

Inclusive determination of $|V_{cb}|$ from semileptonic \bar{B} decays

Gael Finauri

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Milan - 20 December 2024

based on GF, Paolo Gambino 2310.20324, + new results



$|V_{cb}|$ from $\bar{B} \rightarrow X_c \ell^- \bar{\nu}_\ell$

The CKM matrix element V_{cb} is a fundamental input of the Standard Model

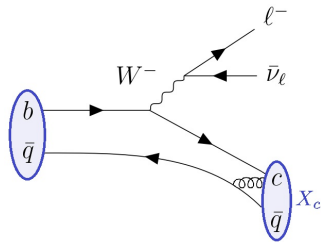


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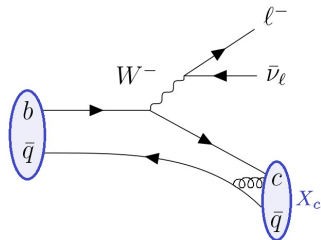
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is computed through a power expansion in $\Lambda_{\text{QCD}}/m_b \sim 0.1$



Heavy Quark Expansion (HQE)

$$f(m_b, m_c, \dots) = f^{\text{LP}} + f^{\text{NLP},\pi} \frac{\mu_\pi^2}{m_b^2} + f^{\text{NLP},G} \frac{\mu_G^2}{m_b^2} + f^{\text{NNLP},D} \frac{\rho_D^3}{m_b^3} + f^{\text{NNLP},LS} \frac{\rho_{LS}^3}{m_b^3} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^4}{m_b^4}\right)$$



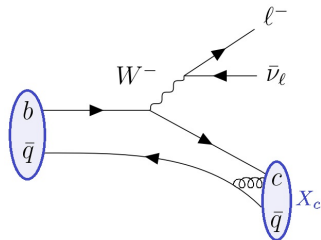
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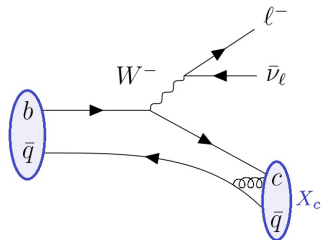
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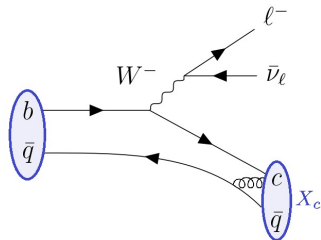
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They can also be extracted from **DATA!**



HQE Parameters from Semileptonic Moments

The **inclusive decay spectrum** is characterized by 3 kinematical variables:
lepton energy (E_ℓ), dilepton invariant mass (q^2), hadronic invariant mass (m_X^2)



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So far only existed fits to (E_ℓ, m_X^2) moments [Bordone et al. '21] or q^2 separately [Bernlochner et al. '22].
First combined (E_ℓ, m_X^2, q^2) fit in [GF, Gambino '23]. Now updated results!



Theory State of the Art in $\bar{B} \rightarrow X_c \ell \bar{\nu}$

	dE_ℓ	dm_X^2	dq^2	Γ
1	α_s^2 [Melnikov 2008] [Pak, Czarnecki 2008]	α_s^2	α_s^2 [Fael, Herren 2024]	α_s^3 [Fael, Schönwald, Steinhauser 2020]
$1/m_b^2$	α_s [Alberti, Ewerth, Gambino, Nandi 2012, 2013]	α_s	α_s	α_s
$1/m_b^3$	1 [Gremm, Kapustin 1997]	1	α_s [Mannel, Moreno Pivovarov 2021]	α_s [Mannel, Pivovarov 2019]
$1/m_b^{4,5}$ $1/(m_b^3 m_c^2)$	1 [Mannel, Turczyk, Uraltsev 2010]	1	1 [Mannel, Milutin, Vos 2023]	1 [Mannel, Turczyk, Uraltsev 2010]



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$1/m_b^{4,5}$ $1/(m_b^3 m_c^2)$	<p>Proliferation of non-perturbative parameters</p> <p>1 RPI can reduce them, but restricted to dq^2</p> <p>[Fael, Mannel, Vos 2018] [Mannel, Turczyk, Uraltsev 2010] [Mannel, Milutin, Vos 2023] [Mannel, Turczyk, Uraltsev 2010]</p>			



q^2 Moments

In practice measured at several lower cuts in q^2

$$M_n(q_{\text{cut}}^2) = \int_{q_{\text{cut}}^2}^{q_{\text{max}}^2} dq^2 \frac{d\Gamma}{dq^2} (q^2)^n, \quad \langle q^{2n} \rangle = \frac{M_n(q_{\text{cut}}^2)}{M_0(q_{\text{cut}}^2)}$$



