2025 Joint Universities Accelerator School

Superconducting Magnets
Section II

Paolo Ferracin

(pferracin@lbl.gov)

Lawrence Berkeley National Laboratory (LBNL)

Outline

- **Section I**
  - Particle accelerators and magnets
  - Superconductivity and practical superconductors
- **Section II**
  - **Magnetic design**
- **Section III**
  - Coil fabrication
  - Forces, stress, pre-stress
  - Support structures
- **Section IV**
  - Quench, training, protection

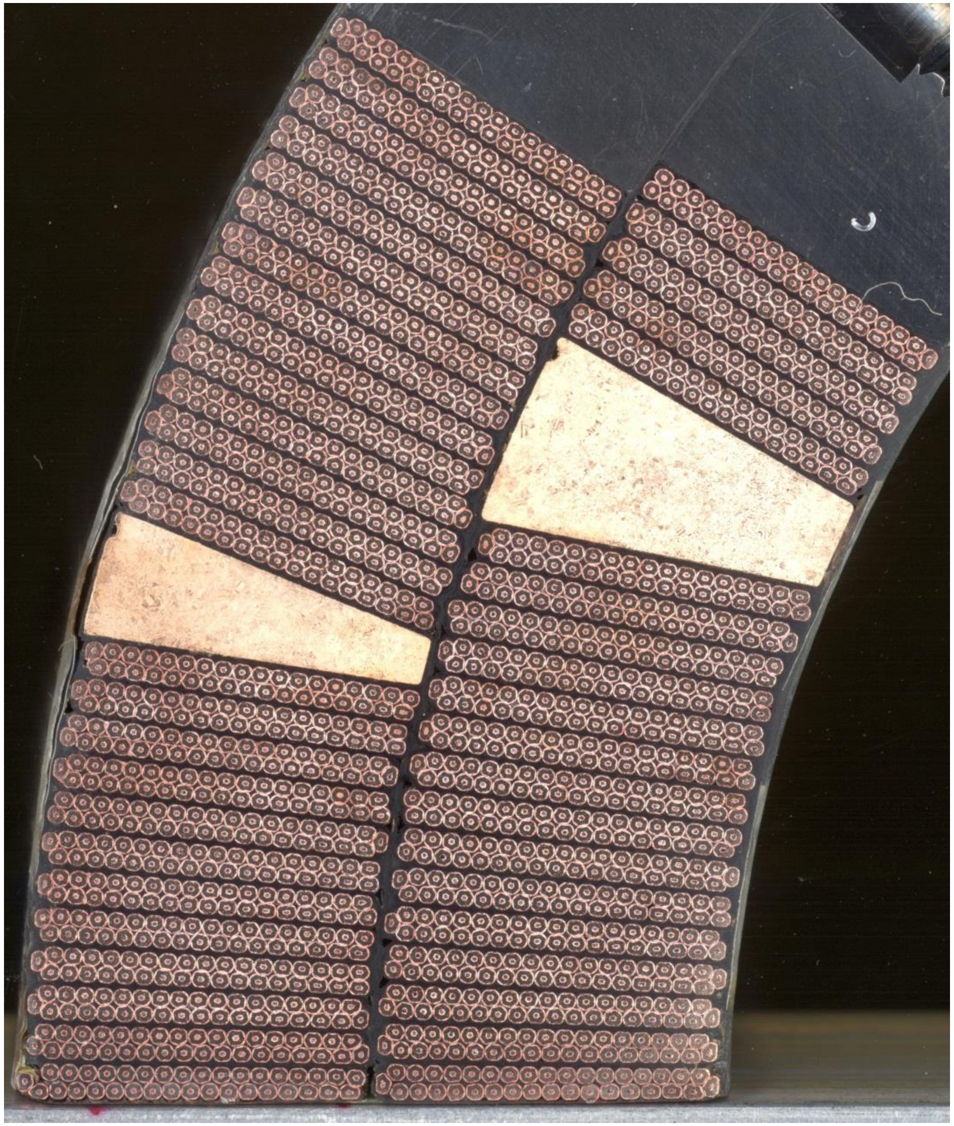
Magnetic design
Introduction

- The magnetic design is one of the first steps in the a superconducting magnet development
- It starts from the **requirements** (from accelerator physicists, researchers, medical doctors…others)
  - A field "**shape**"
    - Dipole, quadrupole, etc
  - A field **magnitude**
    - Usually with low T superconductors from 5 to 20 T
  - A field **homogeneity**
    - Uniformity inside a solenoid, harmonics in a accelerator magnet
  - A given **aperture** (and **volume**)
    - Some cm diameter for accelerator magnets, much more for detectors and fusion magnets

Magnetic design

- **How much** conductor do we need to meet the requirements?
- And in **which configuration**?

- **Outline**
  - How do we create a **perfect field**?
  - How do we express the field and its "**imperfections**"?
  - How do we **design a coil** to minimize field errors?
  - Which is the **maximum field** we can get?
  - **Overview** of different designs



References

- Magnetic design
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  - Martin N. Wilson, "*Superconducting Magnets*", 1983.
  - Fred M. Asner, "*High Field Superconducting Magnets*", 1999.
  - S. Russenschuck, "*Field computation for accelerator magnets*", J. Wiley & Sons (2010).
  - S. Izquierdo Bermudez, "*Field Quality in Nb3Sn Superconducting Accelerator Magnets*", PhD thesis, Universidad Politecnica de Madrid, CERN-THESIS-2023-080.
  - P. Ferracin, E. Todesco, S. Prestemon, "*Superconducting accelerator magnets*", US Particle Accelerator School, www.uspas.fnal.gov.
  - E. Todesco, "Masterclass - Design of superconducting magnets for particle accelerators", https://indico.cern.ch/category/12408/
  - A. Jain, "*Basic theory of magnets*", CERN 98-05 (1998) 1-26
  - L. Rossi, E. Todesco, "*Electromagnetic design of superconducting quadrupoles*", Phys. Rev. ST Accel. Beams 10 (2007) 112401.
  - L. Rossi and Ezio Todesco, "*Electromagnetic design of superconducting dipoles based on sector coils*", Phys. Rev. ST Accel. Beams 9 (2006) 102401.

Perfect dipole field
Intercepting circles (or ellipses)

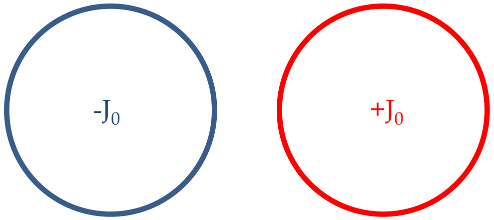
- Within a cylinder carrying j_0, the field is perpendicular to the radial direction and proportional to the distance to the centre r:

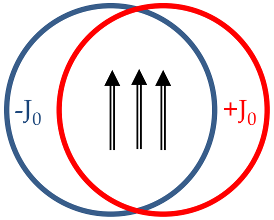
$$B = -\frac{\mu_0 j_0 r}{2}$$

  - Combining the effect of two intersecting cylinders

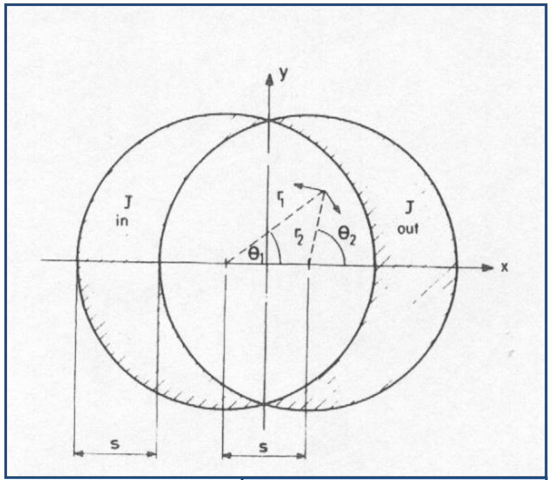
$$B_x = \frac{\mu_0 j_0 r}{2}\{-r_1 \sin\theta_1 + r_2 \sin\theta_2\} = 0$$

$$B_y = \frac{\mu_0 j_0 r}{2}\{-r_1 \cos\theta_1 + r_2 \cos\theta_2\} = -\frac{\mu_0 j_0}{2} s$$



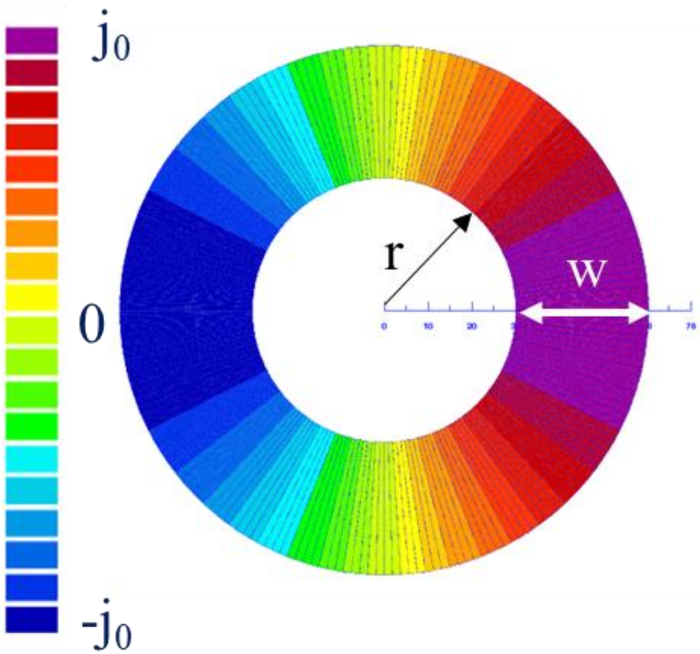


- A uniform current density in the area of two **intersecting circles** produces a pure dipole
  - The aperture is not circular
  - Not easy to simulate with a flat cable
- Similar proof for **intersecting ellipses**



by M. Wilson

- If we assume
  - $J = J_0 \cos\theta$ where J_0 [A/m^2] is ⊥ to the cross-section plane
  - Inner radius of the coil = r
  - Width of the coil = w



by S. Izquierdo Bermudez

- The generated field is a **pure dipole**

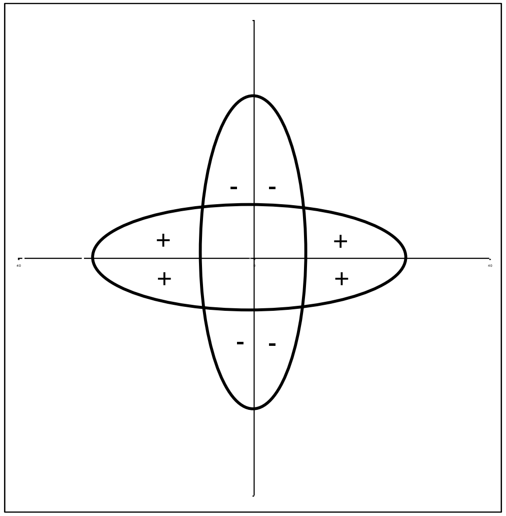
$$B_y = -\frac{\mu_0 J_0}{2} w$$

- Linear dependence on **coil width**

- **Easier** to achieve with a Rutherford cable

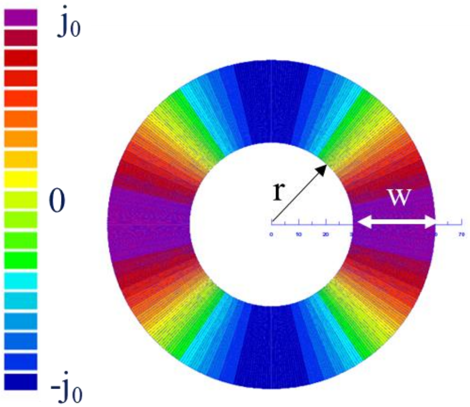
Perfect quadrupole field

- **Intercepting ellipses** or **circles**



- Thick shell with ***cos2θ* current distribution**
- If we assume
  - $J = J_0 \cos 2\theta$ where J_0 [A/m^2] is ⊥ to the cross-section plane
  - Inner radius of the coil = r
  - Width of the coil = w

$$G = \frac{B_y}{r} = -\frac{\mu_0 J_0}{2} \ln\left(1 + \frac{w}{r}\right)$$

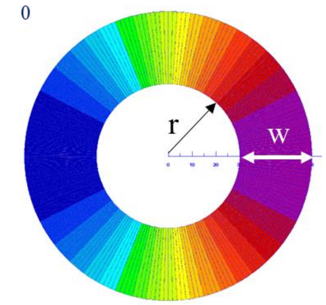


by S. Izquierdo Bermudez

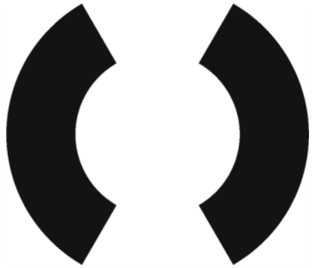
- And so on…
  - Perfect sextupoles: **cos3*θ*** or **3** intersect. ellipses
  - Perfect 2*n*-poles: **cos*nθ*** or ***n*** intersecting ellipses

From ideal to practical configuration

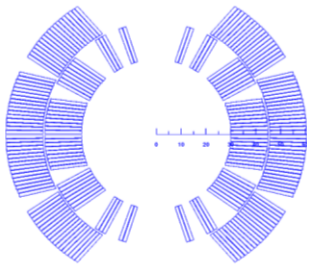
- How can I reproduce **thick shell with a *cosθ*** distribution with a cable?
  - Rectangular cross-section and constant *J*



- First "rough" approximation
  - **Sector dipole**



- Better ones
  - More **layers** and **wedges** to reduce *J* towards 90°

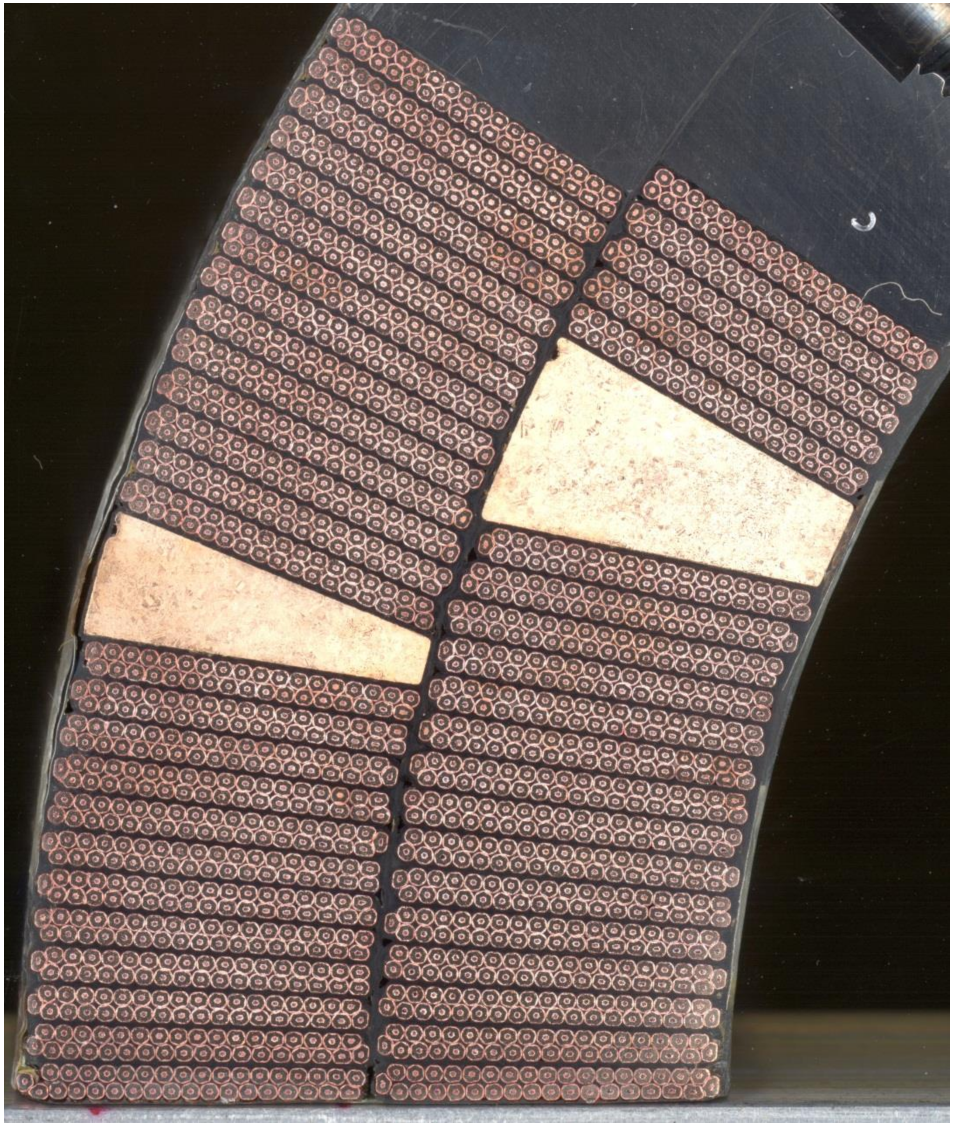


- As a result, the field is **not perfect** anymore
  - How can I express in improve the "imperfect" field inside the aperture?

Magnetic design

- **How much** conductor do we need to meet the requirements?
- And in **which configuration**?

