

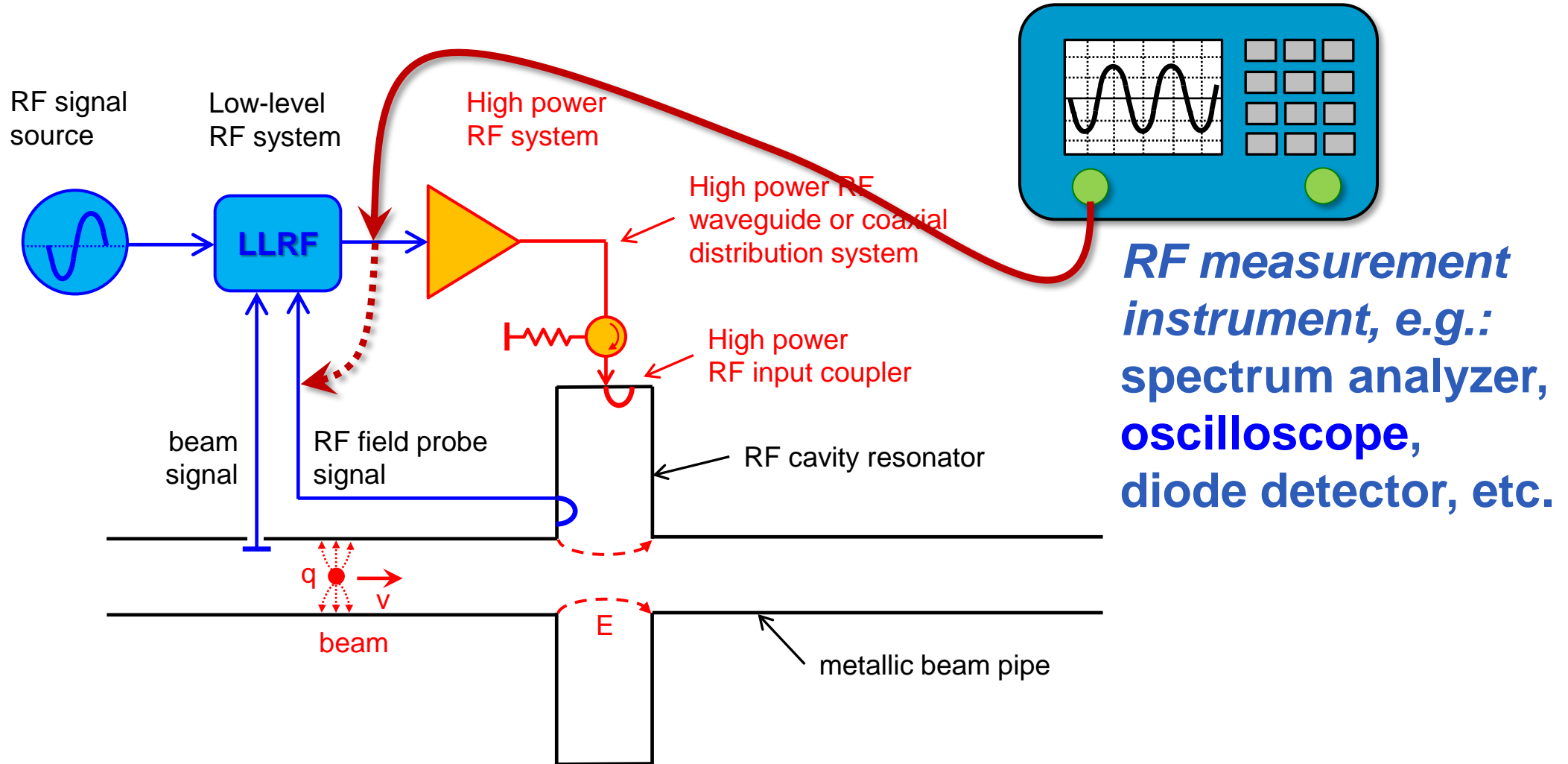
RF Engineering

RF Measurements

Christine Völlinger & Manfred Wendt – CERN

- **Overview of RF measurement instruments**
 - Oscilloscope, spectrum analyzer (SA), signal (FFT) analyzer, slotted measurement line, vector network analyzer (VNA)
- **Reflection measurement with the slotted coaxial air-line**
- **The super-heterodyne receiver principle**
 - Modulation, down-conversion, mixer, spectrum analyzer block schematics
- **S-parameter measurements**
 - Simple measurement setup, VNA block schematics
 - VNA calibration
 - Synthetic pulse measurements with the VNA
 - **Measurement example 1: pillbox resonator characterization**
 - Equivalent circuit parameters, Q-factor measurement in the Smith-chart, Slater's theorem, bead-pull measurement
 - **Measurement example 2: beam coupling impedance**
 - Stretched-wire measurement of the longitudinal coupling impedance

Please note the references to the JUAS proceedings!

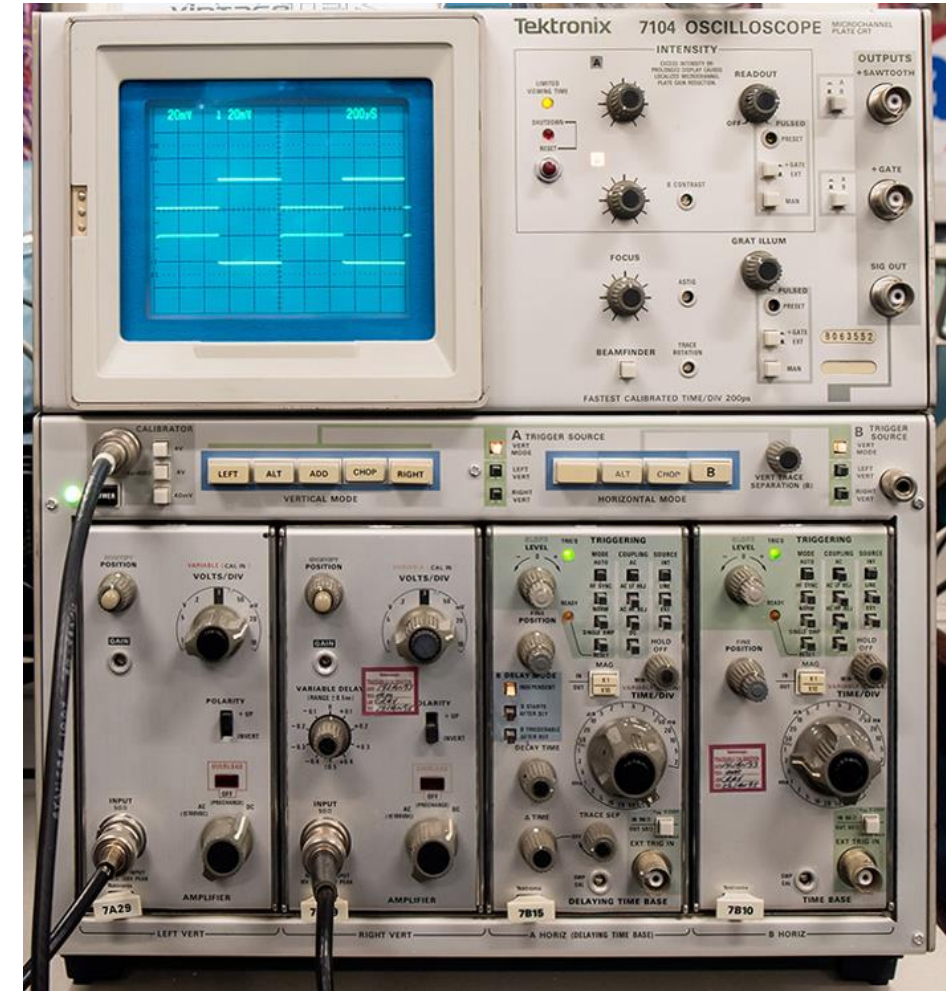
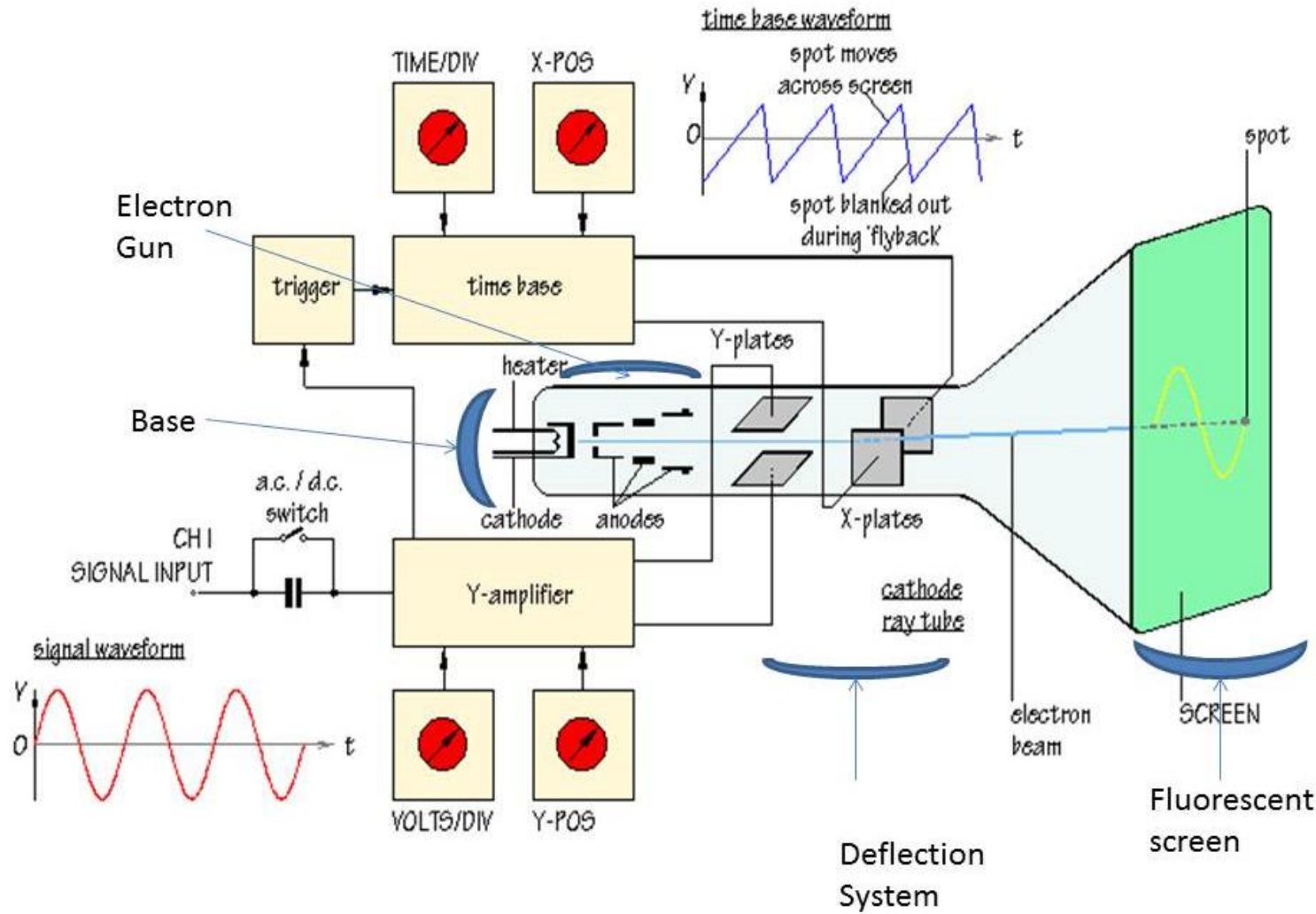


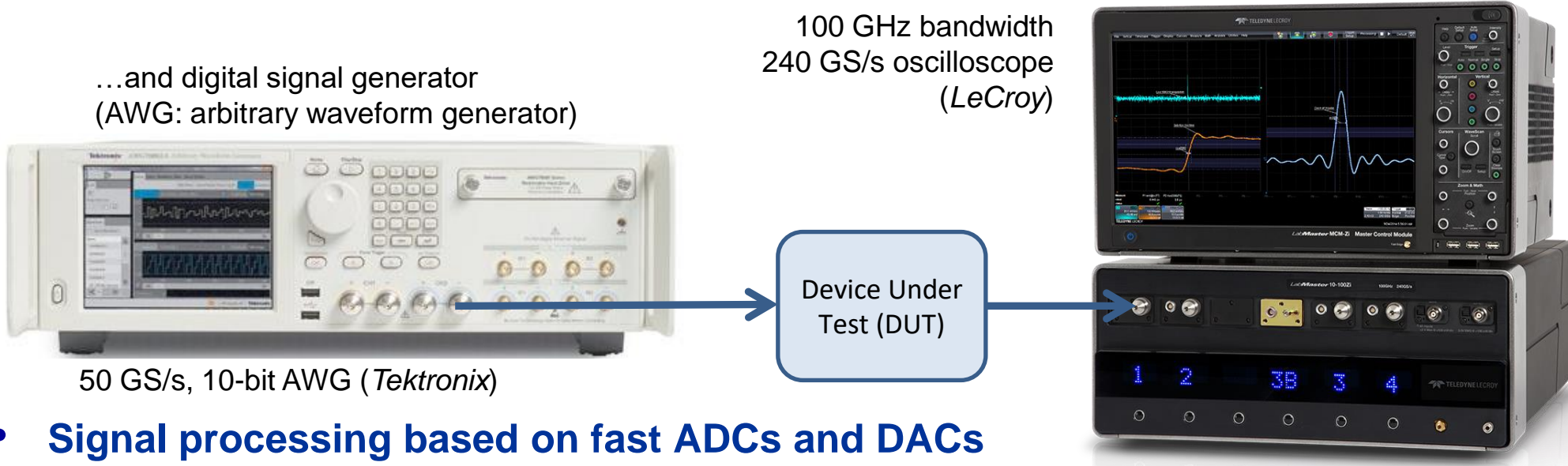
There are different options to observe RF signals

Here some typical measurement tools:

- **Oscilloscope**: to observe signals as **time-domain** waveform
 - periodic signals
 - burst and transient signals with arbitrary waveforms
 - application: direct observation of signals from a beam pick-up, from a test generator, or from other sources
 - visualizes the shape of a waveform, etc.
 - limited performance for the evaluation of non-linear effects.