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coefficient functions have perturbative expansion (NNLO only for leading power $S^{(0)}$)

$$S^{(i)}(q^2) = S^{(i,0)}(q^2) + \frac{\alpha_s}{\pi} S^{(i,1)}(q^2) + \frac{\alpha_s^2}{\pi^2} S^{(i,2)}(q^2) + \mathcal{O}(\alpha_s^3)$$

full two-loop [Fael, Herren, '24] new implementation to the fit, before only BLM $\mathcal{O}(\alpha_s^2\beta_0)$



Kinetic Scheme

To avoid renormalon ambiguities and badly converging perturbative series:
on-shell \rightarrow kinetic scheme ($\mu_k = 1 \text{ GeV}$, $\alpha_s^{(4)}(m_b) = 0.2185$)

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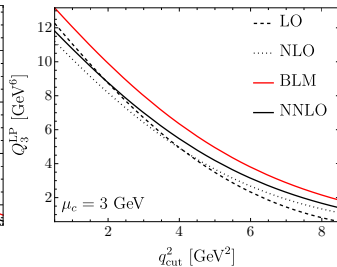
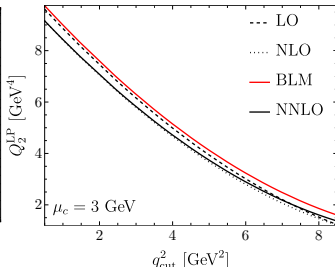
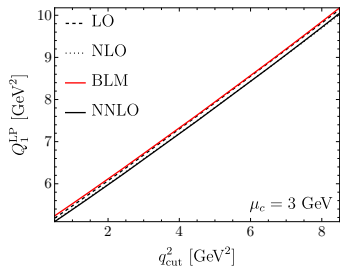
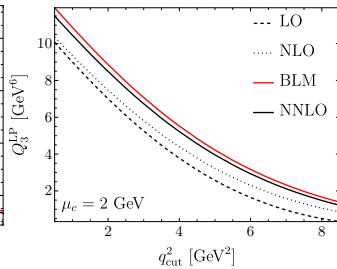
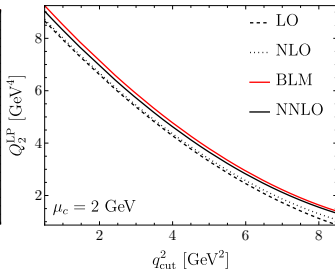
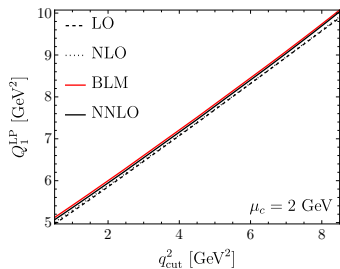
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- m_c : on-shell \rightarrow $\overline{\text{MS}}$ at μ_c

$$m_c^{\text{OS}} = m_c(2 \text{ GeV}) (1 + 0.18\alpha_s + 0.14\alpha_s^2) = m_c(3 \text{ GeV}) (1 + 0.25\alpha_s + 0.18\alpha_s^2)$$

