

From spin correlations to quantum observables in di-boson systems

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mostly based on Grossi GP Vicini 2409.16731

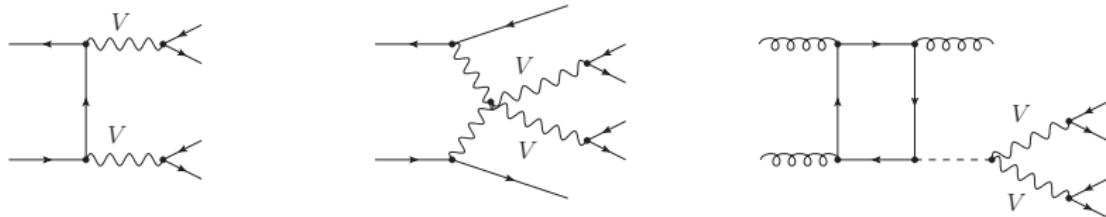
Introduction

Motivations

Precise measurements of multi-boson processes with Run-3 & High-Lumi LHC data.

Spin correlations of EW bosons

- are non trivial to extract
- probe interplay between gauge and scalar SM sectors
- discriminate power between SM and new physics
- allow to construct for entanglement/ Bell non-locality probes



Di-boson: simplest, non-trivial spin correlations with EW bosons

Experimental results

We cannot directly measure the spin state of EW bosons but we can:

- ★ Extract spin-sensitive coefficients from angular distributions

Run-1 analyses mostly relied on angular-coefficient extraction:

- ▶ W/Z+jets [CMS 1104.3829, 1504.03512, 2008.04174, ATLAS 1203.2165, 1606.00689]
- ▶ $t\bar{t}$ [CMS 1605.09047, ATLAS 1612.02577, 2005.03799]

- ★ Perform fits of LHC data with polarised templates

Run-2 analyses mostly rely fits with polarised templates:

- ▶ WZ/ZZ [ATLAS 1902.05759, CMS 2110.11231, ATLAS 2211.09435, 2402.16365, 2310.04350]
- ▶ $W^\pm W^\pm$ scattering [CMS 2009.09429]

What about theory?

Need **proper understanding**, **precise predictions** and **new ideas** to extract polarisations and spin correlations.

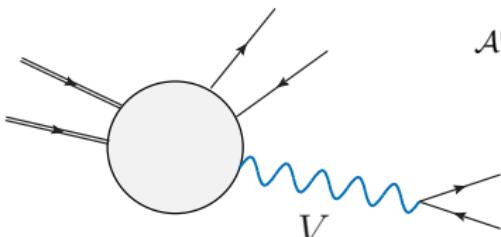
This talk:

spin correlations and quantum observables in $h \rightarrow 4\ell$ with radiative corrections.

General framework

Amplitude structure

Tree-level structure for single resonant boson (in pole/narrow-width approximation):



$$\begin{aligned}\mathcal{A}^{\text{unpol}} &= \mathcal{P}_\mu \frac{-g^{\mu\nu}}{k^2 - M_V^2 + iM_V\Gamma_V} \mathcal{D}_\nu \\ &= \mathcal{P}_\mu \frac{\sum_{\lambda'} \varepsilon_{\lambda'}^\mu \varepsilon_{\lambda'}^{*\nu}}{k^2 - M_V^2 + iM_V\Gamma_V} \mathcal{D}_\nu \\ &= \sum_{\lambda'} \mathcal{P}_\mu \frac{\varepsilon_{\lambda'}^\mu \varepsilon_{\lambda'}^{*\nu}}{k^2 - M_V^2 + iM_V\Gamma_V} \mathcal{D}_\nu = \sum_{\lambda'} \mathcal{A}_{\lambda'}\end{aligned}$$

At the cross section level:

$$|\mathcal{A}^{\text{unpol}}|^2 = \underbrace{\sum_{\lambda} |\mathcal{A}_{\lambda}|^2}_{\text{incoherent sum}} + \underbrace{\sum_{\lambda \neq \lambda'} \mathcal{A}_{\lambda}^* \mathcal{A}_{\lambda'}}_{\text{interference terms}}$$

Polarisation vectors are defined in a specific Lorenzt frame.

