Coaxial-cable as quarter-wave impedance transformer (3 points)



A coaxial cable is utilized providing two-in-one functions, as "balun" (balanced-unbalanced transformer) and simultaneously as $\lambda/4$ impedance transformer in an ADC measurement setup of a 500 MHz LLRF signal, see the illustration above.
(Hint: In this task the balun function of the coaxial cable is of no relevance!)

a) The 500 MHz signal is provided by a LLRF source with the usual 50 Ω reference impedance at the input: $Z_{in} = 50\ \Omega$. The analog-to-digital converter (ADC) has an impedance of $Z = 200\ \Omega$. What is the characteristic impedance Z_0 of the coaxial cable that should perform as $\lambda/4$ impedance transformer? (1 point)

$$Z_0 = \sqrt{Z\, Z_{in}} = \sqrt{200\ 50}\ \Omega = 100\ \Omega$$

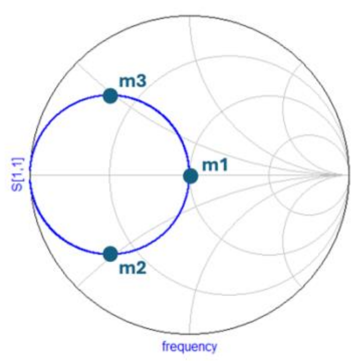
b) The coaxial cable is filled with "Teflon" (PTFE) between inner and outer conductor, which has a permittivity of $\varepsilon_r = 2.1$. At what physical length ℓ the coaxial cable must be cut to operate as $\lambda/4$ impedance transformer for 500 MHz? (2 points)
(Hint: You need to calculate the phase velocity – sometimes called propagation velocity – of the cable.)

$$v_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{2.998 \cdot \frac{10^8 m}{s}}{\sqrt{2.1}} = 2.07 \cdot \frac{10^8 m}{s}$$

$$\beta\ell = \frac{\pi}{2} \Rightarrow \ell = \frac{\pi}{2\beta} = \frac{\pi}{2}\frac{v_p}{\omega} = \frac{\pi}{2}\frac{2.07 \cdot 10^8 \frac{m}{s}}{2\pi \cdot 500 \cdot 10^6\ s^{-1}} = 0.1034\ m$$

3. S11 measurement of a resonant cavity mode (3 points)

A single-cell cavity resonator is under development. After optimization, the R-over-Q of the fundamental accelerating mode was calculated as $R/Q \cong 100\ \Omega$. A prototype was manufactured and a $S_{11}(f)$ measurement was performed for characterization, see the graph.



Three markers, m1, m2, m3, were located on the measured $S_{11}(f)$ trace, formatted as *Smith* chart, showing the following data on the VNA:

m1: frequency = 400.000 MHz, S11 ≈ 0, Z ≈ (50+j0) Ω

m2: frequency = 400.080 MHz, S11 ≈ -0.5-j0.5, Z ≈ (10-j20) Ω

m3: frequency = 399.920 MHz, S11 ≈ -0.5+j0.5, Z ≈ (10+j20) Ω

a) In which kind of coupling is the resonance: under-critical, over-critical or critical? (½ point)

critical coupling

b) Which is the resonance frequency of the measured accelerating mode? (½ point)

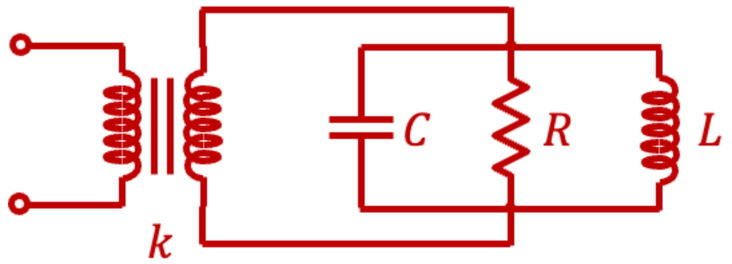
$$f_{res} = 400.000\ MHz$$

c) Calculate the unloaded-Q. (½ point)