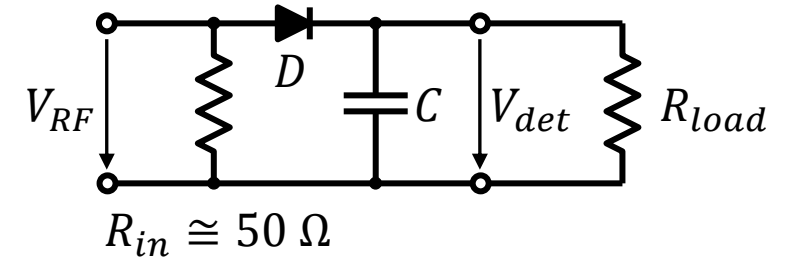
Cathode Ray Tube (CRT) Oscilloscope



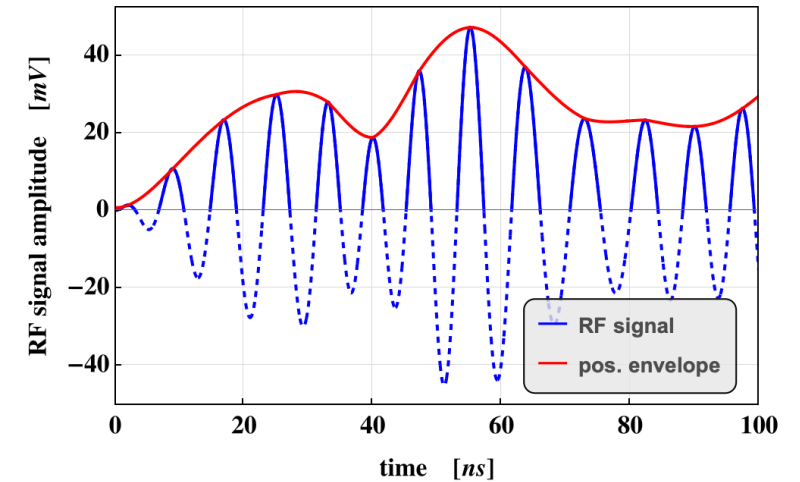
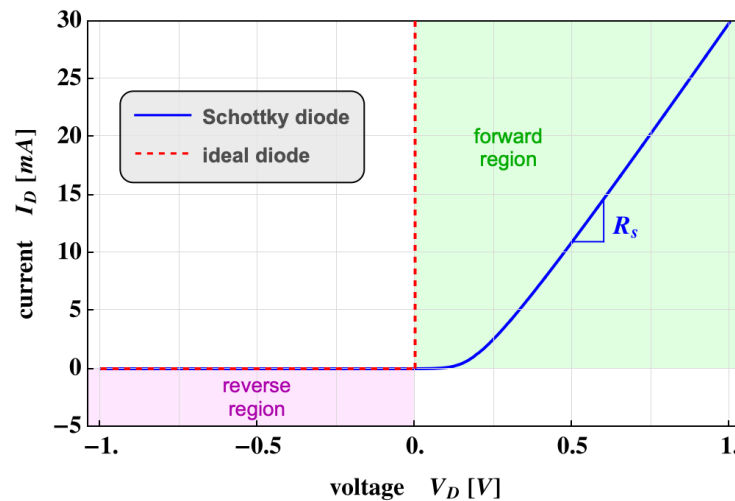
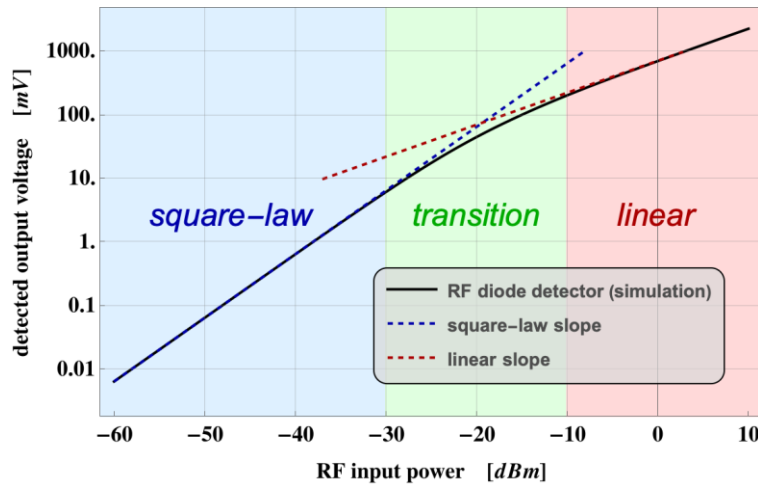


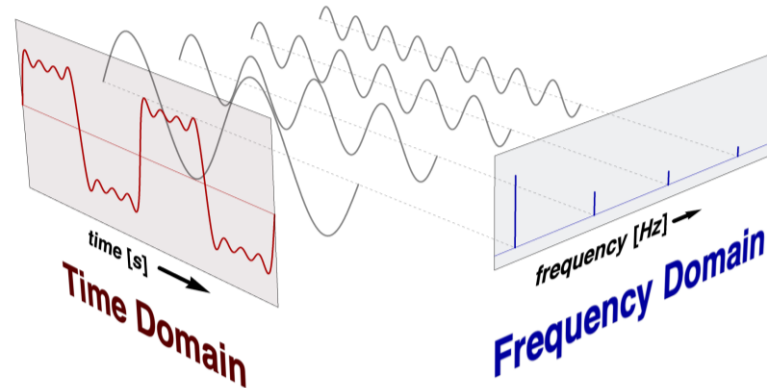
- **Signal processing based on fast ADCs and DACs**
 - **Similar “look and feel” as analog oscilloscopes, but better performance**
 - 8...12-bit multi-GS/s ADCs, still, be aware of aliasing effects!
 - Fast sampling oscilloscope require sufficient memory resources.
- **AWG or pulse generator & digital oscilloscope:**
Time-domain (TD) test setup
 - **Device under test (DUT) characterization and trouble shooting**
 - Impulse, step, or arbitrary waveform (e.g., beam signal) as stimulus signal
 - High impedance probe for measurements on the printed circuit board (PCB)

- **RF detection (Schottky) diode (RF power meter)**
 - Supplies a rectified (video) output signal proportional to the RF signal level
 - Delivers no frequency or phase information, but operates over a very broad frequency range few MHz to many GHz, and up to ~60 dB dynamic range.



$$I_D = I_S \left(e^{\frac{V_D - I_D R_S}{nV_T}} - 1 \right)$$



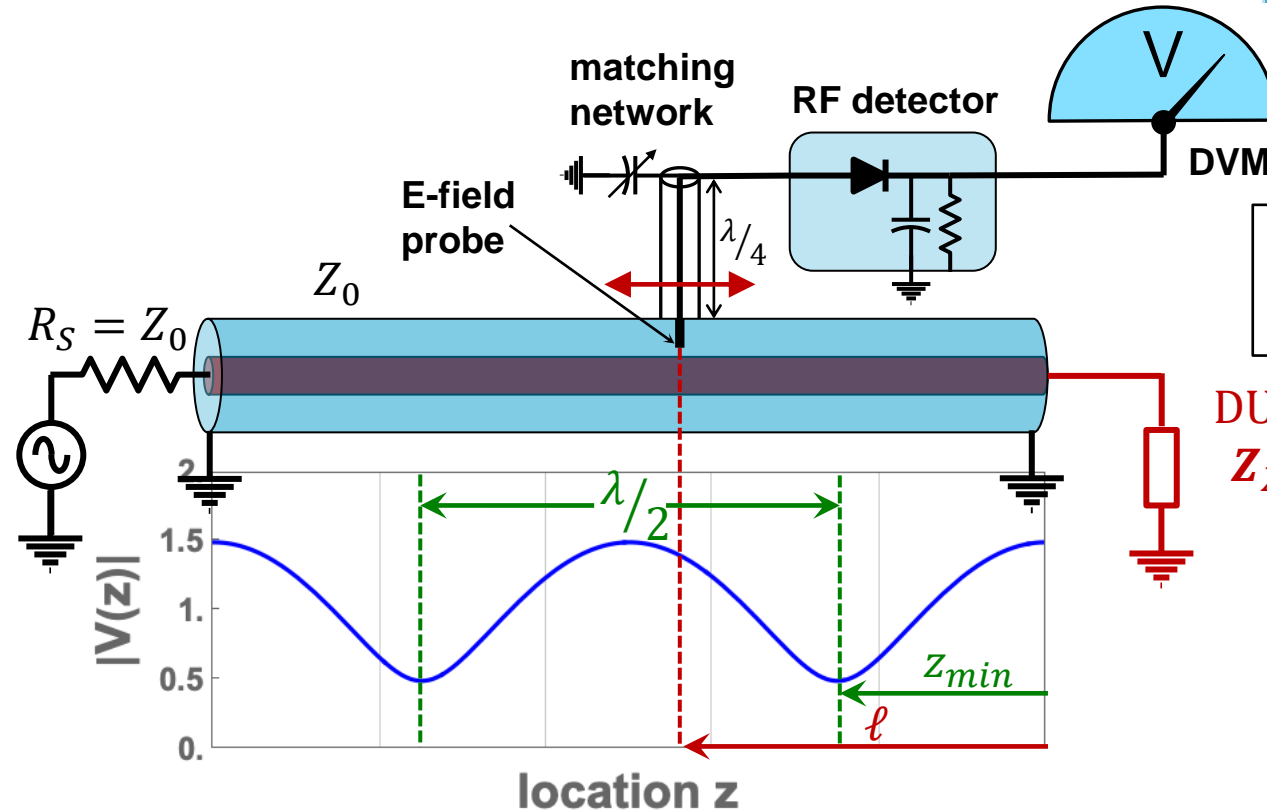
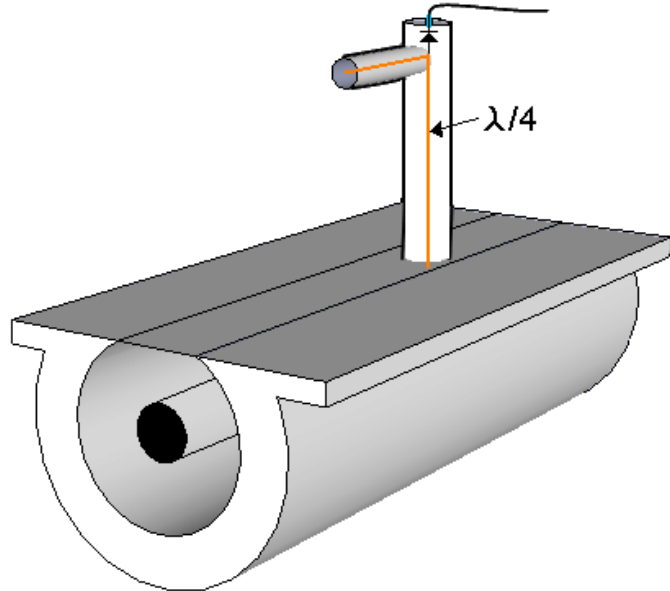


- **Spectrum analyzer: to observe signals in a “frequency-domain like” fashion**
 - sweeps in equidistant steps through a given frequency range
 - application: observation of spectrum from the beam, or from a signal generator or RF source, or the spectrum emitted from an antenna to locate EMI issues in the accelerator tunnel, etc.
 - Also, DUT characterization in the laboratory, e.g., noise figure measurement on amplifiers (requires a noise source), intermodulation measurements on amplifiers (requires two RF generators).
 - Requires periodic signals
 - Assumes **time-invariance** of the measurement object (DUT) throughout the frequency sweep
 - **Large dynamic range!**

- **Vector signal analyzer (VSA), sometimes called FFT analyzer**
 - Acquires the RF signal, after down-conversion to an intermediate (IF) signal, in time-domain by fast sampling
 - Further numerical treatment in digital signal processors (DSPs)
 - Spectrum calculated using Fast Fourier Transform (FFT)
 - Combines **features of an oscilloscope and a spectrum analyzer**: Signals can be observed directly in time-domain, or in a frequency-domain like fashion
 - Contrary to the SA, also the spectrum of non-periodic signals and transients can be measured
 - Application: Observation of tune sidebands, transient behavior of a phase locked loop, single pass beam signal spectrum, etc.
 - **Digital oscilloscopes** and **FFT analyzers** share similar technologies, i.e., fast sampling and digital signal processing, and therefore can provide similar measurement options
 - The digital oscilloscope directly digitizes the RF signal
 - limited dynamic range, large instantaneous bandwidth
 - The FFT analyzer digitizes the down-converted IF signal
 - large dynamic range, but (still) limited instantaneous bandwidth

Tools to characterize RF components and sub-systems:

- **Slotted coaxial (or waveguide) measurement transmission-line**
 - For study and illustration purposes only – not anymore used in today's RF laboratory environment.
- **Vector Network Analyzer (VNA)**
 - Combines the functions of a vector spectrum analyzer (FFT analyzer), a RF sweep generator, and a S-parameter test set (directional coupler)
 - Excites a *Device Under Test* (DUT, e.g., circuit, antenna, amplifier, etc.) network at a given sinusoidal *continuous wave* (CW) frequency, and measures the response in magnitude and phase => **determines the S-parameters**
 - Covers a selectable frequency range by measuring step-by-step at subsequent frequency points (like a spectrum analyzer, again requires the DUT to be time-invariant!)
 - Applications: characterization of passive and active RF components, *Time Domain Reflectometry* (TDR) by Fourier transformation of the reflection response, etc.
 - Also, power sweep measurements (1 dB compression point), 4-port VNAs enable virtual ports: e.g., single-ended / differential port DUT characterization.
 - **The VNA is the most versatile and comprehensive tool in the RF laboratory!**



$$VSWR = \frac{|V_{max}|}{|V_{min}|}$$

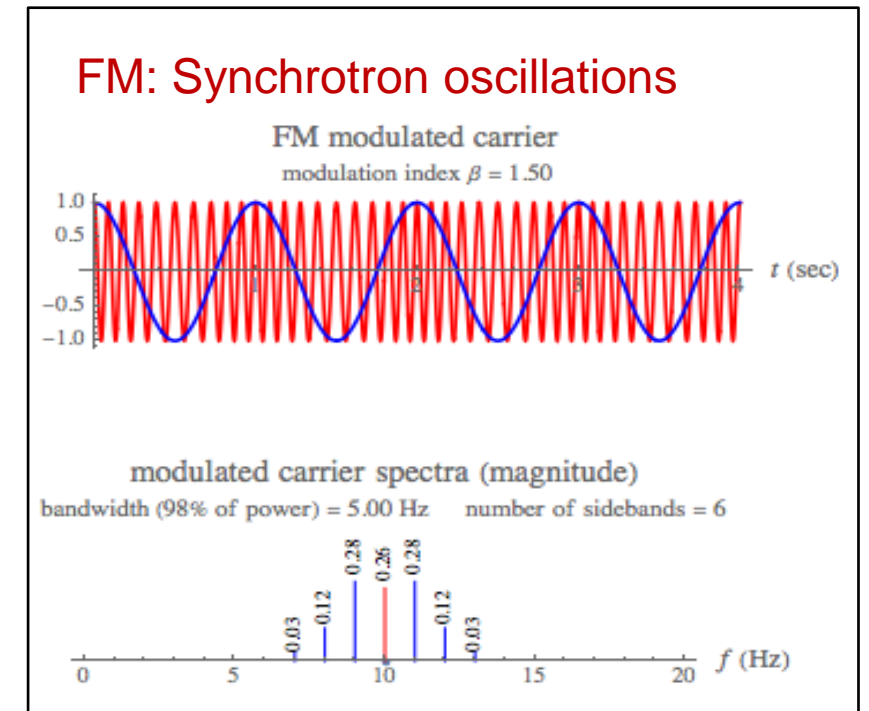
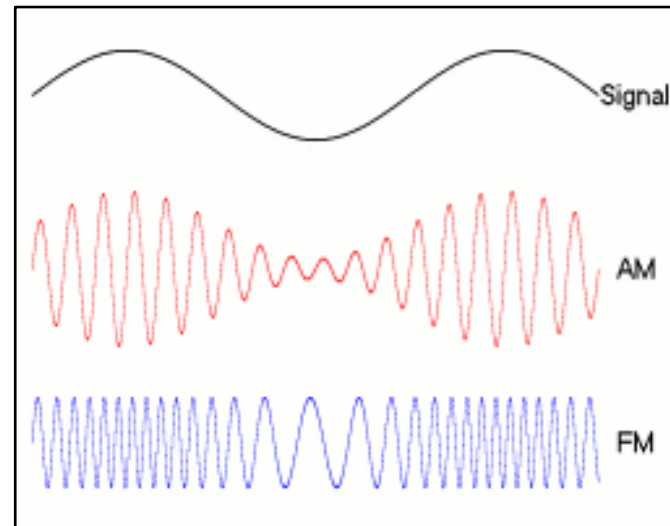
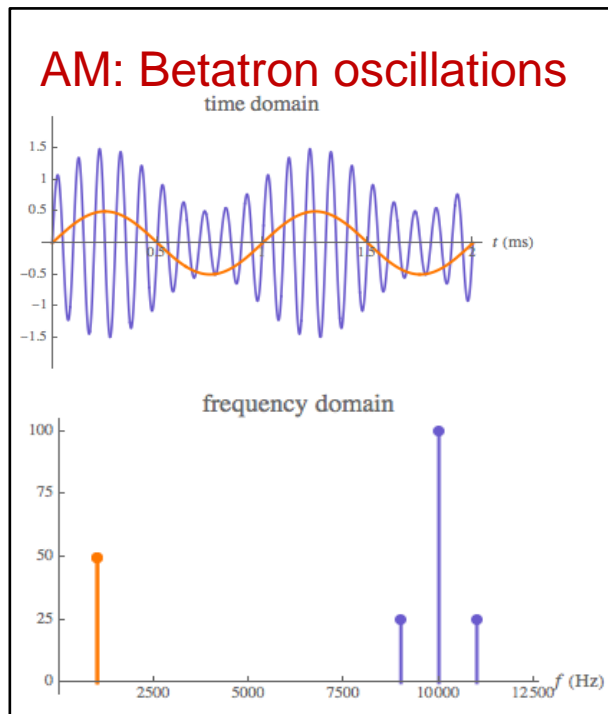
$$|\Gamma| = \frac{VSWR - 1}{VSWR + 1}$$

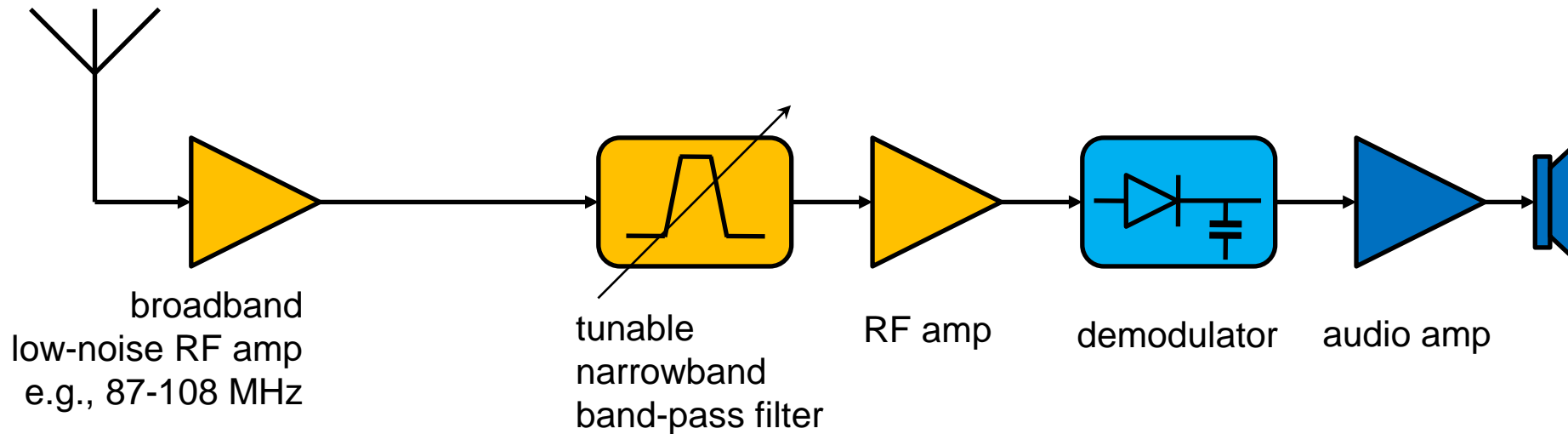
$$\angle \Gamma = \frac{4\pi}{\lambda} z_{min} - \pi$$

$$Z_x = Z_0 \frac{1 - \Gamma}{1 + \Gamma}$$

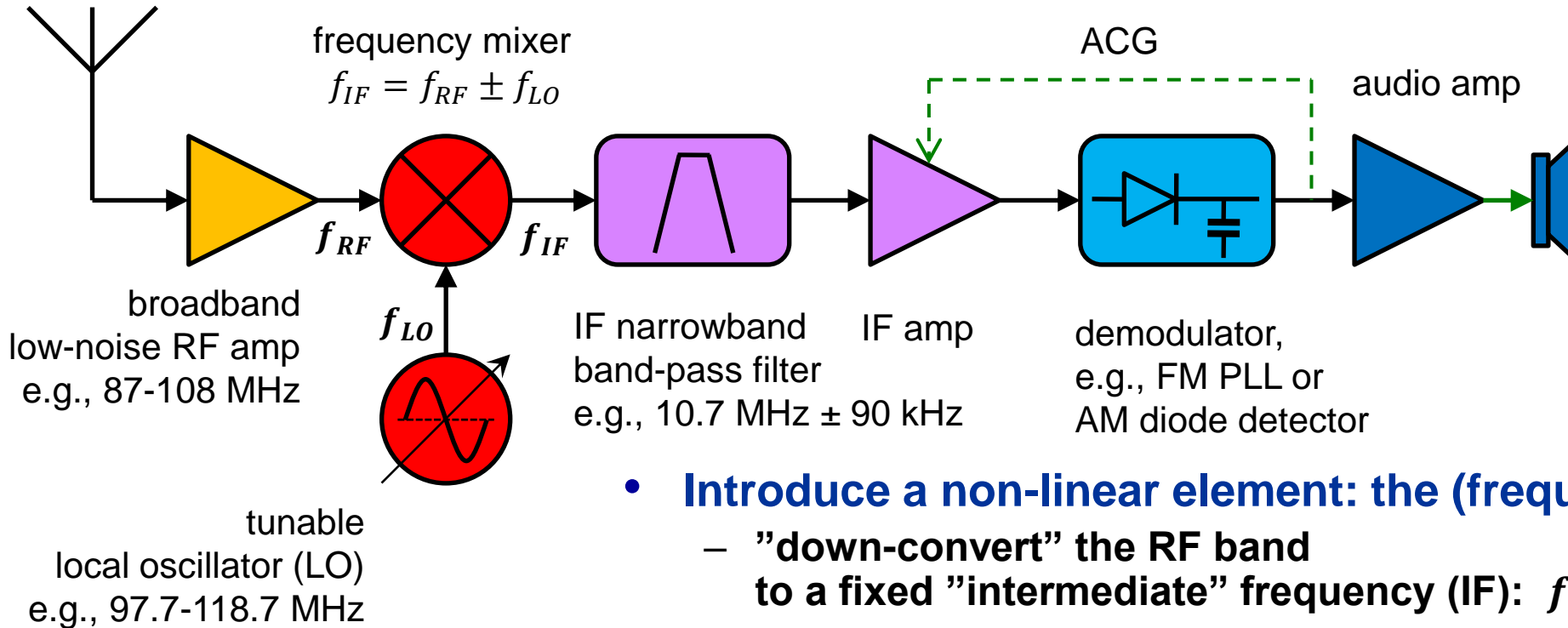
- **Slotted coaxial air-line is used as standing wave detector**
 - Probes the radial **electric field** along the slotted line.
 - Measurement of E-field **minima's** E_{min} and **maxima's** E_{max} with a diode detector, thus detect $|V_{min}|$ and $|V_{max}|$ along the line.
 - Evaluate the **reflection coefficient** Γ of a **DUT of unknown** Z_X at the end of the line

- RF signals are continuous wave (CW), sinusoidal signals
 - Often, a high frequency carrier is **modulated** with low frequency information
 - Modulation appears “naturally” in ring accelerators as:
 - Modulation is also provided through the LLRF system to the accelerating structures





- **...or: How does a "traditional" analog radio works?**
 - It was, and still is, difficult to make precisely tunable narrowband, band-pass filters for high frequencies (~100 MHz)!!
 - high frequency low-noise amplifiers are expensive!
 - high frequency demodulators are not trivial.
 - **direct detection of radio and RF signals is challenging!**



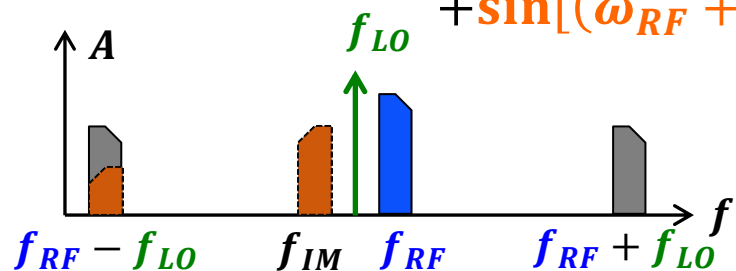
- **Introduce a non-linear element: the (frequency) mixer!**
 - "down-convert" the RF band to a fixed "intermediate" frequency (IF): $f_{IF} = f_{RF} \pm f_{LO}$
 - requires a tunable local oscillator (LO)
 - well manageable IF section:
 - narrowband band-pass filter(s) (BPF) and amplifier(s)
 - RF telecommunication standard
 - Often multiple mixing stages are used in modern RF instruments, e.g., spectrum and network analyzers

$$y_{RF}(t) = A_{RF} \sin(\omega_{RF}t + \varphi_{RF}) \quad RF \rightarrow \otimes \rightarrow IF \quad y_{IF}(t) = y_{RF}(t)y_{LO}(t)$$