- **Outline**
  - How do we create a **perfect field**?
  - How do we express the field and its "**imperfections**"?
  - How do we **design a coil** to minimize field errors?
  - Which is the **maximum field** we can get?
  - **Overview** of different designs



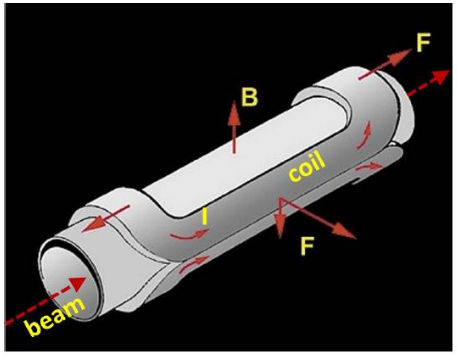
Field representation
Maxwell equations

- **Maxwell equations** for magnetic field

$$\nabla \cdot B = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \qquad \nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

- In absence of charge and magnetized material

$$\nabla \times B = \left(\frac{\partial B_y}{\partial z} - \frac{\partial B_z}{\partial y}, \frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z}, \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} \right) = 0$$



- If $\frac{\partial B_z}{\partial z} = 0$ (constant longitudinal field), then

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \qquad \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} = 0$$

Field representation
Analytic functions

- If $\frac{\partial B_z}{\partial z} = 0$

  Maxwell gives

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = 0$$
$$\frac{\partial B_y}{\partial y} + \frac{\partial B_x}{\partial x} = 0$$

$$\begin{cases} \frac{\partial f_x}{\partial x} - \frac{\partial f_y}{\partial y} = 0 \\ \frac{\partial f_x}{\partial y} + \frac{\partial f_y}{\partial x} = 0 \end{cases}$$

Cauchy-Riemann conditions

and therefore the function B_y+iB_x is analytic

$$B_y(x,y) + iB_x(x,y) = \sum_{n=1}^{\infty} C_n (x+iy)^{n-1}$$

where C_n are **complex coefficients**

$$B_y(x,y) + iB_x(x,y) = \sum_{n=1}^{\infty} C_n (x+iy)^{n-1} = \sum_{n=1}^{\infty} (B_n + iA_n)(x+iy)^{n-1}$$

- Advantage: we reduce the description of the field to a (simple) series of complex coefficients

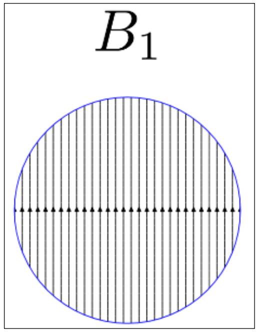
Field representation
Harmonics

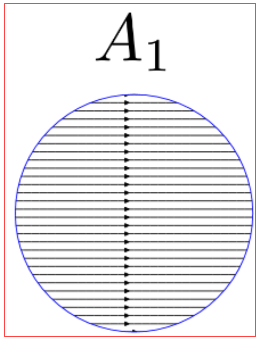
- What are these coefficients (or **harmonics**)?

$$B_y(x,y)+iB_x(x,y)=\sum_{n=1}^{\infty}C_n(x+iy)^{n-1}=\sum_{n=1}^{\infty}(B_n+iA_n)(x+iy)^{n-1}$$

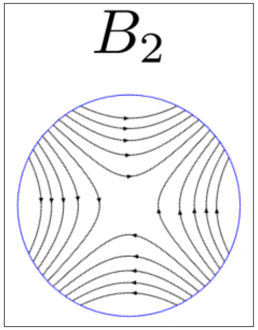
- For *n=1*→ *dipole*

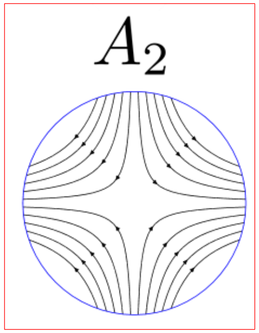
$$B_y+iB_x \Rightarrow (B_1+iA_1)$$





- For *n=2*→ *quadrupole*



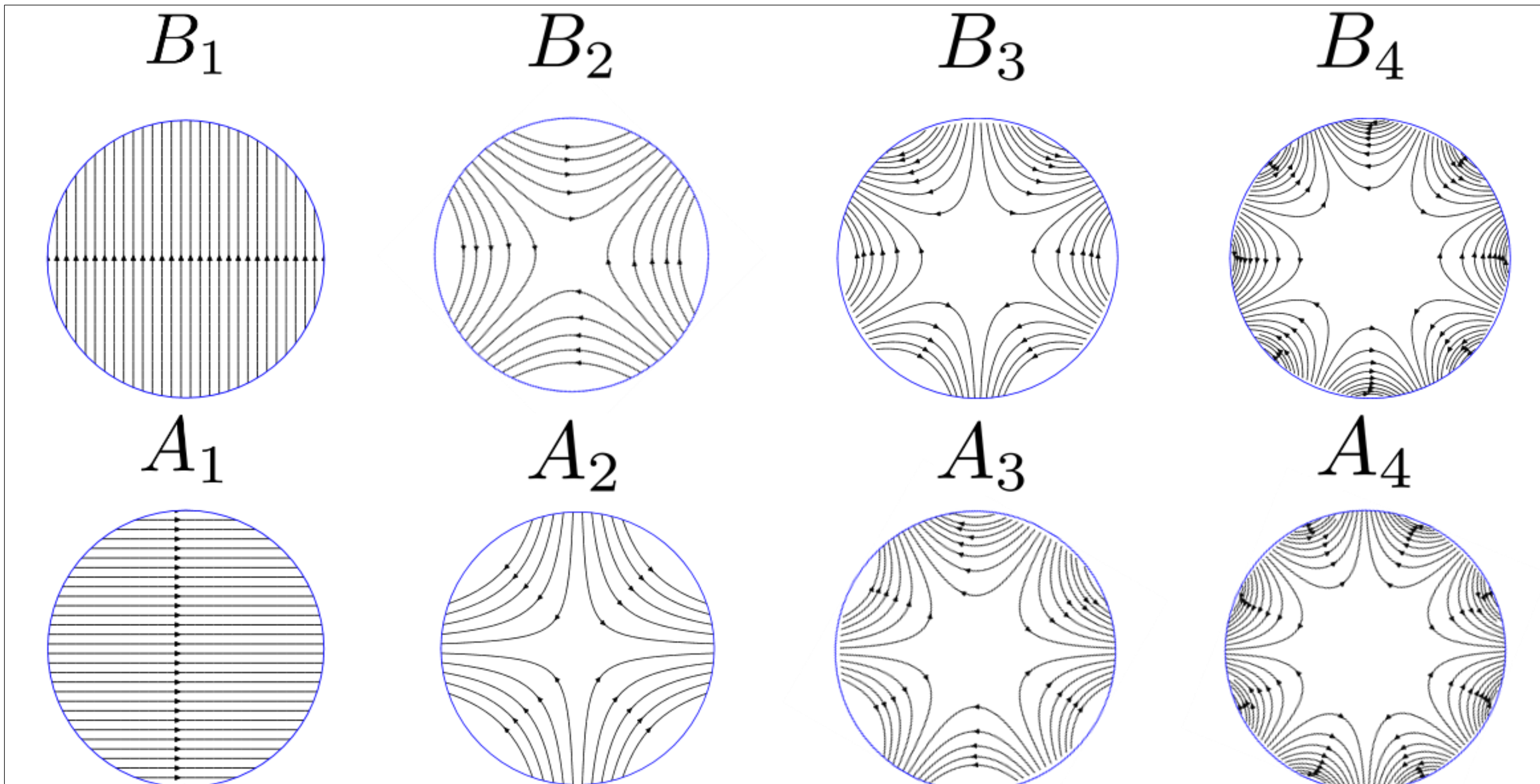


$$B_y+iB_x \Rightarrow (B_2+iA_2)(x+iy)=(B_2x+iB_2y)+(iA_2x-A_2y)$$

by J.L. Rudeiros Fernandez

- So, each coefficient corresponds to a "pure" multipolar field

$$B_y(x,y) + iB_x(x,y) = \sum_{n=1}^{\infty} C_n (x+iy)^{n-1} = \sum_{n=1}^{\infty} (B_n + iA_n)(x+iy)^{n-1}$$



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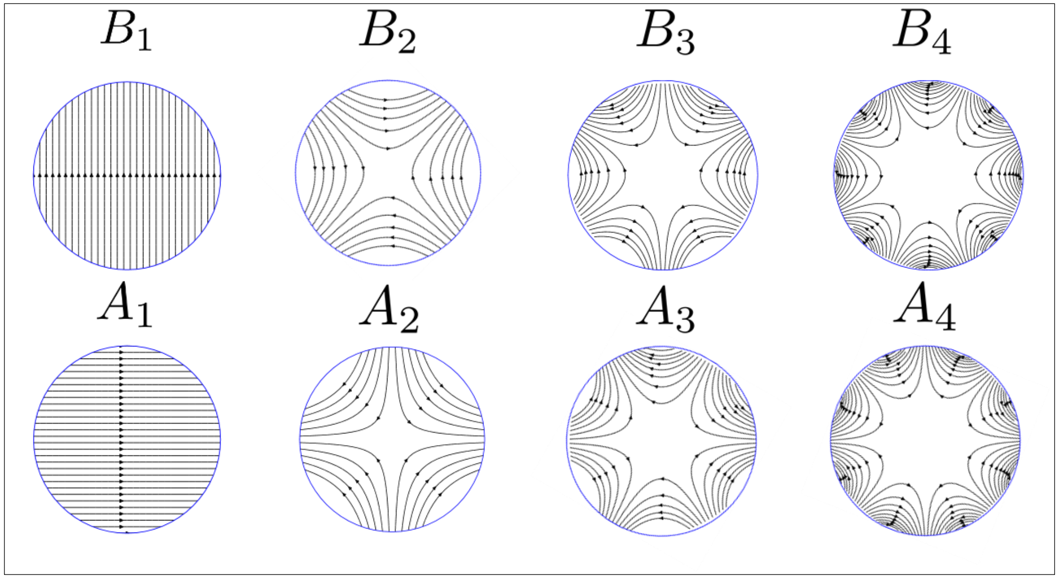
Field representation
Harmonics



- The field harmonics are rewritten as (EU notation)

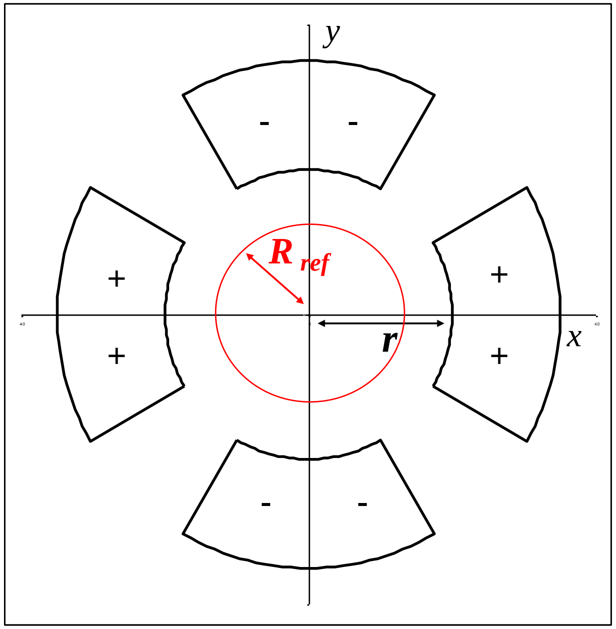
$$B_y + iB_x = 10^{-4} B_1 \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x+iy}{R_{ref}} \right)^{n-1}$$

  - We factorize the main component (B_1 for dipoles, B_2 for quadrupoles)
  - We introduce a reference radius R_{ref} to have dimensionless coefficients
  - We factorize 10^{-4} since the deviations from ideal field are ~0.01%
- The coefficients b_n, a_n are called normalized multipoles
  - b_n are the normal, a_n are the skew (adimensional)

Field representation
Harmonics

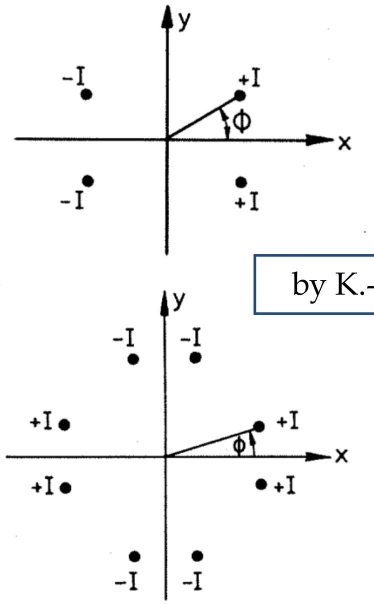
$$B_y + iB_x = 10^{-4} B_1 \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x+iy}{R_{ref}} \right)^{n-1}$$

- The coefficients b_n, a_n are called normalized multipoles
  - b_n are the normal, a_n are the skew (adimensional)
- Reference radius is usually chosen as 2/3 of the aperture radius



Field representation Harmonics

- One can demonstrate that with line currents with a **dipole** or a **quadrupole symmetry**, most of the **multipoles cancelled**
- For $n=1 \rightarrow$ *dipole*
  - Only b_3, b_5, b_7, ….. are present
- For $n=2 \rightarrow$ *quadrupole*
  - Only b_6, b_{10}, b_{14}, ….. are present
- …and so on

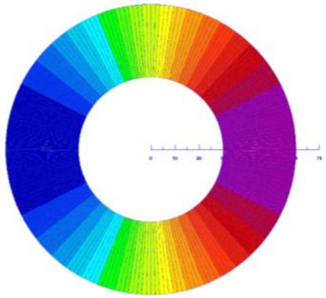


by K.-H. Mess, *et al.*

- These multipoles are called ***allowed multipoles***
- The field quality optimization of a coil lay-out concerns only a **few** quantities
  - For a dipole, usually *b3* , *b5* , *b7* , and possibly *b9* , *b11*

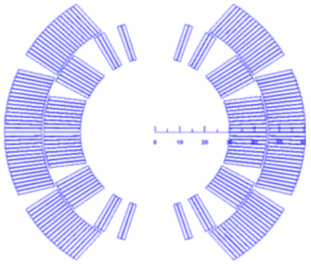
Back to the original issue: From ideal to practical configuration

- How can I reproduce **thick shell with a *cosθ*** distribution with a cable?
  - Rectangular cross-section and constant J



- First "rough" approximation
  - **Sector dipole**

- Better ones
  - More **layers** and **wedges** to reduce J towards 90°



- Now, I can use the multipolar expansion to **optimize** my "practical" **cross-section**

Magnetic design

- **How much** conductor do we need to meet the requirements?
- And in **which configuration**?

- **Outline**
  - How do we create a **perfect field**?
  - How do we express the field and its "**imperfections**"?
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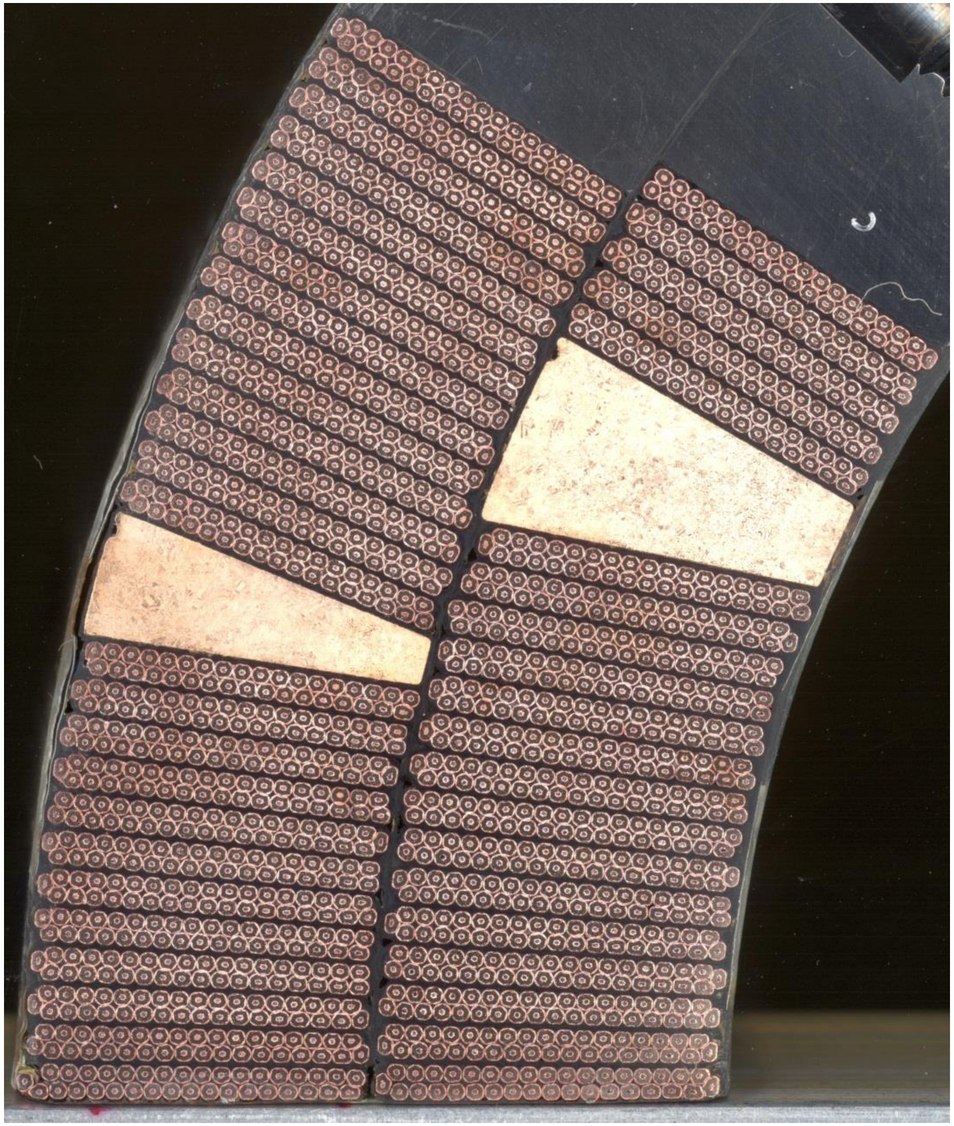
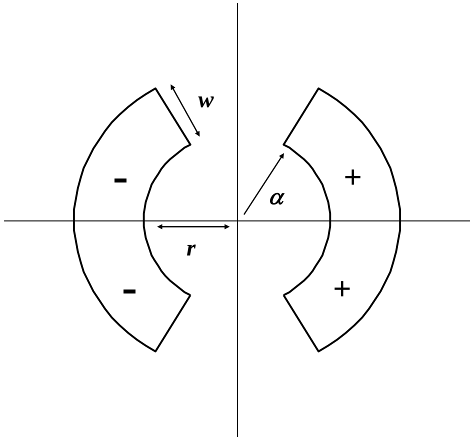


A "good" field quality dipole
Sector dipole

- We compute the central field given by a **sector dipole** with uniform current density j
- We start from **Biot-Savart law** and integrate



$$I \rightarrow j\rho d\rho d\theta$$

- And we obtain

$$B_1 = -2\frac{j\mu_0}{2\pi}\int_{-\alpha}^{\alpha}\int_{r}^{r+w}\frac{\cos\theta}{\rho}\rho d\rho d\theta = -\frac{2j\mu_0}{\pi}w\sin\alpha$$

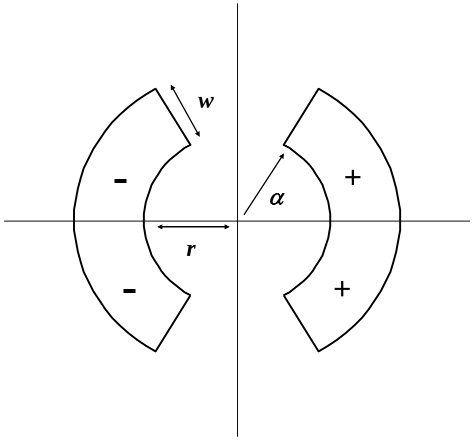
$$B_n = -\frac{j\mu_0 R_{ref}^{n-1}}{\pi}\frac{2\sin(\alpha n)}{n}\frac{(r+w)^{2-n} - r^{2-n}}{2-n}$$

- **Multipoles *n*** are proportional to sin (*n* angle of the sector)
  - They can be made **equal to zero** !

A "good" field quality dipole
Sector dipole

- First allowed multipole **B_3 (sextupole)**

$$B_3 = \frac{\mu_0 j R_{ref}^2}{\pi} \frac{\sin(3\alpha)}{3} \left(\frac{1}{r} - \frac{1}{r+w} \right)$$



for α=π/3 (i.e. a 60° sector coil) one has B_3=0

- Second allowed multipole **B_5 (decapole)**

$$B_5 = \frac{\mu_0 j R_{ref}^4}{\pi} \frac{\sin(5\alpha)}{5} \left(\frac{1}{r^3} - \frac{1}{(r+w)^3} \right)$$

for α=π/5 (i.e. a 36° sector coil) or for α=2π/5 (i.e. a 72° sector coil) one has B_5=0

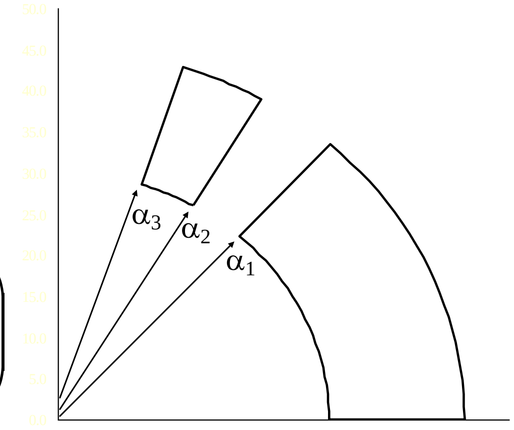
- With one sector one cannot set to zero both multipoles … let us try with more sectors !