As the resonance is in critical coupling:

$$Q_0 = 2\ Q_L = \frac{f_{res}}{f_{BW,3dB,Q_L}} = \frac{2 \cdot 400 \cdot 10^6\ s^{-1}}{(400.08 - 399.92) \cdot 10^6\ s^{-1}} = 5000$$

d) Sketch the lumped-element parallel equivalent circuit of the resonance, including an ideal component for the coupling mechanism. (½ point)



e) Calculate the lumped-element values of the equivalent circuit including the coupling factor. (1 point)

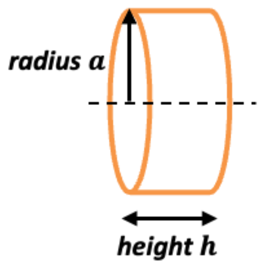
$$R = {}^{R}/_{Q}\, Q_0 = 100\ \Omega \cdot 5000 = 500\ k\Omega$$

$$C = \frac{Q_0}{2\pi f_{res} R} = \frac{5000}{2\pi \cdot 400 \cdot 10^6\ s^{-1} \cdot 500 \cdot 10^3\ \frac{V}{A}} = 3.979\ pF$$

$$L = \frac{R}{2\pi f_{res} Q_0} = \frac{500 \cdot 10^3\ \frac{V}{A}}{2\pi \cdot 400 \cdot 10^6\ s^{-1} \cdot 5000} = 39.79\ nH$$

$$k = \sqrt{\frac{R}{Z_0}} = \sqrt{\frac{500 \cdot 10^3\ \Omega}{50\ \Omega}} = 100$$

4. Resonant Cavity (4 points)



An "empty" (vacuum), single-cell cylindrical cavity is studied to accelerate electrons. The klystron, feeding the cavity with RF power, operates at $f = 500\ MHz$.
We assume a simplified, "ideal" cylindrical resonator (no beam ports) for this analysis!

a) What type of eigenmode is used for particle acceleration? (½ point)

TM_{010} or E_{010}

b) What are the dimensions of the cavity cylinder?

i. What is the radius a? (½ point)

$$f_{TM_{010}} = \frac{c}{2\pi}\frac{p_{01}}{a} \Rightarrow a = \frac{c}{2\pi}\frac{p_{01}}{f_{TM_{010}}} = \frac{2.998\cdot 10^8\,\frac{m}{s}}{2\pi}\frac{2.405}{500\cdot 10^6\ s^{-1}} = 0.2295\ m$$

ii. Calculate the height h of the cavity, such that the frequency of the first higher-order mode is separated by a factor of approximately 1.5 from the fundamental accelerating mode. (½ point)
(Hint: Make use of the mode chart.)
Of what type is this first higher-order eigenmode? (½ point)

For the mode chart

$$f_{TM_{010}} = 500\ MHz \Rightarrow$$
$$(2af)^2\ [MHz\ cm]^2 = (2\cdot 22.95\cdot 500)^2 = 5.266\cdot 10^8\ (MHz\ cm)^2$$
$$f_{HO} = 1.5\cdot 500\ MHz = 750\ MHz \Rightarrow$$
$$(2af)^2\ [MHz\ cm]^2 = (2\cdot 22.95\cdot 750)^2 = 1.185\cdot 10^9\ (MHz\ cm)^2$$

The first mode crossing a horizontal f_{HO}-line in the mode chart is the TE_{111}-mode, approximately at:

$$\left(\frac{2a}{h}\right)^2 \approx 4 \Rightarrow h \cong a = 0.2295\ m$$

The exact value is:

$$\left(\frac{2a}{h}\right)^2 = 3.900 \Rightarrow a = 0.232\ m$$

c) For the prototyping of the cavity two materials are available, stainless-steel with a conductivity of $\sigma_{SS} = 1.3\cdot 10^6\ S/m$ and annealed copper with a conductivity of $\sigma_{Cu} = 5.8\cdot 10^7\ S/m$. Which material should be used to achieve a 3 dB bandwidth of $f_{BW,3dB} = 15\ kHz$ for the unloaded accelerating mode?