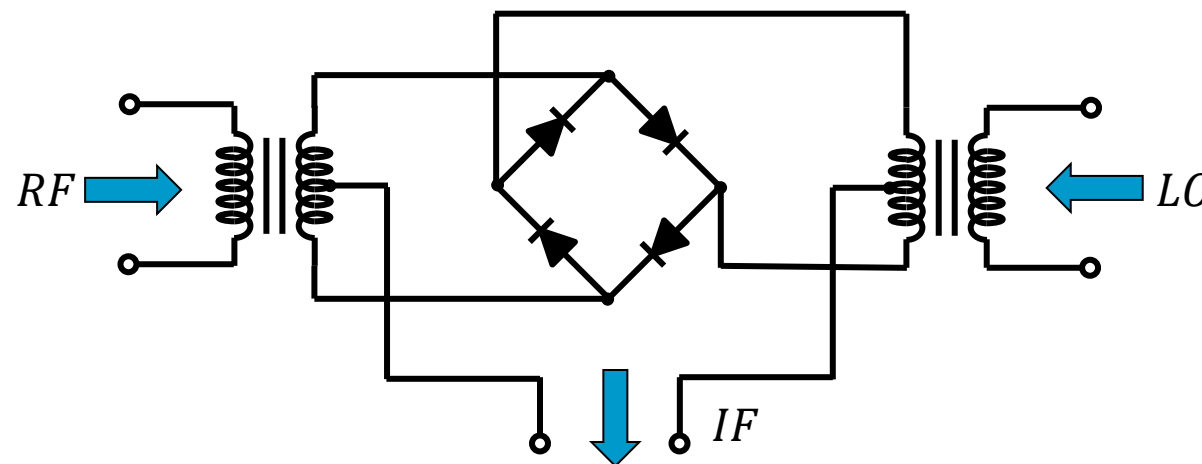
$$LO \quad y_{LO}(t) = A_{LO} \sin(\omega_{LO}t + \varphi_{LO})$$

- Ideal mixer: $f_{IF} = f_{RF} \pm f_{LO}$

$$y_{IF}(t) = \frac{1}{2} A_{LO} A_{RF} \left\{ \sin[(\omega_{RF} - \omega_{LO})t + (\varphi_{RF} - \varphi_{LO})] \text{ upper sideband} \right. \\ \left. + \sin[(\omega_{RF} + \omega_{LO})t + (\varphi_{RF} + \varphi_{LO})] \right\} \text{ lower sideband (also called "base-band")}$$



- The mixer is based on non-linear circuit elements
 - e.g., the diode, popular is the **double-balanced mixer**



- Frequency conversion

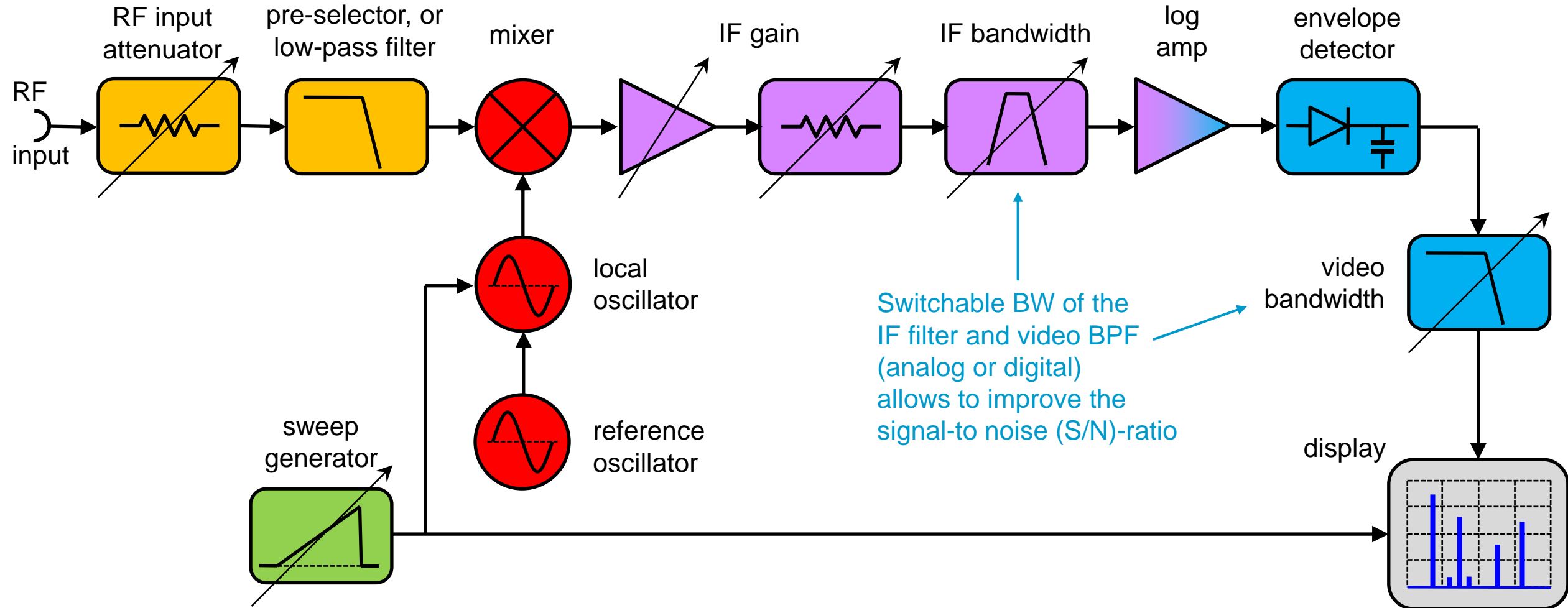
- $f_{RF} \neq f_{LO}$: heterodyne receiver
- $f_{RF} = f_{LO}$: homodyne, demodulator

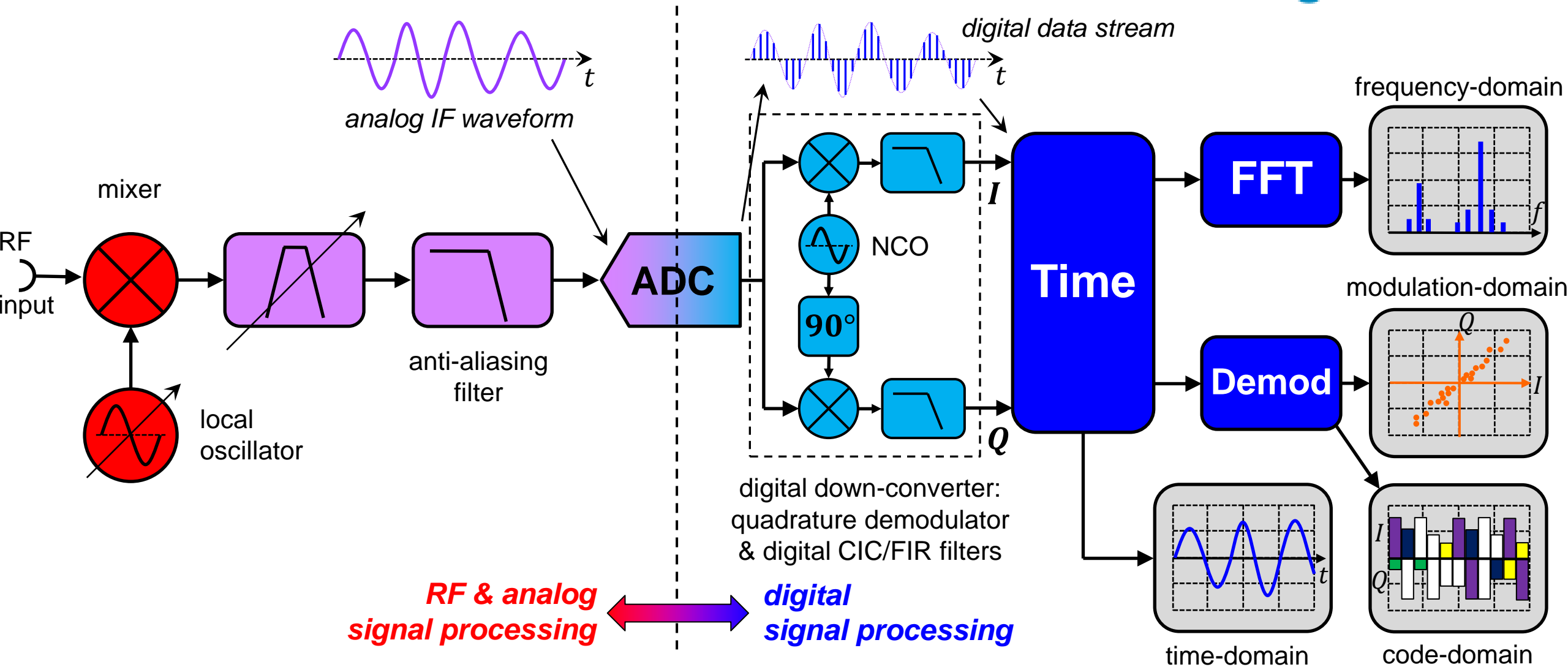
- Real-world mixer: $f_{IF} = m f_{RF} \pm n f_{LO}$