Decay-product angular distributions reflect polarisation state of the decayed V boson
[Bern et al. 1103.5445, Stirling et al. 1204.6427, Belyaev et al. 1303.3297].

Angular dependence for one boson

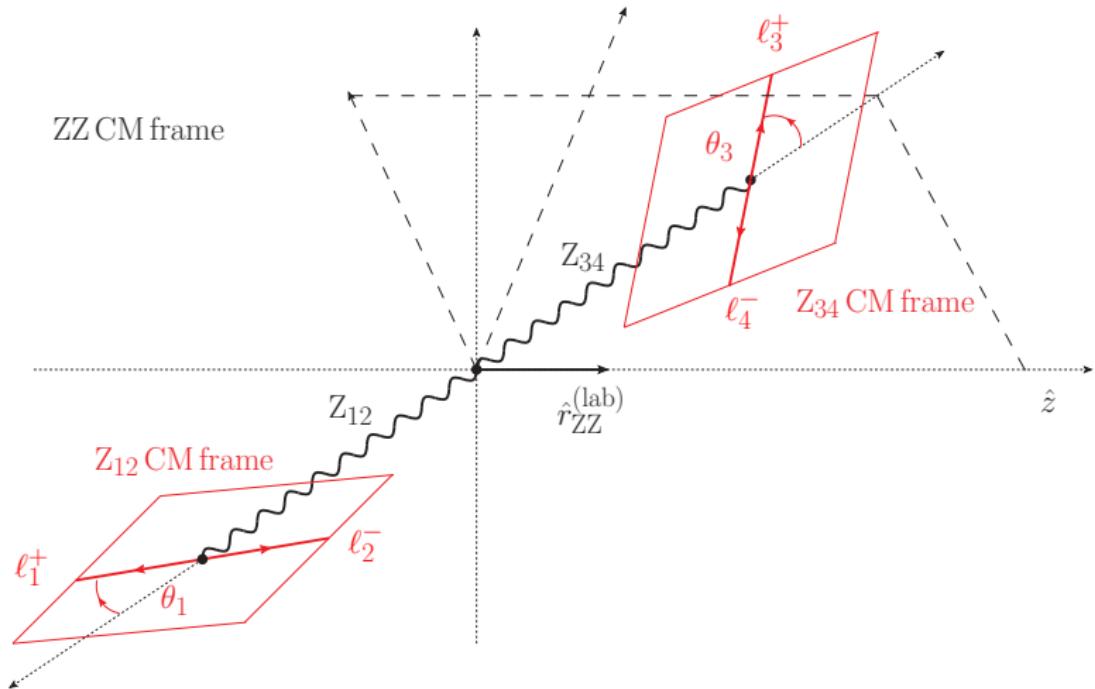
At tree-level, decay of a single resonant boson (θ^*, ϕ^* are ℓ^+ angles in V rest frame, w.r.t. V direction in some Lorentz frame) [Bern et al. 1103.5445, Stirling Vryonidou 1204.6427]:

$$\begin{aligned} \frac{d\sigma}{d\cos\theta^* d\phi^* dX} &= \frac{d\sigma}{dX} \frac{3}{16\pi} \left[(1 + \cos^2\theta^*) + (A_0/2)(1 - 3\cos^2\theta^*) + A_1 \sin 2\theta^* \cos \phi^* \right. \\ &\quad + (A_2/2) \sin^2\theta^* \cos 2\phi^* + A_3 \sin \theta^* \cos \phi^* + A_4 \cos \theta^* \\ &\quad \left. + A_5 \sin \theta^* \sin \phi^* + A_6 \sin 2\theta^* \sin \phi^* + A_7 \sin^2\theta^* \sin 2\phi^* \right] \\ &= \frac{d\sigma}{dX} \left[\frac{1}{4\pi} + \sum_{\ell=1}^2 \sum_{m=-\ell}^{\ell} \alpha_{\ell m} Y_{\ell m}(\theta^*, \phi^*) \right] \end{aligned} \quad (1)$$