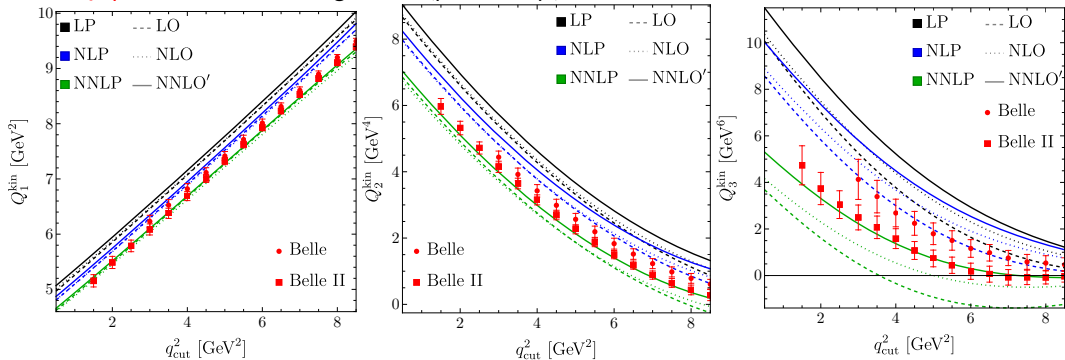


BLM vs Full NNLO



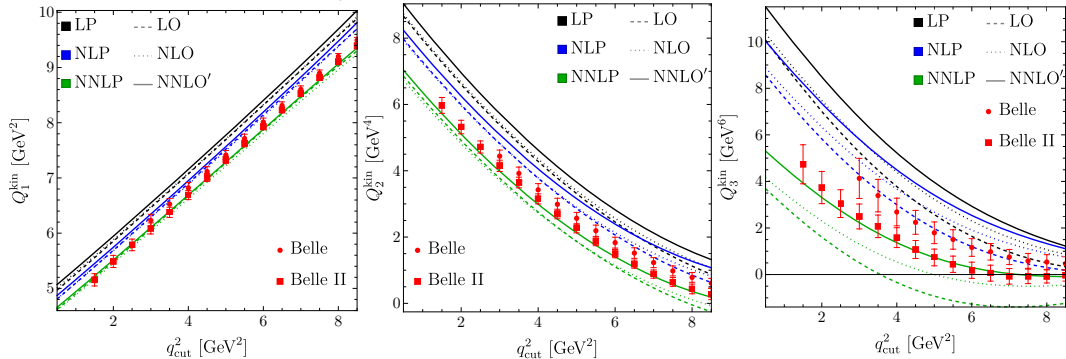
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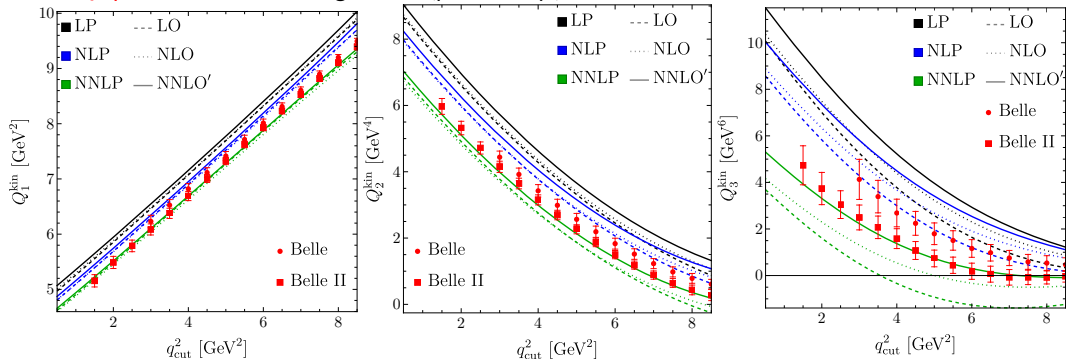


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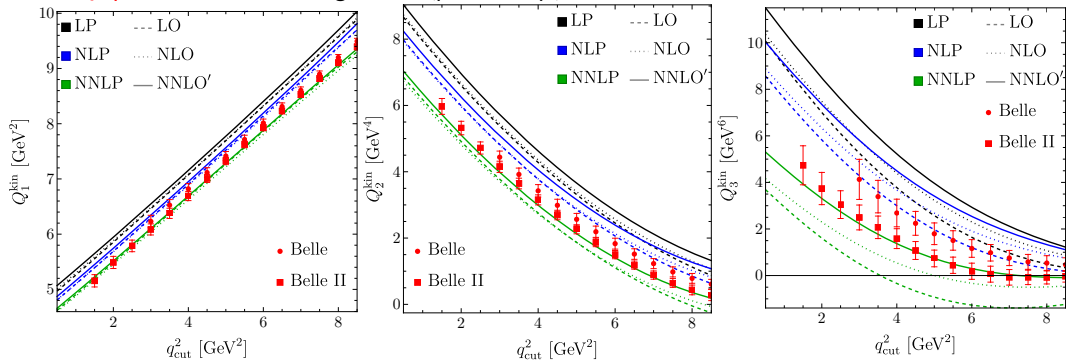
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Systematic shift between Belle and Belle II data $\sim 2\sigma \Rightarrow dE_e$ and dm_X^2 moms. can help!



q^2 Moments

with **HQE parameters** from new **global fit** (preliminary!)