- Image frequency: $f_{IM} = f_{LO} - f_{IF}$

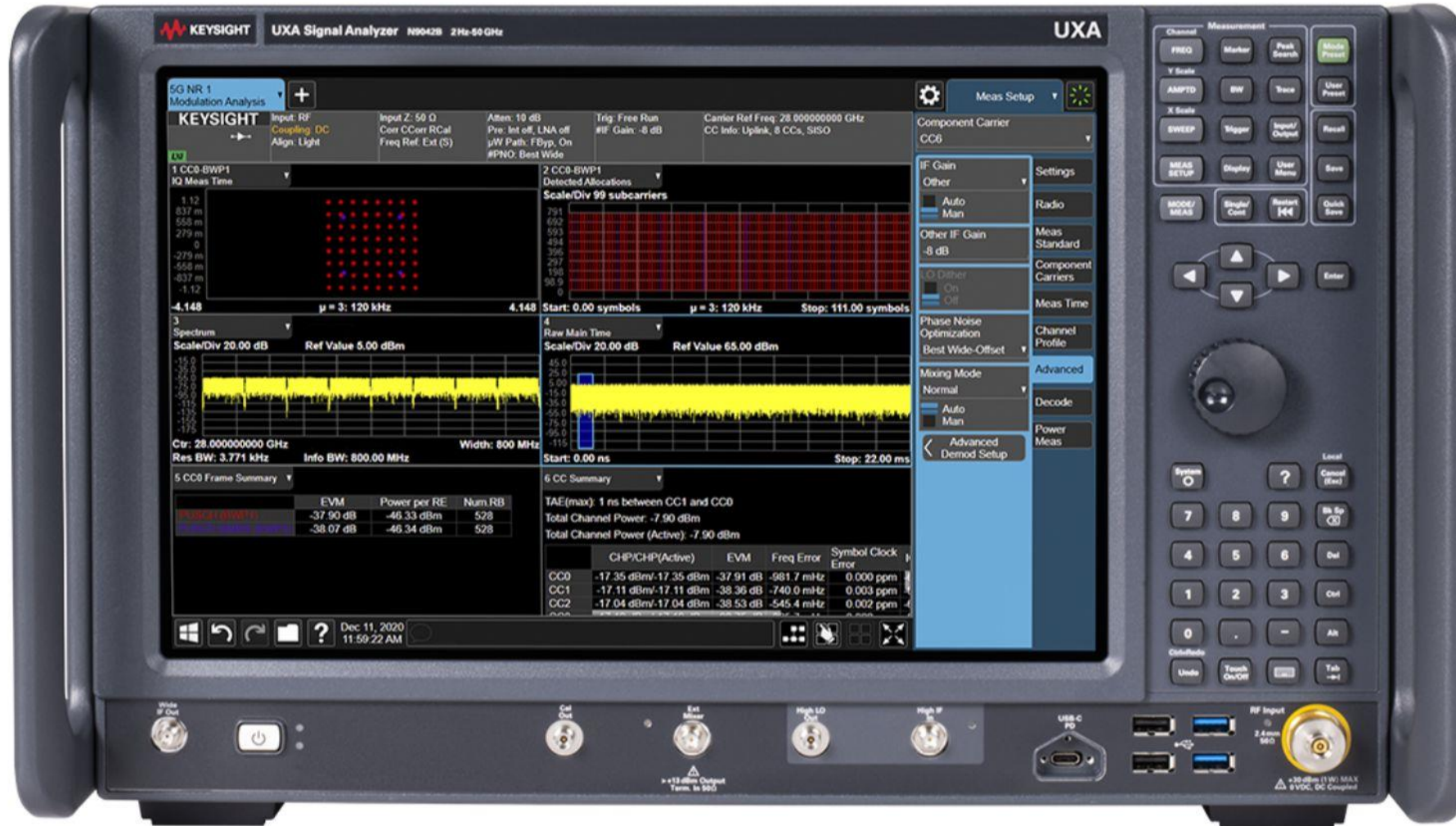
Simplified Spectrum Analyzer

- based on the super-heterodyne principle



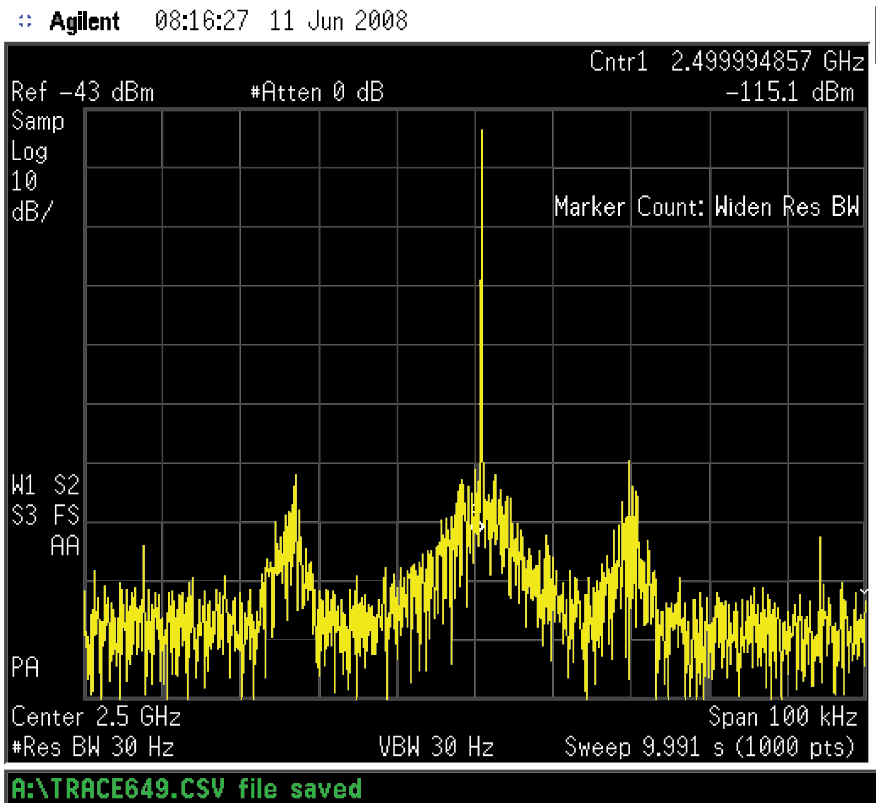


Modern Spectrum (RF Signal) Analyzer

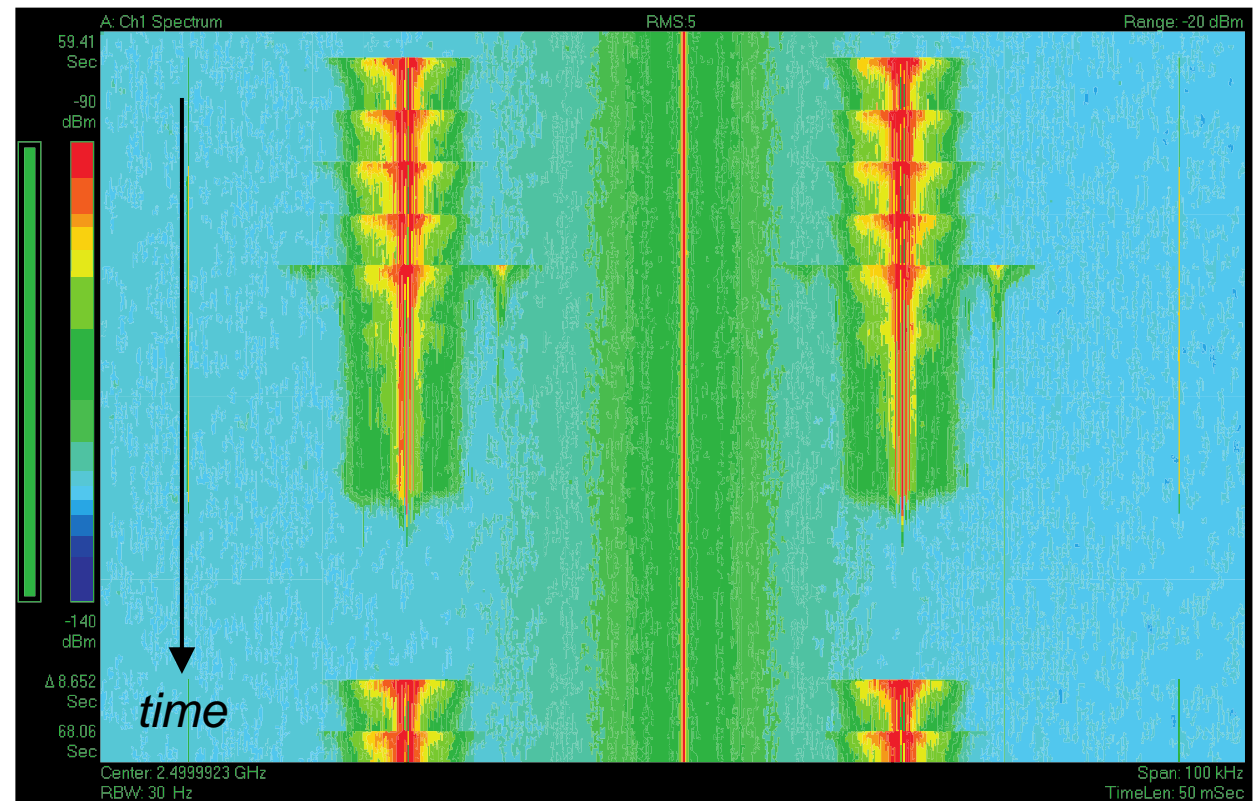


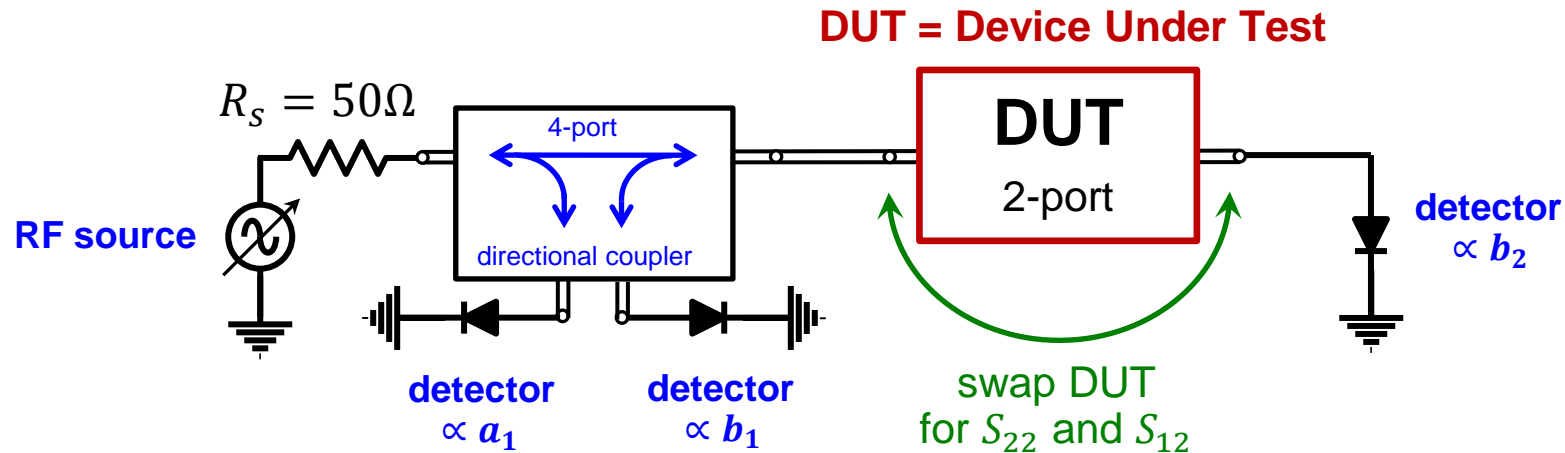
- **Electron cloud beam studies in the SPS (CERN)** – courtesy *F. Caspers*

high-resolution measurement
of the detected beam signal



color-coded spectrogram display
of 200 measurement traces



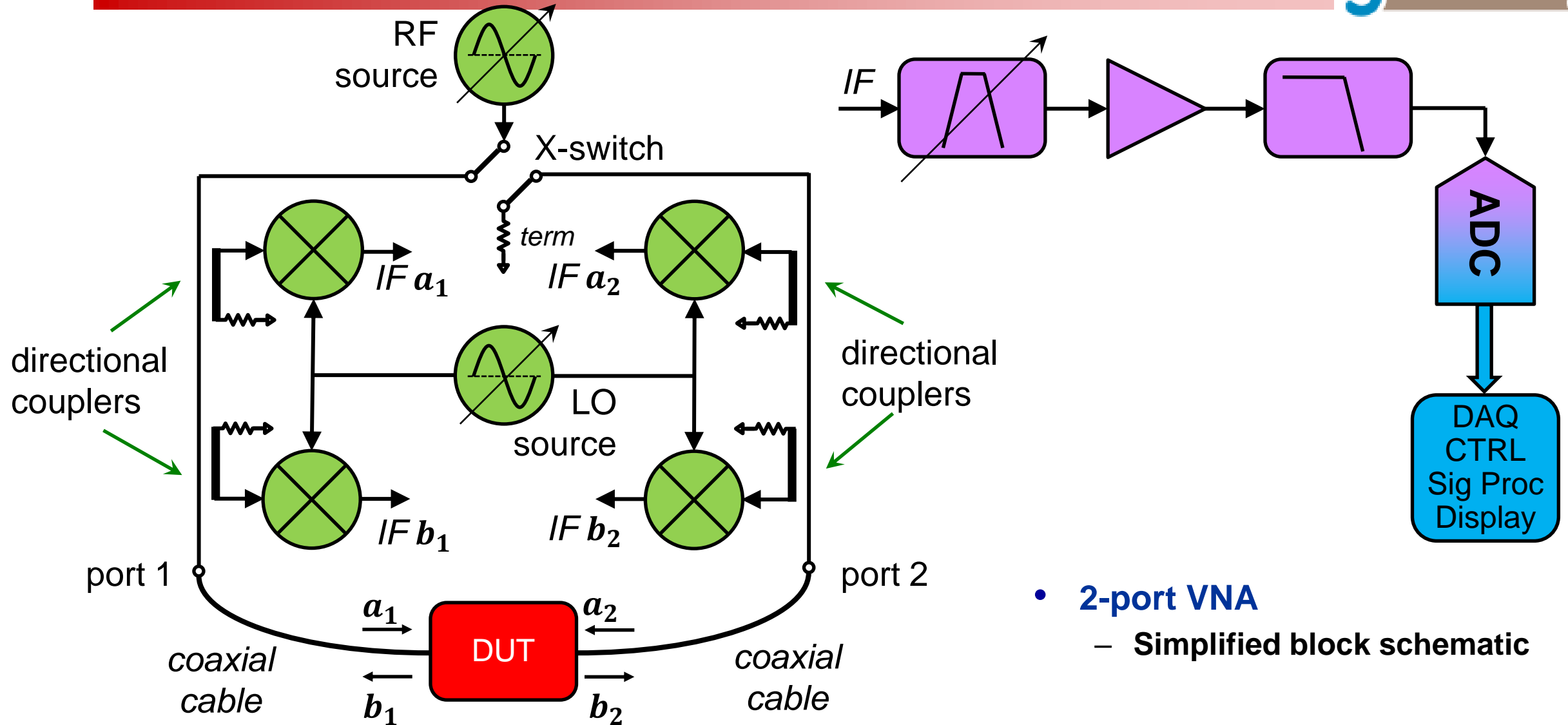


- **Performed in the “frequency domain”**
 - Single or swept frequency generator, stand-alone or as part of a VNA or SA
 - Requires a **directional coupler** and RF detector(s) or receiver(s)
- **Evaluate S_{11} and S_{21} of a 2-port DUT**
 - Ensure $a_2 = 0$, i.e., the detector at port 2 offers a well-matched impedance
 - Measure incident wave a_1 and reflected wave b_1 at the directional coupler ports and compute for each frequency
 - Measure transmitted wave b_2 at DUT port 2 and compute
- **Evaluate S_{22} and S_{12} of the 2-port DUT**
 - Perform the same methodology as above by exchanging the measurement equipment on the DUT ports

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0}$$

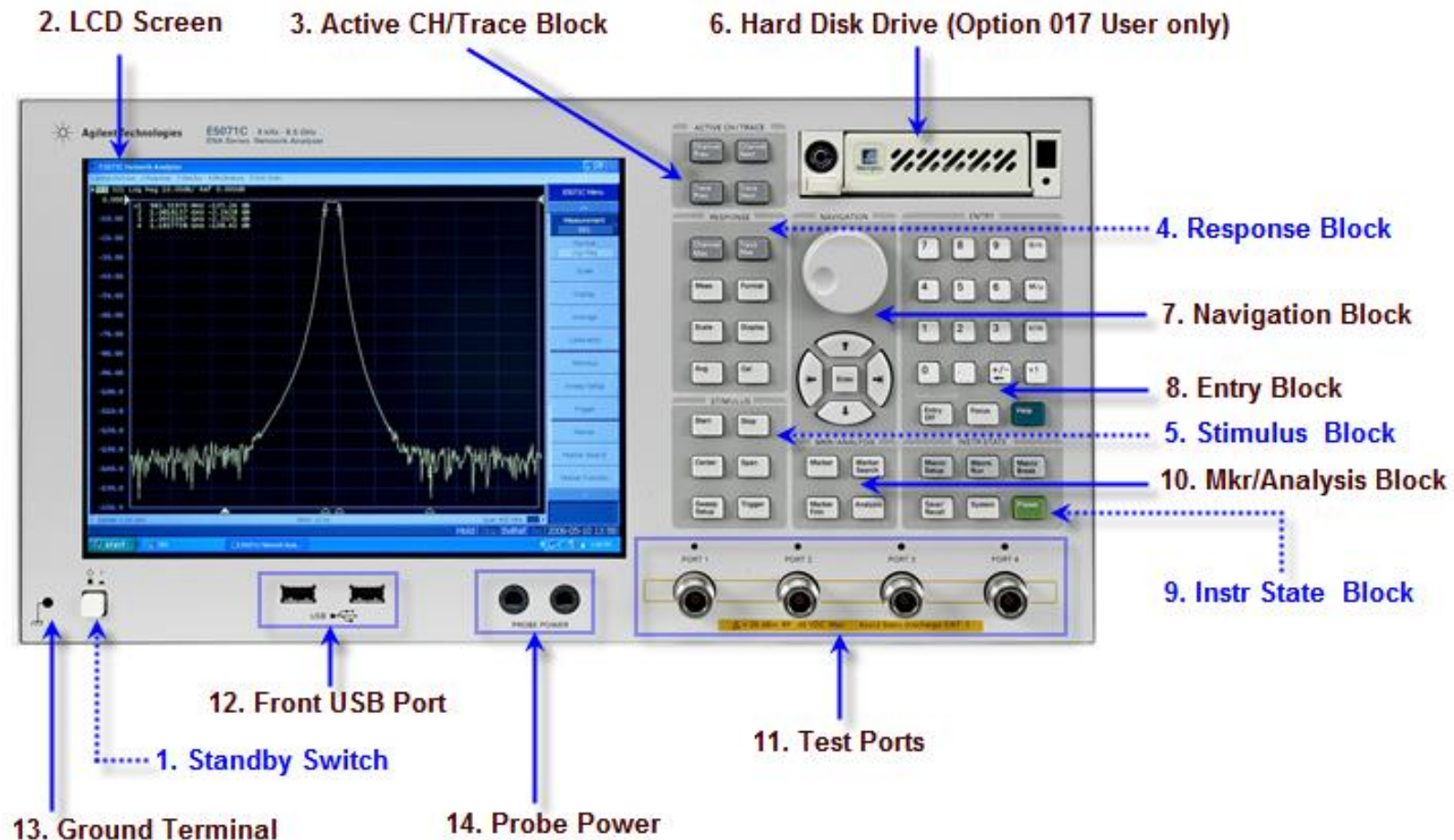
$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0}$$