8 independent coefficients ($\{A_i\}$ or $\{\alpha_{\ell,m}\}$) extracted through projections [Bern et al. 1103.5445, Stirling Vryonidou 1204.6427, Ballestrero Maina GP 1710.09339, Baglio et al. 1910.13746, Frederix Vitos 2007.08867] or asymmetries [Boudjema Singh 0903.4705] e.g.:

$$\int_{-1}^1 d\cos\theta^* \int_0^{2\pi} d\phi^* Y_{\ell m}(\theta^*, \phi^*) \frac{d\sigma}{d\cos\theta^* d\phi^* dX} = \alpha_{\ell m} = \alpha_{\ell m}(X) \quad (2)$$

Two bosons: geometric visualisation



Angular dependence for boson pairs

Two resonant bosons (θ_1, ϕ_1 , and θ_3, ϕ_3) are ℓ_1^+ and ℓ_3^+ angles in each V rest frame, w.r.t. V direction in some Lorentz frame (typical choice: VV-CM frame):

$$\begin{aligned} \frac{d\sigma}{d \cos \theta_1 d\phi_1 d \cos \theta_3 d\phi_3 dX} &= \frac{d\sigma}{dX} \left[\frac{1}{(4\pi)^2} + \right. \\ &+ \frac{1}{4\pi} \sum_{\ell=1}^2 \sum_{m=-\ell}^{\ell} \alpha_{\ell m}^{(1)}(X) Y_{\ell m}(\theta_1, \phi_1) \\ &+ \frac{1}{4\pi} \sum_{\ell=1}^2 \sum_{m=-\ell}^{\ell} \alpha_{\ell m}^{(3)}(X) Y_{\ell m}(\theta_3, \phi_3) \\ &\left. + \sum_{\ell_1=1}^2 \sum_{\ell_3=1}^2 \sum_{m_1=-\ell_1}^{\ell_1} \sum_{m_3=-\ell_3}^{\ell_3} \gamma_{\ell_1 m_1 \ell_3 m_3}(X) Y_{\ell_1 m_1}(\theta_1, \phi_1) Y_{\ell_3 m_3}(\theta_3, \phi_3) \right] \end{aligned} \quad (3)$$

80 independent coefficients [Rahaman Singh 2109.09345, Aguilar-Saavedra et al. 2209.13441], extracted similarly to the single-boson case.

Remark: the two bosons are correlated if $\alpha_{\ell_1, m_1}^{(1)} \alpha_{\ell_3, m_3}^{(3)} \neq \gamma_{\ell_1 m_1 \ell_3 m_3}$.

Issues in angular-coefficient extraction

In principle, coefficients extracted directly from data through projections up to $\ell_{1,3} = 2$ of the angular distributions $d\sigma/d\Omega_1 d\Omega_3$.

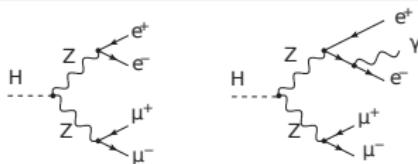
In practice, the analytic structure of $d\sigma/d\Omega_1 d\Omega_3$ in eq. (3):

1. assumes two spin-1 resonances, and subsequent two-body decays,
2. its coefficient values not invariant under Lorentz boosts,
3. is not described by $\ell_{1,3} \leq 2$ if selection cuts/ ν -reconstruction applied,
4. is not fully model independent (if NP in both production and decay)

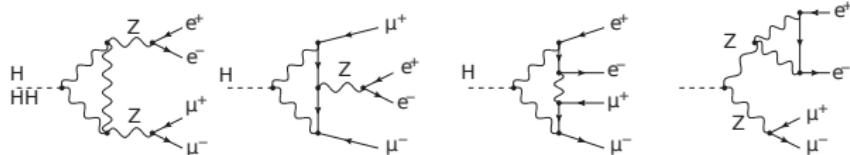
Radiative corrections

EW corrections in $h \rightarrow 4\ell$

LO, NLO EW real:



NLO EW virt.:



$h \rightarrow 4\ell(\gamma)$: one Z boson necessarily off-shell.