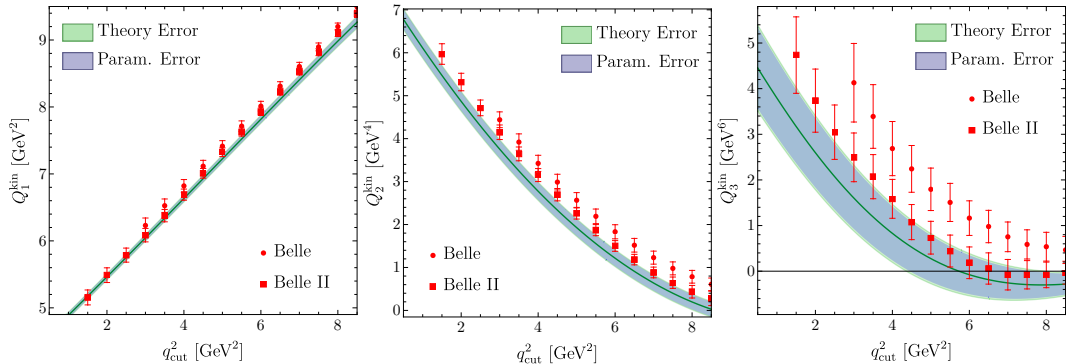
Power corrections are important for higher moments!

Systematic shift between Belle and Belle II data $\sim 2\sigma \Rightarrow dE_\ell$ and dm_X^2 moms. can help!

Inclusion of NLO and NNLO terms can have big impact on **HQE parameters**!



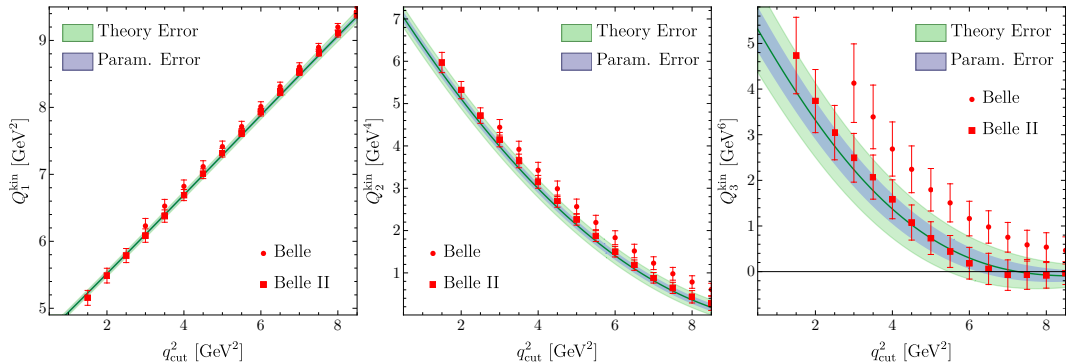
Fit to (E_ℓ, m_X^2) moments [Bordone, Capdevila, Gambino 2021]



q^2 moms probe different direction in parameter space
⇒ reduce parametric uncertainty!



Fit to (E_ℓ, m_X^2, q^2) moments (new, preliminary!)



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Fit Results (PRELIMINARY)

m_b^{kin}	$\overline{m}_c(2 \text{ GeV})$	μ_π^2	μ_G^2	ρ_D^3	ρ_{LS}^3	$\text{BR}_{cl\nu}$	$10^3 V_{cb} $
4.572	1.090	0.430	0.282	0.161	-0.091	10.61	41.83
0.012	0.010	0.040	0.048	0.018	0.089	0.15	0.47
1	0.389	-0.229	0.561	-0.025	-0.181	-0.062	-0.422
	1	0.019	-0.238	-0.030	0.083	0.033	0.076
		1	-0.097	0.536	0.262	0.142	0.334
			1	-0.261	0.006	0.006	-0.260
				1	-0.019	0.022	0.139
					1	-0.011	0.067
						1	0.697
							1

reached a precision of 1.1% on $|V_{cb}|$ ($\chi^2/\text{dof} = 0.59$)