A "good" field quality dipole
Sector dipole

- Coil with **two sectors**

$$B_3 = \frac{\mu_0 j R_{ref}^2}{\pi} \frac{\sin 3\alpha_3 - \sin 3\alpha_2 + \sin 3\alpha_1}{3} \left(\frac{1}{r} - \frac{1}{r+w} \right)$$

$$B_5 = \frac{\mu_0 j R_{ref}^4}{\pi} \frac{\sin 5\alpha_3 - \sin 5\alpha_2 + \sin 5\alpha_1}{5} \left(\frac{1}{r^3} - \frac{1}{(r+w)^3} \right)$$



  - Note: we have to work with **non-normalized multipoles**, which can be added together
- Equations to set to zero B_3 , B_5 and B_5

$$\begin{cases} \sin(3\alpha_3) - \sin(3\alpha_2) + \sin(3\alpha_1) = 0 \\ \sin(5\alpha_3) - \sin(5\alpha_2) + \sin(5\alpha_1) = 0 \end{cases}$$

  - There is a **one-parameter family of solutions**, for instance (48°,60°,72°) or (36°,44°,64°) are solutions

A "good" field quality dipole Sector dipole

- With one wedge one can set to zero three multipoles (B_3, B_5 and B_7)

- What about **two wedges** ?

$$\sin(3\alpha_5)-\sin(3\alpha_4)+\sin(3\alpha_3)-\sin(3\alpha_2)+\sin(3\alpha_1)=0$$

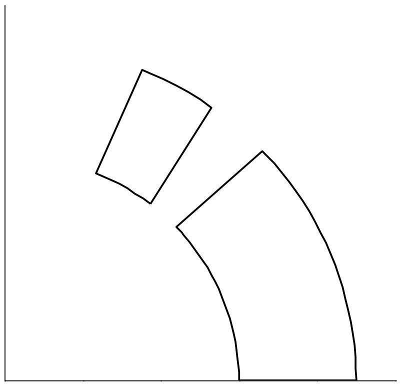
$$\sin(5\alpha_5)-\sin(5\alpha_4)+\sin(5\alpha_3)-\sin(5\alpha_2)+\sin(5\alpha_1)=0$$

$$\sin(7\alpha_5)-\sin(7\alpha_4)+\sin(7\alpha_3)-\sin(7\alpha_2)+\sin(7\alpha_1)=0$$

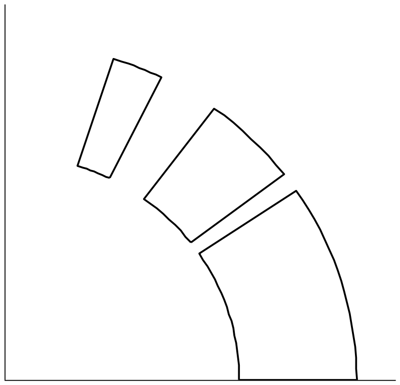
$$\sin(9\alpha_5)-\sin(9\alpha_4)+\sin(9\alpha_3)-\sin(9\alpha_2)+\sin(9\alpha_1)=0$$

$$\sin(11\alpha_5)-\sin(11\alpha_4)+\sin(11\alpha_3)-\sin(11\alpha_2)+\sin(11\alpha_1)=0$$

One can **set to zero five multipoles** (B_3, B_5, B_7, B_9 and B_{11}) ~[0°-33.3°, 37.1°- 53.1°, 63.4°- 71.8°]



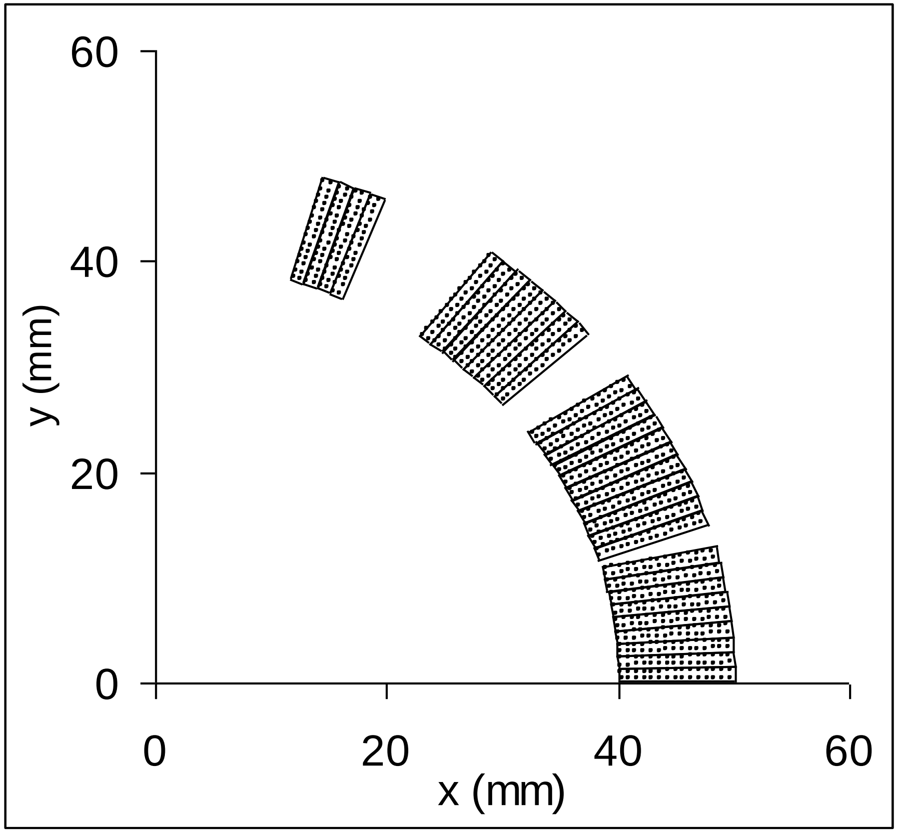
One wedge, $b_3=b_5=b_7=0$ [0°-43.2°,52.2°-67.3°]



Two wedges, $b_3=b_5=b_7=b_9=b_{11}=0$ [0°-33.3°,37.1°-53.1°,63.4°- 71.8°]

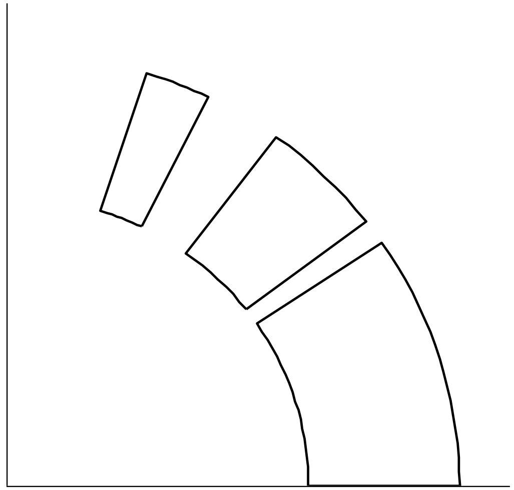
A "good" field quality dipole
Sector dipole

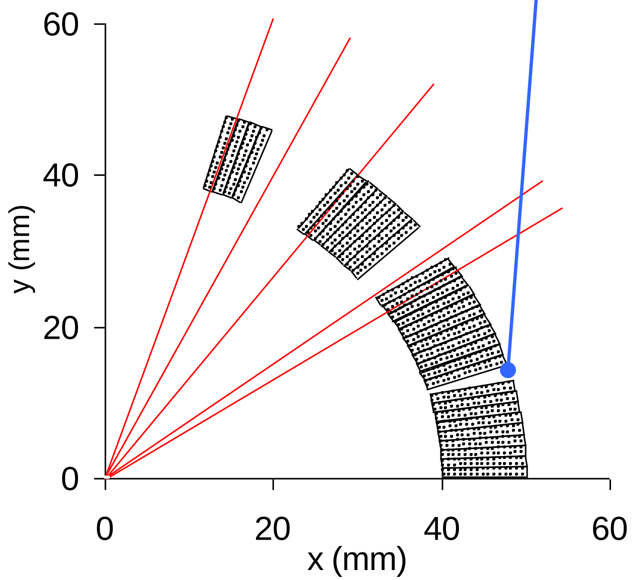
- Let us see two coil lay-outs of real magnets
  - The **RHIC dipole** has **four blocks**



A "good" field quality dipole Sector dipole

- Limits due to the cable geometry
  - Finite thickness → **one cannot produce sectors of any width**
  - Cables cannot be key-stoned beyond a certain angle, **some wedges can be used to better follow the arch**
- One does not always aim at having zero multipoles
  - There are other contributions (iron, persistent currents …)



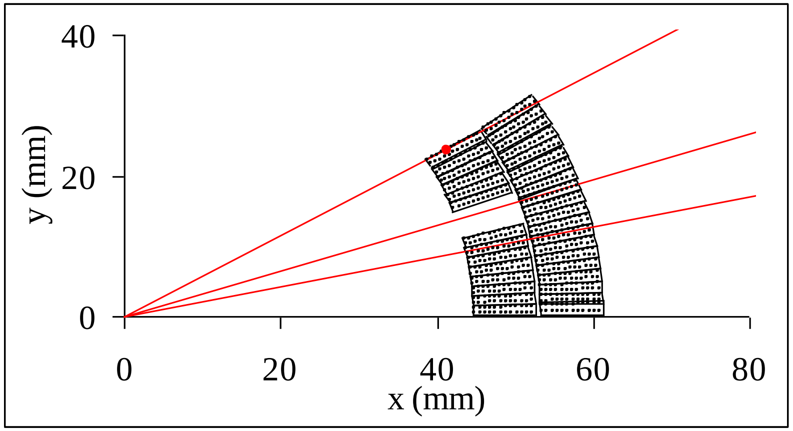


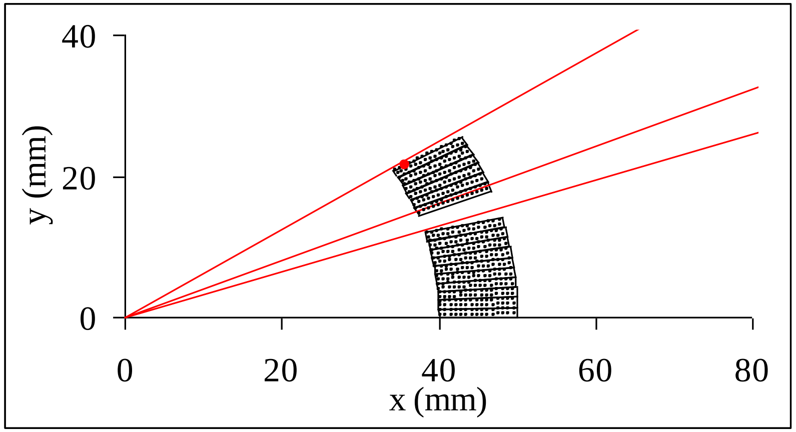
A "good" field quality dipole
Sector quadrupole

- For a sector coil **with one layer**, the same results of the dipole case hold with the following transformation
  - **Angles** have to be **divided by two**
  - **Multipole orders** have to be **multiplied by two**
- First allowed multipole B_6 (dodecapole)

$$B_6 = \frac{\mu_0 j R_{ref}^5}{\pi} \frac{\sin(6\alpha)}{6} \left(\frac{1}{r^4} - \frac{1}{(r+w)^4} \right)$$

for α=π/6 (i.e. a **30° sector coil**) one has $\boldsymbol{B_6=0}$

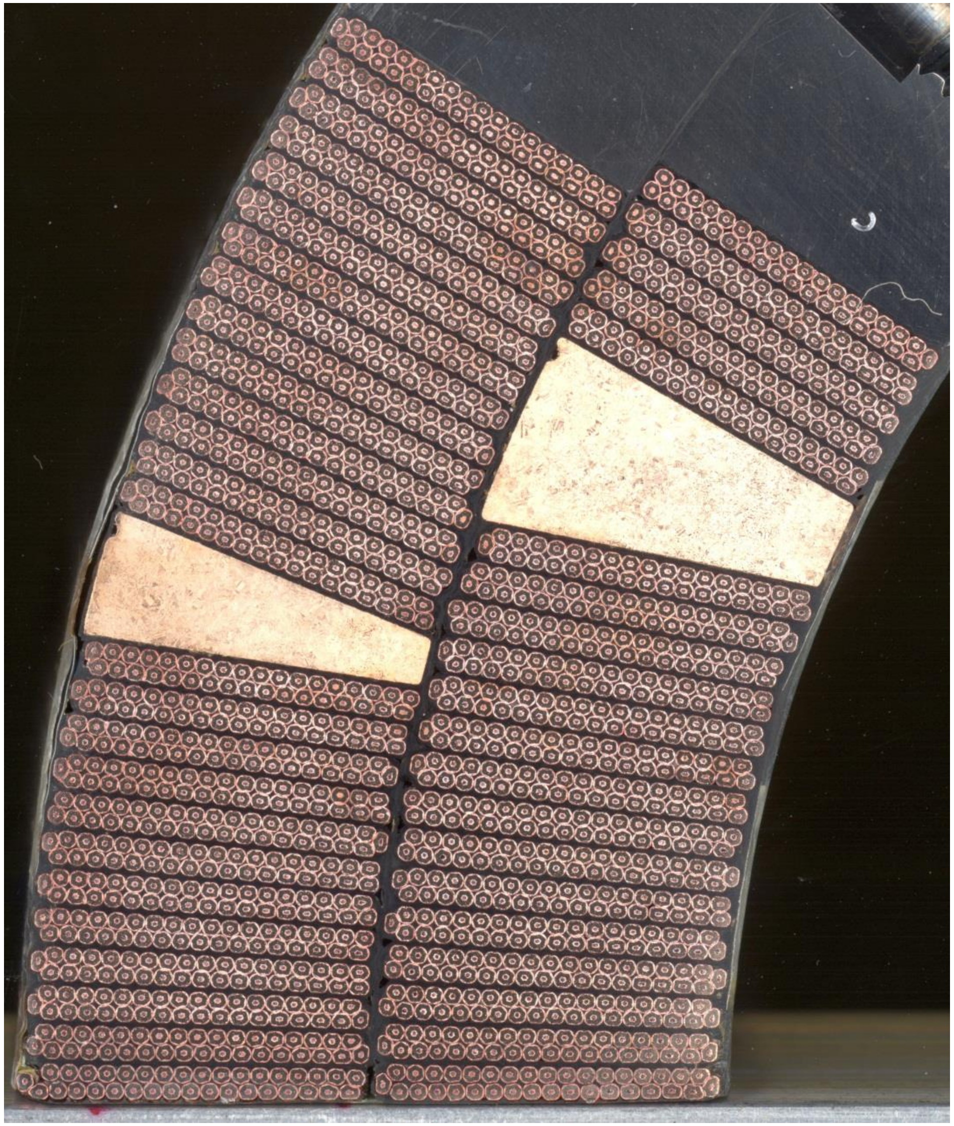




Magnetic design

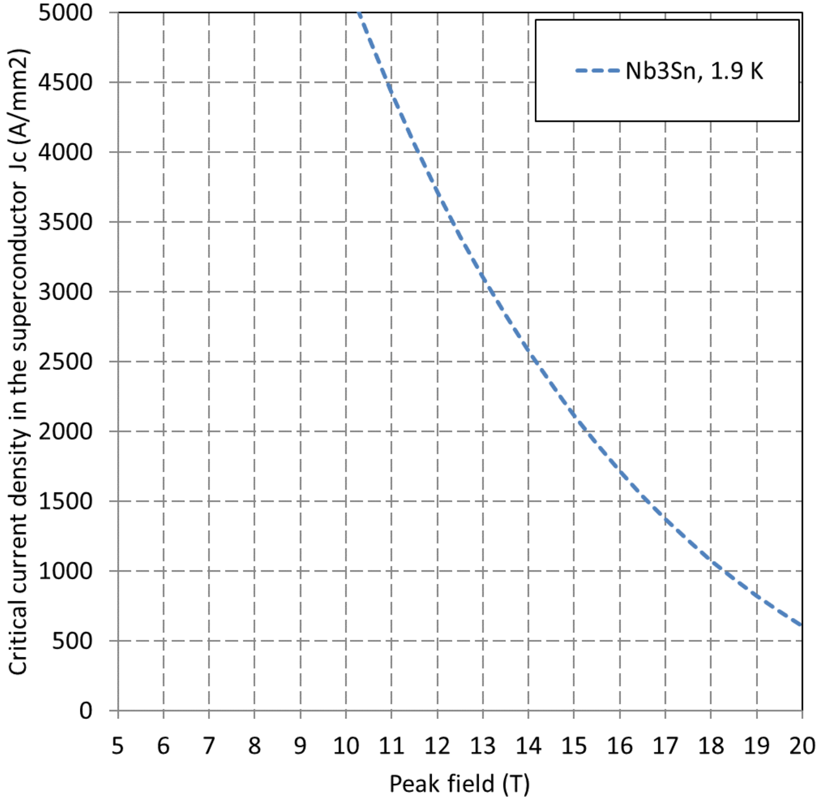
- **How much** conductor do we need to meet the requirements?
- And in **which configuration**?