Non-factorisable virtual and real-photon corrections lead to higher-rank contributions:
tricky interpretation of LO-like angular structure [Grossi GP Vicini 2409.16731].

$M_{\ell^+\ell^-} > 10 \text{ GeV}$	LO	NLO EW (H-rest)	δ_{EW}
$\alpha_{2,0}^{(1)}$	-0.04792(4)	-0.04215(6)	-12.0%
$\alpha_{2,0}^{(3)}$	-0.04792(4)	-0.04224(6)	-11.9%
$\gamma_{1,0,1,0}$	0.00117(1)	0.00011(2)	-91%
$\gamma_{2,0,2,0}$	0.01097(1)	0.01019(2)	-7.1%
$\gamma_{1,1,1,-1}$	-0.00185(2)	-0.00047(2)	-75%
$\gamma_{2,1,2,-1}$	-0.00776(1)	-0.00715(2)	-7.9%
$\gamma_{2,2,2,-2}$	0.00493(2)	0.00481(2)	-2.4%

Frame dependence and real-photon corrections in $h \rightarrow 4\ell$

$h \rightarrow 4\ell + \gamma$ real corrections to (integrated) partial decay rate typically **small**.

Massless charged leptons dressed with real photons to construct **IR-safe** observables.

Quantum tomography can be carried out in **Higgs rest frame** (H-rest) or in **dressed four-lepton CM frame** (4ℓ -CM) [Grossi GP Vicini 2409.16731]:

$M_{\ell^+\ell^-} > 10 \text{ GeV}$	LO	NLO EW (4ℓ -CM)	δ_{EW}	NLO EW (H-rest)	δ_{EW}
$\alpha_{2,0}^{(1)}$	−0.04792(4)	−0.04498(6)	−6.1%	−0.04215(6)	−12.0%
$\alpha_{2,0}^{(3)}$	−0.04792(4)	−0.04506(6)	−6.0%	−0.04224(6)	−11.9%
$\gamma_{1,0,1,0}$	0.00117(1)	0.00012(2)	−90%	0.00011(2)	−91%
$\gamma_{2,0,2,0}$	0.01097(1)	0.01079(2)	−1.6%	0.01019(2)	−7.1%
$\gamma_{1,1,1,−1}$	−0.00185(2)	−0.00048(2)	−74%	−0.00047(2)	−75%
$\gamma_{2,1,2,−1}$	−0.00776(1)	−0.00779(2)	+0.4%	−0.00715(2)	−7.9%
$\gamma_{2,2,2,−2}$	0.00493(2)	0.00489(2)	−0.8%	0.00481(2)	−2.4%

NLO EW corrections to $\ell = 2$ coeff.'s **smaller in the 4ℓ -CM definition**.

Dramatic effects on $\ell = 1$ coeff.'s persists in both definitions: **one-loop weak effects**.