The Vector Network Analyzer (VNA)



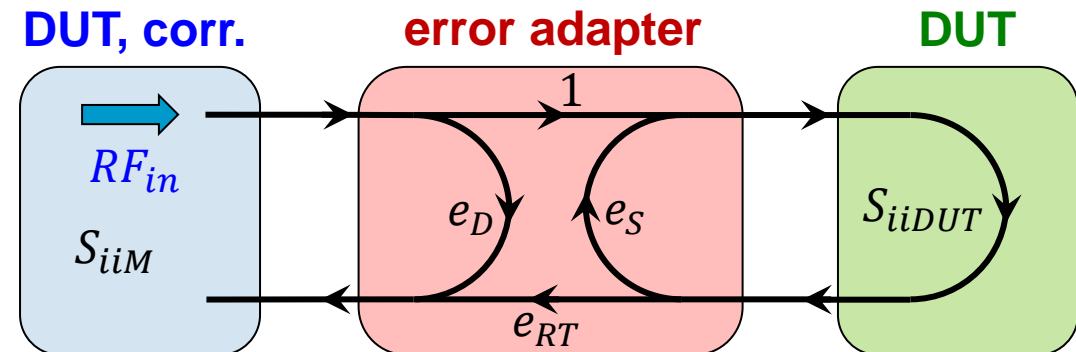
- **2-port VNA**
 - Simplified block schematic

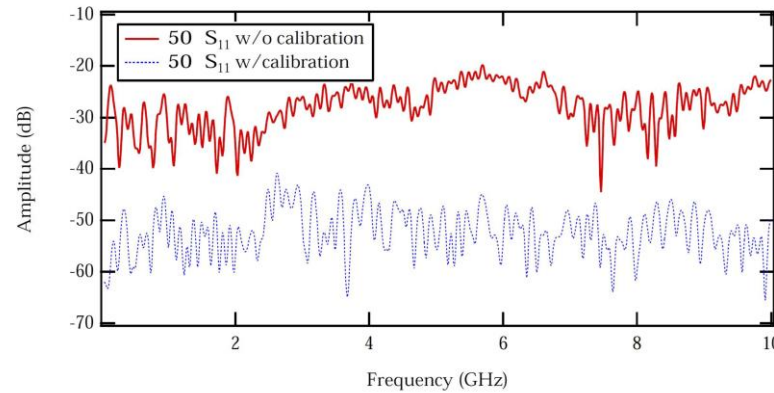
Fun with the VNA!



- The “look and feel” between VNAs vary between manufacturers and models
 - Concepts and operation is still very similar

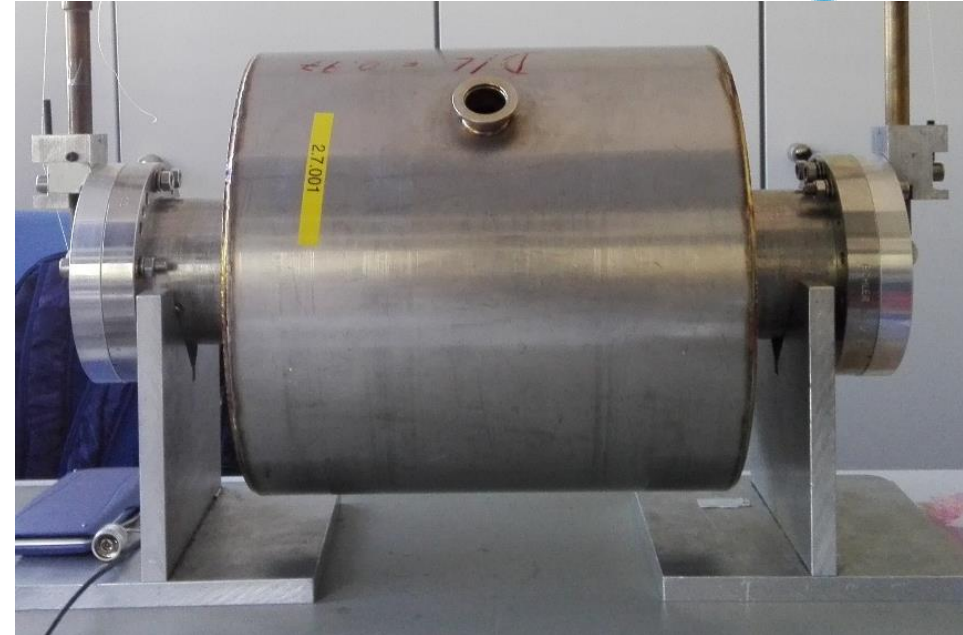
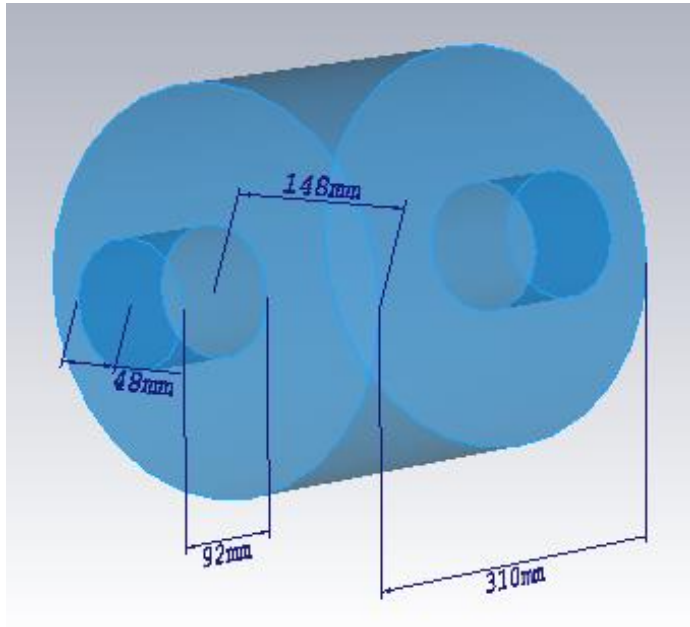
- Calibration is not necessary for pure frequency or phase measurements
- Before calibrating the VNA measurement setup, perform a brief measurement and chose appropriate VNA settings:
 - Frequency range (center, span or start, stop)
 - Number of frequency points
 - Can be sometimes increased by rearranging the VNA memory (# of channels)
 - IF filter bandwidth
 - Output power level
- **Calibrate** the setup, preferable with an **electronic calibration system** if more than 2 ports are used!
 - Each port and combination needs to be calibrated, with the cables attached
 - Choose the appropriate connector type and sex
 - The instrument establishes a correction matrix and displays the "CAL" status.





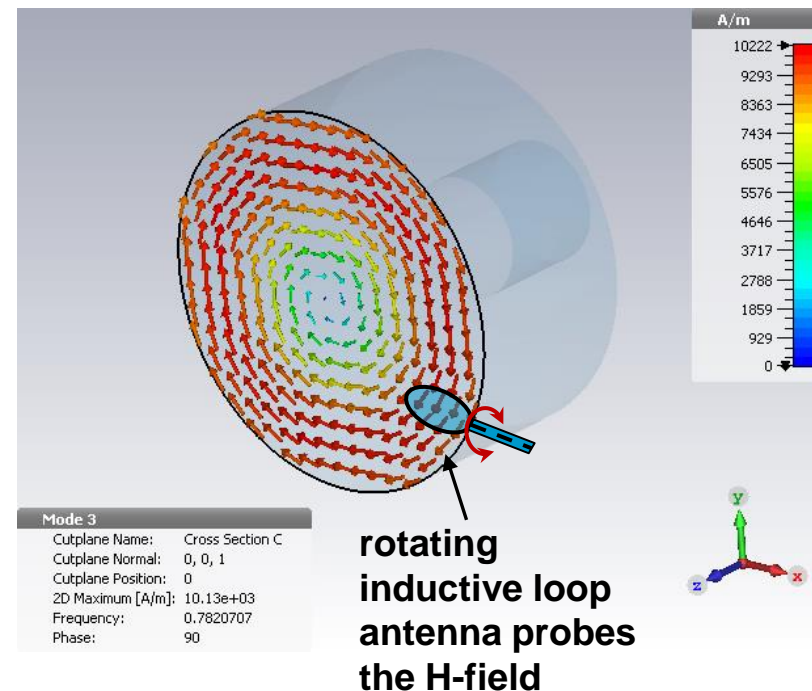
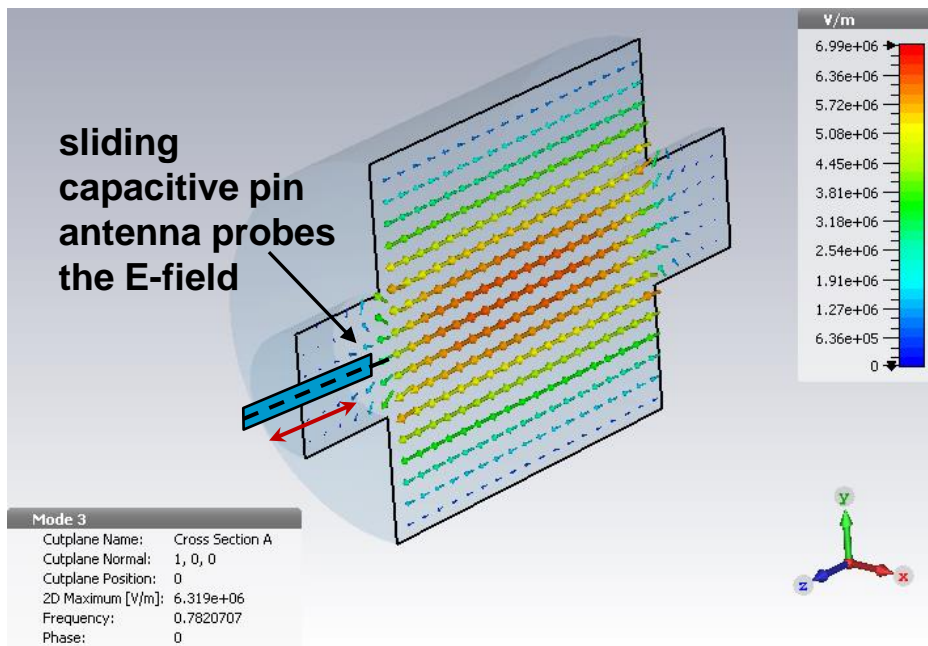
- **Calibration improves the measurement performance**
 - Return loss improvement by typically 20 dB. Enables mdB accuracy measurements!
 - Full 2-port or 4-port calibration with manual calibration kits is prone to errors, better use electronic calibration systems.
 - **Changing VNA settings after calibration** will cause the instrument to inter- and extrapolate, and the **calibration status becomes uncertain!**
- **Cables need to be included in the calibration!**
 - However, changing coaxial connector types usually not!
 - Special VNA cables allows the adaption of different connector types and sex, without requiring a re-calibration of the setup!

