Perturbative Unitarity Violation

in

Non-Relativistic Dark Matter processes

Milan Christmas Meeting 2024

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[2411.08737]

MAX-PLANCK-INSTITUT
FÜR PHYSIK

Dark Matter

Cold Dark Matter

Galaxy Rotation Curve

Detection Experiments

Non-Relativistic today in halos

Non-Relativistic Effective Field Theories

The Model

Dark Matter
fermion
χ

Dark Force
between
DM particles

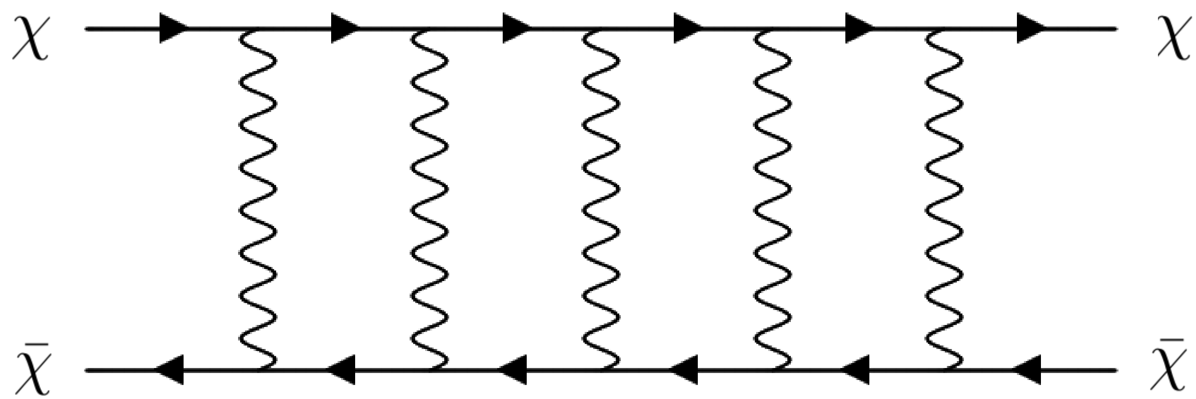
generated by
Light Mediator
A_d^μ

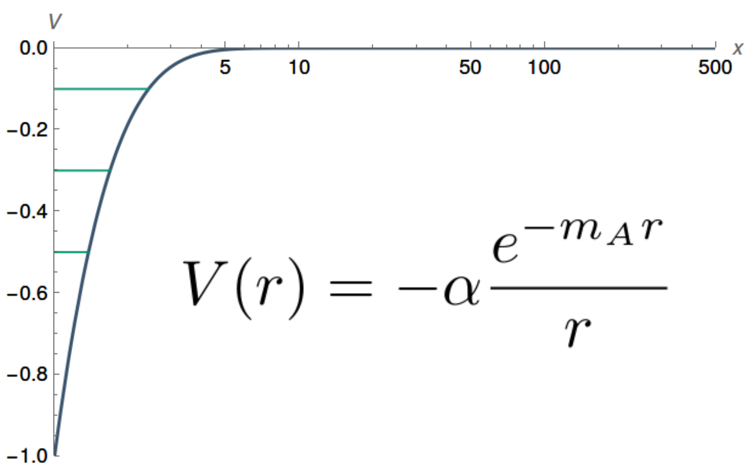
Portal between
Dark Sector
and
Standard Model