NLO EW effects on LO relations amongst coefficients

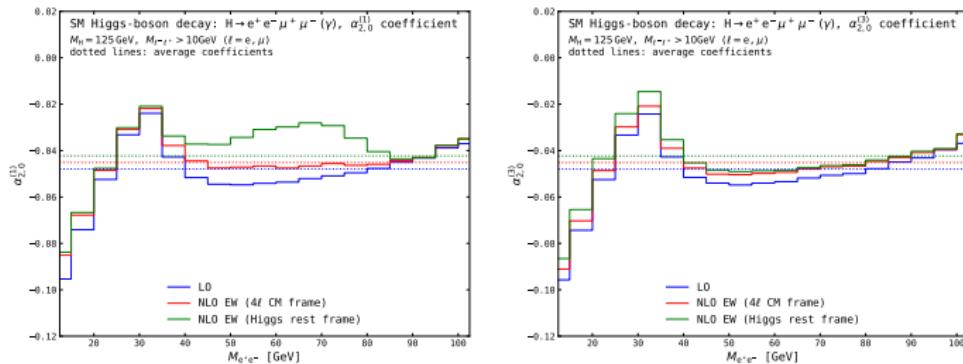
$M_{\ell^+\ell^-} > 10 \text{ GeV}$: NLO EW changes tree structure of the (CP-even) SM amplitude
[Grossi GP Vicini 2409.16731].

	LO	NLO (4 ℓ -CM)	NLO (H-rest)
$\alpha_{2,0}^{(1)}$	-0.04792(4)	-0.04498(6)	-0.04215(6)
$\alpha_{2,0}^{(3)}$	-0.04792(4)	-0.04506(6)	-0.04224(6)
$\gamma_{1,0,1,0}$	0.00117(1)	0.00012(2)	0.00011(2)
$\gamma_{2,0,2,0}$	0.01097(1)	0.01079(2)	0.01019(2)
$\gamma_{1,1,1,-1}$	-0.00185(2)	-0.00048(2)	-0.00047(2)
$\gamma_{1,-1,1,1}$	-0.00186(2)	-0.00047(2)	-0.00047(2)
$\gamma_{2,1,2,-1}$	-0.00776(1)	-0.00779(2)	-0.00715(2)
$\gamma_{2,-1,2,1}$	-0.00778(1)	-0.00783(2)	-0.00709(2)
$\gamma_{2,2,2,-2}$	0.00493(2)	0.00489(2)	0.00481(2)
$\gamma_{2,-2,2,2}$	0.00494(2)	0.00488(2)	0.00479(2)

- * $\alpha_{2,0}^{(1)} = \alpha_{2,0}^{(3)}$
- * $\gamma_{2,m,2,-m} = \gamma_{2,-m,2,m}$ for $m = 1, 2$
- * $\gamma_{1,1,1,-1} = \gamma_{1,-1,1,1}$
- * $20\pi (\gamma_{2,2,2,-2} + \gamma_{2,0,2,0}) = 1$
- * $40\pi \gamma_{2,2,2,-2} = \sqrt{20\pi} \alpha_{2,0}^{(1)} + 1$
- * $5\eta_\ell^2 \gamma_{2,1,2,-1} = \gamma_{1,1,1,-1}$
- * $\eta_\ell^2 (1 - 20\pi \gamma_{2,0,2,0}) = 4\pi \gamma_{1,0,1,0}$

More on off-shell effects $h \rightarrow 4\ell$

Dependence of coefficients on $M_{\ell^+\ell^-}$ [Grossi GP Vicini 2409.16731]. $\alpha_{(2,0)}$ in figure.
 Setup: $M_{e^+e^-}, M_{\mu^+\mu^-} > 10\text{GeV}$.



Strong dependence of coefficients on off-shell-ness already at LO. 4 ℓ -CM definition better behaved at NLO EW.

$\alpha_{2,0}^{(1)} = \alpha_{2,0}^{(3)}$ relation broken at NLO EW if unequal cuts applied on $M_{\ell\ell}$.

Spin-correlations change value with tighter $M_{\ell\ell}$ cuts but preserve symmetric character ($\gamma_{2,+2,2,-2} = \gamma_{2,-2,2,+2}$).

Quantum observables

A fast-growing business

Quantum entanglement between top-quarks (two-qubit system) measured at LHC
[ATLAS 2311.07288, CMS 2406.03976, 2409.11067]

Measuring entanglement and Bell non-locality at colliders possible also for **two-qutrit systems** → two massive weak bosons.