- **Outline**
  - How do we create a **perfect field**?
  - How do we express the field and its "**imperfections**"?
  - How do we **design a coil** to minimize field errors?
  - Which is the **maximum field** we can get?
  - **Overview** of different designs



Maximum field and coil thickness Dipoles

- We recall the **critical surface**
- The current density flowing in the insulated cable is reduced by a factor ***κ*** **(filling ratio)**
  - It ranges from ¼ to 1/3



Maximum field and coil thickness Dipoles

- We characterize the coil by two parameters

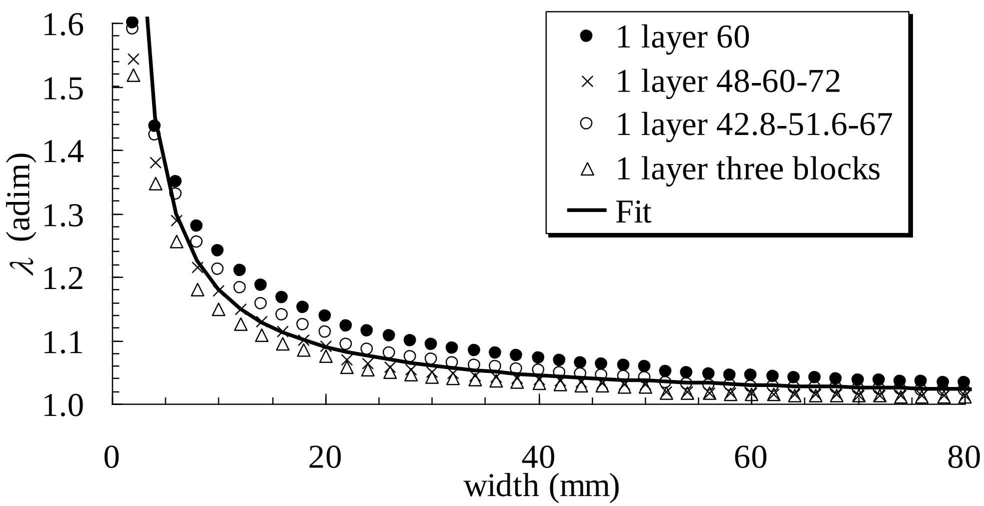
$$B \equiv \gamma_c j \qquad B_p \equiv \lambda B = \lambda \gamma_c j$$

- γ_c: how much **field in the centre** is given **per unit of current density**
  - For a sector dipole

$$B_1 = -\frac{2\mu_0 j}{\pi} w \sin\alpha$$

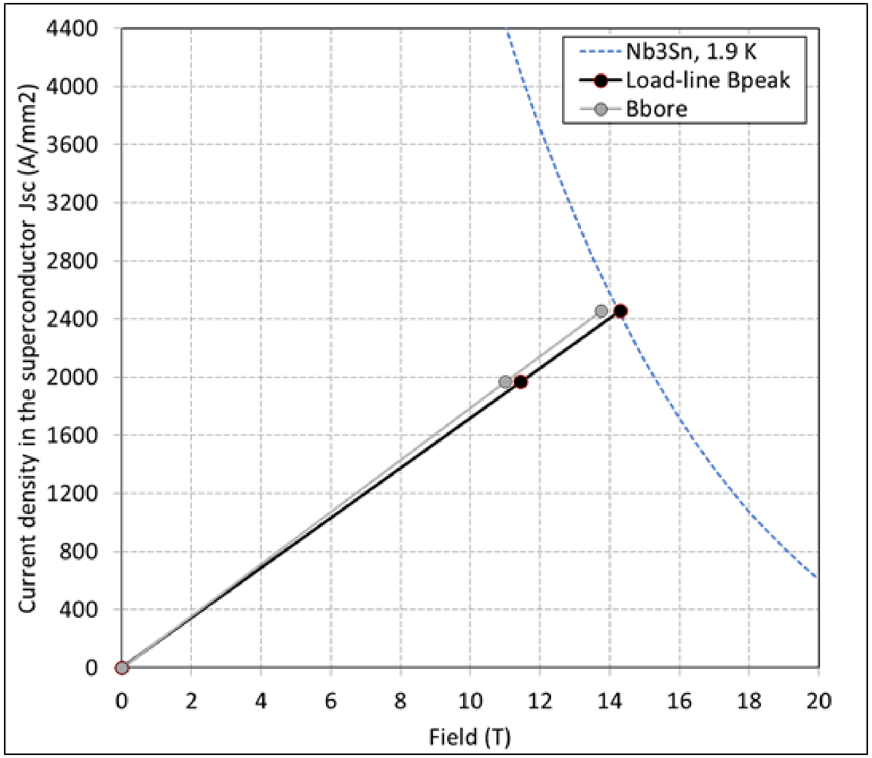
- λ: ratio between **peak field and central field**
  - For a sector and in general is λ= 1.05 - 1.15
  - hyperbolic fit: a~0.045

$$\lambda(w,r) \sim 1 + \frac{ar}{w}$$



Maximum field and coil thickness
Dipoles

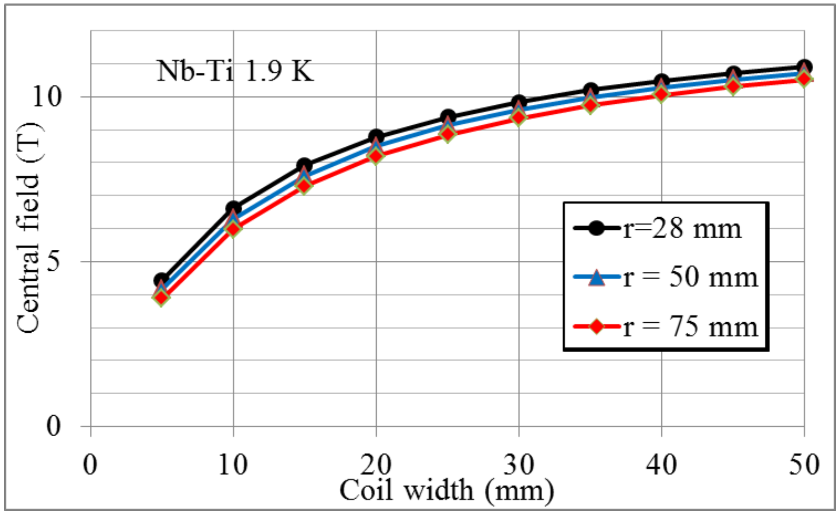
- We can now compute what is the **highest peak field** that can be reached in the dipole

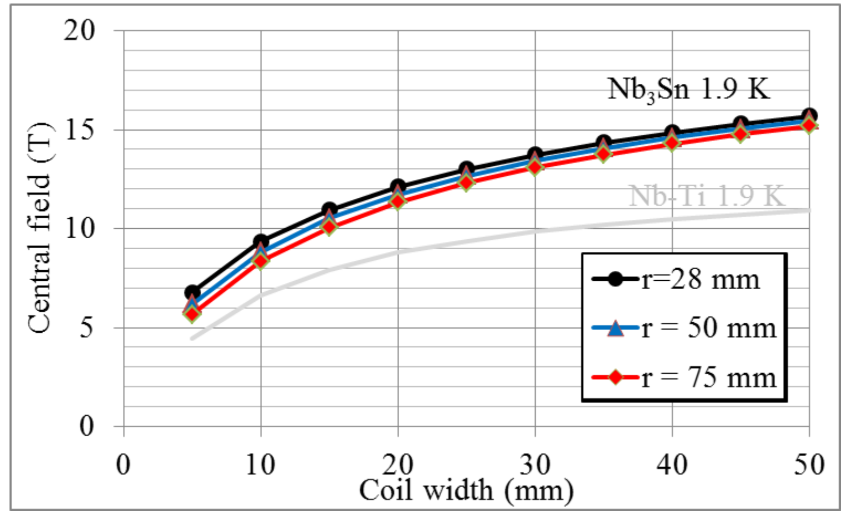


- The **maximum current density** in the superconductor
  - *short sample limit*
- And the **bore short sample field** (in the centre not on the conductor)

Maximum field and coil thickness
Dipoles

- **Maximum bore field**





- Magnets have to work at a given distance from the critical surface, i.e. they are **never operated at short sample** conditions
  - At short sample, any small perturbation quenches the magnet
  - One usually operates at a **fraction of the loadline: 60% to 90%**

Maximum gradient and coil thickness Quadrupoles

- We characterize the coil by two parameters

$$\gamma_c \equiv \frac{G}{j} \qquad \lambda \equiv \frac{B_p}{rG}$$

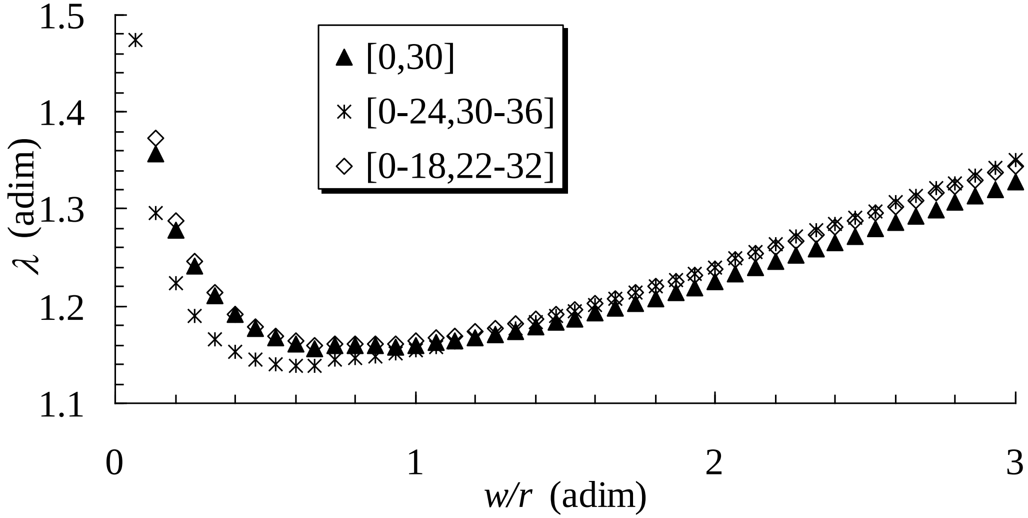
- γ_c: how much **gradient** is given **per unit of current density**
  - For a sector quadrupole

$$G = -\frac{2 j_o \mu_0}{\pi}[\sin 60]\ln\left(1+\frac{w}{r}\right)$$

- λ: ratio between **peak field and gradient · *r***
  - A good fit, with a_{-1}~0.04 and a_1~0.11 is

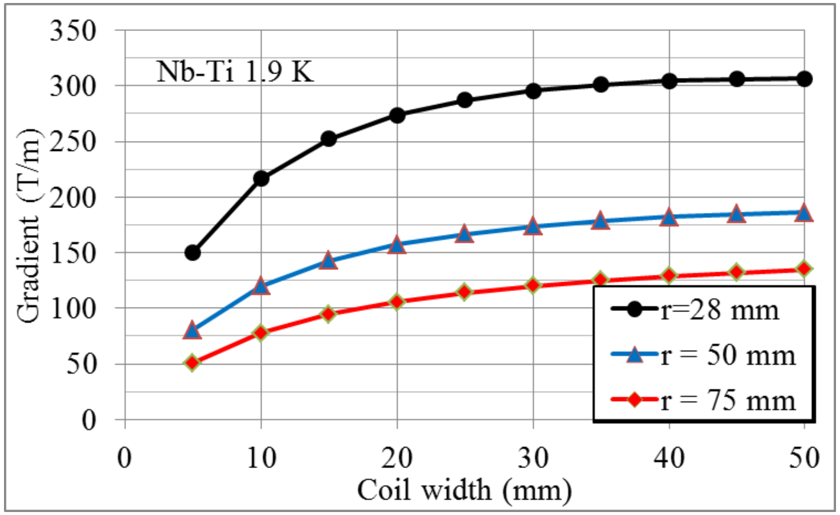
$$\lambda(w,r) = a_{-1}\frac{r}{w} + 1 + a_1\frac{w}{r}$$

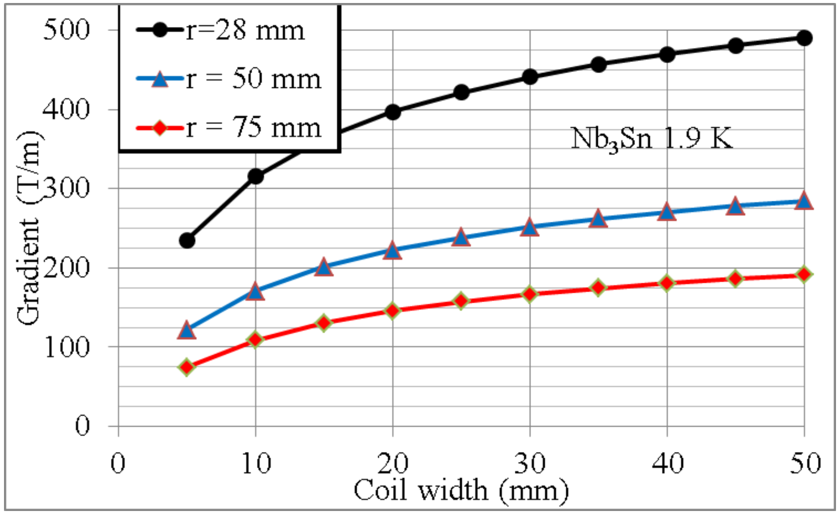
  - **reasonable values is $\lambda \sim \lambda_0 = 1.15$**



Maximum gradient and coil thickness Quadrupoles

- Therefore, the **maximum field, current and gradient**

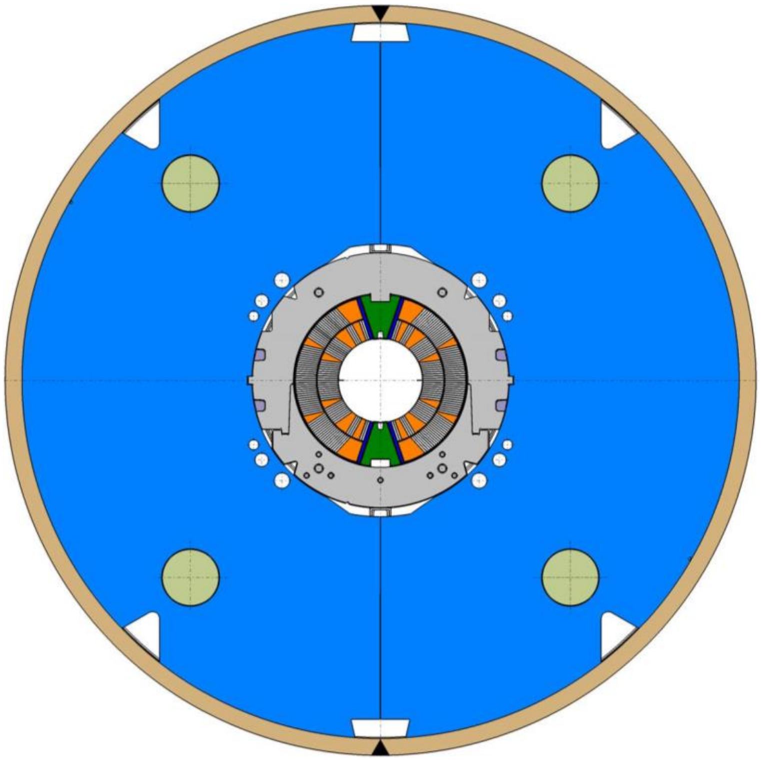




- Unlike dipoles, no point in making coils extremely large!

Iron yoke

- Keep the **return magnetic flux** close to the coils, thus avoiding fringe fields
- In some cases the iron is partially or totally contributing to the **mechanical structure**
- Considerably **enhance the field** for a given current density
  - The increase is relevant (10-30%), getting higher for thin coils
  - This allows using lower currents, easing the protection



Iron yoke

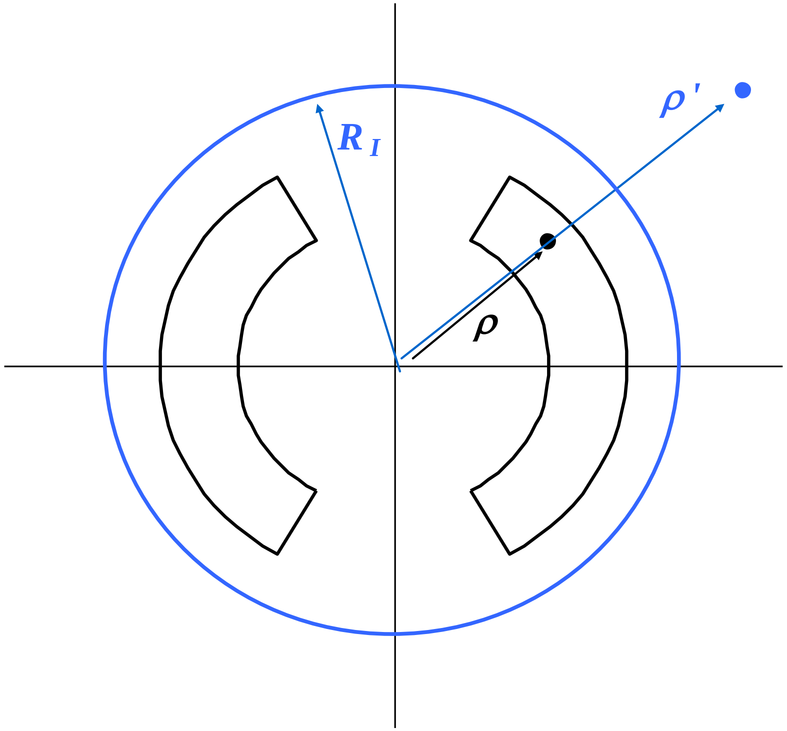
- A **rough estimate** of the **iron thickness** necessary to avoid fields outside the magnet
  - The iron cannot withstand more than 2 T
  - **Shielding condition** for dipoles: $rB \sim t_{iron} B_{sat}$
    - i.e., the iron thickness times 2 T is equal to the central field times the magnet aperture – One assumes that all the field lines in the aperture go through the iron (and not for instance through the collars)
    - Example: in the LHC main dipole **the iron thickness is 150 mm**

$$t_{iron} \sim \frac{rB}{B_{sat}} = \frac{28*9}{2} \sim 130 \text{ mm}$$

  - Shielding condition for quadrupoles: $\frac{r^2 G}{2} \sim t_{iron} B_{sat}$

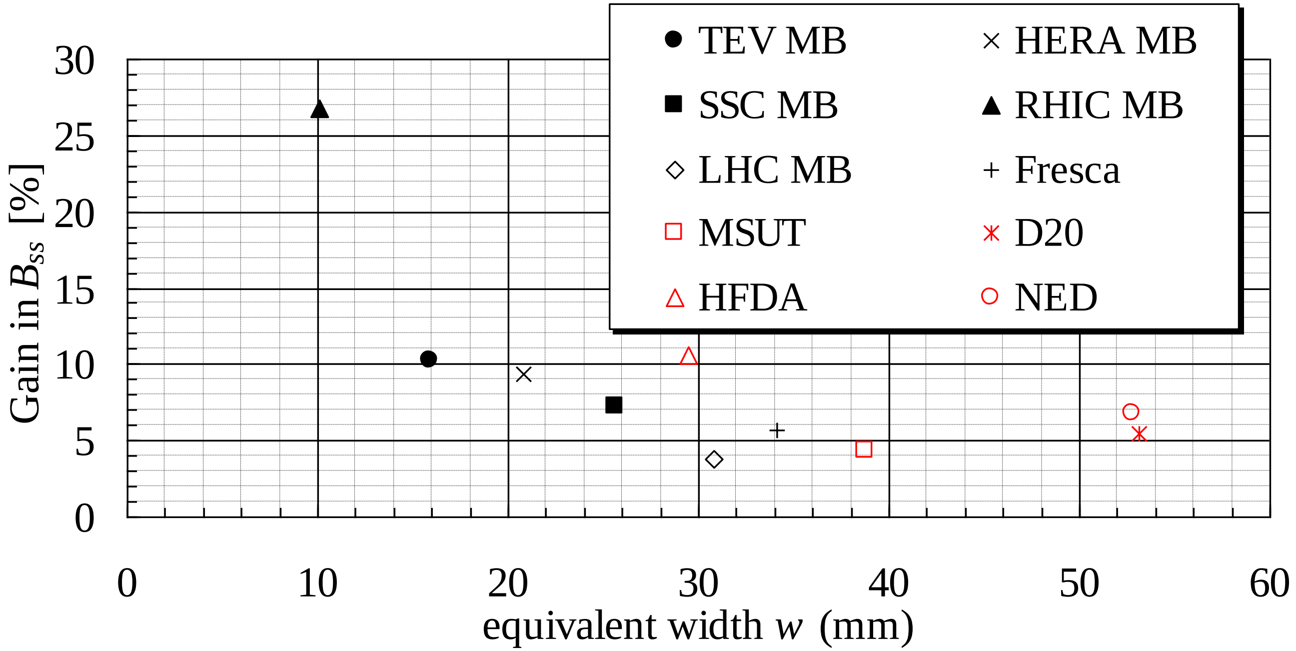
Iron yoke

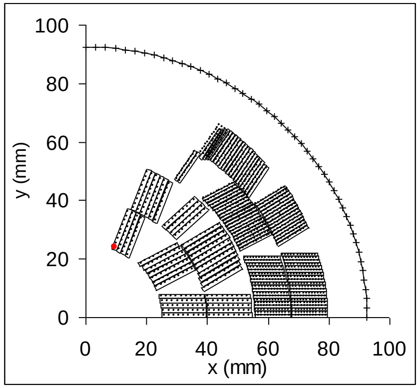
- The iron yoke contribution can be estimated analytically for simple geometries
  - Circular, non-saturated iron: image currents method
  - Iron effect is equivalent to add to each current line a second one
    - at a distance $$\rho' = \frac{R_I^2}{\rho}$$
    - with current $$I' = \frac{\mu - 1}{\mu + 1} I$$
  - Limit of the approximation: iron is not saturated (less than 2 T)