Dark Force

Non-Relativistic limit

Yukawa potential





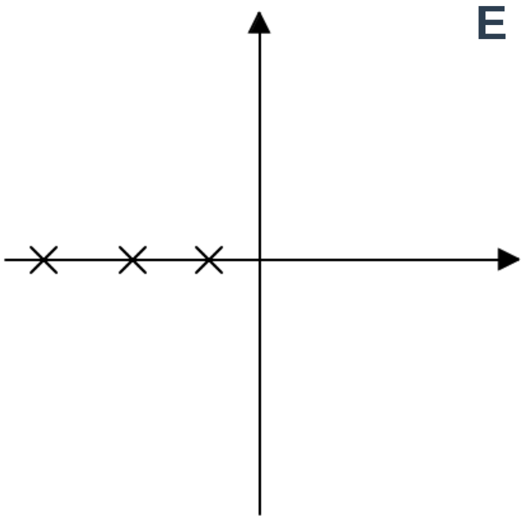
$$V(r) = -\alpha \frac{e^{-m_A r}}{r}$$

Finite number of bound states

$$E_B(m_A)$$

Zero-energy bound states

Scattering Amplitude



$$f_{\ell=0} = -\frac{1}{\sqrt{m_\chi}} \frac{1}{\sqrt{E} - \sqrt{|E_B|}}$$

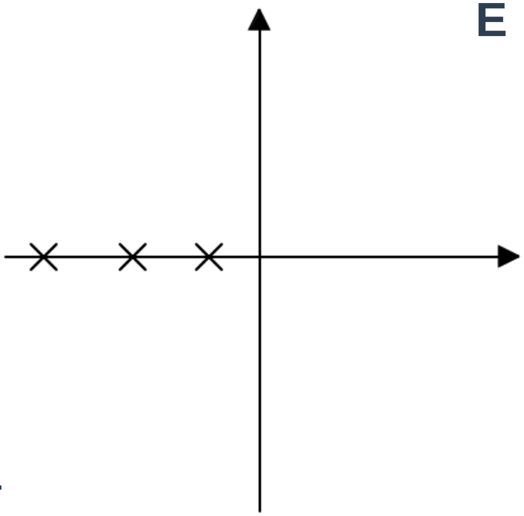
+ non-singular terms

Finite number of bound states

$$E_B(m_A)$$

Zero-energy bound states

Scattering Amplitude



Non-Relativistic limit

$$f_{\ell=0} = \frac{1}{\sqrt{m_\chi}} \frac{1}{\sqrt{|E_B|}} = a$$

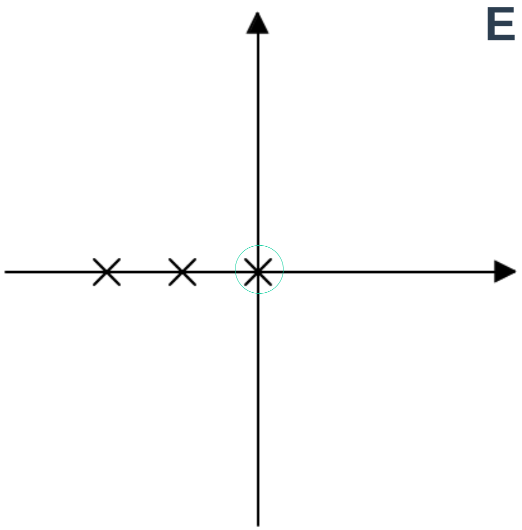
Scattering length

Finite number of bound states

$$E_B(m_A)$$

Zero-energy bound states

Scattering Amplitude



Divergent scattering length

$$a \to \infty$$

Bound-state at zero energy

$$E_B(m_A^*) = 0$$

Zero-energy bound states

Expansion in small momentum

$$f_{\ell=0} = a\left[1 + ia^2k + (ar_0/2 - a^2)k^2 + O(k^3)\right]$$

Non-Relativistic Lagrangian

$$\mathcal{L} = \chi^\dagger\left(i\partial^0 + \frac{\nabla^2}{2m_\chi}\right)\chi + C_0\left(\chi^\dagger\chi\right)^2 + C_{2,1}\left(\chi^\dagger\vec{\nabla}\chi\right)^2 + \ldots$$

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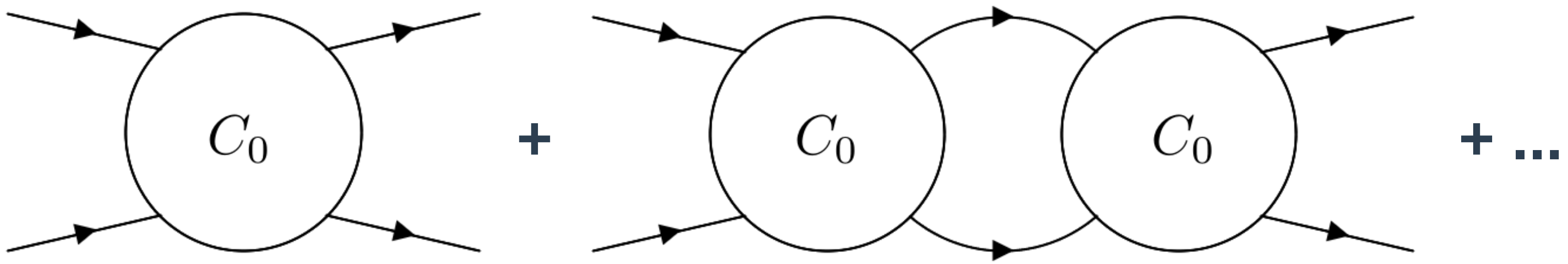
Solution from reorganizing the series

$$f_{\ell=0} = \frac{1}{\frac{1}{a} + ik}\left(1 + \frac{r_0/2}{\frac{1}{a} + ik}k^2 + \ldots\right)$$

Zero-energy bound states

This corresponds to resumming

Kaplan, Savage, Wise: [9802075]