i. What is the unloaded Q-factor Q_0? (½ point)

$$Q_0 = \frac{f_{TM_{010}}}{f_{BW,3dB}} = \frac{500\cdot 10^6\ s^{-1}}{15\cdot 10^3\ s^{-1}} = 33333$$

ii. Calculate the related skin-depth. (½ point)

$$Q_0 = \frac{a}{\delta}\left(1+\frac{a}{h}\right)^{-1} \Rightarrow \delta = \frac{a}{Q_0}\left(1+\frac{a}{h}\right)^{-1} = \frac{0.2295\ m}{33333}(1+1)^{-1} = 3.44\ \mu m$$

iii. Calculate the skin-depth for stainless-steel and for copper, for the frequency of the accelerating mode and decide which material should be used to manufacture the prototype? (½ point)

$$\delta_{SS} = \sqrt{\frac{2}{\omega_{TM_{010}}\sigma_{SS}\mu_0}} = \sqrt{\frac{2}{2\pi \cdot 500 \cdot 10^6\ s^{-1} \cdot 1.3 \cdot 10^6 \frac{S}{m} \cdot 4\pi \cdot \frac{10^{-7} H}{m}}} = 19.74\ \mu m$$

$$\delta_{Cu} = \sqrt{\frac{2}{\omega_{TM_{010}}\sigma_{Cu}\mu_0}} = \sqrt{\frac{2}{2\pi \cdot 500 \cdot 10^6\ s^{-1} \cdot 5.8 \cdot 10^7 \frac{S}{m} \cdot 4\pi \cdot \frac{10^{-7} H}{m}}} = 2.96\ \mu m$$

Copper as material is required!

d) Calculate the R/Q (R-over-Q) value for the accelerating mode of this pillbox resonator? (Hint: Please use the analytical expression for the R/Q given in the lecture.) (½ point)

$$R/_{Q} = 128\ \Omega\ \frac{\sin^2\left(\frac{p_{01}}{2} h/_{a}\right)}{h/_{a}} = 128\ \Omega\ \frac{\sin^2\left(\frac{2.405}{2} 1\right)}{1} = 111.4\ \Omega$$

5. *Smith* chart **(3 points)**

Given are the following complex impedances Z or reflection coefficients Γ:

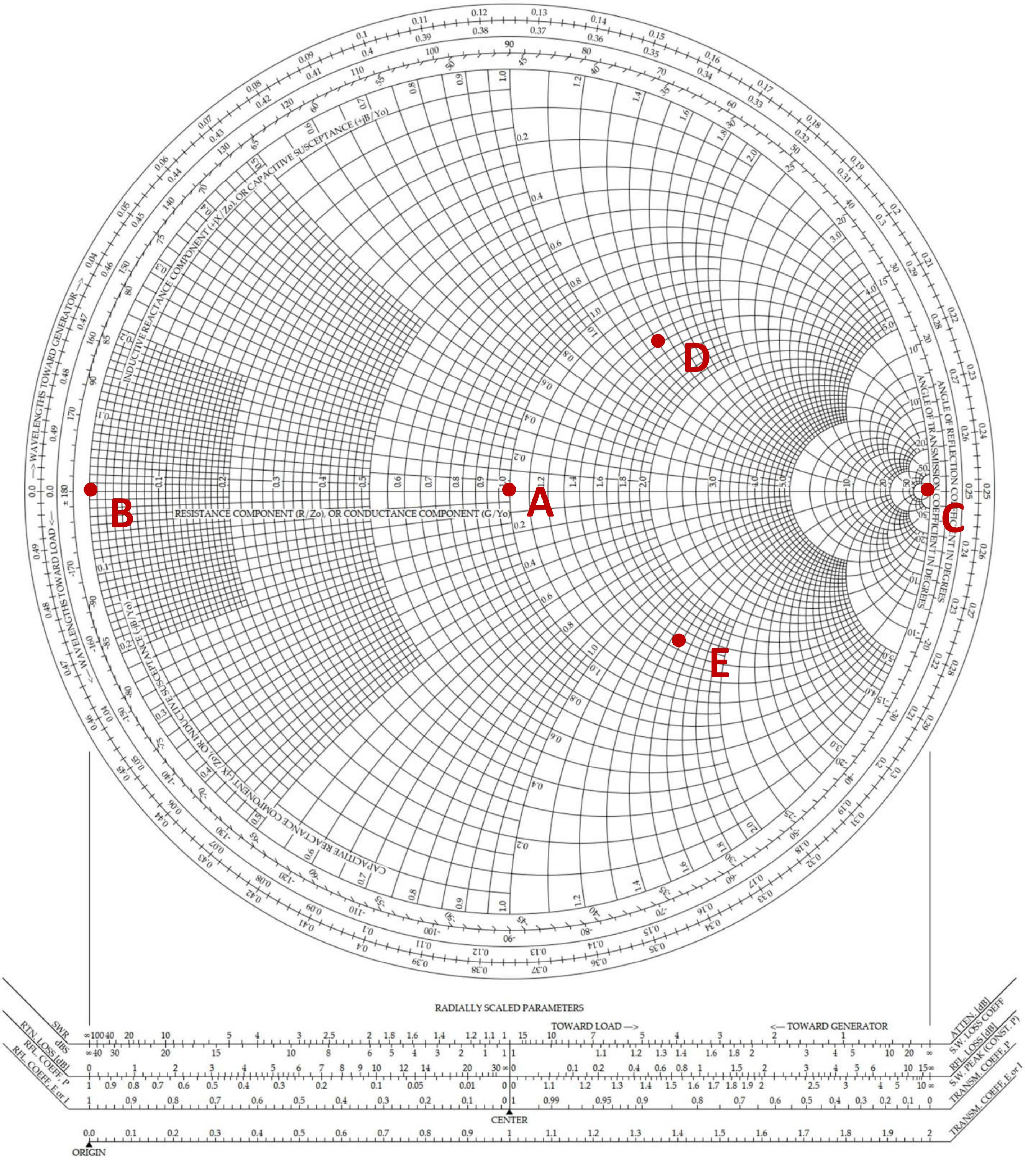
point	A	B	C	D	E
Z	$50\ \Omega$	$0\ \Omega$	$\infty\ \Omega$	$(69.1 + j65.1)\ \Omega$	$R_s = 75\ \Omega, C_s = 4.25\ pF$
Γ	0	-1 or $1\angle-180°$	$1\angle 0°$	$0.5\angle 45°$	$0.542\angle-40.6°$

a) Indicate points A, B, C and D into the "Impedance *Smith* Chart", using $Z_0 = 50\ \Omega$ as reference impedance for the normalization.
With help of the Smith chart, or by calculation, fill the missing information for Z and Γ for each point in the table. (2 points)

b) Calculate $Z(f = 500\ MHz)$ for the series circuit of the lumped element values given under point E and indicate this point into the *Smith* chart. Fill the missing value for the reflection coefficient. (1 point)

$$Z = R_s + \frac{1}{j2\pi f\ C_s} = (75 - j74.9)\ \Omega$$

IMPEDANCE SMITH CHART



6. S-Parameters (3 points)

a) Give the S-parameters in matrix form for an ideal $10\ dB$ TEM directional coupler (operating at the center frequency) (1 point)

$$k = 10^{-\frac{10}{20}} = \frac{1}{\sqrt{10}} = 0.316 \ \Rightarrow t = \sqrt{1-k^2} = \sqrt{0.9} = 0.949$$

$$\begin{bmatrix} 0 & -j0.949 & 0.316 & 0 \\ -j0.949 & 0 & 0 & 0.316 \\ 0.316 & 0 & 0 & -j0.949 \\ 0 & 0.316 & -j0.949 & 0 \end{bmatrix}$$

b) Given are the S-matrices for ideal 3-port RF components:

$$\boldsymbol{S_A} = \frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \boldsymbol{S_B} = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad \boldsymbol{S_C} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

i. What have all components in common? (½ point)

Matched and passive.

ii. What are the names for $\boldsymbol{S_A}$, $\boldsymbol{S_B}$ and $\boldsymbol{S_C}$? (1 point)

S_A: *Wilkinson* power divider / splitter / combiner
S_B: 6 dB resistive power divider / splitter / combiner
S_C: ideal circulator (counterclockwise)

c) Give the S-matrix for the (lossless) coaxial cable transmission-line of question 2. (½ point)

$$\begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix}$$