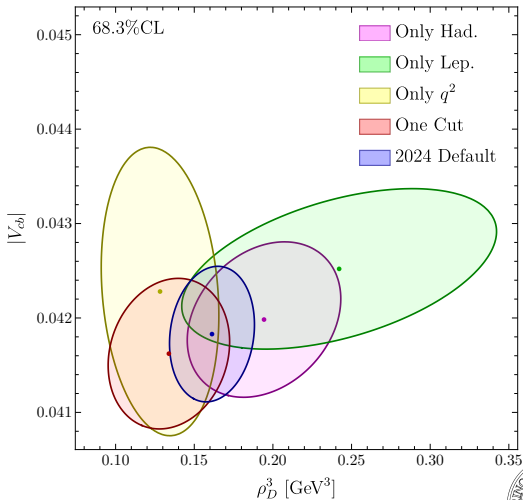
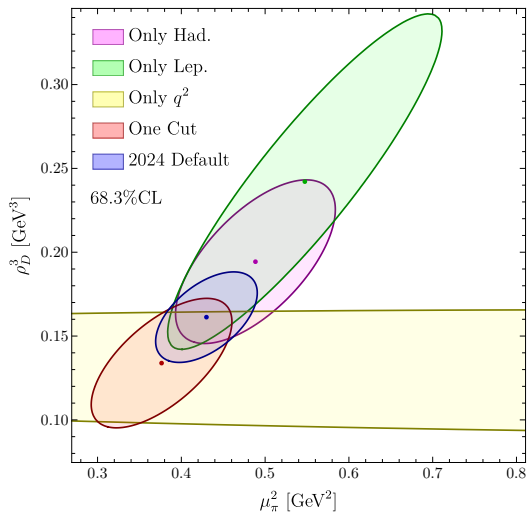
Big improvement in $\sigma_{\mu_\pi^2}$ ($0.056 \rightarrow 0.040$) and $\sigma_{\rho_D^3}$ ($0.031 \rightarrow 0.018$) w.r.t. (E_ℓ, m_X^2) fit

Impact of NNLO q^2 : ρ_D^3 : $0.176 \rightarrow 0.161$ and $10^3|V_{cb}|$: $41.97 \rightarrow 41.83$

*other small improvements to the fit: inclusion of QED effects [Bigi et al. '23], ..., see [2310.20324] for details.



Fit Results (PRELIMINARY)



1σ regions. q^2 moments independent on μ_π^2



Summary & Outlook

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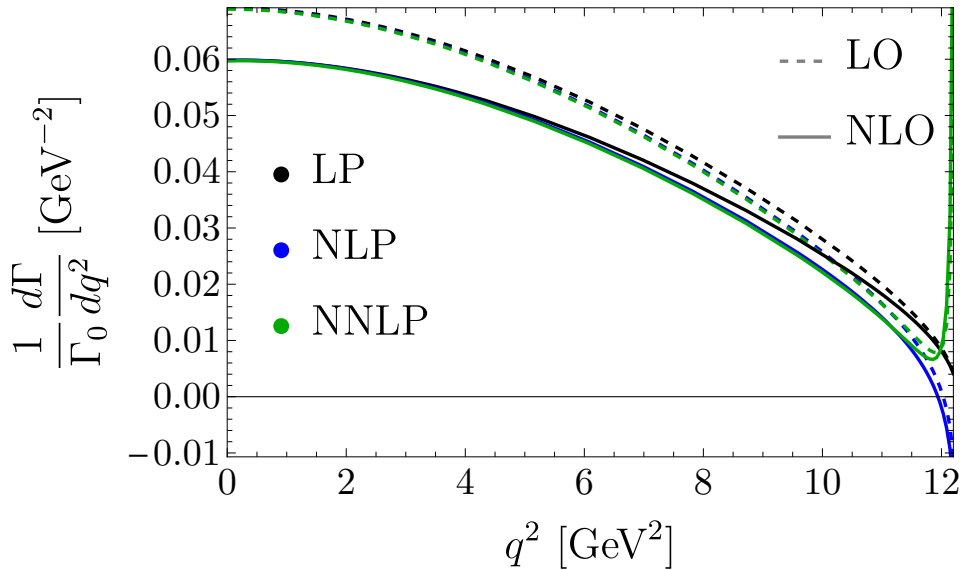
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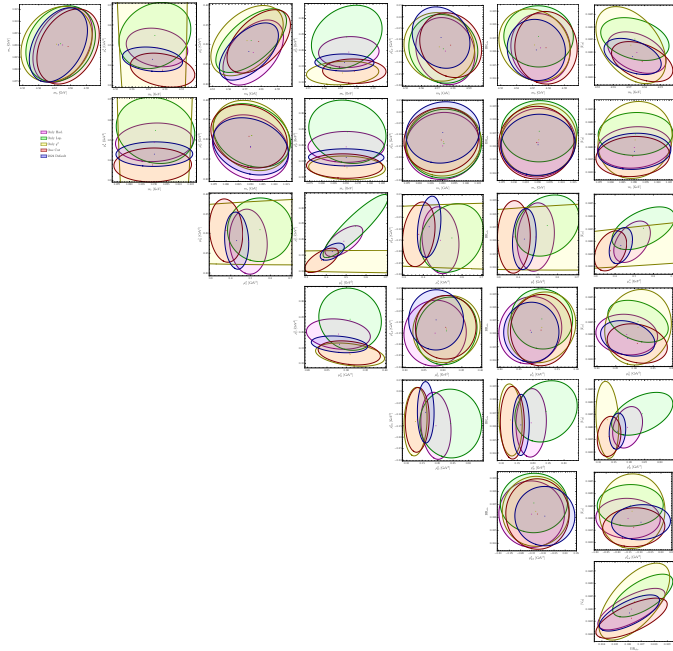
Thank You!