- **Modern VNAs (SAs, oscilloscopes, etc. as well) have many “features”**
- **Hardware features, e.g.**
 - Automatic calibration system, down to DC
 - 4 and more ports
 - Additional 2nd source, for downconverter / mixer measurements
 - Integrated spectrum analyzer function
- **Software, control and data post processing options, e.g.**
 - Far too many to list all
 - Sweep options, e.g., lin., log., segmented, in frequency or power
 - iDFT (or iFFT), gating
 - TDR, TDT for BP or LP step or impulse, segmented (advanced) TDR
 - Only for linear, time-invariant systems!
 - Port extension, virtual ports (4-port VNA), Z_{0e} , Z_{0o} characterization, virtual baluns, etc.
 - Data transformations, e.g., $\Gamma \Rightarrow Z$
 - Noise figure measurements
 - Measurements following telecommunication standards

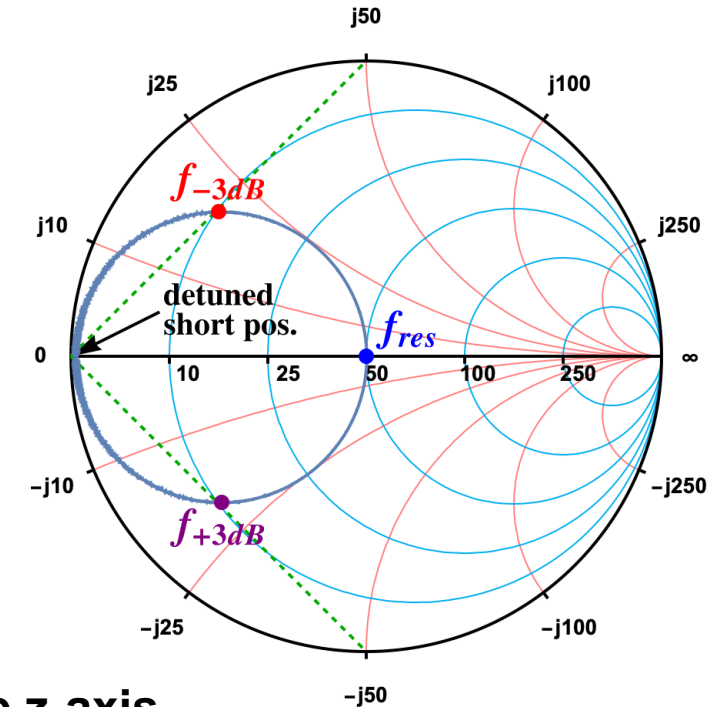
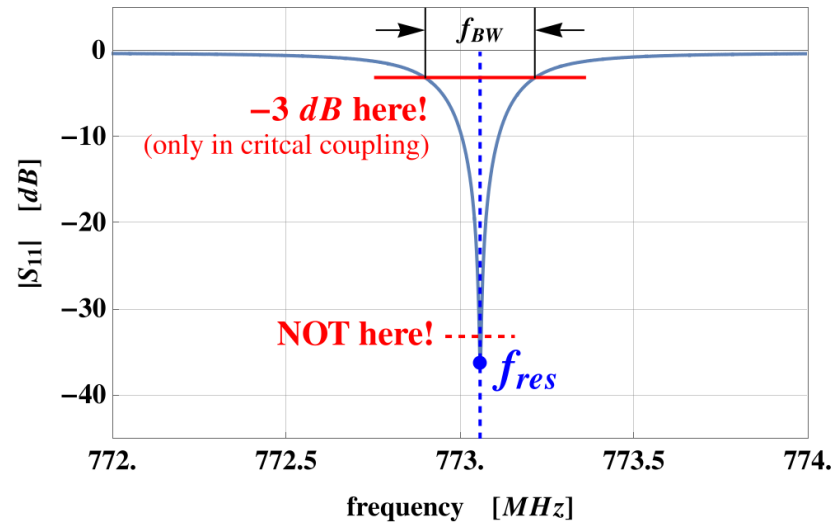
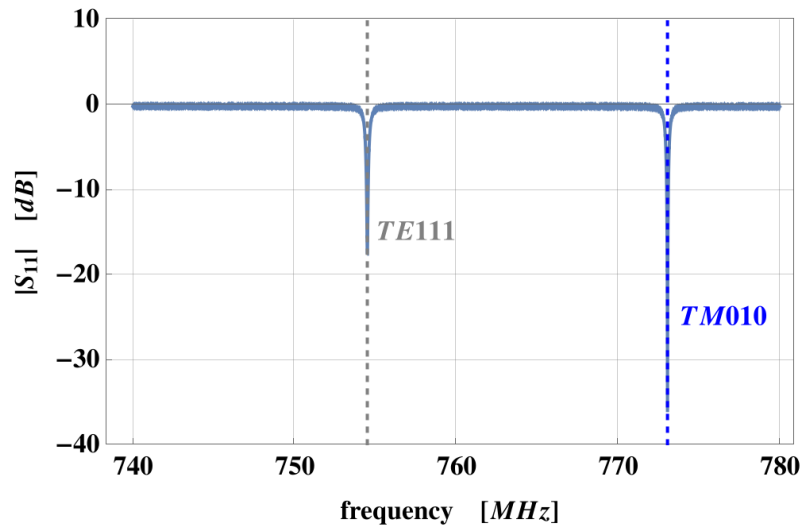


- Characterize the accelerating TM_{010} mode of a cylindrical cavity with beam ports
 - The TM_{010} does not have to be the lowest frequency mode
- Compare the measured values of f_{res} , Q_0 and R/Q
 - with an analytical analysis of a perfect cylinder (no beam ports)
 - with a numerical analysis

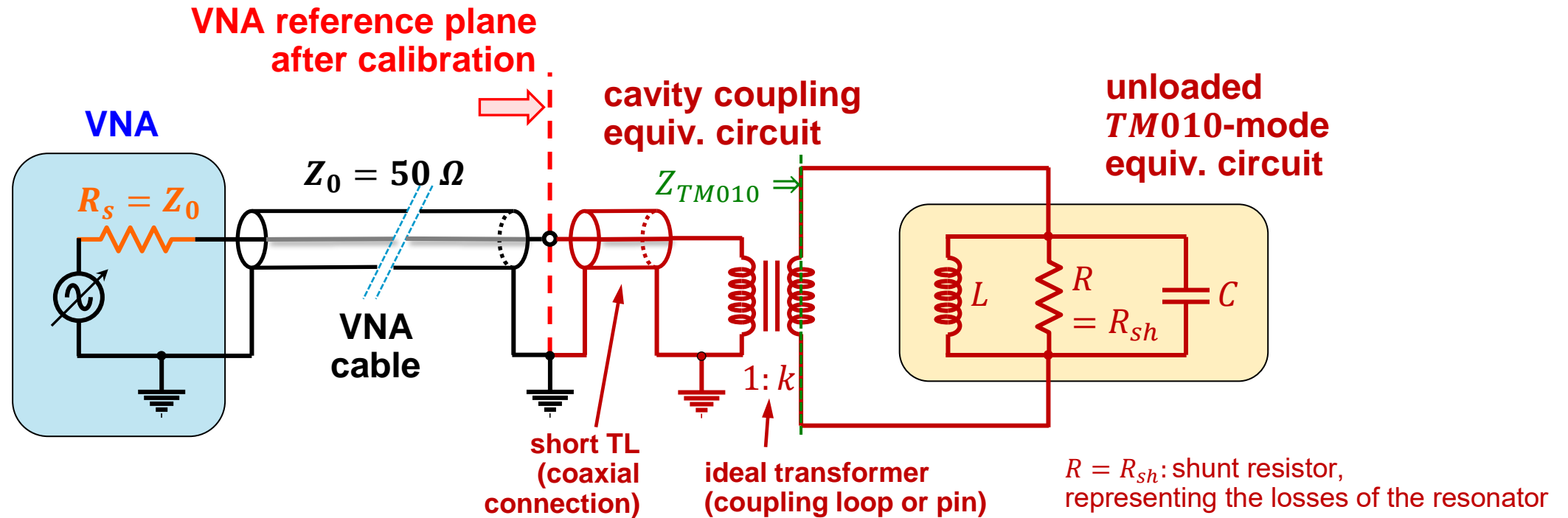
* normal conducting!



- S_{11} measurement with tunable coupling antenna
 - E-field on z-axis using a capacitive coupling pin
 - Center pin, e.g., of semi-rigid coaxial cable
 - H-field on the cavity rim using an inductive coupling loop
 - Bend the center conductor to a closed loop connected to ground



- **Identify the correct (TM_{010}) mode frequency**
 - Introduce a small perturbation, e.g., metallic rod or wire on the z-axis, and observe the shift of the mode frequencies
- **Calibrate the VNA and measure S_{11}**
 - Tune the coupling loop for critical coupling
 - Display the resonant circle in the *Smith* chart using enough points!



we have resonance condition, when:

$$\omega L = \frac{1}{\omega C}$$

at resonance: $Z_{TM010} = R$

→ resonance frequency:

$$\omega_{res} = 2\pi f_{res} = \frac{1}{LC} \Rightarrow f_{res} = \frac{1}{2\pi\sqrt{LC}}$$

- Characteristic impedance "R over Q"
- Stored energy at resonance
- Dissipated power
- Q-factor
- Shunt impedance (circuit definition)
- Tuning sensitivity
- Coupling parameter (shunt impedance over generator or feeder impedance)

$$X = \frac{R}{Q} = \omega_{res} L = \frac{1}{\omega_{res} C} = \sqrt{L/C}$$

$$U = U_e + U_m = \frac{1}{4} |V_C|^2 C + \frac{1}{4} |I_L|^2 L$$

V_C ... Voltage at the capacitor

I_L ... Current in the inductor

$$P = \frac{V^2}{2R}$$

$$Q = \frac{R}{X} = \frac{\omega_{res} U}{P}$$

U ... stored energy
 P ... dissipated power over 1 period

$$R = \frac{V^2}{2P} \quad \frac{R}{Q} = \frac{V^2}{2\omega_{res} U}$$

$$\frac{\Delta f}{f} = \frac{1}{2} \frac{\Delta C}{C} = -\frac{1}{2} \frac{\Delta L}{L}$$

$$k^2 = \frac{R}{R_{input}}$$

tune for critical coupling

- The quality (Q) factor of a resonant circuit is defined as ratio of the stored energy U over the energy dissipated P in one oscillation cycle:

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy dissipated in 1 cycle}} = \frac{\omega_{res}U}{P}$$

- The Q -factor of an impedance loaded resonator:
 - Q_0 : unloaded Q-value of the unperturbed system
 - Q_L : loaded Q-value, e.g., measured with the impedance of the connected generator
 - Q_{ext} : external Q-factor, representing the effects of the external circuit (generator and coupling circuit)

- Q-factor and bandwidth**

- This is how we actually "measure" the Q-factor!

$$Q = \frac{f_{res}}{f_{BW}}$$

with: $f_{BW} = f_{+3dB} - f_{-3dB}$

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

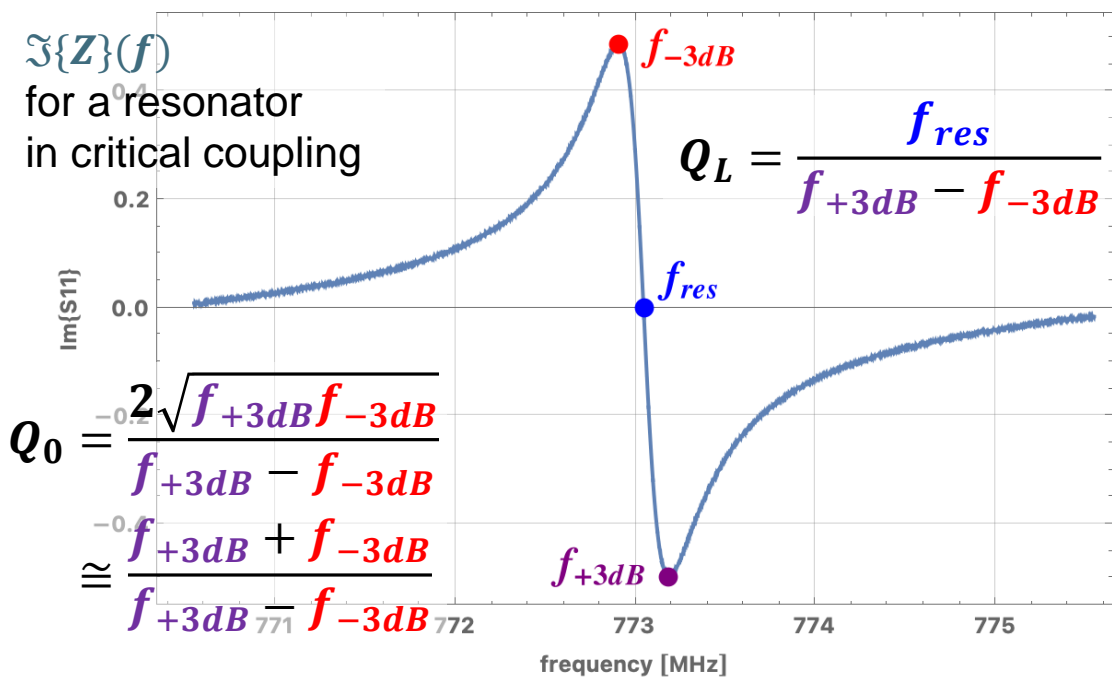
tune k for
critical coupling:

$$Q_0 = Q_{ext}$$

$$\Rightarrow Q_0 = 2 Q_L$$

with Q_L being our
measured Q-value

Q-factor from S_{11} Measurement

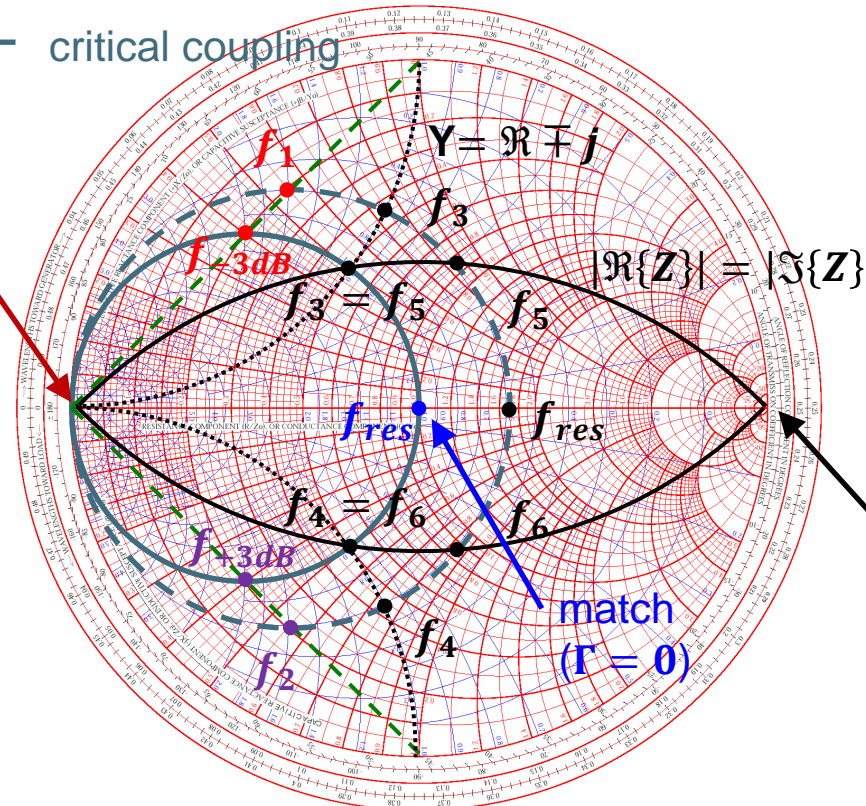


- **Correct for the uncompensated transmission-line effects between calibration reference and the coupling loop**
 - Electrical length adjustment: "straight" $\Im\{Z\}(f)$
- **Adjust the locus circle to the detuned short location**
 - Phase offset
- **Verify no evanescent fields penetrating outside the beam ports**
 - i.e., no frequency shifts if the boundaries at the beam ports are altered