Di-boson EW production at LHC and lepton colliders (generic $\mathbf{1} \otimes \mathbf{1}$) [Barr et al. 2204.11063, Ashby-Pickering et al. 2209.13990, Fabbrichesi et al. 2302.00683, Morales 2306.17247, Aoude et al. 2307.09675, Bernal et al. 2307.13496, Bernal 2310.10838, Barr et al. 2402.07972, Grossi GP Vicini 2409.16731, Grabarczyk 2410.18022]

Di-boson in spin-0 decays ($\mathbf{0} \rightarrow \mathbf{1} \otimes \mathbf{1}$) [Barr 2106.01377, Aguilar-Saavedra et al. 2209.13441, 2209.14033, Ashby-Pickering et al. 2209.13990, Fabbrichesi et al. 2302.00683, 2304.02403, Fabbri et al. 2307.13783, Bernal et al. 2307.13496, Barr et al. 2402.07972, Aguilar-Saavedra 2403.13942, 2411.13464, Bernal et al. 2405.16525, Grossi GP Vicini 2409.16731, Sullivan 2410.10980, Subba Singh 2411.19171]

Entanglement witnesses

For generic **mixed states** of **two subsystems A & B**,

$$\rho = \sum_{i,j,l,k} p_{ij} p_{kl}^* |a_i\rangle \otimes |b_j\rangle \langle a_k| \otimes \langle b_l| \longrightarrow \rho^{T_B} = \sum_{i,j,l,k} p_{il} p_{kj}^* |a_i\rangle \langle a_k| \otimes |b_j\rangle \langle b_l|$$

Peres-Horodecki criterion [Peres 9604005, Horodecki 9703004]: if ρ^{T_B} has a negative eigenvalue, then the system is entangled.

At **LO** two spin-1 bosons from decay of a spin-0 state are entangled **if and only if** $\gamma_{2,1,2,-1} \neq 0$ or $\gamma_{2,2,2,-2} \neq 0$ [Aguilar-Saavedra 2209.13441]. What about **NLO EW**?

$M_{\ell^+\ell^-} > 10 \text{ GeV}$	LO	NLO EW (4 ℓ -CM)	δ_{EW}	NLO EW (H-rest)	δ_{EW}
$\gamma_{2,1,2,-1}$	-0.00776(1)	-0.00779(2)	+0.4%	-0.00715(2)	-7.9%
$\gamma_{2,2,2,-2}$	0.00493(2)	0.00489(2)	-0.8%	0.00481(2)	-2.4%

NLO EW corr. small, many zeros in ρ_{LO} compatible with zero at NLO [Grossi GP Vicini 2409.16731], some get non-zero values.

A priori, no guarantee $\gamma_{2,1,2,-1} \neq 0$ or $\gamma_{2,2,2,-2} \neq 0$ still sufficient and necessary conditions: **full NLO spin-density matrix needed**, then apply Peres-Horodecki

Bell-inequality violation

Quantum state close to maximal entanglement: Bell non-locality can be probed.

CGLMP inequality [Collins et al. 0106024] suitable for two qutrits: violated if

$$\mathcal{I}_3 = \langle \mathcal{O}_{\text{Bell}} \rangle = \text{Tr}(\rho \mathcal{O}_{\text{Bell}}) > 2.$$

For a scalar decaying into two spin-1,

$$\mathcal{I}_3 = \frac{1}{2} + \frac{4\sqrt{3}}{9} - \sqrt{5\pi} \left(1 - \frac{8\sqrt{3}}{9} \right) \alpha_{2,0} - 40\pi \left(\frac{2}{3} + \frac{4\sqrt{3}}{9} \right) \gamma_{2,1,2,-1} + \frac{20\pi}{3} \gamma_{2,2,2,-2}.$$

In our inclusive setup ($M_{\ell^+\ell^-} > 10\text{GeV}$) [Grossi GP Vicini 2409.16731]

$$\mathcal{I}_3^{(\text{LO})} = 2.671(2), \quad \mathcal{I}_3^{(\text{NLO EW}, 4\ell)} = 2.682(4), \quad \mathcal{I}_3^{(\text{NLO EW}, \text{H})} = 2.571(4).$$

\mathcal{I}_3 robust under EW corrections (not guaranteed a priori).