This is needed because

Usually

$$C_{2n} \sim 1$$

$$\left(\chi^\dagger \vec{\nabla}^n \chi\right)^2 \sim k^{2n+1}$$

$$k \gg \frac{1}{a}$$

But now for

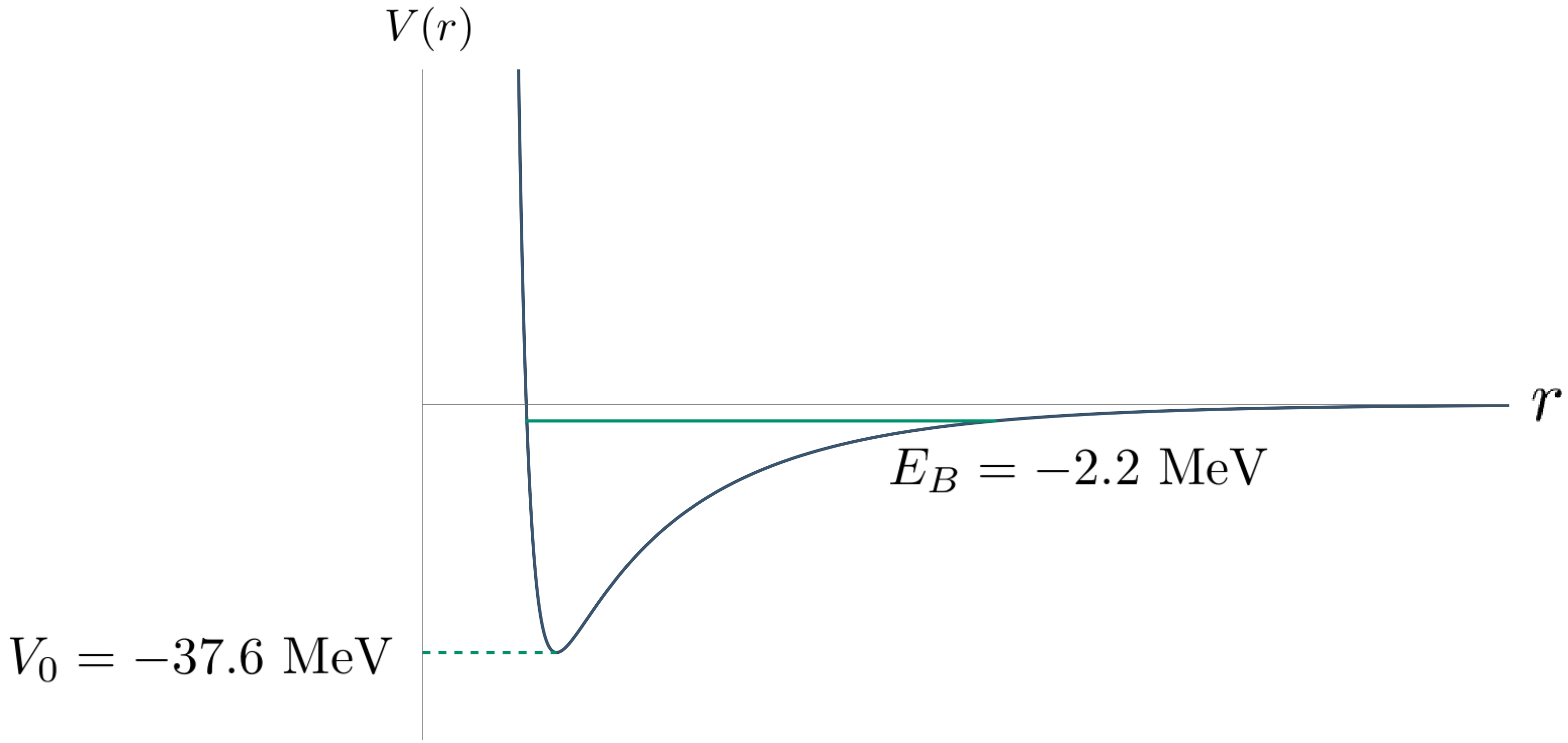
$$C_{2n} \sim k^{-n-1}$$

$$C_{2n} \left(\chi^\dagger \vec{\nabla}^n \chi\right)^2 \sim k^{n}$$

It is a RELEVANT operator for n=0

Deuteron

Deuteron is a shallow bound state



For proton-neutron scattering at low energy the resummation is important

Wino limit of MSSM

Dark Matter is the lightest neutralino χ_0

It has a charged mass splitted component $\chi_\pm$

The massive mediators are $W_\pm$ **and** Z Hisano, Matsumoto, Nojiri, Saito: [0412403]

$$V(r) = \begin{pmatrix} 0 & -\frac{\sqrt{2}\alpha_2}{r} e^{-m_w r} \\ -\frac{\sqrt{2}\alpha_2}{r} e^{-m_w r} & 2\delta m - \frac{\alpha}{r} - \frac{\alpha_2 c_w^2}{r} e^{-m_z r} \end{pmatrix}$$

Zero-energy bound states in the spectrum

Non-Relativistic Annihilation

Non-Relativistic annihilation is usually computed as

$$(\sigma v_{rel})_{pert} = 2\,\mathrm{Im}[C_0] \qquad (\sigma v_{rel}) = 2\,\mathrm{Im}[C_0] \times |\psi(0)|^2$$

$$-\frac{\nabla^2}{2\mu}\psi'' + V(r)\psi + C_0\delta(\vec{r})\psi = \frac{\mu v_{rel}^2}{2}\psi$$

Sommerfeld Factor

$$S(v)$$

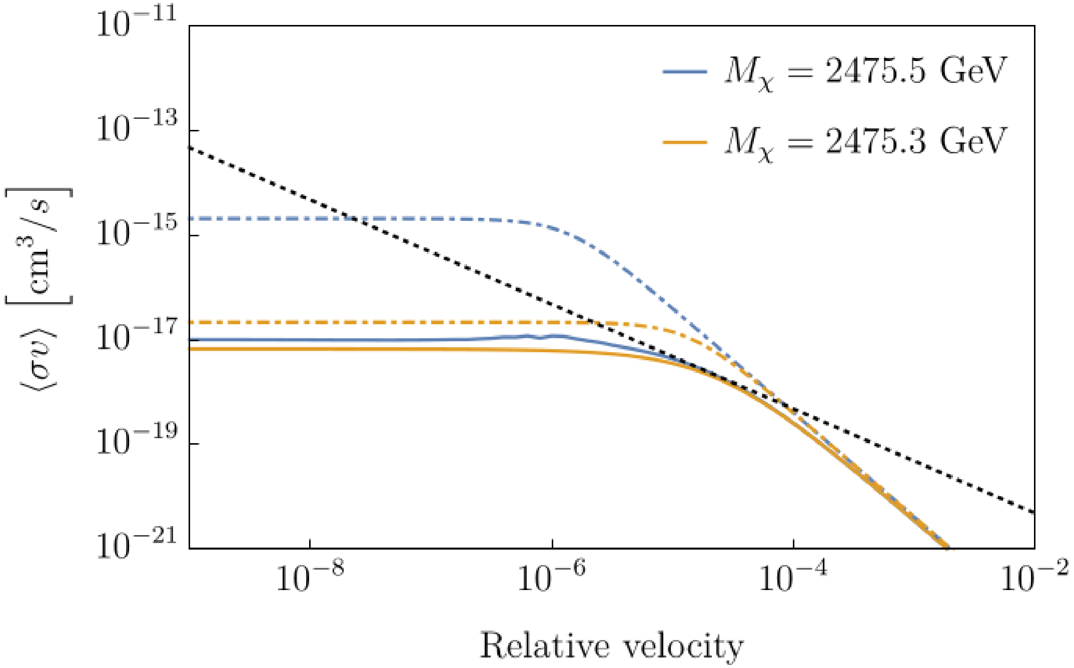
Braaten et al: [1712.07142]

It is not possible to treat $C_0\delta(\vec{r})$ as a perturbation
if there is a zero-energy bound state in the spectrum
Otherwise, it leads to *perturbative unitarity violation*

Perturbative Unitarity Violation

Unitarity Bound

$$(\sigma v_{rel})_\ell \leq (\sigma v_{rel})_{uni,\ell} = \frac{4\pi}{m_\chi}\frac{2\ell+1}{v}$$