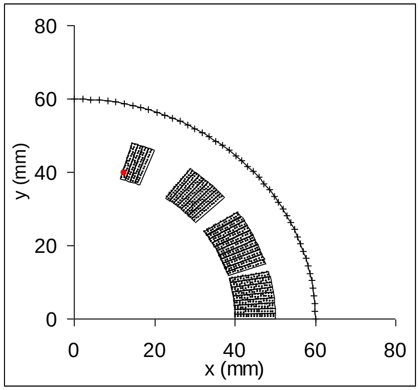
Iron yoke

- Impact of the iron yoke on short sample field
  - Large effect (25%) on RHIC dipoles (thin coil and collars)
  - Between **4% and 10% for most of the others** (both Nb-Ti and Nb_3Sn)





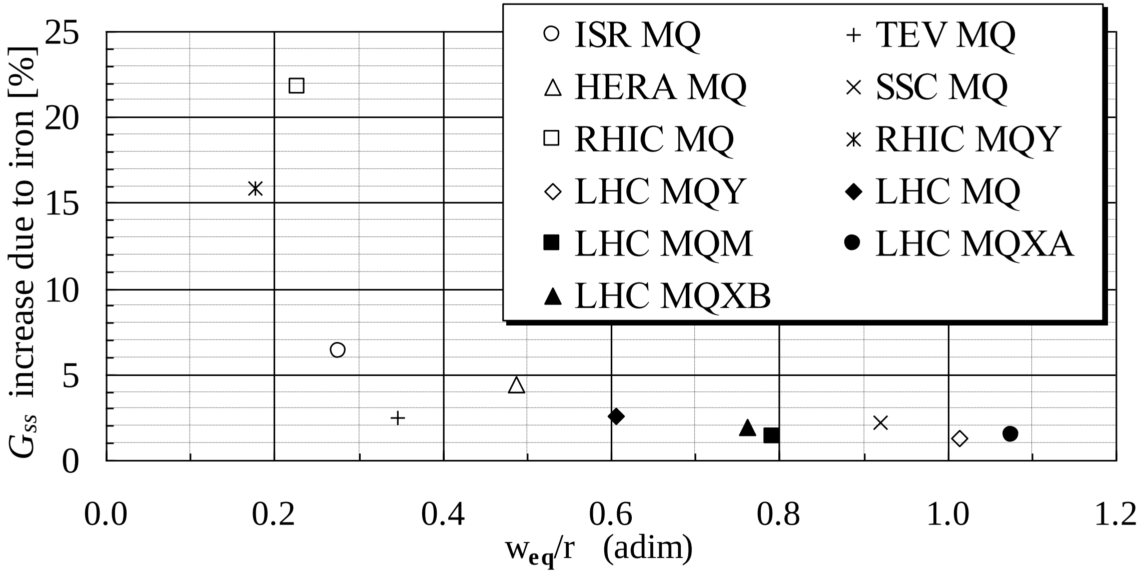
D20 and yoke

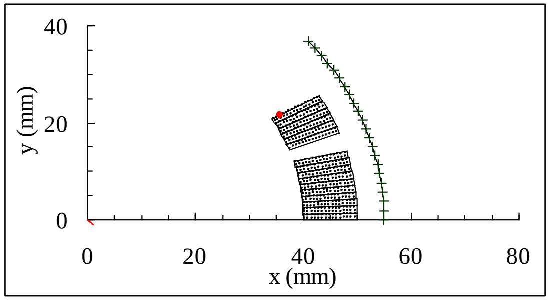


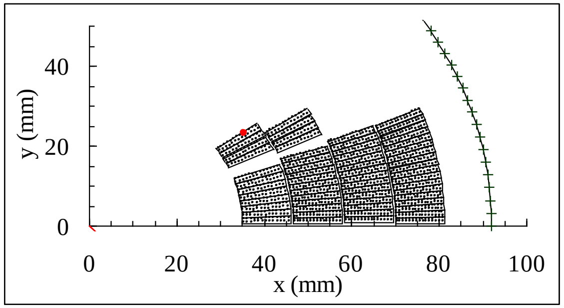
RHIC main dipole and yoke

Iron yoke

- Similar approach can be used in quadrupoles
  - **Large effect on RHIC** quadrupoles (thin coil and collars)
  - Between 2% and 5% for most of the others
  - The effect is smaller than in dipoles since the contribution to B_2 is smaller than to B_1







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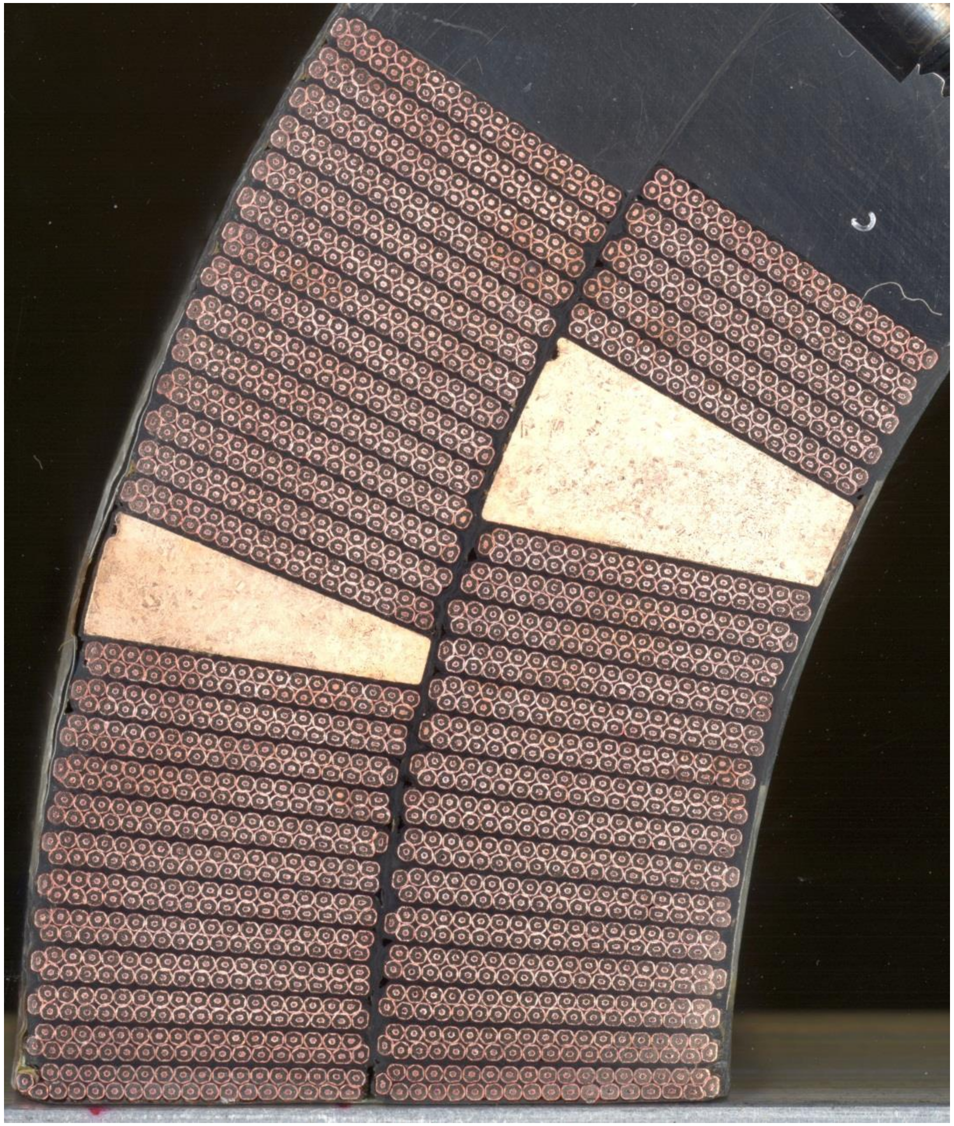
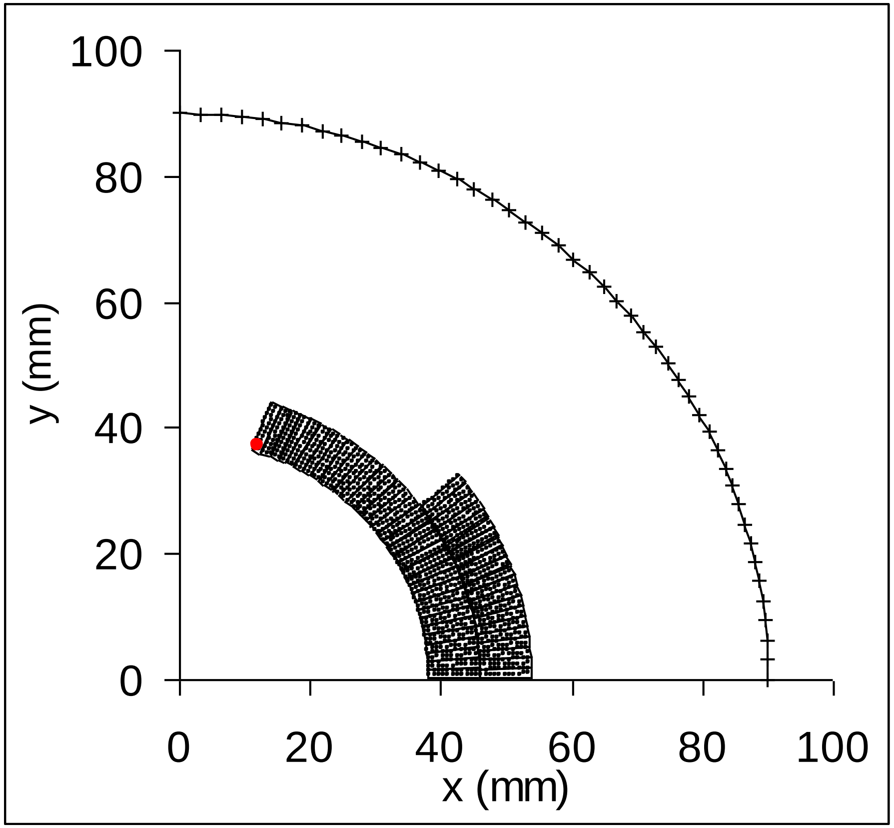
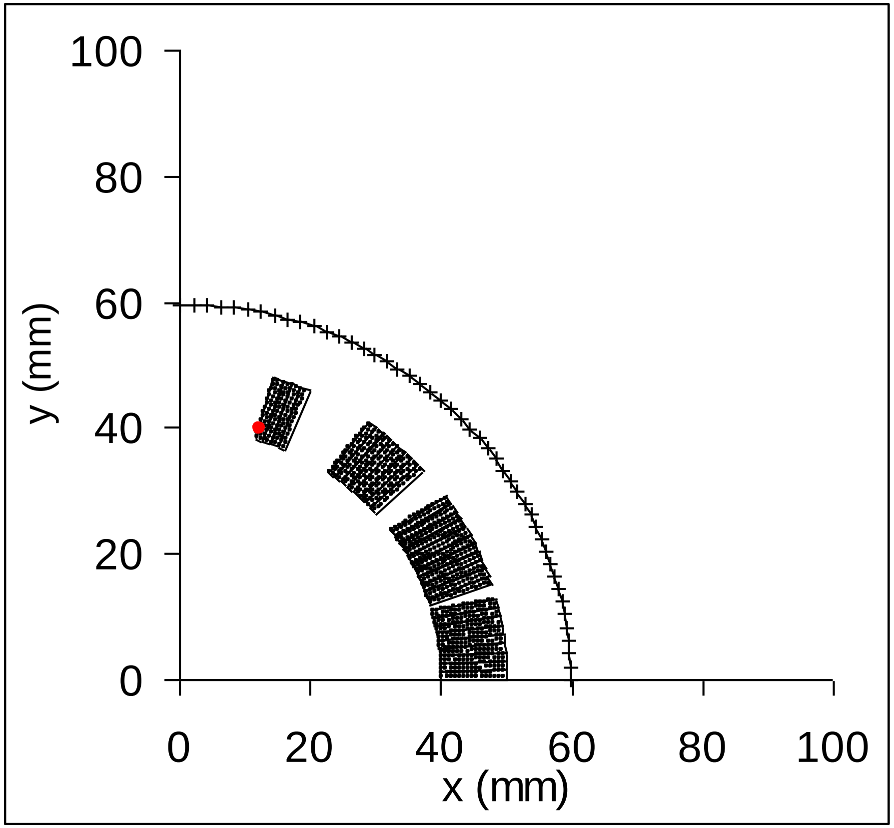


