JUAS 2025 – Tutorial

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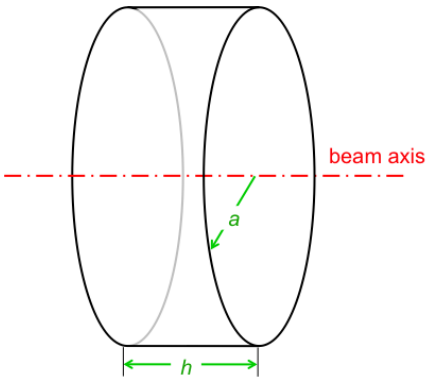
$\mu = \mu_0 \mu_r$
$\mu_0 = 4\pi \cdot 10^{-7}$ Vs/(Am)
$\varepsilon = \varepsilon_0 \varepsilon_r$
$\varepsilon_0 = 8.854 \cdot 10^{-12}$ As/(Vm)
$c_0 = 2.998 \cdot 10^{8}$ m/s

1) Design of a pillbox cavity

Problem: Design a simple "Pillbox" cavity with the following parameters

Frequency:	f = 299.98 MHz (λ = 1.00 m)
Wall material:	Copper (equivalent skin depth δ = 3.8 µm)
Axial length:	h = 0.2 m

For this example, we ignore beam ports, i.e. vacuum chamber stubs required for the beam passage, so that all analytical formulas describing the pillbox cavity apply.



Questions:

1. Find from the analytical formulas:
   - Cavity radius a
   - Cavity quality factor Q
   - "Geometry factor", also known as "characteristic impedance" R/Q
   - Is the cavity completely determined?

2. Sketch the equivalent circuit of the cavity and compute the values of the circuit elements.

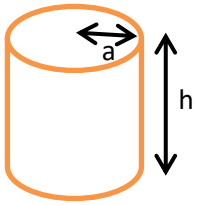
3. Calculate the 3-dB bandwidth of the intrinsic (not connector to any generator) cavity.

4. Calculate the necessary RF power (RMS) for a gap peak voltage of $\hat{V} = 100\ kV$, assuming critical coupling.

5. The cavity is critically coupled, fed by an amplifier, designed for a load impedance of 50 Ω. Determine:

   - The peak voltage at the amplifier output.
   - The necessary transformer ratio k of the input coupler.

2.) "Pillbox" Cavity



Design a pillbox cavity. The $E_{010} = TM_{010}$ mode shall resonate at $f_{res} = 2.95$ GHz. The aspect ratio shall be $a/h = 0.5$.

1. What is the radius a of the cavity?
2. What is the height h of the cavity?
3. The 3 dB bandwidth of the unloaded resonance shall be 150 kHz, how big does the unloaded Q_0 of the cavity need to be?
4. What is the maximum tolerable surface resistance ρ of the cavity walls to get $Q_0 = 20\,000$.
5. What is the R/Q for this cavity geometry?
6. Derive the equivalent circuit parameters R, L and C for the cavity with
   $Q_0 = 20\,000$ and R/Q = 30
7. The cavity is critically coupled to an RF power amplifier and driven by 50 W of input power on its resonant frequency. The loaded Q_L is 10 000. What is the stored energy U_{CAV} in the cavity?
8. Determine the peak gap voltage V_{gap}?