In the full treatment the Sommerfeld factor saturates

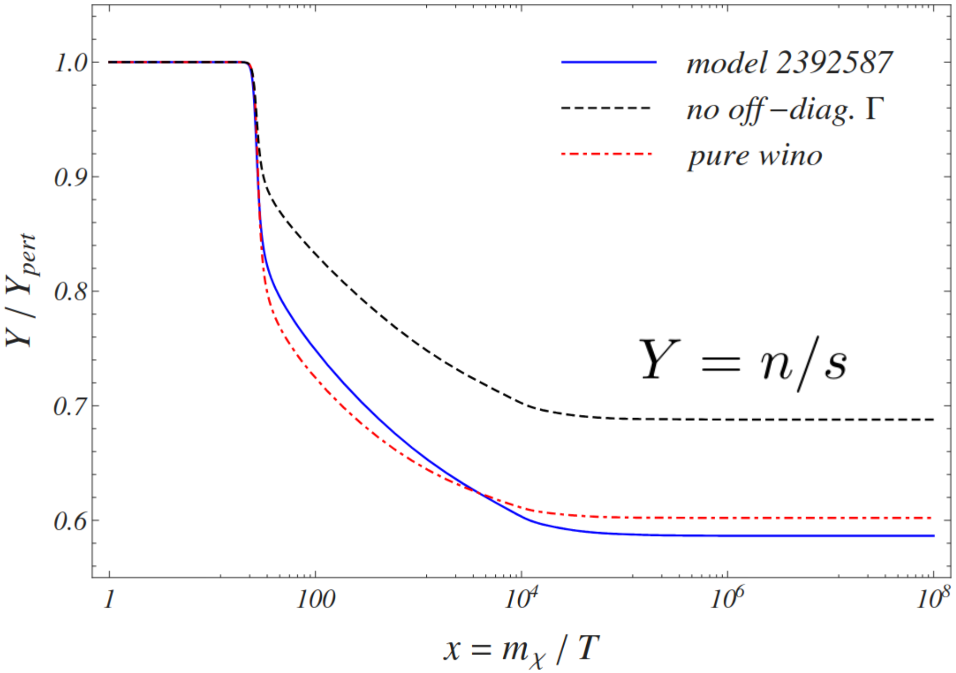
$$S(v) \rightarrow S\left(v + v_c\right)$$

Parikh, Sato, Slatyer: [2410.18168]

Relic Abundance

Why computing Dark Matter annihilation rates?

They set the Dark Matter relic abundance



Beneke, Hellmann, Ruiz-Femenia: [1411.6930]

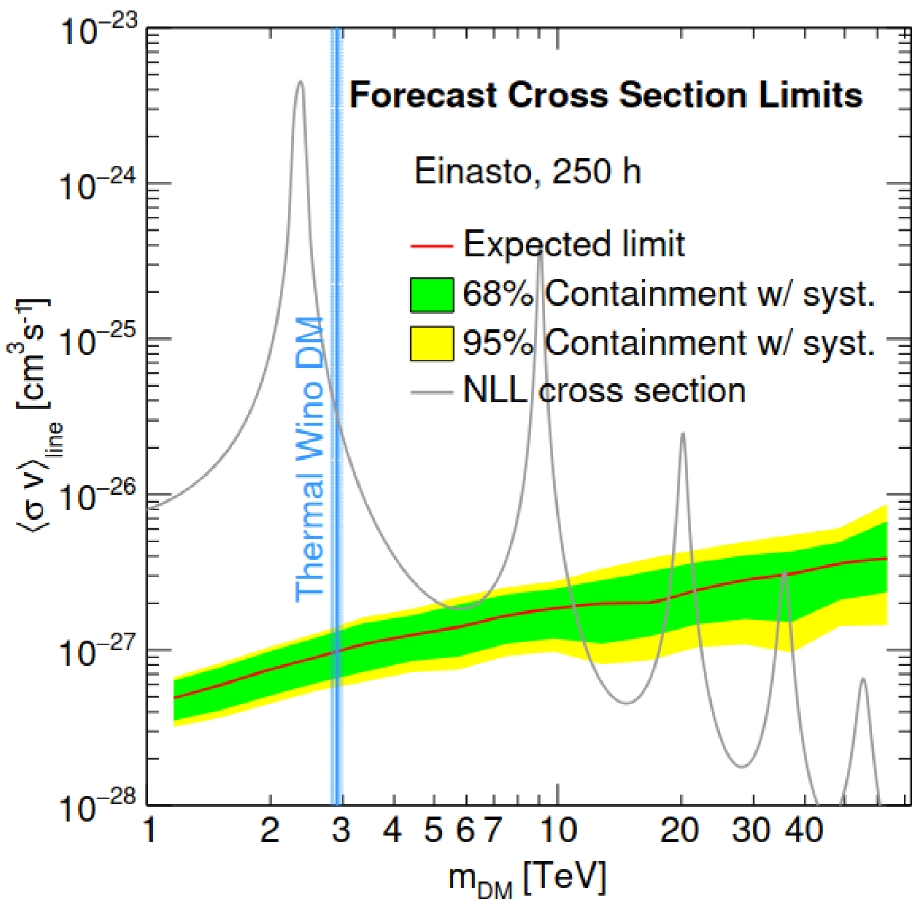
At high temperature, Dark Matter is at chemical equilibrium with the SM