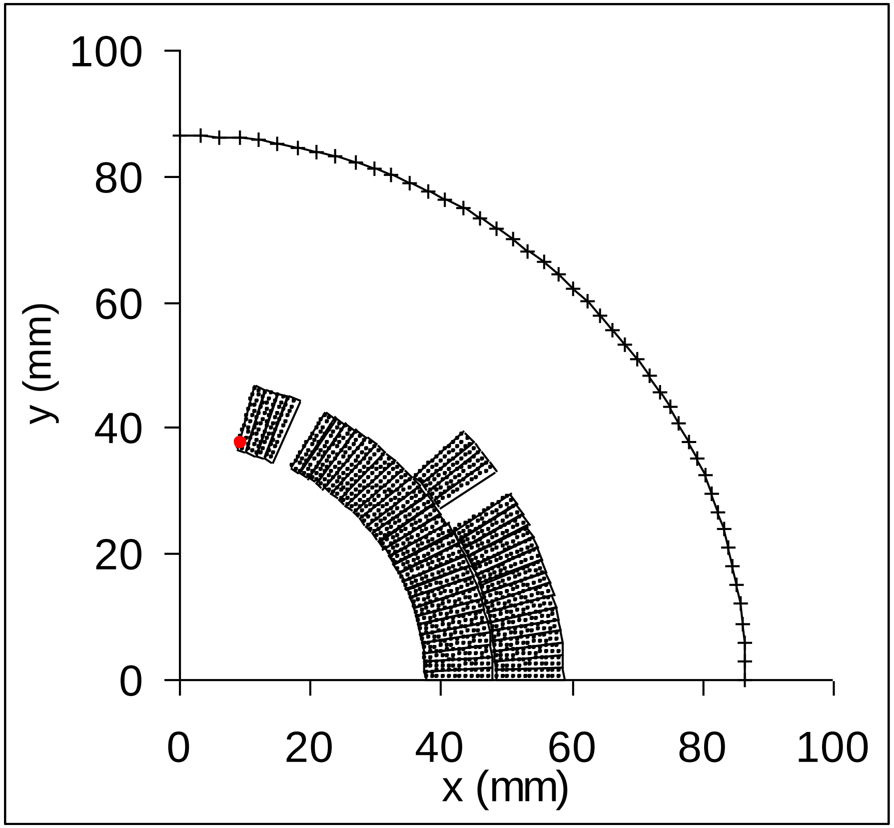
- Tevatron MB



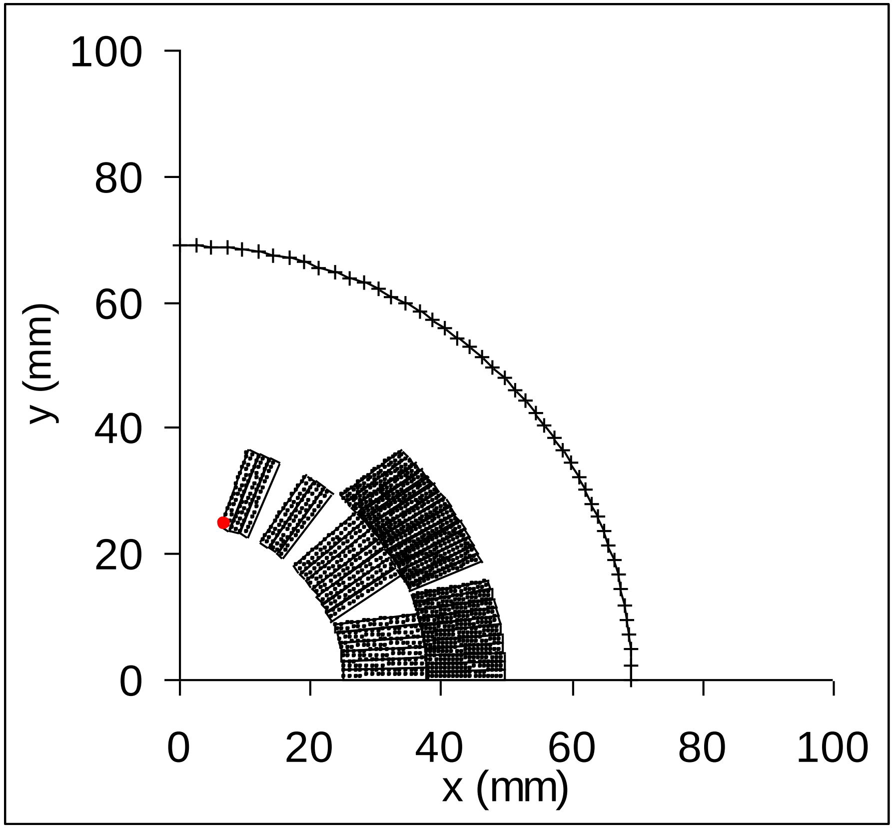
- RHIC MB



- HERA MB

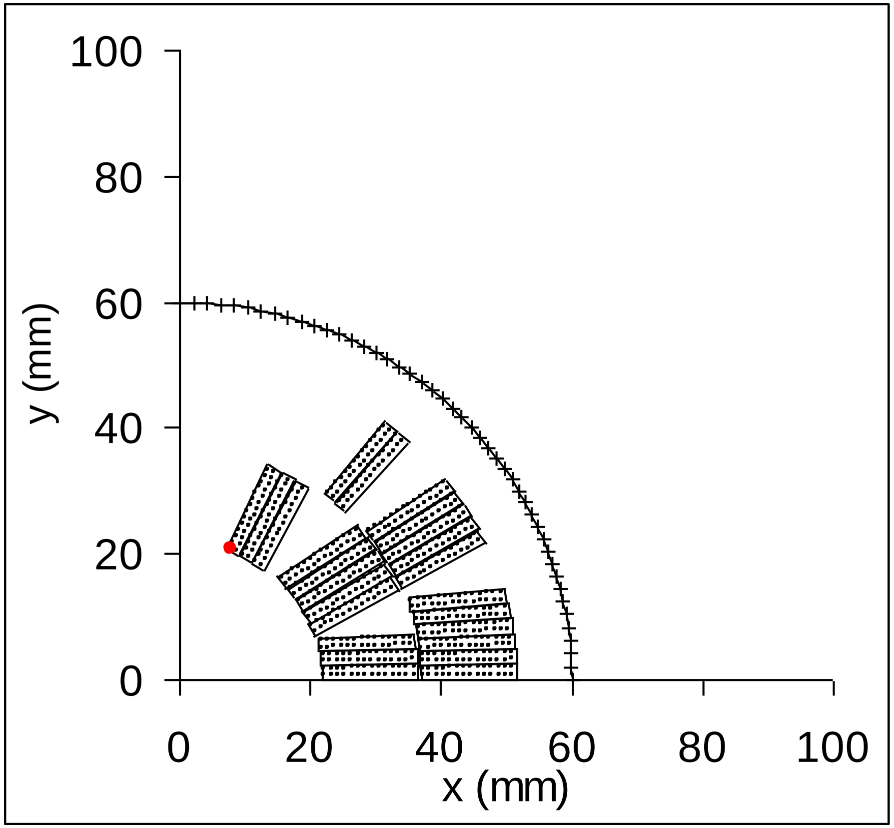


- SSC MB



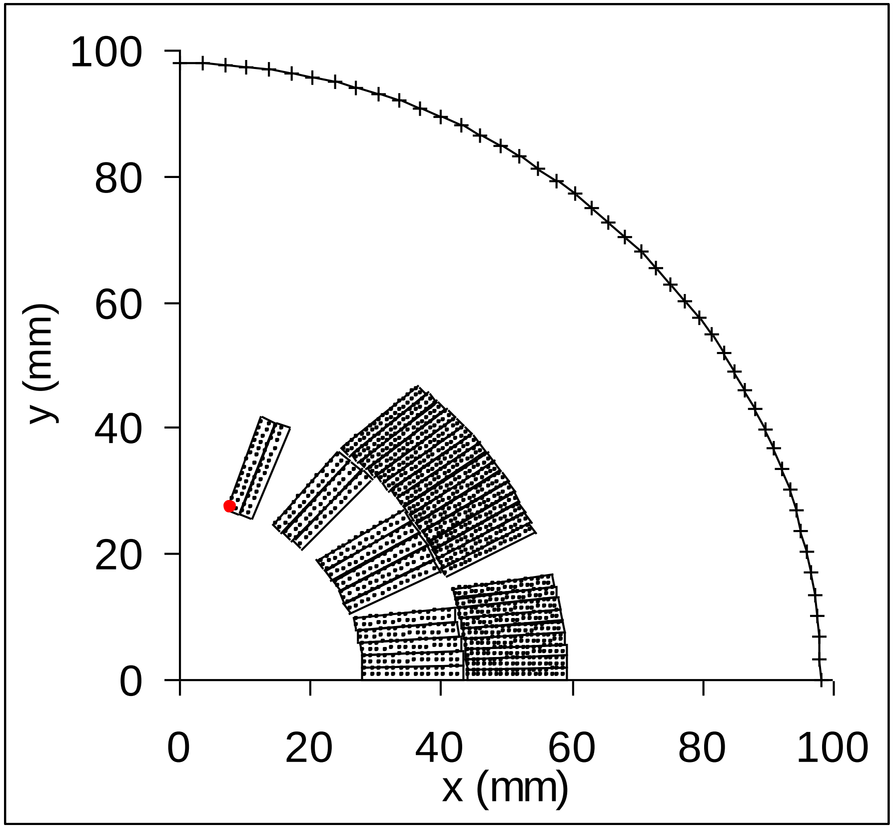
A review of dipole lay-outs

- HFDA dipole

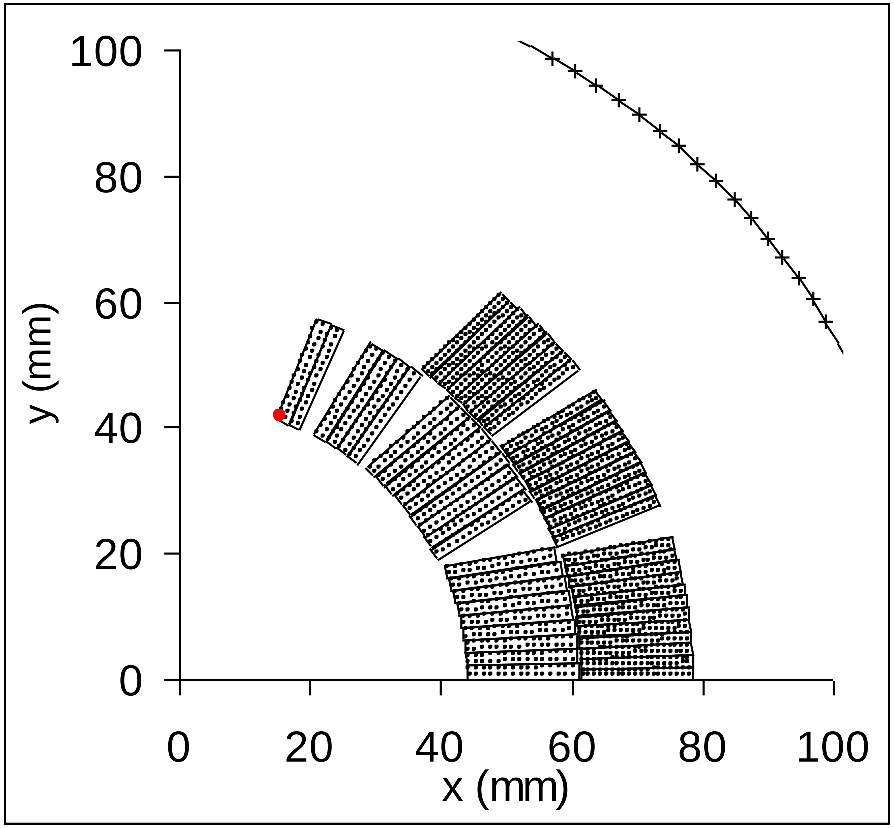


A review of dipole lay-outs

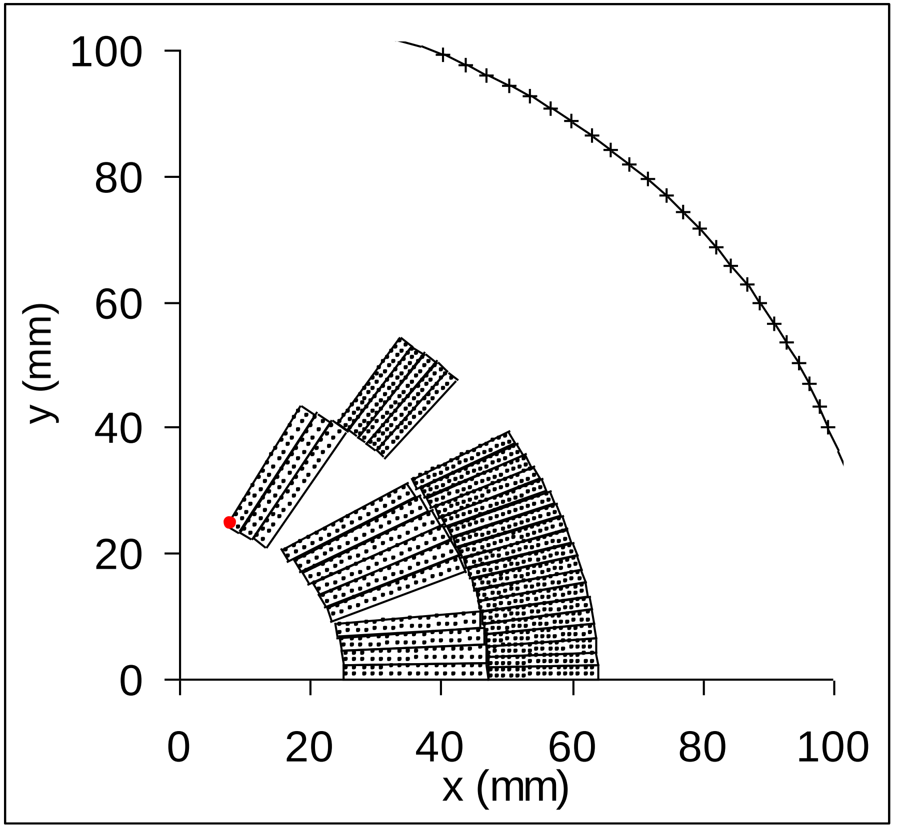
- LHC MB



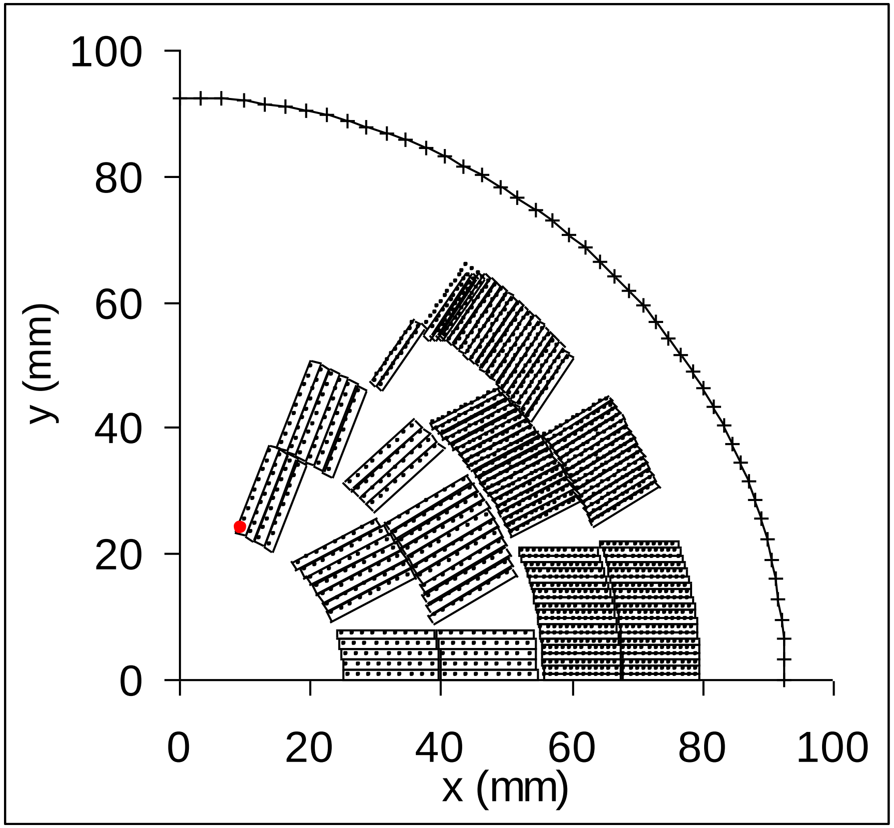
- FRESCA



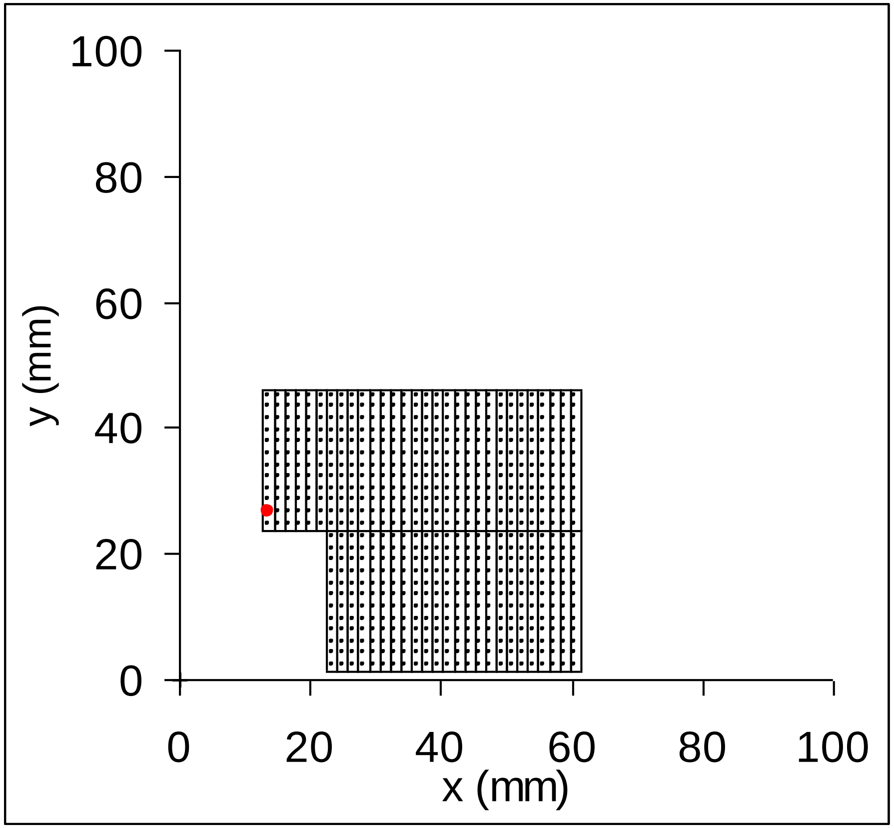
- MSUT



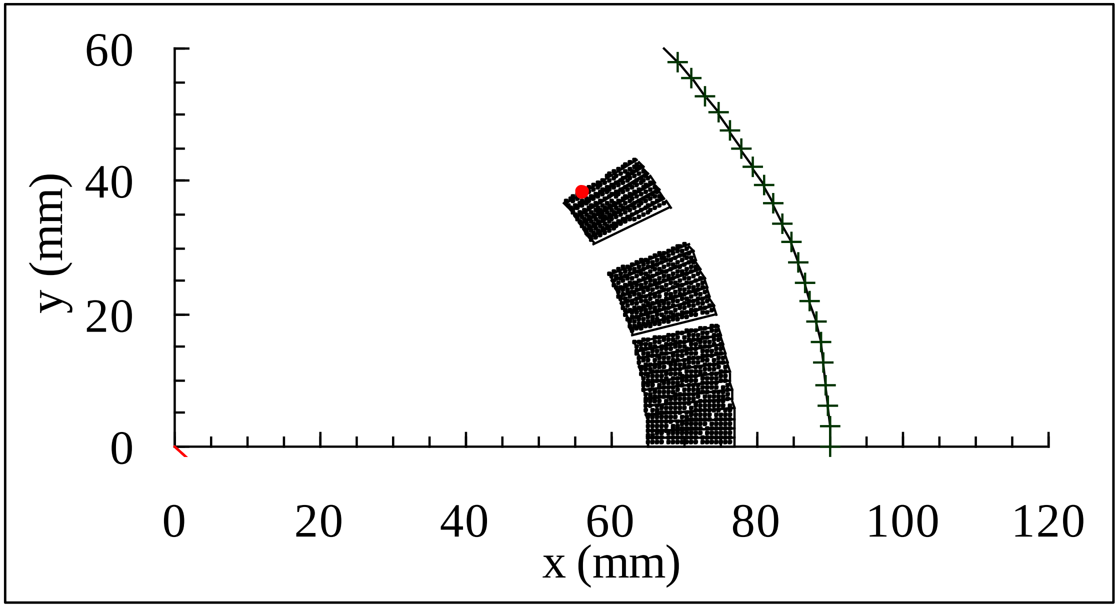
- D20



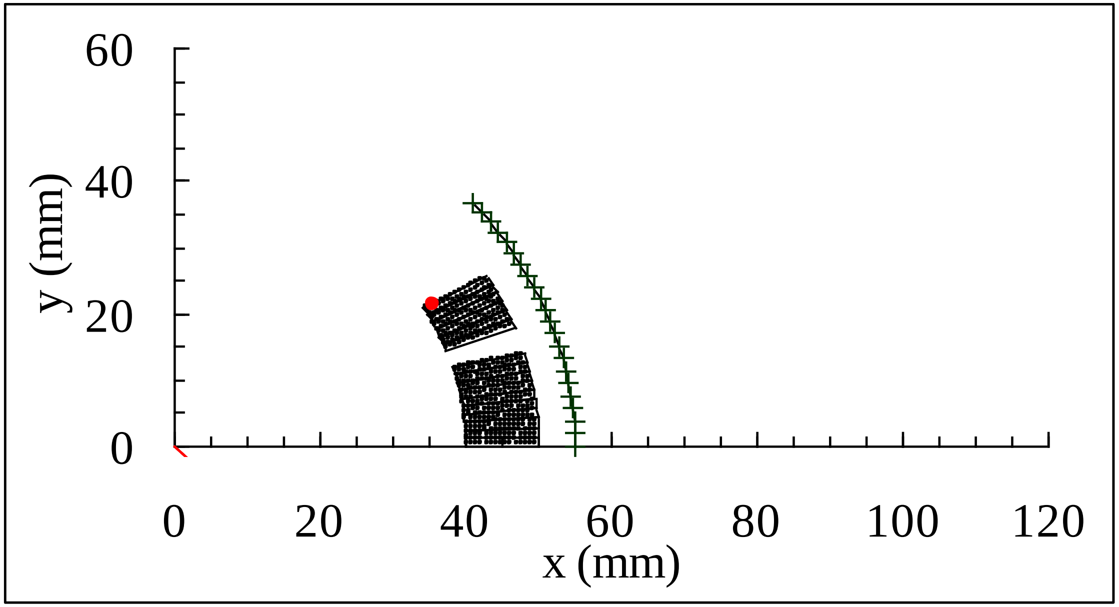
- HD2



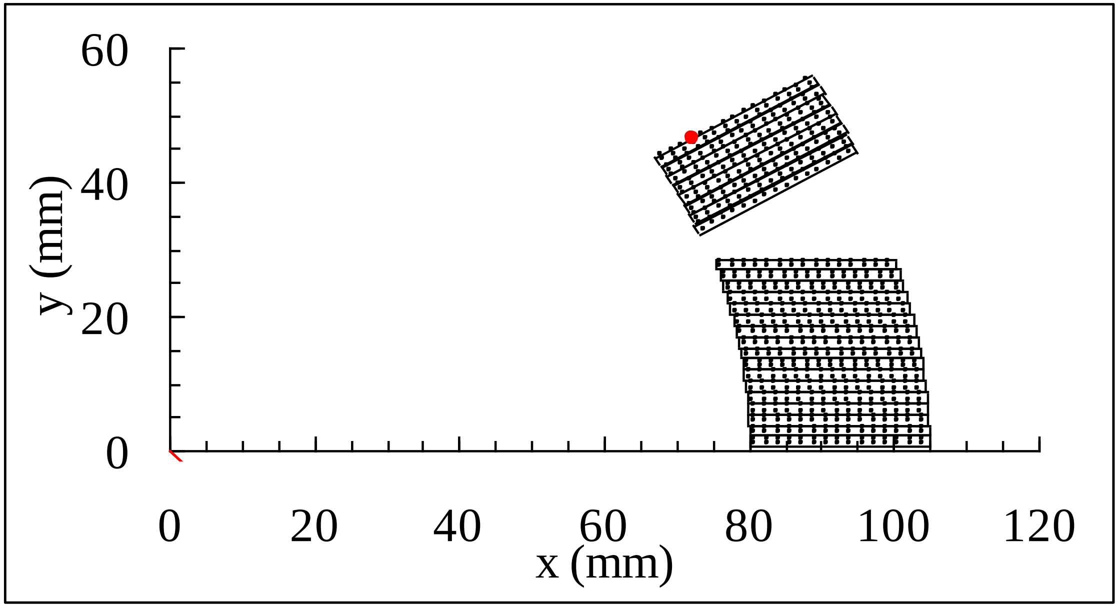
RHIC MQX



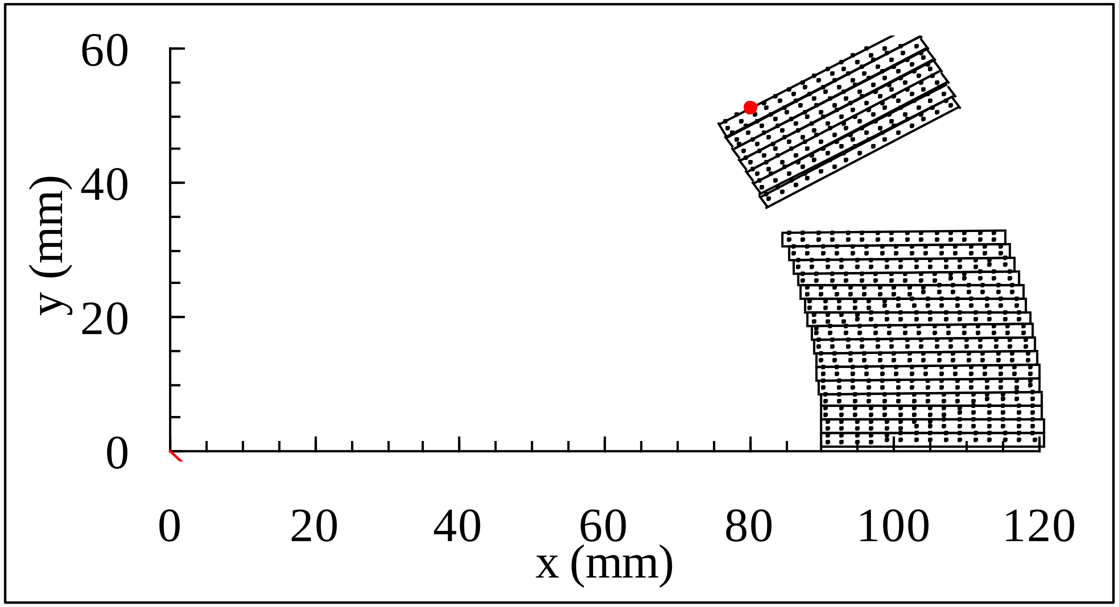
RHIC MQ



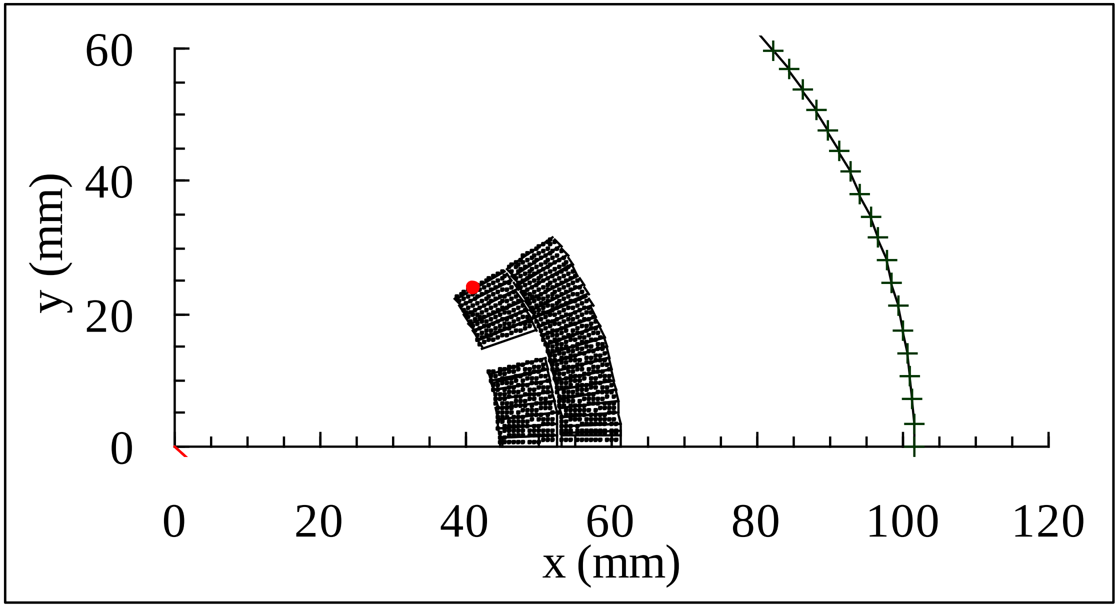
LEP II MQC



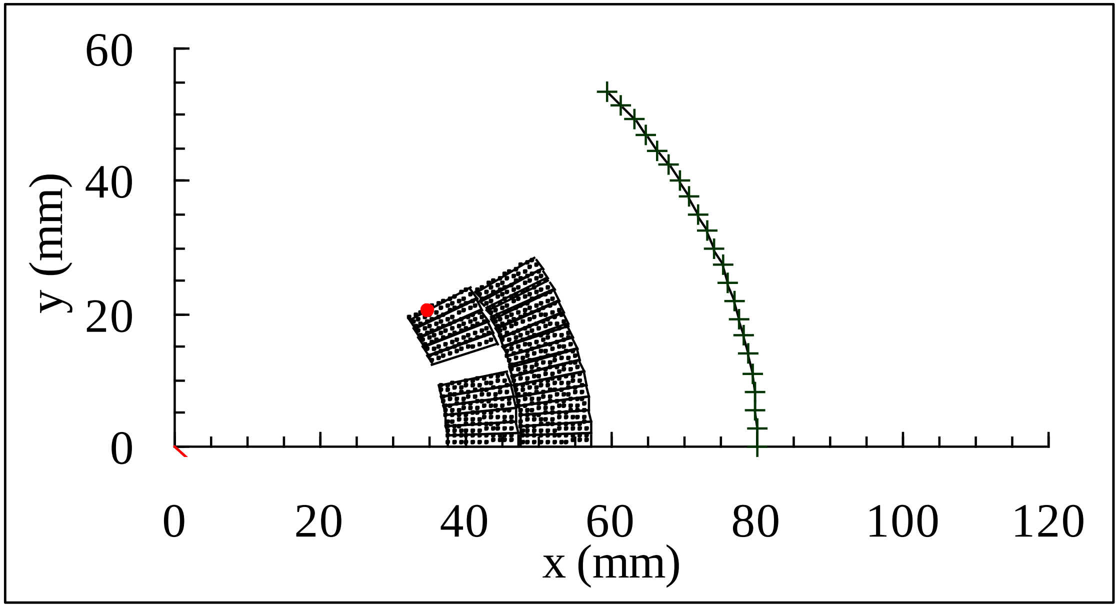
LEP I MQC



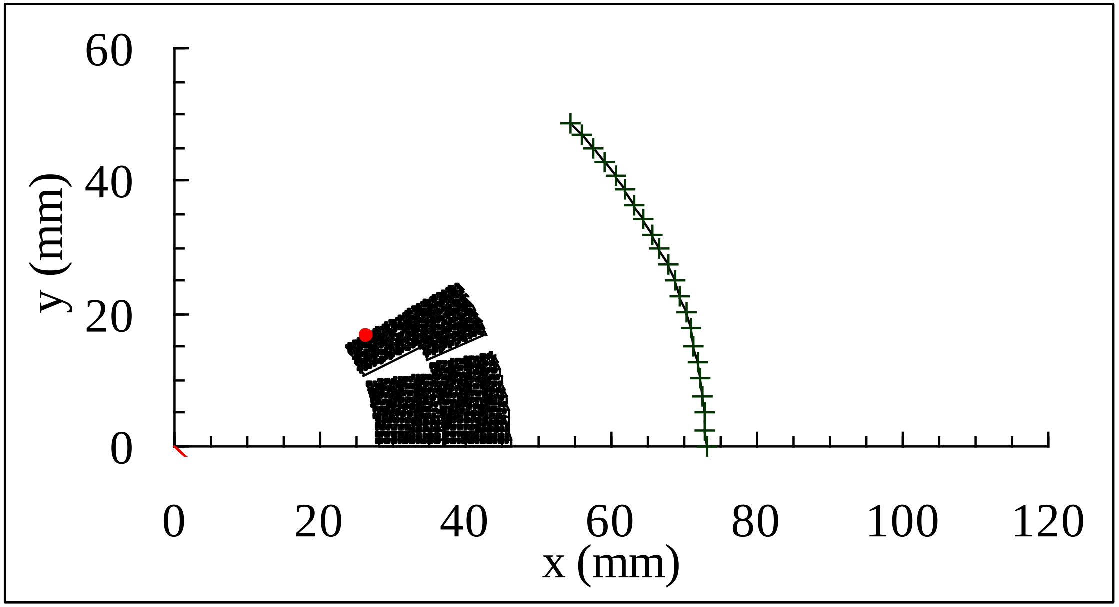
Tevatron MQ



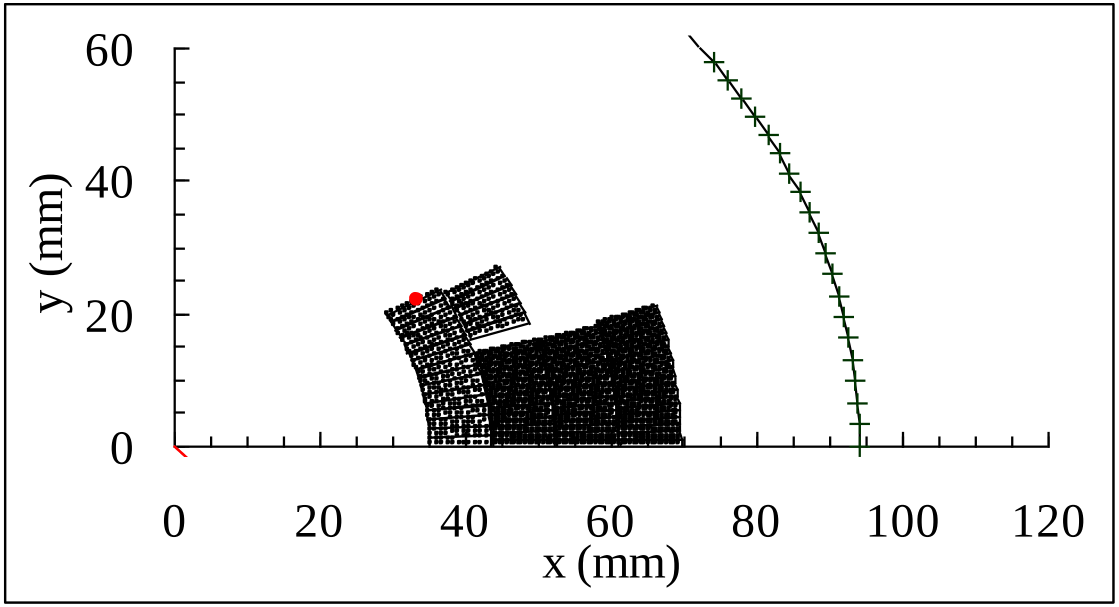
HERA MQ



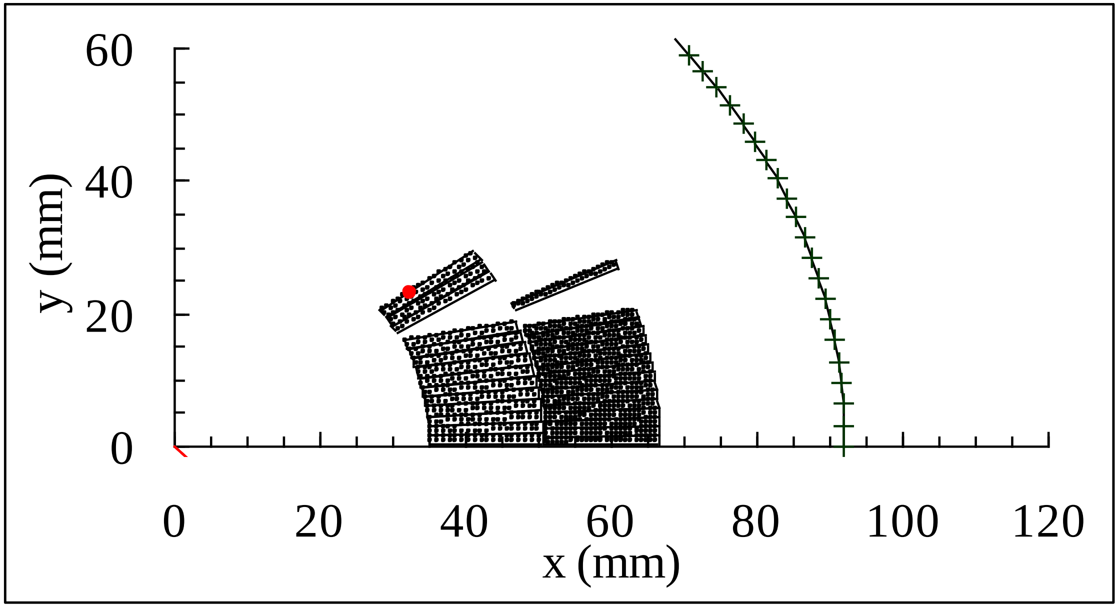
LHC MQM



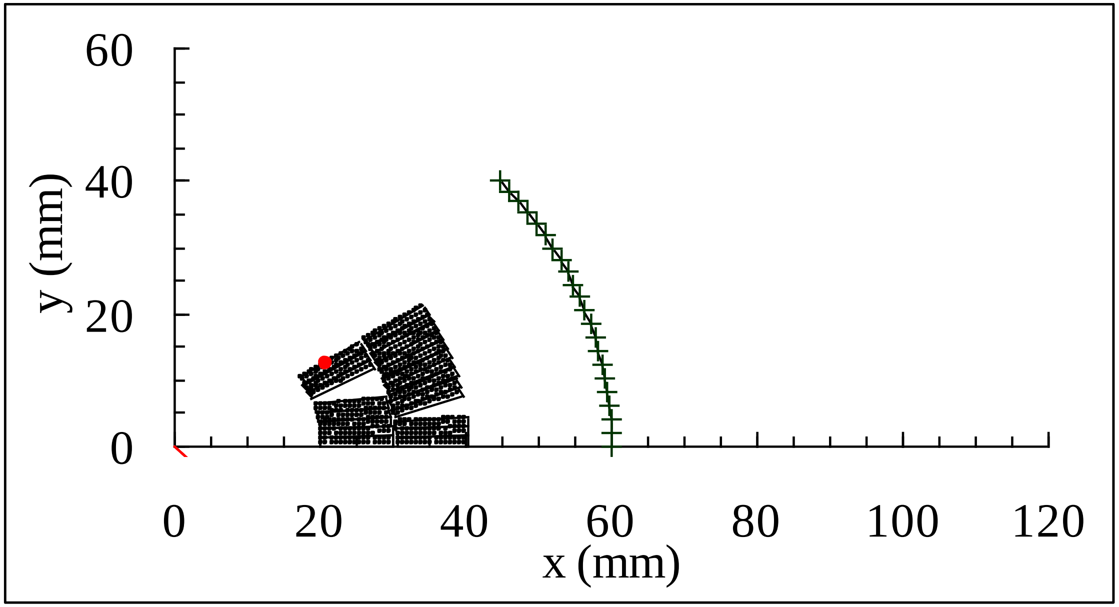
LHC MQY



LHC MQXB

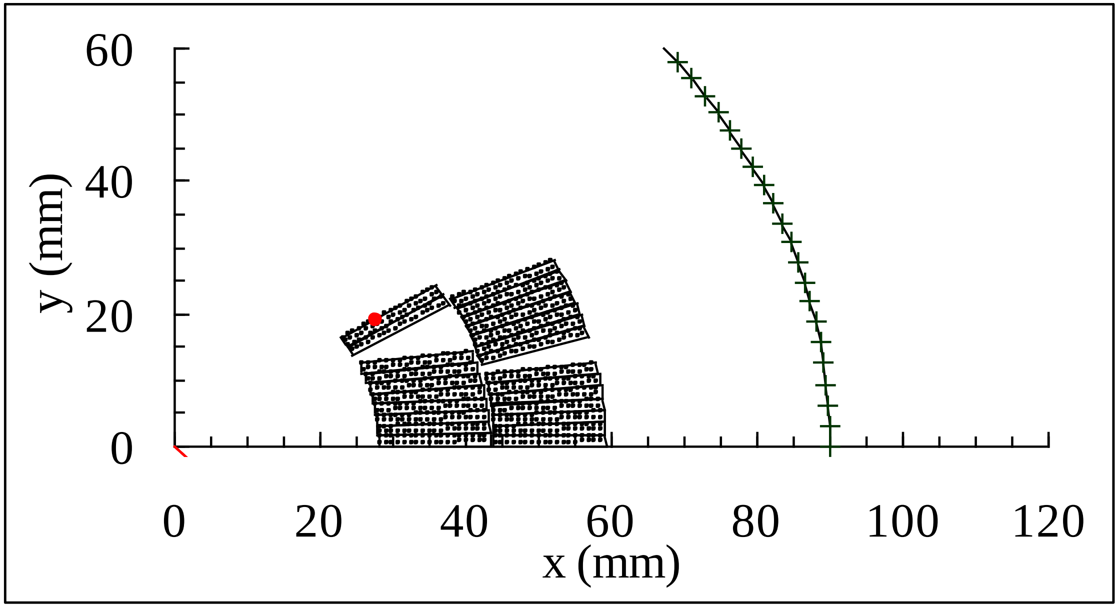


SSC MQ

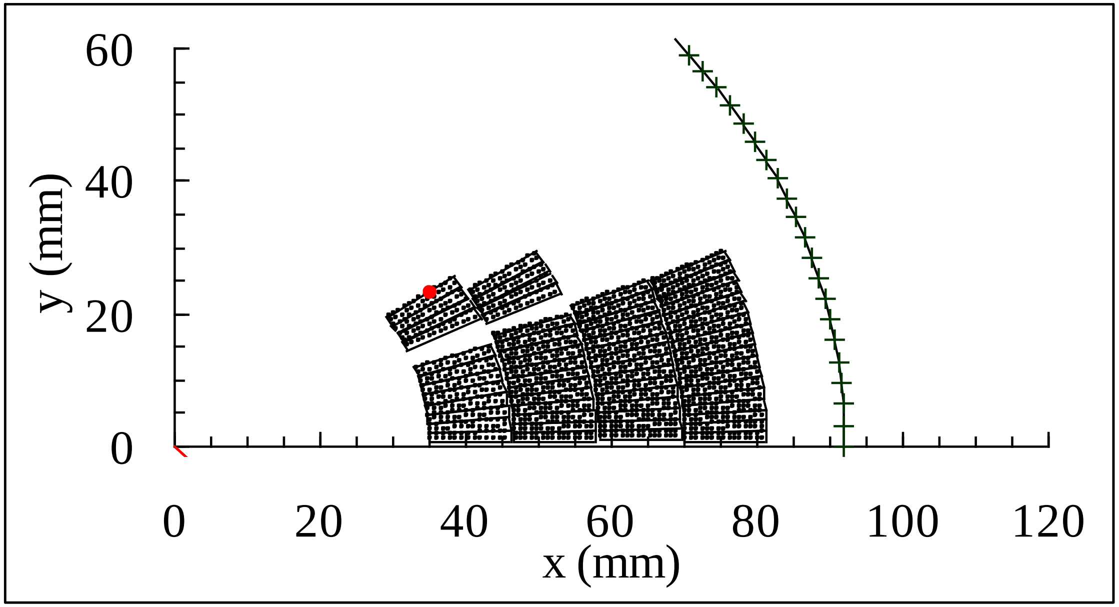


Review of quadrupole lay-outs

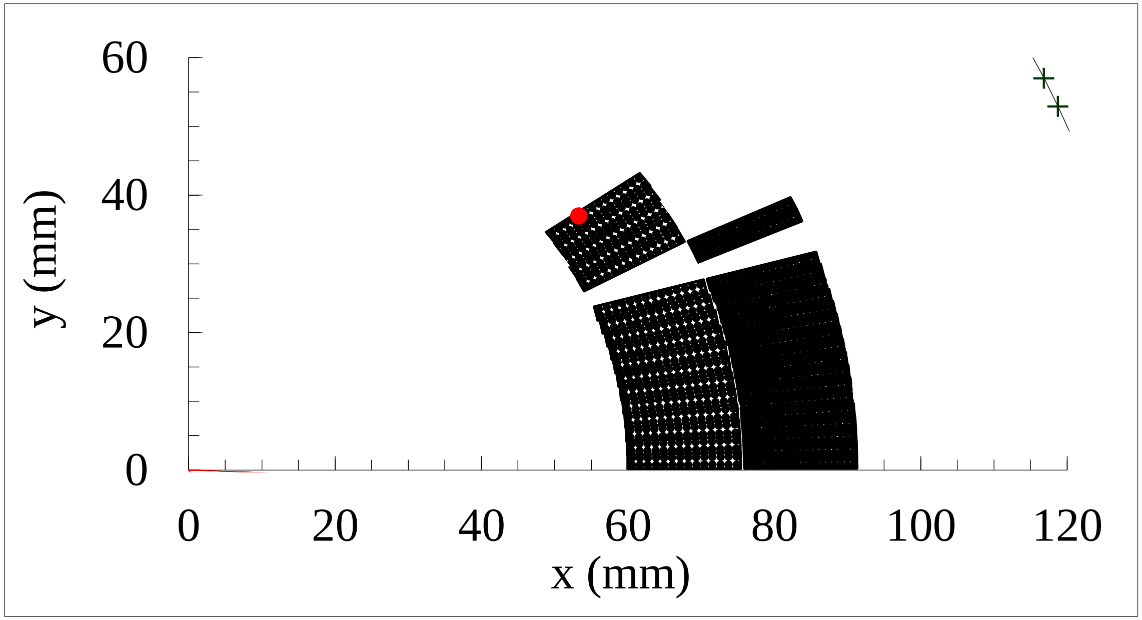
LHC MQ



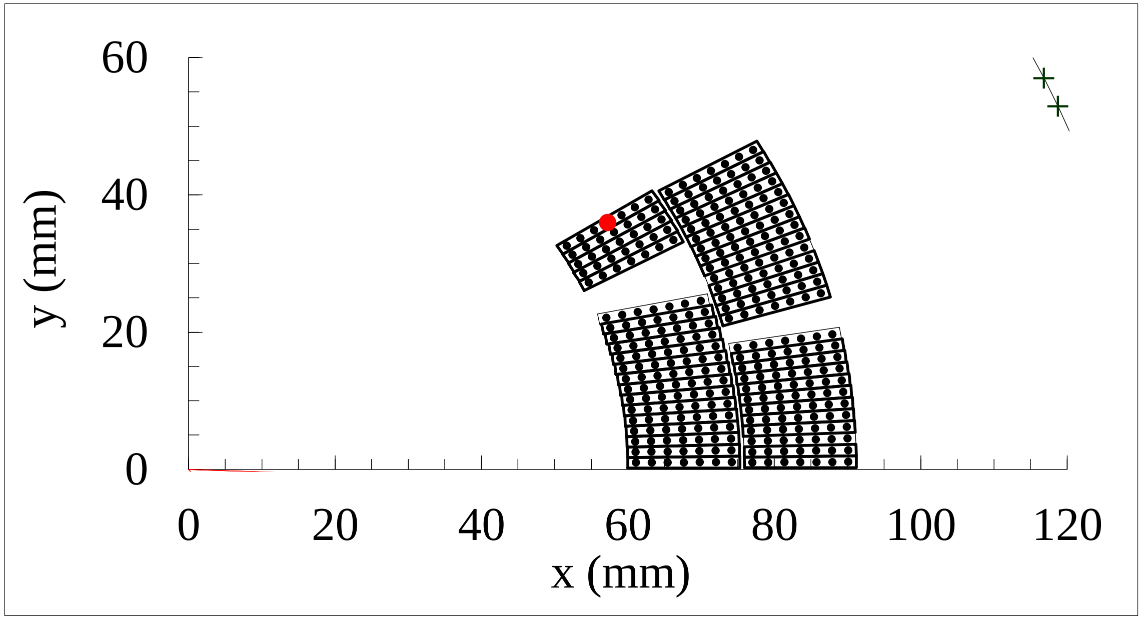
LHC MQXA



LHC MQXC

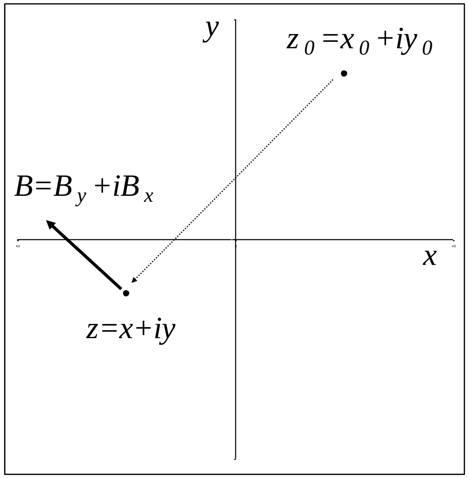


LARP HQ



Appendix

- Important property: starting by the multipolar expansion of a current line (Biot-Savart law)



$$B(z) = B_y(z) + iB_x(z)$$

$$B(z) = \frac{I\mu_0}{2\pi(z - z_0)} = -\frac{I\mu_0}{2\pi z_0}\frac{1}{1 - \frac{z}{z_0}}$$