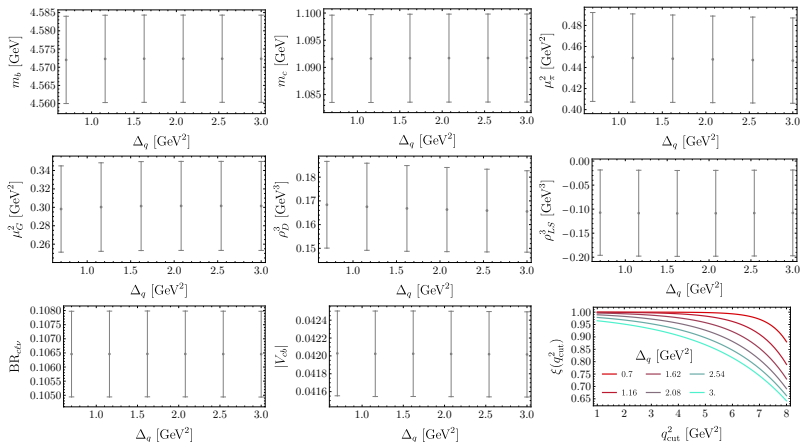
Backup Slides







Theoretical Correlations



Correlations between different central moments set to 0

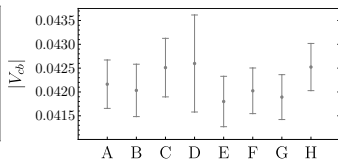
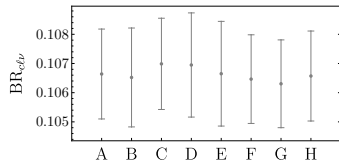
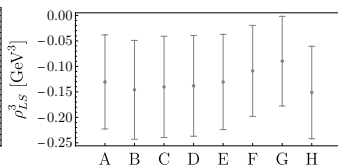
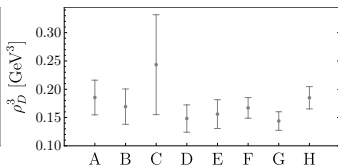
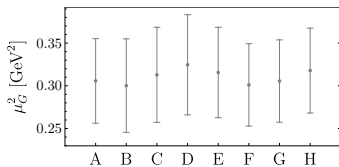
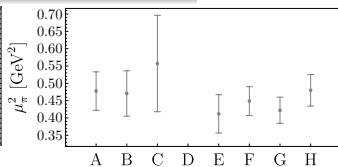
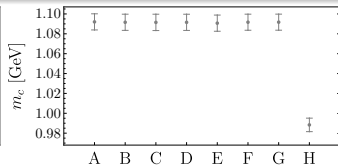
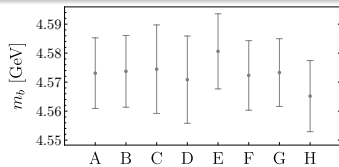
Correlations between same moments at 0.5 GeV² distance in q_{cut}^2 :

$$\xi(q_{cut}^2) = 1 - \frac{1}{2} e^{-\frac{9\text{GeV}^2 - q_{cut}^2}{\Delta_q}}$$

q_{cut}^2 dependent to take into account spectrum endpoint



Fit Variations



- A: 2021 Default
- B: Only Hadronic
- C: Only Leptonic
- D: Only q^2
- E: One Cut
- F: All Data
- G: $\mu_s = m_b$
- H: $\mu_c = 3$ GeV