As the Universe expands, temperature decreases

When Dark Matter becomes Non-Relativistic, it starts depleting by annihilating into SM

$$\rho_0^{DM} = m_\chi s_0 Y_0$$

Relic Abundance

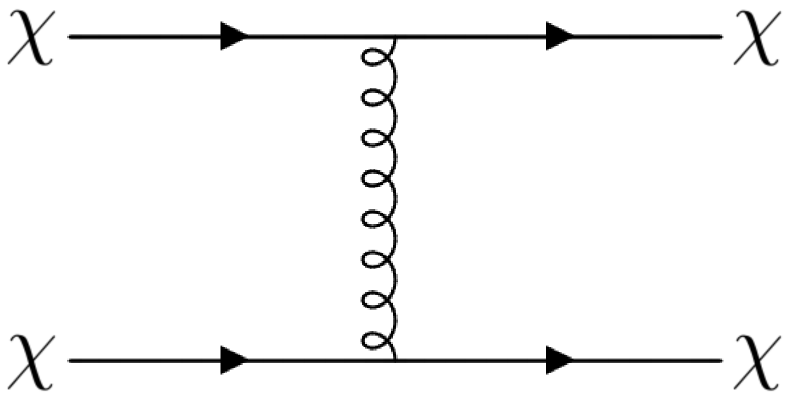


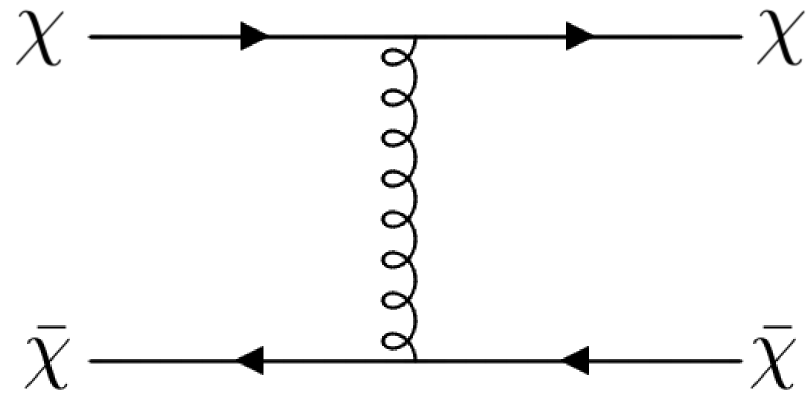
Rinchiuso et al: [1808.04388]

Bound-state formation

Dark SU(N) with a heavy fermion χ

The Non-Relativistic Potentials are



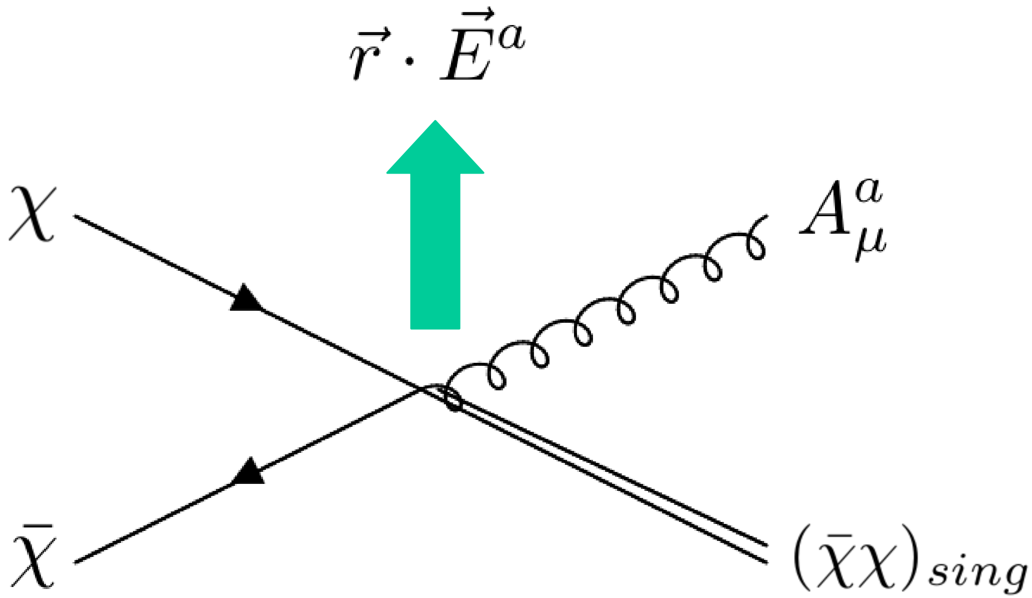


$$V_{adj}(r) = \frac{1}{2N}\frac{\alpha}{r}$$

$$V_{sing}(r) = -C_F\frac{\alpha}{r}$$

Bound-state formation

We look at the bound-state formation rate of the color singlet bound state $(\bar{\chi}\chi)_{sing}$



Bound-state formation

We look at the bound-state formation rate of the color singlet bound state $(\bar{\chi}\chi)_{sing}$

$$Q_{LM,fi} \equiv \langle n\ell m | r^L Y^*_{LM}(\hat{\boldsymbol{r}}) | \boldsymbol{p} \rangle = \int d^3\boldsymbol{r}\, \mathcal{B}^*_{n\ell m}(\boldsymbol{r}) r^L Y^*_{LM}(\hat{\boldsymbol{r}}) \mathcal{S}_{\boldsymbol{p}}(\boldsymbol{r})$$

$$(\sigma v)^{(L)}_{i\to f} = g^2 \frac{2(L+1)}{L[(2L+1)!!]^2}\, \omega^{2L+1} \sum_{M=-L}^{L} |Q_{LM,fi}|^2$$