$$B(z) = -\frac{I\mu_0}{2\pi z_0}\sum_{n=1}^{\infty}\left(\frac{z}{z_0}\right)^{n-1} = -\frac{I\mu_0}{2\pi z_0}\sum_{n=1}^{\infty}\left(\frac{R_{ref}}{z_0}\right)^{n-1}\left(\frac{x+iy}{R_{ref}}\right)^{n-1}$$

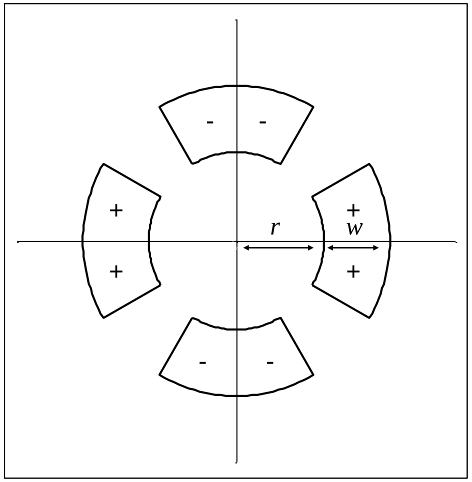
$$B_y + iB_x = 10^{-4}B_1\sum_{n=1}^{\infty}(b_n + ia_n)\left(\frac{x+iy}{R_{ref}}\right)^{n-1} \qquad b_n + ia_n = -\frac{I\mu_0 10^4}{2\pi z_0 B_1}\left(\frac{R_{ref}}{z_0}\right)^{n-1}$$

A "good" field quality dipole
Sector quadrupole

- Let's look at the quadrupoles
- First allowed multipole B_6 (dodecapole)

$$B_6 = \frac{\mu_0 j R_{ref}^5}{\pi} \frac{\sin(6\alpha)}{6} \left(\frac{1}{r^4} - \frac{1}{(r+w)^4} \right)$$

for α=π/6 (i.e. a **30° sector coil**) one has $\boldsymbol{B_6=0}$



- Second allowed multipole B_{10}

$$B_{10} = \frac{\mu_0 j R_{ref}^8}{\pi} \frac{\sin(10\alpha)}{10} \left(\frac{1}{r^8} - \frac{1}{(r+w)^8} \right)$$

for α=π/10 (i.e. a **18° sector coil**) or for α=π/5 (i.e. a **36° sector coil**) one has $\boldsymbol{B_{10}=0}$

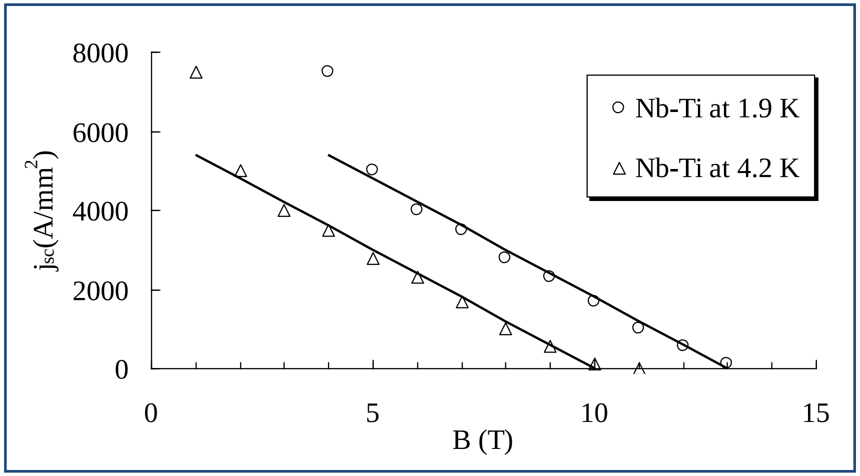
- The conditions look similar to the dipole case …

Maximum field and coil thickness Dipoles

- We recall the equations for the **critical surface**
  - Nb-Ti (linear approximation)

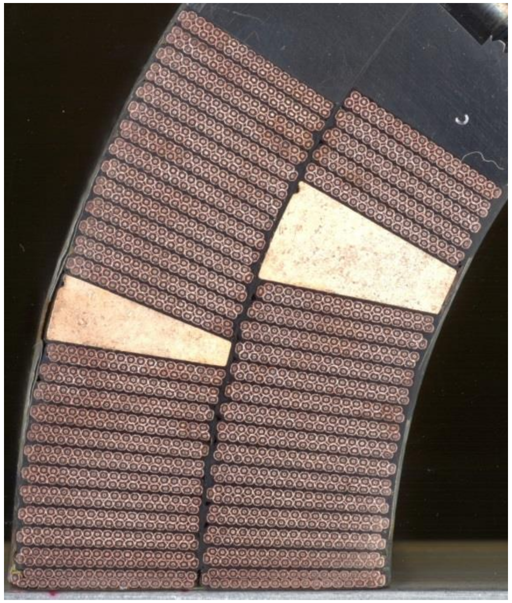
$$j_{sc,c}(B) = s(B^*_{c2} - B),$$

    - with s~6.0×10^8 [A/(T m^2)] and B^*_{c2}~10 T at 4.2 K or 13 T at 1.9 K



- The current density flowing in the insulated cable is reduced by a factor **κ (filling ratio)**