--- arbitrary coupling (here: over-critical)

— critical coupling

detuned short position ($Z = 0$)



detuned open position ($Y = 0$)

Frequency marker points in the *Smith* chart:
 $f_{1,2}$ (f_{-3dB}, f_{+3dB}): $|\Im\{S_{11}\}| = \mathbf{max}$. to calculate Q_L
 $f_{3,4}$: $Y = \Re + j$ to calculate Q_{ext}
 $f_{5,6}$: $|\Re\{Z\}| = |\Im\{Z\}|$ to calculate Q_0

- Remember from the equivalent circuit:

$$\frac{R}{Q} = \frac{V_{acc}^2 / 2P_d}{\omega_{res} U / P_d} = \frac{V_{acc}^2}{2\omega_{res} U} \quad \text{with: } V_{acc} = \left| \int E_z(z) \cos\left(\frac{\omega z}{\beta c_0}\right) dz \right|$$

transit time related

- V_{acc} is based on the integrated longitudinal E-field component E_z along the z-axis ($x = y = 0$)
- Based on Slater's perturbation theorem:

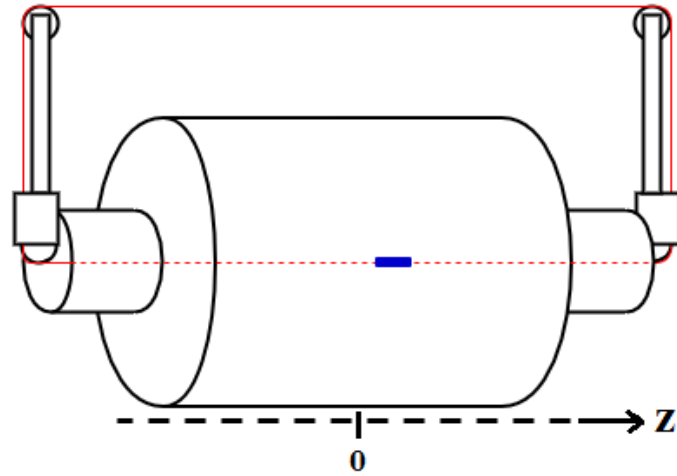
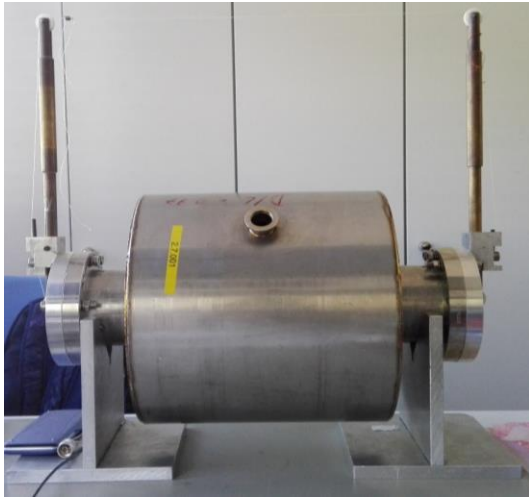
$$\frac{\Delta f}{f_{res}} = \frac{1}{U} \left[\mu_0 \left(k_{\parallel}^H |H_{\parallel}|^2 + k_{\perp}^H |H_{\perp}|^2 \right) - \epsilon_0 \left(k_{\parallel}^E |E_{\parallel}|^2 + k_{\perp}^E |E_{\perp}|^2 \right) \right]$$

- Resonance frequency shift due to a small perturbation object, expressed in longitudinal and transverse E and H field components
- k : coefficients proportional to the electric or magnetic polarizability of the perturbation object (here: only k_{\parallel}^E for a longitudinal metallic object)

- E-field characterization along the z-axis

$$E(z) = E_{\parallel}(z) = \sqrt{U \frac{\Delta f(z)}{f_{res}} \cdot \frac{-1}{k_{\parallel}^E \epsilon_0}}$$

with: $k_{\parallel}^E = \frac{\pi}{3} l^3 \left[\sinh^{-1} \left(\frac{2l}{3\pi a} \right) \right]^{-1}$
(metallic ellipsoid, e.g., syringe needle of half length l and radius a)



- **E-field characterization by evaluating**

- The frequency shift Δf (S_{11} reflection measurement with a single probe) or

- The phase shift ϕ at f_{res} (S_{21} transmission measurement with 2 probes)

$$\frac{\Delta f}{f_{res}} = \frac{1}{2 Q_0} \tan \phi$$

- **Exercise with a manual bead-pull through a known cavity**

- requires: fishing wire, syringe needle, ruler and VNA
- Compare the measured E_z at the maximum f or ϕ shift (in the center of the cavity) with the theoretical estimation (e.g., numerical computed value)

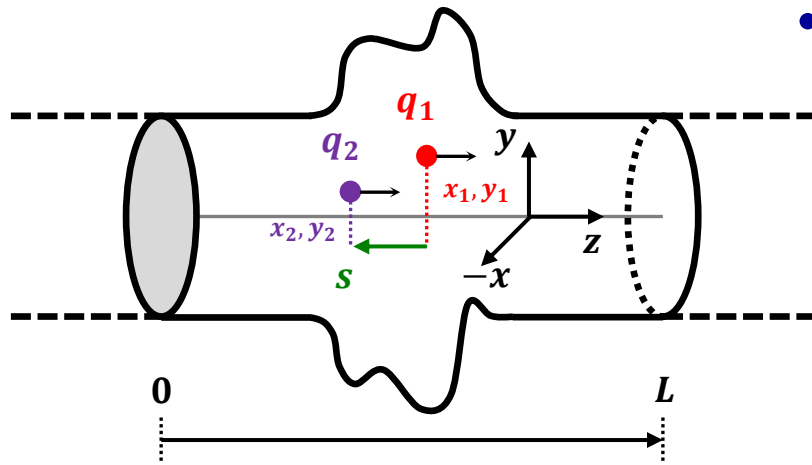
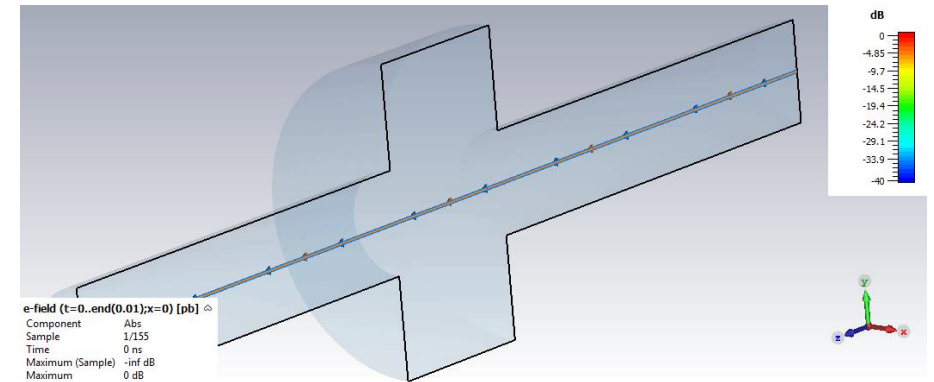
- The wake potential

- Lorenz force on q_2 by the wake field of q_1 :

$$\vec{F} = \frac{d\vec{p}}{dt} = q_2(\vec{E} + c_0\vec{e}_z \times \vec{B})$$

- Wake potential of a structure, e.g., a discontinuity driven by q_1

$$\vec{w}(x_1, y_1, x_2, y_2, s) = \frac{1}{q_1} \int_{-\infty}^{+\infty} dz [\vec{E}(x_2, y_2, z, t) + c_0\vec{e}_z \times \vec{B}(x_2, y_2, z, t)]_{t=(s+z)/c}$$



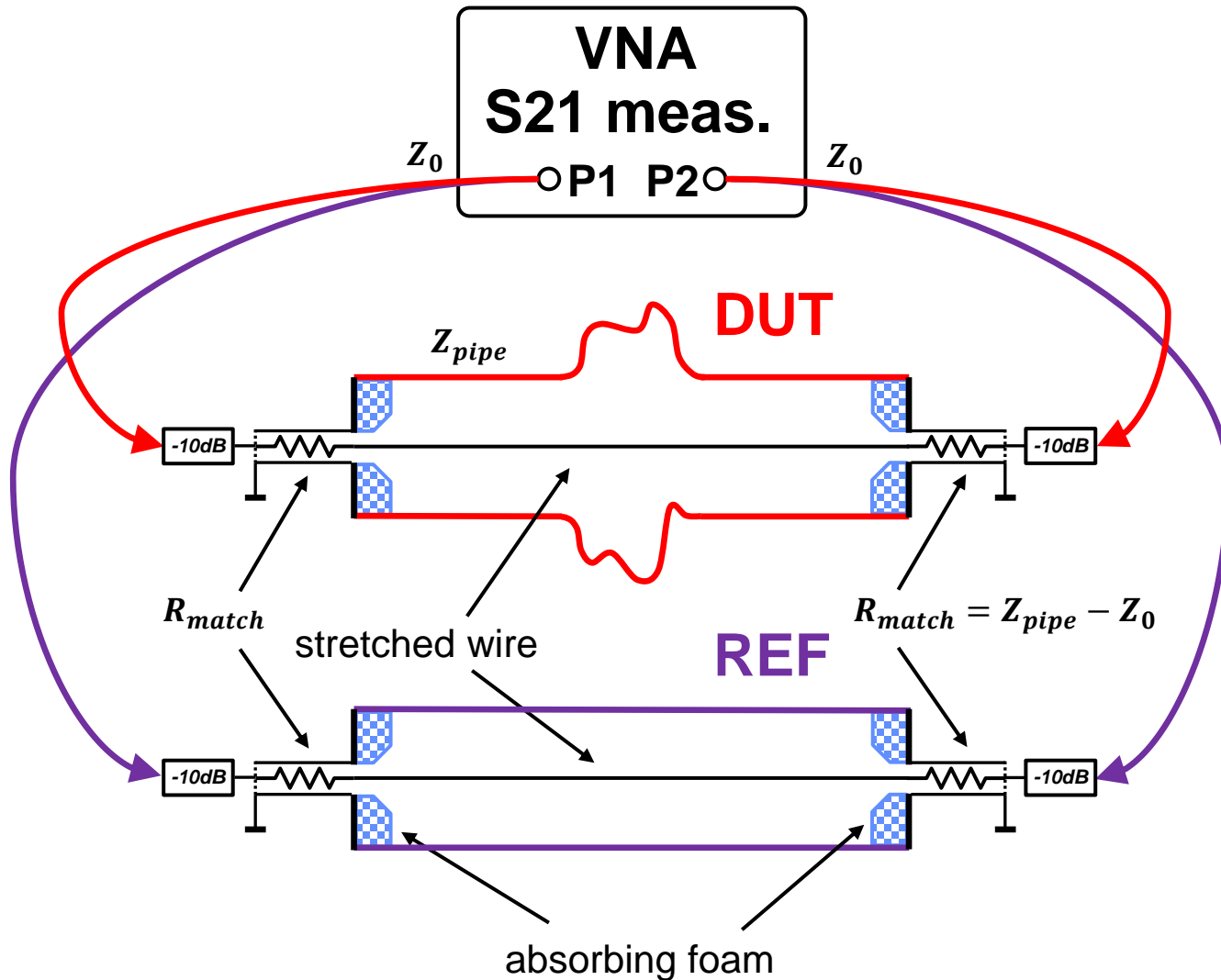
- Beam coupling impedance

- Frequency domain representation of the wake potential

$$Z(x_1, y_1, x_2, y_2, \omega) = -\frac{1}{c_0} \int_{-\infty}^{+\infty} ds \vec{w}(x_1, y_1, x_2, y_2, s) e^{-j\omega s/c_0}$$

- Can be decomposed in **longitudinal Z_{\parallel}** and **transverse Z_{\perp}** components (*Panofsky-Wenzel* theorem)

- Resonant structures, i^{th} mode: $R_{sh,i} = Z_{\parallel,i} = \frac{2k_{loss,i}Q_i}{\omega_i}$



- Formulas:

- Normalized electrical length: $\theta = 2\pi \frac{L}{\lambda}$
- Lumped impedance formula

$$Z_{\parallel} = 2Z_{pipe} \frac{1 - S_{21}}{S_{21}} \quad \begin{matrix} \theta \leq 1 \\ L < D_{pipe} \end{matrix}$$

- Log formula

$$Z_{\parallel} = -2Z_{pipe} \ln S_{21}$$

- Improved log formula

$$Z_{\parallel} = -2Z_{pipe} \ln S_{21} \left(1 + j \frac{\ln S_{21}}{2\theta} \right)$$

- Transmission coefficient

$$S_{21} = \frac{S_{21,DUT}}{S_{21,REF}}$$

- Circular beam pipe impedance

$$Z_{pipe} = \frac{\eta_0}{2\pi\sqrt{\epsilon_r}} \ln \frac{D}{d} \cong 60 \Omega \ln \frac{D_{pipe}}{d_{wire}}$$

- No summary, just

Thank you!