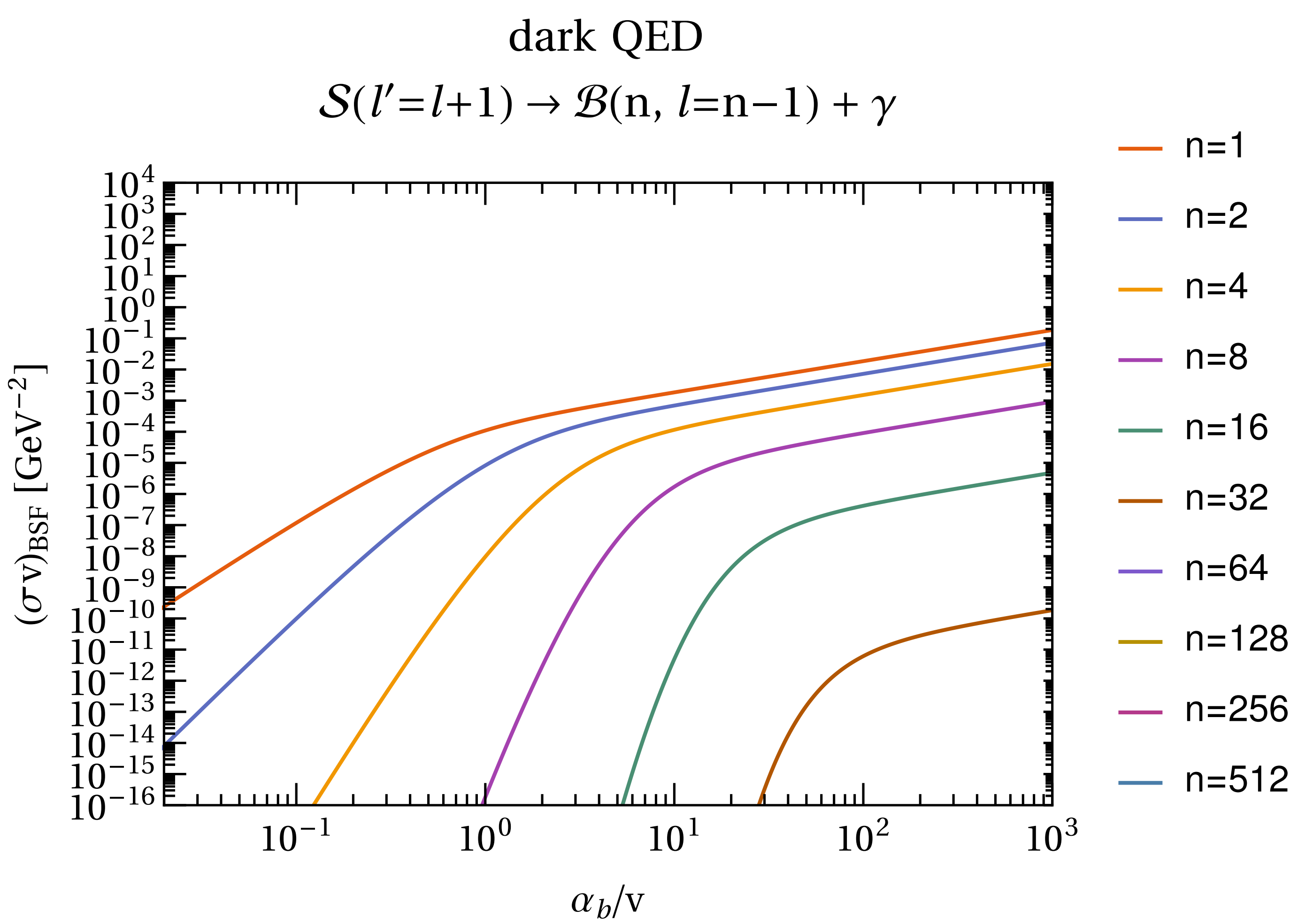
$$J_p^{L,\Delta}(n_s, n, \ell) =$$

$$\frac{(-1)^{1+L}}{2^{2\Delta}\, p^{L+2\ell+\Delta+3}} \,(\partial_s)^{1+L-\Delta}\Big|_{s=0} \frac{\Gamma(2\ell+2\Delta+2)}{(s+\zeta_n+i)^{2\ell+2}} \left(\frac{s-\zeta_n+i}{s+\zeta_n+i}\right)^{\bar{n}} \left(\frac{s+\zeta_n-\mathrm{i}}{s+\zeta_n+\mathrm{i}}\right)^{\bar{n}_s^*}$$

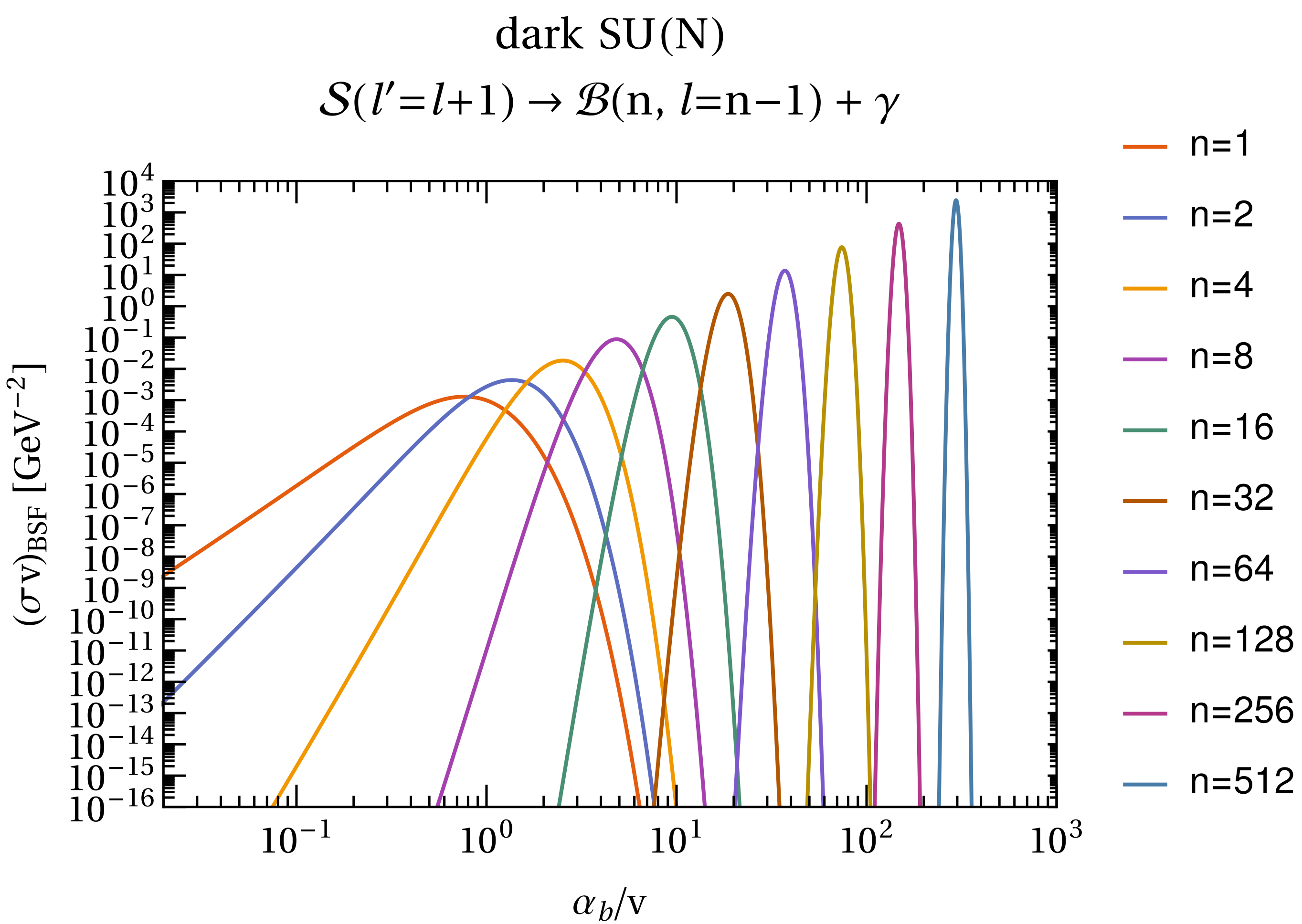
$$\times \sum_{j=0}^{2\Delta} (-1)^j \binom{2\Delta}{j} \left(\frac{s+\zeta_n-\mathrm{i}}{s+\zeta_n+\mathrm{i}}\right)^j {}_2F_1\left(-\bar{n}, -\bar{n}_s^*-j, 2\ell+2; \frac{-4i\zeta_n}{(\zeta_n-i)^2-s^2}\right)$$