  - It ranges from ¼ to 1/3

$$j_c(B) \equiv \kappa j_{sc,c}(B) \qquad j_c(B) = \kappa s(B^*_{c2} - B)$$



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$$B_1 = -\frac{2\mu_0 j}{\pi} w \sin\alpha$$

  - λ: ratio between **peak field and central field**
    - For a sector and in general is λ= 1.05 - 1.15
    - hyperbolic fit: a~0.045

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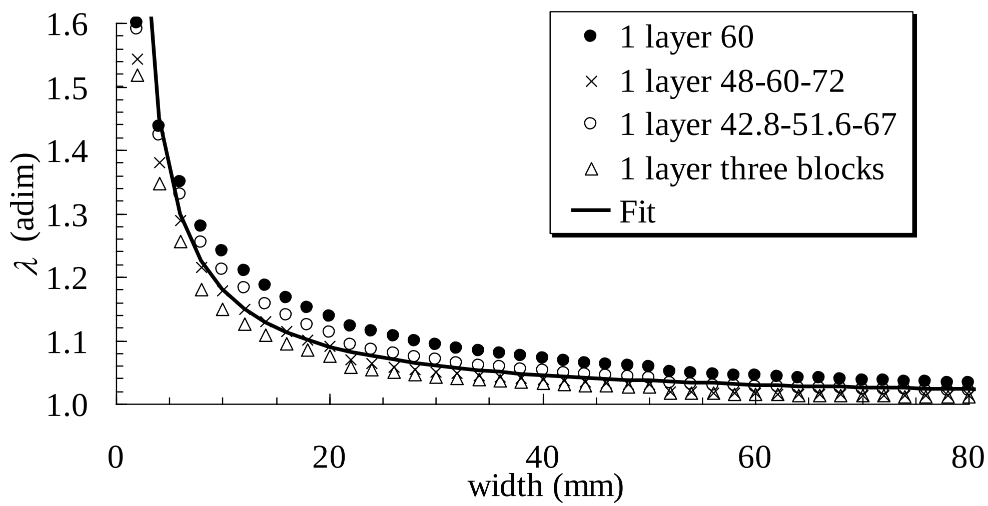
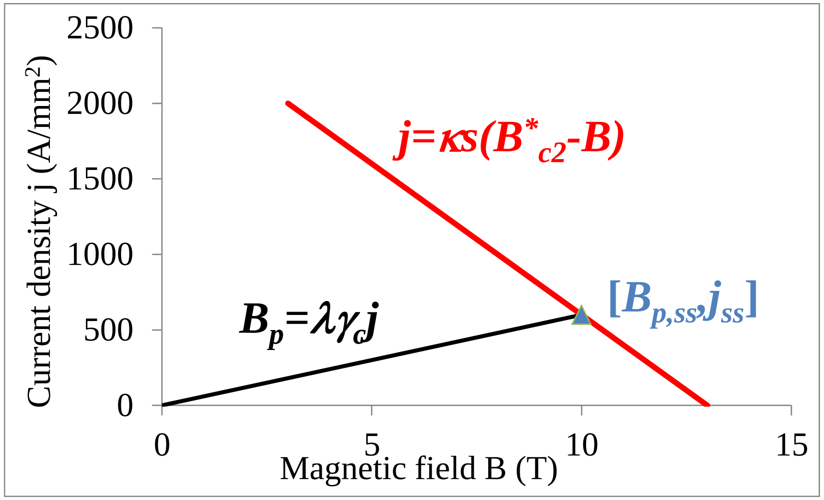


Maximum field and coil thickness
Dipoles

- We can now compute what is the **highest peak field** that can be reached in the dipole

$$B_{p,ss} = \frac{\lambda \gamma_c \kappa s}{1 + \lambda \gamma_c \kappa s} B^*_{c2}$$



- The **maximum current density** in the superconductor
  - *short sample limit*

$$j_{ss} = \frac{\kappa s}{1 + \lambda \gamma_c \kappa s} B^*_{c2}$$

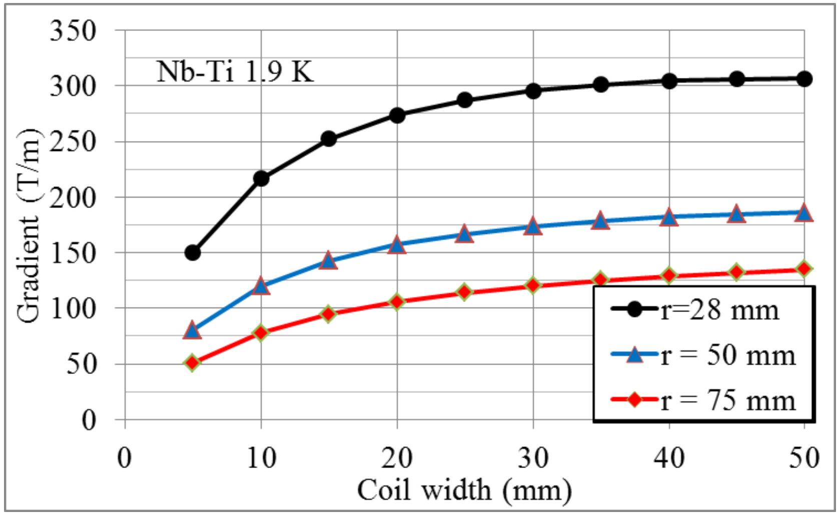
- And the **bore short sample field** (in the centre not on the conductor)

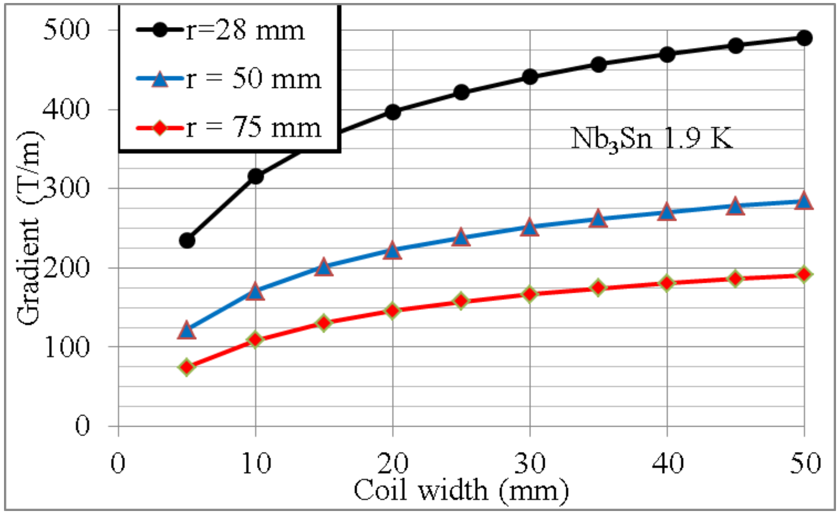
$$B_{ss} = \frac{\gamma_c \kappa s}{1 + \lambda \gamma_c \kappa s} B^*_{c2}$$

Maximum gradient and coil thickness Quadrupoles

- Therefore, the **maximum field, current and gradient**

$$B_{p,ss} = \frac{\lambda r \gamma_c \kappa s}{1+\lambda r \gamma_c \kappa s} B_{c2}^{*} \qquad j_{ss} = \frac{\kappa s}{1+\lambda r \gamma_c \kappa s} B_{c2}^{*} \qquad G_{ss} = \frac{\gamma_c \kappa s}{1+\lambda r \gamma_c \kappa s} B_{c2}^{*}$$





- Unlike dipoles, no point in making coils extremely large!