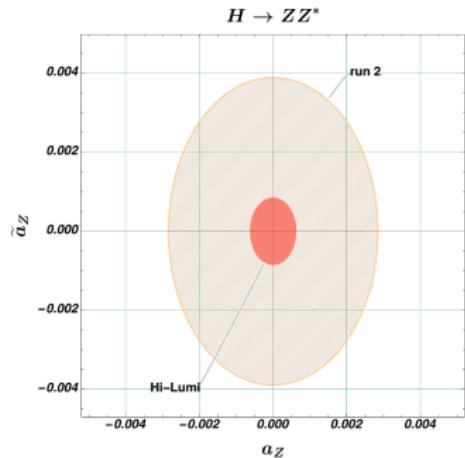
LO spin-density matrix seems sufficient at LHC [Aguilar-Saavedra et al. 2209.13441].

Importance of reference-frame choice for the spin quantisation.

Quantum observables and new physics in Higgs decays

Quantum observables with new-physics in $h \rightarrow 4\ell$ decay:

- ★ anomalous couplings [Bernal et al. 2307.13496, Fabbrichesi et al. 2304.02403]



$$\begin{aligned}\mathcal{L}_{hVV} = & g_W M_W W_\mu^+ W^{-\mu} h + \frac{g_W}{2 \cos \theta_W} M_Z Z_\mu Z^\mu h \\ & - \frac{g_W}{M_W} \left[\frac{a_W}{2} W_{\mu\nu}^+ W^{-\mu\nu} + \frac{\tilde{a}_W}{2} W_{\mu\nu}^+ \tilde{W}^{-\mu\nu} \right] h \\ & - \frac{g_W}{M_W} \left[\frac{a_Z}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{\tilde{a}_Z}{4} Z_{\mu\nu} \tilde{Z}^{\mu\nu} \right] h\end{aligned}$$

Obtained limits on anomalous couplings at **HL-LHC**, through **3 entanglement witnesses** ($\mathcal{E}_{\text{entropy}} = \text{Tr}(\rho_A \log \rho_A)$, $\mathcal{C}[\rho] \geq \mathcal{C}_2$, $\mathcal{C}_{\text{odd}} = \frac{1}{2} \sum_{i < j} |\rho_{ij}^{\text{off}} - \rho_{ji}^{\text{off}}|$)

- ★ SMEFT [Sullivan 2410.10980, Subba Singh 2411.19171]: determining $c_{VH}, c_{V\tilde{H}}$ challenging also at **HL-LHC**.

Conclusion

Conclusions

Rich phenomenology from **decay angles** in di-boson (or **two-qutrit**) systems at LHC.

Aimed at **measuring polarisation properties** to pin down the **EWSB** mechanism realised in nature, and accessing **quantum entanglement** at high energies.

Several aspects discussed for $h \rightarrow 4\ell$:

- ★ **off-shell effects** and **radiative decays** have strong impact
- ★ **NLO corrections** sizeably change the angular coefficients
- ★ important to choose suitable **Lorentz frame**, especially with rad. corr.
- ★ **LO picture** may be enough for some entanglement/Bell non-locality witnesses
- ★ better to scrutinise **NLO effects**

Backup

Two-qutrit formalism

Generic spin-density matrix:

$$\rho = \frac{1}{9} \left[(\mathbb{I}_3 \otimes \mathbb{I}_3) + A_{l_1, m_1}^{(1)} \left(T_{m_1}^{l_1} \otimes \mathbb{I}_3 \right) + A_{l_3, m_3}^{(3)} \left(\mathbb{I}_3 \otimes T_{m_3}^{l_3} \right) + C_{l_1, m_1, l_3, m_3} \left(T_{m_1}^{l_1} \otimes T_{m_3}^{l_3} \right) \right]$$

T_m^l : irreducible tensor representations of the boson spin.