Flores, Petraki: [2405.02222], Beneke, Binder, DR, Garny, Lederer: [2411.08737]

Bound-state formation

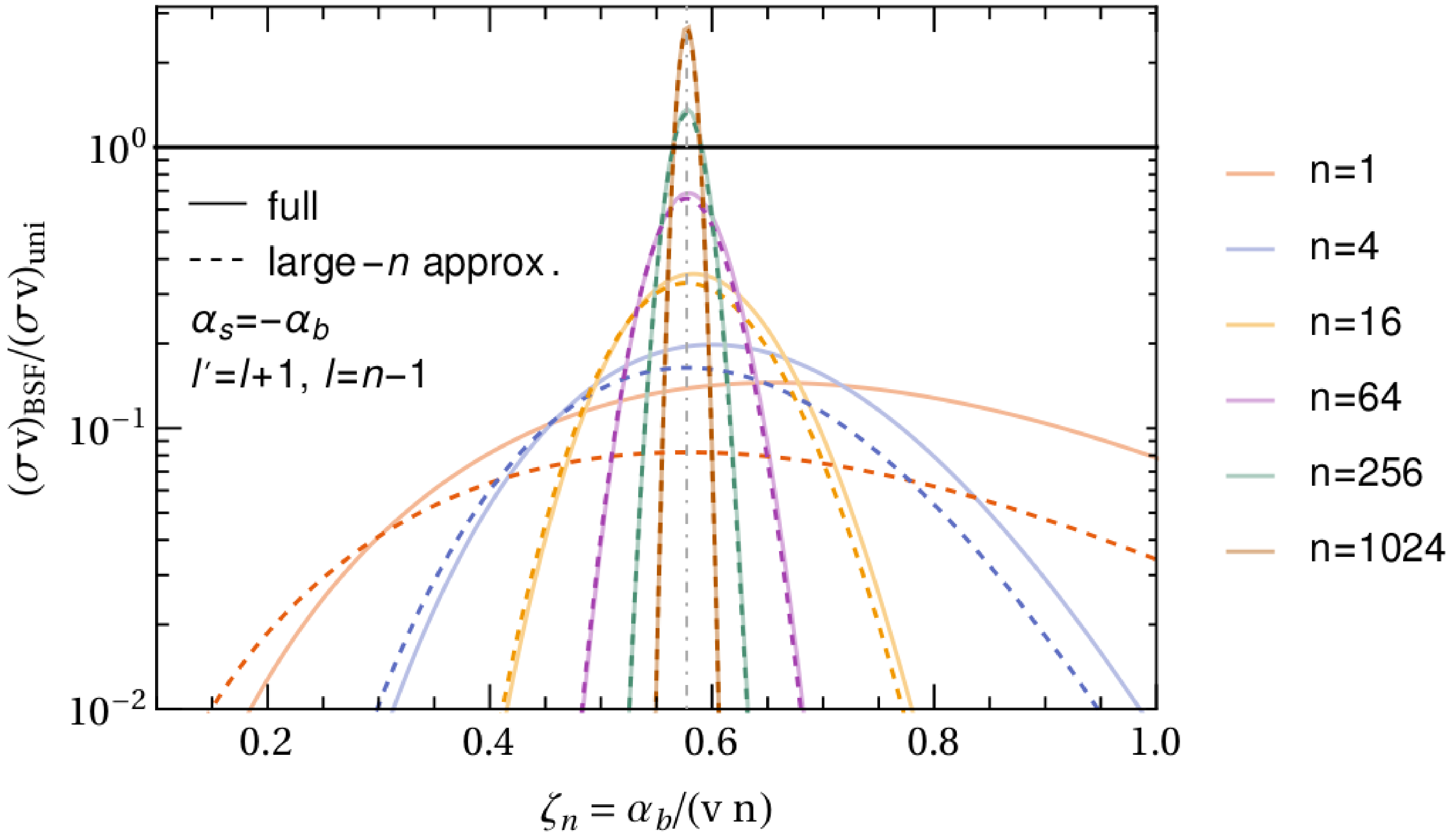


Bound-state formation

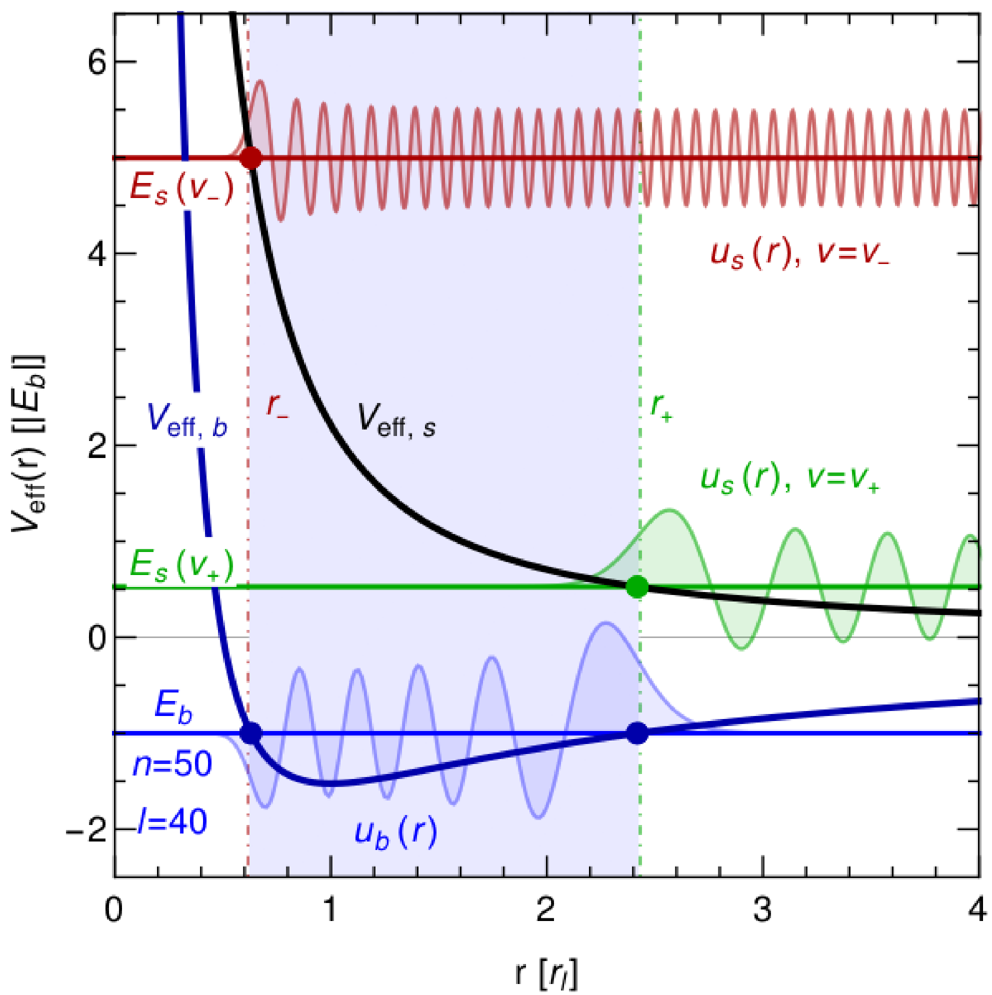


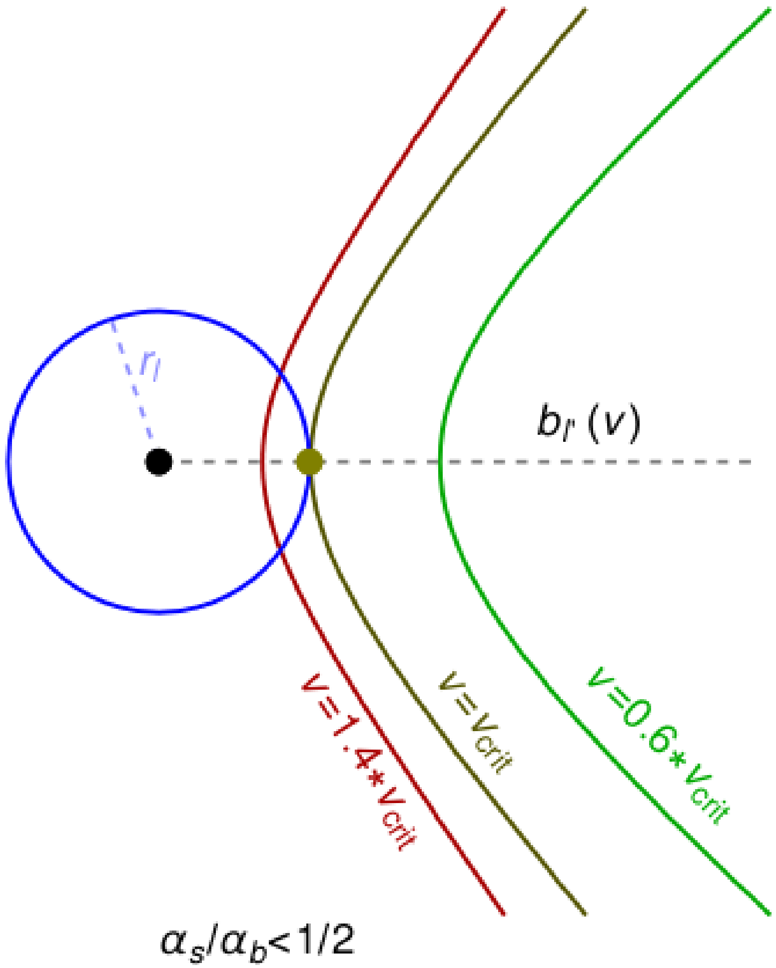
Bound-state formation

Perturbative unitarity violation can be shown analytically



Bound-state formation





The threshold corresponds to $T_{\mathcal{B}}^{(r)} \simeq T_{\mathcal{S}}^{(r)} \to 0$

Conclusion

- Non-Perturbative effects may alter the standard power counting
- The EFT in the new power counting reorganizes the series and restores perturbative unitarity
- For Bound-State Formation processes this understanding is still missing