Our original, fully decayed formula is nothing but:

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega_1 d\Omega_3} = \left(\frac{3}{4\pi} \right)^2 \text{Tr} \{ \rho (\Gamma^{(1)} \otimes \Gamma^{(3)})^\top \} .$$

$\Gamma^{(1,3)}$: decay matrices.

$$\sqrt{40\pi} \alpha_{2, m_i}^{(i)} = A_{2, m_i}^{(i)}, \quad 40\pi \gamma_{2, m_1, 2, m_3} = C_{2, m_1, 2, m_3}, \quad 8\pi \gamma_{1, m_1, 1, m_3} / \eta_\ell^2 = C_{1, m_1, 1, m_3}.$$

$$\text{Tr} \left(\mathbb{I}_3 \Gamma^\top \right) \propto Y_{00}(\theta, \phi), \quad \text{Tr} \left(T_m^l \Gamma^\top \right) \propto Y_{lm}(\theta, \phi)$$

$$\begin{aligned} \mathcal{O}_{\text{Bell}} &= \frac{2}{3\sqrt{3}} (T_1^1 \otimes T_1^1 - T_0^1 \otimes T_0^1 + T_1^1 \otimes T_{-1}^1) + \frac{1}{12} (T_2^2 \otimes T_2^2 + T_2^2 \otimes T_{-2}^2) \\ &+ \frac{1}{2\sqrt{6}} (T_2^2 \otimes T_0^2 + T_0^2 \otimes T_2^2) - \frac{1}{3} (T_1^2 \otimes T_1^2 + T_1^2 \otimes T_{-1}^2) + \frac{1}{4} T_0^2 \otimes T_0^2, + \text{h.c.} \end{aligned}$$

Polar-angle correlations

$$\begin{aligned}\alpha_{1,0}^{(i)} &= \frac{1}{4} \sqrt{\frac{3}{\pi}} \eta_i (f_+^{(i)} - f_-^{(i)}) , \\ \alpha_{2,0}^{(i)} &= \frac{1 - 3 f_L^{(i)}}{4\sqrt{5\pi}} , \\ \gamma_{1,0,1,0} &= 3 \eta_1 \eta_3 \frac{f_{--} + f_{++} - f_{-+} - f_{+-}}{16\pi} , \\ \gamma_{2,0,2,0} &= \frac{1 - 3 f_L^{(1)} - 3 f_L^{(3)} + 9 f_{LL}}{80\pi} , \\ \gamma_{1,0,2,0} &= \sqrt{\frac{3}{5}} \eta_1 \frac{3 (f_{-L} - f_{+L}) - (f_-^{(1)} - f_+^{(1)})}{16\pi} , \\ \gamma_{2,0,1,0} &= \sqrt{\frac{3}{5}} \eta_3 \frac{3 (f_{L-} - f_{L+}) - (f_-^{(3)} - f_+^{(3)})}{16\pi} .\end{aligned}$$

The 4ℓ channel: EW production and Higgs decays

Four-charged-lepton is promising channel to measure entanglement and Bell-inequality violation at hadron/lepton colliders [Ashby-Pickering et al. 2209.13990, Aguilar-Saavedra et al. 2209.13441, Fabbrichesi et al. 2302.00683, Aoude et al. 2307.09675, Aguilar-Saavedra 2403.13942, 2411.13464, Bernal et al. 2405.16525, Grossi GP Vicini 2409.16731, Sullivan 2410.10980, Grabarczyk 2410.18022, Subba Singh 2411.19171]:

- clean signature, no ν reconstruction
- ZZ CM frame precisely reconstructed
- EW production has lower sensitivity to NP than WZ/WW
- off-shell bosons make two-boson angular structure not well defined in Higgs-boson decays
- fiducial cuts disrupt angular integration
- higher-order corrections

Goodness of entanglement witnesses (sufficient conditions for system to be entangled) depend on how well we extract spin-density-matrix entries from (LHC) data.